NavSoft's
CELESTIAL NAVIGATION TUTORIAL

Contents

The Concept of Using a Sextant Altitude

Using the altitude of a celestial body is similar to using the altitude of a lighthouse or similar object of known height, to obtain a distance.

Height

Altitude

Distance = Height/ Tan(Altitude)
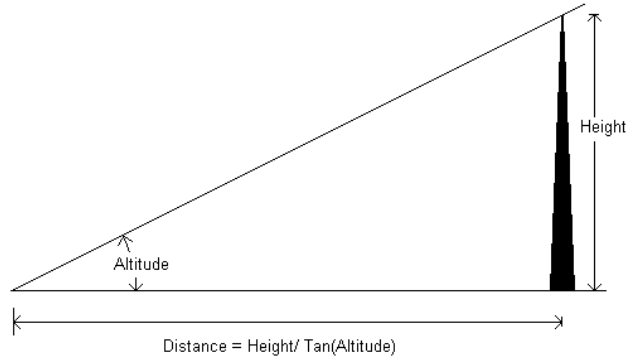
One object or body provides a distance but the observer can be anywhere on a circle of that radius away from the object. At least two distances/ circles are necessary for a position. (Three avoids ambiguity.)

A

B

C

Observed
Position
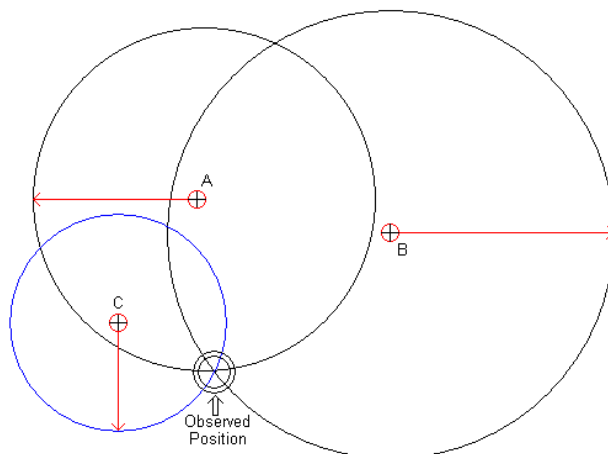
In practice, only that part of the circle near an assumed position would be drawn.

Using a Sextant for Celestial Navigation

After a few corrections, a sextant gives the true distance of a body if measured on an imaginary sphere surrounding the earth.

Using a Nautical Almanac to find the position of the body, the body's position could be plotted on an appropriate chart and then a circle of the correct radius drawn around it. In practice the circles are usually thousands of miles in radius therefore distances are calculated and compared with an estimate.

Working on this sphere, the distance becomes [90° - Altitude.] The point on the sphere corresponding to the Observer is known as his **Zenith**.
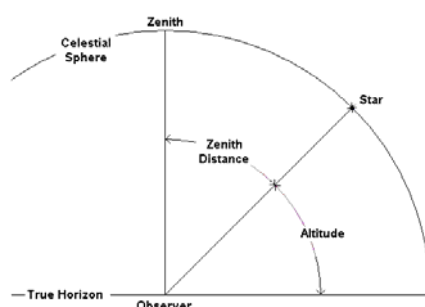
Position Lines

Because of the immense radius, the short length of interest can be considered a straight line.

Comparing the observed distance to the body and the calculated distance from an assumed position provides the distance towards or away from the body. The bearing is found by calculation and then it can be a simple matter of marking the assumed position on a chart, drawing a line in the direction of the body and marking off the difference between the observed and calculated distances.
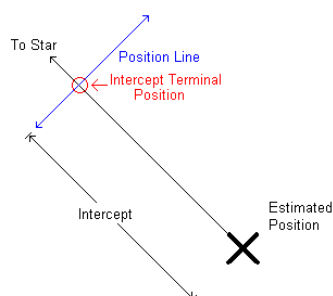
The observed distance is known as the **True Zenith Distance** (TZD.) The value based on the assumed position is the **Calculated Zenith Distance** (CZD.) The difference between the two is known as the **Intercept**.

The closest point on this circle is known as the **Intercept Terminal Position** (ITP) and the line representing the circle at that position is called a **Position Line**.

Additional sights provide additional position lines that intersect to provide a **Fix**.

A Running Fix

A vessel is usually moving between sights therefore they are combined "on the run."

The position line from a first sight must be moved to allow it to be combined with another position line for a different time.
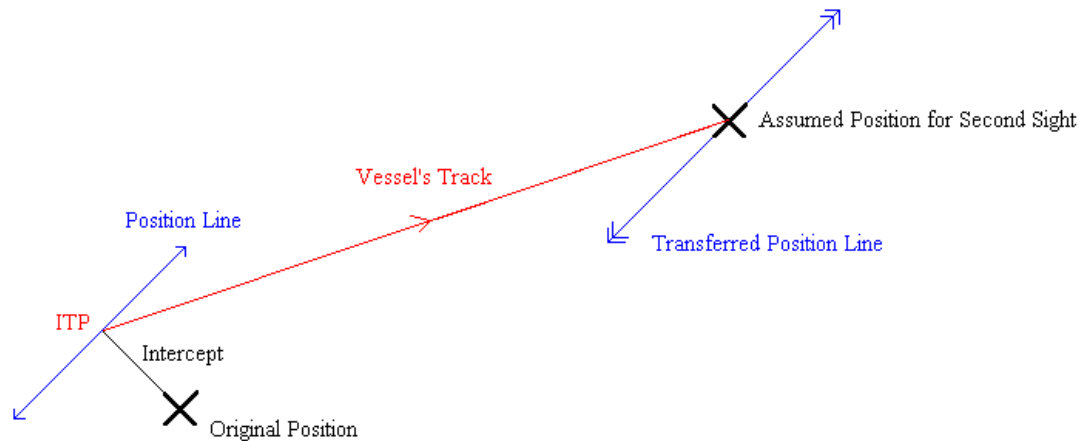
A double-headed arrow identifies a **Transferred Position Line**.

