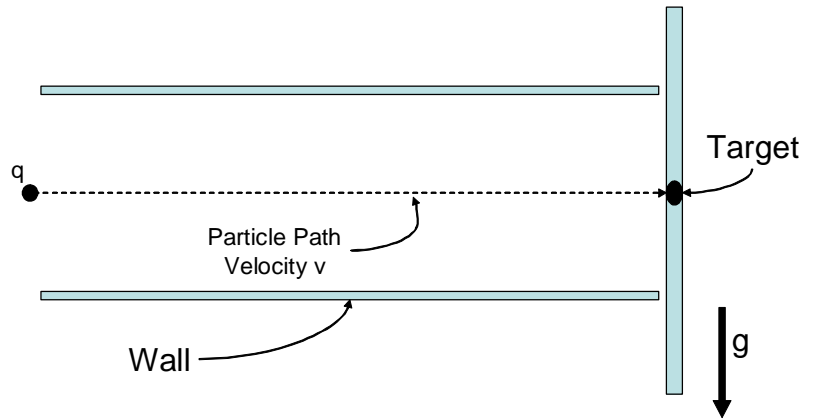


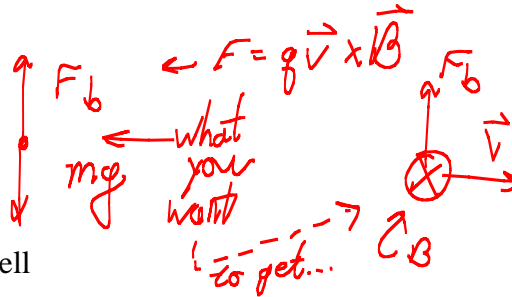
Name Solutions! Student ID \_\_\_\_\_ Score \_\_\_\_\_  
 last first

Part I. [27 points] The figure at right shows a particle about to enter an area of uniform magnetic field (field is not drawn). The magnetic field has a magnitude of 1.2 T, and is oriented such that the particle follows the shown path. Gravity points vertically down, as indicated. Just before it enters the magnetic field, the particle has zero vertical velocity, and an initial horizontal velocity  $v$ , to the right. The charge of the particle is  $+13.0 \mu\text{C}$  and it has a mass of 2.5 grams. The walls are 25 cm above and 25 cm below the particle's shown trajectory. The particle is 1.5 m from the target when it enters the magnetic field. Ignore air friction.



1. [4 points] The arrows on the magnetic field lines should point:

- A. Up
- B. Down
- C. Into page
- D. Out of page
- E. Not enough information to tell



2. [5 points] In order for charge to hit the point on the target what must its velocity be?

- A.  $2.8 \times 10^{-2} \text{ m/s}$
- B. 1.6 m/s
- C. 6.6 m/s
- D.  $1.6 \times 10^3 \text{ m/s}$
- E.  $6.6 \times 10^3 \text{ m/s}$

$$F_b = F_g \quad qvB = mg$$

$$v = \frac{mg}{qB} = \frac{(2.5 \times 10^{-3} \text{ kg})(9.8)}{(13 \times 10^{-6} \text{ C})(1.2 \text{ T})} = 1570 \text{ m/s}$$

3. [4 points] What is the charge's final velocity when it hits the target?

- A. Larger than the answer to Problem 2.
- B. The same as the answer to Problem 2.  $F_{\text{net}} = 0$
- C. Smaller than the answer to Problem 2.

For the rest of this problem, assume that gravity is turned off, and plays no part in the particle's trajectory.

4. [4 points] Upon entering the magnetic field the particle will

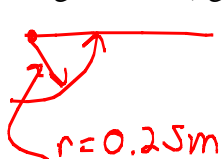
- A. Curve out of the page and circle back out of the field region
- B. Curve into the page and circle back out of the field region
- C. Curve up and possibly hit the upper wall.
- D. Curve down and definitely hit the lower wall.
- E. Move in a straight line and hit the target.

so only  $F_b$  is going to be in effect!?

