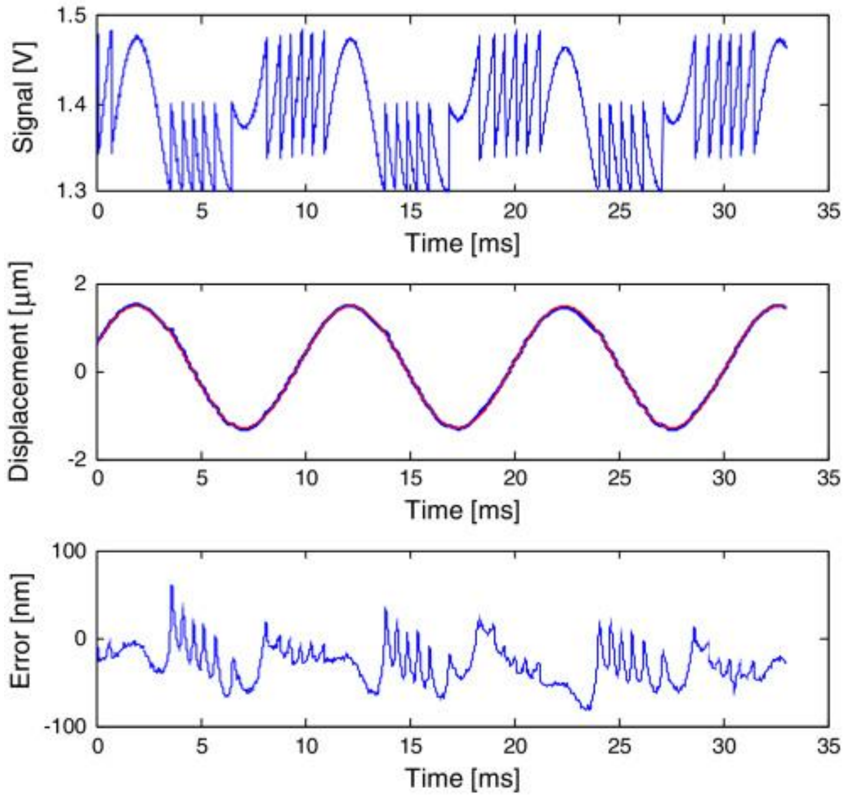
Algebra 2/Trig: Trigonometric Graphs
(SHORT VERSION)

In this unit, we will…

- Learn the properties of sine and cosine curves: amplitude, frequency, period, and midline.
- Determine what the parameters a, b, and d of the function $y = a \sin bx + d$ and $y = a \cos bx + d$ do to the basic graph of sine and cosine
- Determine the relationship between the period and the frequency of a trig function
- Identify the graph of tangent
- Identify the graphs of secant, cosecant, and cotangent
- Graph (for real) sine and cosine curves and a system of those equations.



Name:________________________________

Teacher:______________________________

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Day 1: Graphs of Sine and Cosine Functions

Warm - Up

Solve for *all* values of $2 \sin \theta + \sqrt{3} = 0$ when $0° \leq \theta \leq 360°$.

A) 150°, 210°

B) 240°, 300°

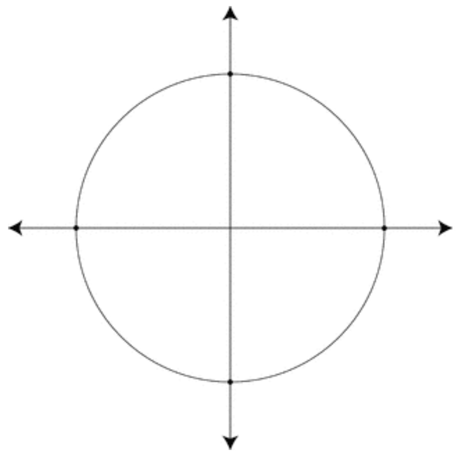
C) 60°, 120°

D) 60°, 300°

Periodic Functions

- Sin and Cos repeat their values in a regular pattern so they are called periodic functions
- We will often just graph ***one period*** for the function
- Regular Period is [0, 2π]
- The graphs will have the angle measures (in radians) along the x-axis and the value of the function at that angle along the y-axis

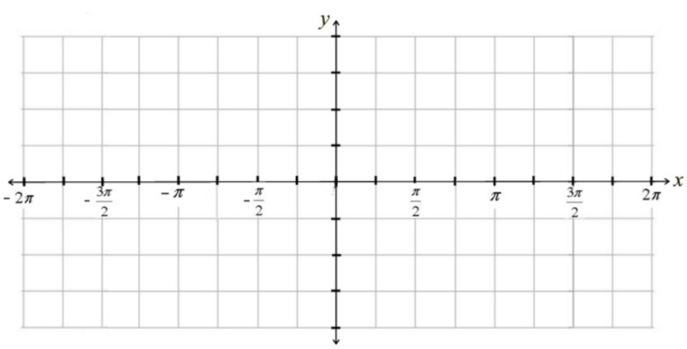
Unit Circle



Sine Function

- Domain $(-\infty, \infty)$ and Range is $[-1, 1]$
- Graph is symmetrical with the origin
  - Odd Function which means $\sin(-x) = -\sin(x)$

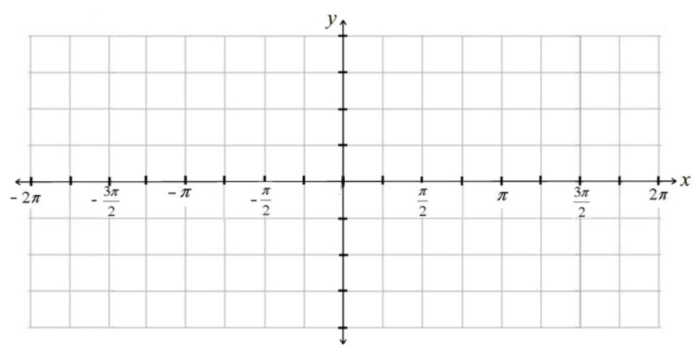
$$y = \sin\theta$$



Cosine Function

- Domain $(-\infty, \infty)$ and Range is $[-1, 1]$
- Graph is symmetrical with the y-axis
  - Even Function which means $\cos(-x) = \cos(x)$

$$y = \cos\theta$$



Similarities:

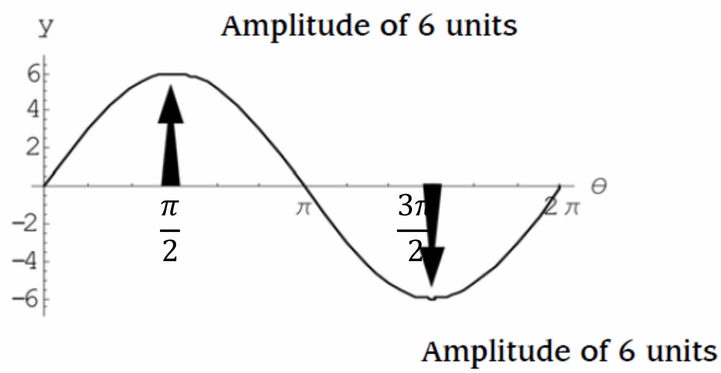
- Look at the y-axis. Both graphs have a maximum at 1, and a minimum at -1.
- Look at the x-axis. Both graphs start at 0 and end at 2π.
  (One complete revolution of the unit circle)
- The midline for each graph is at $y = 0$.

Differences:

- Notice how the sine pattern looks like a wave, while the cosine pattern looks like a U.

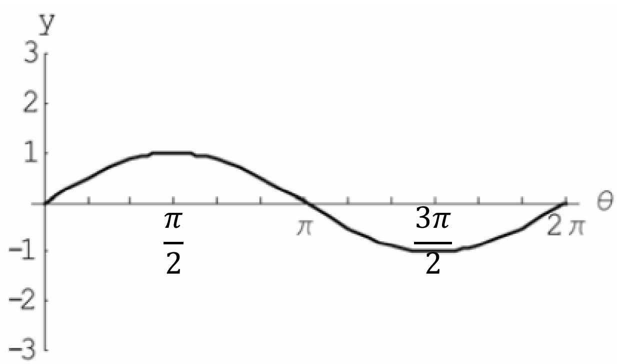
Amplitude ($|a|$) $y = a\cos x$ or $y = a\sin x$

The distance from the midline of a trig graph to the maximum or minimum is called the amplitude.



- The amplitude (a) is the "height" of your graph.
- Think of it like a "y-multiplier"
- Range changes to become [-|a|, |a|]

The amplitude is the number you multiply all y-values by to do a vertical stretch.



The amplitude may be found from a graph by using the formula:

$$\text{Amplitude} = \frac{|\text{max - min}|}{2}$$

This formula is useful in application questions where a graph is hard to read.

For example, the calculation in the previous graph is:

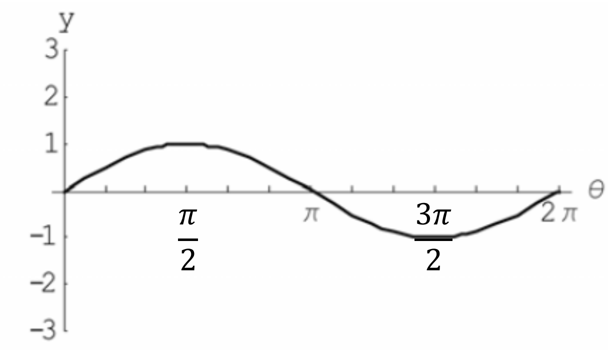
Amplitude =

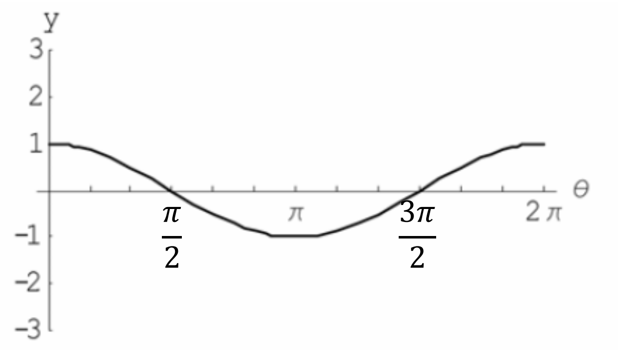
Memorize the amplitude formula, as it is not on your formula sheet!

Coefficients with a magnitude greater than one will stretch the graph, making it taller. Coefficients with a magnitude less than one will compress the graph, making it shorter.

$y = 3\sin\theta$

$y=\frac{1}{2}\cos\theta$

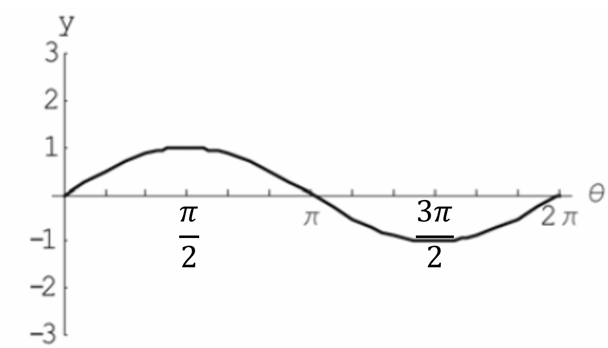


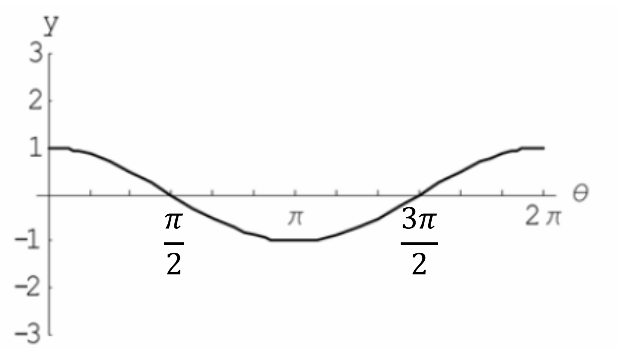


If there is a negative sign in front of a trig function, this will flip the graph upside down in addition to whatever stretch is required.

$y = -3\sin\theta$

$y=-\frac{1}{2}\cos\theta$





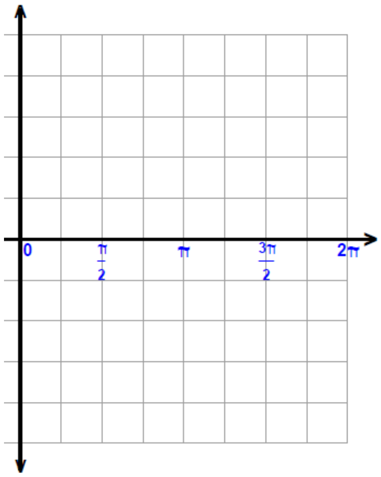
Terminology Alert

The term amplitude is defined as the absolute value of the coefficient.
For $y = -3\sin\theta$, we would say *"the graph has an amplitude of 3, and is reflected in the x-axis."* It would be incorrect to say *"the graph has an amplitude of -3."*

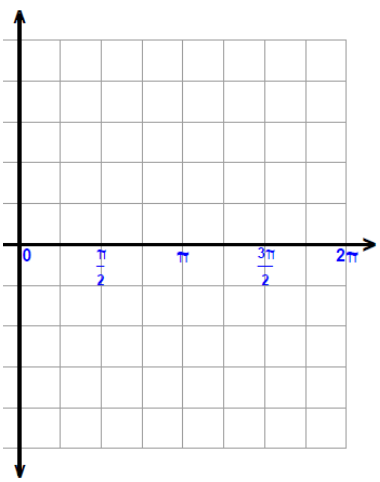
Part II: Vertical Shifts $y = \sin\theta + d$ or $y = \cos\theta + d$

The vertical translation of a trig graph is how far up or down we shift the midline.

Example 1: Graph $y = \sin\theta + 3$



Example 2: Graph $y = \cos\theta - 5$



There is an equation you can use for finding the equation of the midline:

$$y = \frac{\min + \max}{2}$$

From the graph in Example 2: ($y = \cos\theta - 5$), The minimum is -6 and the maximum is -4.

Using the formula on the graph verifies that the midline equation is y =

Part III Graphing a and d

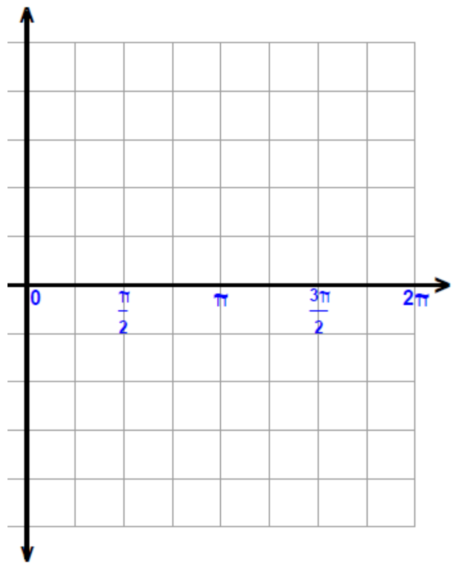
We will now look at trig graphs with the form: $y = a\sin\theta + d$ **or**

$$y = a\cos\theta + d$$

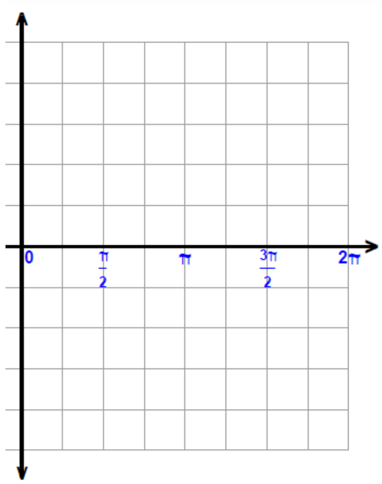
"**a**" is the letter used to represent amplitude.
"**d**" is the letter used to represent vertical translation.

When doing the transformation, we should follow a particular order. First apply the amplitude, then the vertical translation.

Example 1: Graph $y = 2\cos\theta + 1$

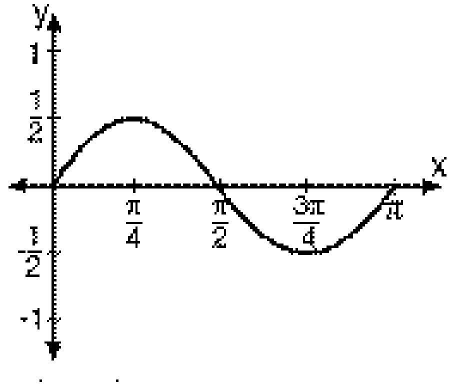


Example 2: Graph $y = -\frac{1}{2}\sin\theta - 1$



Challenge

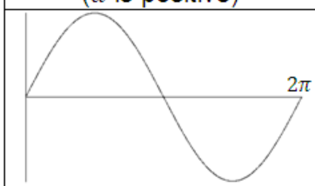
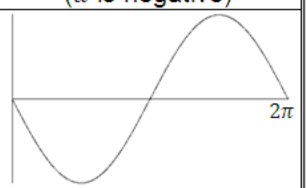
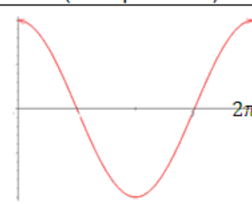
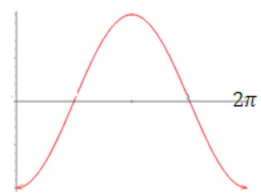
The equation of the graph shown is



SUMMARY

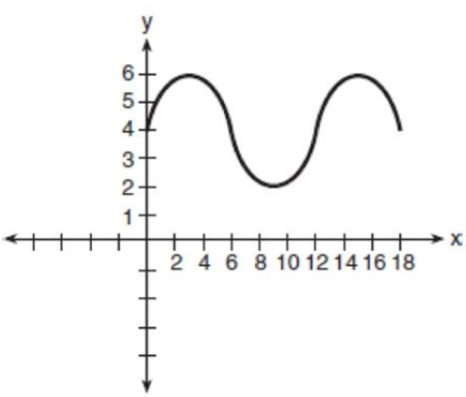
Algebra2/Trig: Identifying the Equation of a Graph
$y = a \sin bx + d$ or $y = a \cos bx + d$

One cycle of a sine or cosine curve.