After a second sight has been calculated, its position line can be plotted and combined with the first to provide a fix.
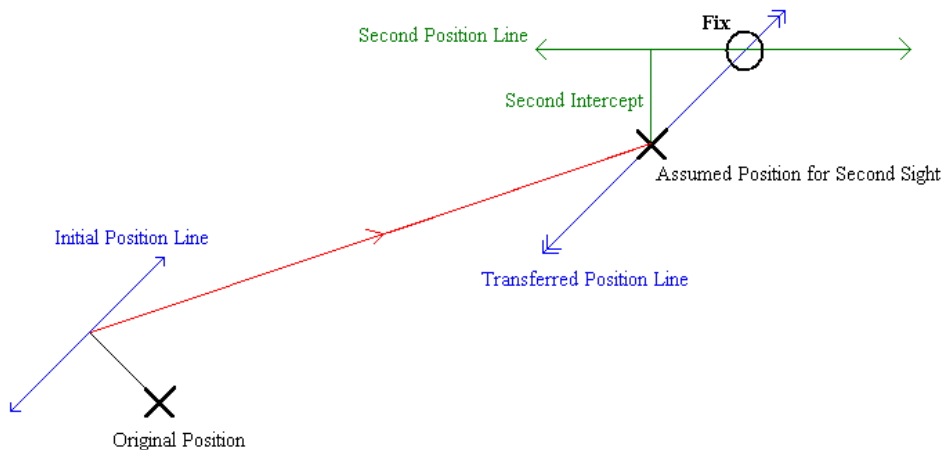
The shorter the "run" between the two observations, the more accurate will be the final result. This is why star sights are usually better than sun sights. The opportunity to obtain more position lines of different directions is an additional benefit.

Notes on Running Fixes

Under normal conditions, one would expect an error of +/- 0'.3 in the Position Lines. (This error is mainly due to the time recorded under practical conditions.) Land Surveyors achieve accuracy comparable to a GPS using more sophisticated instruments but the same calculations/ method.

Final accuracy is obviously improved by taking more observations. Six star sights will typically provide a fix within 0'.2 of the true position.

Most people adopt some shortcuts in the interest of speed. These have a cost in terms of accuracy. The Sun's Total Correction Tables assume that the Sun's semi-diameter is either 15'.9 or 16'.2. A Sun Sight in April (SD = 16'.0) is immediately in error by 0'.2. Tables are rounded to the nearest 0'.1 which could introduce a cumulative error of 0'05 for every item.

With Star sights, the short interval between the first and last sight means that many people use a single position for all the sights and plot the results without allowing for the vessel's movement. The error is larger than above, but quite acceptable in mid-ocean.

Before GPS and Calculators

The method used until the 1980s was the Haversine Formula and Log Tables. A few commercial navigators used Sight Reduction Tables but most preferred the longer method in the interests of accuracy and flexibility.

The Haversine formula is a rearrangement of the Cosine formula (above) substituting Haversines for the Cosine terms. ($Hav(\theta) = \frac{1}{2} \times [1 - Cos(\theta)]$). This makes a calculation using logarithms slightly easier, as the terms are always positive.

Hav(CZD) = Hav(Lat difference Dec) + Hav(LHA) x Cos(Lat) x Cos Dec

Sight Calculations and obtaining a Position

The stages to resolving a sight are;

> Correct the Sextant Altitude to find the true distance of the body
> Calculate the bearing and distance from an assumed position
> Using the difference in distances to obtain a Position Line

Finally Position Lines are combined to provide a fix.

Correcting a Sextant Altitude

An explanation of the corrections is found in the next section under "Corrections to a Sextant Altitude." All of these, except Index Error, are found in Nautical Tables.

Example for the Sun

Sextant Altitude	31° 22'.0	
Index Error	2'.0	Assuming "Off the Arc"
Observed Altitude	31° 24'.0	
Dip	-3'.0	Subtract
Apparent Altitude	31° 21'.0	
Refraction	- 1'.6	Subtract
True Altitude	31° 19'.4	
Semi-Diameter	16'.5	Add for Lower Limb
True Altitude	32° 26'.8	
	90° 00'.0	
True Zenith Distance	57° 33'.2	

Altitudes of Stars do not need a Semi-Diameter correction while the Moon needs more corrections. See examples at the end of the next section.

Calculating the Bearing and Distance

Positions for the observer and position lines can be plotted on a chart or calculated. The section on "Sailings" deals with mathematical calculations.

The other terms in the following formulae are derived from a Nautical Almanac. (See Nautical Almanac Information.)

The formulae for calculating the distance of the body and its altitude are

$$\text{Cos(Zenith Distance)} = \text{Sin(Lat)} \times \text{Sin(Dec)} + \text{Cos(Lat)} \times \text{Cos(Dec)} \times \text{Cos(LHA)}$$
and
$$\text{Tan(Azimuth)} = \text{Sin(LHA)} / (\text{Cos(Lat)} \times \text{Tan(Dec)} - \text{Sin(Lat)} \times \text{Cos(LHA)})$$

These formulae can be used without further knowledge however the section on "Celestial Navigation Calculations" provides an introduction to spherical trigonometry.

ABC Tables

ABC tables are very easy to use and more than adequate for the bearing of a celestial body.

These tables avoid the need to use a calculator or Log tables but are based on the previous formulae.

These transpose the Azimuth formula so that

A = Tan(Lat) / Tan(LHA)
B = Tan(Dec) / Sin(LHA)
C = Difference A ~ B = 1/ [Tan(Azimuth) x Cos(Lat)]

ABC Tables Example

Latitude		20° N
Declination		45° S
LHA		30°

A	0.63	S	Opposite to Latitude unless LHA > 180°
B	2.00	S	Same as Declination

C	2.63	S	Same name; Sum. Different names; Difference

The C Table gives a bearing of 22°.0. The sign of C means that this bearing is south. It is west because the LHA is less than 180.

