

# Machine Learning for Natural Language Processing

## Language Modeling

### Lecture 5

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- ① The Why and What of Natural Language Processing
- ② Representing text with vectors
- ③ Task specific Modeling of Text
- ④ Neural Natural Language Processing
- ⑤ Language Modeling
- ⑥ Transfer Learning with Neural Modeling for NLP

- Language Model
- Conditioned Language Model : focus on Sequence to Sequence

# Language Model

- What is a Language Model ?
- Modeling language with n-grams
- Modeling language with a LSTM
- The Transformer Architecture

# Language modeling

# What is language modeling?

- **Language modeling** corresponds to assigning a probability to a text
- A text is a **sequence of tokens**, or characters
- Tokens can be words, sub-words,
- For example:

$$\{\text{a cat}\} = \{\text{a, cat}\},$$

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# What is language modeling?

- Given a sequence  $\{w_1, \dots, w_T\}$  of tokens, a language model estimates its probability:

$$P(w_1, \dots, w_T)$$

- $P$  depends on a **vocabulary**, i.e., the set of unique tokens.
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- Causal Language Model
- Mask Language Model

Language models are applied in several fields:

- Speech recognition:

$$P(\text{"Vanilla, I scream"}) < P(\text{"Vanilla ice cream"}).$$

- Machine translation:

$$P(\text{"D\u00e9\u00e7u en bien"} \mid \text{"Pleasantly surprised"}) < \\ P(\text{"Agr\u00e9ablement surpris"} \mid \text{"Pleasantly surprised"})$$

- Optical Character Recognition:

$$P(\text{"m0ve fast"}) < P(\text{"move fast"})$$

- Sequence probability as a product of token probabilities:

$$P(w_1, \dots, w_T) = \prod_{t=1}^T P(w_t \mid w_{t-1}, \dots, w_1)$$

## Probabilistic *Causal* language model

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$$\begin{aligned} P(w_1, w_2, w_3) &= P(w_1)P(w_2, w_3 \mid w_1) \\ &= P(w_1)P(w_2 \mid w_1)P(w_3 \mid w_2, w_1). \end{aligned}$$



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- Causal Language models estimate probability of upcoming token given past:

$$P(w_t \mid w_{t-1}, \dots, w_1).$$

# Estimating Language Models

- Causal Language Model
- Mask Language Model

**Sentence** The cat is drinking milk in the kitchen

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<sup>1</sup> Devlin et al. (2018)

<sup>2</sup> also referred as Cloze Task

**Sentence** The cat is drinking milk in the kitchen

**input** The cat <MASK> drinking <MASK> in the kitchen

- Randomly replace 15% of words in sentence with a <MASK> token

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**Sentence** The cat is drinking milk in the kitchen

**input** The cat <MASK> drinking <MASK> in the kitchen

**targets** {"is", "milk"}

- Randomly replace 15% of words in sentence with a <MASK> token
- Take the masked words as targets for the model to predict

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<sup>2</sup> also referred as Cloze Task

**Sentence** The cat is drinking milk in the kitchen

**input** The cat **mushroom** drinking **shoes** in the kitchen

**targets** {"is", "milk"}

- Randomly replace 15% of words in sentence with a <MASK> token
- Take the masked words as targets for the model to predict
- Extension: use random words from vocabulary instead of <MASK>

---

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Masked Language Modeling estimates the probability of sequence tokens of length  $T$  with:

$$P(w_i | w_1, \dots, w_{i-1}, w_i, \dots, w_T)$$

- a Language Model is a model that predicts a **token** based on its surrounding linguistic **context**
- Tokens can be words, sub-words or characters
- Context can be the *left sequence*, *left and right sequence*, the sentence, a window around the words, the paragraph...
- We saw two way of defining language models: Causal Language Model and Mask Language Model



## Estimating language models

- Statistical approach: N-Gram model
- Neural Language Models
  - Recurrent Neural Networks (LSTM)
  - The Transformer Architecture