$y = \sin x$ (a is positive)	$y = -\sin x$ (a is negative)	$y = \cos x$ (a is positive)	$y = -\cos x$ (a is negative)
			

Exit Ticket

1 What is the amplitude of the function shown in the accompanying graph?



1) 1.5
2) 2
3) 6
4) 12

2 What is the amplitude of the function $y = \frac{2}{3} \sin 4x$?

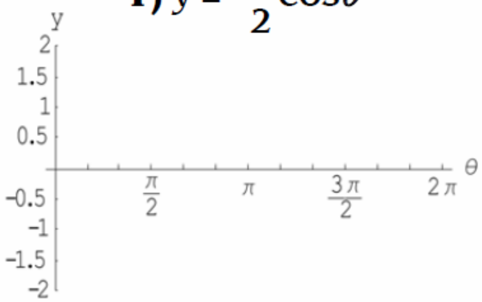
1) $\frac{\pi}{2}$
2) $\frac{2}{3}$
3) 3π
4) 4

Day 1 – Homework

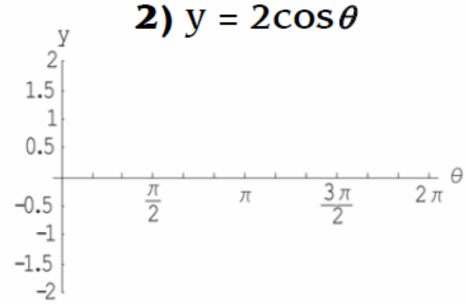
PART I   AMPLITUDE

QUESTIONS: *Draw the following graphs by hand, then graph them in your calculator to check.*

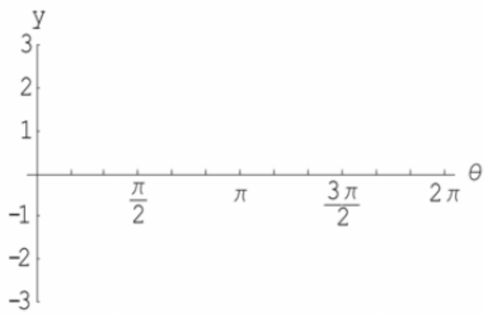
1) $y = -\frac{1}{2}\cos\theta$



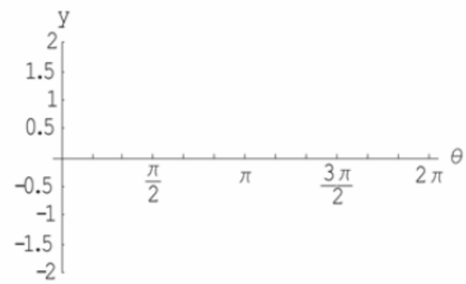
2) $y = 2\cos\theta$



3) $y = -3\sin\theta$

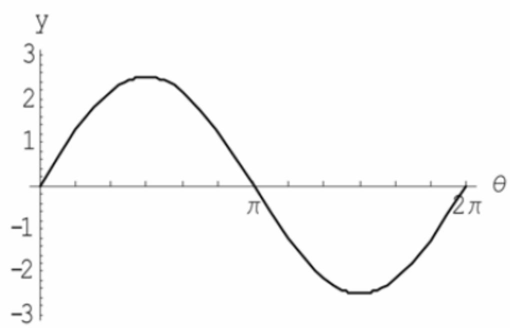


4) $y = -\frac{1}{2}\sin\theta$



For each of the following graphs, write the equation:

5)



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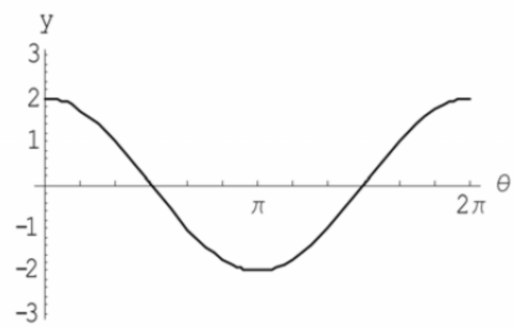

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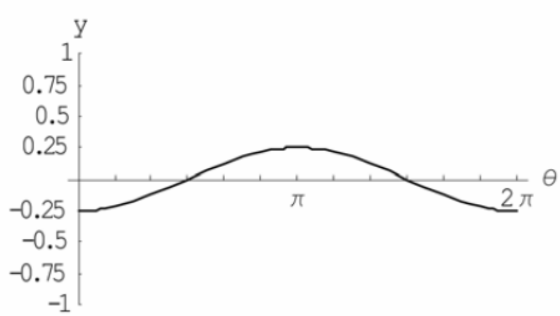

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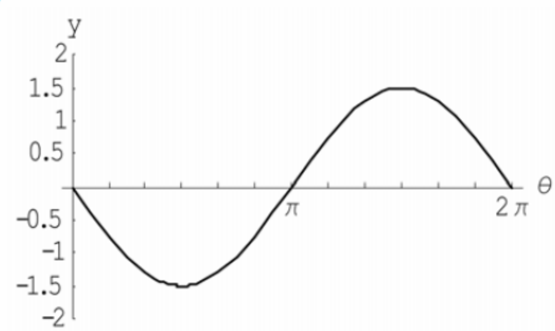
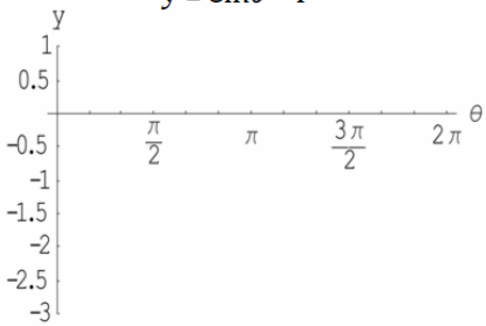


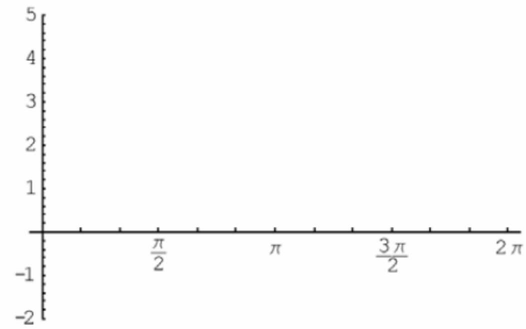
PART II VERTICAL TRANSLATION

Questions: *Draw the graph:*

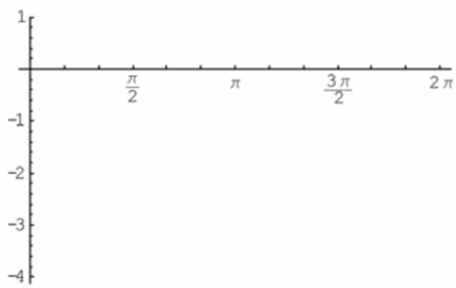
1) $y = \sin\theta - 1$



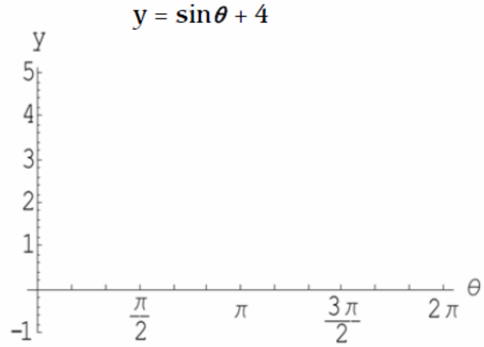
2) $y = \cos\theta + 2$



3) $y = \cos\theta - 3$



4) $y = \sin\theta + 4$



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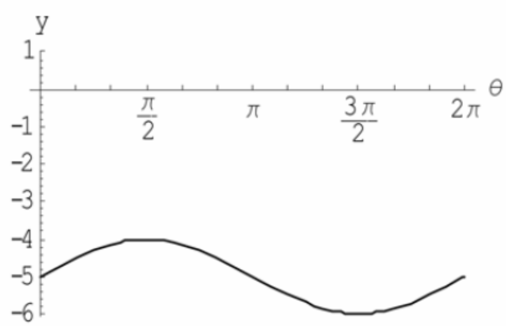

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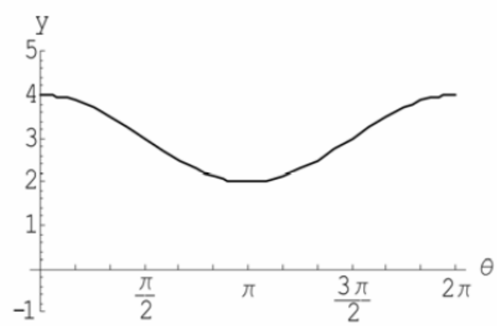

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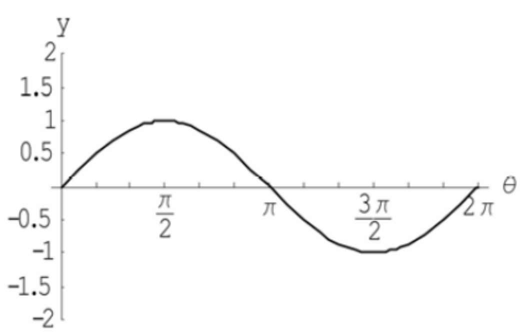

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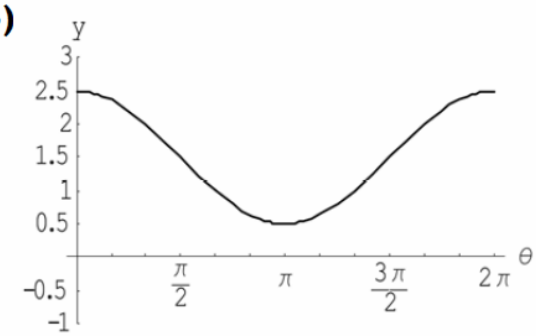
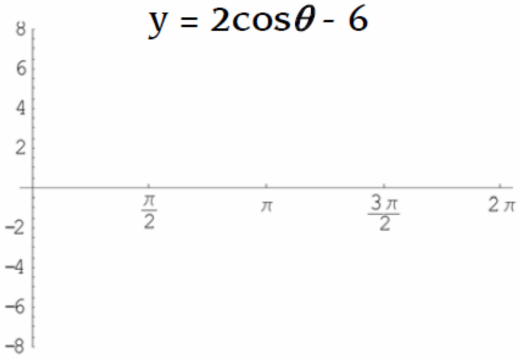
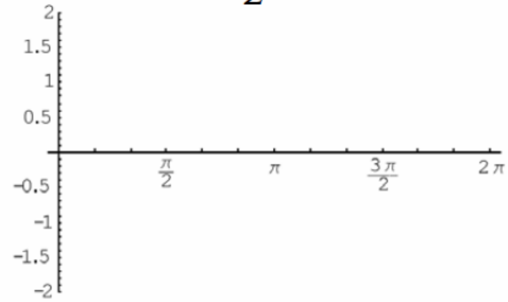


PART III GRAPHING A AND D

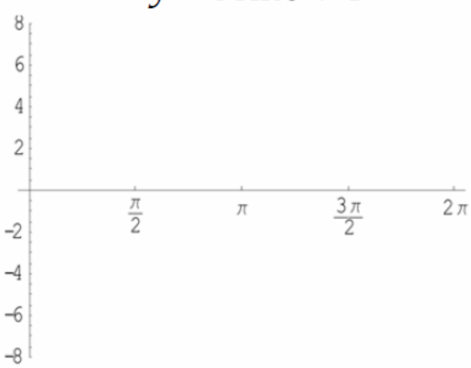
1) $y = 2\cos\theta - 6$



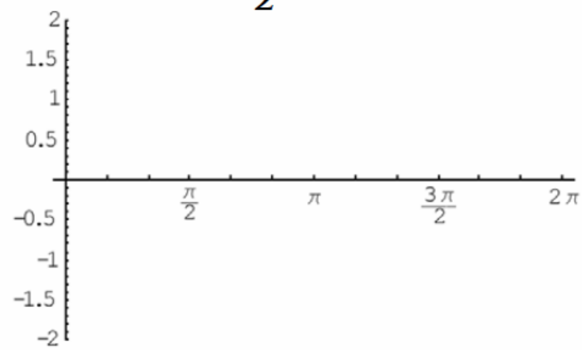
2) $y = -\frac{1}{2}\cos\theta - 1$



3) $y = 3\sin\theta + 1$

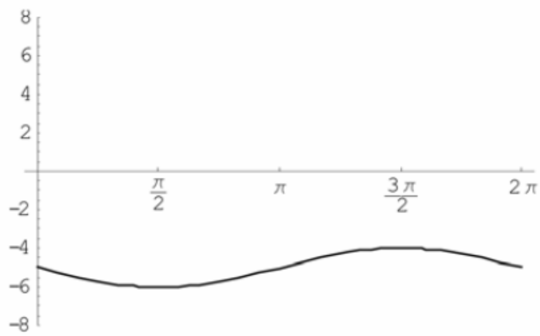


4) $y = -\frac{1}{2}\sin\theta + 1$



For each of the following graphs, write the equation.

5)



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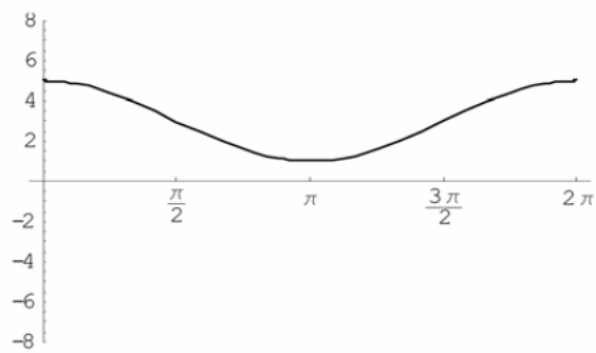

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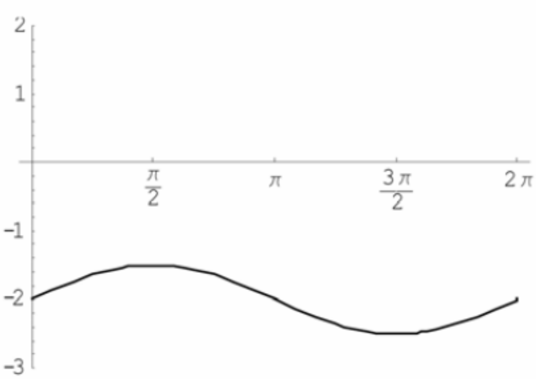

Part I: Answers

Answers:

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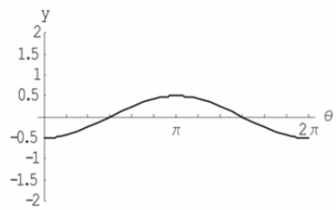

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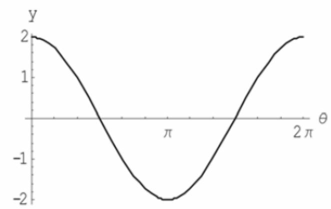

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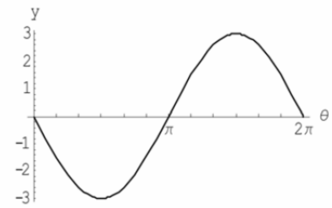

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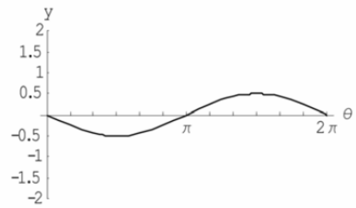


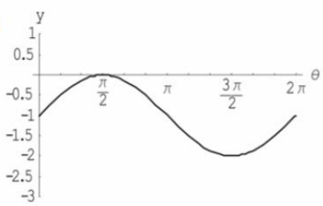
5) $y = 2.5\sin\theta$

6) $y = 2\cos\theta$

7) $y = -0.25\cos\theta$

8) $y = -1.5\sin\theta$

Part II: Answers

1)


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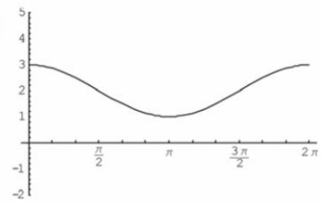

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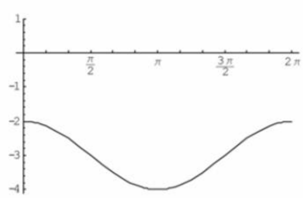

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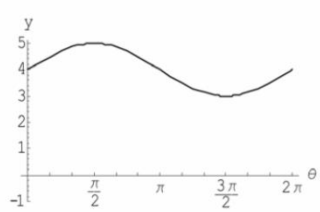


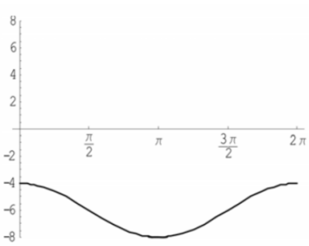
5) $y = \sin\theta - 5$

6) $y = \cos\theta + 3$

7) $y = \sin\theta$

8) $y = \cos\theta + \frac{3}{2}$

Part III: Answers

1)


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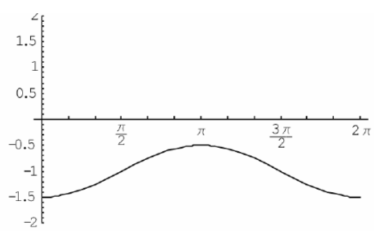

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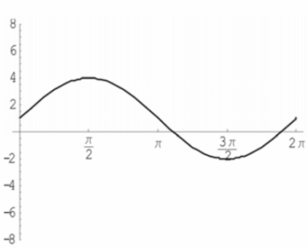

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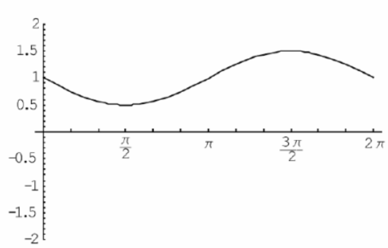


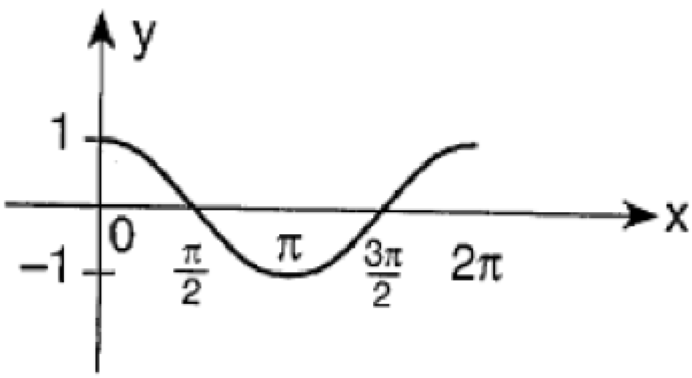
5) $y = -\sin\theta - 5$

6) $y = 2\cos\theta + 3$

7) $y = \frac{1}{2}\sin\theta - 2$

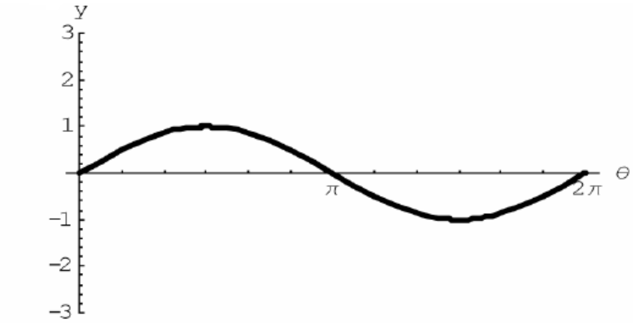
<u>Warm – Up</u>

The graph below *incorrectly* represents the equation $y = 2\cos x$. Write a mathematical explanation of why this graph is incorrect.

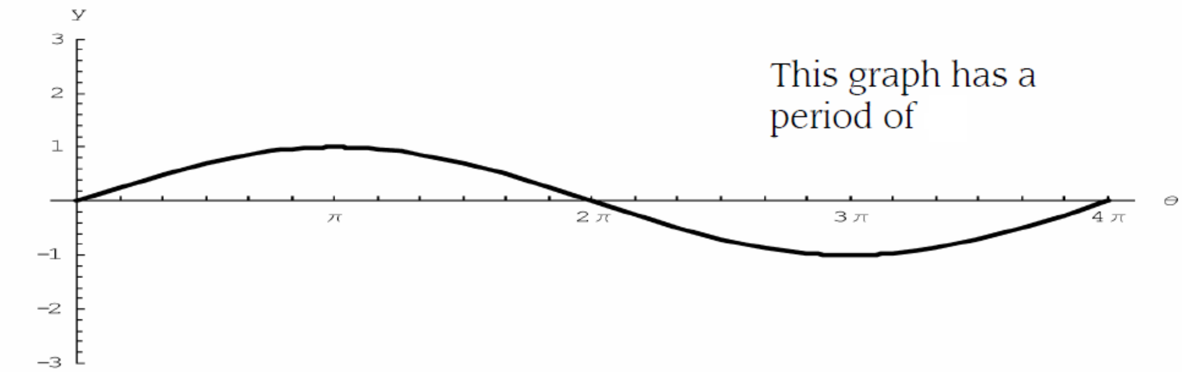


Part I: Period

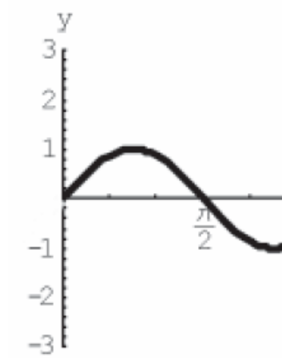
The ______ of a graph is defined as the length of one complete cycle.



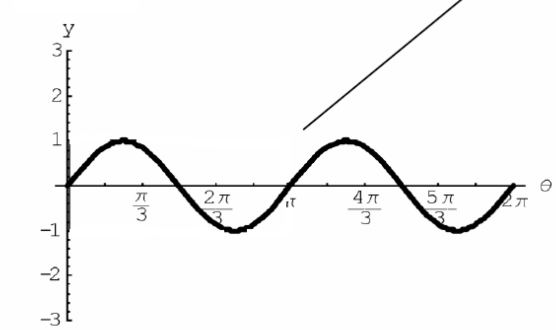
This graph has a period of



This graph has a period of



This graph has a period of



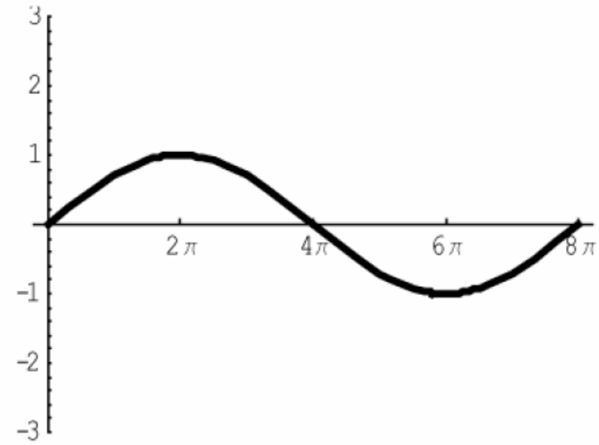
Most graphs given to you won't be as simple as the first three. In trig graphs that are continuous, you will have to first identify a sine or cosine pattern before you can determine the period.

The easiest way to do this is to draw a square around either pattern and look at the length.