The C result would normally be written as "S 22.0 W" or 202°.

The effect of rounding ABC Tables' values is negligible (+/- 0°1.) This is not true of the older Sight Reduction Tables where the calculated altitude is rounded to the nearest minute. Furthermore the need to use a plotting sheet with a rounded, estimated position provides considerable scope for inaccuracy. (Sight Reduction Tables were known as the Air Navigation Tables until 2003.)

The author's preferred manual method is to use a calculator for the Zenith Distance and ABC tables for Azimuths. Without a calculator he would still use the Cosine formula but with log tables.

<u>Obtaining a Position Line</u>

The difference between the True (TZD) and Calculated (CZD) Zenith Distances is the Intercept.

TRUE, TINY, TOWARDS

If the TZD is less than the CZD then the assumed position must be moved in the direction of the body by the amount of the Intercept. This gives a position of the correct distance from the body. It is known as the Intercept Terminal Position or ITP.

As the radius of the circle is normally very large, it is considered to be a straight line near this point. A line at 90° to the direction of the body is the Position Line.

<u>Combining Position Lines</u>

A single Position Line must be combined with other observations for a fix. This can be achieved using a plotting sheet and then transferring the ITP by the distance to the next sight and redrawing the Transferred Position Line in the same direction as the original.

For Sun sights, it is more usual to calculate the ITP of a morning sight and then calculate the transferred position for the Sun's Meridian Passage (Noon.) The difference between calculated and observed latitudes provides a longitude using "Plane Sailing." With a little practice, this will be found to be a faster, not to mention more accurate method.

For Star Sights, many people use a single position and then plot the Position Lines without allowing for the vessel's movement. This may appear a sloppy practice but a few miles error mid-ocean is usually irrelevant. Even if the position at sunset was perfect, there is no guarantee that the position an hour later is within a mile. Indeed even if the position agrees perfectly with a GPS position, there is no guarantee that an intervening military operation has not thrown the GPS position out let alone a fault in the equipment/ aerial. "I am about here," is a far safer assumption than "My wheelhouse is/ was within 10m of this position."

Corrections to a Sextant Altitude

Index Error
This error can be found using the horizon. The sextant's altitude is set to zero and then the two images of the horizon are aligned. The Index Error can then be read off.

If the sextant altitude reads high, the correction is subtractive and termed "On the Arc." "Off the Arc" is the opposite.

After Index Error has been applied, the Sextant Altitude it is referred to as the Observed Altitude.

Dip/ Height of Eye
The True Horizon is at 90° to the Earth's gravitational field. It coincides with the apparent horizon at sea level. However the Apparent Horizon starts to dip below the horizontal plane as the height of (the observer's) eye increases.

Dip includes an allowance for Refraction below the horizontal plane.
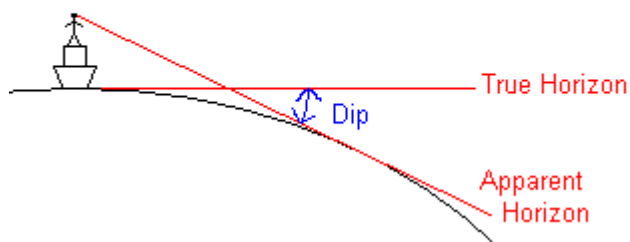
The formulae are;
      Dip = 0.97 x Square Root (Ht of Eye in feet)
      Dip = 1.76 x Square Root (Ht of Eye in meters)

Dip is subtracted from the Observed Altitude to give Apparent Altitude.

Refraction
The deflection of light as it enters/ passes through the atmosphere is known as Refraction.

Refraction is stable and therefore predictable above about 15°, below that one needs to consider the characteristics of the atmospheric layers through which the light passes at that time. (Taking the altitude of bodies at less than 15° is usually avoided for this reason.)

For altitudes above 15°, a simplified formula is adequate (± 0'.02)
      Refraction = 0.96/ Tan (Altitude)

Refraction tables make assumptions on the layers for low altitudes and should be treated with caution. +/- 2° is not uncommon at an altitude of 2°.

Refraction is subtracted from the Apparent Altitude to obtain the True Altitude.

<u>Temperature and Pressure Correction for Refraction</u>
The correction for Refraction assumes a temperature of 10° C and pressure of 1010mb. This may be modified for actual temperature and pressure. A temperature difference of 10° C will alter Refraction by 3% and a 10mb pressure difference will change Refraction by 1%. (0'05 and 0'.02 for an altitude of 30°)

The multiplier to correct for Temperature (°C) and Pressure (mb)
$$= \text{Pressure}/ 1010 * 283/ (\text{Temperature} + 273)$$

<u>Semi-Diameter</u>
When measuring the altitudes of the Sun, Moon, Venus and Mars, it is usual to use either the top (Upper Limb) or bottom (Lower Limb) of the body. This offset must then be removed before comparison with the calculated value.

The angular diameter of a body depends on its distance from the Earth. Thus for the Sun the Semi-Diameter varies between 16'.3 in January, when the Sun is closest and 15'.7 in June when it is furthest away.

For a lower limb observation, the Semi-Diameter should be added to the True altitude.

<u>Augmentation of the Moon's Semi-Diameter</u>
The Earth's radius is about 1/ 60th of the distance to the Moon. The reduction in distance compared to when on the horizon, has a measurable effect on its size. In contrast the Sun's distance is 23,000 times the Earth's radius and the effect is negligible.

Augmentation = Sin (Altitude) x Horizontal Parallax

Horizontal Parallax is used in the formula as the lunar distance is not provided in a Nautical Almanac.

