

# **New Brunswick Mathematics Grade 3**

# **CURRICULUM**

*New Brunswick Department of Education  
Educational Programs and Services*

---

## Acknowledgements

The Department of Education of New Brunswick gratefully acknowledges the contributions of the following groups and individuals toward the development of the *New Brunswick Grade 3 Mathematics Curriculum Guide*:

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education: *The Common Curriculum Framework for K-9 Mathematics*, May 2006. Reproduced (and/or adapted) by permission. All rights reserved.
- Alberta Education (Department of Education)
- Newfoundland and Labrador Department of Education
- Prince Edward Island Department of Education
- The Elementary Mathematics Curriculum Development Advisory Committee
- The Grade 3 Curriculum Development Team:
  - Diana Bourque, School District 2
  - Kathy Goudreau, School District 8
  - Sylvia McConkey, School District 10
  - Karen Mifflin-Pierce, School District 14
  - Michelle Trynor, School District 17
  - Kathy Wallace, School District 6
- Cathy Martin, Learning Specialist, K-8 Mathematics and Science, NB Department of Education
- Mathematics Learning Specialists, Numeracy Leads, and mathematics teachers of New Brunswick who provided invaluable input and feedback throughout the development and implementation of this document.

---

## Table of Contents

### Curriculum Overview for K-9 Mathematics

<b>Background and Rationale</b>	<b>2</b>
<b>Beliefs about Students and Mathematics Learning</b>	<b>2</b>
Goals for Mathematically Literate Students	3
Opportunities for Success	3
Diverse Cultural Perspectives	4
Adapting to the Needs of All Learners	4
Connections Across the Curriculum	4
<b>Assessment</b>	<b>5</b>
<b>Conceptual Framework for K – 9 Mathematics</b>	<b>6</b>
<b>Mathematical Processes</b>	<b>7</b>
Communication	7
Connections	7
Reasoning	7
Mental Mathematics and Estimation	8
Problem Solving	8
Technology	9
Visualization	9
<b>Nature of Mathematics</b>	<b>10</b>
Change	10
Constancy	10
Number Sense	10
Relationships	10
Patterns	11
Spatial Sense	11
Uncertainty	11
<b>Structure of the Mathematics Curriculum</b>	<b>12</b>
<b>Curriculum Document Format</b>	<b>13</b>
<b>Specific Curriculum Outcomes</b>	<b>14</b>
Number	14
Patterns and Relations	62
Shape and Space	70
Statistics and Probability	94
<b>Appendix A: Glossary of Models</b>	<b>102</b>
<b>Appendix B: List of Grade 3 Specific Curriculum Outcomes</b>	<b>109</b>
<b>Appendix C: References</b>	<b>110</b>

## BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, the Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

## BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are

engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

### **GOALS FOR MATHEMATICALLY LITERATE STUDENTS**

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

### **OPPORTUNITIES FOR SUCCESS**

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices. Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals. Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

**DIVERSE CULTURAL PERSPECTIVES**

Students attend schools in a variety of settings including urban, rural and isolated communities. Teachers need to understand the diversity of cultures and experiences of all students.

Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components. Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. It is important to note that these general instructional strategies may not apply to all students.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

**ADAPTING TO THE NEEDS OF ALL LEARNERS**

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

**CONNECTIONS ACROSS THE CURRICULUM**

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

## ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & Wiliam, 1998; OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

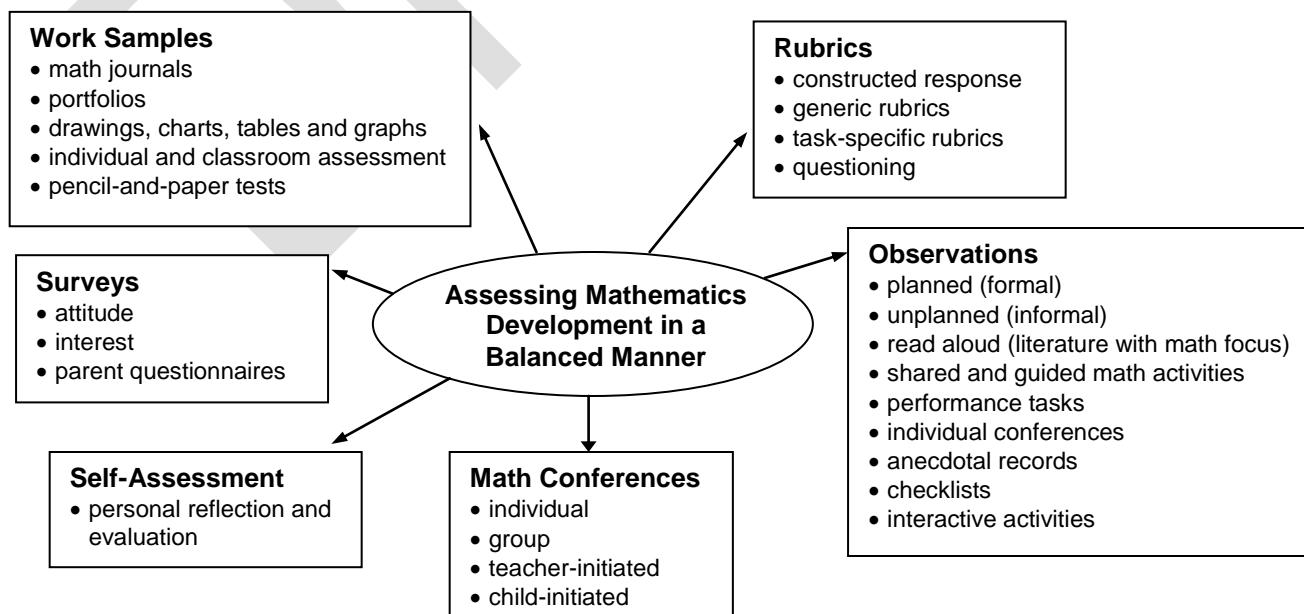
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment*, or assessment of learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



## CONCEPTUAL FRAMEWORK FOR K – 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

GRADE		K	1	2	3	4	5	6	7	8	9
STRAND											
<b>Number</b>											
<b>Patterns and Relations</b> <ul style="list-style-type: none"><li>• Patterns</li><li>• Variables and Equations</li></ul>											
<b>Shape and Space</b> <ul style="list-style-type: none"><li>• Measurement</li><li>• 3-D Objects and 2-D shapes</li><li>• Transformations</li></ul>											
<b>Statistics and Probability</b> <ul style="list-style-type: none"><li>• Data Analysis</li><li>• Chance and Uncertainty</li></ul>											
<b>MATHEMATICAL PROCESSES – COMMUNICATION, CONNECTIONS, REASONING, MENTAL MATHEMATICS AND ESTIMATION, PROBLEM SOLVING, TECHNOLOGY, VISUALIZATION</b>											

**NATURE OF MATHEMATICS**

Change  
Constancy  
Number Sense  
Patterns  
Relationships  
Spatial Sense  
Uncertainty

### NATURE OF MATHEMATICS

Change  
 Constancy  
 Number Sense  
 Patterns  
 Relationships  
 Spatial Sense  
 Uncertainty

### INSTRUCTIONAL FOCUS

The New Brunswick Curriculum is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.
- There is a greater emphasis on mastery of specific curriculum outcomes.

The mathematics curriculum describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.



## MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

### Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

### Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

*“Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p. 5).*

### Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these

generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

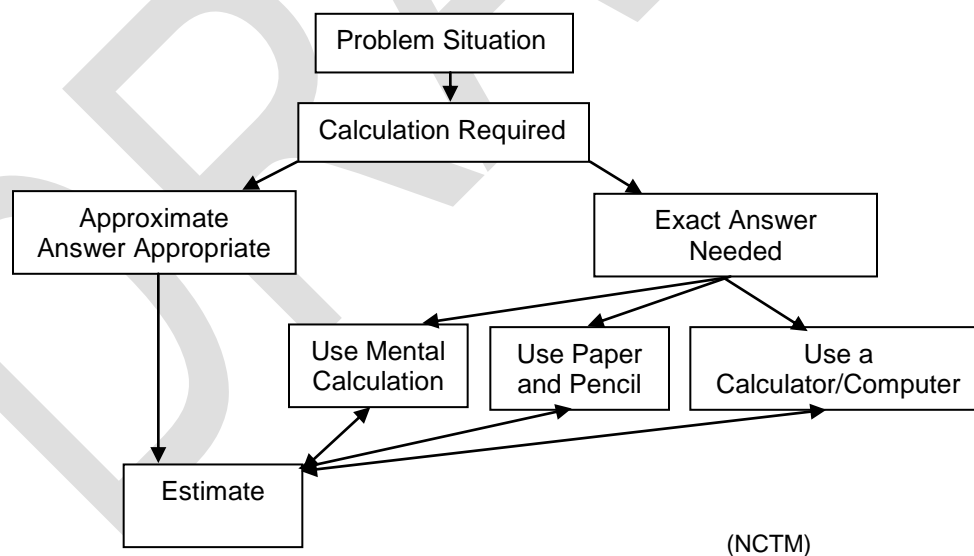
### **Mental Mathematics and Estimation [ME]**

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*” (Rubenstein, 2001). Mental mathematics “*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below.



### **Problem Solving [PS]**

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “*How would you...?*” or “*How could you...?*” the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

### **Technology [T]**

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

### **Visualization [V]**

Visualization “*involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world*” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

## NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change, constancy, number sense, relationships, patterns, spatial sense and uncertainty.**

### Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

### Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is  $180^\circ$
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

### Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

### Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

**Patterns**

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

**Spatial Sense**

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four.

**Uncertainty**

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

## STRUCTURE

### STRANDS

The learning outcomes in the New Brunswick Curriculum are organized into four strands across the grades, K–9. Strands are further subdivided into sub-strands which are the general curriculum outcomes.

### OUTCOMES AND ACHIEVEMENT INDICATORS

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

**General Curriculum Outcomes (GCO)** are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

**Specific Curriculum Outcomes (SCO)** are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

**Achievement Indicators** are one example of a representative list of the depth, breadth and expectations for the outcome. Achievement indicators are pedagogy and context free.

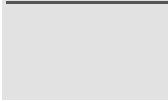
Strand	General Curriculum Outcome (GCO)
<b>Number (N)</b>	<b>Number:</b> Develop number sense
<b>Patterns and Relations (PR)</b>	<b>Patterns:</b> Use patterns to describe the world and solve problems
	<b>Variables and Equations:</b> Represent algebraic expressions in multiple ways
<b>Shape and Space (SS)</b>	<b>Measurement:</b> Use direct and indirect measure to solve problems
	<b>3-D Objects and 2-D Shapes:</b> Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them
	<b>Transformations:</b> Describe and analyze position and motion of objects and shapes
<b>Statistics and Probability (SP)</b>	<b>Data Analysis:</b> Collect, display and analyze data to solve problems
	<b>Chance and Uncertainty:</b> Use experimental or theoretical probabilities to represent and solve problems involving uncertainty

## CURRICULUM FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The specific curriculum outcomes (SCOs) are presented on individual four-page spreads as illustrated below.

GCO:
SCO: (specific curriculum outcome and mathematical processes)
Key for mathematical processes
<b><u>Scope and Sequence</u></b>
Current Grade

<b><u>Elaboration</u></b>
<u>Guiding Questions</u>
(Describes the “big ideas” and what the students should learn this year in regards to this concept.)

Page 1

GCO:
SCO:
<b><u>Achievement Indicators</u></b>
<u>Guiding Questions</u>
(Describes what could be observed to determine whether students have met the specific outcome.)

Page 2

GCO:
SCO:
<b><u>Planning for Instruction</u></b>
<u>Guiding Questions</u>
<u>Choosing Instructional Strategies</u>
(Lists general strategies to assist in teaching this outcome.)
<u>Suggested Activities</u>
(Lists possible specific activities to assist students in learning this concept.)
<u>Possible Models</u>

Page 3

GCO:
SCO:
<b><u>Assessment Strategies</u></b>
<u>Guiding Questions</u>
(Overview of assessment)
<u>Whole Class/Group/Individual Assessment</u>
(Lists sample assessment tasks.)
<b><u>Follow-up on Assessment</u></b>
<u>Guiding Questions</u>

Page 4

SCO: N1: Say the number sequence forward and backward from 0 to 1000 by:

- 5s, 10s or 100s using any starting point
- 3s using starting points that are multiples of 3
- 4s using starting points that are multiples of 4
- 25s using starting points that are multiples of 25.

[C, CN, ME]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>N1</b> Say the number sequence from 0 to 100 by: <ul style="list-style-type: none"> <li>• 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively</li> <li>• 10s using starting points from 1 to 9</li> <li>• 2s starting from 1.</li> </ul>	<b>N1</b> Say the number sequence forward and backward from 0 to 1000 by: <ul style="list-style-type: none"> <li>• 5s, 10s, or 100s, using any starting point</li> <li>• 3s using starting points that are multiples of 3</li> <li>• 4s using starting points that are multiples of 4</li> <li>• 25s, using starting points that are multiples of 25.</li> </ul>	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students are continuing to develop an understanding of **number** and **counting**. A focus on skip counting in the early years helps students see the patterns in our place value system as well as prepare students for work with money (Small, 2008, p. 86). Skip counting by 2's, 3's, 4's and 25's, is a cornerstone for later multiplicative understanding.

Students will extend their experience with **counting patterns** to 1000. The development of number sense beyond 100 is often challenging for students. Extending the place value pattern beyond 100 is not necessarily evident for all students. It is, therefore, crucial to provide multiple opportunities to work with numbers bridging the decades through the hundreds (e.g., 98, **99, 100, 101**, 102, 103, 104, 105, 106, 107, 108, **109, 110, 111**...).

Students will develop the following **skip counting** skills:

- counting by 5s, 10s and 100s backwards and forwards using any starting point
- counting by 3s starting at a given multiple of 3
- counting by 4s starting at a given multiple of 4
- counting by 25s using starting points that are multiples of 25

Students will be able to identify and correct errors and omissions in a given skip counting sequence and identify and explain the skip counting pattern for a given number sequence. They will be able to count a collection of coins consisting of nickels, dimes, quarters and loonies using efficient strategies.



SCO: N1: Say the number sequence forward and backward from 0 to 1000 by:

- 5s, 10s or 100s using any starting point
- 3s using starting points that are multiples of 3
- 4s using starting points that are multiples of 4
- 25s using starting points that are multiples of 25.

[C, CN, ME]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Extend a given skip counting sequence by 5s, 10s or 100s, forward and backward, using a given starting point.
- Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3.
- Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4.
- Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25.
- Identify and correct errors and omissions in a given skip counting sequence.
- Determine the value of a given set of coins (nickels, dimes, quarters, loonies) by using skip counting.
- Identify and explain the skip counting pattern for a given number sequence.

SCO: N1: Say the number sequence forward and backward from 0 to 1000 by:

- 5s, 10s or 100s using any starting point
- 3s using starting points that are multiples of 3
- 4s using starting points that are multiples of 4
- 25s using starting points that are multiples of 25.

[C, CN, ME]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Give students frequent opportunities to count materials (large quantities) in a variety of ways.
- Highlight the numbers on a number line or hundred charts that occur when skip counting and have students describe the patterns they see. Ensure that the numbers extend beyond 100.
- Provide students with multiple experiences counting both forward and backward, with various starting points.
- Use the calculator constant feature (described below) to count by 3s, 4s, 5s, 10s, 25s, and 100s. Ask the students to predict what number will come next, before it appears on the display.

### **Suggested Activities**

- Provide students with a hundred chart and have them colour in the pattern for a given skip counting sequence.
- Provide students with many number patterns to encourage skip counting; for example:
  - 25, 50, \_\_, \_\_, 125, \_\_, \_\_
  - 652, \_\_, 452, \_\_, 252, \_\_
  - 95, 90, \_\_, \_\_, 75, \_\_, \_\_, \_\_
- Have students identify and correct the error in a given skip counting sequence, such as:
  - 12, 16, 21, 24, 28, 32
  - 27, 30, 33, 35, 39, 42
- Provide coins for the students. Ask: Can you use 6 coins to make \$1.00? Can you make a total of \$1.45 with only 6 coins? What are the coins? This activity could be extended to use bills.
- Provide students with play coins. Tell them that you have, for example, 5 coins in your hand that total 81 cents. Ask: What coins am I holding? (This is a problem situation and may require time.)
- Have students count beans in a jar. Ask them how they grouped the beans (e.g., by 2s, 5s, 10s) for ease of counting.
- Use the constant (repeat) function (press 0, +, 25, =, =, =,...) on the calculator to skip count to a target number. For example, if you start at 0 and want to end at 400, by which number(s) could you skip count? (4, 5, 10, 25, 100) What if you started at a different point? What if you wanted to end at a different point?
- Play "What's in the Can?". Tell the student that you are going to drop nickels (or dimes or quarters) into a can. Have the student listen as the coins drop and count to find the total. As an extension, tell the student that there is, for example, 45 cents in the can. Tell him/her that you are going to add nickels (or dimes) and ask him/her to keep track to find the total.

**Possible Models:** number line, hundred chart, calculator, various objects for counting (e.g. beans, counters), money (coins and bills)

SCO: N1: Say the number sequence forward and backward from 0 to 1000 by:

- 5s, 10s or 100s using any starting point
- 3s using starting points that are multiples of 3
- 4s using starting points that are multiples of 4
- 25s using starting points that are multiples of 25.

[C, CN, ME]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Begin to count, “25, 50, 75, 100, 125, 150.” Ask the student to continue to count to 500 by 25s. Be sure to use many examples including numbers greater than 100.
- Provide students with a number of beans (e.g., 60). Ask the student to separate the beans from the pile as he/she counts them by 3s, then by 4s.
- Ask: Why do you say fewer numbers when counting to 100 by 10s than when counting by 5s?
- Tell the student to decide which starting point, 6 or 7, is easier when counting by 3s. Ask the student to explain his/her choice.
- Write and say, “25, 50, 60, 65, 70.” Ask: What coins am I counting?
- Give the student 4 quarters, 3 dimes, 2 nickels and 6 pennies. Ask him/her to count the coins to find if an item costing \$2 can be bought.
- Have students count by 5s until they reach sixty. Ask: What other numbers can you count by and still land on 60?
- Have students skip count following directions, such as:
  - Start at 95 and count forward by 5s to 140.
  - Start at 349 and count by 100s without going over 1000 but get as close as you can.
  - Start at 450 and count backward by 25s until you reach 250.
  - Start at 635 and count forward by 10s to 725.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: <b>N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.</b> [C, CN, V]			
[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>N4:</b> Represent and describe numbers to 100, concretely, pictorially and symbolically.	<b>N2</b> Represent and describe numbers to 1000, concretely, pictorially and symbolically.	<b>N1</b> Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

When representing numbers students must be provided with a variety of opportunities to use concrete materials such as base ten blocks, number lines, hundred frames, money and place value charts. It is important to provide practice modelling numbers containing zeros. Interactive whiteboards are excellent for modelling pictorially, however are not considered a concrete model.

Students who have a deep understanding of numbers up to 1000 will be able to **partition numbers** in a variety of ways. For example: students will know that 15 tens is equivalent to 150. They will also know that they can build 15 tens in a variety of ways (e.g., 8 tens and 7 more tens, 10 tens and 5 more tens, 9 tens and 6 tens, etc.).

Students will also represent and describe numbers as **expressions**. An expression is a mathematical phrase made up of numbers connected by an operation. For example,  $60 + 40$  and  $150 - 50$  are both expressions to represent 100.