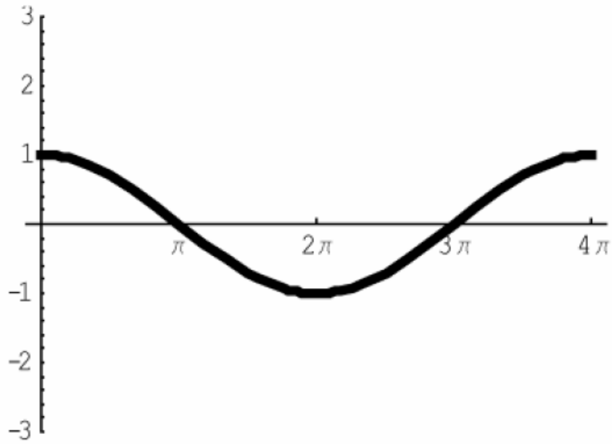
The graph on the left has a period of

Questions: *For each of the following graphs, draw a rectangle around the indicated pattern and state the period.*

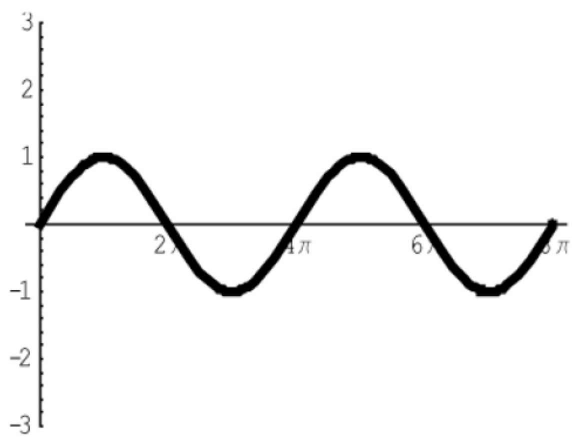
1) Draw a rectangle around a **sine** pattern.



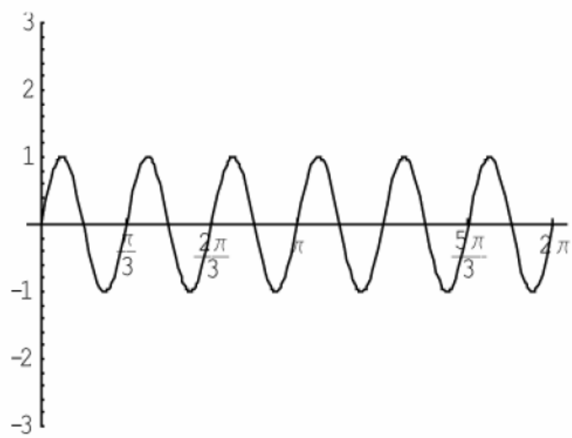
2) Draw a rectangle around a **cosine** pattern.



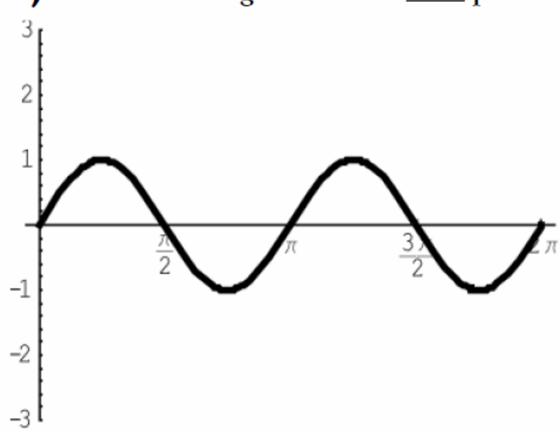
3) Draw a rectangle around a **sine** pattern.



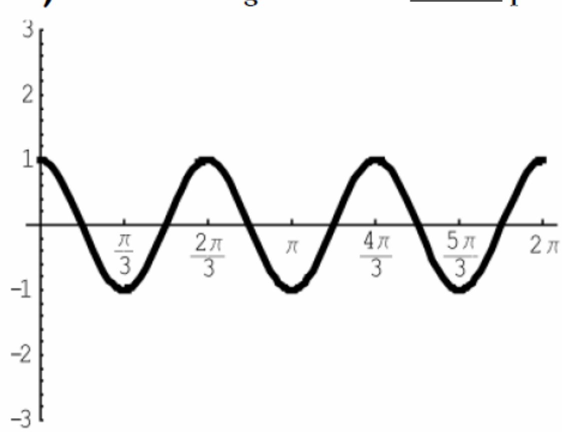
4) Draw a rectangle around a **sine** pattern.



5) Draw a rectangle around a **sine** pattern.



6) Draw a rectangle around a **cosine** pattern.



Part II: Graphing with Amplitutde, Frequency,and Period

$y = a\cos bx \quad or \quad y = a\sin bx$

Frequency "b"

The "b" value represents the number of cycles a trig graph has within a span of 2π.
It is the number that you see in a trig function right beside θ. ($y = \sin b\theta$)
The b value is <u>NOT</u> the period.

The b-value and period (for radians) are related by the formula: $\text{Period} = \frac{2\pi}{\text{b}}$ *or* $\text{b} = \frac{2\pi}{\text{Period}}$

Period $(\frac{2\pi}{b})$

- Think of it like an "x-multiplier"
- Will shrink or stretch the period along the x-axis
- For b > 0, period will be $\frac{2\pi}{b}$

- First find length of period
- Then divide by four to find each of the five tick marks for the x-axis
- Plot as usual

Concept 1: Graph the following functions for the interval $0 \leq x \leq 2\pi$.

Ex 1: y = cos2x

What is the frequency? ______

What does the frequency tell you?

Ans: There are _____ ___________ curves from $0 \leq x \leq 2\pi$

Original Pattern: __, __, __, __, __

Amplitude = ___

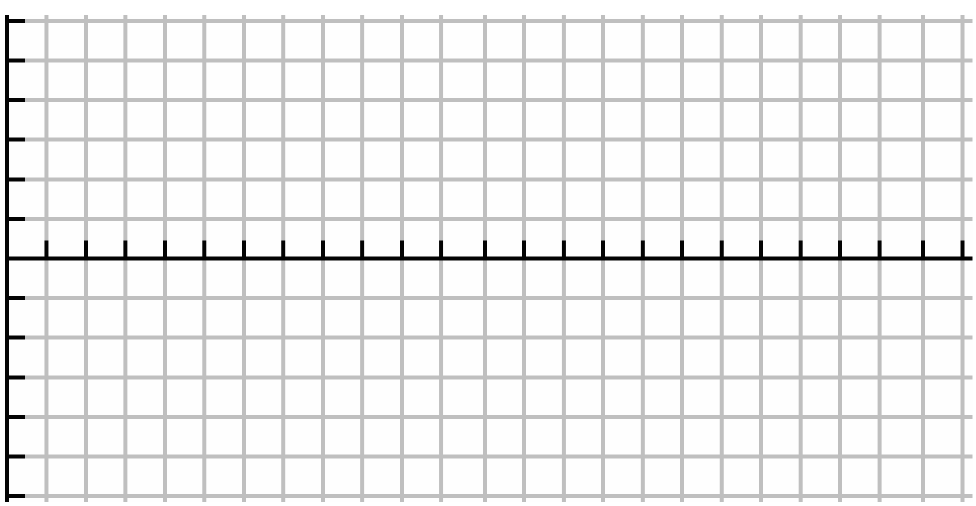
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __



Ex 2: $y = 3\sin\frac{2}{3}x$

What is the frequency? ______

What does the frequency tell you?

Ans: There are _____ ___________ curves from $\mathbf{0} \leq \boldsymbol{x} \leq \mathbf{2}\,\boldsymbol{\pi}$

Original Pattern: __, __, __, __, __

Amplitude = ___

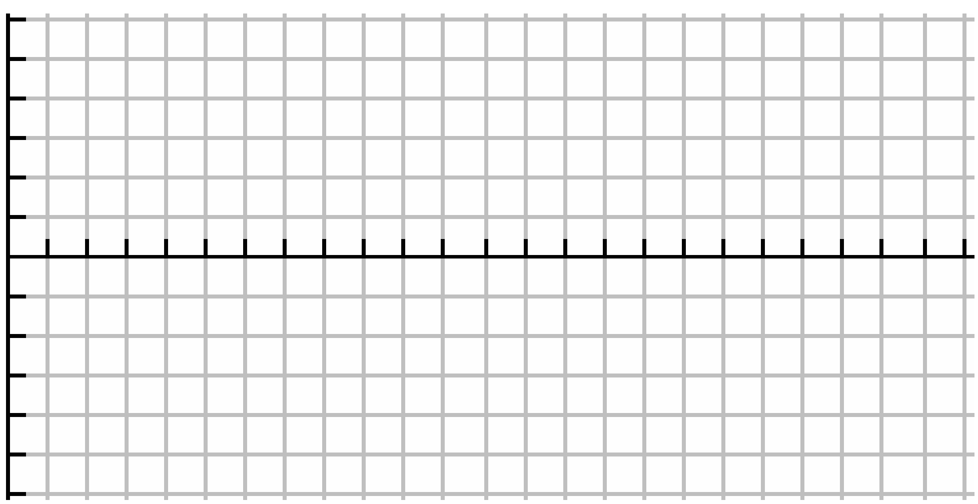
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __



Ex 3: y = -4cos4x

What is the frequency? ______

What does the frequency tell you?

Ans: There are _____ ___________ curves from $\mathbf{0 \leq x \leq 2\pi}$

Original Pattern: __, __, __, __, __

Amplitude = ___

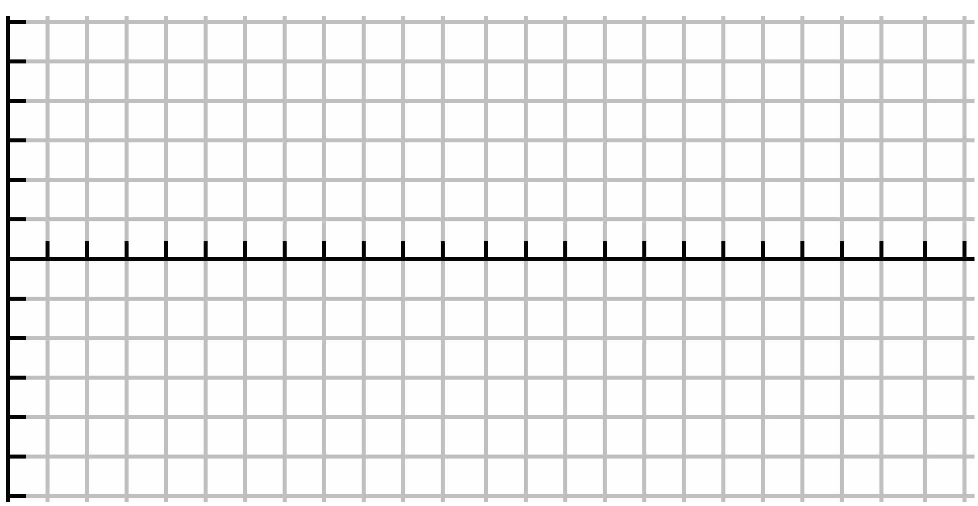
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b} =$

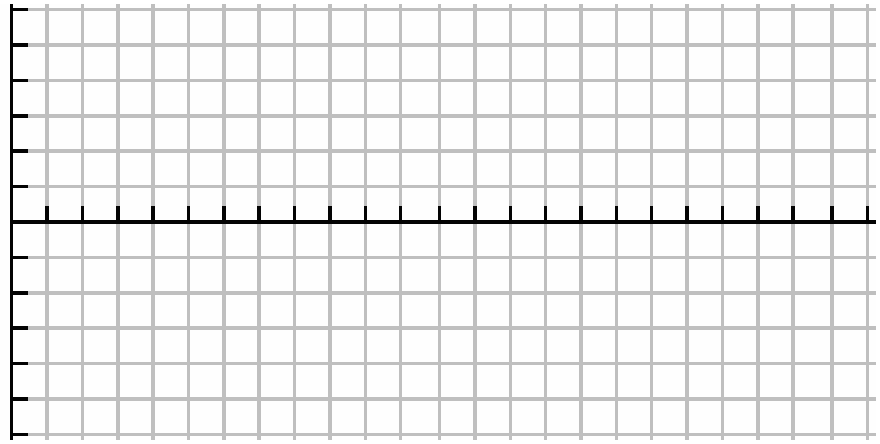
Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __



Practice:

1. $y = \frac{1}{2}\sin\frac{1}{2}x$



Original Pattern: __, __, __, __, __

Amplitude = ___

New Pattern: __, __, __, __, __

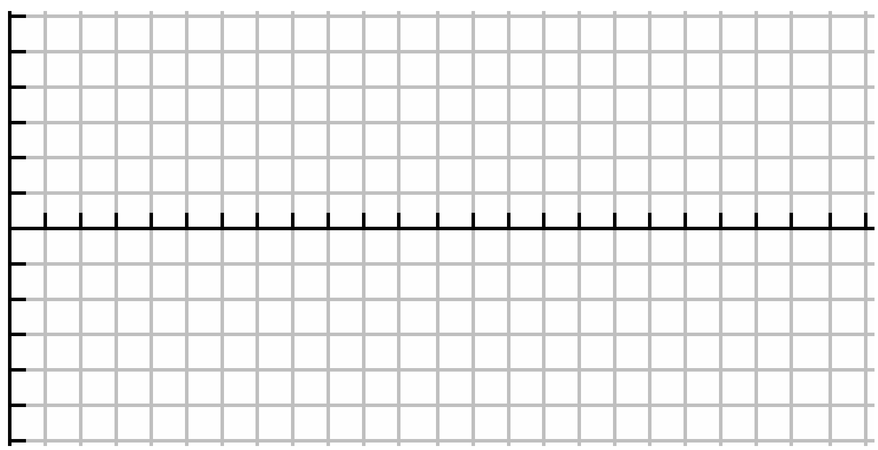
Frequency = __

Period = $\frac{2\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __

2. $y = -4\cos \pi x$



Original Pattern: __, __, __, __, __

Amplitude = ___

New Pattern: __, __, __, __, __

Frequency = __

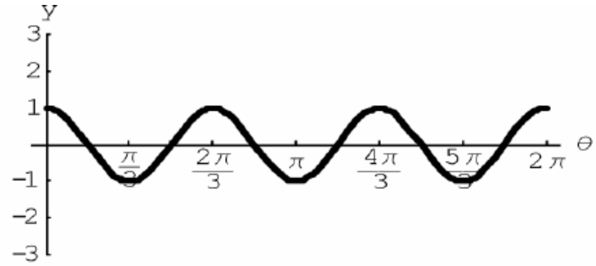
Period = $\frac{2\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __

Part III: Writing Equations of Trig Graphs

Example 1: Find the cosine equation of the following graph:

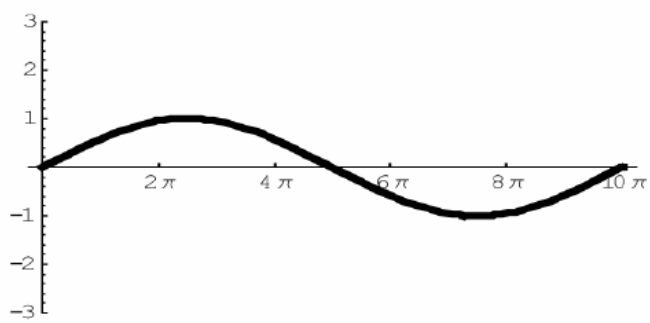


Step 1: First you need to draw a rectangle around the cosine pattern. In this graph,

Step 2: Once you identify the period, find b by performing the following calculation:

Step 3: Now that we have the b value, and a cosine pattern is identified, we can write the equation :

Example 2: Find the sine equation of the following graph:



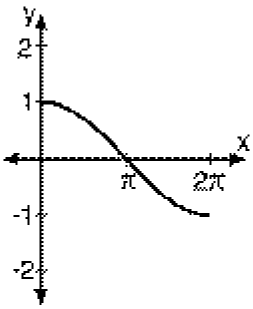
Step 1: First you need to draw a rectangle around the sine pattern you want to use. In this graph,

Step 2: Once you identify the period, find b by performing the following calculation:

Step 3: Now that we have the b value, and we identified a sine pattern, we can write the equation:

Practice: Write the following trig equations for the graphs below.

Ex 3:



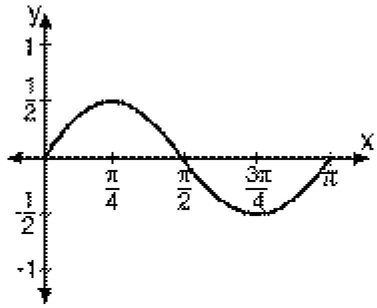
Amplitude = ____

Frequency = ____

Period = ____

Equation: ________________

Ex 4:



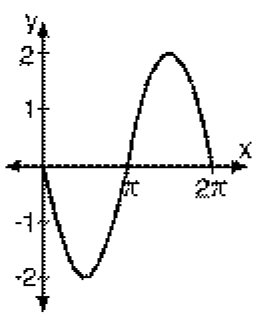
Amplitude = ____

Frequency = ____

Period = ____

Equation: ________________

5)



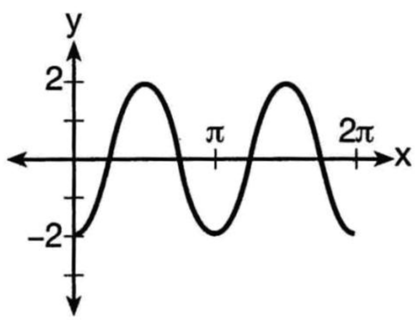
Amplitude = ____

Frequency = ____

Period = ____

Equation: ________________

6)



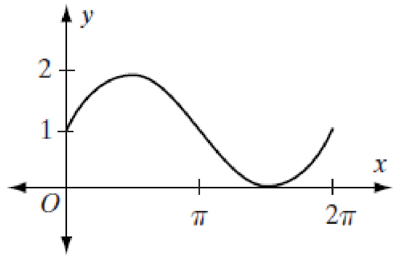
Amplitude = ____

Frequency = ____

Period = ____

Equation: ________________

Challenge

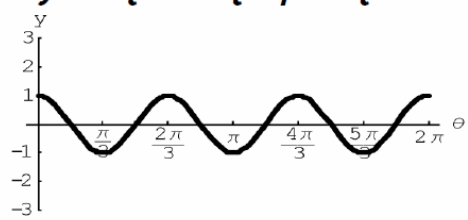


The equation of the graph shown is

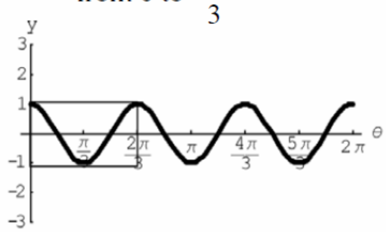
Summary:

Given a graph, you must find the b value before you can write the equation.

Example 1: Find the cosine equation of the following graph:



Step 1: First you need to draw a rectangle around the cosine pattern. In this graph, we can easily see a cosine patten going from 0 to $\frac{2\pi}{3}$



Step 2: Once you identify the period, find *b* by performing the following calculation:

$$b = \frac{2\pi}{\text{Period}}$$

$$b = \frac{2\pi}{\frac{2\pi}{3}}$$

$$b = 2\pi \times \frac{3}{2\pi}$$

$$b = 3$$

Step 3: Now that we have the *b* value, and a cosine pattern is identified, we can write the equation :

$$y = \cos 3\theta$$

Exit Ticket:

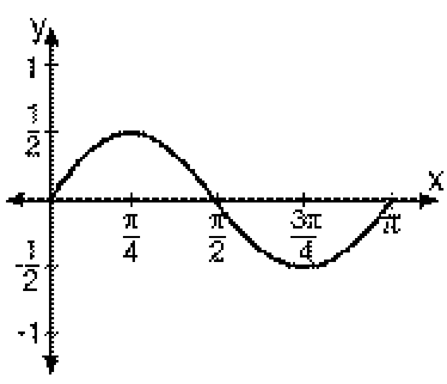
What equation is represented by the graph below?

A) $y = \frac{1}{2}\sin\frac{1}{2}x$

B) $y = 2\sin\frac{1}{2}x$

C) $y = -\frac{1}{2}\cos 2x$

D) $y = \frac{1}{2}\sin 2x$



Day 2 – Homework

Determine the amplitude and period of each function.

