The North Carolina High School Collaborative Instructional Framework

The NC High School Collaborative Instructional Framework was created in a collaborative process involving higher education, the Department of Public Instruction, district leadership, and teachers. As with the collaborative pacing guide, the Instructional Framework is optional. The framework includes instructional support and suggested pacing and sequencing of units. More instructional support has been provided by merging and adapting the Math Resource for Instruction (MRI) to the Collaborative Pacing Guides Units for Instruction. Based on feedback, the order of the units have changed in each course.

The Instructional Framework is a work in progress and will be updated throughout this school year.

The September 2017 Update has been completed. Please provide feedback through the links found in the footer of each page of the Instructional framework.

Tips for use:around common themes.

- ***Do not use Google Chrome to view the docs.*** *(Chrome sometimes adds extra spacing into equations.)*
- *The order of the standards in each unit does not imply the order the standards should be taught. The standards are organized*

The North Carolina High School Collaborative Instructional Framework

NC Math 1	NC Math 2	NC Math 3
Unit 1 - Equations and Introduction to Functions	Unit 1 - Transformations of Functions and Geometric Objects	Unit 1 - Functions and their Inverses
Unit 2 - Linear Functions	Unit 2 - Similarity and Congruency	Unit 2 - Exponential and Logarithmic Functions
Unit 3 - Introduction to Exponential Functions	Unit 3 - Quadratic Functions	Unit 3 - Polynomial Functions
Unit 4 - Introduction to Quadratic Functions and Equations	Unit 4 - Square Roots and Inverse Variation Functions	Unit 4 -Modeling with Geometry
Unit 5 - Systems of Equations and Inequalities	Unit 5 - Relationships in Triangles	Unit 5 - Reasoning with Circles, Parallelograms, and Triangles
Unit 6 - Descriptive Statistics	Unit 6 - Probability	Unit 6 - Introduction to Rational Functions
		Unit 7 - Introduction to Trigonometric Functions
		Unit 8 - Statistics

The North Carolina High School Collaborative Instructional Framework

NC Math 1

Unit 1: Equations & Introduction to Functions

10 Days Block Schedule **September 2017 Update** 20 Days Traditional Schedule

RESEARCH BRIEF: **Unit 1: Equations & Introduction to Functions**

Essential Questions:

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables change?
- How can equations and inequalities be used to model real world situations?

Learning Outcomes	Student Objectives
• Given an equation students will solve and justify their method and steps of solving. • Students will be able to interpret key features of expressions, equations, graphs, tables, and verbal descriptions in context. • Create an equation or inequality and interpret reasonable solutions in context. • Given a formula students will solve for a specified variable. • Given a function students will determine domain and range. • Given a function create an equation from various representations and use them to solve problems. • Given a function in function notation students will evaluate and interpret results in context. • Understand what it takes to be a function in categorical, numerical, and graphical scenarios. • Students should be able to understand functions as a correspondence between inputs and outputs.	• I will **solve** an equation or inequality and **justify** my steps. NC.M1.A-REI.3, NC.M1.A-REI.1, NC.M1.A.REI.12 • I will be able to **interpret** key features and solutions. NC.M1.A-SSE.1a, NC.M1.F-IF.4 • I will be able to **create** an equation or inequality from a word problem. NC.M1.A-CED.1 • I will **manipulate** a formula to **solve** for a specific variable. NC.M1.A-CED.4 • I will be able to **recognize** domain and range values in a function. NC.M1.F-IF.1 • I will **evaluate** a function for a given value. NC.M1.F-IF.2 • I will **understand** what a function is. NC.M1.F-IF.1 • I will **identify** the relationship between input and output. NC.M1.F-IF.1

Standards Addressed in this Unit

Construct expressions, equations, and inequalities from a given context and determine the appropriateness of the solution.

- **NC.M1.A-SSE.1a** : Interpret expressions that represent a quantity in terms of its context. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

- **NC.M1.A-REI.3**: Solve linear equations and inequalities in one variable.

