NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2013

MATHEMATICS: PAPER I

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 9 pages, an Answer Sheet of 2 pages (i – ii) and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions. Question 5 should be answered in the Answer Sheet provided. Ensure that you write your examination number on this Answer Sheet and submit it with your other answers.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a) Solve for x:

(1) $(x+2)^2 = 3x(x-2)$
giving your answers correct to one decimal digit. (5)

(2) $x^2 - 9x \geq 36$. (4)

(3) $3^x - 3^{x-2} = 72$. (4)

(b) Given: $(2m-3)(n+5) = 0$.

Solve for:

(1) n if $m = 1$. (1)

(2) m if $n \neq -5$. (1)

(3) m if $n = -5$. (2)

[17]

QUESTION 2

(a) Evaluate: $\sum_{k=2}^{6} \frac{2^{k-1}}{k}$. (3)

(b) The number of members of a new social networking site doubles every day.
On day 1 there were 27 members and on day 2 there were 54 members.

(1) Calculate the number of members there were on day 12. (2)

(2) The site earns half a cent per member per day. Calculate the amount of money that the site earned in the first 12 days.
Give your answer to the nearest Rand. (4)

(c) Gina plans to start a fitness programme by going for a run each Sunday. On the first Sunday she runs 1 km and plans to increase the distance by 750 m each Sunday. When Gina reaches 10 km, she will continue to run 10 km each Sunday thereafter.

(1) Calculate the distance that Gina will run on the 9^{th} Sunday. (3)

(2) Determine on which Sunday Gina will first run 10 km. (2)

(3) Calculate the total distance that Gina will run over the first 24 Sundays. (4)

[18]

QUESTION 3

(a) Given $f(x)=6x^2$, determine $f'(x)$ from first principles. (4)

(b) Determine $f'(x)$ given $f(x)=\dfrac{3x^4+7x^2-5x}{2x^2}$.
Leave your answer with positive exponents. (4)

(c) Given: $f(x)=x^3-7x^2+7x+15$
Determine the average gradient of the curve between the points where $x=-1$ and $x=1$. (3)

[11]

QUESTION 4

(a) (1) Joe invested a sum of R50 000 in a bank. The investment remained in the bank for 15 years, earning interest at a rate of 6% p.a., compounded annually. Calculate the amount at the end of 15 years. (2)

(2) Financial gain is defined as the difference between the final value of an investment and the contribution. Determine the financial gain of Joe's investment. (1)

(b) Pumla took a mortgage loan of R850 000 to buy a house and was required to pay monthly instalments for 30 years. She was charged interest at 8% p.a., compounded monthly.

(1) Show that her monthly instalment was R6 237. (4)

(2) Calculate the outstanding balance on her loan at the end of the first year. (3)

(3) Hence calculate how much of the R74 844 that she paid during the first year, was taken by the finance company as payment towards the interest it charged. (3)

[13]

QUESTION 5

ANSWER THIS QUESTION ON THE ANSWER SHEET

A situation involving two types of bookings for hotel accommodation can be summarised as follows, where x represents the number of booked rooms of Type A and y represents the number of booked rooms of Type B.

$$x \geq 1$$
$$y \geq 2$$
$$x + y \geq 5$$
$$x + 3y \leq 18$$

(a) Draw the constraints on the set of axes provided, clearly indicating the feasible region. (7)

(b) Write down the fewest number of rooms (both types together) that can be booked so that all the constraints are satisfied. (1)

(c) Suppose that the hotel makes a profit of R500 per night for each type of room that is booked. Calculate their maximum possible profit. (3)

(d) The hotel manager considers an option where he reduces the price for a Type A room and increases the price for a Type B room. He would then make a profit of R200 on each of the Type A rooms and a profit of R600 for Type B rooms. Write down 4 combinations of room bookings that will ensure the manager of a maximum profit. (4)

[15]