When reading numbers the word “**and**” refers to the decimal. The number 205 is read “two hundred five”. Students will also need to be able to write the number words for the **multiples of ten** and **multiples of a hundred**. While correct spelling of these words should be encouraged, it is not the goal of this outcome. It should also be noted that when writing 4-digit numbers symbolically, there is no space or comma between the thousands and hundreds place (e.g., 1000 not 1 000, nor 1,000). Although the focus in grade 3 is numbers up to 1000, students may explore numbers with five or more digits and should note that these numbers require a space between the thousands and hundreds place (e.g., 10 000).

Students must be able to record numbers heard, read written numbers, and write numbers in words. They must be able to translate a number from the written to the oral. Representing written numbers symbolically is also an expectation (e.g., write one thousand two as 1002).

SCO: **N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.**  
[C, CN, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Read a given three-digit numeral without using the word “and,” e.g., 321 is three hundred twenty one, NOT three hundred AND twenty one.
- Read a given number word (0 to 1000).
- Represent a given number as an expression, e.g.,  $300 - 50$  for 250 or  $230 + 20$
- Represent a given number using manipulatives, such as base ten materials, in multiple ways.
- Represent a given number pictorially.
- Write number words for given multiples of ten to 90.
- Write number words for given multiples of 100 to 900.

SCO: **N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.**  
[C, CN, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Ensure students have many opportunities to use a variety of concrete materials.
- Have a math word wall available for students to assist with the correct spelling of number words.
- Provide students with frequent opportunities to represent numbers using words, pictures and symbols.
- Have students create different expressions for the same numbers.

### **Suggested Activities**

- Have students create a “thousand” chart using ten hundred charts.
- Have students find numbers up to 1000 from different sources, such as newspapers, the Internet, signs, bulletin boards, etc., and read and model pictorially.
- Have students rename a number less than 1000 as the sum of other numbers.
- Have students place benchmark numbers on a number line, for example 250, 500, 750.
- Have students create and solve number riddles such as “I have written a secret number between 600 and 800. It is an odd number. What might it be?”
- Use literature to provide a context for the number 1000. For example, “*How Much, How Many, How Far, How Heavy, How Long, How Tall, is 1000?*” by Helen Nolan.
- Ask students to record a series of numbers that are read to them. Include examples such as “ten less than 652” and numbers that contain a zero.
- Ask students to represent the number of students in their school as many different ways as they can.
- Model a number using base ten materials in an unconventional order and have students say the number.

**Possible Models:** hundred frame, base ten blocks, number lines (including walk-on and open number line), Rekenrek®, place value chart, hundred chart, counters, money, cards with digits, place value dice

SCO: **N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.**  
[C, CN, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask students to rearrange the digits 1, 4 and 5 to create the number that is closest to 500.
- Have students show a number less than 1000 in 2 ways.
- Provide students with a number line (0-1000) and have them place the benchmarks 250, 500, 750.
- Ask students, "How many numbers can you make using 3 digits (for example 2, 3 and 4) if you only use each digit once in each number".
- Ask students to record numbers read orally, both symbolically and with words, making sure to include numbers that have a zero.
- Have students work with a partner and record a number with words, exchange with their partner, record that number symbolically then say that number to their partner.
- Ask students, "Which of the expressions in the box represents 360?"

200+160	380-30	400-40
260+75+25	357+4	260+100

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N3: Compare and order numbers to 1000.**  
[CN, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

<u>Grade Two</u>	<u>Grade Three</u>	<u>Grade Four</u>
<b>N5</b> Compare and order numbers up to 100.	<b>N3</b> Compare and order numbers to 1000.	<b>N2</b> Compare and order numbers to 10 000.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should be able to **compare** two or more numbers, each less than 1000, to determine relative sizes. Include situations in which numbers are located on hundreds charts and/or a number line. Emphasis remains on using words (**greater than / less than**) rather than symbols ( $<$ ,  $>$ ) to compare numbers. Students should be able to name numbers greater than, less than, or between given numbers. Given a set of numbers, students should be able to place the numbers in **ascending** or **descending order**, and verify the result using a hundred chart or by drawing number line. Students need to be able to identify missing numbers and errors in hundred charts and number lines that go beyond a hundred.

An understanding of **place value** (explored in greater depth in N5) and number patterns is essential for students to compare and order numbers. For example, to compare 667 and 607, students should notice that both numbers have 6 hundreds, but that the 667 is greater than 607 because it has more tens in the tens place. The numbers could also be compared by considering their relative position in the counting sequence: 667 comes after 607, so 667 is greater than 607.



SCO: N3: Compare and order numbers to 1000.  
[CN, R, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Place a given set of numbers in ascending or descending order and verify the result by using a hundred chart, e.g., a one hundred chart, a two hundred chart, a three hundred chart, a number line or by making references to place value.
- Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
- Identify errors in a given ordered sequence.
- Identify missing numbers in isolated parts of a given hundred chart (include charts beyond 100).
- Identify errors in a given hundred chart (include charts beyond 100).

SCO: **N3: Compare and order numbers to 1000.**  
[CN, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skill.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Have students plot numbers on an open number line (horizontal and vertical) to show their relative positions. Have students share their thinking.
- Give students some prepared cards with 3-digit numbers on them. Ask him/her to order the number cards in ascending and descending order.
- Show the students two numbers (e.g. 501 and 398) and ask which is greater. Have the students explain their answers. Encourage them to use a variety of models in their explanations.
- Give students many opportunities to explore the magnitude of similar digits. Example: How are the digits in 777 similar? How are they different?

### **Suggested Activities**

- Prepare a deck of number cards that contain both 2- and 3-digit numbers. Have the students deal all the cards face down to the players. Have each player turn the top card over; the one who has the greater (greatest) number "wins" both or all the cards in play. The winner is the one who has collected the most cards when all the cards are turned over.
- Provide a set of cards (10 to 15) with each card having a 2- or 3-digit number on it. Ask the student to order the number cards from least to greatest and to explain how he/she determined the relative number size.
- Have students use a prepared deck of 40 number cards (4 sets of 0 to 9). Have the student select three or four of the cards and arrange them to make the greatest possible number and the least possible number. Ask the student to model these numbers.
- As a class activity, repeatedly roll a die and have the students fill in the digits, one at a time, on a place-value chart. Alternate by having them try to make the greatest number or the least number. Model the task by placing your digits on an overhead chart or interactive whiteboard. Regularly ask questions such as, "What do you need? What don't you want me to roll?"
- Play "Guess My Number", in partners, with numbers less than 1000. Use greater than, less than, is equal to in the response (e.g., "Is your number 489?" "No. My number is greater than that."). Continue the game until the number is guessed and then change roles and have the other partner guess.
- Give each of two students a spinner with 10 numbers that are in the hundreds. Have them spin at the same time. The one who spins the higher number gets a token. The students play until someone has gathered 10 tokens. Select numbers according to the students' level of understanding. (Numbers could be: 345, 354, 381, 309, 608, 680, 853, 835, 903, and 930.)

**Possible Models:** number lines, open number lines, hundred charts (100s, 200s, 300s, etc.), base ten blocks, spinners, dice (including place value dice), number cards, calculator, Rekenrek<sup>®</sup>

SCO: N3: Compare and order numbers to 1000.  
[CN, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask the student to use models to show why 243 is less than 324.
- Ask the student to find a number between 312 and 387 that can be represented using 8 base ten blocks.
- Ask: What do you do to compare the size of two numbers?
- Ask: Why are there more numbers greater than 123 than less than 123?
- Ask: If  $\square 39$  is greater than 422, what do you know about  $\square$  (the missing digit)?  
If  $\square 39$  is greater than  $\square 87$ , what do you know about the missing digits?
- Ask the student to write a number that is:
  - greater than 165 but less than 200      - a little less than 300
  - between 463 and 474                      - greater than 348 but less than 360, etc.
- Ask the student to explain why a 3-digit whole number is always greater than a 2-digit whole number. Provide students with specific examples to use in their explanation (e.g., 560 and 56)
- Ask the student to select five numbers between 600 and 630, and to write them in increasing order.
- Provide number lines for students and ask them to estimate where some numbers might lie, and to give their reasoning; for example, 400.



- Ask students to make as many numbers as they can using the digits 2, 3 and 4, but using each digit only once. Have them list them in order from least to greatest or greatest to least.
- Provide students with an ordered sequence of numbers that contain an error. Have students identify and correct the error (e.g., 123, 132, 213, 231, 321, 312).

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N4: Estimate quantities less than 1000 using referents.**  
[ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

<u>Grade Two</u>	<u>Grade Three</u>	<u>Grade Four</u>
<b>N6:</b> Estimate quantities to 100 using referents.	<b>N4</b> Estimate quantities less than 1000 using referents.	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students use **reasoning** skills to estimate a total using a **visual referent**. A referent can be created for any known quantity to help determine an unknown quantity that is larger or smaller. Students may create a mental picture (**visualization**) of an amount and use that picture to estimate a total. This process of making connections between similar visuals will lead to greater proportional understanding and reasoning. It is essential that students develop referents in order to be effective at estimating.

Knowing how much 10 stars is, helps  
estimate the larger group of stars.

☆☆☆☆☆  
☆☆☆☆☆

☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆  
☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆  
☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆  
☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆  
☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆  
☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆  
☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆

Students must build on prior strategies developed in earlier grades involving ten to include a sense of one hundred. Using their knowledge of one hundred, students can then estimate larger quantities. For example, a bag of one hundred pennies may be used to determine how many pennies are in a larger pile, by estimating how many groups of 100 are in the pile.

Through the process of selecting and using referents, students will be able to justify a referent for determining an estimate for a given quantity. For example, when asked to determine the number of jellybeans in a jar, students select a useful visual referent to determine a reasonable estimate.

SCO: **N4: Estimate quantities less than 1000 using referents.**  
[ME, PS, R, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Estimate the number of groups of ten in a given quantity (using 10 as a referent) (known quantity).
- Estimate the number of groups of a hundred in a given quantity (using 100 as a referent).
- Estimate a given quantity by comparing it to a referent.
- Select an estimate for a given quantity by choosing among three possible choices.
- Select and justify a referent for determining an estimate for a given quantity.

SCO: N4: Estimate quantities less than 1000 using referents.  
[ME, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide students with many opportunities to count groups of 10 and 100 objects in order to develop a sense of these benchmarks. Students should discover that these quantities are not always the same size (e.g., 100 raisins versus 100 oranges).
- Estimate a given quantity by comparing it to a referent (known quantity).
- Estimate the number of groups of 10 and 100 in a given quantity using 10 and 100 as a referent.
- Select between three possible estimates for a given quantity and explain the choice.
- Use children's literature as a context for estimation such as *Counting on Frank* by Rod Clement and *Betcha!* By Stuart Murphy. Discuss how the characters in the stories used estimation.

### **Suggested Activities**

- Show 100 paperclips as a visual referent for the students. Next display a larger group of paper clips. Ask students to estimate how many paper clips there are. Have students explain their thinking.
- Do activity 2.13 (Van de Walle & Lovin, vol. 2, 2006, p. 50). Ask students how many:
  - Candy bars would cover the floor of your room
  - Steps an ant would take to walk around the school building
  - Grains of rice would fill a cup
  - Quarters could be stacked in one stack floor to ceiling
  - Pennies can be laid side by side down an entire room or hallway
  - Pieces of notebook paper would cover the gym floor
  - Minutes you have lived
- Collect some type of object as a class, with the object of reaching 1000 (stickers, pennies, marbles, toy cars, rocks, leaves, buttons, etc.).
- Show a quantity of objects such as linking cubes. Ask: "If this is equal to 10 linking cubes, what might 143 linking cubes look like?"
- Say; 100 counters takes up this much space. How much space would 783 counters take up if you placed them flat on the table? If they were in a milk jug? Explain your thinking?

**Possible Models:** variety of containers and objects (baggies, buckets, beads, marbles, cubes, paper clips, linking cubes, 500 sheet package of photocopy paper, etc.)

SCO: **N4: Estimate quantities less than 1000 using referents.**  
[ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Show students a group of items and ask them to choose between three given estimates. Have students explain their reasoning.
- Place a pile of objects on a desk (e.g., paper clips, linking cubes, base ten units, buttons). Ask students to estimate the number. Observe and interview students to determine if they are using a referent. Guiding questions should include, "How did you pick that number?"
- Ask students to describe a strategy used to find an estimate.
- Show students 25 buttons; all buttons touching sides. Say: Susan said; "643 buttons will fit on top of a desk." Do you agree or disagree? Explain.
- Place a pile of objects on a desk (e.g., paper clips, linking cubes, base ten blocks, etc.). Ask students to estimate the number of objects. Observe how students approach this task and ask questions to determine if they are using a referent. Guiding questions should include, "How did

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.**

[C, CN, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

<b>Grade Two</b>	<b>Grade Three</b>	<b>Grade Four</b>
<b>N7</b> Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	<b>N5</b> Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.	<b>N1</b> Represent and describe whole numbers to 10 000, pictorially and symbolically.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students must extend their knowledge of **unitizing** (the concept that ten ones equals one ten) to develop an understanding that:

- 10 tens is one hundred;
- 10 hundreds is one thousand;
- 100 tens is one thousand.

Students must have a deep understanding of numbers up to 1000 and be able to rename numbers in a variety of ways. For example, 842 is the same as 84 tens and 2 ones or 8 hundreds and 42 ones or 8 hundreds, 4 tens and 2 ones. They will also understand that the position of a digit determines its value.

Students will need many opportunities to explore the value of the digits in a number using proportional and non-proportional materials. Proper introduction and use of these materials will move student thinking from low-level counting strategies to a deeper understanding of numbers. It is important to understand that students must construct their own understanding of number; the math is not in the manipulative. This is best accomplished through a variety of materials and through the introduction of those materials as a representation of student thinking. **Proportional** concrete models include base ten blocks, Rekenreks®, ten frames, and hundred frames as these models show the magnitude of the number. For example, a base ten rod is ten times the size of a unit. **Non-proportional** models include money and an abacus. For example, a loonie is not ten times the size of a dime, even though it is ten times greater in value.

It is important that this stage not be rushed. Many problems that children later encounter with place value and calculations stem from inadequate attention to early



SCO: **N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.**  
[C, CN, R, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Record, in more than one way, the number represented by given proportional and non-proportional concrete materials.
- Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent, e.g., 351 can be represented as three 100s, five 10s and one 1s, or two 100s, fifteen 10s and one 1s, or three 100s, four 10s and eleven 1s.
- Explain, and show with counters, the meaning of each digit for a given 3-digit numeral with all digits the same, e.g., for the numeral 222, the first digit represents two hundreds (two hundred counters) the second digit represents two tens (twenty counters) and the third digit represents two ones (two counters).
- Record a number represented by base ten blocks arranged in a non-conventional format

SCO: **N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.**  
[C, CN, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

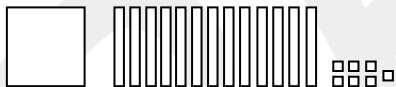
### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Ensure students have opportunities to use both proportional and non-proportional concrete materials.
- Have students represent the same number with different partitions. For example, 254 can be represented using 2 hundred dollar bills and 54 loonies or 1 hundred dollar bill, 15 ten dollar bills and 4 loonies.
- Provide multiple opportunities for students to show they understand that the position of a digit within a number determines its value.

### **Suggested Activities**

- Ask student to record the number that is made up of 15 tens and 15 ones.
- Ask students to record the value of the base ten blocks shown:



- Ask students to build a model or draw a picture using base ten blocks. Ask: What is the value of the drawing or model?
- Have students create non-proportional concrete models and explain their values.
- Ask students to write as many numbers as possible with an 8 in the tens place.
- Give each group of students 12 base ten rods and 16 units. Have them record the number the blocks represent.
- Ask the students to model 507 in a variety of ways.
- Have students model numbers such as 421 and 139. Discuss which number has more tens, and how they know. Students should recognize that 421 has more tens, although has a smaller digit in the tens place.
- Ask the students to enter a certain number on a calculator (e.g., 235). Ask: How can you, without clearing the calculator, make the number 255? (35? 205? 261?).
- Ask students to record a specific 4-digit number, with all digits different. Ask students to remove the value of one of the digits or the value of one of the places with only one operation. For example: to remove the value of the 2 from the number 1234, the student would need to subtract 200.

**Possible Models:** hundred frame, base ten blocks, number lines, Rekenrek®, place value chart, hundred chart, counters, money, digit cards

SCO: N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.

[C, CN, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.



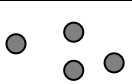
### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask the student to describe 324 without using the word “hundred” (e.g., thirty-two tens, four).
- Ask the student to show that 132 is 13 tens and 2 ones.
- Tell students they have 780 dollars. What bills could they have? (can use denominations of \$5, \$10, \$20, \$50, \$100)
- Ask the student to describe 1000 in as many ways as they can.
- Ask the student to explain using words, numbers, and/or pictures how they know that 1000 is the same as 100 tens.
- Tell students that pencils can be bought in packages of 1, 10, and 100. Show as many ways as you can to buy 132 pencils.
- Ask how 480 and 680 are the same and how they are different (focus should be on place value).
- Ask students what number is represented in the chart below.

hundreds	tens	ones
		

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: <b>N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:</b> <ul style="list-style-type: none"> <li>• adding from left to right</li> <li>• taking one addend to the nearest multiple of ten and then compensating</li> <li>• using doubles.</li> </ul> SCO: <b>N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:</b> <ul style="list-style-type: none"> <li>• taking the subtrahend to the nearest multiple of ten and then compensating</li> <li>• thinking of addition</li> <li>• using doubles.</li> </ul> [C, ME, PS, R, V]			
[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>N10</b> Apply mental mathematics strategies, such as: using doubles; making 10; one more, one less; two more, two less; building on a known double; addition for subtraction to determine basic addition facts to 18 and related subtraction facts.	<b>N6</b> Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: <ul style="list-style-type: none"> <li>• adding from left to right</li> <li>• taking one addend to the nearest multiple of ten and then compensating</li> <li>• using doubles.</li> </ul> <b>N7</b> Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: <ul style="list-style-type: none"> <li>• taking the subtrahend to the nearest multiple of ten and then compensating</li> <li>• thinking of addition</li> <li>• using doubles.</li> </ul>	<b>N3</b> Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: using personal strategies for adding and subtracting; estimating sums and differences; solving problems involving addition and subtraction.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

When a problem requires an exact answer, students should first determine if they are able to calculate it mentally. This should become an automatic response. Situations must be regularly provided to ensure that students have sufficient practice with **mental math strategies** and that they use their skills as required. Using mental math will focus a student on the relationships between numbers and operations rather than relying on completing a traditional algorithm. For example, students might solve  $49 + 99$  mentally by adding 100 to 49, then subtracting 1. This method involves using benchmark numbers then compensating by adding or subtracting, whichever operation is necessary.

Most mental math strategies provide a more efficient **computation** method as opposed to pencil and paper. When **decomposing** (breaking the number apart) one of the **addends** (numbers being added) or **subtrahends** (numbers that are being subtracted), students need to consider which decomposition of a number is easiest for them to use. Students should be exposed to the **horizontal form** of an equation to encourage the use of mental math.

Students will develop, apply and describe mental math strategies to add or subtract two 2-digit numbers. These include, but are not limited to those listed in this outcome. Strategies explored should also include students' **personal strategies** as well as other mental math strategies suggested in other grades.