- **NC.M1.A-REI.1**: Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

- **NC.M1.A-REI.12**: Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

- **NC.M1.A-CED.1**: Create equations that describe numbers or relationships. Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

- **NC.M1.A-CED.4**: Create equations that describe numbers or relationships. Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

Distinguish key features of a function given multiple representations.

- **NC.M1.F-IF.1**: Understand the concept of a function and use function notation. Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that:
    - if *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*
    - the graph of f is the graph of the equation $y = f(x)$.

- **NC.M1.F-IF.2**: Understand the concepts of a functions and use function notation. Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **NC.M1.F-IF.4**: Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

- **NC.M1.F-IF.6**: Interpret functions that arise in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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The Math Resource for Instruction - Customized for the Content of this Unit

NC.M1.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify parts of an expression using precise vocabulary (6.EE.2b)
- Interpret numerical expressions written in scientific notation (8.EE.4)
- For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)

Connections

- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)
- Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)
- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Quadratic term, exponential term

Mastering the Standard for this Unit

Comprehending the Standard

This set of standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

For this part of the standards, students recognize that the linear expression $mx + b$ has two terms, m is a coefficient, and b is a constant.

Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression.

Assessing for Understanding

Students should recognize that in the expression $2x + 1$, “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “$2x$” and “1” being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

Example: The height *(in feet)* of a balloon filled with helium can be expressed by $5 + 6.3s$ where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.

NC.M1.A-REI.3

Solve equations and inequalities in one variable.

Solve linear equations and inequalities in one variable.

Concepts and Skills

Pre-requisite

- Solving multi-step equations (8.EE.7)
- Solving two-step inequalities (7.EE.4)

Connections

- Create one variable linear equations and inequalities (NC.M1.A-CED.1)
- Justify a solution methods and the steps in the solving process (NC.M3.A-REI.1)
- Solve systems of linear equations (NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.

Mastering the Standard for this Unit

Comprehending the Standard

Students are taught to solve multi-step equations in 8th grade. Students should become fluent solving multi-step equations in Math 1.

Students were taught to solve two-step inequalities in 7th grade. In Math 1 students extend this skill to multi-step inequalities.

This should be taught with the mathematical reasoning found in NC.M1.A-REI.1. Students should not be presented with a list steps to solve a linear equation/inequalities. Like many purely procedural practices, such steps are only effective for linear equations. It is more effective for students to be taught the mathematical reasoning for the solving process as these concepts can be applied to all types of equations.

Assessing for Understanding

Students should be able to solve multistep linear equations and inequalities.

Example: Solve:

a) $\frac{7}{3}y - 8 = 111$

b) $3x - 2 > 9 + 5x$

c) $\frac{3+x}{7} = \frac{x-9}{4}$

d) $\frac{2}{3}x + 9 < 8(\frac{1}{3}x - 2)$

e) $\frac{1}{5}(10 - 20x) \leq - 14$

NC.M1.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.
- Solve multi-step equations (8.EE.7)

Connections

- Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)
- Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)
- Solve for a quantity of interest in a formula (NC.M1.A-CED.4)
- Solve linear and quadratic equations and systems of linear equations (NC.M1.A-REI.3, NC.M1.A-REI.4, NC.M1.A-REI.5, NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to defend their method of solving an equation and each step of the solving process.

New Vocabulary: quadratic equation

Mastering the Standard for this Unit

Comprehending the Standard

When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Properties of operations can be used to change expressions on either side of the equation to equivalent expressions.

In the properties of equality, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions.

Students do not have to name the property, but can describe property using mathematical reasoning.

Assessing for Understanding

Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency.

Example: To the right are two methods to solve the same equation. Justify each step in the solving process. Which method do you prefer? Why?