This correction is typically 0'.15 and should be added to the Moon's Semi-Diameter before applying the Semi-Diameter to the True Altitude.

Parallax in Altitude

The Parallax correction allows for the difference in the altitude measured on the Earth's surface versus the altitude that would be measured at the centre of the Earth.
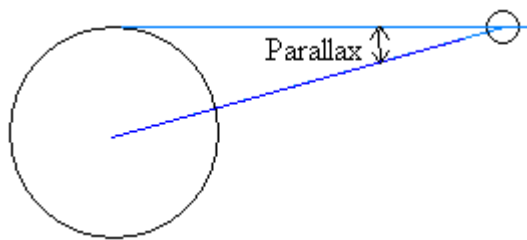
The effect of parallax reduces with altitude. It is greatest when the body is at the horizon (Horizontal Parallax) and declines to zero when the body is overhead.

The effect is also proportional to the distance of the body. Thus the Horizontal Parallax for the Moon is about 1° but only 0'.15 for the Sun. This correction must be included for the Moon but is usually ignored for the Sun. It can be significant for Venus and Mars, depending on their distance, but is always insignificant for Jupiter and Saturn. (< 0'.05)

Sin (Horizontal Parallax) = Earth's Radius/ Distance of Body

After correcting for altitude, the correction is known as Parallax in Altitude.

Parallax in Altitude = Horizontal Parallax x Cos (Altitude)

Parallax in Altitude should be added to the True Altitude.

Reduction of the Moon's Horizontal Parallax

Horizontal Parallax is proportional to the Earth's radius. Therefore as the Earth's radius declines with latitude, so does Horizontal Parallax.

Correction = Horizontal Parallax * [Sin (Lat) ^ 2] / 298.3

This should be subtracted from Horizontal Parallax before calculating Parallax in Altitude.

<u>Further Examples of Corrections to a Sextant Observation</u>

Add or Subtract

<u>For a Star</u>

Sextant Altitude	31° 22'.0	
Index Error	2'.0	Depends on the error
Observed Altitude	31° 24'.0	
Dip	-3'.0	Subtract
Apparent Altitude	31° 21'.0	
Refraction	- 1'.6	Subtract
True Altitude	31° 19'.4	
	90° 00'.0	
True Zenith Distance	58° 40'.6	

<u>For the Moon</u>

Sextant Altitude	31° 22'.0	
Index Error	2'.0	Depends on the error
Observed Altitude	31° 24'.0	
Dip	-3'.0	Subtract
Apparent Altitude	31° 21'.0	
Refraction	- 1'.6	Subtract
True Altitude	31° 19'.4	
Semi-Diameter	16'.5	Add for Lower Limb
Parallax (in Altitude)	51'.1	Add
True Altitude	32° 26'.8	
	90° 00'.0	
True Zenith Distance	57° 33'.2	

<u>Moon's Additional Corrections</u>

Tabulated Horizontal Parallax	59'.9	
Latitude Correction	- 0'.1	e.g. 52° N
Horizontal Parallax	59'.8	
Tabulated Semi-Diameter	16'.3	
Augmentation of Semi-Diameter	+ 0'.15	
Moon's Semi-Diameter	16'.5	

Nautical Almanac Information

Greenwich Hour Angle (GHA)

This is the body's angular distance west of the Greenwich meridian.

The GHA for the Sun, Moon and Planets is tabulated in a Nautical Almanac for each hour.

2011   January   19, 20, 21   (Wed, Thur, Fri)

SATURN	+0.8	STARS			SUN		MOON				
GHA	Dec	Name	SHA	Dec	GHA	Dec	GHA	v	Dec	d	HP
281° 15'.2	4° 26'.6 S	Acamar	315° 19'.5	40° 15'.8 S	177° 22'.8	20° 27'.0 S	10° 09'.4	6'.1	21° 08'.5 N	7'.6	59'.3
296° 17'.6	26'.6	Achernar	335° 28'.0	57° 11'.1 S	192° 22'.6	26'.5	24° 34'.5	6'.0	00'.9	7'.7	59'.4
311° 20'.0	26'.6	Acrux	173° 11'.1	63° 09'.5 S	207° 22'.4	26'.0	38° 59'.5	6'.1	20° 53'.2 N	7'.8	
326° 22'.5	26'.6	Adhara	255° 13'.5	28° 59'.4 S	222° 22'.2	25'.5	53° 24'.6	6'.1	45'.4	8'.0	
341° 24'.9	26'.6	Aldebaran	290° 51'.1	16° 31'.9 N	237° 22'.0	25'.0	67° 49'.7	6'.2	37'.4	8'.1	59'.5
356° 27'.3	26'.6				252° 21'.8	24'.5	82° 14'.9	6'.1	29'.3	8'.3	
11° 29'.8	26'.6	Alioth	166° 21'.9	55° 53'.6 N	267° 21'.6	23'.9	96° 40'.0	6'.2	21'.0	8'.4	
26° 32'.2	26'.6	Alkaid	153° 00'.1	49° 15'.1 N	282° 21'.4	23'.4	111° 05'.2	6'.3	12'.6	8'.5	
41° 34'.6	26'.6	Alnair	27° 46'.2	46° 54'.5 S	297° 21'.3	22'.9	125° 30'.5	6'.3	04'.1	8'.7	59'.6
56° 37'.0	26'.6	Alnilam	275° 47'.8	1° 11'.8 S	312° 21'.1	22'.4	139° 55'.8	6'.3	19° 55'.4 N	8'.8	
71° 39'.5	26'.6	Alphard	217° 57'.5	8° 42'.6 S	327° 20'.9	21'.9	154° 21'.1	6'.3	46'.6	9'.0	
86° 41'.9	26'.6				342° 20'.7	21'.3	168° 46'.4	6'.4	37'.6	9'.1	
101° 44'.3	4° 26'.6 S	Alphecca	126° 12'.6	26° 40'.4 N	357° 20'.5	20° 20'.8 S	183° 11'.8	6'.4	19° 28'.5 N	9'.2	59'.7
116° 46'.8	26'.6	Alpheratz	357° 45'.5	29° 09'.3 N	12° 20'.3	20'.3	197° 37'.2	6'.5	19'.3	9'.4	
131° 49'.2	26'.6	Altair	62° 10'.2	8° 53'.9 N	27° 20'.1	19'.8	212° 02'.7	6'.5	09'.9	9'.4	
146° 51'.6	26'.6	Ankaa	353° 17'.5	42° 14'.9 S	42° 19'.9	19'.2	226° 28'.2	6'.5	00'.5	9'.7	
161° 54'.1	26'.6	Antares	112° 28'.6	26° 27'.3 S	57° 19'.7	18'.7	240° 53'.7	6'.6	18° 50'.8 N	9'.7	59'.8
176° 56'.5	26'.6				72° 19'.5	18'.2	255° 19'.3	6'.6	41'.1	9'.9	