**Automaticity** (see N10) with the basic facts to 18 is essential for success with mental computations.

**SCO: N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:**

- adding from left to right
- taking one addend to the nearest multiple of ten and then compensating
- using doubles.

**SCO: N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:**

- taking the subtrahend to the nearest multiple of ten and then compensating
- thinking of addition
- using doubles.

[C, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

#### **N6**

- Add two given 2-digit numbers using a mental mathematics strategy and explain or illustrate the strategy.
- Explain how to use the “adding from left to right” strategy, e.g., to determine the sum of  $23 + 46$ , think  $20 + 40$  and  $3 + 6$ .
- Explain how to use the “taking one addend to the nearest multiple of ten” strategy, e.g., to determine the sum of  $28 + 47$ , think  $30 + 47 - 2$  or  $50 + 28 - 3$ .
- Explain how to use the “using doubles” strategy, e.g., to determine the sum of  $24 + 26$ , think  $25 + 25$ ; to determine the sum of  $25 + 26$ , think  $25 + 25 + 1$  or doubles plus 1.
- Apply a mental mathematics strategy for adding two given 2-digit numerals.

#### **N7**

- Subtract two given 2-digit numerals using a mental mathematics strategy and explain or model the strategy used.
- Explain how to use the “taking the subtrahend to the nearest multiple of ten” and then compensating strategy, e.g., to determine the difference of  $48 - 19$ , think  $48 - 20 + 1$ .
- Explain how to use the “thinking of addition” strategy, e.g., to determine the difference of  $62 - 45$ , a student might think  $45 + 5$ , then  $50 + 12$  and then  $5 + 12$ . Using an open number line is helpful with this strategy.
- Explain how to use the “using doubles” strategy, e.g., to determine the difference of  $24 - 12$ , think  $12 + 12$ .
- Apply a mental mathematics strategy for subtracting two given 2-digit numerals.

SCO: N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:

- adding from left to right
- taking one addend to the nearest multiple of ten and then compensating
- using doubles.

SCO: N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:

- taking the subtrahend to the nearest multiple of ten and then compensating
- thinking of addition
- using doubles.

[C, ME, PS, R, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

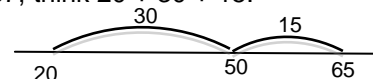
### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students recognize that mental math is an approach that they should use daily whenever they are required to calculate.
- Require that students know addition and subtraction facts to 18 with automaticity as described in outcome N10.
- Review “making a ten” with students. For example, for  $28 + 4$ , one might think 28 and 2 make 30, and 2 more is 32. This can be extended to 2-digit numbers. For example, for  $38 + 24$ , 38 plus 20 is 58, and 2 more is 60, plus 2 is 62.
- Have students add two 2-digit numbers using the front-end approach, and explain their thinking. For example,  $24 + 31$ . Students might say, “20 and 30 makes 50. 4, 55 - the answer is 55.”
- Relate addition to subtraction regularly, so students are better able to use this understanding to solve addition and subtraction problems and to check their work. Use missing addends to encourage this.
- Have students explain thinking using number lines. For example, to solve  $28 + 37$ , think  $20 + 30 + 15$ .

### Suggested Activities:

- Ask students to circle the computation questions, from a group of provided questions that they could solve mentally and describe the strategy they would use.
- Have students explain how they would use a calculator to solve 2-digit addition questions, such as  $34 + \square = 69$  or  $39 + \square = 64$ .
- Have the student make a list of calculations involving 2-digit numbers which would be quicker to do mentally than on paper or with a calculator.
- Present calculations, such as the following, orally (or on an overhead), and ask the student to write only the answer. Allow only a few seconds for each question (e.g.,  $300 + 600$ ,  $200 - 40$ ,  $200 + 80 + 30$ ,  $220 - 40$ ).
- Ask: How many different ways can you add 19 to 63 in your head?
- Ask: How many different ways can you subtract 19 from 63 in your head? Which way was easiest?
- Have the student list the doubles facts that might help him/her solve expressions such as  $28 + 29$  and  $40 - 20$  or  $57 - 29$ .



**Possible Models:** calculator, number cards, Rekenrek® (100 frame)

SCO: N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:

- adding from left to right
- taking one addend to the nearest multiple of ten and then compensating
- using doubles.

SCO: N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:

- taking the subtrahend to the nearest multiple of ten and then compensating
- thinking of addition
- using doubles.

[C, ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask the student to add mentally as you draw numbers from a bag, and to stop you when the sum has passed 40. Have the student explain their thinking process to reach the sum of 40 or more.
- Tell the student that to subtract 7 from 51, Jon said that he would rather subtract 6 from 50. Ask him/her if this works and why.
- Ask the student to describe a strategy for solving  $48 - 9$  (or  $76 + 11$ ) mentally using models, numbers, words, or pictures.
- Give the student this problem:  $62 + 63$  and say: Dave said he could use the doubles plus/minus one strategy to solve this problem. Explain Dave's thinking.
- Have students explain what is wrong with Lisa's method to solve  $45 - 26$ . "Lisa said,  $45 - 25 = 20$  and  $20 + 1 = 21$ . The answer is 21."
- Ask students how to solve  $59 + 13$  mentally and explain their thinking.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*



**SCO: N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**

[C, ME, PS, R]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
	<b>N8</b> Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

**Estimation** strategies in Grade 3 focus on predicting sums and differences with 2-digit numerals. This skill will be extended in later grades to numbers with more than two digits. Estimation is a skill that should be part of every problem solving process and these problems should be in a relevant context.

It is important that students understand that the problem solving situation will dictate how accurate the estimate needs to be. Estimation assists students in predicting answers and checking their calculations. In some situations, an estimate is all that is required. When using estimation to check calculations, students should ask themselves, “Does my answer make sense?” Teachers can help students develop this skill by frequently asking questions such as:

- Is your answer reasonable?
- How do you know?

Estimation is a mental activity that becomes more precise with practice. Regular attention to estimation activities and the sharing of strategies is necessary and promotes the use of mental math.

Students will estimate the solution for given story problems involving sums and differences of two 2-digit numbers. Estimation strategies include: **rounding** to the nearest multiple of ten or 100, estimating using **compatible numbers**, left to right or **front-end** method, or a combination of strategies. It should be noted that it can be just as easy and appropriate to get the actual answer as it is to estimate (e.g., when using the front-end method to estimate).

Students need to consider the numbers and the operation used to determine how to get the best estimate. For example, it would be best to round both numbers up or down to get the answer to  $84 - 27$  but to add  $84 + 27$  it would be better to round one up and one down. Exploring the proximity of the estimates to the exact answers and making comparisons to the numbers and operation used will enable students to become more efficient in their ability to estimate.



**SCO: N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**

[C, ME, PS, R]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Estimate the solution for a given story problem involving the sum of two 2-digit numerals, e.g., to estimate the sum of  $44 + 57$ , use  $40 + 60$ ; the sum is close to 100.
- Estimate the solution for a given story problem involving the difference of two 2-digit numerals, e.g., to estimate the difference of  $53 - 28$ , use  $50 - 30$ ; the difference is close to 20.

**SCO: N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**

[C, ME, PS, R]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

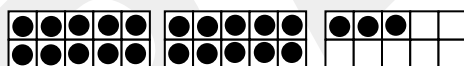
Consider the following strategies when planning lessons:

- Ensure students recognize that estimation should be used daily whenever they are required to solve a problem, make predictions, or check answers.
- Use number line models to solve problems, such as: Gas stations are imagined at each multiple of 10. To estimate  $83 + 78$ , you would place your "car" on 83 and decide that it is closer to the 80 gas station, and then add on 78 to get 158. It is not always necessary to round both numbers.
- Use left-to-right or front-end method: The following is an example in which this method makes sense.

$$\begin{array}{r} 38 \\ + 49 \\ \hline \end{array}$$

( $30 + 40$  is 70,  $8 + 9$  is close to 20, for an estimate of 90)

- Use ten frames for relatively small numbers. Displaying 23 on ten frames, for example, clearly shows that it is closer to 20 than to 30.



- Use base ten blocks or a hundreds chart to help students as they begin estimating. For example, if the student thinks of the estimate of a two-digit number as a set of rods only, he/she might see that 37 (3 rods and 7 ones) is closer to 4 rods than to 3 rods. Eventually, students should realize that estimating can easily be performed without the base ten blocks.
- Use quarters as a model to think of multiples of 25 as compatible numbers. Have students consider multiples of 25 when estimating numbers.

### **Suggested Activities**

- Have students use estimation in story problem situations such as:
  - Tali baked 49 cookies and Miranda baked 58. Do they have enough to feed the hundred parents coming to Math Night?
- Tell the student that to estimate the sum of 36 and 29, Jake said, "30 and 20 are 50, and 6 and 9 are more than 10, so the answer is more than 60, but less than 70." Ask him/her to describe Jake's thinking.
- Play "A Fast Ten" with students. Students turn over two playing cards (a deck of cards numbered 1-9 only) to build a two-digit number. The student who determines which multiple of ten that number is closest to gets the cards. This game could be extended to add or subtract estimates of two pairs of cards.
- Tell the student that two numbers have been added and the sum is about, but not exactly, 120. Ask the student to list four possible pairs of numbers that might have been added.

**Possible Models:** base ten blocks, number lines, ten frames, playing cards

**SCO: N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**

[C, ME, PS, R]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask him/her to identify situations in which an exact answer would be required and some in which an estimate is sufficient.
- Have the student explain two different ways to solve  $54 - 23$ .
- Tell the students that a number between 30 and 40 is added to a number between 40 and 50. Ask: What might be a good estimate for the answer? Why?
- Ask: Do you think that 48 might ever be used as an estimate for a sum or difference? Explain your answer.
- Ask the student to explain why a good estimate for a subtraction might be greater than the actual answer sometimes, but less other times. Encourage the student to use examples to help her/him explain.
- Tell the student that José thinks she will need about 33 hot dogs for her party. Ask: If the hot dogs come in packages of ten, how many packages should José buy? Explain your answer.
- Tell the student that Jason knew there were 35 members in his Karate Club and about 28 in the club in the neighbouring town. When asked to estimate the number of name tags to make for members of both clubs, Jason said, "I think I should make 65." Ask: How do you think Jason estimated? Was it a good estimate?
- Have the student toss three dice and list all of the six possible 2-digit numbers that can be formed. Ask the student to give an estimate of how much should be added to each number to get a sum of about 100. How much could be subtracted to get a difference of about 10?
- Tell the student that  $4\square + \square 8$  is about 70. Ask what digits might go in the blanks.
- Ask the student which of the following solutions is close to 150 and explain his/her thinking.  
 $92 + 37$        $69 + 82$        $77 + 87$
- Show the student the number of sports cards in James' collection.  
 Baseball: 48      Football: 19      Hockey: 84  
 Ask the student to estimate the total number of cards in the collection and to describe the strategy used.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

<b>SCO: N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:</b> <ul style="list-style-type: none"> <li>• using personal strategies for adding and subtracting with and without the support of manipulatives</li> <li>• creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.</li> </ul> [C, CN, ME, PS, R]			
<b>[C]</b> Communication <b>[T]</b> Technology	<b>[PS]</b> Problem Solving <b>[V]</b> Visualization	<b>[CN]</b> Connections <b>[R]</b> Reasoning	<b>[ME]</b> Mental Math and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>N9:</b> Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by: using personal strategies for adding and subtracting with and without the support of manipulatives; creating and solving problems that involve addition and subtraction; explaining that the order in which numbers are added does not affect the sum explaining that the order in which numbers are subtracted may affect the difference.	<b>N9:</b> Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by: <ul style="list-style-type: none"> <li>• using personal strategies for adding and subtracting with and without the support manipulatives;</li> <li>• creating and solving problems that involve addition and subtraction concretely, pictorially and symbolically</li> </ul>	<b>N3</b> Demonstrate an understanding of addition of numbers with sums to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by: using personal strategies for adding and subtracting; estimating sums and differences; solving problems involving addition and subtraction.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students are expected to apply what they know about the addition and subtraction of single digit numbers and the meanings of those operations to 2- and 3-digit numbers. It is at this point that it becomes important to develop **algorithms** or procedures to add and subtract greater numbers (Small, 2008, p. 162).

Students need to develop and use a variety of strategies, including **personal strategies** (both mental and paper-and-pencil) to solve problems. These alternatives are equally as valid as a traditional algorithm. Frequent use of the horizontal form encourages the use of a variety of strategies, instead of just the traditional algorithm. Regardless of which strategy students select and use, it is critical that the emphasis is on students understanding the mathematics and that the approach is accurate and efficient. The most appropriate strategy used may vary depending on the student and the numbers involved in the problem. Students should be able to explain their strategy and whether their solution makes sense.

It is important to ensure students examine each question before selecting a strategy. Connecting and encouraging students to use their conceptual understanding of 5 and 10 will make the addition and subtraction of “messy” numbers easier for students to understand. Students should be encouraged to use models to support and develop their understanding of addition and subtraction. From there, students move to pictorial then symbolic representations of both operations. If students develop a personal strategy involving **regrouping**, then the use of the words regroup or trade should be used to describe the action. Students should be encouraged to use estimation as described in outcome N8.

**SCO: N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:**

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.

[C, CN, ME, PS, R]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.
- Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.
- Create an addition or subtraction story problem for a given solution.
- Determine the sum of two given numbers using a personal strategy, e.g., for  $326 + 48$ , record  $300 + 60 + 14$ .
- Determine the difference of two given numbers using a personal strategy, e.g., for  $127 - 38$ , record  $38 + 2 + 80 + 7$  or  $127 - 20 - 10 - 8$ .
- Solve a given problem involving the sum or difference of two given numbers.

**SCO: N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:**

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.

[C, CN, ME, PS, R]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide examples of situations in which students will have to devise some method for regrouping. For example, tell them that one student found that there were 55 M & M's in one bag and 58 in another. Ask them to determine how many there were in the two bags. Ask them to model the question and to explain how they solved it.
- Create a sheet with 10 ten frames to represent 100. Provide each student with two sheets and have him/her show each of the two addends (e.g.,  $67 + 76$ ). This will help students visualize how the numbers could be combined (e.g.,  $60 + 70$ ,  $7 + 6$ ). Students may need to explore moving the amounts to be on the same sheet if they are having trouble finding the sum.
- Examine number patterns to help students understand the connection between addition and subtraction facts and two-digit plus two-digit and three-digit plus three-digit addition and subtraction. For example,  $6 + 7 = 13$  so  $60 + 70 = 130$  and  $600 + 700 = 1300$  and  $13$  tens minus  $6$  tens =  $7$  tens.

### **Suggested Activities**

- Tell students that Fran had 25 cents. She spent 16 cents. How much change does she get back? Encourage the students to explain how they go about solving the problem; for instance: "16 and 4 more are 20, plus 5 is 25. She gets 9 cents change." Or, "16 cents and 10 cents are 26 cents, so she gets only 9 cents back."
- Set up a "store" within the classroom and have the students take turns being the cashier. Model for them how to "count on" when making change.
- Tell students that Paul has 78 cents, and his brother has 92 cents. How much more money does Paul's brother have? (Providing a visual aid for the students—such as a number line—may be helpful.)
- Use the following digits to create two, 2-digit numbers that have the greatest possible sum: 2, 3, 4, 5. Use the same digits to create the greatest difference. The diagrams below may be helpful to some students.

$$\begin{array}{r} \square \square \\ + \square \square \\ \hline \end{array} \qquad \begin{array}{r} \square \square \\ - \square \square \\ \hline \end{array}$$

**Possible Models:** base ten blocks, money, place value mats, calculator, number line, hundred chart

SCO: **N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:**

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.

[C, CN, ME, PS, R]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Tell a student that someone told you that you do not have to learn to subtract if you know how to add. Ask: Do you agree? Why or why not?
- Observe the student as he/she adds 125 and 134 or subtracts 134 from 217 using base-ten materials.
- Provide the following addition or subtraction calculations for the student to complete. Ask the student to explain his/her strategy.
  - $38 + 97$
  - $98 - 44$
  - $400 - 255$
- Show the student a number of addition and subtraction questions, some of which require regrouping and some of which do not. Ask him/her to circle the questions they could do quickly and explain why they made those choices.
- Have the student explain in writing why someone might first subtract 30 from 74 in order to calculate  $674 - 26$ . Ask what would be done next.
- Display the numbers 124 and 75 with base ten blocks. Ask the student to describe the addition process as he/she manipulates the models.
- Tell the student that Sue was to add  $36 + 59$  and said, "36, 96, 95." Have the student explain Sue's thinking.
- Ask why someone might find it easier to subtract  $123 - 99$  than  $123 - 87$ .
- Ask the student to prepare a display showing a variety of ways to calculate  $57 - 18$ , indicating his/her preference and the reason for it.
- Ask the students to use a sales flyer to create some problems for his/her classmates. Have them record both problems and solutions.
- Using the numbers 62 and 25, create a subtraction problem that can be solved using addition. Solve.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*



SCO: <b>N10: Apply mental mathematics strategies and number properties, such as:</b> <ul style="list-style-type: none"> <li>• using doubles</li> <li>• making 10</li> <li>• using the commutative property</li> <li>• using the property of zero</li> <li>• thinking addition for subtraction</li> </ul> <b>to recall basic addition facts to 18 and related subtraction facts.</b> [C, CN, ME, R, V]			
<b>[C]</b> Communication <b>[T]</b> Technology	<b>[PS]</b> Problem Solving <b>[V]</b> Visualization	<b>[CN]</b> Connections <b>[R]</b> Reasoning	<b>[ME]</b> Mental Math and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>N10</b> Apply mental mathematics strategies, such as: using doubles; making 10; one more, one less; two more, two less; building on a known double addition for subtraction to determine basic addition facts to 18 and related subtraction facts.	<b>N10</b> Apply mental mathematics strategies and number properties, such as: <ul style="list-style-type: none"> <li>• using doubles; making 10;</li> <li>• using the commutative property;</li> <li>• using the property of zero;</li> <li>• thinking addition for subtraction to determine answers for basic addition facts and to 18 related subtraction facts.</li> </ul>	<b>N5</b> Describe and apply mental mathematics strategies, such as: skip counting from a known fact; using doubling or halving; using doubling or halving and adding or subtracting one more group; using patterns in the 9s facts; using repeated doubling to determine basic multiplication facts to $9 \times 9$ and related division facts.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

It is expected that by the end of Grade 3 students will have achieved **computational fluency** with the addition and subtraction facts to 18. Understanding how addition and subtraction are related is the key to computational fluency. To establish this relationship, students must be able to visualize the operations as connected to each other. This is best accomplished through the use of concrete materials. Once this connection has been established children must then commit the basic facts to memory.

It is important that students are able to demonstrate **automaticity** (recall sums and differences in a few seconds without counting) with all of these basic facts. Students may continue to use personal strategies developed in Grades 1 and 2, but the goal is for all students to know all of the basic facts before Grade 4. It is important to provide opportunities for this practice, using games and meaningful contexts as much as possible, rather than only requiring the memorization of facts in isolation. Experience exploring and building an understanding of the relationships between numbers, number facts and operations will increase flexibility, leading to faster recall of number facts. Having a strong number sense and proficiency with the facts will enable students to develop and use a variety of mental math strategies.