- Example:

$$\begin{aligned}P(\text{English} \mid \text{The moment one learns}) &= \frac{c(\text{The moment one learns English})}{c(\text{The moment one learns})} \\ &= \frac{35}{73} = 0.48\end{aligned}$$

Sentence “The moment one learns English” appears 35 in dataset

Sentence “The moment one learns” appears 75 in dataset

## Limitations of count based language model

- Number of unique sentences increases with dataset size,
  - Long sentences are rare: no good statistics for them
- Too many sentences with not enough statistics  
(Sparsity due to combinatorial structure of language)

# Count based language model

- **Solution** truncate past to a fixed size window
- For example:

$$P(\text{English} \mid \text{The moment one learns}) \approx P(\text{English} \mid \text{one learns})$$

- Implicit assumption: **most important information about a word is in its recent history**
- **Beware!** In general:

$$P(w_1, \dots, w_T) \neq \prod_{t=1}^T P(w_t \mid w_{t-1}, \dots, w_{t-n+1})$$

- **Truncated count based models =  $n$ -gram models**
- “ $n$ ” refers to the size of past
- Examples:
  - Unigram:

$$P(w_1, \dots, w_T) = \prod_{t=1}^T P(w_t)$$

- Bigram:

$$P(w_1, \dots, w_T) = \prod_{t=1}^T P(w_t | w_{t-1})$$

- Probability of a sentence with a unigram model:

$$P_U(w_1, \dots, w_T) = \prod_{t=1}^T P(w_t) = \prod_{t=1}^T \frac{c(w_t)}{N}$$

$N$  = total number of tokens in dataset

$c(w_t)$  = number of occurrences of  $w_t$  in dataset

- Unigram only uses word frequency
- Example of text generation with this model:

*the or is ball then car*

- Probability of a sentence with a bigram model:

$$P_U(w_1, \dots, w_T) = \prod_{t=1}^T P(w_t | w_{t-1}) = \prod_{t=1}^T \frac{c(w_{t-1}w_t)}{c(w_{t-1})}$$

$c(w_{t-1}w_t)$  = number of occurrences of sequence  $w_{t-1}w_t$

- Predict a word just with the previous word



- Example of text generation with bigram model:

*new car parking lot of the*

- “car” is generated from “new”, “parking” from “car”...
- But “new” has no influence on “parking”

- Simple to extend to longer dependencies: trigrams, 4-grams...
- $n$ -grams can be “good enough” in some cases
- But  $n$ -grams cannot capture long term dependencies required to truly model language

## Estimating $n$ -gram probabilities: an example

- bigram:

$$P(w_t | w_{t-1}) = \frac{c(w_{t-1}w_t)}{c(w_{t-1})}$$

- Dataset:

*<s>we sat in the house*

*<s>we sat here we two and we said*

*<s>how we wish we had something to do*

- Extract some probabilities:

$$P(\text{sat} | \text{we}) = 0.33, \quad P(\text{wish} | \text{we}) = 0.17, \quad P(\text{in} | \text{sat}) = 0.5$$

- $\langle s \rangle$  = token for beginning of sentence;  $P(\langle s \rangle) = 1$ .
- Compute sentence probability with them

## Estimating $n$ -gram probabilities: an example

- Extract count from Berkeley Restaurant dataset (9222 sentences)
- Unigram counts:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Bigram counts:

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

## Estimating $n$ -gram probabilities: an example

- The bigram probabilities are obtained by dividing the bigram counts with the unigram counts:

$$P(w_2 | w_1) = \frac{c(w_1 w_2)}{c(w_1)}$$

- Resulting bigram probabilities:

	i	want	to	eat	chinese	food	lunch	spend
i	0.022	0.33	0	0.036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

## Estimating $n$ -gram probabilities: an example

- Example:

$$P(\langle s \rangle \text{ i want chinese food})?$$

$\langle s \rangle$  = token for beginning of sentence;  $P(\langle s \rangle) = 1$ .