74 marks

SECTION B

QUESTION 6

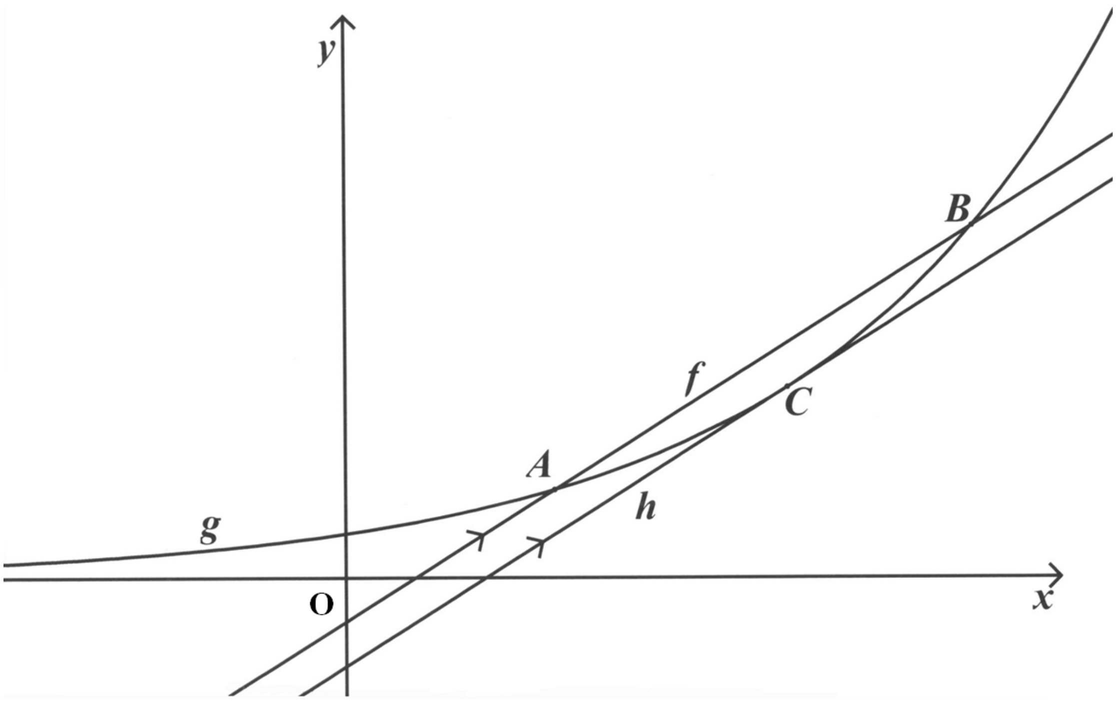
(a) $\sqrt[12]{2}$ is a special number in music.

On an idealised piano, the frequency $f(n)$ of the n^{th} key, in Hertz, is given by $f(n) = \left(\sqrt[12]{2}\right)^{n-49} \times 440$.

(1) Calculate the frequency of the 73^{rd} key. (2)

(2) Determine which key has a frequency of 3 520 Hz. (5)

(b) Refer to the figure showing the graphs of $f(x) = 3x - 1$ and $g(x) = 2^x$ intersecting at A(1 ; 2) and B(3 ; 8). C(2,1 ; 4,3) is a point on *g*, coordinates rounded to one decimal digit, such that the tangent to *g* at C is parallel to *f*.



(1) Determine the equations of $y = f^{-1}(x)$ and $y = g^{-1}(x)$. (4)

(2) **Use the above graphs** to determine the values of *x* for each of the following:

(i) $f(x) < g(x)$ (2)

(ii) $g^{-1}(x) < 0$ (2)

(iii) $f^{-1}(x) = g^{-1}(x)$ (2)

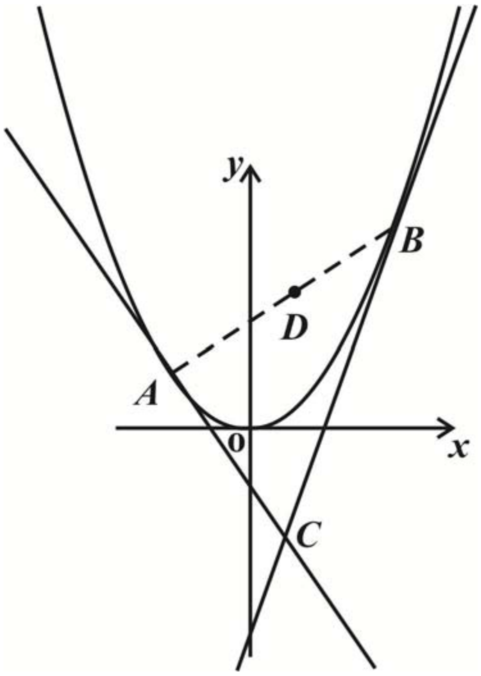
(iv) $g'(x) > f'(x)$ (2)

[19]

QUESTION 7

Refer to the figure showing the graph of $f(x) = x^2$. A and B are any two different points on the parabola. The tangents at A and B intersect at C.