Students will build on the mental math strategies learned in Grade 2. These include using doubles, making ten, and thinking addition for subtraction. Understanding the basic principles behind addition and subtraction allows children greater flexibility in their thinking. In Grade 3, students are introduced to the **order property** or **commutative property**. While it is not important that students know the name of this property, they should be able to recognize that the order for addition does not affect the sum. Another important principle for students to learn is the **zero property** (e.g.,  $0 + 6 = 6$  and  $4 - 0 = 4$ ). Students should recognize that adding or subtracting 0 has no effect on the value of a number. Both of these concepts should be developed through the use of concrete materials in a problem solving context. It is important that mental math is developed through meaningful context and not in isolation. Students should be encouraged to use their strategies whenever they need a calculation to solve problems.



**SCO: N10: Apply mental mathematics strategies and number properties, such as:**

- using doubles
  - making 10
  - using the commutative property
  - using the property of zero
  - thinking addition for subtraction
- to recall basic addition facts to 18 and related subtraction facts.**

[C, CN, ME, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Describe a mental mathematics strategy that could be used to determine a given basic fact, such as:
  - doubles, e.g., for  $6 + 8$ , think  $7 + 7$
  - doubles plus one, e.g., for  $6 + 7$ , think  $6 + 6 + 1$
  - doubles take away one, e.g., for  $6 + 7$ , think  $7 + 7 - 1$
  - doubles plus two, e.g., for  $6 + 8$ , think  $6 + 6 + 2$
  - doubles take away two, e.g., for  $6 + 8$ , think  $8 + 8 - 2$
  - making 10, e.g., for  $6 + 8$ , think  $6 + 4 + 4$  or  $8 + 2 + 4$ ;  $17 - 9$ , think 9 and one more is 10, and 10 and 7 more is 17, so  $17 - 9 = 8$
  - commutative property, e.g., for  $3 + 9$ , think  $9 + 3$
  - addition to subtraction, e.g., for  $13 - 7$ , think  $7 + ? = 13$ .
- Provide a rule for determining answers for adding and subtracting zero.
- Recall basic addition facts to 18 and related subtraction facts to solve problems.

**SCO: N10: Apply mental mathematics strategies and number properties, such as:**

- using doubles
  - making 10
  - using the commutative property
  - using the property of zero
  - thinking addition for subtraction
- to recall basic addition facts to 18 and related subtraction facts.**

[C, CN, ME, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide students with a variety of models to practice and help visualize the basic facts. Students can also use other strategies, such as drawing pictures and role playing various sums and differences in a problem solving context.
- Use mathematical vocabulary with students, including sum, difference, and number sentence.
- Use the open number line to explore “making ten” or “bridging through 10”.
- Ensure students have the opportunity to discuss their strategies with others. The focus should be on the efficiency of the strategy.
- Use practice and drills that are short with immediate feedback over an extended period of time.
- Use the addition table to explore patterns and help students identify the facts which they have mastered. The known facts can be coloured in with the goal of having the entire table coloured.
- Have students create realistic word problems related to addition and subtraction.
- Provide many opportunities where the focus is on the relationship between the numbers.

$$7 + 8 = 8 + 7 \text{ or } 7 + 7 + 1 + 8 + 8 - 1 \text{ or } 10 + 5$$

### **Suggested Activities**

- Have students roll 2 number cubes (dice). They either add or subtract these values. For example: if a 5 and a 2 are rolled, they will work with either 5, 2, 7 or 5, 2, 3. Ask the students to make up a subtraction story based on these numbers, and write the corresponding number sentence. (Note: Ten-sided dice work well, as do prepared number cards.) If students are proficient with the addition facts, adapt this activity so that they must focus on subtraction.
- Play “Missing Part” game for two students to practice their fact recall. One student places a number of counters in front of them (e.g., 16) and then the student covers some of the counters with their hand. The other student must determine how many counters are hidden as quickly as possible.
- Use a “Looping Activity” where every student is given a card with a basic fact number sentence in which one of the numbers is missing; written as “Who has...?” (e.g., Who has  $5 + \square = 11$ ). The card also has the answer from someone else’s card written as “I have...” Students take turns reading their cards in sequence by responding when their card answers someone else’s.

“I have 12.”  
“Who has  $7 + 0$ ?”

“I have 7.”  
“Who has  $8 + 6$ ?”

- Provide students with cards with a subtraction number sentence (e.g.,  $13 - 7 =$ ). Have students rewrite the sentence as a missing addend number sentence (e.g.,  $7 + \square = 13$ ) and solve it.

**Possible Models:** number cubes, 10-sided dice, Rekenrek®, double ten frames, counters, number cards, open number line, addition chart, linking cubes, dominoes (double nine)

SCO: N10: Apply mental mathematics strategies and number properties, such as:

- using doubles
  - making 10
  - using the commutative property
  - using the property of zero
  - thinking addition for subtraction
- to recall basic addition facts to 18 and related subtraction facts.

[C, CN, ME, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask students to explain, using a model, why he/she knows that  $3 + 4$  has to equal  $4 + 3$  even before finding the total.
- Ask: Why is it easy to add or subtract 0 to numbers? (interview student or write response in a journal)
- Ask: Why is it easy to add the numbers  $5 + 5 + 6 + 4 + 8 + 2$ ?
- Ask: How can you use addition to solve  $16 - 7$ ?
- Have students write all of the number sentences they can for a provided sum or difference (e.g., 6 as a difference:  $6 - 0$ ,  $7 - 1$ ,  $8 - 2$ ,  $9 - 3$ ,  $10 - 4$ ,  $11 - 5$ ,  $12 - 6$ ,  $13 - 7$ ,  $14 - 8$ ,  $15 - 9$ ).
- Ask: How does knowing  $8 + 8 = 16$ , help you solve  $58 + 8$ ?
- Ask: How does knowing  $13 - 6 = 7$ , help you solve  $53 - 6$ ?
- Ask students to describe as many different ways as possible to solve  $8 + 9$ .

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: N11: Demonstrate an understanding of multiplication to  $5 \times 5$  by:

- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involve multiplication
- modelling multiplication using concrete and visual representations, and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division.

[C, CN, PS, R]

[C] Communication

[T] Technology

[PS] Problem Solving

[V] Visualization

[CN] Connections

[R] Reasoning

[ME] Mental Math

and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
	<p><b>N11</b> Demonstrate an understanding of multiplication to <math>5 \times 5</math> by:</p> <ul style="list-style-type: none"> <li>• representing and explaining multiplication using equal grouping and arrays</li> <li>• creating and solving problems in context that involve multiplication</li> <li>• modelling multiplication using concrete and visual representations, and recording the process symbolically</li> <li>• relating multiplication to repeated addition</li> <li>• relating multiplication to division.</li> </ul>	<p><b>N6</b> Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: using personal strategies for multiplication with and without concrete materials; using arrays to represent multiplication; connecting concrete representations to symbolic representations; estimating products.</p>

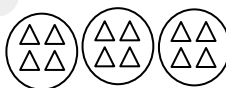
### ELABORATION

#### Guiding Questions:

- What do I want my students to learn?
- What do I want my students to understand and be able to do?

In **multiplication**, the numbers being multiplied are called **factors**. The answer is the **product**. It is important that students understand that there are different ways of looking at the concept of multiplication. They need to be aware that the product may be determined by:

- **repeated addition**: for example,  $4 + 4 + 4$  can be written as  $3 \times 4$  ;
- making **sets of equal groups**: for example, students can create equal-sized groups with actual items;
- the total number in an **array**.



3 groups of 4 make 12 ( $3 \times 4 = 12$ )



3 rows of 4 make 12 ( $3 \times 4 = 12$ )

Repeated addition is a helpful introduction to understanding multiplication, but students need to move beyond this strategy as their knowledge develops and other models become more efficient and appropriate. The array is a powerful tool to illustrate the order or **commutative property** in multiplication.



$$4 \times 2 = 2 \times 4$$



It is important that students come to understand the **inverse relationship** of multiplication and division (see N12). It is recommended that teachers “combine multiplication and **division** soon after multiplication has been introduced in order to help students see how they are related” (Van de Walle & Lovin, vol. 2, 2006, p. 60). The formal writing of multiplication sentences should be delayed until students understand the meaning of the operation.

SCO: N11: Demonstrate an understanding of multiplication to  $5 \times 5$  by:

- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involve multiplication
- modelling multiplication using concrete and visual representations, and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division.

[C, CN, PS, R]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

(It is not intended that students recall the basic facts but become familiar with strategies to mentally determine products. Practicing these strategies should lead to increased efficiency, necessary for ease in calculations.)

- Identify events from experience that can be described as multiplication.
- Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
- Represent a given multiplication expression as repeated addition.
- Represent a given repeated addition as multiplication.
- Create and illustrate a story problem for a given number sentence, e.g., given  $2 \times 3$ , create and illustrate a story problem.
- Represent, concretely or pictorially, equal groups for a given number sentence.
- Represent a given multiplication expression using an array.
- Create an array to model the commutative property of multiplication.
- Relate multiplication to division by using arrays and writing related number sentences.
- Solve a given problem in context involving multiplication.

SCO: N11: Demonstrate an understanding of multiplication to  $5 \times 5$  by:

- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involve multiplication
- modelling multiplication using concrete and visual representations, and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division.

[C, CN, PS, R]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### Choosing Instructional Strategies

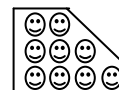
Consider the following strategies when planning lessons:

- Use contexts when creating multiplication problems in which things actually come in groups, for example, packaged foods - cans of pop ( $2 \times 3$ ), juice packs ( $1 \times 3$ ), hamburger buns ( $2 \times 4$ ).
- Play "Broken Calculator" to relate multiplication and addition. Students use the constant feature of the calculator to find various products without using the multiplication key. Challenge the students to model their product using counters.
- Explore the distributive property by displaying a  $5 \times 4$  array of objects and place a ruler on the line shown. Ask how this shows that  $5 \times 4 = 5 \times 3 + 5 \times 1$ . Then ask the student to move the ruler to show another way to find  $5 \times 4$  and explain his/her thinking, and/or provide the student with a piece of paper upon which an array ( $5 \times 5$ ) has been drawn. Ask him/her to fold the paper to show different ways the multiplication can be expressed.
- Show an array. Ask students to write the fact family illustrated in the array (multiplication and division).
- Ask students to show multiple representations of a given multiplication fact.
- Ask the student to draw pictures showing various situations in which multiplication might be used.
- Give students many opportunities to solve missing factor problems. Example: It takes 4 toothpicks to build a square. How many of the same sized squares can be built with 16 toothpicks?



### Suggested Activities

- Using counters, have students build as many arrays as possible for a given number, and write its corresponding equation.
- Create arrays on cards and cut off a corner so that some counters are missing but the intended number of rows and columns remains clear. Show cards to students and ask them how many counters the card had initially if all the rows and columns had the same number of counters.
- Have students investigate what happens when you multiply a number by 0, 1, 2, 3, 4, 5. Is there a pattern in the products?
- Invite a group of students to act out a skit modelling either a multiplication or division situation. Ask other students to guess the number sentence being dramatized.
- Have the students create a realistic story problem to go with a given number sentence (e.g.,  $4 \times 5$ ) or describe a situation for which you might have to find the answer to  $5 \times 3$ .
- Ask students how they might use a hundred chart to find the product of  $5 \times 4$ .



**Possible Models:** square tiles, counters, ruler, calculator, array examples, hundred chart

SCO: N11: Demonstrate an understanding of multiplication to  $5 \times 5$  by:

- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involve multiplication
- modelling multiplication using concrete and visual representations, and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division.

[C, CN, PS, R]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Have students represent story problems using models or diagrams and record the corresponding number sentences.
- Have students create a real-life story problem that involve multiplication and solve it.
- Have students represent a given multiplication sentence, such as  $5 \times 3$ , using an array.
- Have students represent a given repeated addition as multiplication.
- Have students represent equal groups for a given number sentence concretely or pictorially.
- Ask students to model as many arrays as possible with 16 counters. Have them write the related multiplication and division facts for each array.
- Solve comparison problems such as: One tower is 8 blocks high. The other is 2 blocks high. How many times higher is the first tower?
- Ask students to put ten tiles, into rows of five. Ask how many rows are there.
- Show students an array and have them provide the related multiplication and division sentences.
- Ask students to model ten stamps in two rows. Ask how many stamps are in each row.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*



SCO: N12: Demonstrate an understanding of division by:

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.

(limited to division related to multiplication facts up to  $5 \times 5$ )

[C, CN, PS, R]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
	<p><b>N12</b> Demonstrate an understanding of division by:</p> <ul style="list-style-type: none"> <li>• representing and explaining division using equal sharing and equal grouping</li> <li>• creating and solving problems in context that involve equal sharing and equal grouping</li> <li>• modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically</li> <li>• relating division to repeated subtraction</li> <li>• relating division to multiplication.</li> </ul> <p>(limited to division related to multiplication facts up to <math>5 \times 5</math>)</p>	<p><b>N7</b> Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: using personal strategies for dividing with and without concrete materials; estimating quotients; relating division to multiplication.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In division, the number being divided is called the **dividend**, and the other number by which the dividend is to be divided is called the **divisor**. The answer is called the **quotient**. It is important that students see that division can mean:

- **equal sharing**:  $16 \div 4 = 4$  is the amount each person gets if 16 items are shared equally among 4 people
- **equal grouping**:  $16 \div 4 = 4$  is the number of equal groups of 4 you can make with 16 items
- **repeated subtraction**:  $16 \div 4 = 4$  is the number of times you can subtract 4 from 16 before you reach zero.

Repeated subtraction, like repeated addition, is a helpful introduction to understanding division, but students need to move beyond this strategy as their knowledge develops and other models become more efficient and appropriate.

Multiplication and division are inverse operations; for example, if  $20 \div 5 = 4$ , then  $5 \times 4 = 20$ . Note: while you can multiply numbers in any order (commutative property), the order in which numbers are divided matters. Problems should be worded in such a way as to develop this part-part- whole understanding. If one of the factors is unknown then the problem can be solved using division. To develop this vital link between the two operations, problems highlighting the inverse relationship between multiplication and division should be integrated regularly into mathematics lessons.



SCO: N12: Demonstrate an understanding of division by:

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.  
(limited to division related to multiplication facts up to  $5 \times 5$ )

[C, CN, PS, R]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify events from experience that can be described as equal sharing.
- Identify events from experience that can be described as equal grouping.
- Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading and solve the problem.
- Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
- Listen to a story problem, represent the numbers using manipulatives or a sketch and record the problem with a number sentence.
- Create and illustrate with counters, a story problem for a given number sentence, e.g., given  $6 \div 3$ , create and illustrate a story problem.
- Represent a given division expression as repeated subtraction.
- Represent a given repeated subtraction as a division expression.
- Relate division to multiplication by using arrays and writing related number sentences.
- Solve a given problem involving division.

SCO: N12: Demonstrate an understanding of division by:

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.

(limited to division related to multiplication facts up to  $5 \times 5$ )

[C, CN, PS, R]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Use concrete materials to help students understand the relationship between the meanings of division. Demonstrate that, in sharing 12 items among 3 people, for example, the actual giving of 1 item to each person is the same as creating a group of 3. In other words, sharing among 3 people is equivalent to finding how many groups of 3 can be formed.
- Play "Broken Calculator". Students work in groups to find ways to use the calculator to solve division exercises without using the divide key.
- Provide problem solving situations in which solutions can be found using either multiplication or division.
- Explore various types of division problems: equal share, equal groups, and comparison.

### **Suggested Activities**

- Provide the student with some toothpicks and ask him/her to use 12 to make 4 identical shapes. Ask the student what division and multiplication sentences could describe the creation of the shapes.
- Set up a  $3 \times 4$  array and ask the student to give two multiplication and two division sentences that describe it by looking at the array from different perspectives.
- Invite a group of students to act out a skit modelling either a multiplication or division situation. Ask other students to guess the number sentence being dramatized.
- Ask the student to write problems in which one has to multiply or divide to find the answer. Have him/her illustrate the solutions and describe the multiplication/division relationship.
- Ask students to skip count backwards by 2's, 4's, 5's from a given multiple of that number.
- Ask students to solve a division problem in as many ways as possible (including multiplication).

**Possible Models:** number lines, counters, Rekenrek<sup>®</sup> (100 frame), arrays

SCO: N12: Demonstrate an understanding of division by:

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.

(limited to division related to multiplication facts up to  $5 \times 5$ )

[C, CN, PS, R]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask the student to write a division story about  $25 \div 5$ .
- Have the student draw a picture or use counters to show what  $12 \div 3$  means.
- Ask the student to describe a situation for which you might have to find the answer to  $15 \div 3$ .
- Ask the student to draw pictures showing various situations in which either multiplication or division might be used.
- Show the following number line. Ask the student to record what multiplication and division sentences it might be showing.



- Show the student the multiplication sentence  $5 \times 4 = 20$ . Ask the student to write related division sentences.
- Tell the students that amusement park rides are priced as follows:
  - \$1 for the Ferris wheel,
  - \$2 for the Bullet, and
  - \$3 for the Twister.
 Ask: How many rides, and of which kind, can you have for \$13? Are there other possibilities?
- Show students an array of up to 25 counters. Ask students which multiplication and division family is shown by the array.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N13: Demonstrate an understanding of fractions by:**

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators.

[C, CN, ME, R, V]

[C] Communication

[T] Technology

[PS] Problem Solving

[V] Visualization

[CN] Connections

[R] Reasoning

[ME] Mental Math

and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
	<b>N13</b> Demonstrate an understanding of fractions by: <ul style="list-style-type: none"> <li>• explaining that a fraction represents a part of a whole;</li> <li>• describing situations in which fractions are used;</li> <li>• comparing fractions of the same whole with like denominators.</li> </ul>	<b>N8</b> Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: name and record fractions for the parts of a whole or a set; compare and order fractions; model and explain that for different wholes, two identical fractions may not represent the same quantity; provide examples of where fractions are used.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In grade three, the focus is on students developing a beginning understanding of fractions less than one, relating fractions to authentic situations, and comparing fractions with the same denominator.

Children seem to understand the idea of separating a quantity into two or more parts to be shared fairly among friends. They eventually make connections between the idea of **fair share** and **fractional parts**. Sharing tasks are, therefore, good places to begin the development of fractions (Van de Walle & Lovin, vol. 1, 2006, p. 252).

Students are introduced to the concept that there are numbers to represent parts of a whole. They need to experience and discuss activities that promote the following understandings:

- Fractional parts are equal shares or equal sized portions of a whole.
- A fraction is a number that describes a relationship between a part (represented by the **numerator**) and a whole (represented by the **denominator**) (Small, 2008, p. 196).
- A fraction can be read/written/modeled in different ways but still have the same meaning.

$$\text{one fourth} = \text{one quarter} = \frac{1}{4} = \text{one part of every four parts} = 25\text{¢}, \dots$$

It is important that students represent fractions with concrete models. Students often find fractions challenging if they are only presented as symbols. Representing fraction concepts with a variety of materials and drawings and within real life situations is essential to the development of fractional understanding so that fractions don't simply become associated with pieces of a chocolate bar or pizza. It is important that students develop visual images for fractions and be able to tell "about how much" a particular fraction represents and have exposure to common **benchmarks**, such as one half.

The meaning of the numerator (top number) and the denominator (bottom number) needs to be emphasized as these can be misleading to students. This is best accomplished by introducing and focusing on the numerator or denominator separately and using visual models linked to the symbols.

SCO: N13: Demonstrate an understanding of fractions by:

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators.