1. $y = \sin 4x$

Amplitude = __________

Period = ____________

2. $y = \cos 5x$

Amplitude = __________

Period=____________

3. $y = \sin x$

Amplitude=____________

Period=____________

4. $y = 4\cos x$

Amplitude = __________

Period = ____________

5. $y = -2\sin x$

Amplitude = __________

Period=____________

6. $y = 2\sin(-4x)$

Amplitude=____________

Period=____________

7. $y = 3\sin\frac{2}{3}x$

Amplitude = __________

Period = ____________

8. $y = -4\cos 5x$

Amplitude = __________

Period=____________

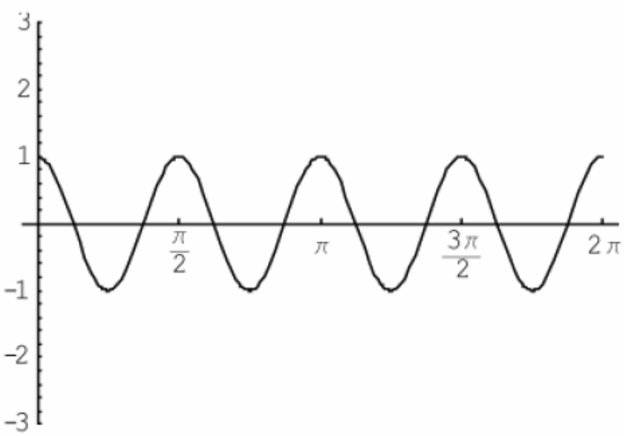
9. $y = 3\cos(-2x)$

Amplitude=____________

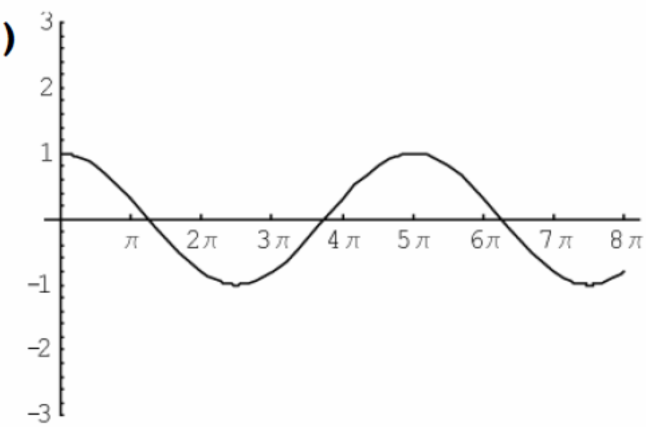
Period=____________

Questions: For each of the following graphs, write the equation:

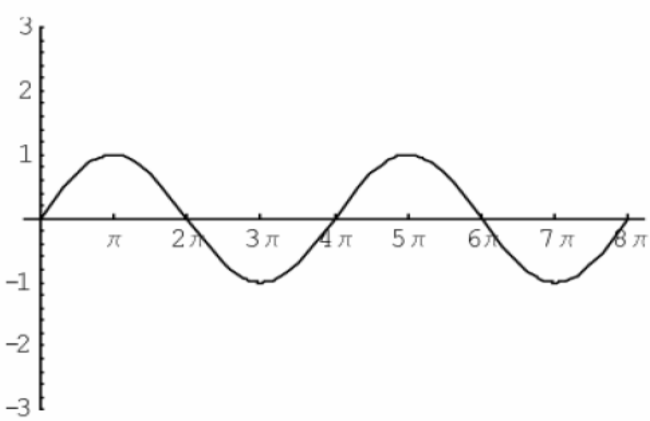
1)



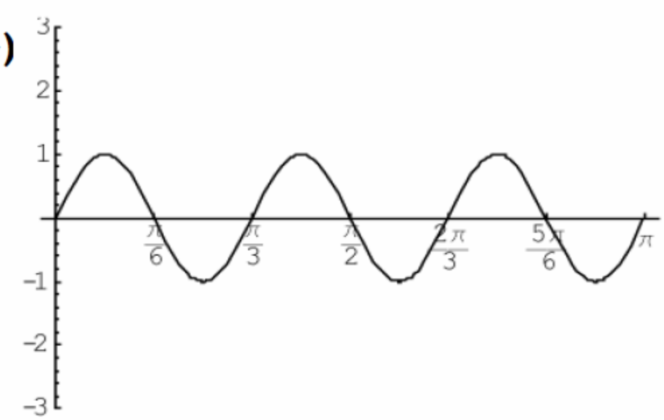
2)



3)

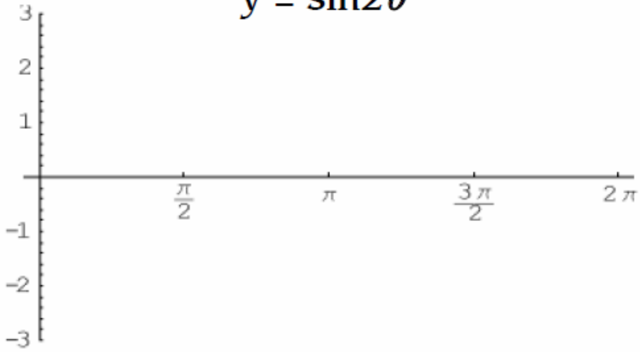


4)

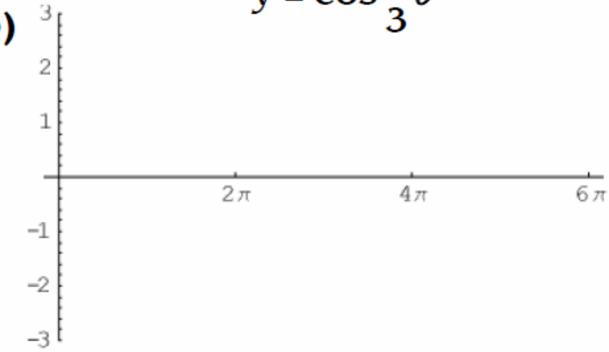


For each of the following equations, draw the graph:

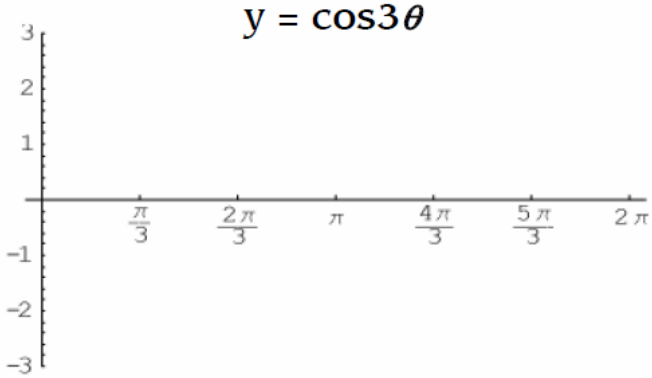
5) $y = \sin 2\theta$



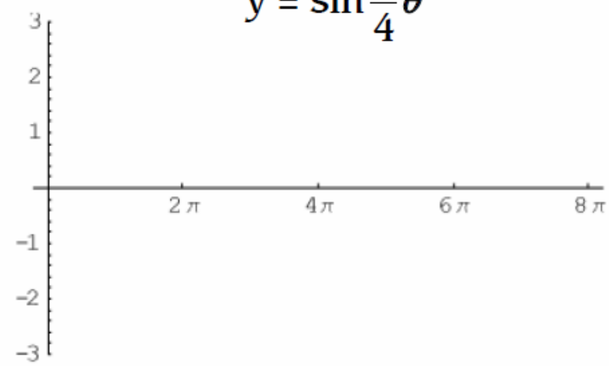
6) $y = \cos \frac{1}{3}\theta$



7) $y = \cos 3\theta$



8) $y = \sin \frac{1}{4}\theta$

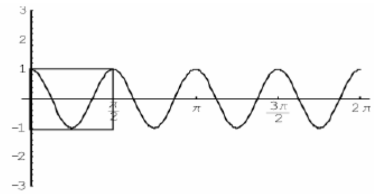


Day 2 - Answers

1. A = 1; P = $\frac{\pi}{2}$	2. A = 1; P = $\frac{2\pi}{5}$	3. A = 1; P = 2π	4. A = 4; P = 2π	
5. A = 2; P = 2π	6. A = 2; P = $\frac{\pi}{2}$	7. A = 3; P = 3π	8. A = 4; P = $\frac{2\pi}{5}$	9. A = 3; P = π

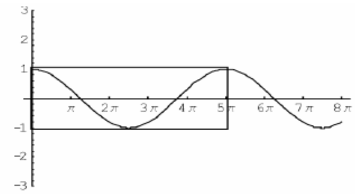
Answers:

1) $\cos 4\theta$



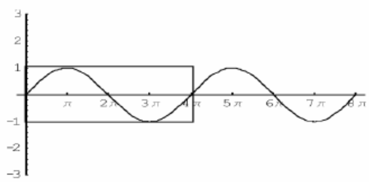
$$b=\frac{2\pi}{P}=\frac{2\pi}{\frac{\pi}{2}}=2\not\pi\times\frac{2}{\not\pi}=4$$

2) $\cos\frac{2}{5}\theta$



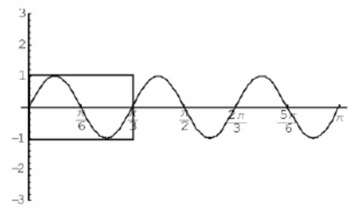
$$b=\frac{2\pi}{P}=\frac{2\not\pi}{5\not\pi}=\frac{2}{5}$$

3) $\sin\frac{1}{2}\theta$



$$b=\frac{2\pi}{P}=\frac{2\not\pi}{4\not\pi}=\frac{1}{2}$$

4) $\sin 6\theta$



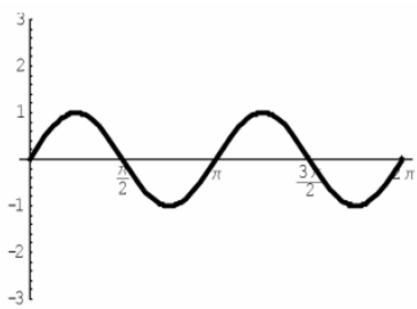
$$b=\frac{2\pi}{P}=\frac{2\pi}{\frac{\pi}{3}}=2\not\pi\times\frac{3}{\not\pi}=6$$

5)

$P=\frac{2\pi}{b}$

$P=\frac{\not2\pi}{\not2}$

$P=\pi$



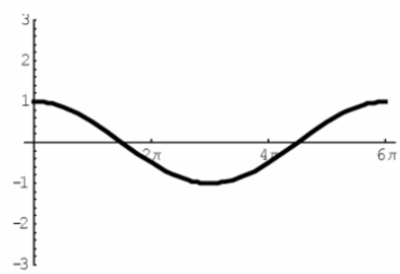
6)

$P=\frac{2\pi}{b}$

$P=\frac{2\pi}{\frac{1}{3}}$

$P=2\pi\times\frac{3}{1}$

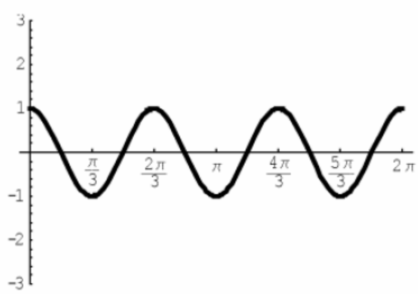
$P=6\pi$



7)

$P=\frac{2\pi}{b}$

$P=\frac{2\pi}{3}$



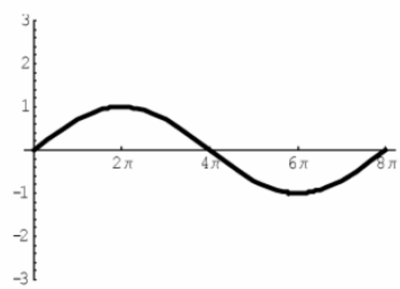
8)

$P=\frac{2\pi}{b}$

$P=\frac{2\pi}{\frac{1}{4}}$

$P=2\pi\times\frac{4}{1}$

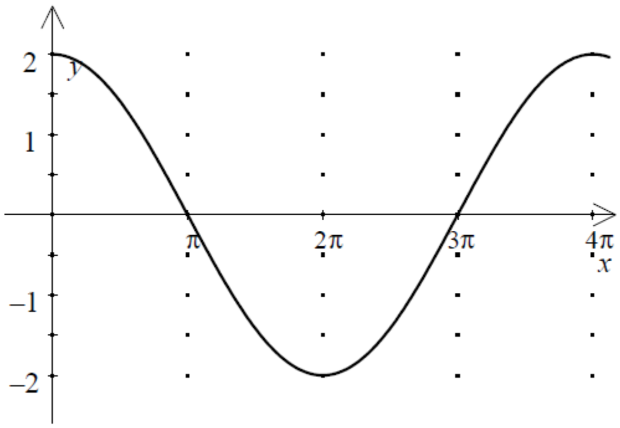
$P=8\pi$



Algebra2/Trig: Day 3 – Phase Shifts of Sine & Cosine Functions

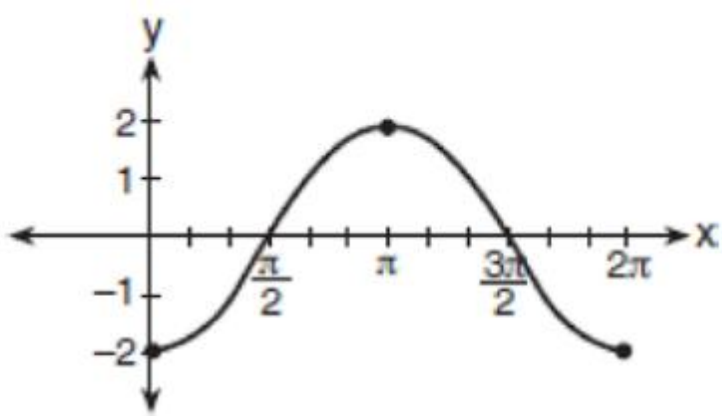
Warm - Up

1. Which is the equation of the function graphed below?



[A] $y = 2\cos\frac{x}{2}$ [B] $y = 2\cos 2x$ [C] $y = 2\cos x$ [D] $y = \cos 2x$

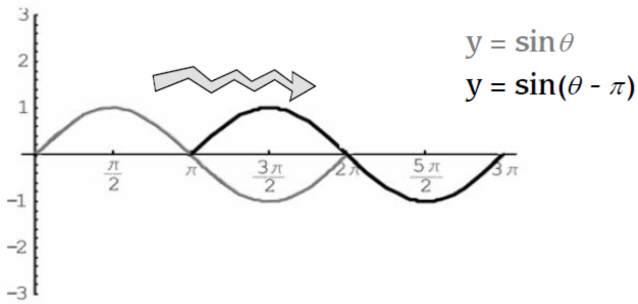
2 The accompanying graph shows a trigonometric function. State an equation of this function.



The phase shift is the horizontal translation applied to a trig graph. It is the number added or subtracted to θ inside the equation.

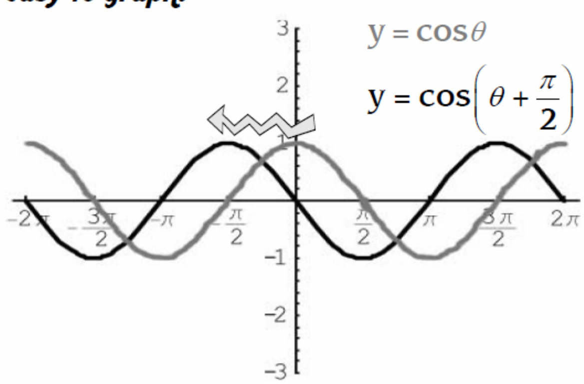
Phase shift is represented by the letter "c" in $y = \sin(\theta \pm c)$

Notice in the following graphs that you will do the opposite of what the sign is.
The + will move the graph *left*, and the – will move the graph *right*.



The $-\pi$ means we move the graph ***right*** by π units.

Not all graphs are going to be given as one cycle, since trig graphs can go forever in both directions! A phase shift will shift everything horizontally by the same amount, so it's still easy to graph.



The $+\frac{\pi}{2}$ means we move the graph ***left*** by $\frac{\pi}{2}$ units.

A **phase shift** is a horizontal shift of a sine or cosine curve.

$$y = a\sin b(x \pm c) + d$$
$$y = a\cos\ b(x \pm c) + d$$

WHERE

- $|a|$ is still the amplitude,
- $|b|$ is still the frequency
- d is the vertical shift or midline.

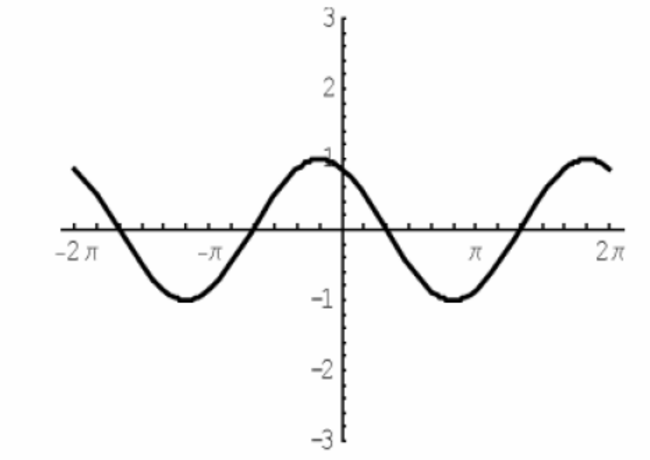
AND

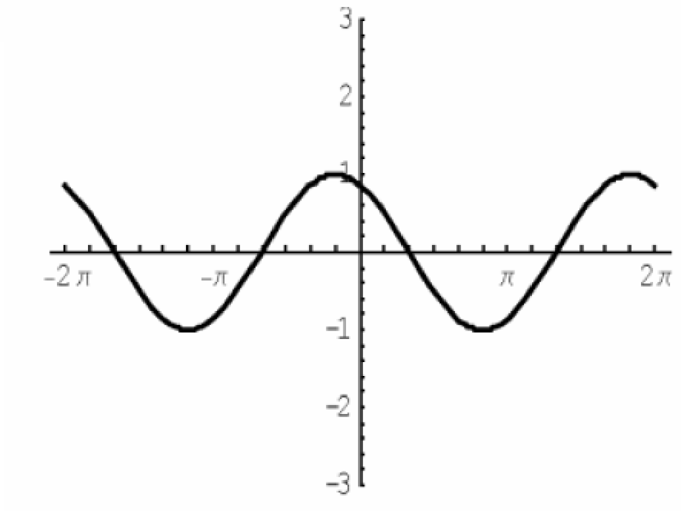
- c is the phase (horizontal) shift

Examples:

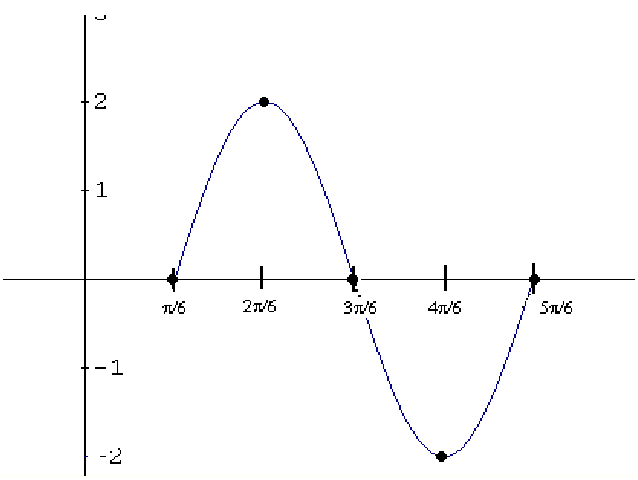
- $y = 5\sin 2\left(x - \frac{\pi}{2}\right) + 3$ is the graph $y = 5\sin 2x$ shifted 3 units up and $\frac{\pi}{2}$ units to the **right**.
- $y = -2\cos 4\left(x + \frac{\pi}{3}\right) + 4$ is the graph $y = -2\cos\ 4x$ shifted 4 units ___ and $\frac{\pi}{3}$ units to the ______.
- $y = \cos\frac{1}{2}(x + \pi) - 2$ is the graph $y =$ ____________ shifted __units ____ and _____ units to the ______.
- $y = -3\sin(x - \frac{\pi}{4}) + 1$ is the graph $y =$ ____________ shifted __ units ____ and _____ units to the ______.

It is always possible to write at least one sine equation and one cosine equation for the same trig graph.





Write a trig equation for this graph below.