Given the *x*-coordinate of A is *k* and the *x*-coordinate of B is *m*.



(a) Show that the equation of the tangent at A can be written as: $y = 2kx - k^2$. (5)

(b) Hence write down the equation of the tangent at B. (1)

(c) Determine a simplified expression for the *x*-coordinate of C. (5)

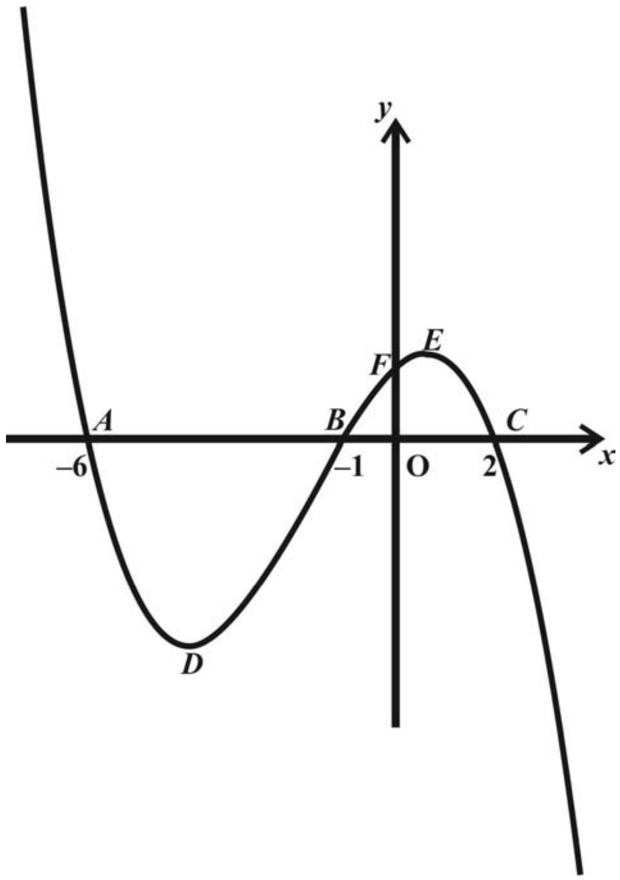
(d) D is the midpoint of the line segment between A and B.
Show that CD is parallel to the *y*-axis. (2)

[13]

QUESTION 8

Refer to the figure showing the graph of a cubic function $f(x) = ax^3 + bx^2 + cx + d$.

A(–6 ; 0), B(–1 ; 0), C(2 ; 0) and F(0 ; 24) are intercepts with the axes, with D and E as turning points.



(a) Show that $a = -2$, $b = -10$, $c = 16$ and $d = 24$. (5)

(b) Determine the coordinates of D. (6)

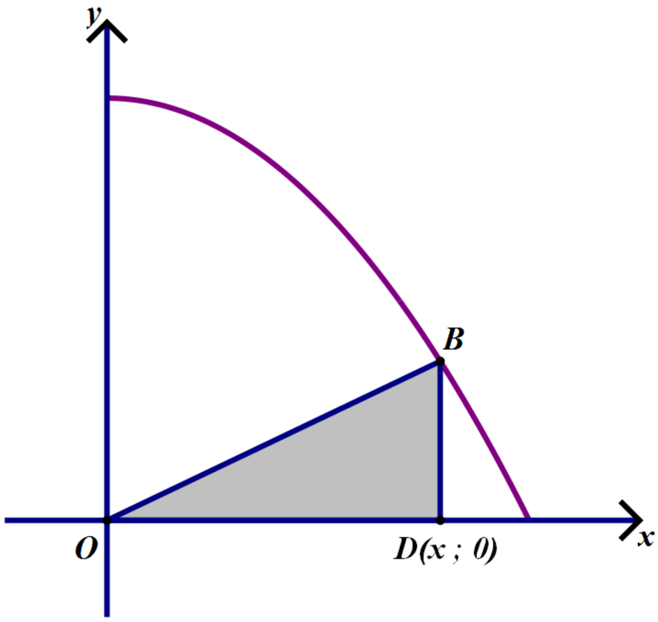
(c) Suppose that the graph is translated in such a way that the point D is moved to the origin. That is, the new graph has equation $y = f(x - p) + q$, where p and q are constants.
Write down the values of p and q. (2)

[13]

QUESTION 9

Refer to the figure showing the parabola given by $f(x) = 4 - \frac{x^2}{4}$ with $0 \leq x \leq 4$.