*last**first*

5. [5 points] You would like to have the particle hit either the upper or lower wall, at a point 25 cm from the entrance (which is where the magnetic field starts, as shown in figure on previous page). What horizontal velocity should the particle enter the region with (again, assume initial vertical velocity is zero).

- A.  $1.2 \times 10^{-5}$  m/s  
 B.  $1.1 \times 10^{-3}$  m/s  
 C.  $1.6 \times 10^{-3}$  m/s  
 D. 1.6 m/s  
 E.  $2.1 \times 10^2$  m/s



$qvB = mv^2/r$   
 $v = \frac{qB}{m}r = 0.00156$

6. [5 points] What is the time between the particle entering the magnetic field and slamming into the wall?

- A.  $1.9 \times 10^{-3}$  s  
 B.  $2.5 \times 10^{-1}$  s  
 C.  $3.6 \times 10^2$  s  
 D.  $2.5 \times 10^2$  s  
 E.  $3.3 \times 10^4$  s

velocity must be constant.

$$x = vt$$

$\uparrow$   $\frac{1}{4}$  the circumference

$$= \frac{2\pi r}{4} = \frac{\pi r}{2}$$

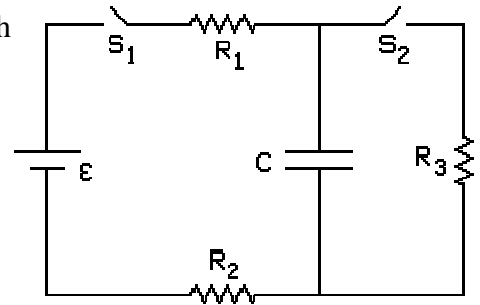
$$t = \frac{\pi r}{2v} = \frac{\pi(0.25)}{(2)(0.00156)}$$

$$= 251 \text{ sec}$$

last

first

Part II. [28 Points] The circuit shown at right contains two switches; both are initially open and the capacitor is uncharged. The battery has zero internal resistance. The parameters of the circuit elements are:



$E = 14 \text{ V}$

$R_1 = 3 \times 10^4 \Omega$

$R_2 = 3 \times 10^4 \Omega$

$R_3 = 30 \times 10^4 \Omega$

$C = 5 \mu\text{F}$

At  $t = 0$  switch  $S_1$  is closed ( $S_2$  remains open), and the capacitor starts to charge.

7. [5 points] Calculate the time constant that applies to *charging* of the capacitor.

- A.  $5 \times 10^{-6} \text{ s}$
  - B.  $1.7 \times 10^{-5} \text{ s}$
  - C.  $1.5 \times 10^{-1} \text{ s}$
  - D.  $3.0 \times 10^{-1} \text{ s}$**
  - E.  $3.3 \text{ s}$
- $\tau = RC = (R_1 + R_2) C$   
 $= (6 \times 10^4 \Omega)(5 \times 10^{-6} \text{ C})$   
 $= 0.30$

8. [5 points] Calculate the charge on the capacitor after a very long time (many time constants).

- A.  $9.4 \times 10^{-6} \text{ C}$
  - B.  $1.3 \times 10^{-5} \text{ C}$
  - C.  $7.0 \times 10^{-5} \text{ C}$**
  - D.  $3.1 \times 10^{-3} \text{ C}$
  - E.  $1.1 \text{ C}$
- $Q = CV = (5 \times 10^{-6} \text{ F})(14 \text{ V})$   
 $= 0.00007 \text{ C}$

9. [5 points] Calculate the charge on the capacitor at  $t = 0.26 \text{ s}$ .

- A.  $5.3 \times 10^{-6} \text{ C}$
  - B.  $4.1 \times 10^{-5} \text{ C}$**
  - C.  $5.8 \times 10^{-5} \text{ C}$
  - D.  $7.0 \times 10^{-5} \text{ C}$
  - E.  $8.2 \times 10^{-5} \text{ C}$
- $Q = Q_0 (1 - e^{-t/\tau}) = (0.00007)(1 - e^{-0.26/0.3})$   
 $= 4.057 \times 10^{-5}$

10. [5 points] Calculate the current flowing into the positive plate of the capacitor at  $t = 0.26 \text{ s}$ .