How to scale the x-axis

1. Factor out any coefficient on x, this is your “b”
2. Find the period and divide by 4 to find spacing for x-axis
3. Solve for the new start and end point of the period
4. Label axis with five tick marks using spacing found in step 2

Level A: Example 1: $y = \sin\left(x - \frac{\pi}{3}\right)$ **Inequality:** $\mathbf{0 \le b(x \pm c) \le 2\pi}$

Original Pattern: ___, ___, ___, ___, ___

Amplitude = ___

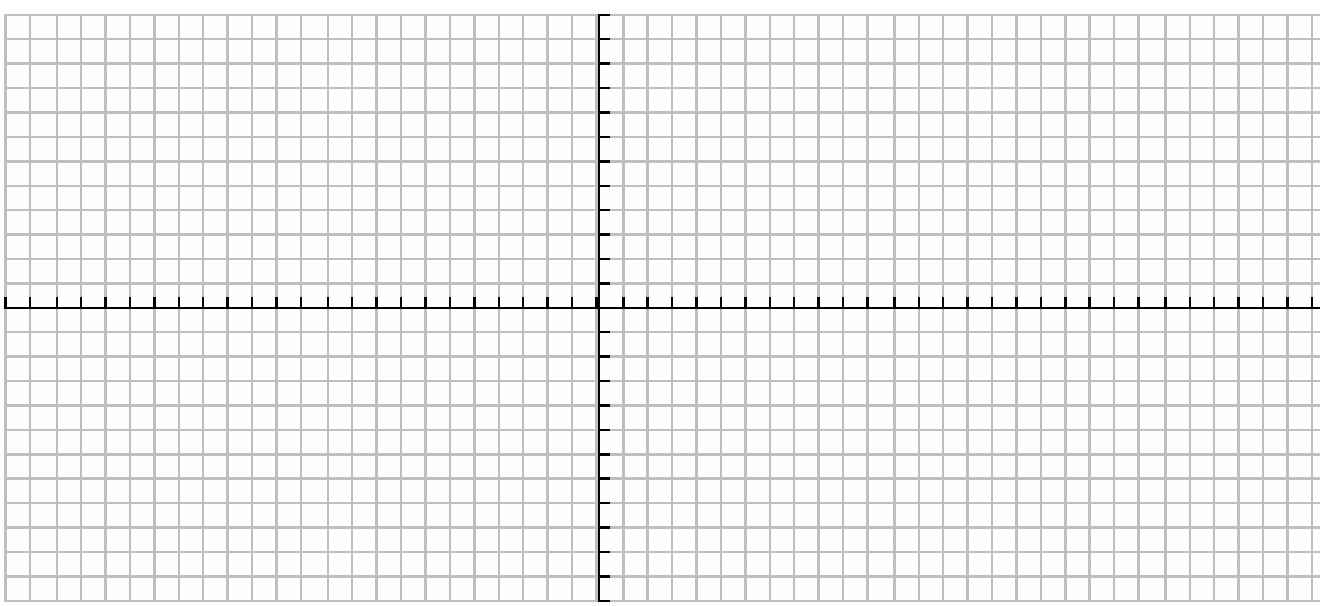
New Pattern: ___, ___, ___, ___, ___

Frequency = ___

Period = $\frac{2\pi}{b} =$

Scale = _____

Critical Points (scale for x-axis): ___, ___, ___, ___, ___



Level A: Example 2: $y = 3\cos\left(x + \frac{\pi}{4}\right)$ **Inequality:** $\mathbf{0} \leq \boldsymbol{b}(\boldsymbol{x} \pm \boldsymbol{c}) \leq \mathbf{2\pi}$

Original Pattern: ___, ___, ___, ___, ___

Amplitude = ___

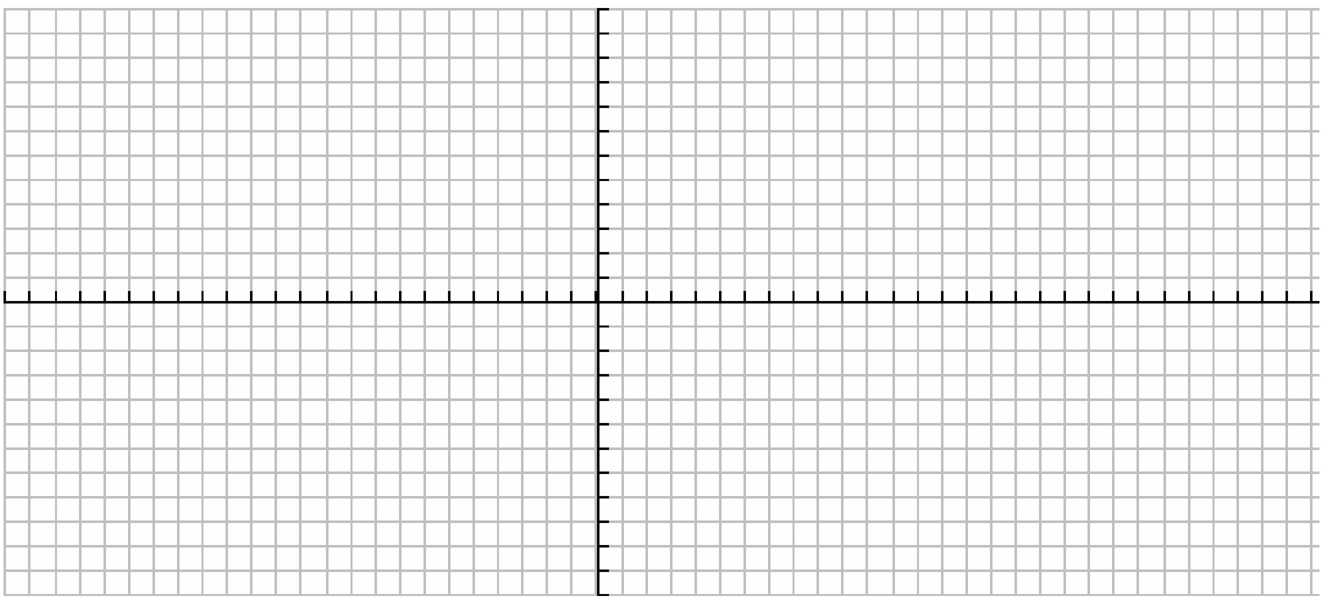
New Pattern: ___, ___, ___, ___, ___

Frequency = ___

Period = $\frac{2\pi}{b} =$

Scale = _____

Critical Points (scale for x-axis): ___, ___, ___, ___, ___



Level B.

Example 3: $y = 4\cos(3x - \frac{\pi}{2}) + 2$

Factored Form:

Inequality: $0 \leq b(x \pm c) \leq 2\pi$

Original Pattern: __, __, __, __, __

Amplitude = ___

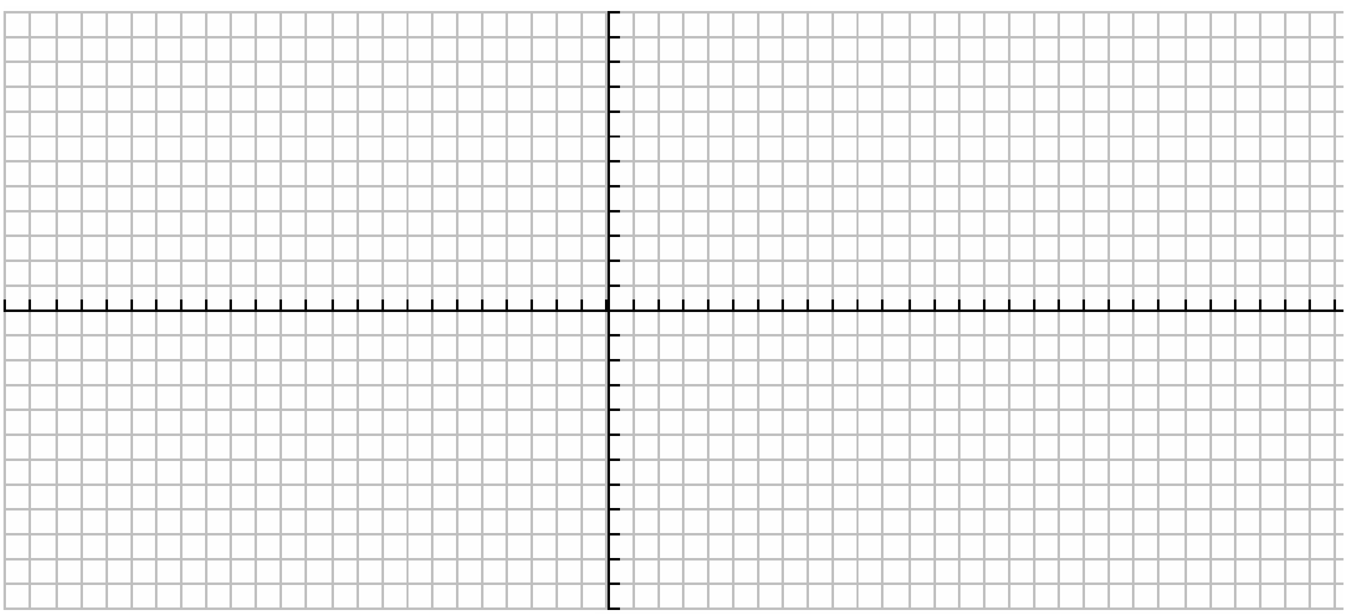
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __



Example 4: $y = \frac{1}{2}\sin(2x + \frac{\pi}{2})$ - 1

Factored Form:

Inequality: $0 \leq b(x \pm c) \leq 2\pi$

Original Pattern: __, __, __, __, __

Amplitude = ___

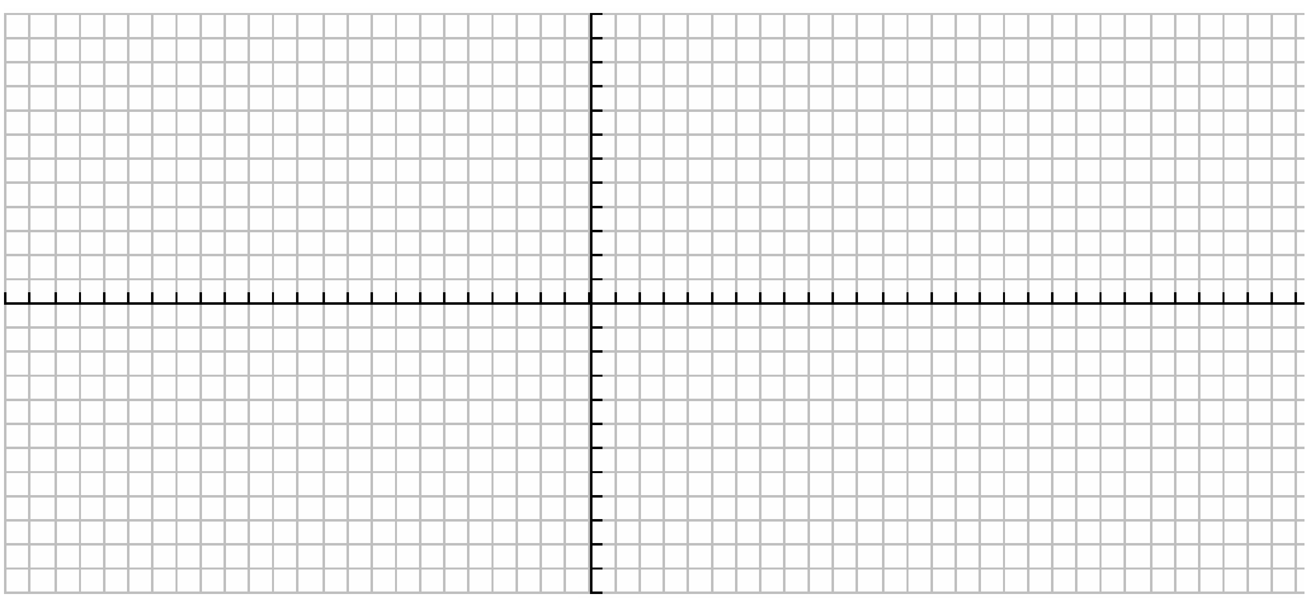
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __



SUMMARY:

Analyze the graph of $y = \frac{1}{2}\sin\left(x - \frac{\pi}{3}\right)$.

Algebraic Solution

The amplitude is $\frac{1}{2}$ and the period is 2π. By solving the equations

$$x - \frac{\pi}{3} = 0 \quad \text{and} \quad x - \frac{\pi}{3} = 2\pi$$

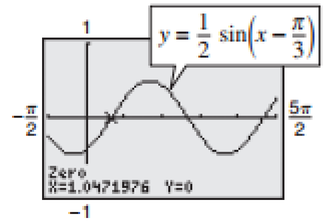
$$x = \frac{\pi}{3} \qquad\qquad x = \frac{7\pi}{3}$$

you see that the interval $[\pi/3, 7\pi/3]$ corresponds to one cycle of the graph. Dividing this interval into four equal parts produces the following key points.

Intercept	*Maximum*	*Intercept*	*Minimum*	*Intercept*
$\left(\frac{\pi}{3}, 0\right)$,	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$,	$\left(\frac{4\pi}{3}, 0\right)$,	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$,	$\left(\frac{7\pi}{3}, 0\right)$

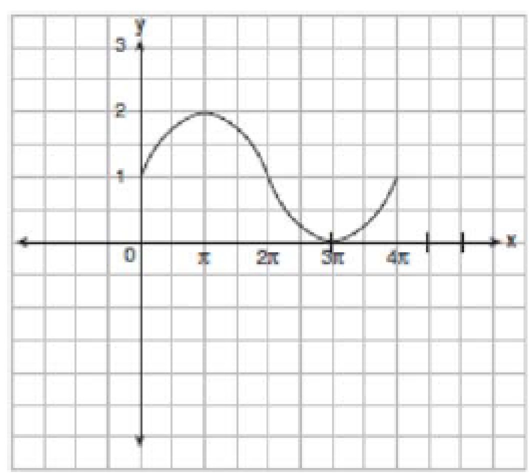
Graphical Solution

Use a graphing utility set in *radian* mode to graph $y = (1/2)\sin(x - \pi/3)$, as shown in Figure 4.49. Use the *minimum*, *maximum*, and *zero* or *root* features of the graphing utility to approximate the key points $(1.05, 0)$, $(2.62, 0.5)$, $(4.19, 0)$, $(5.76, -0.5)$, and $(7.33, 0)$.



Exit Ticket:

In physics class, Eva noticed the pattern shown in the accompanying diagram on an oscilloscope.



Which equation best represents the pattern shown on this oscilloscope?

1) $y = \sin\left(\frac{1}{2}x\right) + 1$

2) $y = \sin x + 1$

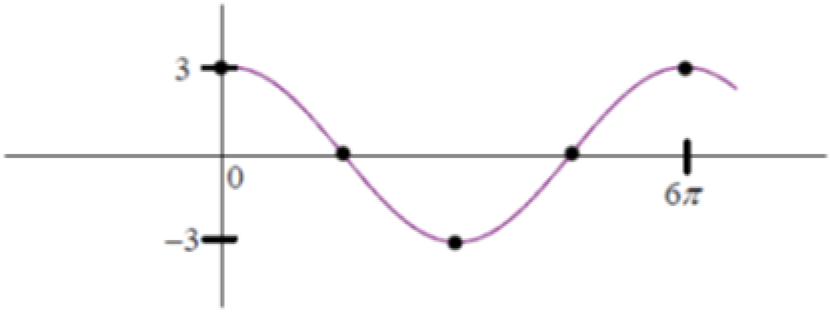
3) $y = 2\sin x + 1$

4) $y = 2\sin\left(-\frac{1}{2}x\right) + 1$

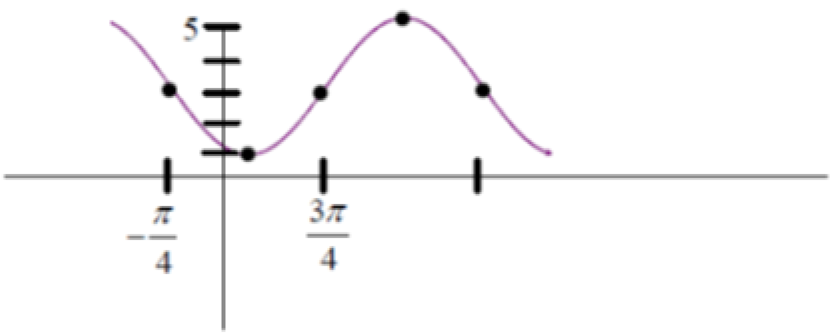
For each function below, list the amplitude, period, vertical shift and phase shift.

Function	**Amplitude**	**Period**	**Vertical Shift**	**Phase Shift**
1.) $y = -5\cos 3\left(x - \frac{\pi}{6}\right)$				
2.) $y = -1 + \frac{1}{3}\sin\left(x + \frac{3\pi}{4}\right)$				
3.) $y = 6 - \sin\left(2x - \frac{\pi}{2}\right)$				
4.) $y = 2\cos\frac{1}{4}x - \pi$				
5.) $y = \frac{4}{3}\sin\left(3x + \frac{7\pi}{6}\right) - \frac{3}{2}$				
6.) $y = -5 + 7\cos\frac{1}{2}\left(x + \frac{\pi}{3}\right) + 3$				

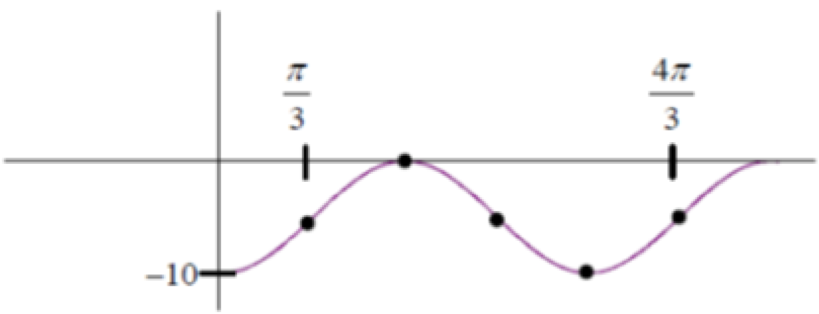
7.) Write the equation of the cosine function displayed in the graph below.



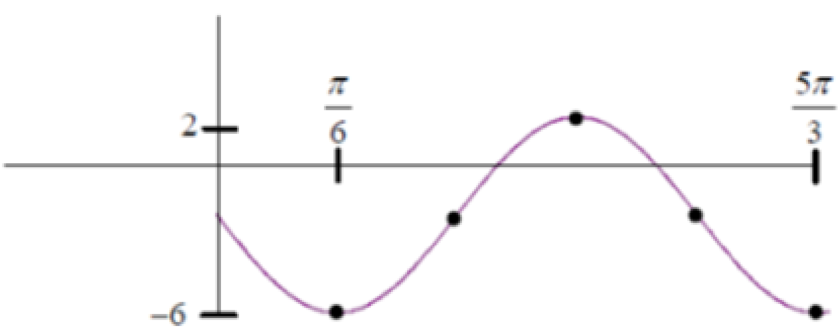
8.) Write the equation of the sine function displayed in the graph below.



9.) Write the equation of the sine function displayed in the graph below.



10.) Write the equation of the cosine function displayed in the graph below.