[C, CN, ME, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify common characteristics of a given set of fractions.
- Describe everyday situations where fractions are used.
- Cut or fold a whole into equal parts, or draw a whole in equal parts; demonstrate that the parts are equal and name the parts.
- Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.
- Represent a given fraction concretely or pictorially.
- Name and record the fraction represented by the shaded and non-shaded parts of a given region.
- Compare given fractions with the same denominator using models.
- Identify the numerator and denominator for a given fraction.
- Model and explain the meaning of numerator and denominator.

SCO: N13: Demonstrate an understanding of fractions by:

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators.

[C, CN, ME, R, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

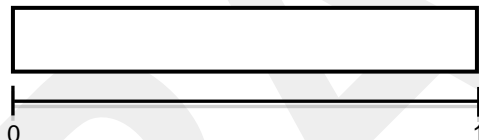
### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students explore various models for fractions: part of a region and part of a length.
- Ensure students develop an understanding that a fraction is not meaningful without knowing what the “whole” that it is part of.
- Provide students with rectangles and number lines that are the same length. Ask students to color half of a rectangle, and indicate where half is on the number line. Once students understand the concept of half, this activity could be extended to fourths (quarters) and thirds, etc.



### Suggested Activities

- Ask students to fold a strip of paper into equal parts.
- Give students some pattern blocks or Cuisenaire® rods. Have them model  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{3}$  using various blocks or rods.
- Show students three pictures of varying sizes of the same item, all items cut into the same number of pieces. Ask students which of the pieces they would like to have. Explain why they made that choice.
- Have students model a specific fraction using at least one of each of the pattern blocks. Ask students to rearrange the blocks and show the same fraction. How many different ways can this be done?
- Ask students to model a specific fraction using five pattern blocks. Draw their model on isometric grid paper and color the fractional part they have represented with their model.
- Have students model on a number line to 100 where  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{3}{4}$  would be. Read a number and have students make connections to these benchmarks by estimating where in relation to the benchmarks that number would go. Have them explain their thinking.

**Possible Models:** number lines, fraction pieces, Cuisenaire® rods, pattern blocks, colour tiles, isometric grid paper

SCO: N13: Demonstrate an understanding of fractions by:

- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole with like denominators.

[C, CN, ME, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask students: “Is half a lot or a little?” Have them explain their thinking.
- Ask students to identify the numerator and denominator of a given fraction.
- Provide students with fractions with the same denominator and have them identify the larger (or smaller) and explain their reasoning using models.
- Ask students: “If you are really hungry and want a large piece of cake, would you cut the cake into thirds, fourths, or tenths?”
- Provide students with paper shapes. Have students cut or fold the shape into halves and/or fourths (quarters).
- Show students a fraction of a region with a shaded part. Ask students to name and record the fraction represented.
- Provide students with a five frame, and ask them to place a counter on  $\frac{1}{5}$  of the squares.
- Ask students to draw a picture to represent a given fraction such as  $\frac{1}{2}$ , or  $\frac{3}{4}$ .
- Have students place the following fractions on the number line below:  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{3}{4}$ .



## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*



SCO: PR1: Demonstrate an understanding of increasing patterns by: <ul style="list-style-type: none"> <li>• describing</li> <li>• extending</li> <li>• comparing</li> <li>• creating</li> </ul> patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).			
SCO: PR2: Demonstrate an understanding of decreasing patterns by: <ul style="list-style-type: none"> <li>• describing</li> <li>• extending</li> <li>• comparing</li> <li>• creating</li> </ul> patterns using manipulatives, diagrams, sounds and actions (numbers to 1000). [C, CN, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math
[T] Technology	[V] Visualization	[R] Reasoning	and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>PR1</b> Demonstrate an understanding of repeating patterns (three to five elements) by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions. <b>PR2</b> Demonstrate an understanding of increasing patterns by: describing; reproducing; extending; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 100).	<b>PR1</b> Demonstrate an understanding of <u>increasing</u> patterns; and <b>PR2</b> Demonstrate an understanding of <u>decreasing</u> patterns by: <ul style="list-style-type: none"> <li>• describing;</li> <li>• extending;</li> <li>• comparing;</li> <li>• creating</li> </ul> patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	<b>PR1</b> Identify and describe patterns found in tables and charts, including a multiplication chart. <b>PR2</b> Reproduce a pattern shown in a table or chart using concrete materials. <b>PR3</b> Represent and describe patterns and relationships using charts and tables to solve problems.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Patterns are the foundation for many mathematical concepts. They should be taught throughout the year in situations that are meaningful to students, as patterns are embedded in all areas of mathematics. For example, our place value system is based on an increasing pattern. Providing students with the opportunity to discover and create patterns then describe and extend those patterns will result in more flexible thinking across strands.

It is essential that students are provided with many different concrete materials to build **increasing** (growing) and **decreasing** (shrinking) patterns. Students should also be able to represent these patterns pictorially and symbolically.

Students must be able to explain the **pattern rule** in written, oral and symbolic forms. Students need to recognize and extend different forms of the same pattern, including those constructed or in their environment. They must identify the **core** or sequence, and be able to predict an element in increasing and decreasing patterns using many strategies within a variety of contexts. Students should also be able to apply their knowledge to identify errors and fill in missing elements in a pattern.



SCO: PR1: Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

SCO: PR2: Demonstrate an understanding of decreasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

[C, CN, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

### **PR1**

- Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- Identify the pattern rule of a given increasing pattern and extend the pattern for the next three terms.
- Identify and explain errors in a given increasing pattern.
- Locate and describe various increasing patterns found on a hundred chart, such as horizontal, vertical and diagonal patterns.
- Compare numeric patterns of counting by 2s, 5s, 10s, 25s and 100s.
- Create a concrete, pictorial or symbolic representation of an increasing pattern for a given pattern rule.
- Create a concrete, pictorial or symbolic increasing pattern and describe the pattern rule.
- Solve a given problem using increasing patterns.
- Identify and describe increasing patterns in the environment.
- Identify and apply a pattern rule to determine missing elements for a given pattern.
- Describe the strategy used to determine missing elements in a given increasing pattern.

### **PR2**

- Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- Identify the pattern rule of a given decreasing pattern and extend the pattern for the next three terms.
- Identify and explain errors in a given decreasing pattern.
- Identify and describe various decreasing patterns found on a hundred chart, such as horizontal, vertical and diagonal patterns.
- Compare decreasing numeric patterns of counting backward by 2s, 5s, 10s, 25s and 100s.
- Create a concrete, pictorial or symbolic decreasing pattern for a given pattern rule.
- Create a concrete, pictorial or symbolic decreasing pattern and describe the pattern rule.
- Solve a given problem using decreasing patterns.
- Identify and describe decreasing patterns in the environment.
- Identify and apply a pattern rule to determine missing elements for a given pattern.
- Describe the strategy used to determine missing elements in a given decreasing pattern.

SCO: PR1: Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

SCO: PR2: Demonstrate an understanding of decreasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

[C, CN, PS, R, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with a variety of concrete and pictorial materials to create and extend increasing and decreasing patterns.
- Expect students to discuss and write about how patterns increase or decrease, and be extended.
- Encourage students to identify the attributes of different increasing/decreasing patterns (e.g., increases by the same amount).
- Have students describe errors or missing elements within an increasing or decreasing pattern.
- Create a box of patterns that contains symbolic representation of patterns that students have created for other students to name the pattern rule.
- Provide students with materials such as linking cubes, colour tiles or pattern blocks and have them create increasing and decreasing patterns.
- Expect students to demonstrate their understanding of patterns by representing the same pattern in many different ways; concretely, pictorially, symbolically, orally, rhythmically, physically.

### Suggested Activities

- Have students explore the hundred charts that go up to 1000. Look for patterns when counting by 2's, 5's, 10's, 25's, and 100.
- Give students one of the elements of a pattern (not necessarily the first element). Ask students to model as many possible ways to extend the pattern as they can (e.g., if the third element is 12, possible solutions could be: 4, 8, 12, 16... 3, 7, 12, 18... 2, 6, 12, 20... 6, 9, 12, 15...).
- Have students go on a "Pattern Hunt" around the school identifying increasing and decreasing patterns in their school environment. Have them record the patterns they find with a description of each.
- Have students add 2, 10 and/or 25 to a number. Ask students to describe what they notice.
- Provide students with the first 3 or 4 elements of a pattern. Have them use appropriate materials to extend and explain the pattern.



**Possible Models:** linking cubes, coloured tiles, hundred charts (up to 1000), pattern blocks, grid paper

SCO: PR1: Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

SCO: PR2: Demonstrate an understanding of decreasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

[C, CN, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Give students a diagram showing a square table with 4 chairs (one on each side). Tell students that if 2 tables were put together, it would seat 6. Ask: how many can we seat with 6 tables? 8? 10? What if we started with a table of 6? Have students explain their reasoning.
- Ask students to show you different ways these patterns could be extended.
  - 20, 40, \_\_\_\_, \_\_\_\_, \_\_\_\_
  - 1, 4, \_\_\_\_, \_\_\_\_
  - 1000, 500, \_\_\_\_, \_\_\_\_
- Tell students: "I am thinking of a pattern. I have landed on 50. What could I be counting by?" Accept any reasonable answer that includes an explanation.
- Ask the student to say a number that is 100 more (100 less, 10 more, 10 less) than a 2- or 3-digit number that is provided.
- Give students a pattern modelled with tiles and ask them to describe, recreate, and extend the pattern in another way.
- Have students identify the errors in the following patterns and correct them:
  - 3, 6, 9, 12, 15, 19, 21, 24, 28, 30
  - 40, 35, 29, 25, 20, 15, 10, 5

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.**  
[C, CN, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>PR3</b> Demonstrate and explain the meaning of equality and inequality, concretely and pictorially. <b>PR4</b> Record equalities and inequalities symbolically, using the equal symbol or the not equal symbol.	<b>PR3</b> Solve one-step addition and subtraction equations involving symbols representing an unknown number.	<b>PR5</b> Express a given problem as an equation in which a symbol is used to represent an unknown number. <b>PR6</b> Solve one-step equations involving a symbol to represent an unknown number.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In Grade 2, students learned the concepts of equality and inequality and the meaning of the symbols (= and  $\neq$ ). This knowledge is extended in Grade 3 to solving **equations** that include **symbols** that represent **unknowns**. An equation is a mathematical statement that includes an equal sign and may have been called a **number sentence**. The **equal sign** tells us that the quantity on the left is the same as the quantity on the right (balanced).

The unknown value in an equation can be shown using a variety of symbols (e.g., box, circle, triangle). It is important that a variety of symbols are used so students do not develop the misconception that an unknown can only be represented by a box (open frame). Students should explore creating equations to help solve story problems (e.g., “Josh has some marbles and he bought 12 more. Now he has 33 marbles. How many marbles did he have at the start?” This can be represented with the equation:  $\triangle + 12 = 33$ ). Ensure that students have opportunity to explore both **combining and separating situations** in the problems they are solving. Students should initially use concrete materials and a variety of strategies, such as “guess and test” and mental math. As they solve equations, students should develop the understanding that the unknown in the equation can only have one value.

Students should solve open sentences of the following 6 forms with the equal sign in different locations:

$a + b = \triangle$	(e.g., $6 + 3 = \triangle$ )	or	$(\triangle = a + b)$
$a + \bigcirc = c$	(e.g., $2 + \bigcirc = 8$ )	or	$(c = a + \bigcirc)$
$\square + b = c$	(e.g., $\square + 4 = 5$ )	or	$(c = \square + b)$
$c - a = \diamond$	(e.g., $7 - 2 = \diamond$ )	or	$(\diamond = c - a)$
$c - \nabla = b$	(e.g., $4 - \nabla = 2$ )	or	$(b = c - \nabla)$
$\square - a = b$	(e.g., $\square - 8 = 1$ )	or	$(b = \square - a)$

Students may find some of these equations difficult to solve and need to have many opportunities to explore all of the different forms. Provide students with equations like those shown in the examples above.

SCO: PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.  
[C, CN, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain the purpose of the symbol, such as a triangle or a circle, in a given addition and in a given subtraction equation with one unknown.
- Create an addition or subtraction equation with one unknown to represent a given combination or separation action.
- Provide an alternative symbol for the unknown in a given addition or subtraction equation.
- Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
- Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and test.
- Explain why the unknown in a given addition or subtraction equation has only one value.

SCO: **PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.**  
[C, CN, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Ensure students see and use a variety of symbols representing the unknown.
- Re-emphasize the link between addition sentences and subtraction sentences. This will help students solve a variety of equations by thinking of them in a different way (e.g.,  $12 - \bigcirc = 8$  can be thought of as  $8 + \bigcirc = 12$ ).
- Provide students with equations that include more than basic fact knowledge to solve (e.g.,  $\triangle + 15 = 36$ ). Have them explore the use of "guess and test" as a strategy to solve for the unknown.
- Use models such as counters to help students solve equations.
- Have students solve equations that originate from word problems. Ensure that students are able to explain how to find the unknown in a variety of equations.

### **Suggested Activities**

- Have students match equations with word problems where the unknown is in different locations.

Word problem	Equation
Mia has 15 cherries and eats some. Now she has 6. How many did she eat?	$15 - 6 = \bigcirc$
Edmond has 6 hockey cards, but he would like to have 15. How many more does he need?	$15 - \triangle = 6$
Zane has 15 markers, but 6 of them no longer work. How many does he have that work?	$6 + \square = 15$
Some cookies are on a plate. Six cookies are in a jar making 15 cookies altogether. How many cookies are on the plate?	$\square + 6 = 15$

- Have students create problems to represent equations such as the following:  
 $4 + 7 = \triangle$                        $\nabla - 8 = 8$                        $\square + 4 = 13$
- Show the students a balance scale with linking cubes to represent an equation. Represent the unknown with a piece of paper with a question mark. Have the student write the equation and solve it. Students can replace the paper with linking cubes to help solve the equation or to check their answer.



**Possible Models:** counters, balance scales, linking cubes

SCO: PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.  
[C, CN, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask the student to find the number that makes each equation true.  
 $5 + \square = 13$   
 $16 - \square = 7$   
 $\square = 6 + 4$   
Have the student explain the strategy they used to solve it.  
Ask: Can there be more than one answer for each? Why or why not?
- Ask: How might you use counters to find the number to make this equation true?  
 $\triangle + 18 = 25$   
Have the student write a story problem for this equation.
- Ask the student to create four different equations with unknowns for which the solution is 9. Ensure he/she uses the same fact and includes both addition and subtraction equations.
- Have students write the corresponding equation for a word problem and solve it. For example: "Gabrielle had some stickers and gave her friend 9. Now she has 8 left. How many did she have at the start?" ( $\star - 9 = 8$ )
- Ask students to explain what a symbol represents in an equation (e.g., it represents an unknown).
- Show students a math fact that includes a symbol for the missing number. Ask students to record their answers on individual white boards or paper. Have students share their strategy for solving the question.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*



SCO: <b>SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).</b> [CN, ME, R]			
SCO: <b>SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.</b> [C, CN, PS, R, V]			
[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation

### Scope and Sequence of Outcomes

<u>Grade Two</u>	<u>Grade Three</u>	<u>Grade Four</u>
<b>SS1</b> Relate the number of days to a week and the number of months to a year in a problem-solving context.	<b>SS1</b> Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). <b>SS2</b> Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.	<b>SS1</b> Read and record time using digital and analog clocks, including 24-hour clocks.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Time is about the **duration** of an event from beginning to end. Understanding duration, or the **passage of time**, helps students to understand and describe the world around them. Students develop **personal referents** such as television shows to understand duration, but they also learn to use measurement tools such as sand timers, etc. The goal is for students to be able to determine the unit of time appropriate to describe an event developing the understanding that the length of a unit of time is directly proportional to the amount of time it takes for the event to occur.

Experiences measuring time in a variety of ways helps students to better understand what measuring time really means. Personal referents, such as the length of a favourite television show or how long it takes to properly wash their hands, allow students to better estimate time. Students can also use activities with known durations as referents for estimating the duration of other activities.

Students will learn to identify activities that can or cannot be completed in a given amount of time (**minutes, hours, days, weeks, months** and **years**). Students will solve problems that associate the number of **seconds** to a minute; minutes to an hour; number of days to a month using calendars and personal events. When measuring using non-standard units, a student should select and justify their own methods (rather than teacher-prescribed).

Students will be exploring the concepts of the passage of time and will use time vocabulary (seconds, minutes, hours, days, weeks, months, years). Although it is not an expectation that students will be assessed on their ability to use a clock to tell time, clocks may be referred to during the learning of this outcome.



SCO: **SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).**

[CN, ME, R]

SCO: **SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.**

[C, CN, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

### **SS1**

- Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
- Identify activities that can or cannot be accomplished in minutes, hours, days, months and years.
- Provide personal referents for minutes and hours.

### **SS2**

- Determine the number of days in any given month using a calendar.
- Solve a given problem involving the number of minutes in an hour or the number of days in a given month.
- Create a calendar that includes days of the week, dates and personal events.

SCO: **SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).**

[CN, ME, R]

SCO: **SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.**

[C, CN, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students create to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

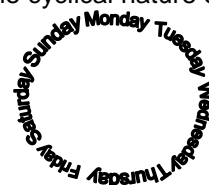
Consider the following strategies when planning lessons:

- Ask students to identify events that take exactly one minute. More than a minute? Less than a minute? This could be extended to other durations of time.
- Have students create their own non-standard unit timers to compare durations, such as plastic water bottles to create a water timer (see Van de Walle & Lovin, vol. 1, 2006, p. 242, Fig. 8.14) or a pendulum by using a tennis ball suspended on a long string.
- Discuss the duration of various school events occurring throughout the school day and year.
- Use children's literature such as *Counting Crocodiles* by Judy Sierra, *Time Flies* by Ellen Goodenow or *Time* by Nina Filipek to provide connections for students with this outcome.
- Provide a calendar for the year, and have the students figure out how many school days each month will have. How many Friday 13ths are there in the year? On what days do the birthdays of friends and family fall? Ask the students to write about their findings.

### **Suggested Activities**

- Ask the student to estimate how many times one can count to ten, while walking heel-to-toe across the classroom. Have the student verify his/her estimate. Ask why another student might get a different result.
- Have students work in pairs to predict which of two specified activities will take longer. One student times the other performing the two activities, then roles are reversed. Activities could include:
  - printing their name five times
  - walking the length of the classroom heel to toe
  - making a chain of 25 links or paperclips or linking cubes
  - completing 10 jumping jacks
  - singing "Happy Birthday"
- Ask pairs of students to predict how many weeks there are in a year. Have them use a calendar for the year to check their prediction.
- Show the student a calendar for the year. Ask him/her to point out the day's date and to find out what date it will be in six weeks. Seven weeks?
- Ask students to build a timeline reflecting the time and duration of a sequence of events, then make comparisons between the events.
- Create time circles for days of the week and months of the year to demonstrate the cyclical nature of the passage of time (Small, 2008, p. 441).

**Possible Models:** calendar, sand timer, clocks, links, linking cubes



SCO: **SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).**

[CN, ME, R]

SCO: **SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.**

[C, CN, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Have students describe the duration of something (e.g., physical education class) using their personal referent.
- Ask: How might you use a calendar to help subtract 14 from a number?
- Tell the student to find the 11<sup>th</sup> day in a month on a calendar. Ask: how might you use it to add 16, and what is the date?
- Show the student a calendar for the year and ask him/her to identify ways in which months are the same and ways in which they differ.
- Ask: What is something you can do about 10 times in a minute? In an hour?
- Have students create a calendar and include their birthday and 3 other important dates for them.
- Ask students to tell how many:
  - seconds in a minute
  - minutes in an hour
  - days in a given month
- Give students a set of time cards (minutes, hours, days, months, years) and have students hold up the appropriate card to describe the duration of an event said by the teacher (i.e. recess – student holds up minutes).