- A. No Current Is Flowing (0.0 A)
  - B.  $8.3 \times 10^{-5} \text{ A}$
  - C.  $9.8 \times 10^{-5} \text{ A}$**
  - D.  $2.1 \times 10^{-4} \text{ A}$
  - E.  $3.8 \times 10^{-4} \text{ A}$
- $I = I_0 e^{-t/\tau}$   
 $I_0 = \frac{V}{R} = \frac{14 \text{ V}}{6 \times 10^3 \Omega} = 0.0023 \text{ A}$   
 $I = 0.0023 \text{ A} e^{-0.26/0.3} = 9.81 \times 10^{-5}$

After a very long time, switch  $S_1$  is opened and following that, switch  $S_2$  is closed.

11. [4 points] Calculate the time constant that applies to *discharging* of the capacitor.

- A.  $1.5 \times 10^{-1} \text{ s}$
  - B.  $3.0 \times 10^{-1} \text{ s}$
  - C.  $1.5 \text{ s}$**
  - D.  $1.7 \text{ s}$
  - E.  $1.8 \text{ s}$
- $\tau = RC = R_3 C = (30 \times 10^4)(5 \times 10^{-6} \text{ F})$   
 $= 1.5 \text{ s}$

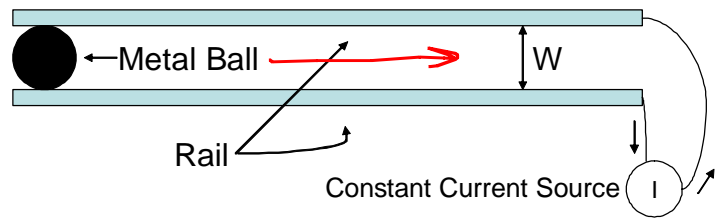
12. [4 points] Calculate the charge on the capacitor at 1.6 s after switch  $S_2$  is closed.

- A.  $1.6 \times 10^{-9}$  C
- B.  $3.4 \times 10^{-7}$  C
- C.  $2.4 \times 10^{-5}$  C
- D.  $2.7 \times 10^{-5}$  C
- E.  $2.9 \times 10^{-5}$  C

$$Q = Q_0 e^{-t/\tau}$$
$$= (0.00007) e^{-1.6/1.5}$$
$$= 2.409 \times 10^{-5} \text{ C}$$

last first

Part III. [25 groovy points] A constant current rail gun is shown at right. The rail gun has been configured to shoot a steel metal ball of mass  $M$  at a speed of 50 m/s. The rails are of length  $L$ , and the diameter of the ball is  $W$ . In the space between the rails is a large constant magnetic field of magnitude  $B$ . A constant current source is connected between the two rods, supplying a current  $I$ . This was used by Austin Powers to shoot down Dr. Evil's death ray satellite in Episode XV.



13. [5 points] In order for the rail gun to work properly, and accelerate the ball to 50 m/s, the magnetic field must be properly oriented. Is the magnetic field pointing into or out of the page? What force accelerates the ball? Explain.

Current flows through the ball, down.  $F_B = i\vec{l} \times \vec{B}$   
 Launch to the right,  $\vec{v} \rightarrow F_b \therefore \vec{B}$  is into page  
 Magnetic force accelerates the ball.

14. [7 points] The ball accelerates from rest to 50 m/s along the length of the rails. Calculate the value of the current  $I$  in terms of specified quantities.

50 m/s when it hits the end of the rod.  $I$  is constant.  
 $F_B = IWB$   
 $a = \frac{IWB}{M}$   
 $v_f^2 - v_0^2 = 2aL$   
 $v_f^2 = \frac{2IWB}{M} L \Rightarrow I = \frac{v_f^2 M}{2WB}$   $\leftarrow v_f = 50 \text{ m/s}$

15. [7 points] Calculate the kinetic energy of the projectile as it leaves the gun in terms of the quantities specified. What component of the system did the work to give it this energy?