- Result:

$$\begin{aligned} P(\langle s \rangle \text{ i want chinese food}) &= P(\langle s \rangle)P(i|\langle s \rangle)P(\text{want}|i)P(\text{chinese}|\text{want})P(\text{food}|\text{chinese}) \\ &= 1 \times .25 \times 0.33 \times 0.0065 \times 0.52 \\ &= 0.00027885 \end{aligned}$$

## Estimating $n$ -gram probabilities: an example

	i	want	to	eat	chinese	food	lunch	spend
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...								
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

- Example:

$$P(\langle s \rangle \text{ i bring my lunch to work})?$$

- Result:

$$\begin{aligned} P(\langle s \rangle \text{ i bring my lunch to work}) &= P(\langle s \rangle) \dots P(\text{to}|\text{lunch}) \dots \\ &= 1 \times \dots \times 0 \times \dots \\ &= \mathbf{0} \end{aligned}$$

- **Does not generalize well!**

## Good-Turing estimation

- **Idea** reallocate probability mass of  $n$ -grams that occur exactly  $c + 1$  times to  $n$ -grams that occur exactly  $c$  times
  - reallocate mass of  $n$ -grams appearing once to unseen  $n$ -grams
- **alternative to Add-1**



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→ **alternative to Add-1**

- the adjusted count:

$$c^* = (c + 1) \frac{N_{c+1}}{N_c}$$

where  $N_c$  is the number of  $n$ -grams that appears exactly  $c$  times

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- $n$ -gram probability depends on  $c^*$  instead of  $c$
- **Problem** What if  $N_{c+1} = 0$  (but  $N_c > 0$ )?

- If no good statistics on long context: use shorter context
- **Backoff**: use trigram if enough data, else backoff to bigram.
- **Interpolation**: mix statistics of trigram, bigram and unigram.

## Pros

- Fast at training and inference
- Can reach good accuracy if lots of data

## Cons

- Impossible to model very long dependencies (simplistic assumptions done)
- Generalization limited
- Not Deep Learning compatible

- Statistical approach: N-Gram model
- Neural Language Models
  - Recurrent Neural Networks (LSTM)
  - The Transformer Architecture

How to frame language modeling in a deep learning compatible way ?  
What neural architecture/objective ?

- Neural Language Model objective and Training
- Architectures
  - Recurrent Network
  - Transformer

# Neural Language Model

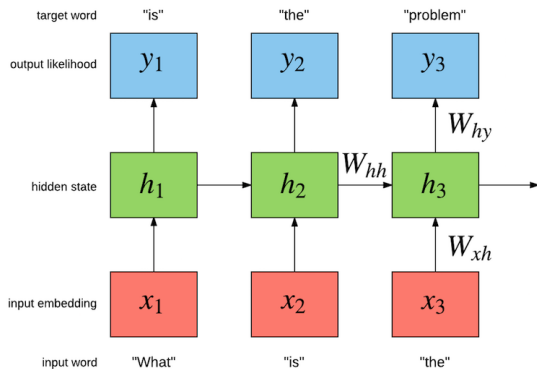


Figure: Neural Language modeling schema view <sup>3</sup>

<sup>3</sup><http://torch.ch/blog/2016/07/25/nce.html>



# Neural Language Model training and inference

Let  $(x^1, \dots, x^T)_i$  sequence of tokens (1-hot encoded),  $E$  embedding layer,  
 $NN_\theta$  a sequential model (e.g. LSTM)  $\{f, W\}$  dense layer

We present forward/backward step to predict token  $x^{t+1}$  with  $x^1, \dots, x^t$

$$e_t = Ex^t \quad \forall t \leq T \quad \textit{Embedding layer}$$

$$h_t = NN_\theta(e_1, \dots, e_{t-1}, e_t) \quad \textit{sequential layers with weights } \theta$$

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**Train time**

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Compute  $\nabla \text{loss}$  backprop  
(update E,  $\theta$ , W)