11) $y = 3\cos 2\left(x - \frac{\pi}{12}\right)$

Original Pattern: __, __, __, __, __

Amplitude = ___

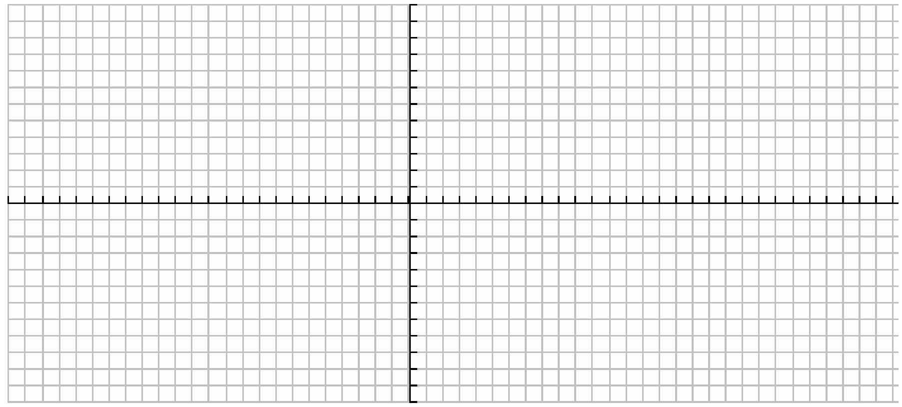
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b}$ =

Scale = _____

Critical Points (scale for x-axis): __, __, __, __, __



12) $y = -\sin\left(x + \frac{\pi}{3}\right) - 4$

Original Pattern: __, __, __, __, __

Amplitude = ___

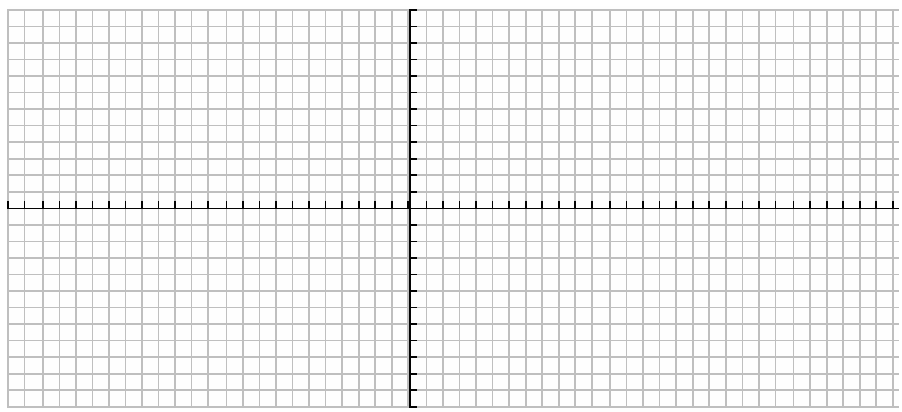
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b}$ =

Scale = _____

Critical Points (scale for x-axis): __, __, __, __, __



13) $y = 2\sin\left(\frac{3}{2}x - \pi\right) + 2$

Original Pattern: __, __, __, __, __

Amplitude = ___

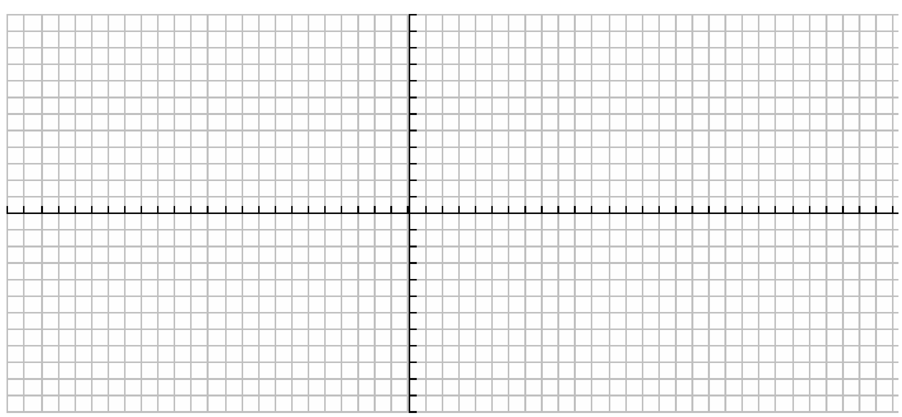
New Pattern: __, __, __, __, __

Frequency = __

Period = $\frac{2\pi}{b}$ =

Scale = _____

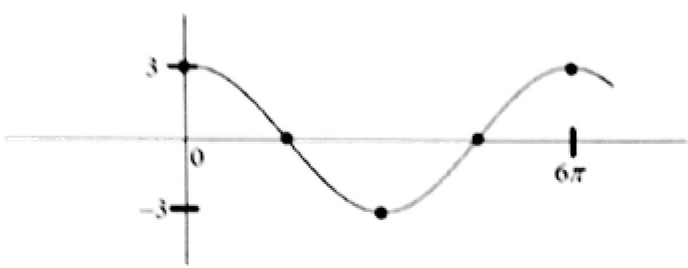
Critical Points (scale for x-axis): __, __, __, __, __



Answers – Day 3

Function	Amplitude	Period	Vertical Shift	Phase Shift
1.) $y=-5\cos 3\left(x-\frac{\pi}{6}\right)$	$\lvert -5\rvert=5$	$b=3$ $P=\frac{2\pi}{3}$	none	right $\frac{\pi}{6}$
2.) $y=-1+\frac{1}{3}\sin\left(x+\frac{3\pi}{4}\right)$	$\frac{1}{3}$	$b=1$ $P=\frac{2\pi}{1}=2\pi$	down 1	left $\frac{3\pi}{4}$
3.) $y=6-\sin\left(2x-\frac{\pi}{2}\right)$	$\lvert -1\rvert=1$	$b=2$ $P=\frac{2\pi}{2}=\pi$	up 6	$\frac{2x}{2}-\frac{\pi/2}{2}$ $2(x-\pi/4)$ right $\pi/4$
4.) $y=2\cos\frac{1}{4}x-\pi$	2	$b=1/4$ $P=\frac{2\pi}{\frac{1}{4}}=2\pi\cdot\frac{4}{1}$ $P=8\pi$	down π	none
5.) $y=\frac{4}{3}\sin\left(3x+\frac{7\pi}{6}\right)-\frac{3}{2}$	$\frac{4}{3}$	$b=3$ $P=\frac{2\pi}{3}$	down $3/2$	$\frac{3x}{3}+\frac{7\pi/6}{3}$ $3(x+7\pi/18)$ left $7\pi/18$
6.) $y=-5+7\cos\frac{1}{2}\left(x+\frac{\pi}{3}\right)+3$	7	$b=1/2$ $P=\frac{2\pi}{1/2}=2\pi\cdot\frac{2}{1}$ $P=4\pi$	down 2 $(-5+3)$	left $\pi/3$

7.) Write the equation of the cosine function displayed in the graph below.



$a=3$

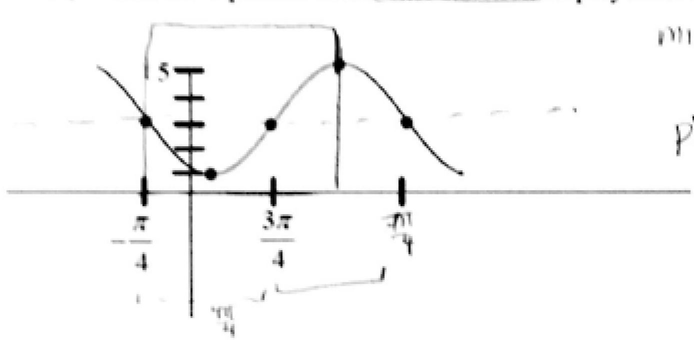
$P=\frac{6\pi}{1}=\frac{2\pi}{b}$

$6b=2$, $\frac{2}{6}$

$b=\frac{1}{3}$

Ans: $y=3\cos\frac{1}{3}x$

8.) Write the equation of the sine function displayed in the graph below.



midline = 3

$a=-2$

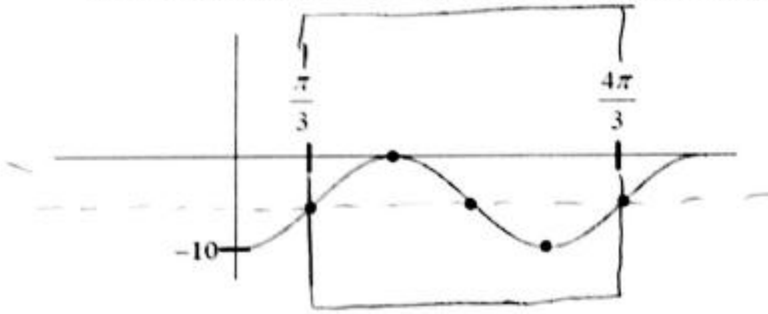
phase shift = $-\pi/4$

$y=-2\sin\left(x+\frac{\pi}{4}\right)+3$

Period = $\frac{7\pi}{4}-\left(-\frac{\pi}{4}\right)=\frac{8\pi}{4}=2\pi$

$b=1$

9.) Write the equation of the sine function displayed in the graph below.



midline = −5

$a = 5$

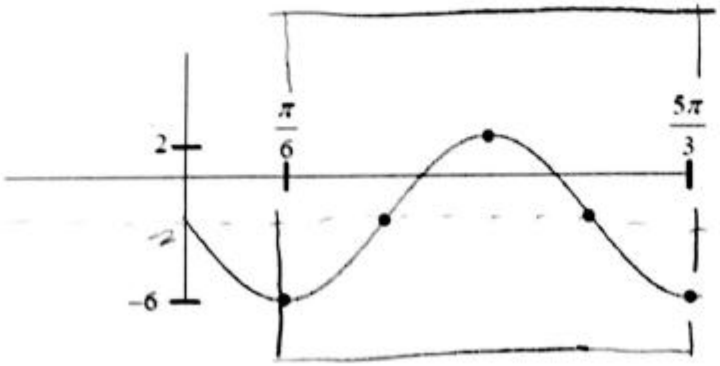
phase shift = $\pi/3$

Period = $\frac{4\pi}{3} - \frac{\pi}{3} = \frac{3\pi}{3} = \pi$

$\frac{\pi}{1} = \frac{2\pi}{b} \rightarrow \frac{\pi}{1} = \frac{2\pi}{b} \rightarrow b = 2$

$y = 5\sin 2(x - \pi/3) - 5$

10.) Write the equation of the cosine function displayed in the graph below.



midline = $\frac{min + max}{2} = \frac{-6+2}{2} = -2$

$a = -4$

Period = $\frac{5\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{2}$

$\frac{P}{1} = \frac{2\pi}{b} \rightarrow \frac{\frac{3\pi}{2}}{1} = \frac{2\pi}{b} \rightarrow \frac{3b}{2} = \frac{2}{1}$

$3b = 4 \rightarrow b = \frac{4}{3}$

$y = -4\cos\frac{4}{3}(x - \pi/6) - 2$

36

11) $y = 3\cos 2\left(x - \frac{\pi}{12}\right)$

$\frac{0}{2} \le \frac{2(x - \pi/12)}{2} \le \frac{2\pi}{2}$

$0 \le x - \pi/12 \le \pi$

$\pi/12 \le x \le 13\pi/12$

Original Pattern: 1, 0, −1, 0, 1

Amplitude = 3

New Pattern: 3, 0, −3, 0, 3

Frequency = 2

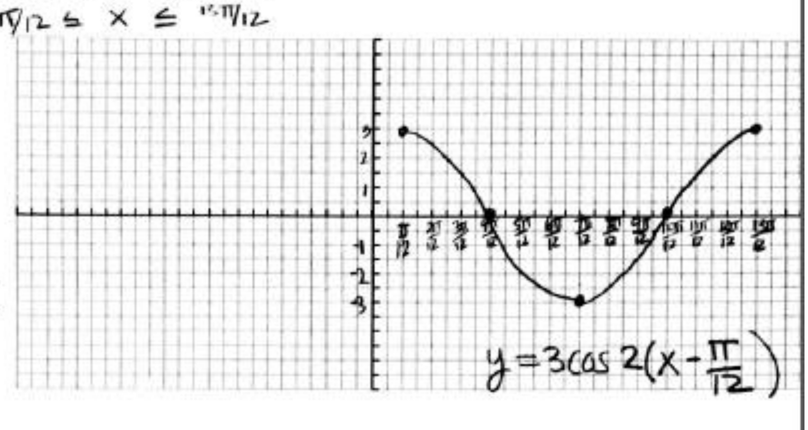
Period = $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Scale = (45°) $\pi/4 = \frac{3\pi}{4}$

Critical Points (scale for x-axis): $\frac{\pi}{12}$, $\frac{4\pi}{12}$, $\frac{7\pi}{12}$, $\frac{10\pi}{12}$, $\frac{13\pi}{12}$

$\left(\frac{\pi}{3}\right)$ $\left(\frac{5\pi}{6}\right)$

GCF of 15 & 45 = 15°



$0 \le x + \pi/3 \le 2\pi$

$-\frac{\pi}{3} \le x \le 5\pi/3$

12) $y = -\sin\left(x + \frac{\pi}{3}\right) - 4$

Original Pattern: 0, 1, 0, -1, 0 → $\cdot -1, -4$

Amplitude = $|-1| = 1$; midline $= -4$

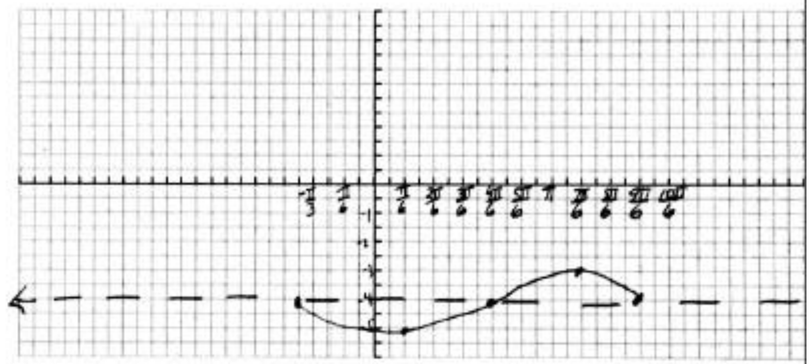
New Pattern: -4, -5, -4, -3, -4

Frequency = 1

Period = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Scale = $2\pi/4 = \frac{\pi}{2}$

Critical Points (scale for x-axis): $-\frac{\pi}{3}$, $\frac{\pi}{6}$, $\frac{4\pi}{6}$ $\left(\frac{2\pi}{3}\right)$, $\frac{7\pi}{6}$, $5\pi/3$



GCF of 60° and 90° = 30° = $\pi/6$

$0 \le \frac{3}{2}x - \pi \le 2\pi$ → $\frac{2\pi}{3} \le \frac{3x}{3} \le \frac{6\pi}{3}$

$\pi \le \frac{3x}{2} \le 3\pi$

$\frac{2\pi}{3} \le x \le 2\pi$

13) $y = 2\sin\left(\frac{3}{2}x - \pi\right) + 2$

Original Pattern: 0, 1, 0, -1, 0 → $\cdot 2 + 2$

Amplitude = 2; midline = 2

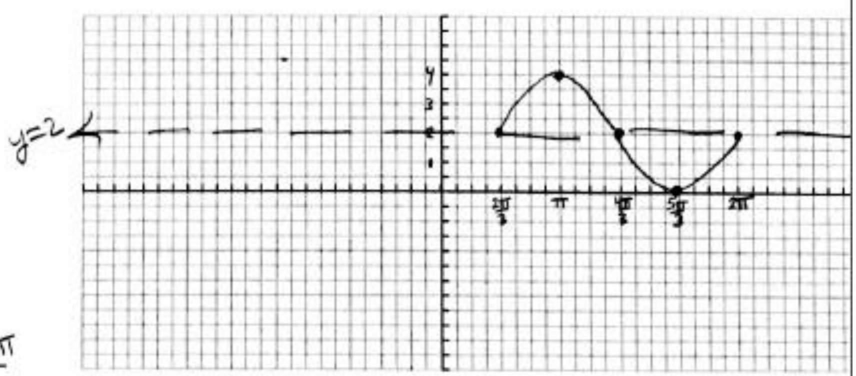
New Pattern: 2, 4, 2, 0, 2

Frequency = $3/2$

Period = $\frac{2\pi}{b} = \frac{2\pi}{3/2} = 2\pi \cdot \frac{2}{3} = 4\pi/3$

Scale = $\frac{4\pi/3}{4} = \frac{4\pi}{12} = \pi/3$

Critical Points (scale for x-axis): $\frac{2\pi}{3}$, $\frac{3\pi}{3}$ (π), $\frac{4\pi}{3}$, $\frac{5\pi}{3}$, 2π



Day 4: Algebra2/Trig: The Graph $\boldsymbol{Tan\, x\,, Cotx, Secx, and\ Cscx}$

θ (in degrees)	0°	30°	45°	60°	90°	180°	270°
θ (in radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$ (0.5)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{\sqrt{3}}{2}$ (0.866)	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$ (0.866)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{1}{2}$ (0.5)	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ (0.577)	1	$\sqrt{3}$ (1.732)	undefined	0	undefined

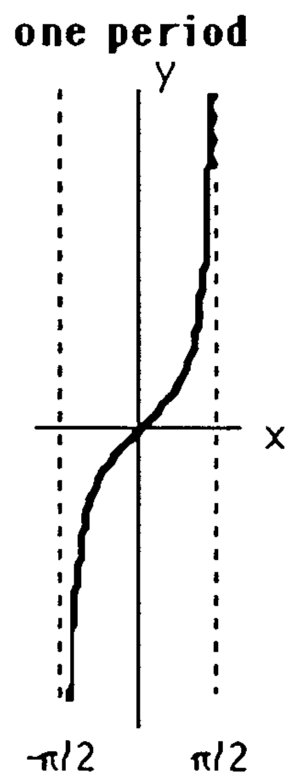
Tangent is undefined at regular intervals. In the chart above, tangent is undefined at x = ____ and x = _____.

This means the domain of y = tan x is:_______________

$Y = tan\ x$ covers ever y value there is, so the range of y=tan x is:

How to graph $\boldsymbol{y = atan(bx)}$

- **Period** length is $\frac{\pi}{b}$
- For your scale we will still use 5 numbers
- First and Last are Vertical Asymptotes (VA; x = #)
- To find the VA use the following inequality: $-\frac{\pi}{2} \le bx \le \frac{\pi}{2}$
- Amplitude = none, but there is an "a" value.
- Pattern of three points: y-values at **–a, 0, a**



Example 1: $y = \tan x$ **on** $-2\pi \le x \le 2\pi$

Inequality for VA: $-\frac{\pi}{2} \le bx \le \frac{\pi}{2}$

Original Pattern: __, __, __,

a = ___

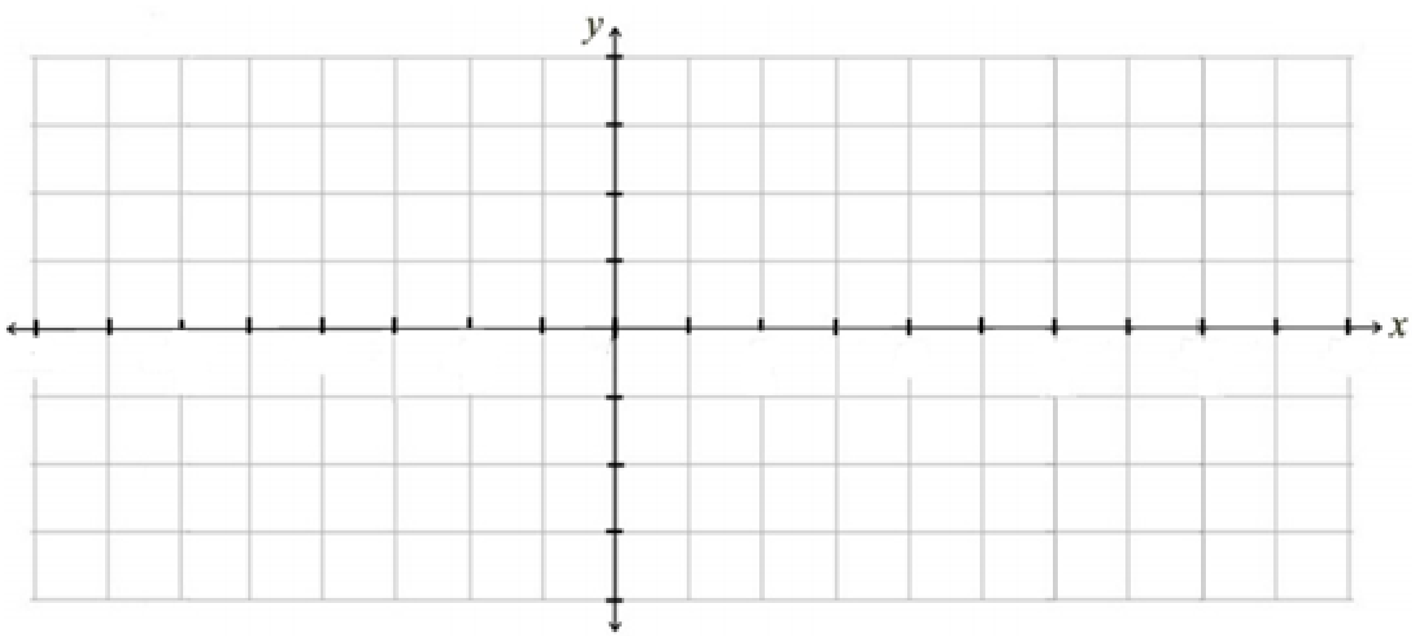
New Pattern: __, __, __,

Frequency = __

Period = $\frac{\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __



The graph of $y = cot\ x$

θ (in degrees)	0°	30°	45°	60°	90°	180°	270°
θ (in radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$ (0.5)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{\sqrt{3}}{2}$ (0.866)	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$ (0.866)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{1}{2}$ (0.5)	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (0.577)	1	$\sqrt{3}$ (1.732)	undefined	0	undefined

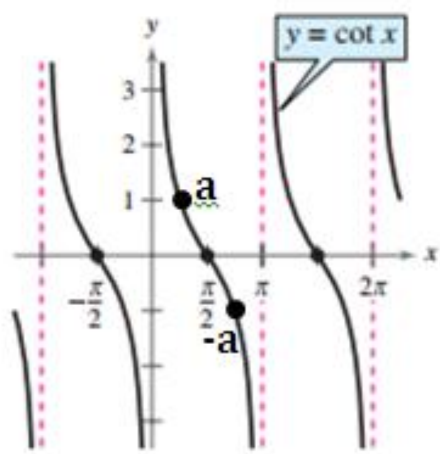
Cotangent is undefined at regular intervals. In the chart above, cotangent is

undefined at x = ____ and x = _____.