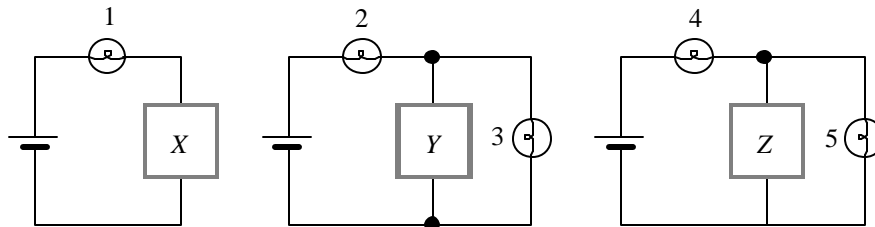
$E = \frac{1}{2} M v_f^2$   
 The constant current source does the work.

16. [6 points] Remove the current source and instead directly connect the two rails. What other modification would you have to make in order to make this device continue to function as a rail gun? Describe its operation, though do no calculations.

So, if  $I$  is not there anymore, we could vary  $B(t)$ .  
 This will cause current to flow, and we will be back to  $F = i\vec{l} \times \vec{B} \rightarrow$  This is hard to calculate because the  $\frac{d\vec{B}}{dt}$  varies as a position of the ball.

IV. [20 pts total] In the circuits below, bulbs 1-5 are identical, and the batteries are identical and ideal. Boxes X, Y, and Z contain unknown arrangements of bulbs.

The resistances of boxes X and Y are equal, and greater than the resistance of Box Z,  $R_x = R_y > R_z$ .



17. [4 pts] Rank the brightness of bulbs 1, 2 and 4 in order of increasing brightness. Explain.

$1 < 2 < 4$  because if I look at bulb 1, and add a bulb in parallel to element X, I would decrease the total resistance, thus increase the overall current through the battery. Since elements X and Y have the same resistance, this is equivalent to bulb 2, thus  $2 > 1$ . Replacing element X in bulb 2's circuit with element Z which has less resistance, will again decrease the total resistance, increasing the current through the battery, since this is equivalent to the circuit bulb 4 is in, bulb 4 must be brighter than bulb 2.

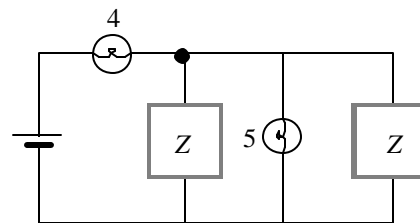
18. [7 pts] Rank the potential difference across box X, box Y, and box Z in order of increasing absolute value. Explain your reasoning.

$X > Y > Z$  since the bulbs brightness are ranked  $4 > 2 > 1$ , and I know that the brightness is an indicator of the potential difference across the bulb, the potential difference across the bulbs must also be ranked  $V_4 > V_2 > V_1$ . I also know that  $V_{batt} = V_1 + V_x = V_2 + V_y = V_3 + V_z$ , so the voltages across X, Y, and Z must be ranked:  $V_z < V_y < V_x$ .

19. [3 pts] Is the brightness of bulb 3 *greater than*, *less than*, or *equal to* the brightness of bulb 5? Explain.

Since bulb 3 is in parallel with Y and bulb 5 is in parallel with Z, this means that  $V_3 = V_y$  and  $V_5 = V_z$ , so since  $V_y > V_z$  then  $V_3 > V_5$  and since brightness is an indicator of the potential difference across a bulb, bulb 3 is brighter than bulb 5

20. [6 pts] Suppose a second box identical to box Z is connected to the circuit as shown. Determine whether the brightness of each of the bulbs below will *increase*, *decrease*, or *remain the same*. Explain.



- bulb 4  
By adding in this 2nd element Z in parallel, we have decreased the equivalent resistance of the circuit, so there is more current flowing through the battery, all of which flows through bulb 4, so it will get brighter.
- bulb 5  
 $V_4 + V_5$  still equals  $V_{batt}$ , and since bulb 4 is brighter,  $V_4$  must be larger than before we added in the 2nd element Z. If  $V_4$  is larger, then in order to have them sum to  $V_{batt}$ ,  $V_5$  must be smaller, so bulb 5 will get dimmer.