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## Inference/Prediction Time

$$x_{t+1} = \text{argmax}_{v \in 1, \dots, V} (s_{tv})$$

# Neural Language Model with LSTM cell

In this case,  $NN_\theta$  is defined as (seen in lecture 4):  
( $\theta$  is equal to  $W_{p \in \{C, f, i, o\}}$ )

Based on  $e_1, \dots, e_t$  we compute iteratively  $h_1, \dots, h_t$

$$\tilde{C}^t = \tanh(W_C[e_t, h_{t-1}] + b_C) \quad \text{candidate cell}$$

$$f^t = \sigma(W_f[x_t, h_{t-1}] + b_f) \quad \text{forget gate}$$

$$i^t = \sigma(W_i[x_t, h_{t-1}] + b_i) \quad \text{input gate}$$

$$o^t = \sigma(W_o[x_t, h_{t-1}] + b_o) \quad \text{ouput gate}$$

$$C^t = i^t \star \tilde{C}^t + f^t \star C^{t-1} \quad \text{new cell state}$$

$$h_t = o^t \star \tanh(C_t) \quad \text{new hidden vector}$$

- Language Models are evaluated with perplexity

$$\text{perplexity} = 2^{-p_i \log(\hat{p}_i)}$$

- It is a measure of "surprise" of the model



## Comparing various language Models

Model	Perplexity
Kneser-Ney 5-gram	141
Neural $n$ -gram	140
RNN	125
LSTM	<b>115</b>

- Penn Treebank dataset
- LSTM outperforms RNN

## Limits of LSTM-based architectures

- LSTM models are widely used in NLP for their ability to model sequential data
- In theory, they are able to model sequences of infinite length (Siegelmann and Sontag, 1992)
- In practice, until recently LSTM based models were State-of-the-Art (SOTA) for language modeling (Rae et al., 2018)
  
- In practice, the recurrent nature of LSTM limits the possibility to scale the training process to more data (we cannot parallelize LSTM easily!)
- → **Transformer** were recently shown to work better for a great variety of tasks including Language Model (Radford et al., 2019)

# The Transformer Architecture<sup>5</sup>

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<sup>5</sup>Vaswani et al. (2017)

## Combining vectors with attention

- Use (self) attention mechanism
- Given a set of vectors  $\mathbf{w}_1, \dots, \mathbf{w}_T \in \mathbb{R}^d$  representing words

$$\mathbf{h}_t = \sum_{i=1}^T a_{it} \mathbf{V} \mathbf{w}_i$$

where  $\sum_{i=1}^T a_{it} = 1$ .

- We could use  $a_{it} = \frac{1}{T}$  and get bag of words
- We can also learn  $a_{it}$  based on the input and output as we did for the standard attention mechanism

- Introducing matrix  $\mathbf{W} \in \mathbb{R}^{d \times T}$  where columns correspond to  $\mathbf{w}_i$ ,

$$\mathbf{h}_t = \mathbf{VW}\mathbf{a}_t$$

- And finally

$$\mathbf{H} = \mathbf{VWA}$$

- How to compute the matrix  $\mathbf{A}$ ?

$$\mathbf{A} = \text{softmax}(\mathbf{W}^\top \mathbf{K}^\top \mathbf{Q} \mathbf{W})$$

where the softmax is applied column-wise.

- Why softmax? to get positive entries, and columns summing to 1.
- Why  $\mathbf{W}^\top \mathbf{K}^\top \mathbf{Q} \mathbf{W}$ ? Bilinear form over the input

- Putting everything together:

$$\mathbf{H} = \mathbf{V}\mathbf{W}\text{softmax}(\mathbf{W}^T\mathbf{K}^T\mathbf{Q}\mathbf{W})$$

where  $\mathbf{H}, \mathbf{W} \in \mathbb{R}^{d \times T}$  and  $\mathbf{V}, \mathbf{K}, \mathbf{Q} \in \mathbb{R}^{d \times d}$