This means the domain of y=cot x is:______________

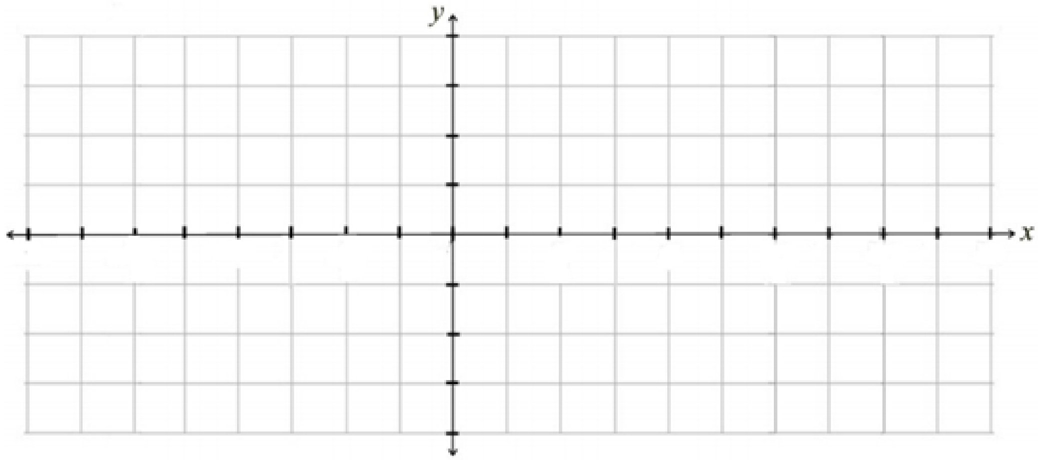
$Y = cot\ x$ covers ever y value there is, so the range of $y = cot\ x$ is: ______

How to graph $\boldsymbol{y = acot(bx)}$



- **Period** length is $\frac{\pi}{b}$
- For your scale we will still use 5 numbers
- First and Last are Vertical Asymptotes (VA; x = #)
- To find the VA use the following inequality:
  $0 \leq bx \leq \pi$
- Amplitude = none
- Pattern of three points: y-values at **a, 0, -a**

Example 2: Graph the functions $y = 2\cot x$ and $y = \sin x$ on the same set of axes over the interval $-2\pi \leq x \leq 2\pi$.



Work:

$y = \sin x$

Original Pattern: __, __, __,

a = ___

New Pattern: __, __, __,

Frequency = __

Period = $\frac{2\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __

$y = 2\cot x$

Original Pattern: __, __, __,

a = ___

New Pattern: __, __, __,

Frequency = __

Period = $\frac{\pi}{b} =$

Scale = ____

Critical Points (scale for x-axis): __, __, __, __, __

Part a: How many full sine curves are present over this domain?

Part b: For how many value of x does $2\cot x = \sin x$?

How to graph the Sec/Csc Graph

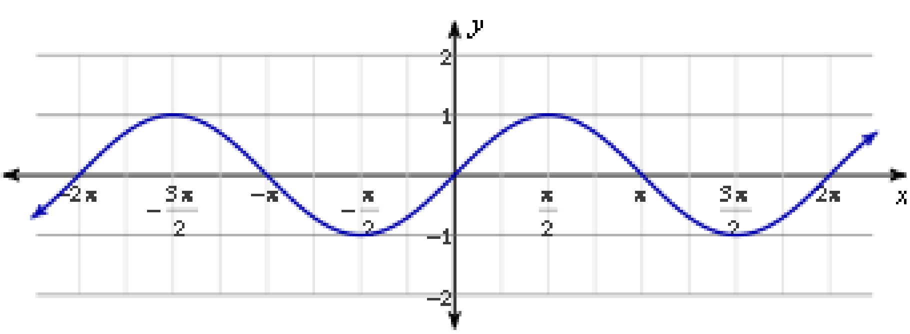
- Mirror image of the corresponding cos/sin graph
- To Graph:
  - First graph the cos/sin graph with dotted lines
  - Sec/Csc graph will have **vertical asymptotes** (VA; x = #) where the cos/sin graph has **x-intercepts**
  - Graph the mirror image
  - Find domain by excluding values at VA
  - Find range by excluding the range of the cos/sin graph

θ (in degrees)	0°	30°	45°	60°	90°	180°	270°
θ (in radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$ (0.5)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{\sqrt{3}}{2}$ (0.866)	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$ (0.866)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{1}{2}$ (0.5)	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ (0.577)	1	$\sqrt{3}$ (1.732)	undefined	0	undefined

Example 1: Graph each of the functions below.

$$y = Cscx$$

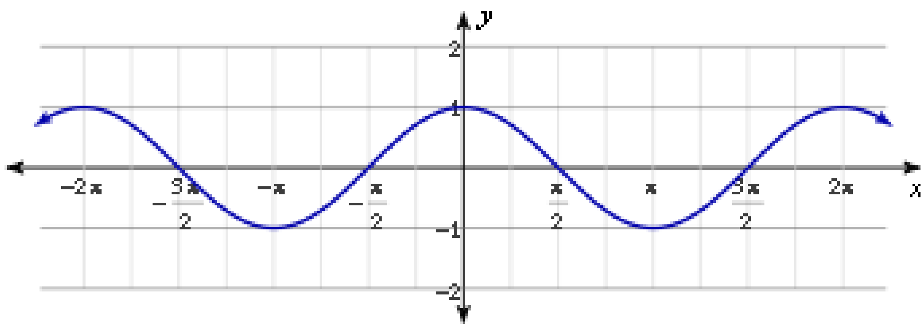
Think:



θ (in degrees)	0°	30°	45°	60°	90°	180°	270°
θ (in radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$ (0.5)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{\sqrt{3}}{2}$ (0.866)	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$ (0.866)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{1}{2}$ (0.5)	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ (0.577)	1	$\sqrt{3}$ (1.732)	undefined	0	undefined

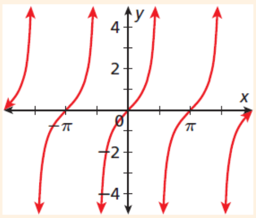
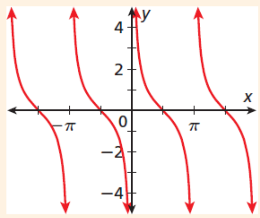
$$y = Secx$$

Think:

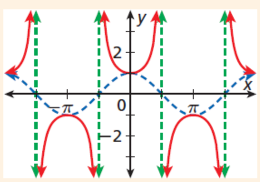
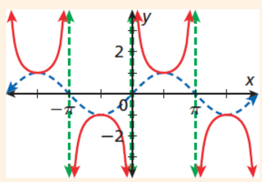


SUMMARY

Characteristics of the Graphs of Tangent and Cotangent

FUNCTION	$y = \tan x$	$y = \cot x$
GRAPH		
DOMAIN	$\left\{x \mid x \neq \frac{\pi}{2} + \pi n, \text{ where } n \text{ is an integer}\right\}$	$\left\{x \mid x \neq \pi n, \text{ where } n \text{ is an integer}\right\}$
RANGE	$\{y \mid -\infty < y < \infty\}$	$\{y \mid -\infty < y < \infty\}$
PERIOD	π	π
AMPLITUDE	undefined	undefined

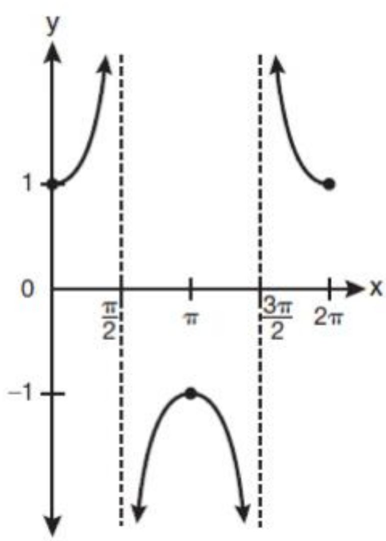
Characteristics of the Graphs of Secant and Cosecant

FUNCTION	$y = \sec x$	$y = \csc x$
GRAPH		
DOMAIN	$\left\{x \mid x \neq \frac{\pi}{2} + \pi n, \text{ where } n \text{ is an integer}\right\}$	$\left\{x \mid x \neq \pi n, \text{ where } n \text{ is an integer}\right\}$
RANGE	$\{y \mid y \leq -1, \text{ or } y \geq 1\}$	$\{y \mid y \leq -1, \text{ or } y \geq 1\}$
PERIOD	2π	2π
AMPLITUDE	undefined	undefined

Exit Ticket

Which equation is represented by the graph below?

1) $y = \cot x$
2) $y = \csc x$
3) $y = \sec x$
4) $y = \tan x$



Homework

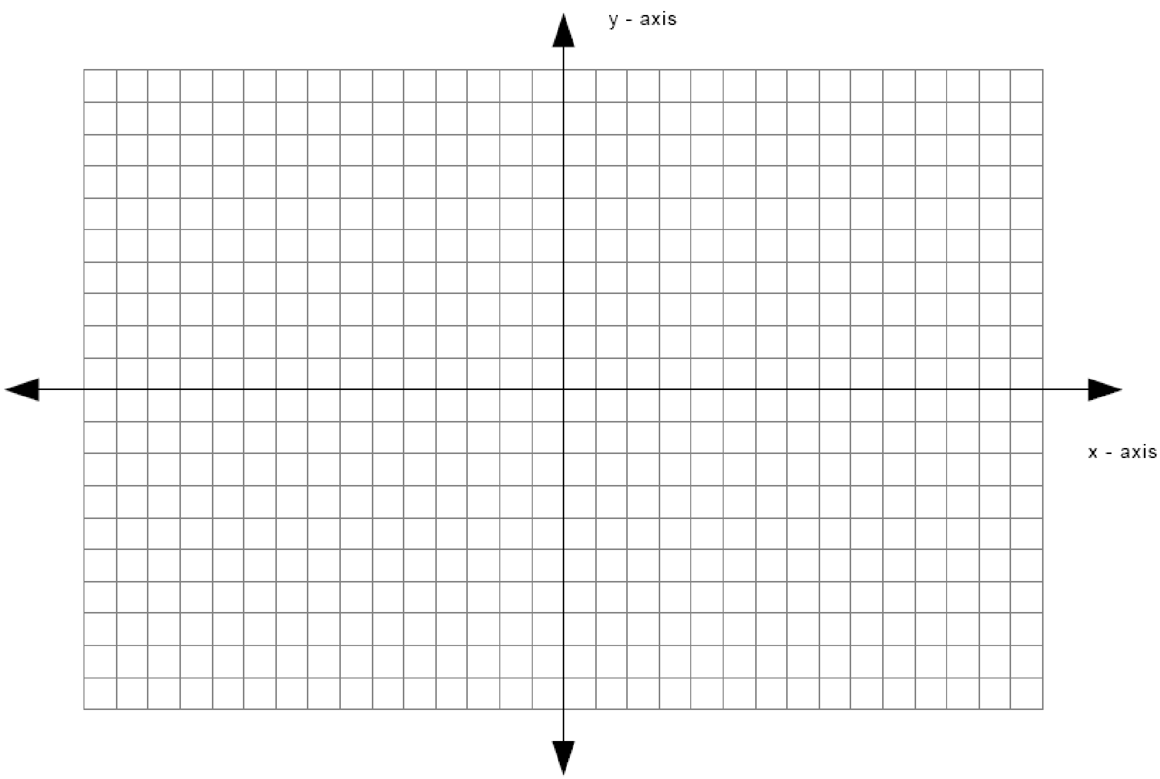
1. What is the domain of y = tan x?
2. What is the range of y = tan x?
3. What is the period of y = tan x?

4. Which is not an element of the domain of y = tan x?

   (1) π (2) 2π (3) $\frac{\pi}{2}$ (4) $-\pi$

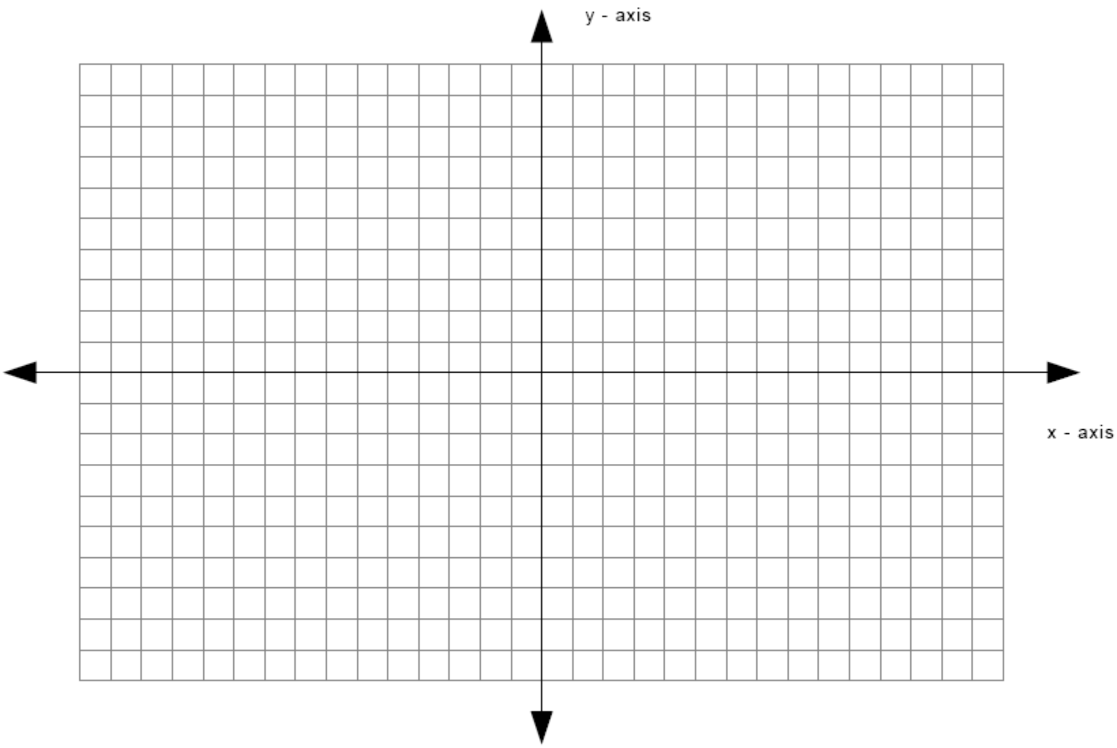
5. a. On the same set of axes, sketch the graphs of y = 2sin x and y = tan x for values of x in the interval $0 \le x \le 2\pi$.

   b. State how many values of x in the interval $0 \le x \le 2\pi$ are solutions of tan x = 2 sin x.



6. a. On the same set of axes, sketch and label the graphs of the equations
y = -3cos x and y = tan x in the interval $-\pi \le x \le \pi$.

b. Using the graph sketched in part a, find the number of values of x in the interval $-\pi \le x \le \pi$ that satisfy the equation -3cos x = tan x



Example7-8: Graph each of the functions below.

$y = 2\,csc\frac{2}{3}x$

Pattern for _____:

___, ____, ____, ____, ____

Midline/V.S. = _____

P.S. = _____

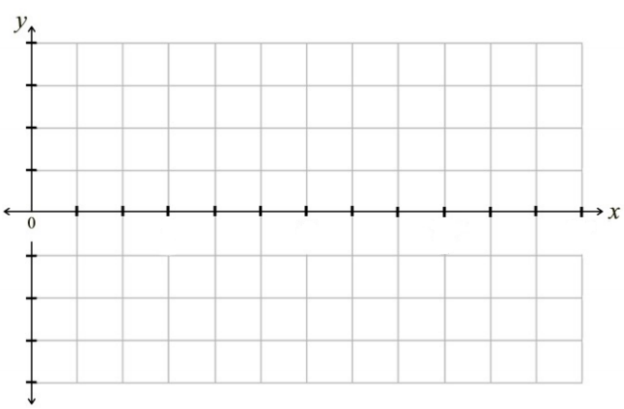
a = _____

b = ______

Period = ______

Scale = ______

_____, _____, _____, _____, _____



$y = sec2x + 1$

Pattern for _____:

___, ____, ____, ____, ____

Midline/V.S. = _____

P.S. = _____

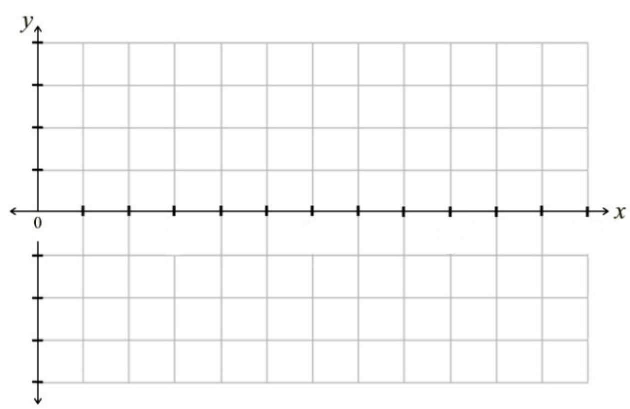
a = _____

b = ______

Period = ______

Scale = ______

_____, _____, _____, _____, _____



Day 5: Algebra2/Trig: The Inverse Trigonometric Functions

Warm – Up

1) Liam's grandfather clock has a pendulum that moves from its central position at rest according to the trigonometric function $P(t) = -3.5\sin\left(\frac{\pi}{2}t\right)$ where t represents the time in seconds. How many seconds does it take the pendulum to complete one full cycle from rest at the center to the left and then to right and back to rest?

(1) 1 second (2) 2 seconds (3) 3.5 seconds (4) 4 seconds

2) **Graph the function below.**

$y = 2\,csc\frac{1}{2}x - 2$

Pattern for _____:

___, ____, ____, ____, ____

Midline/V.S. = ______

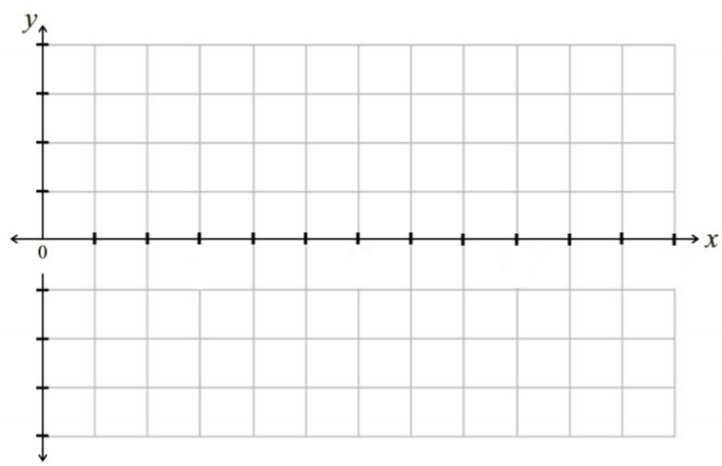
P.S. = ______

a = ______

b = _______

Period = _______

Scale = _______



_____, _____, _____, _____, _____

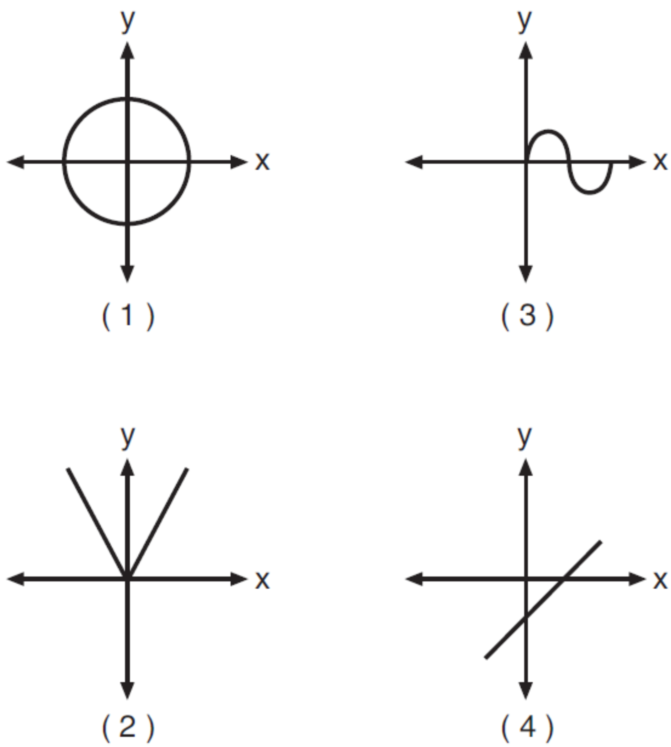
Graphs of Inverse Trig Functions

Mini – Lesson:

1) How do we find the ***inverse*** of a function?

2) What notation do we use to represent the inverse of a function f(x)?

3) Which graph has an inverse that is a function?



Four Facts About Functions and Their Inverse Functions:

1. A function must be one-to-one (any horizontal line intersects it at most once) in order to have an inverse function.

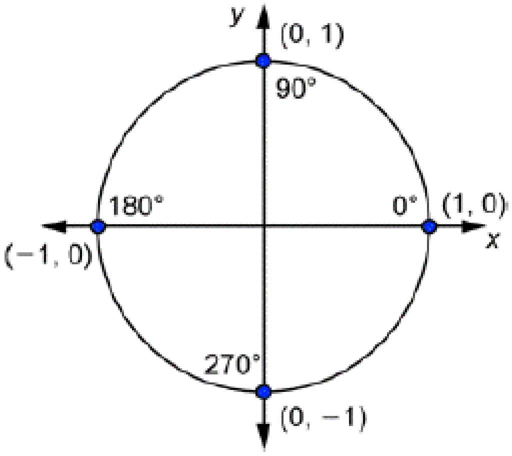
2. The graph of an inverse function is the reflection of the original function about the line $y = x$.