An increment is applied to allow for the minutes and seconds. These assume that the GHAs change by a uniform amount per hour. This assumption is corrected using a "v" correction.

30ᵐ   INCREMENTS AND CORRECTIONS

S	Sun Planets	Aries	Moon	v or d	Corr'n	v or d	Corr'n	v or d	Corr'n	S	Sun Planets	Aries	Moon
00	7° 30'.0	7° 31'.2	7° 09'.5	0.0	0.0	6.0	3.1	12.0	6.1	00	7° 45'.0	7° 46'.3	7° 23'.8
01	7° 30'.2	7° 31'.5	7° 09'.7	0.1	0.1	6.1	3.1	12.1	6.2	01	7° 45'.3	7° 46'.5	7° 24'.1
02	7° 30'.5	7° 31'.8	7° 10'.0	0.2	0.1	6.2	3.2	12.2	6.2	02	7° 45'.5	7° 46'.8	7° 24'.3
03	7° 30'.8	7° 32'.0	7° 10'.2	0.3	0.2	6.3	3.2	12.3	6.3	03	7° 45'.7	7° 47'.0	7° 24'.5
04	7° 31'.0	7° 32'.3	7° 10'.5	0.4	0.2	6.4	3.3	12.4	6.3	04	7° 46'.0	7° 47'.3	7° 24'.8
05	7° 31'.2	7° 32'.5	7° 10'.7	0.5	0.3	6.5	3.3	12.5	6.4	05	7° 46'.3	7° 47'.5	7° 25'.0
06	7° 31'.5	7° 32'.8	7° 10'.9	0.6	0.3	6.6	3.4	12.6	6.4	06	7° 46'.5	7° 47'.8	7° 25'.2
07	7° 31'.8	7° 33'.0	7° 11'.2	0.7	0.4	6.7	3.4	12.7	6.5	07	7° 46'.7	7° 48'.0	7° 25'.5
08	7° 32'.0	7° 33'.3	7° 11'.4	0.8	0.4	6.8	3.5	12.8	6.5	08	7° 47'.0	7° 48'.3	7° 25'.7
09	7° 32'.2	7° 33'.5	7° 11'.6	0.9	0.5	6.9	3.5	12.9	6.6	09	7° 47'.2	7° 48'.5	7° 26'.0

To find the Increment, the Increments and Corrections pages are entered with the minutes and seconds of the sight. For a Sun sight taken at exactly 30 minutes past the hour, the increment value would be 7° 30'.0,

On the left hand side of the Increments page for the appropriate minutes, there is the correction to apply based on the tabulated "v" value for the body. The "v" corrections are considered to be linear but are actually tabulated for halfway through the minute. This is why a "v" of 12.0 produces 6.1 for 30 minutes. The "v" adjustment is always positive for the Moon but can be negative for the planets.

The "v" correction for the planets is found at the bottom of the daily pages. The rapid changes in the motion of the Moon mean that a "v" value is tabulated for each hour. No "v" correction is supplied for the Sun. Instead the GHA values for each hour in the tables are massaged.

The GHA for stars are treated differently. Here the SHA (Sidereal Hour Angle) or angular distance west of Aries is tabulated for each three day period. The GHA of Aries is then tabulated for each hour and the Increment value for minutes and seconds is found in exactly the same way as for the Sun, Moon and planets. No "v" value is given for simplicity. The GHA for a star is the sum of the GHA for Aries and the SHA.

Local Hour Angle (LHA)
This is the angular distance west of the observer. Thus we find the GHA of a body and then adjust it for longitude. An easterly longitude is added to the GHA and a westerly longitude is subtracted.

LHA = GHA +/- Longitude

Declination (Dec)
The Declination for most bodies is tabulated for each hour. Due to the very small movement of stars, it is only provided once for each three day period.

The hourly change of Declination is usually small therefore the adjustment is found using the "d" value at the bottom of the page for the Sun and planets. (It is negligible for stars.) The Moon however moves rapidly which means that its' "d" value is provided for each hour.

The "d" correction is found in the same manner as for "v" by going to the appropriate Increments and Corrections page to obtain the appropriate correction from the right hand "v and d" section.

The direction in which to apply the "d" correction (North or South) is determined by examining the next hourly value.

Simplifications Vs Accuracy in Nautical Almanacs
In the explanation section at the back of the UK/ US Governments' "The Nautical Almanac," paragraph 24 provides details of the expected errors in the values of GHA and Declination. The maximum error in each is 0'.2 for the planets, 0'.25 for the Sun and 0'.3 for the Moon. These errors are caused entirely by the need to keep the presentation as simple as possible.

It goes on to say; "In practice it may be expected that only one third of the values of GHA and Dec taken out will have errors larger than 0'.05 and less than one tenth will have errors larger than 0'.1."