- $\mathbf{V}, \mathbf{K}, \mathbf{Q}$  are parameters to be learned.
- This operation is called **self-attention**

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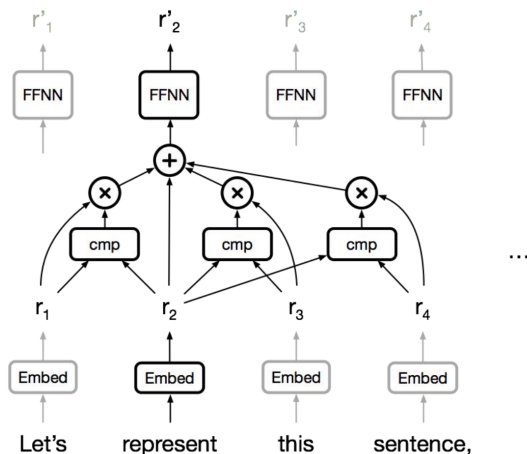
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- $\mathbf{V}, \mathbf{K}, \mathbf{Q}$  are parameters to be learned.
- This operation is called **self-attention**
  
- It can be generalized to **multiple heads**:
  - Split input vectors into  $n$  subvectors of dimension  $d/n$ ,
  - Apply self attention (with different  $\mathbf{V}, \mathbf{K}, \mathbf{Q}$ ) over these smaller vectors
  - Concatenate the results to get back  $d$  dimensional vectors



# Combining vectors with attention



from Vaswani and Huang:

<http://web.stanford.edu/class/cs224n/slides/>

# Transformer network

Transformer block:

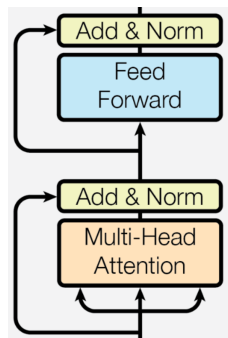
- Multi-head attention layer with skip connection and normalization
- Followed by feed forward with skip connection and normalization

Skip connection+normalization:

- Given a network block **nn** and input **x**
- The output **y** is computed as

$$\mathbf{y} = \mathbf{norm}(\mathbf{x} + \mathbf{nn}(\mathbf{x}))$$

where **norm** normalize the input



Vaswani et al.  
(2017)

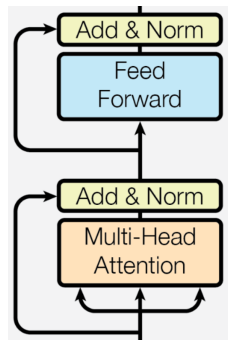
# Transformer network

## Feed forward block

- Two layer network, with ReLU activation

$$\mathbf{y} = \mathbf{W}_2 \text{ReLU}(\mathbf{W}_1 \mathbf{x})$$

- Usually,  $\mathbf{W}_1 \in \mathbb{R}^{4d \times d}$  and  $\mathbf{W}_2 \in \mathbb{R}^{d \times 4d}$
- i.e. hidden layer of dimension  $4d$ .



Vaswani et al.  
(2017)

## Position embeddings

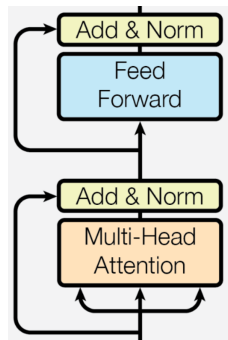
- **Limitation:** self attention does not take position into account!
- Indeed, shuffling the input gives the same results
  
- **Solution:** add position encodings.
- Replace the matrix  $\mathbf{W}$  by  $\mathbf{W} + \mathbf{E}$ , where  $\mathbf{E} \in \mathbb{R}^{d \times T}$
  
- $\mathbf{E}$  can be learned, or defined using sin and cos:

$$e_{2i,j} = \sin\left(\frac{j}{10000^{2i/d}}\right)$$
$$e_{2i+1,j} = \cos\left(\frac{j}{10000^{2i/d}}\right)$$