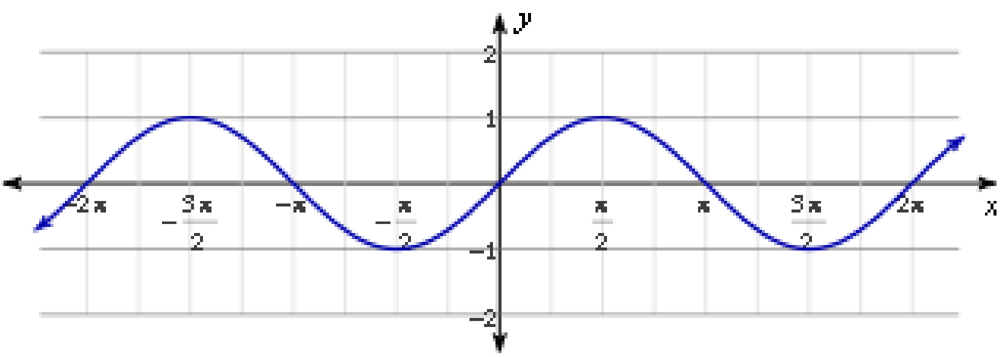
3. If (x, y) is a point on the graph of the original function, then (y, x) is a point on the graph of the inverse function.

4. The domain and range of a function and it's inverse are interchanged.

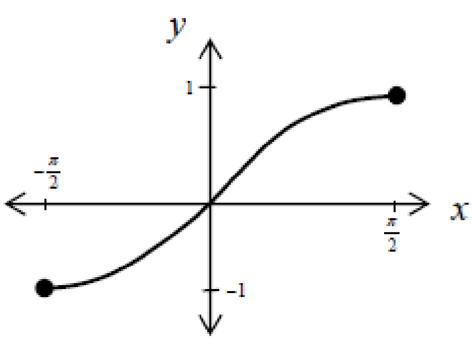
Inverse Sine Function

Definition: The inverse sine function, denoted by ________________ or _________________ is defined to be the inverse of the **restricted** sine function.

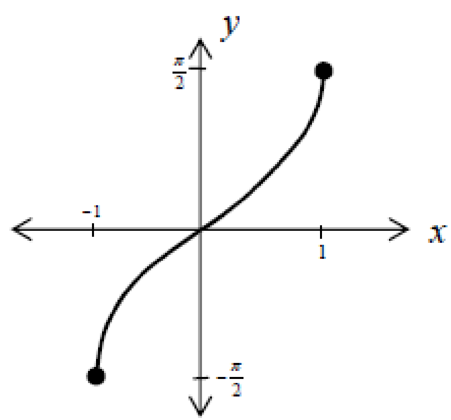




$y = \sin x:$

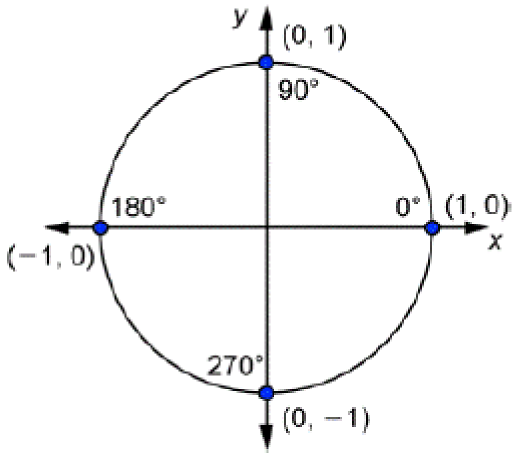


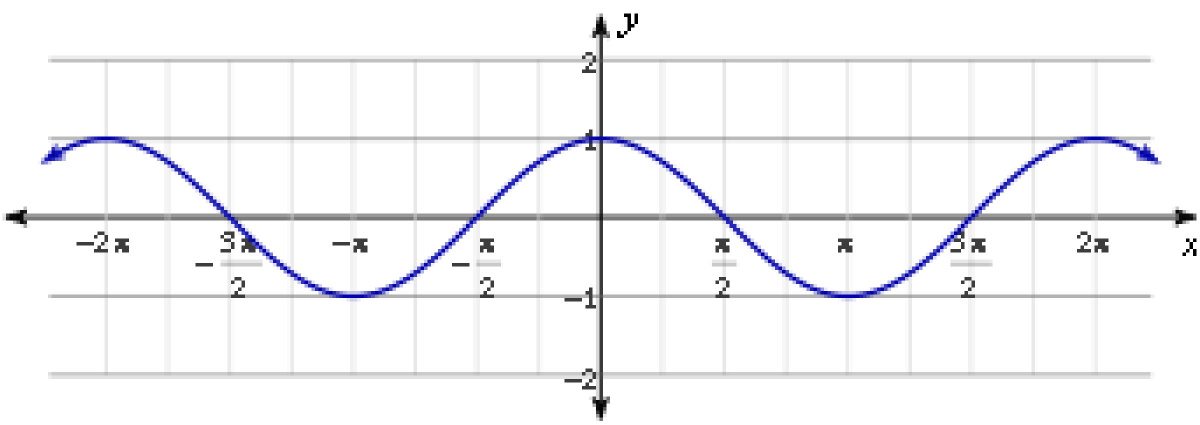
$y = \arcsin x = \sin^{-1} x:$



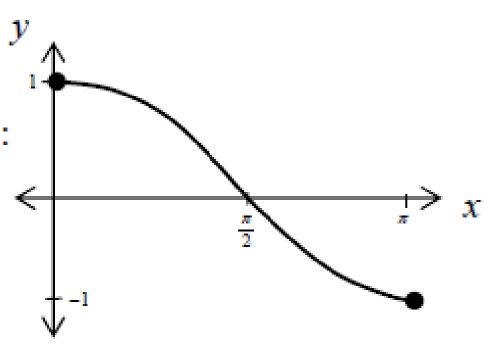
Inverse Cosine Function

Definition: The inverse cosine function, denoted by ________________ or _________________ is defined to be the inverse of the **restricted** cosine function.

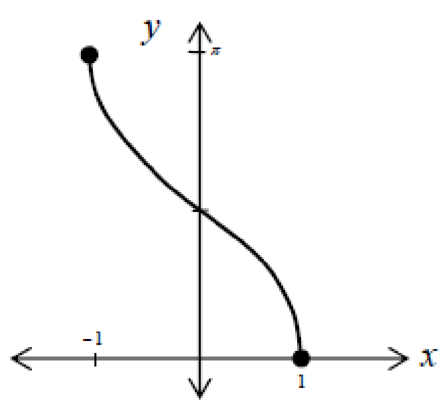




$y = \cos x$:



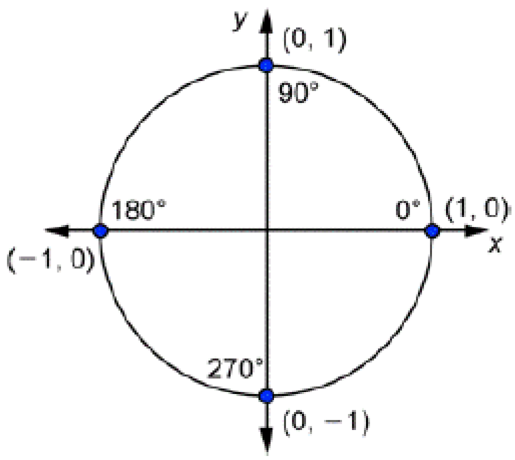
$y = \arccos x = \cos^{-1} x$:

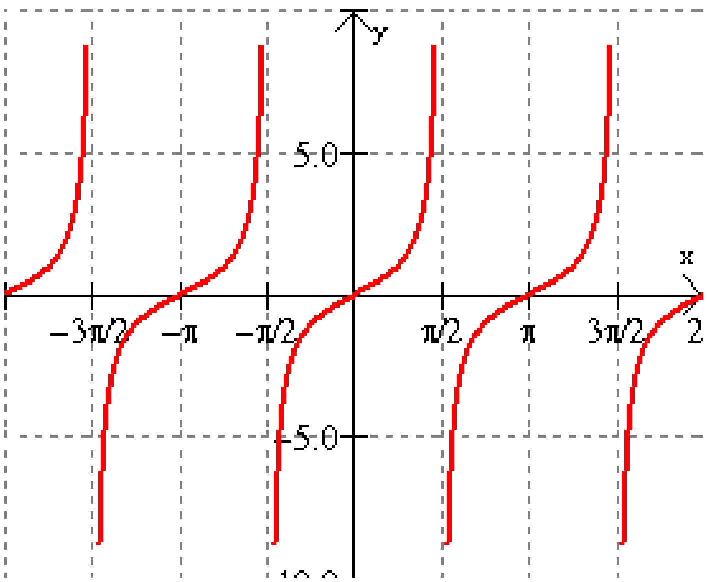


y

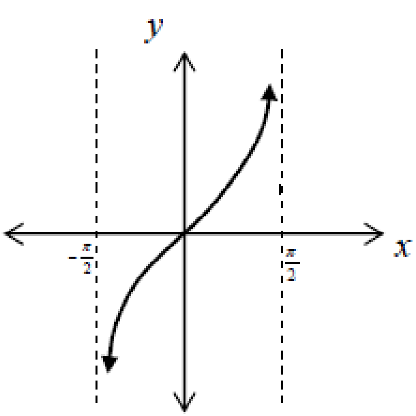
Inverse Tangent Function

Definition: The inverse tangent function, denoted by ________________ or _________________ is defined to be the inverse of the **<u>restricted</u>** tangent function.

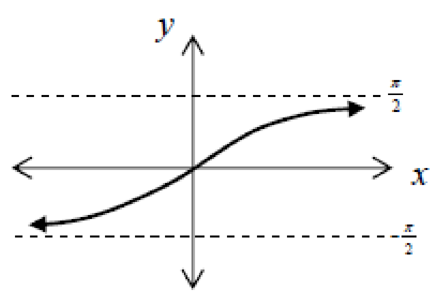




$y = \tan x$:



$y = \arctan x = \tan^{-1} x$:



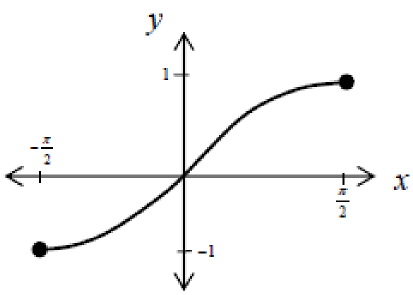
SUMMARY

Each trigonometric function has a restricted domain for which an inverse function is defined. The restricted domains are determined so the trig functions are one-to-one.

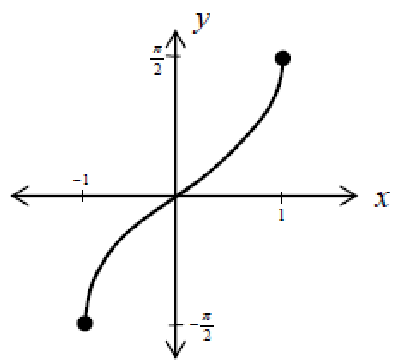
Trig function	Restricted domain	Inverse trig function	Principle value range
$y = \sin x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$y = \arcsin x$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos x$	$0 \le x \le \pi$	$y = \arccos x$	$0 \le y \le \pi$
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$y = \arctan x$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Graphs:

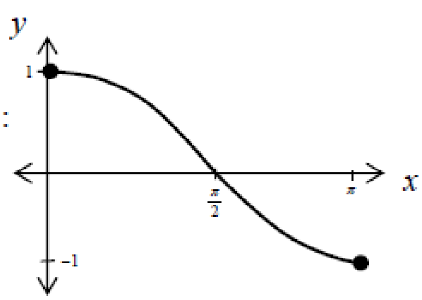
$y = \sin x$:



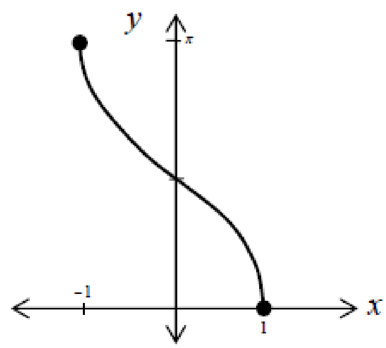
$y = \arcsin x = \sin^{-1} x$:



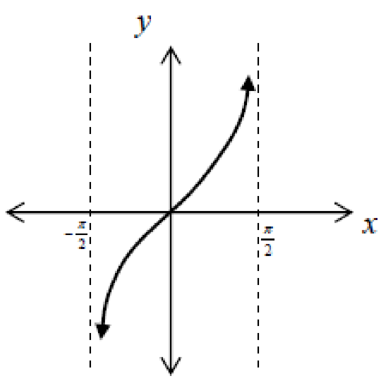
$y = \cos x$:



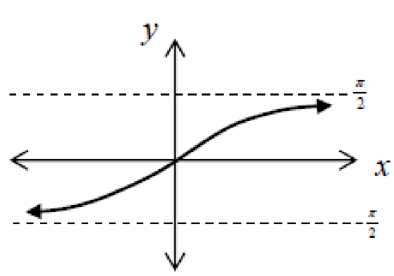
$y = \arccos x = \cos^{-1} x$:



$y = \tan x$:



$y = \arctan x = \tan^{-1} x$:



If you're given the value of $\sin\theta$, $\cos\theta$, $\tan\theta$, or any of their reciprocals, there is a function that "undoes" the function so that you can isolate θ.
These functions are called the INVERSE TRIGONOMETRIC FUNCTIONS.
The notation looks like it is a power, but it is not a power.

$\sin^{-1}\theta$	Inverse sine or Arcsin	$\csc^{-1}\theta$	Inverse cosecant or Arccsc
$\cos^{-1}\theta$	Inverse cosine or Arccos	$\sec^{-1}\theta$	Inverse cosine or Arcsec
$\tan^{-1}\theta$	Inverse tangent or Arctan	$\cot^{-1}\theta$	Inverse tangent or Arccot

PROBLEMS:

1. Evaluate:

a) $\arccos(0)$ b) $\arccos\left(\frac{\sqrt{2}}{2}\right)$ c) $\cos^{-1}(-1)$ d) $\cos^{-1}(1)$

e) $\sin^{-1}(0)$ f) $\arcsin(-\frac{\sqrt{3}}{2})$ g) $\arctan(\sqrt{3})$ h) $\tan^{-1}(-\sqrt{3})$

In 2-5, find the exact value of the given expressions.

2. $\cos(Arc\ \sin 1)$

3. $\tan\left(Arc\ \cos -\frac{\sqrt{2}}{2}\right)$

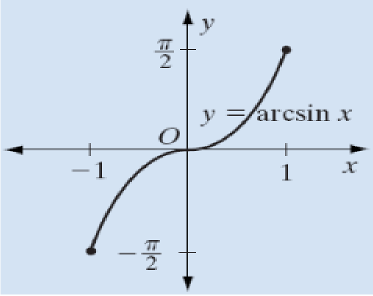
4. $\cos\left(Arc\ \tan\frac{12}{5}\right)$

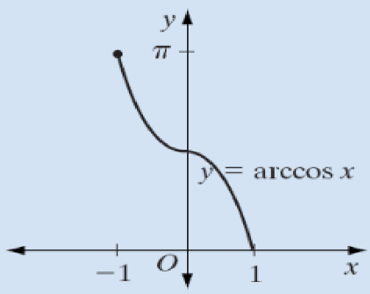
5. $\sin\left(Arc\ \cos -\frac{15}{17}\right)$

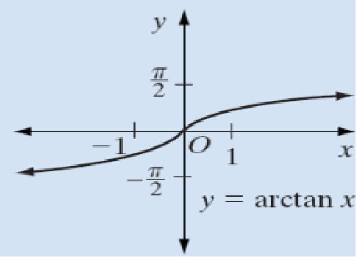
Summary

What You Need to Know

Function	Restricted Domain	Inverse Function
$y = \sin x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$y = \arcsin x$ or $y = \sin^{-1} x$
$y = \cos x$	$0 \le x \le \pi$	$y = \arccos x$ or $y = \cos^{-1} x$
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$y = \arctan x$ or $y = \tan^{-1} x$

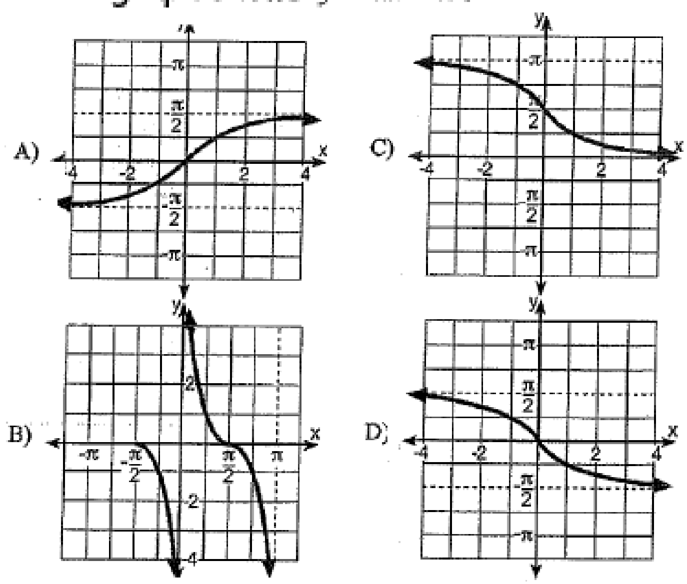






Exit Ticket

1. Which graph shows $y = \tan^{-1} x$?



2. If $x = \text{Arc}\cos\left(-\frac{1}{2}\right)$, then x is equal to

1) 120° 3) 210°

2) 150° 4) 300°

Day 5 - Homework

1. When the function y = cos x is reflected in the line y = x, the new function is

(1) y = sin x (2) y = arcsin x (3) y = arccos x (4) x = arccos y

2. In which quadrant would θ appear if θ = arctan (-1)?

(1) I (2) II (3) III (4) IV

3. The value of $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)-\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ is

(1) $\frac{-2\sqrt{3}-3\sqrt{2}}{6}$ (2) -105° (3) $-75°$ (4) $255°$

4. Which of the following is a true statement with regard to the reflection of the graph of y = sin x in the line y = x?

(1) Unless the domain of y = sin x is restricted, the reflection is not a function.

(2) The graph of the reflection of y = sin x in the line y = x is always a function.

(3) The equation of the graph of the reflection of y = sin x in the line y = x is y = cos x.

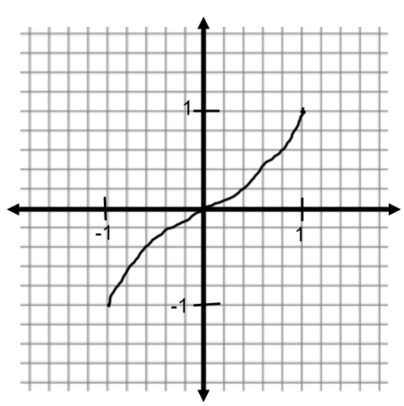
(4) The equation of the graph of the reflection of y = sin x in the line y = x is y = sin (-x).

5. To obtain its inverse function, the domain of y = tan x must be restricted to which quadrants?

(1) I and IV (2) II and III (3) III and IV (4) I and III

6. The graph shows which of the following?

(1) y = arcsin x (2) y = arccos x
(3) y = arctan x (4) y = arccos (-x)



7. The value of arcsin $\left(-\frac{\sqrt{3}}{2}\right)$ + arcsin $\frac{1}{2}$ is

(1) $-\frac{\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{3\pi}{2}$

8. Find the value of cos (arctan (-1)).

(1) 1 (2) $\frac{\sqrt{2}}{2}$ (3) $\frac{1}{2}$ (4) $-\frac{\sqrt{2}}{2}$

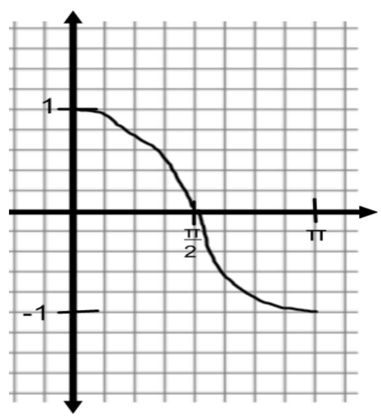
9. If θ = arctan $\left(-\sqrt{3}\right)$, the value of θ is

(1) $-60°$ (2) $-30°$ (3) $120°$ (4) $150°$

10. Evaluate $\cos^{-1}(-1)+\sin^{-1}\frac{1}{2}$.

(1) $90°$ (2) $120°$ (3) $210°$ (4) $240°$

11. What is the inverse of the function shown?



(1) x = arcsin y (2) y = arcsin x

(3) x = arccos y (4) y = arccos x

12.

What is the value of $\tan(\text{Arc}\cos\frac{5}{13})$?

1) $\frac{12}{13}$

2) $\frac{5}{12}$

3) $\frac{12}{5}$

4) $\frac{13}{5}$