The superficial attraction of using data from a source such as the Astronomical Almanac in the interests of accuracy is illusory. RA and Declination may be quoted in arc-seconds to several decimal places but the bodies, particularly the Moon, do not necessarily move in a linear manner during the day. Similarly the Hour Angle of Aries does not change linearly. Irregular motions are included in Nautical Almanac data.

<u>Example of Calculations for the Moon ~ 00:30:05 GMT, 19th January 2011</u>

GHA at 00:00	10° 09'.4
Increment for 30m 05s	+ 7° 10'.7
"v" (6'.1 at 00:00)	+ 3'.1
GHA at 00:30:05	17° 23'.2
Longitude	10° 00'.0 E
LHA	27° 23'.2

Declination at 00:00	21° 08'.5 S	
"d" (7'.6 at 00:00)	3'.8 S	S by inspection
Declination at 00:30:05	21° 04'.7 S	

<u>Example of Calculations for Acamar ~ 00:30:05 GMT, 19th January 2011</u>

GHA of Aries at 00:00	118° 02'.7	(Not visible below)
Increment for 30m 05s	+ 7° 32'.5	
SHA	315° 19'.5	
GHA of Acamar at 00:30:05	80° 54'.7	(440° 54' - 360°)
Longitude	10° 00'.0 W	
LHA	70° 54'.7	

The Declination value of 40° 15'.8 taken from the day's pages is not adjusted.

2011   January   19, 20, 21   (Wed, Thur, Fri)

SATURN	+0.8	STARS			SUN		MOON				
GHA	Dec	Name	SHA	Dec	GHA	Dec	GHA	v	Dec	d	HP
281° 15'.2	4° 26'.6 S	Acamar	315° 19'.5	40° 15'.8 S	177° 22'.8	20° 27'.0 S	10° 09'.4	6'.1	21° 08'.5 N	7'.6	59'.3
296° 17'.6	26'.6	Achernar	335° 28'.0	57° 11'.1 S	192° 22'.6	26'.5	24° 34'.5	6'.0	00'.9	7'.7	59'.4
311° 20'.0	26'.6	Acrux	173° 11'.1	63° 09'.5 S	207° 22'.4	26'.0	38° 59'.5	6'.1	20° 53'.2 N	7'.8	
326° 22'.5	26'.6	Adhara	255° 13'.5	28° 59'.4 S	222° 22'.2	25'.5	53° 24'.6	6'.1	45'.4	8'.0	
341° 24'.9	26'.6	Aldebaran	290° 51'.1	16° 31'.9 N	237° 22'.0	25'.0	67° 49'.7	6'.2	37'.4	8'.1	59'.5
356° 27'.3	26'.6				252° 21'.8	24'.5	82° 14'.9	6'.1	29'.3	8'.3	
11° 29'.8	26'.6	Alioth	166° 21'.9	55° 53'.6 N	267° 21'.6	23'.9	96° 40'.0	6'.2	21'.0	8'.4	
26° 32'.2	26'.6	Alkaid	153° 00'.1	49° 15'.1 N	282° 21'.4	23'.4	111° 05'.2	6'.3	12'.6	8'.5	
41° 34'.6	26'.6	Alnair	27° 46'.2	46° 54'.5 S	297° 21'.3	22'.9	125° 30'.5	6'.3	04'.1	8'.7	59'.6
56° 37'.0	26'.6	Alnilam	275° 47'.8	1° 11'.8 S	312° 21'.1	22'.4	139° 55'.8	6'.3	19° 55'.4 N	8'.8	
71° 39'.5	26'.6	Alphard	217° 57'.5	8° 42'.6 S	327° 20'.9	21'.9	154° 21'.1	6'.3	46'.6	9'.0	
86° 41'.9	26'.6				342° 20'.7	21'.3	168° 46'.4	6'.4	37'.6	9'.1	
101° 44'.3	4° 26'.6 S	Alphecca	126° 12'.6	26° 40'.4 N	357° 20'.5	20° 20'.8 S	183° 11'.8	6'.4	19° 28'.5 N	9'.2	59'.7
116° 46'.8	26'.6	Alpheratz	357° 45'.5	29° 09'.3 N	12° 20'.3	20'.3	197° 37'.2	6'.5	19'.3	9'.4	
131° 49'.2	26'.6	Altair	62° 10'.2	8° 53'.9 N	27° 20'.1	19'.8	212° 02'.7	6'.5	09'.9	9'.4	
146° 51'.6	26'.6	Ankaa	353° 17'.5	42° 14'.9 S	42° 19'.9	19'.2	226° 28'.2	6'.5	00'.5	9'.7	
161° 54'.1	26'.6	Antares	112° 28'.6	26° 27'.3 S	57° 19'.7	18'.7	240° 53'.7	6'.6	18° 50'.8 N	9'.7	59'.8
176° 56'.5	26'.6				72° 19'.5	18'.2	255° 19'.3	6'.6	41'.1	9'.9	