Method 1:

$$\begin{aligned}5(x+3)-3x&=55\\5x+15-3x&=55\\2x+15&=55\\2x+15-15&=55-15\\2x&=40\\\frac{2x}{2}&=\frac{40}{2}\\x&=20\end{aligned}$$

Method 2:

$$\begin{aligned}5(x+3)-3x&=55\\\frac{5(x+3)}{5}-\frac{3x}{5}&=\frac{55}{5}\\x+3-\frac{3}{5}x&=11\\\frac{2}{5}x+3&=11\\\frac{2}{5}x+3-3&=11-3\\\frac{2}{5}x&=8\\\frac{5}{2}\left(\frac{2}{5}\right)x&=\frac{5}{2}(8)\\x&=20\end{aligned}$$

For example: Transforming $2x-5=7$ to $2x=12$ is possible because $5=5$, so adding the same quantity to both sides of an equation makes the resulting equation true as well.	Students should be able to critique the solving process of others, recognize incorrect steps and provide corrective action to the process. **Example:** The following is a student solution to the inequality $\frac{5}{18}-\frac{x-2}{9}\leq\frac{x-4}{6}$. a) There are two mathematical errors in this work. Identify at what step each mathematical error occurred and explain why it is mathematically incorrect. b) How would you help the student understand his mistakes? c) Solve the inequality correctly. $\frac{5}{18}-\frac{x-2}{9}\leq\frac{x-4}{6}$ $\frac{5}{18}-\left(\frac{2}{2}\right)\frac{x-2}{9}\leq\left(\frac{3}{3}\right)\frac{x-4}{6}$ $\frac{5}{18}-\frac{2x-2}{18}\leq\frac{3x-4}{18}$ $5-(2x-2)\leq 3x-4$ $5-2x+2\leq 3x-4$ $7-2x\leq 3x-4$ $-5x\leq -11$ $x\leq\frac{11}{5}$ (https://www.illustrativemathematics.org/content-standards/HSA/REI/A/1/tasks/807) Note: While this standard does not cover inequalities, this could be a good extension.

NC.M1.A-REI.12

Represent and solve equations and inequalities graphically

Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Concepts and Skills

Pre-requisite

- Solve two-step linear inequalities (7.EE.4b)
- Solve linear inequalities in one variable (NC.M1.A-REI.3)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1)
- Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.

Mastering the Standard for this Unit

Comprehending the Standard

Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically.

It is an American tradition to shade the region that represent the solutions of the inequality. In other countries, they shade regions of the plane that do not contain solutions, marking that region out. This results in an unmarked solution region making it easier to identify and work with points in the solution region. This means that it is important for students to understand what the shaded region represents according to the context of the problem.

Assessing for Understanding

Students should be able to represent and interpret solutions to one variable inequalities on a number line.

Example: Elvira, the cafeteria manager, has to be careful with her spending and manages the cafeteria so that they can serve the best food at the lowest cost. To do this, Elvira keeps good records and analyzes all of her budgets. Elvira's cafeteria has those cute little cartons of milk that are typical of school lunch. The milk supplier charges \$0.35 per carton of milk, in addition to a delivery charge of \$75. What is the maximum number of milk cartons that Elvira can buy if she has budgeted \$500 for milk?

a) Write and solve an inequality that models this situation, then graph the solution on a number line.
b) Describe in words the quantities that would work in this situation.

(www.mathematicsvisionproject.org)

Example: Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 900 pounds of weight for safety reasons.

a) Let p represent the total number of people. Write an inequality to describe the number of people that a boat can hold. Draw a number line diagram that shows all possible solutions.
b) Let w represent the total weight of a group of people wishing to rent a boat. Write an inequality that describes all total weights allowed in a boat. Draw a number line diagram that shows all possible solutions.