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**

- selecting and justifying referents for the units cm and m
- modelling and describing the relationship between the units cm and m
- estimating length using referents
- measuring and recording length, width and height.

[C, CN, ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>SS2:</b> Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).	<b>SS3:</b> Demonstrate an understanding of measuring length (cm, m) by: <ul style="list-style-type: none"> <li>• selecting and justifying referents for the units cm and m</li> <li>• modelling and describing the relationship between the units cm and m</li> <li>• estimating length using referents</li> <li>• measuring and recording length, width and height.</li> </ul>	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Prior to Grade 3, students have become familiar with **measurement attributes** using direct comparison and nonstandard units. Students will now be introduced to two basic **standard units** of length: **centimetre** and **metre**. Students should have the opportunity to explore why standard units are necessary to ensure consistency when measuring and communicating measurements. Students will be familiar with measuring the attributes of **length** and **height** from previous grades. In Grade 3, students will add the attribute of **width** which will enable them to measure 3-D objects as well as 2-D shapes.

When introducing centimetres and metres, it is important for students to have the opportunity to discover **personal referents** for these standard units of length (e.g., a centimetre is about the width of a finger, a metre is about the height of a doorknob). Having these personal referents helps students visualize measurements and estimate more accurately. Personal referents also make the units easier for students to remember.

**Estimation** in measurement is an essential part of the measurement process that has applications to real world situations. Estimates are sometimes all that is needed and at other times reassure us about the reasonableness of our answers. Through estimation, students become more familiar with the standard units. It is also engaging for students to challenge themselves to have their estimates as close as possible to the actual measurement.

Students should recognize that a metre is 100 centimetres long. Although many metre sticks are marked up to 100, it is often still not clear to students that lining up 100 centimetres really produces a metre. Students will also become familiar with the abbreviations for these two units: centimetre - **cm**; metre - **m**. This is the first year where students will begin to use a standard tool to measure length. It is valuable to use simpler **rulers** that are created by the students. Manufactured rulers can be confusing to students as they often include more than one unit. Emphasis should be placed on counting the number of units (the interval between numbers) rather than looking at the number on the ruler which is aligned with the end of the object.

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**

- selecting and justifying referents for the units cm and m
- modelling and describing the relationship between the units cm and m
- estimating length using referents
- measuring and recording length, width and height.

[C, CN, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide a personal referent for one centimetre and explain the choice.
- Provide a personal referent for one metre and explain the choice.
- Match a given standard unit to a given referent.
- Show that 100 centimetres is equivalent to 1 metre by using concrete materials.
- Estimate the length of an object using personal referents.
- Determine and record the length and width of a given 2-D shape.
- Determine and record the length, width or height of a given 3-D object.
- Draw a line segment of a given length using a ruler.
- Sketch a line segment of a given length without using a ruler.

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**

- selecting and justifying referents for the units cm and m
- modelling and describing the relationship between the units cm and m
- estimating length using referents
- measuring and recording length, width and height.

[C, CN, ME, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide opportunities for students to discover and share their personal referents for centimetres and metres. They should be able to explain their choices and recognize that there are many appropriate referents for each unit.
- Include measurement situations which are of interest to the students and that provide useful information, such as measuring book heights for a new bookcase or determining if a large piece of furniture can fit through the door.
- Have students create their own rulers. Initially numbers should not be included so students need to count the number of units, rather than looking at the number on the ruler. As they become more familiar with its use, numbers can be added.
- Present situations requiring students to choose the most appropriate unit of measure.

### **Suggested Activities**

- Have students relate lengths to their own bodies. For example: "My legs are about half a metre long, my nose is 4 cm long, and 8 of my footprints would make a metre."
- Compare connected 100 centimetre cubes (base ten blocks) to a metre.
- Have students use the same ruler to measure the same object in different ways (e.g., changing the start points or measuring different parts of the object and combining results (Van de Walle & Lovin, vol. 1, 2006, p. 233).
- Read the book, *How Big is a Foot?* by Rolf Myller, and relate the story to why standard units of measurement are valuable. As a follow up, discuss why it is not a good idea to tell someone how long a table is by using pieces of paper as a measurement unit.
- Have students develop a book on measurement that they can add to over time. This could include drawings of their personal referents, pictures of objects that they have estimated and measured, and descriptions of length, width and height.
- Give each student a metre-long piece of twine and ask them to use it to measure objects in their homes. Have them make lists of items that are almost a metre, one metre, or a little more than a metre. Have the students enter their findings in a table such as the one shown below.

almost a metre	one metre	more than a metre

How could one use the twine to identify objects that are about half a metre?

- Set up a mini-Olympics in which students compete in events such as a tissue kick, a penny thumb toss, and cotton ball puffing. Have students measure results to the nearest centimetre or metre, and then record and compare them.

**Possible Models:** centimetre cubes, metre sticks, rulers, Cuisenaire rods®, string

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**

- selecting and justifying referents for the units cm and m
- modelling and describing the relationship between the units cm and m
- estimating length using referents
- measuring and recording length, width and height.

[C, CN, ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask students to estimate the length of a book using a personal referent for centimetres.
- Ask students to estimate the length of the classroom using a personal referent for metres.
- Ask students to cut a length of about 1 m from a ball of string. Have them verify their estimates.
- Have students draw a line segment that is about 7 cm long without using a ruler.
- Show students a line segment that is 95 cm and have students estimate its length and then measure it with a ruler.
- Have students use materials to show that a metre is the same as 100 centimetres.
- Provide students with a shoebox or other box and have them measure the length, width, and height.
- Have students use a ruler to measure the length of a pencil or other object without using zero as the starting point.
- Provide students with a photograph and have the student measure the length and width of the picture.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*



SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**

- selecting and justifying referents for the units g and kg
- modelling and describing the relationship between the units g and kg
- estimating mass using referents
- measuring and recording mass.

[C, CN, ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>SS2:</b> Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).	<b>SS4:</b> Demonstrate an understanding of measuring mass (g, kg) by: <ul style="list-style-type: none"> <li>• selecting and justifying referents for the units g and kg</li> <li>• modelling and describing the relationship between the units g and kg</li> <li>• estimating mass using referents</li> <li>• measuring and recording mass.</li> </ul>	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In previous grades, students have investigated **mass** using non-standard units. They will now begin to **estimate** and **measure** masses, using the **gram (g)** and **kilogram (kg)**. Estimating mass is more difficult than estimating other measures, as the object's size and shape is not directly related to its mass. They should develop a sense of what a gram and kilogram "feel" like.

Students need to understand that grams are used to measure very light objects and kilograms are more appropriate units for heavier objects. It is also important for student to know that 1000 grams is equal to a kilogram. As with all measurement units, it is valuable for students to have a **personal referent** for:

- a gram (e.g. a raisin, paper clip, unit cube);
- 100 grams (e.g., individual size yogurt, 40 pennies, a 3-D base ten flat, a granola bar);
- a kilogram (e.g., a bag of sugar, 1L of water).

Measuring and comparing items with different masses will help students understand the necessity for using the same unit of measurement when comparing the amount of matter those objects contain. Having opportunities to compare objects that are alike will strengthen the understanding that the same object rearranged will maintain its original mass.

There are different types of scales that students can use to measure mass. These include pan balance scales, beam balance scales, and kitchen scales. Ensure all scales are accurate prior to having students use them.



SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**

- selecting and justifying referents for the units g and kg
- modelling and describing the relationship between the units g and kg
- estimating mass using referents
- measuring and recording mass.

[C, CN, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide a personal referent for one gram and explain the choice.
- Provide a personal referent for one kilogram and explain the choice.
- Match a given standard unit to a given referent.
- Explain the relationship between 1000 grams and 1 kilogram using a model.
- Estimate the mass of a given object using personal referents.
- Determine and record the mass of a given 3-D object.
- Measure, using a scale, and record the mass of given everyday objects using the units g and kg.
- Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g and 1 kg.
- Determine the mass of two given similar objects with different masses and explain the results.
- Determine the mass of an object, change its shape, re-measure its mass and explain the results.

SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**

- selecting and justifying referents for the units g and kg
- modelling and describing the relationship between the units g and kg
- estimating mass using referents
- measuring and recording mass.

[C, CN, ME, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Have students compare the mass of objects to an established gram, 100 gram and 1 kg mass.
- Have students create masses of 1 g, 100 g, 1 kg (e.g., ask students to fill containers with various materials until they think a mass of 1 kg is reached).
- Have students find common items that are measured in grams and kilograms. Create a classroom display.
- Have students measure mass on a balance scale or other more accurate scales, as opposed to a bathroom scale with which mass is harder to interpret.
- Ensure students estimate and measure mass, using grams and kilograms as the units.
- Provide situations in which students make comparisons between the masses of two objects, one in grams; the other in kilograms.
- Provide opportunities for students to explore what happens to the mass of the same object if the shape of the object changes.

### **Suggested Activities**

- Have students estimate and then measure the mass of different objects in the classroom.
- Ask the student to predict, from a collection of objects, which one has a mass of about 1 kilogram.
- Ask the student to choose a small item. Next have the student estimate and determine how many of the items would be required to make a mass of a kilogram.
- Ask the student to find something which has the same mass as two bags of marbles.
- Ask the student to find the number of potatoes in 2 kg. Ask: Will the number always be the same? Why or why not?
- Have students predict and measure the number of pennies needed for a mass of 100 grams. Repeat with other coins. Ask students how much 1 kg of that coin would be worth.
- Have students measure 20 g of unpopped popcorn. Have students predict, if the mass will be greater, the same, or less after it has been popped? Have students compare how much space is taken up by the popped versus unpopped popcorn.
- Investigate the number of kilograms students could comfortably carry in their backpack or the total number of kilograms of a group of books on a shelf, etc.
- Use balance scales to have students investigate the mass of different kinds of balls (e.g., ping pong ball vs. golf ball).
- Have students write what they know about the relationship between 1000 grams and a kilogram.
- Have the students select a personal referent for 1 g and 1 kg and explain their choice.

**Possible Models:** pan balance or beam balance, sets of standard weights, base ten blocks, variety of objects to weigh

SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**

- selecting and justifying referents for the units g and kg
- modelling and describing the relationship between the units g and kg
- estimating mass using referents
- measuring and recording mass.

[C, CN, ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask: Could you eat a 1 kg cantaloupe? 1 kg of popcorn? Have students explain their thinking.
- Have students discuss which unit (g or kg) is more likely to be used in measuring:
  - a bag of potatoes
  - a box of paper clips
  - an apple
  - a bicycle
- Have students measure the mass of a ball of modeling clay. Have them use all of the clay to make a new object. Ask them to predict the mass of the new object and verify their prediction.
- Provide students with a collection of objects. Ask students to predict which have a mass of about 100 g and 1 kg. Have students explain their choices.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**

- estimating perimeter using referents for centimetre or metre
- measuring and recording perimeter (cm, m)
- constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.

[C, ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<p><b>SS4:</b> Measure length to the nearest non-standard unit by: using multiple copies of a unit; using a single copy of a unit (iteration process).</p> <p><b>SS5:</b> Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.</p>	<p><b>SS5</b> Demonstrate an understanding of perimeter of regular and irregular shapes by:</p> <ul style="list-style-type: none"> <li>• estimating perimeter, using referents for cm or m;</li> <li>• measuring and recording perimeter (cm, m);</li> <li>• constructing different shapes for a given perimeter (cm, m); to demonstrate that many shapes are possible for a perimeter.</li> </ul>	<p><b>SS3</b> Demonstrate an understanding of area of regular and irregular 2-D shapes by: recognizing that area is measured in square units; selecting and justifying referents for the units <math>\text{cm}^2</math> or <math>\text{m}^2</math>; estimating area by using referents for <math>\text{cm}^2</math> or <math>\text{m}^2</math>; determining and recording area' constructing different rectangles for a given area (<math>\text{cm}^2</math> or <math>\text{m}^2</math>) in order to demonstrate that many different rectangles may have the same area.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students extend their knowledge of measuring length to measuring a distance that is not a straight line.

**Perimeter** is a linear measure of a distance that is a continuous line and is often referred to as the “distance around” an object. Constructing this meaning for perimeter will enable students to recognize the outside of any object as its perimeter. This is a foundation for a later understanding of dimensions, area, and the area model for multiplication. The standard units used to measure perimeter, in Grade 3, are centimetres and metres. Students should also use **personal referents** when estimating perimeter. Through estimation, students can verify whether their measurements are reasonable. Estimation may also be the only measurement necessary.

Teachers should provide investigations with a variety of concrete materials to help students develop strategies for finding perimeter. These strategies should include those involving direct and indirect measurement. Students should also be given opportunities to construct shapes with a given perimeter. It is important students find perimeter of many different regular and irregular shaped objects before being introduced to pictorial and symbolic forms. The intent of this outcome is for students to explore their own methods for determining the perimeter of a shape and not developing or following a formula for perimeter.

SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**

- estimating perimeter using referents for centimetre or metre
- measuring and recording perimeter (cm, m)
- constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.

[C, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Measure and record the perimeter of a given regular shape, and explain the strategy used.
- Measure and record the perimeter of a given irregular shape, and explain the strategy used.
- Construct a shape for a given perimeter (cm, m).
- Construct or draw more than one shape for the same given perimeter.
- Estimate the perimeter of a given shape (cm, m) using personal referents.

SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**

- **estimating perimeter using referents for centimetre or metre**
- **measuring and recording perimeter (cm, m)**
- **constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.**

[C, ME, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Ask students to predict the perimeter prior to making their measurements.
- Provide students with frequent opportunities to construct, measure and record perimeter of regular and irregular shapes.
- Ask students to construct or draw more than one shape for the same given perimeter.
- Use perimeter problem solving situations that provide a context for students (e.g., border around rooms or bulletin boards, frames, fences, trim, etc.).
- Provide many opportunities for students to measure the perimeter of irregular shapes using indirect measure with materials such as a string and ruler,
- Ask students to make comparisons between quantities of one object and the perimeter of another to make estimations concerning one fitting the dimensions of the other.

### **Suggested Activities**

- Ask students "How can we find the distance around an object?"(Provide regular and irregular shapes).
- Give each group a metre stick, tape measure and a 30 cm ruler, and string. Ask them to figure out how to find the perimeter of objects around the classroom. Discuss different results.
- Give students pieces of string (different lengths) and ask, "How many different objects can you find with a perimeter that is equal to the length of your string?"
- Ask students "How many objects can you find with a perimeter of 10 cm? 30 cm? 1 m? 3 m?"
- Provide students with geoboards or grid paper and ask "How many different shapes can you make with a given perimeter?"
- Tell students "I have drawn a shape in grid paper with a perimeter of 24 cm. What might my shape look like?"
- Have students trace the outline of their bodies with sidewalk chalk and then estimate and measure the perimeter of their bodies.
- Use a trundle wheel to find the perimeter of the gym or playground.
- Ask students to estimate if a roll of crepe paper would be sufficient to go around the outside of their desk, a window, the door, the outside of the room...Explain why or why not?

**Possible Models:** string, 30 cm rulers, metre sticks, tape measure, Cuisenaire® rods, geoboards, grid paper, base ten units and rods, pentominoes, trundle wheel

- SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**
- **estimating perimeter using referents for centimetre or metre**
  - **measuring and recording perimeter (cm, m)**
  - **constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.**
- [C, ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Give students regular and irregular shapes and have them find the perimeter.
- Ask students to construct two different shapes with the same given perimeter.
- Ask students to estimate the perimeter of a given shape. Have them measure and record the actual length.
- Ask students to solve the following problem: "Farmer Bill has 48 metres of fencing. How many different rectangular chicken coops can he make?"

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**  
[C, CN, PS, R, V]

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>SS6:</b> Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.	<b>SS6:</b> Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.	<b>SS4</b> Describe and construct rectangular and triangular prisms.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

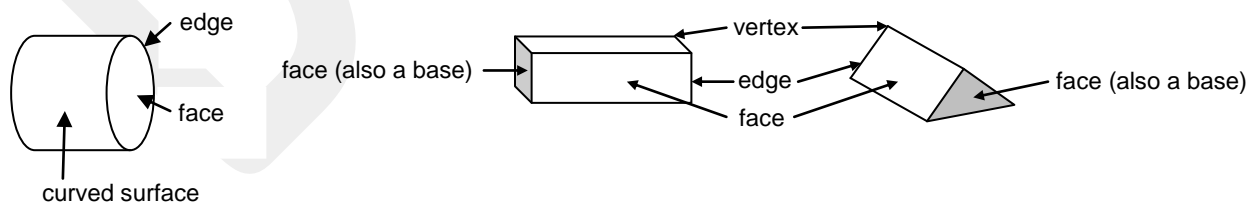
In Grade three, instruction is focused on describing and sorting 3-D objects according to their geometric **attributes**. Students will identify properties of objects and use proper mathematical vocabulary to describe them.

The 3-D objects used in Grade 3 include:

- **cubes**
- **spheres**
- **cones**
- **cylinders**
- **prisms**
- **pyramids**

The geometric attributes of 3-D objects are:

- **face**: a 2-D shape that forms part of a 3-D object. It is a flat surface that can be traced. Both the shape of the face and the number of faces should be considered as an attribute.
- **edge**: occurs where two surfaces of a 3-D object join.
- **vertex (vertices)**: a point where 3 or more edges meet. Note: on a cone, a vertex is the highest point above the base.
- **curved surface**: is a surface that is not flat, and not typically described as a face. These are found on cylinders, cones, and spheres.



Provide students with opportunities to explore these attributes through sorting and construction activities. Building **skeletons** for these objects will highlight characteristics within the categories of objects and of the objects themselves. As they become more familiar with identifying the attributes, students can determine the number of faces, edges, and vertices.



SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**

[C, CN, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify the faces, edges and vertices of given 3-D objects, including cubes, spheres, cones, cylinders, pyramids and prisms.
- Identify the shape of the faces of a given 3-D object.
- Determine the number of faces, edges and vertices of a given 3-D object.
- Construct a skeleton of a given 3-D object and describe how the skeleton relates to the 3-D object.
- Sort a given set of 3-D objects according to the number of faces, edges or vertices.

SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**

[C, CN, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide students with concrete models of given 3-D objects (geometric solids or other objects), including cubes, spheres, cones, cylinders, pyramids, and prisms.
- Identify and sort 3-D objects according to the number of faces, edges and vertices.
- Construct and describe skeletons of 3-D objects.
- Use cross-curricular opportunities to explore 3-D objects in Art and Science classes.
- Ask students to identify particular 3-D objects in their environment and in pictures and to justify their answers.

### **Suggested Activities**

- Ask students "In a bag I have an object that has flat faces, and straight edges. What might this object be?" (Other attributes should be used to extend this activity.)
- Tell students, "The object behind my back is able to roll." Ask what might it be? (Other attributes should be used to extend this activity.)
- Have the students create a mini book about 3-D objects that includes a picture of each and its attributes.
- Ask students to build a wall using 3-D objects. Discuss what 3-D objects could and could not be used.
- Ask students, "What can you tell me about a pyramid? A cone? A prism?" Have them focus on the attributes.
- Have students build skeletons of 3-D objects using toothpicks and marshmallows, the toothpicks are the edges, the marshmallows are the vertices. Have students describe their objects.
- Give each group a collection of 3-D objects. Have them sort the objects according to the geometric attributes and provide the sorting rule.
- Have students choose two different 3-D shapes. Have them write and illustrate three ways they are the same, and different.
- Read *Sir Cumference and the Sword in the Cone* by Cindy Neuschwander. Discuss.
- Ask students to play a game of "Name That 3-D Shape". Students must determine the object from the clues given.
- Cut out and label pictures from magazines to build a collage and to identify 3-D objects in the environment.