30^m   INCREMENTS AND CORRECTIONS

S	Sun Planets	Aries	Moon	v or d	Corr'n	v or d	Corr'n	v or d	Corr'n	S	Sun Planets	Aries	Moon
00	7° 30'.0	7° 31'.2	7° 09'.5	0.0	0.0	6.0	3.1	12.0	6.1	00	7° 45'.0	7° 46'.3	7° 23'.8
01	7° 30'.2	7° 31'.5	7° 09'.7	0.1	0.1	6.1	3.1	12.1	6.2	01	7° 45'.3	7° 46'.5	7° 24'.1
02	7° 30'.5	7° 31'.8	7° 10'.0	0.2	0.1	6.2	3.2	12.2	6.2	02	7° 45'.5	7° 46'.8	7° 24'.3
03	7° 30'.8	7° 32'.0	7° 10'.2	0.3	0.2	6.3	3.2	12.3	6.3	03	7° 45'.7	7° 47'.0	7° 24'.5
04	7° 31'.0	7° 32'.3	7° 10'.5	0.4	0.2	6.4	3.3	12.4	6.3	04	7° 46'.0	7° 47'.3	7° 24'.8
05	7° 31'.2	7° 32'.5	7° 10'.7	0.5	0.3	6.5	3.3	12.5	6.4	05	7° 46'.3	7° 47'.5	7° 25'.0
06	7° 31'.5	7° 32'.8	7° 10'.9	0.6	0.3	6.6	3.4	12.6	6.4	06	7° 46'.5	7° 47'.8	7° 25'.2
07	7° 31'.8	7° 33'.0	7° 11'.2	0.7	0.4	6.7	3.4	12.7	6.5	07	7° 46'.7	7° 48'.0	7° 25'.5
08	7° 32'.0	7° 33'.3	7° 11'.4	0.8	0.4	6.8	3.5	12.8	6.5	08	7° 47'.0	7° 48'.3	7° 25'.7
09	7° 32'.2	7° 33'.5	7° 11'.6	0.9	0.5	6.9	3.5	12.9	6.6	09	7° 47'.2	7° 48'.5	7° 26'.0

Sailings

Plane (or Plain) Sailing
The relationships can be laid out as two triangles. The sides of the top triangle are Distance, difference in latitude (dLat) and Departure. Departure is the longitude distance in miles. The lower triangle relates Departure to the difference in longitude (dLong.) The angle labelled mLat stands for Mean Latitude.
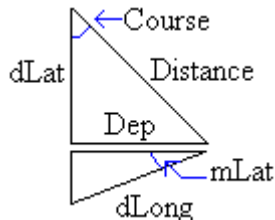
From the top triangle:-
      dLat = Distance x Cos (Course)   Eq 1
      Departure = Distance x Sin (Course)   Eq 2

From the lower triangle:-
      dLong = Departure/ Cos(Mean Latitude)   Eq 3

Tabulated values are found in Nautical Tables as "Traverse Tables." However the simplicity of the formulae are ideal for calculators.

Example
Initial Position   45° N 30° W
Course   045° T
Distance   100'

Using the first formula (Eq 1):
dLat =   100 x Cos(45°)
dLat =   70'.7 = 1° 10'.7
Final Latitude   45° + 1° 10'.7 = 46° 10'.7 N

From the second formula (Eq 2);
Departure = 100 x Sin (045°) = 70.71

From the third formula (Eq 3)
      dLong = 70.71/ Cos (45° + 70.7/ 2)
      dLong = 101'.04 = 1° 41'.0
Therefore the final longitude = 30° + 1° 41'.0 = 31° 41'.0 W

For a Noon calculation, dLat is the difference between the calculated and observed latitudes. The "Course" becomes the direction of the Position Line.

Distance = dLat/ Cos(P/L Dirn)
Departure = Distance x Sin(P/L Dirn)
dLong = Departure/ Cos(Mean Latitude)

Plane Sailing is adequate for distances up to about 60'. If the example is worked using Mercator Sailing, there is a difference of 0'.3 in the final longitude.

Mercator Sailing.
Mercator Sailing allow for the oblate shape of the Earth. (A squashed sphere.)

The formulae are
         dLat = Dist x Cos(Course)   As for Plane Sailing - Eq 1
and
         dLong = Tan(Course) x DMP   Eq 4

DMP stands for Difference in Meridian Parts. Meridian Parts are the distance in nautical miles from the equator to the required latitude. These are tabulated in Nautical Tables.

Example
Initial Position   45° N   30° W
Course   045° T
Distance   100'

Using the first formula (Eq 1):
dLat =   100 x Cos(45°)
dLat =   70'.7 = 1° 10'.7 N   because the track is northerly
Final Latitude   45° + 1° 10'.7 = 46° 10'.7 N

Meridian Parts
         for 45°   3013.38
         for 46° 10'.7   3114.08
Difference in Meridian Parts   100.7

From the second formula (Eq 4);
         dLong = Tan (045°) x 100.7   = 100'.7 = 1° 40'.7

Original Longitude   30° 00'.0 W
dLong   1° 40'.7 West   because the course is westerly
Final Longitude   31° 40'.7 W

The drawback with Mercator Sailing is the need to refer to tables. If you use a programmable calculator then this is the formula to calculate Meridian Parts. (The infinite series of terms in Bowditch et al is simply an expansion of this.)

   A x Ln(Tan(45° + Lat/ 2)/ ((1 + e x Sin(Lat))/ (1 – e x Sin(Lat)) ^ (e/ 2)

         For WGS84;   A = 3437.74677 and e = 0.08182

Note that most nautical tables are based on the Clarke 1880 spheroid that uses a different compression to WGS84. The difference is small but noticeable (0.27 at 45°.) For Clarke 1880 use e = 0.08248.

Celestial Navigation Calculations

An imaginary sphere surrounding the Earth is used for calculations. This is known as the Celestial Sphere. Declination corresponds to Latitude and Hour Angles to Longitudes.

Solving a sight uses spherical trigonometry. The triangle is known as the PZX triangle.
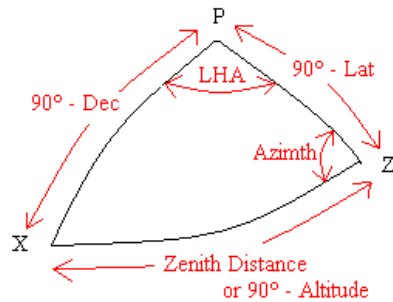

P is the Pole, Z, the Observer's Zenith and X is the body.

Altitude Vs Zenith Distance

An Altitude is a terrestrial measurement while calculations are carried out on the Celestial sphere.

The reason that the calculations do not allow for the shape of the Earth is because the calculations are performed on the Celestial Sphere. Gravity ensures that the horizon (and thus Altitude,) corresponds with the equivalents on the celestial Sphere.