(https://www.illustrativemathematics.org/content-standards/tasks/642)

NC.M1.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

Concepts and Skills

Pre-requisite

- Create two-step linear equations and inequalities from a context (7.EE.4)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)
- Solve linear and quadratic equations and linear inequalities (NC.M1.A-REI.3, NC.M1.A-REI.4)
- Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)
- Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.
New Vocabulary: exponential function, quadratic function

Mastering the Standard for this Unit

Comprehending the Standard

Students create equations and inequalities in one-variable and use them to solve the problems.
In Math I, focus on linear, quadratic, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can also be created from an associated function.
After the students have created an equation, they can use other representations to solve problems, such as graphs and tables
Students in Math I are not responsible for interval notation as a solution. They are to write answers to these inequalities using inequality notation.

Assessing for Understanding

Students should be able to create an equation from a function and use the equation to solve problems.

Example: A government buys x fighter planes at z dollars each, and y tons of wheat at w dollars each. It spends a total of B dollars, where $B = xz + yw$. In (a)–(c), write an equation whose solution is the given quantity.

a) The number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each.
b) The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 a ton, for a total of $50 million.
c) The price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million.

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/580)

Students should be able to create equations and inequalities from various representations, such as verbal descriptions, and use them to solve problems.

Example: Mary and Jeff both have jobs at a baseball park selling bags of peanuts. They get paid $12 per game and $1.75 for each bag of peanuts they sell. Create equations, that when solved, would answer the following questions:

a) How many bags of peanuts does Jeff need to sell to earn $54?
b) How much will Mary earn if she sells 70 bags of peanuts at a game?
c) How many bags of peanuts does Jeff need to sell to earn at least $68?

NC.M1.A-CED.4

Create equations that describe numbers or relationships.

Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

Concepts and Skills

Pre-requisite

- Solve linear equations in one variable (8.EE.7 and NC.M1.A-REI.3)
- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number (8.EE.2)
- Justify a solution method and each step in the solving process (NC.M1.A-REI.1)

Connections

- Create an equation in two variables that represent a relationship between quantities (NC.M1.A-CED.2)
- Justify a solving method and each step in the solving process (NC. M1.A-REI.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify the steps in their solving process.

Mastering the Standard for this Unit

Comprehending the Standard

Students should be able to solve an equation for a given variable. In Math 1, focus on real mathematical and scientific formulas. This may be a good opportunity to talk with the science teachers and ask them for formulas that they use often.

This standard also covers solve for variables in mathematical forms as well as formulas.

(Students are <u>not</u> expected to write linear equation into "proper" standard form.)

This standards should be taught in conjunction with NC.M1.A-REI.1 in which students have to justify each step of the solving process and justify a particular solving method.

Assessing for Understanding

Students should be able to solve for variables in mathematical forms as well as formulas.

Example: Solve $(y - y_1) = m(x - x_1)$ for m.

Students should be able to solve for variable in science and math formula.

Example: (NCDPI Math I released EOC #18) Energy and mass are related by the formula $E = mc^2$.

- m is the mass of the object
- c is the speed of light

Which equation finds m, given E and c?

A) $m = E - c^2$

B) $m = Ec^2$

C) $m = \frac{c^2}{E}$

D) $m = \frac{E}{c^2}$

Example: In each of the equations below, rewrite the equation, solving for the indicated variable.

a) If F denotes a temperature in degrees Fahrenheit and C is the same temperature measured in degrees Celsius, then F and C are related by the equation, $F = 95C + 32$. Rewrite this equation to solve for C in terms of F.

b) The surface area S of a sphere of radius r is given by $S = 4\pi r^2$. Solve for r in terms of S.

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/4/tasks/1828)

Example: The equation for an object that is launched from the ground is given by $h(t) =- 16t^2 + v_0 t$ where h is the height, t is the time, and v_0 is the initial velocity. What is the initial velocity of an object that is one-hundred feet off the ground four

	seconds after it is launched?

NC.M1.F-IF.2

Understand the concept of a function and use function notation.

Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Concepts and Skills

Pre-requisite

- Use substitution to determine if a number if a solution (6.EE.5)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Define a function and use functions notation (NC.M1.F-IF.1)

Connections

- Creating and solving one variable equations (NC.M1.A-CED.1)
- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Function standards that relate domain and range (NC.M1.F-IF.3, NC.M1.F-IF.4, NC.M1.F-IF.5, NC.M1.F-IF.7)
- Comparing the end behavior of functions (NC.M1.F-LE.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the domain, range, input, output and the relationship between the variables of a function in context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard for this Unit

Comprehending the Standard

Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function.

Students should be able to interpret statements in function notation in contextual situations.

Assessing for Understanding

Students should be able to use evaluate functions written in function notation.

Example: Evaluate $f(2)$ for the function $f(x) = 5(x-3)+17$.

Evaluate $f(2)$ for the function $f(x) = 1200(1+.04)^x$.

Evaluate $f(2)$ for the function $f(x) = 3x^2 + 2x - 5$.

Students should be able to evaluate functions and interpret the result in a context.

Example: You placed a yam in the oven and, after 45 minutes, you take it out. Let f be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.

a) $f(0) = 65$

b) $f(5) < f(10)$

c) $f(40) = f(45)$

d) $f(45) > f(60)$

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/625)

Example: The rule $f(x) = 50(0.85)^x$ represents the amount of a drug in milligrams , $f(x)$, which remains in the bloodstream after x hours. Evaluate and interpret each of the following:

a) $f(0)$

b) $f(2) = k \cdot f(1)$. What is the value of k?

Example: Suppose that the function $f(x) = 2x + 12$ represents the cost to rent x movies a month from an internet movie club. Makayla now has \$10. How many more dollars does Makayla need to rent 7 movies next month?

(NCDPI Math 1 released EOC #12)

Example: Let $f(t)$ be the number of people, in millions, who own cell phones t years after 1990. Explain the meaning of the following statements.

a) $f(10) = 100.3$

b) $f(a) = 20$

c) $f(20) = b$

d) $n = f(t)$

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/634)

NC.M1.F-IF.1

Understand the concept of a function and use function notation.

Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that:

- if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.
- the graph of f is the graph of the equation $y = f(x)$.

Concepts and Skills

Pre-requisite

- Understand that a function is a rule that assigns to each input exactly one output (8.F.1)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)

Connections

- Create and graph two variable equations (NC.M1.A-CED.2)
- All other function standards

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to accurate describe a function in their own terms.

New Vocabulary: notation

Mastering the Standard for this Unit

Comprehending the Standard

Students should understand the definition of a function. It is deeper than just "x" cannot repeat or the vertical line test. Students should understand what it takes to be a function in categorical, numerical, and graphical scenarios.

In 8th grade, students studied the definition of a function. In Math 1, function notation is introduced. While this standard places a focus of the definition of a function on the correspondence of input and output values, a function can also be defined by how one variable changes in relation to another variable. This view of a function is highlighted in other standards throughout Math 1 when students are asked to identify, interpret, and use the rate of change.

For this unit, domain and range may be given in

Assessing for Understanding

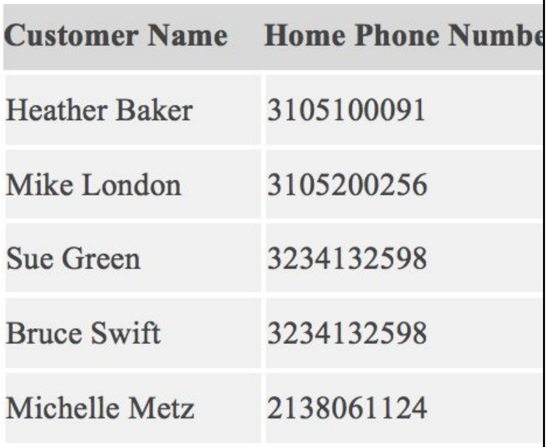
Students should be able to understand functions in categorical scenarios.