# Transformer network

Transformer network:

- Word embeddings + Position embeddings
- Then  $N$  transformer blocks (e.g.  $N = 12$ )
- Softmax classifier (e.g. for language modeling)



Vaswani et al.  
(2017)

# Masking for Transformer Language Models

- In transformer,  $\mathbf{h}_t$  depends on **all** inputs
- Could not be used as such for causal language modeling
- Solution: use mask in attention, to only use past
  
- Reminder:

$$\begin{aligned}\mathbf{H} &= \mathbf{VW} \cdot \text{softmax}(\mathbf{W}^\top \mathbf{K}^\top \mathbf{QW}) \\ &= \mathbf{VWA}\end{aligned}$$

Hence,  $\mathbf{a}_{it}$  is weight of input  $i$  in representation of position  $t$

- We want representation at time  $t$  to only depends on  $i \leq t$
- We could enforce  $\mathbf{a}_{it} = 0$  for  $i \geq t$

- We introduce the masked softmax operator
- Given an input  $\mathbf{x}$  and a binary mask  $\mathbf{m}$ ,

$$[\text{masked\_softmax}(\mathbf{x}, \mathbf{m})]_i = \frac{m_i \exp(x_i)}{\sum_{i=1}^d m_i \exp(x_i)}$$

- Still sums to one,  $m_i = 0$  implies  $[\text{masked\_softmax}(\mathbf{x}, \mathbf{m})]_i = 0$
- Sometimes implemented as:

$$\text{softmax}(\mathbf{x} + \log(\mathbf{m}))$$

- **Beware:** do not learn the mask (e.g. PyTorch: `register_buffer`)

# Training of a Transformer

- In practice, transformers are very unstable during training
- If the learning rate is too large, it diverges
- However if the learning rate is too small, it does not learn well



# Transformer network for Language Modeling: Results

Model	bpc
LSTM	1.25
Transformer	<b>1.07</b>

- Text8
- Character level language modeling
- bpc = bit per character.

## Why are language model useful?

- Standard Language Models are not that useful as such
- For specific-tasks we will see that they can be useful in Lecture 6
- For controlled generation (Machine Translation, Speech to Text, Question Answering...) we need more.
- **How to build a "controllable" text generation system using a language model ?**

- Language Model
- **Conditioned Language Model** : focus on Sequence to Sequence

- Problematically, controllable text generation can be seen as estimating:

$P(w_t | w_1, \dots, w_{t-1}, C)$  where  $C$  is a conditioning variable

## Sequence to Sequence Architecture

## Direct modeling of translation

We have:

a sentence  $S = (x_1, \dots, x_m)$  in a **Source** language (e.g. French)

its translation  $T = (y_1, \dots, y_n)$  in a **Target** language (e.g. English)

We directly work on the probability of a translation given a source sentence by expressing translation as conditional language modeling:

$$P(T | S) = \prod_{t=1}^n P(y_t | y_{t-1}, \dots, y_1, S)$$

**Goal** Learn a translation model where  $T$  is the most probable sentence given  $S$ :

$$T = \underset{T' \text{ in Target language}}{\operatorname{argmax}} P(T' | S)$$

**Challenge** How to encode the source sentence  $S$  ?

# Sequence to Sequence: Machine Translation

We want to condition a language model of the target language (e.g English) on a source sentence

- 1 Encode source sentences
- 2 Generate the target sentence based on the encoded source and a language target language model

# Sequence to Sequence: Machine Translation

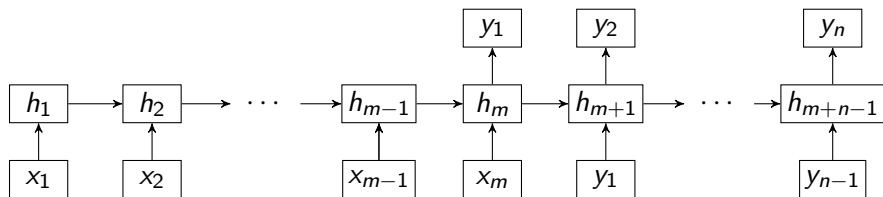
We want to condition a language model of the target language (e.g English) on a source sentence