Zenith Distance is the correct term and helps avoid confusion.

Parts of the PZX Triangle
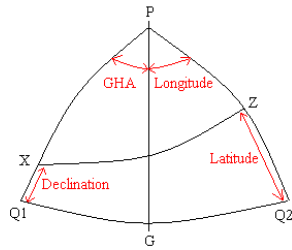
Q1 to Q2 is the Equator and the Vertical line from P to G is the Greenwich Meridian (0° GHA and Longitude.)

The compliment of an angle is 90° - the angle. This is also true of the sides in a spherical triangle.

The distance from P to Q1 is 90 ° therefore PX = 90° - Declination or
PX = co-Declination.

Compliments enable formulae to be simplified because the Sine of an angle equals the Co-Sine of the compliment of that angle. This also applies to Tangents and Cotangents, Secants and Cosecants.

Sin(60°) = Co-Sine(90° - 60°) = Cosine(30°)

The compliment of a compliment is the same as the original angle;
Cos(co-30°) = Sin(90° - (90° - 30°)) = Sin(30°)

Simplifying the diagram and adding some labels:-
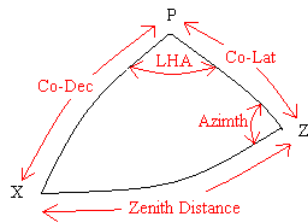
SPHERICAL FORMULAE
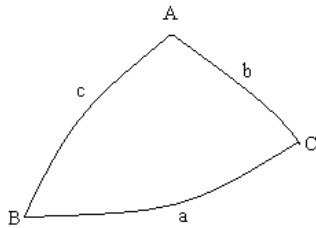
To Calculate a Side - The Cosine Formula

$Cos(a) = Cos(b) \times Cos(c) + Sin(b) \times Sin(c) \times Cos(A)$

Applying this to the PZX triangle we get:-

$Cos(\text{Zenith Distance}) = Cos(\text{co-Lat}) \times Cos(\text{co-Dec}) + Sin(\text{co-Lat}) \times Sin(\text{Co-Dec}) \times Cos(LHA)$

Because $Sin(\text{co-A}) = Cos(A)$ and $Cos(\text{co-A}) = Sin(A)$

$Cos(\text{Zenith Distance}) = Sin(Lat) \times Sin(Dec) + Cos(Lat) \times Cos(Dec) \times Cos(LHA)$

If Altitude is preferred; Zenith Distance = co-Altitude thus

$Sin(\text{Altitude}) = Sin(Lat) \times Sin(Dec) + Cos(Lat) \times Cos(Dec) \times Cos(LHA)$

For an Angle

$Tan(C) = Sin(A)/ [Sin(b)/ Tan(c) – Cos(b) \times Cos(A)]$

Inserting terms from the PZX triangle this becomes

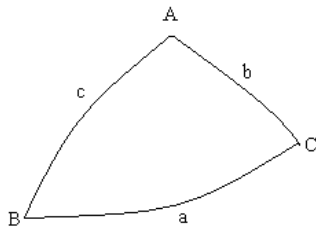$Tan(Az) = Sin(LHA)/ [Sin(\text{co-Lat})/ Tan(\text{co-Dec}) – Cos(\text{co-Lat}) \times Cos(LHA)]$

Or

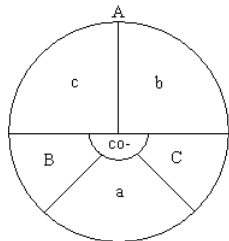$Tan(Az) = Sin(LHA)/ (Cos(Lat) \times Tan(Dec) – Sin(Lat) \times Cos(LHA))$

The Spherical Sine Formulae

$Sin(a)/ Sin(A) = Sin(b)/ Sin(B) = Sin(c)/ Sin(C)$

Napier's Rules
These can be used when one of the sides or angles is 90°.

If angle A is 90° then a diagram is drawn with A above the circle and the sectors filled with the adjoining sides.

Notice that the three sectors of the lower half are marked "co-" In other words the compliment of these angles is used.

The two formulae are;

Sin(Mid Part) = Tan (Adjacent Parts)
e.g. Sin(c) = Tan(co-B) x Tan(b)
or Sin(c) =Cot(B) x Tan(b)

      And

Sin (Mid Part) = Cos(Opposite Parts)
e.g. Sin(c) = Cos(co-a) x Cos(co-C)
or Sin(c) = Sin(a) x Sin(C)

Example using Napier's Rules

Assume that the True Altitude is 0° therefore the Zenith Distance is 90°.
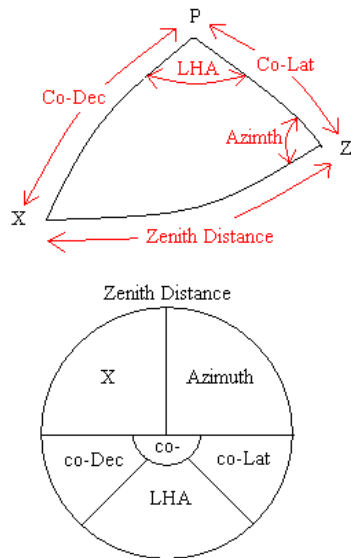
If co-Dec is the mid-part then the two opposites are Azimuth and co-Lat.

Sin (Mid Part) = Cos(Opposites)
Sin(co-Dec) = Cos(Azimuth) x Cos(co-Lat)

Cos(Dec) = Cos(Azimuth) x Sin(Lat)

Cos(Azimuth) = Sin(Dec)/ Sin(Lat)

The Amplitude of a body is measured from East or West rather than North. In other words Amplitude = 90° - Azimuth = co-Azimuth.

Sin(Amplitude) = Sin(Dec)/ Sin(Lat)

This gives the formula that many readers will be familiar with of
Sin(Amplitude) = Sin(Dec) x Sec(Lat)