**Possible Models:** toothpicks, straws, geometric solids, Polydrons, modelling clay

SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**  
[C, CN, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask students to describe objects according to their attributes, making sure correct mathematical terms are used for names of objects and faces, edges and vertices.
- Have 3-D objects sorted by attribute, and ask students for the sorting rule.
- Have students make “Wanted” posters for 3-D objects, describing number of faces, edges and vertices, and shapes of faces.
- Have students construct a skeleton of a 3-D object and describe it using correct terminology.
- Have students sort a group of objects according to the number of faces, edges and vertices.
- Have students identify the shape of the faces of a given 3-D object.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS7: Sort regular and irregular polygons, including:**

- triangles
- quadrilaterals
- pentagons
- hexagons
- octagons

according to the number of sides.

[C, CN, R, V]

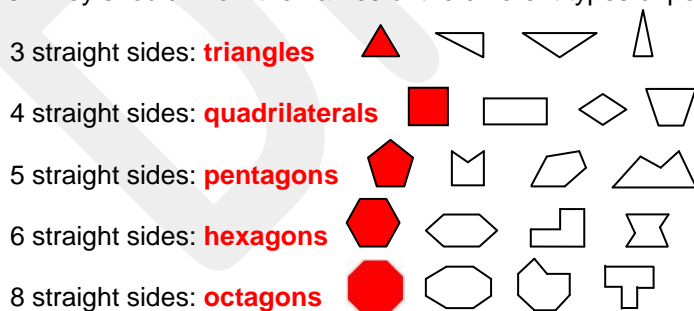
[C] Communication  
[T] Technology[PS] Problem Solving  
[V] Visualization[CN] Connections  
[R] Reasoning[ME] Mental Math  
and Estimation**Scope and Sequence of Outcomes**

Grade Two	Grade Three	Grade Four
<b>SS7</b> Describe, compare and construct 3-D objects, including: cubes; spheres; cones; cylinders' pyramids. <b>SS8</b> Describe, compare and construct 2-D shapes, including: triangles; squares; rectangles; circles.	<b>SS7</b> Sort regular and irregular polygons, including: <ul style="list-style-type: none"> <li>• triangles</li> <li>• quadrilaterals</li> <li>• pentagons</li> <li>• hexagons</li> <li>• octagons</li> </ul> according to the number of sides.	

**ELABORATION****Guiding Questions:**

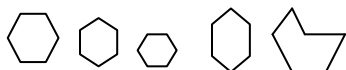
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

During their prior schooling, students will have had many opportunities to explore 2-D shapes through sorting, patterning, and building activities. Students' previous experiences with describing and comparing **polygons** included squares, triangles, and rectangles. Polygons are 2-D shapes with three or more straight sides. In Grade 3, students will extend their knowledge to include both regular and irregular polygons. **Regular polygons** have all equal sides and angles (e.g., equilateral triangles, squares, yellow hexagon pattern block). **Irregular polygons** do not have all sides the same size. Students should focus comparing the number of sides as the key attribute for classifying different polygons. They should know the names of the different types of polygons.



**Regular** polygons  
are shaded red

Students should be given opportunities to explore both regular and irregular polygons. Although pattern blocks are regularly used for geometric inquiry, most of the shapes are regular. Students may develop the misconception that only certain familiar polygons meet the criteria for these shapes. For example, students may not initially recognize all of the shapes below as hexagons.



Varying the orientation, form, configuration and size of shapes helps students focus on the number of sides as the key attribute for classifying polygons.

SCO: **SS7: Sort regular and irregular polygons, including:**

- triangles
- quadrilaterals
- pentagons
- hexagons
- octagons

**according to the number of sides.**

[C, CN, R, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Classify a given set of regular and irregular polygons according to the number of sides.
- Identify given regular and irregular polygons having different dimensions.
- Identify given regular and irregular polygons having different orientations.

SCO: SS7: Sort regular and irregular polygons, including:

- triangles
- quadrilaterals
- pentagons
- hexagons
- octagons

according to the number of sides.

[C, CN, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Use questioning to assist student focusing on the attributes of polygons. For example: What other shapes look like this one? In what way are the shapes alike? In which ways are they different?
- Provide opportunities for students to develop their own definitions for the different types of polygons. Have students sort shapes such as those included as black line masters in *Teaching Student-Centered Mathematics K-3*. Have students explain their reasoning to others.
- Have students create different polygons on geoboards or dot paper. Challenge the students to create different types of triangles (quadrilaterals, pentagons, etc.).
- Have students create a book of polygon shapes. Include a variety of examples (regular and irregular) for each type of polygon included in this outcome. This could be extended to include other types of polygons.
- Use geo-strips or strips of paper of different lengths to create various polygons.
- Use children's literature, such as *The Greedy Triangle* by Marilyn Burns and *The Warlord's Puzzle* by Virginia Walton Pilegard to further explore the attributes of polygons.
- Integrate art activities using these shapes. For example, create a piece of art using only a single 2-D shape, but changing the other attributes (size, orientation, length of sides, colour, etc.).

### **Suggested Activities**

- Ask the students to make a triangle on a geoboard that has 2 pegs inside, then one that has three. Ask: What is the greatest number of pegs that can be inside a triangle on a geoboard? Repeat this activity with other shapes.
- Have students sort a collection of pattern blocks by the type of polygon.
- Provide students with sets of tangrams and pentominoes. Have them sort the shapes into triangles, quadrilaterals, pentagons, hexagons, and octagons. Note: there are no pentagons and there are some shapes that have more than 8 sides.
- Have groups of students create a "path of polygons" using sidewalk chalk and drawing a sequence of different polygons.

**Possible Models:** geoboard, geo-strips, tangrams, pentominoes, pattern blocks, dot paper

SCO: **SS7: Sort regular and irregular polygons, including:**

- triangles
- quadrilaterals
- pentagons
- hexagons
- octagons

**according to the number of sides.**

[C, CN, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Provide students with a sheet that includes a number of different polygons (regular and irregular) that are different sizes, forms, configurations, and/or orientations. Have the students sort and name the polygons.
- Have students create two different pentagons (or other polygons) on a geoboard.
- Have students explain how an octagon and a hexagon (or other shapes) look similar and different.
- Ask: if you draw a pentagon (or other shape) and your friend draws a pentagon, will the two shapes look exactly the same? Why or why not?

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: SP1: Collect first-hand data and organize it using:

- tally marks
  - line plots
  - charts
  - lists
- to answer questions.  
[C, CN, V]

[C] Communication  
[T] Technology[PS] Problem Solving  
[V] Visualization[CN] Connections  
[R] Reasoning[ME] Mental Math  
and Estimation**Scope and Sequence of Outcomes**

Grade Two	Grade Three	Grade Four
<b>SP1</b> Gather and record data about self and others to answer questions.	<b>SP1</b> Collect first-hand data and organize it using: <ul style="list-style-type: none"> <li>• tally marks</li> <li>• line plots</li> <li>• charts</li> <li>• lists</li> </ul> to answer questions.	<b>SP1</b> Demonstrate an understanding of many-to-one correspondence.

**ELABORATION****Guiding Questions:**

- What do I want my students to learn?
- What do I want my students to understand and be able to do?

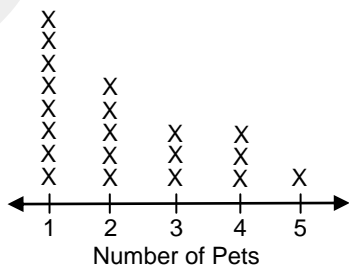
Students should develop strategies to collect and record information. This **first-hand data** (data that students have collected themselves) should relate to the students themselves, their school or community, and/or other topics that are meaningful to them. If students are collecting data through a survey, attention should be paid the questioning method used.

It is important to use real-world contexts in order to establish a purpose for collecting data, and to use this data to solve problems. Students can compare and analyze data then make predictions by arranging the data in a graph. The expectation is that students will communicate their understanding by recording data in an organized manner and by writing, asking and answering questions concerning data.

Students should be encouraged to organize and record their data using a **tally system**, **line plots**, **charts** and **lists** to solve problems. A line plot is a graph that uses a number line or words as its base and shows each piece of data with an “x”. Line plots are useful counts of things along a numeric scale. One advantage of a line plot is that every piece of data is shown on the graph (Van de Walle & Lovin, vol. 2, 2006, p. 333).

Number of Pets	Number of Students
1	III
2	
3	III
4	III
5	I

Tally system



Line plot

Number of Pets	Number of Students
1	8
2	5
3	3
4	3
5	1

Chart/List



SCO: SP1: Collect first-hand data and organize it using:

- tally marks
  - line plots
  - charts
  - lists
- to answer questions.  
[C, CN, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Record the number of objects in a given set using tally marks.
- Determine the common attributes of line plots by comparing line plots in a given set of data displays.
- Organize a given set of data using tally marks, line plots, charts or lists.
- Collect and organize data using tally marks, line plots, charts and lists.
- Answer questions arising from a given line plot, chart or list.
- Answer questions using collected data.

SCO: **SP1: Collect first-hand data and organize it using:**

- tally marks
  - line plots
  - charts
  - lists
- to answer questions.**  
[C, CN, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**



- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Have pairs of students decide on the procedure they will use to collect and display data that will show interesting information about class members.
- Have students plan and conduct an in-class survey about a favourite \_\_\_\_\_ (e.g., toy, television program, hockey player). Ask them to present the results of the survey in an organized chart or list.
- Have students conduct a survey to find out what objects 8- and 9-year-olds like to collect. They will need to decide who to survey and how to organize and present their data.
- Ask small groups of students to brainstorm an interesting list of questions for a possible survey.

### **Suggested Activities**

- Ask students why it is easier to count the “yes” responses when they are shown like this, , rather than like this, .
- Have students collect, record and organize data in a line plot, chart, or list to describe the favourite books of their classmates (or other relevant topic).
- Model recording a set of data in a line plot, list and chart format and discuss the advantages and disadvantages of each type of data display.
- Collect and display data that represent:
  - accomplishments of favourite sport figures or friends (e.g., the number of goals, hits, points)
  - the distance class members can throw a ball
  - prices of board games
  - mass of various fruits or vegetables
  - mass of subject textbooks
  - mass of different breeds of dogs
- Give students a list of questions and have them identify the questions that might be used for a particular graph or set of data.
- Ask students to build a list of criteria they would expect to find in every “well built” line plot.
- Give students a graph that has no labels nor name and ask them to label and name it. Then have them analyze the data by answering a given set of questions.

**Possible Models:** paper clips, linking cubes, craft sticks

SCO: SP1: Collect first-hand data and organize it using:

- tally marks
  - line plots
  - charts
  - lists
- to answer questions.  
[C, CN, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

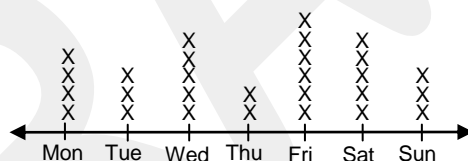
### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

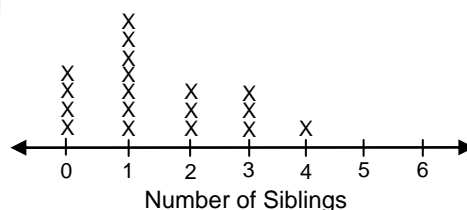
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/One-on-One Assessment**

- Ask students to select a topic, survey family members and/or neighbours, and present their findings to the class in an organized chart, list, or line plot.
- Ask the students to keep track of weather conditions over the period of one month and to design a way to present the information in an organized chart, list, or line plot.
- Show students a line plot such as the one below and ask what it may represent.



- Ask students how they would represent the sports the children in their class play and how many students play each sport?
- Show the student the following line plot and ask questions such as:  
What is the most common number of siblings?  
How many students have two siblings or less? How many students have 4 siblings?  
(Ensure students know that siblings are what their brothers and sisters are called.)



## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**  
[PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Two	Grade Three	Grade Four
<b>SP2</b> Construct and interpret concrete graphs and pictographs to solve problems.	<b>SP2</b> Construct, label and interpret bar graphs to solve problems.	<b>SP2</b> Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

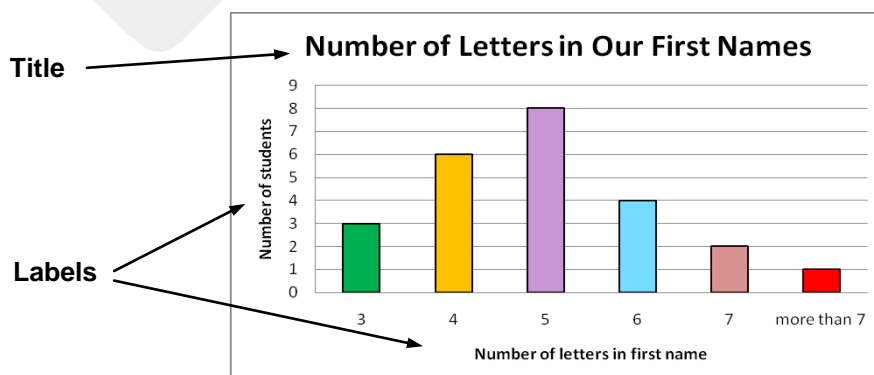
**Bar graphs** use the lengths or heights of bars to represent quantities. This is an extension of the Grade 2 outcome, where students created concrete graphs using models such as linking cubes. In Grade 3 it is helpful for students to work on grid paper to ensure the squares are all equal in size. **Bar graphs** can be constructed as **vertical** and **horizontal** displays. It is important for their displays to include **labels** and a **title**. Bar graphs in Grade 3 should be limited to a one-to-one correspondence (i.e., the number scale uses 1, 2, 3, ... and not multiples of 2, 5, 10, etc.).

Once students have constructed a graph, it is important for students to have an opportunity to make observations and interpret the data. They should also be given experiences discussing other graphs that they can find, such as in newspapers and magazines, and on television and the Internet.

Questioning should be ongoing whenever students use graphs to encourage students to interpret the data presented and to draw inferences. It is important to ask questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be posed. For example:

- What can you tell about.....by looking at this graph?
- How many more/less than....?
- Based on the information presented in the graph, what other conclusions can you make?
- Why do you think . . . ?

Sample bar graph:



SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**  
[PS, R, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine the common attributes, title and axes, of bar graphs by comparing bar graphs in a given set.
- Create bar graphs from a given set of data including labelling the title and axes.
- Draw conclusions from a given bar graph to solve problems.
- Solve problems by constructing and interpreting a bar graph.

SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**  
[PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Emphasize using real data when constructing graphs.
- Use grid paper to ensure bar graphs are as accurate as possible.
- Determine common attributes of bar graphs by examining examples from various sources.
- Make use of opportunities to integrate graphing concepts in other areas, such as science, morning message, social studies, etc.

### **Suggested Activities**

- Ask students to create a bar graph to show the kinds of pets students in the class have at home. Have them write two questions about their graph.
- Provide several bar graphs. Have students compare and determine the common attributes, making sure title, and axes and labels are included.
- Provide several bar graphs. Have students draw conclusions and answer questions about the graphs.
- Provide students with a real-life problems to solve such as "What game should we play in Phys Ed?" or "What special activity should be at the Celebration Assembly?" or "What book should be read during Literacy time?" Create a bar graph from collected data, and use it to make decisions or solve problems.
- Create a bar graph for a set of data on a grid on the floor or use a Learning Carpet®. (Categories cannot include more than 10 items if using the Learning Carpet®.)

**Possible Models:** grid paper, Learning Carpet®, pre-made bar graphs

SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**  
[PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

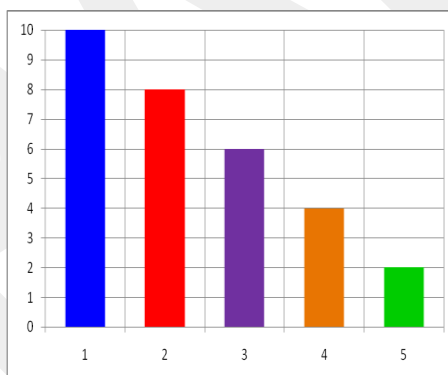
### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/One-on-One Assessment**

- Show students a bar graph on a topic of interest to students. Have them answer questions about the graphs and have them make up questions about the graph.
- Provide students with data. Have them construct a bar graph on grid paper.
- Have students answer the following: “This is a graph of a survey I did with my Grade 3 class. What might the survey be about? Label the graph, make up a title, and then, make up 3 questions that could be answered with this graph.”





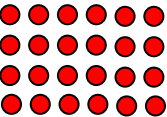
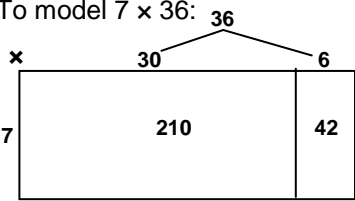


## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**






- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*



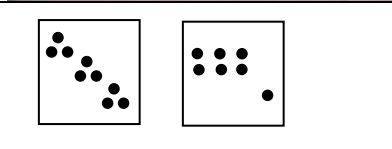
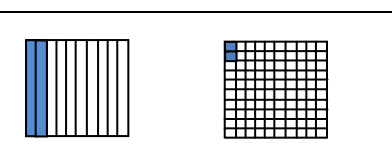


### GLOSSARY OF MODELS

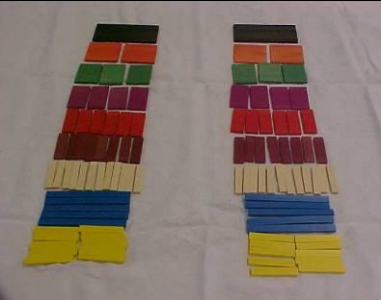
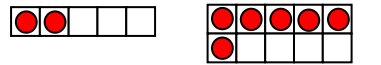



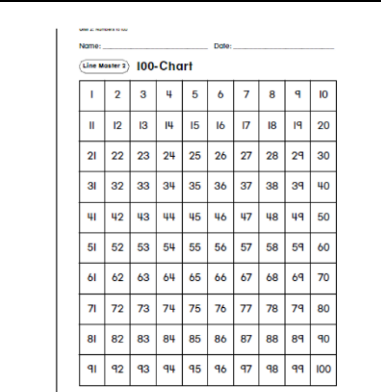
This glossary is identical for all grade levels (kindergarten to grade 8). Most of the models have a variety of uses at different grade levels. More information as to which models can be used to develop specific curriculum outcomes is located on the *Instructional Strategies* section of each four-page spread in this curriculum document. The purpose of this glossary is to provide a visual of each model and a brief description of it.