- 1 Encode source sentences
  - 2 Generate the target sentence based on the encoded source and a language target language model
- We have seen that Neural Networks are good Language Models (i.e. can generate proper sentences)
  - We have seen that Neural Networks are good at modeling sequence.
    - We are going to combine two network
      - An **encoder** for encoding source sentences
      - A **decoder** for conditioned language modeling

→ This new architecture is referred to as an *encoder-decoder* or *sequence to sequence model*



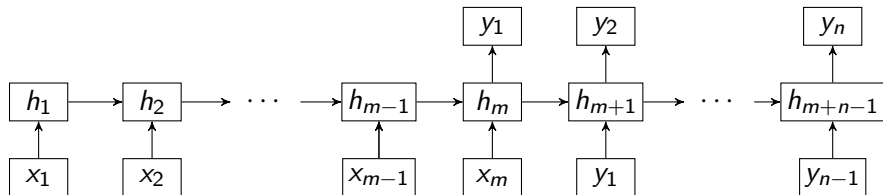
## Simple approach: sequence-to-sequence (seq2seq)



Equivalent to:

1. Build a representation of the source sentence by taking last hidden layer  $\mathbf{h}_T$  of LSTM applied to the source sentence
2. Use this representation as initialization of the hidden variables of LSTM applied to the target sentence

# Simple approach: sequence-to-sequence (seq2seq)



- Pro:
  - Very simple to implement:
    - input: Concatenation of source and target sentence.
    - output: target sentence
- Cons:
  - Needs large hidden layer to store everything about source sentence
  - Does not work on very long sentences
  - Same conditioning for the whole target sentence

## Architecture

- Encoder and Decoder can be any NN architecture seen so far
- In practice, LSTMs and Transformer are the most efficient (in most cases)

## Decoding

- At inference, we can improve performance by using *beam-search* instead of *greedy* decoding

- As any Neural Networks, we train a seq2seq architecture with backpropagation
- Using pairs of source-target aligned sentences we train the model to generate the target language based on source language

Source: *This week we'll continue to try to close a deal to purchase a dairy farm.*

Target: *Cette semaine, nous allons continuer d'essayer de signer un contrat d'achat d'une exploitation laitière.*

- To overcome the main encoding issue, the sequential attention on the source sentence improve importantly the performance

- How good is a given machine translation system?
- Hard problem, since many different translations acceptable
- Evaluation metrics:
  - subjective judgments by human evaluators
  - automatic evaluation metrics
  - task-based evaluation (how much post-editing effort? does information come across?)

NB: Evaluating sequence generation model is never easy (subjectivity!)

6

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<sup>6</sup>from Philipp Koehn: <http://mt-class.org/jhu/>

Measure  $n$ -gram overlap between machine translation output and reference translation

Compute precision for  $n$ -grams of size 1 to 4

Add brevity penalty to avoid too short translations

$$\text{BLEU} = \min \left( 1, \frac{\text{output\_length}}{\text{reference\_length}} \right) \left( \prod_{i=1}^4 \frac{C_i}{N_i} \right)^{\frac{1}{4}}$$

where  $C_i$  is the number of correct  $n$ -gram of size  $i$  and  $N_i$  is the total number of  $n$ -grams in the output of the system

Computed over full corpora, not just a sentence from Philipp Koehn: <http://mt-class.org/jhu/>

- (Abstractive) Summarization
  - Input:** Document
  - Output:** Summary
- Text Simplification
  - Input:** Complex sentence
  - Output:** Simplified sentence
- Multi-Modal tasks
  - Speech To Text
  - Caption Generation (Image to Text)



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