D is the point (x ; 0) and DB is parallel to the y-axis, with B on the graph of f.



(a) Write down the coordinates of B in terms of x. (2)

(b) Show that the area, A, of ΔOBD is given by: $A = 2x - \frac{x^3}{8}$. (3)

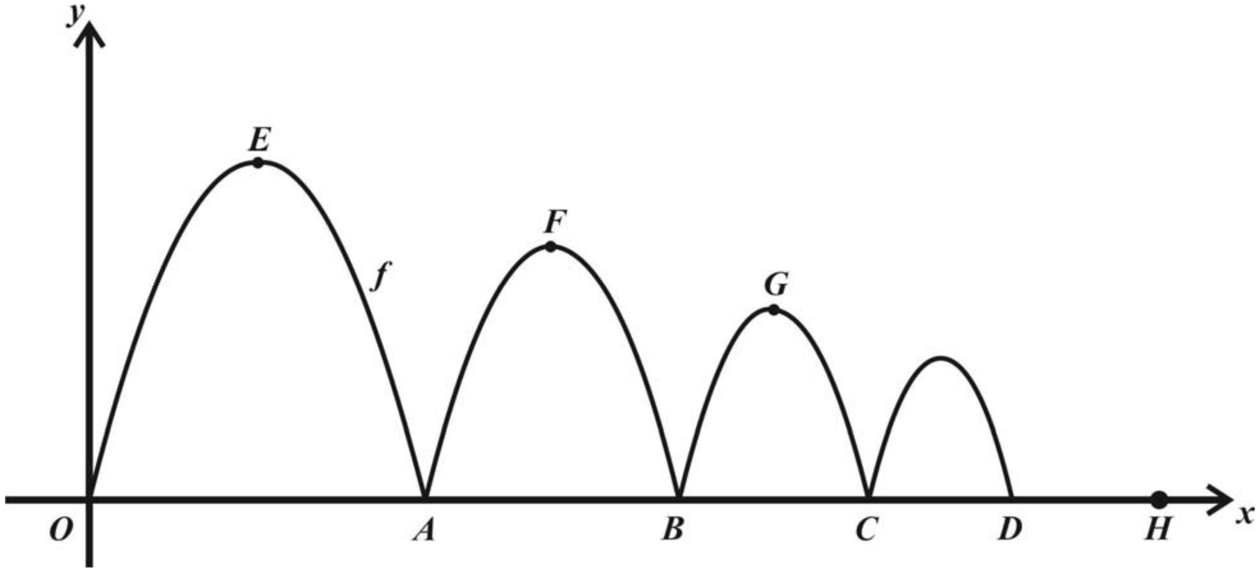
(c) Determine how far D should be from O in order that the area of ΔOBD is as large as possible. (5)

(d) Hence, calculate the area of ΔOBD when D is at the point determined in (c). (2)

[12]

QUESTION 10

Refer to the figure showing the graph of $f(x) = -x^2 + 4x$ followed by a number of decreasing sized parabolas. The height of each turning point as well as the difference between the x-intercepts of each parabola is $\frac{3}{4}$ of that of the previous parabola.



(a) Determine the coordinates of A and E. (6)

(b) Show that the coordinates of G are $\left(\frac{65}{8}; \frac{9}{4}\right)$. (6)

(c) Determine the equation of the third parabola passing through B, G and C, leaving your answer in the form $y = a(x-p)^2 + q$. (4)

(d) Suppose that decreasing parabolas are constructed indefinitely in the same way as the first few that are shown. Determine whether all the parabolas will fit on OH, where $\text{OH} = 15$. (3)

[19]

76 marks

Total: 150 marks