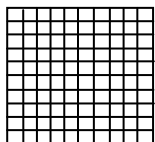
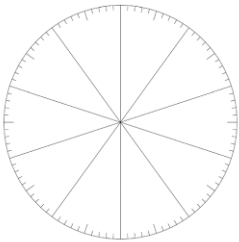


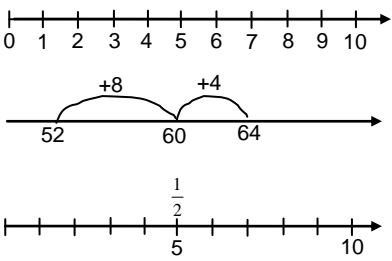
Name	Picture	Description
<b>Algebra tiles</b>		<ul style="list-style-type: none"> <li>Sets include “X” tiles (rectangles), “X<sup>2</sup>” tiles (large squares), and integer tiles (small squares).</li> <li>All tiles have a different colour on each side to represent positive and negative. Typically the “X” tiles are green and white and the smaller squares are red and white.</li> <li>Some sets also include “Y” sets of tiles which are a different colour and size than the “X” tiles.</li> </ul>
<b>Area Model</b>	<p>To model <math>12 \times 23</math>:</p> 	<ul style="list-style-type: none"> <li>Use base ten blocks to represent the parts of each number that is being multiplied.</li> <li>To find the answer for the example shown, students can add the various parts of the model: <math>200 + 30 + 40 + 6 = 276</math>.</li> <li>This model can also be used for fraction multiplication.</li> </ul>
<b>Arrays and Open Arrays</b>	<p>To model <math>4 \times 6</math>:</p>  <p>To model <math>7 \times 36</math>:</p> 	<ul style="list-style-type: none"> <li>Use counters arranged in equal rows or columns or a Black line Master with rows and columns of dots.</li> <li>Helpful in developing understanding of multiplication facts.</li> <li>Grids can also be used to model arrays.</li> <li>Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property.</li> </ul>
<b>Attribute Blocks</b>		<ul style="list-style-type: none"> <li>Sets of blocks that vary in their attributes: <ul style="list-style-type: none"> <li>5 shapes: circle, triangle, square, hexagon, rectangle</li> <li>2 thicknesses</li> <li>2 sizes</li> <li>3 colours</li> </ul> </li> </ul>
<b>Balance (pan or beam) scales</b>		<ul style="list-style-type: none"> <li>Available in a variety of styles and precision.</li> <li>Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units.</li> <li>Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance.</li> </ul>




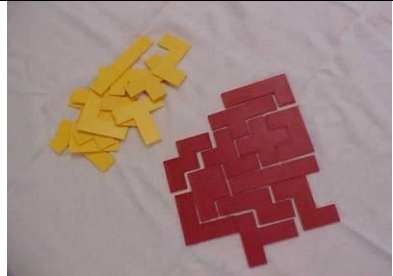

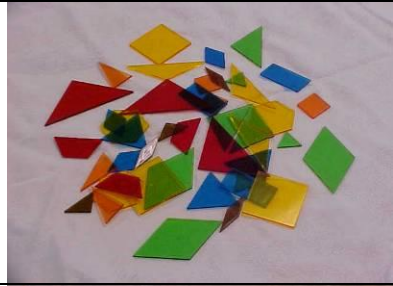
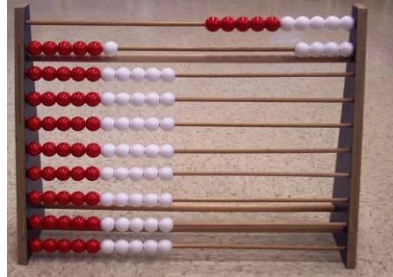
<b>Base Ten Blocks</b>		<ul style="list-style-type: none"> <li>• Include unit cubes, rods, flats, and large cubes.</li> <li>• Available in a variety of colours and materials (plastic, wood, foam).</li> <li>• Usually 3-D.</li> </ul>									
<b>Beam Balance</b>	☞ see Balance (pan or beam)										
<b>Carroll Diagram</b>	<p>Example:</p> <table border="1"> <tr> <td></td><td>1-digit</td><td>2-digit</td></tr> <tr> <td>Even</td><td>2, 4, 6, 8</td><td>26, 34</td></tr> <tr> <td>Odd</td><td>1, 3, 5, 7, 9</td><td>15, 21</td></tr> </table>		1-digit	2-digit	Even	2, 4, 6, 8	26, 34	Odd	1, 3, 5, 7, 9	15, 21	<ul style="list-style-type: none"> <li>• Used for classification of different attributes.</li> <li>• The table shows the four possible combinations for the two attributes.</li> <li>• Similar to a Venn Diagram</li> </ul>
	1-digit	2-digit									
Even	2, 4, 6, 8	26, 34									
Odd	1, 3, 5, 7, 9	15, 21									
<b>Colour Tiles</b>		<ul style="list-style-type: none"> <li>• Square tiles in 4 colours (red, yellow, green, blue).</li> <li>• Available in a variety of materials (plastic, wood, foam).</li> </ul>									
<b>Counters (two colour)</b>		<ul style="list-style-type: none"> <li>• Counters have a different colour on each side.</li> <li>• Available in a variety of colour combinations, but usually are red &amp; white or red &amp; yellow.</li> <li>• Available in different shapes (circles, squares, bean).</li> </ul>									
<b>Cubes (Linking)</b>		<ul style="list-style-type: none"> <li>• Set of interlocking 2 cm cubes.</li> <li>• Most connect on all sides.</li> <li>• Available in a wide variety of colours (usually 10 colours in each set).</li> <li>• Brand names include: Multilink, Hex-a-Link, Cube-A-Link.</li> <li>• Some types only connect on two sides (brand name example: Unifix).</li> </ul>									
<b>Cuisenaire Rods®</b>		<ul style="list-style-type: none"> <li>• Set includes 10 different colours of rods.</li> <li>• Each colour represents a different length and can represent different number values or units of measurement.</li> <li>• Usual set includes 74 rods (22 white, 12 red, 10 light green, 6 purple, 4 yellow, 4 dark green, 4 black, 4 brown, 4 blue, 4 orange).</li> <li>• Available in plastic or wood.</li> </ul>									

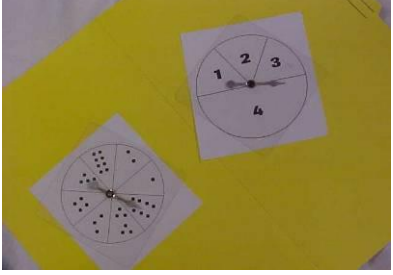
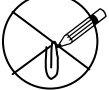


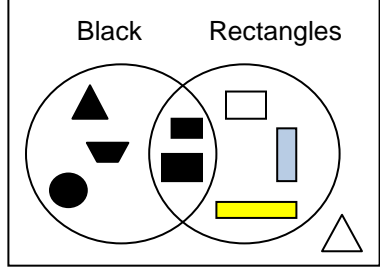
<b>Dice (Number Cubes)</b>		<ul style="list-style-type: none"> <li>Standard type is a cube with numbers or dots from 1 to 6 (number cubes).</li> <li>Cubes can have different symbols or words.</li> <li>Also available in: <ul style="list-style-type: none"> <li>4-sided (tetrahedral dice)</li> <li>8-sided (octahedral dice)</li> <li>10-sided (decahedra dice)</li> <li>12-sided, 20-sided, and higher</li> <li>Place value dice</li> </ul> </li> </ul>
<b>Dominoes</b>		<ul style="list-style-type: none"> <li>Rectangular tiles divided in two-halves.</li> <li>Each half shows a number of dots: 0 to 6 or 0 to 9.</li> <li>Sets include tiles with all the possible number combinations for that set.</li> <li>Double-six sets include 28 dominoes.</li> <li>Double-nine sets include 56 dominoes.</li> </ul>
<b>Dot Cards</b>		<ul style="list-style-type: none"> <li>Sets of cards that display different number of dots (1 to 10) in a variety of arrangements.</li> <li>Available as free Black line Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8).</li> </ul>
<b>Decimal Squares®</b>		<ul style="list-style-type: none"> <li>Tenths and hundredths grids that are manufactured with parts of the grids shaded.</li> <li>Can substitute a Black line Master and create your own class set.</li> </ul>
<b>Double Number Line</b>	☞ see Number lines (standard, open, and double)	
<b>Five-frames</b>	☞ see Frames (five- and ten-)	
<b>Fraction Blocks</b>		<ul style="list-style-type: none"> <li>Also known as Fraction Pattern blocks.</li> <li>4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle.</li> <li>Use with basic pattern blocks to help study a wider range of denominators and fraction computation.</li> </ul>
<b>Fraction Circles</b>		<ul style="list-style-type: none"> <li>Sets can include these fraction pieces: <math display="block">1, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}</math> </li> <li>Each fraction graduation has its own colour.</li> <li>It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole).</li> </ul>

<b>Fraction Pieces</b>		<ul style="list-style-type: none"> <li>Rectangular pieces that can be used to represent the following fractions:  <math>\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}</math></li> <li>Offers more flexibility as different pieces can be used to represent 1 whole.</li> <li>Each fraction graduation has its own colour.</li> <li>Sets available in different quantities of pieces.</li> </ul>
<b>Frames (five- and ten-)</b>		<ul style="list-style-type: none"> <li>Available as a Black line Master in many resources or you can create your own.</li> <li>Use with any type of counter to fill in the frame as needed.</li> </ul>
<b>Geoboards</b>		<ul style="list-style-type: none"> <li>Available in a variety of sizes and styles. <ul style="list-style-type: none"> <li>5 x 5 pins</li> <li>11 x 11 pins</li> <li>Circular 24 pin</li> <li>Isometric</li> </ul> </li> <li>Clear plastic models can be used by teachers and students on an overhead.</li> <li>Some models can be linked to increase the size of the grid.</li> </ul>
<b>Geometric Solids</b>		<ul style="list-style-type: none"> <li>Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres.</li> <li>The number of pieces in a set will vary.</li> <li>Available in different materials (wood, plastic, foam) and different sizes.</li> </ul>
<b>Geo-strips</b>		<ul style="list-style-type: none"> <li>Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes.</li> <li>Strips come in 5 different lengths. Each length is a different colour.</li> </ul>
<b>Hundred Chart</b>		<ul style="list-style-type: none"> <li>10 x 10 grid filled in with numbers 1-100 or 0 - 99.</li> <li>Available as a Black line Master in many resources or you can create your own.</li> <li>Also available as wall charts or "Pocket" charts where cards with the numbers can be inserted or removed.</li> </ul>

<b>Hundred Grid</b>		<ul style="list-style-type: none"> <li>• 10 × 10 grid.</li> <li>• Available as Black line Master in many resources.</li> </ul>
<b>Hundredths Circle</b>	<p>Percent Circles</p> 	<ul style="list-style-type: none"> <li>• Circle divided into tenths and hundredths.</li> <li>• Also known as “percent circles”.</li> </ul>
<b>Learning Carpet®</b>	 <p><a href="http://www.thelearningcarpet.ca">http://www.thelearningcarpet.ca</a></p>	<ul style="list-style-type: none"> <li>• 10 × 10 grid printed on a floor rug that is six feet square.</li> <li>• Number cards and other accessories are available to use with the carpet.</li> </ul>
<b>Linking Cubes</b>	☞ see Cubes (Linking)	
<b>Mira®</b>		<ul style="list-style-type: none"> <li>• Clear red plastic with a bevelled edge that projects reflected image on the other side.</li> <li>• Other brand names include: Reflect-View and Math-Vu™.</li> </ul>
<b>Number Cubes</b>	☞ see Dice (Number Cubes)	
<b>Number Lines (standard, open, and double)</b>		<ul style="list-style-type: none"> <li>• Number lines can begin at 0 or extend in both directions.</li> <li>• Open number lines do not include pre-marked numbers or divisions. Students place these as needed.</li> <li>• Double number lines have numbers written above and below the line to show equivalence.</li> </ul>
<b>Open Arrays</b>	☞ see Arrays and Open Arrays	
<b>Open Number Lines</b>	☞ see Number Lines (standard, open, and double)	
<b>Pan Balance</b>	☞ see Balance (pan or beam)	



<b>Pattern Blocks</b>		<ul style="list-style-type: none"> <li>Standard set includes: Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms.</li> <li>Available in a variety of materials (wood, plastic, foam).</li> </ul>
<b>Pentominoes</b>		<ul style="list-style-type: none"> <li>Set includes 12 unique polygons.</li> <li>Each is composed of 5 squares which share at least one side.</li> <li>Available in 2-D and 3-D in a variety of colours.</li> </ul>
<b>Polydrons</b>		<ul style="list-style-type: none"> <li>Geometric pieces snap together to build various geometric solids as well as their nets.</li> <li>Pieces are available in a variety of shapes, colours, and sizes: Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons</li> <li>Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames™.</li> </ul>
<b>Power Polygons™</b>		<ul style="list-style-type: none"> <li>Set includes the 6 basic pattern block shapes plus 9 related shapes.</li> <li>Shapes are identified by letter and colour.</li> </ul>
<b>Rekenrek</b>		<ul style="list-style-type: none"> <li>Counting frame that has 10 beads on each bar: 5 white and 5 red.</li> <li>Available with different number of bars (1, 2, or 10).</li> </ul>

<b>Spinners</b>		<ul style="list-style-type: none"> <li>• Create your own or use manufactured ones that are available in a wide variety:               <ul style="list-style-type: none"> <li>○ number of sections;</li> <li>○ colours or numbers;</li> <li>○ different size sections;</li> <li>○ blank.</li> </ul> </li> <li>• Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins.</li> </ul> 
<b>Tangrams</b>		<ul style="list-style-type: none"> <li>• Set of 7 shapes (commonly plastic):               <ul style="list-style-type: none"> <li>○ 2 large right-angle triangles</li> <li>○ 1 medium right-angle triangle</li> <li>○ 2 small right-angle triangles</li> <li>○ 1 parallelogram</li> <li>○ 1 square</li> </ul> </li> <li>• 7-pieces form a square as well as a number of other shapes.</li> <li>• Templates also available to make sets.</li> </ul>
<b>Ten-frames</b>	☞ see Frames (five- and ten-)	
<b>Trundle Wheel</b>		<ul style="list-style-type: none"> <li>• Tool for measuring longer distances.</li> <li>• Each revolution equals 1 metre usually noted with a click.</li> </ul>
<b>Two Colour Counters</b>	☞ see Counters (two colour)	
<b>Venn Diagram</b>		<ul style="list-style-type: none"> <li>• Used for classification of different attributes.</li> <li>• Can be one, two, or three circles depending on the number of attributes being considered.</li> <li>• Attributes that are common to each group are placed in the interlocking section.</li> <li>• Attributes that don't belong are placed outside of the circle(s), but inside the rectangle.</li> <li>• Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted.</li> <li>• Similar to a Carroll Diagram.</li> </ul>

**List of Grade 3 Specific Curriculum Outcomes****Number (N)**

1. Say the number sequence forward and backward from 0 to 1000 by:  
5s, 10s, or 100s, using any starting point; 3s using starting points that are multiples of 3; 4s using starting points that are multiples of 4; 25s, using starting points that are multiples of 25
2. Represent and describe numbers to 1000, concretely, pictorially and symbolically.
3. Compare and order numbers to 1000.
4. Estimate quantities less than 1000 using referents.
5. Illustrate, concretely & pictorially, the meaning of place value for numerals to 1000
6. Describe and apply mental mathematics strategies for adding two 2-digit numerals.
7. Describe and apply mental mathematics strategies for subtracting two 2-digit numerals.
8. Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.
9. Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals).
10. Apply mental mathematics strategies and number properties, such as: using doubles; making 10; using the commutative property; using the property of zero; thinking addition for subtraction to determine answers for basic addition facts and related subtraction facts (to 18).
11. Demonstrate an understanding of multiplication to  $5 \times 5$ .
12. Demonstrate an understanding of division (limited to division related to multiplication facts up to  $5 \times 5$ ).
13. Demonstrate an understanding of fractions by: explaining that a fraction represents a part of a whole; describing situations in which fractions are used; comparing fractions of the same whole with like denominators.

**Patterns & Relations (PR)****(Patterns)**

1. Demonstrate an understanding of increasing patterns by: describing, extending, comparing, creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).
2. Demonstrate an understanding of decreasing patterns by: describing, extending, comparing, creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

**(Variables and Equations)**

3. Solve one-step addition and subtraction equations involving symbols representing an unknown number.

**Shape and Space (SS)****(Measurement)**

1. Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).
2. Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.
3. Demonstrate an understanding of measuring length (cm, m) by: selecting and justifying referents for the units cm and m; modelling and describing the relationship between the units cm and m; estimating length using referents; measuring and recording length, width and height.
4. Demonstrate an understanding of measuring mass (g, kg).
5. Demonstrate an understanding of perimeter of regular and irregular shapes.

**(3-D Objects and 2-D Shapes)**

6. Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.
7. Sort regular and irregular polygons, including: triangles, quadrilaterals, pentagons, hexagons, octagons, according to the number of sides.

**(Transformations)****Statistics and Probability (SP)****(Data Analysis)**

1. Collect first-hand data and organize it using: tally marks, line plots, charts, lists to answer questions.
2. Construct, label and interpret bar graphs to solve problems.

**(Chance and Uncertainty)**

## REFERENCES

- Alberta Education. *LearnAlberta.ca: Planning Guides K, 1, 4, and 7*, 2005-2008.
- American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.
- Banks, J.A. and C.A.M. Banks. *Multicultural Education: Issues and Perspectives*. Boston: Allyn and Bacon, 1993.
- Black, Paul and Dylan Wiliam. "Inside the Black Box: Raising Standards Through Classroom Assessment." *Phi Delta Kappan*, 20, October 1998, pp.139-148.
- British Columbia. Ministry of Education. *The Primary Program: A Framework for Teaching*, 2000.
- Caine, Renate Numella and Geoffrey Caine. *Making Connections: Teaching and the Human Brain*. Menlo Park, CA: Addison-Wesley Publishing Company, 1991.
- Computation, Calculators, and Common Sense. May 2005, NCTM.
- Davies, Anne. *Making Classroom Assessment Work*. British Columbia: Classroom Connections International, Inc., 2000.
- Hope, Jack A. et.al. *Mental Math in the Primary Grades* (p. v). Dale Seymour Publications, 1988.
- National Council of Teachers of Mathematics (NCTM). *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence*. Reston, VA: NCTM, 2006.
- \_\_\_\_\_, *Mathematics Assessment Sampler, Grades 3-5*. edited by Jane Reston, VA: NCTM, 2000.
- \_\_\_\_\_, *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.
- OECD Centre for Educational Research and Innovation. *Formative Assessment: Improving Learning in Secondary Classrooms*. Paris, France: Organization for Economic Co-operation and Development (OECD) Publishing, 2006.
- Rubenstein, Rheta N. *Mental Mathematics beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.
- Shaw, J.M. and Cliatt, M.F.P. (1989). "Developing Measurement Sense." In P.R. Trafton (Ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: National Council of Teachers of Mathematics.
- Small, M. *Making Math Meaningful to Canadian Students, K-8*. Toronto: Nelson Education Ltd., 2008.
- Steen, L.A. (ed.). *On the Shoulders of Giants – New Approaches to Numeracy*. Washington, DC: National Research Council, 1990.
- Stenmark, Jean Kerr and William S. Bush, Editor. *Mathematics Assessment: A Practical Handbook for Grades 3-5*. Reston, VA: National Council of Teachers of Mathematics, Inc., 2001.
- Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades K-3*. Boston: Pearson Education, Inc. 2006.
- Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 3-5*. Boston: Pearson Education, Inc. 2006.



---

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 5-8*. Boston: Pearson Education, Inc. 2006.

Western and Northern Canadian Protocol. *Common Curriculum Framework for K-9 Mathematics*, 2006.

DRAFT