Example: A certain business keeps a database of information about its customers.

a) Let C be the rule which assigns to each customer shown in the table his or her home phone number. Is C a function? Explain your reasoning.

b) Let P be the rule which assigns to each phone number in the table above, the customer name(s) associated with it. Is P a function? Explain your reasoning.

c) Explain why a business would want to use a person's social security number as a way to identify a particular customer instead of their phone number.



Customer Name	Home Phone Numbe
Heather Baker	3105100091
Mike London	3105200256
Sue Green	3234132598
Bruce Swift	3234132598
Michelle Metz	2138061124

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624)

inequality notation. Students in Math I are not responsible for interval or set notation as a solution. They are to write answers to these inequalities using inequality notation.	Students should be able to understand functions as a correspondence between inputs and outputs. **Example:** A pack of pencils cost \$0.75. If n number of packs are purchased, then the total purchase price is represented by the function $t(n) = 0.75n$. a) Explain why t is a function. b) What is a reasonable domain and range for the function t? **Example:** Suppose f is a function. a) If $10 = f(-4)$, give the coordinates of a point on the graph of f. b) If 6 is a solution of the equation $f(w) = 1$, give a point on the graph of f. (https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/630)

NC.M1.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Concepts and Skills

Pre-requisite

- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

Connections

- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identification of key features and interpret those key features in context.

New Vocabulary: maximum, minimum

Mastering the Standard for this Unit

Comprehending the Standard

Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that "tell a story" hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions.

Assessing for Understanding

Students should be able to identify and interpret key features of functions.

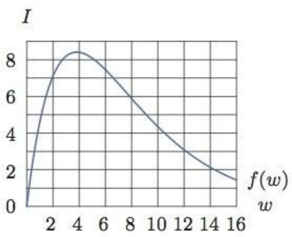
Example: An epidemic of influenza spreads through a city. The figure below is the graph of $I = f(w)$, where I is the number of individuals (in thousands) infected w weeks after the epidemic begins.

a) Estimate $f(2)$ and explain its meaning in terms of the epidemic.

b) Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form $f(a) = b$.

c) For approximately which w is $f(w) = 4.5$; explain what the estimates mean in terms of the epidemic.

d) An equation for the function used to plot the image above is $f(w) = 6w(1.3)^{-w}$. Use the graph to estimate the solution of the inequality $6w(1.3)^{-w} \geq 6$. Explain what the solution means in terms of the epidemic. *(This would make a great Honors level extension to this standard)*



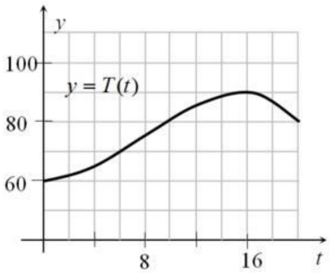
Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and maximum/minimum) for any given graph, not just “function families”. This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulation algebraically.

By contrast, NC.M1.F-IF.7, has students work with specific functions in which students have the ability to use algebraic manipulation to identify additional key features.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/637)

Example: The figure shows the graph of T, the temperature (in degrees Fahrenheit) over one particular 20-hour period in Santa Elena as a function of time t.

a) Estimate $T(14)$.
b) If $t = 0$ corresponds to midnight, interpret what we mean by $T(14)$ in words.
c) Estimate the highest temperature during this period from the graph.
d) When was the temperature decreasing?
e) If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?
(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/639)



Example: Eliana observed her dog, Lola, running around the yard and recorded the time and distance that Lola was away from her dog house in the table below.

a) Sketch a graph of Lola’s play time away from her dog house.
b) Describe what is happening between minutes 2 & 3.

Time (minutes)	**Distance (feet)**
0	0
1	5
2	30
3	15
4	25
5	50

NC.M1.F-IF.6

Interpret functions that arise in applications in terms of the context.

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Concepts and Skills

Pre-requisite

- Determine and interpret the rate of change of a linear function (8.F.4)
- Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)

Connections

- Interpret key features of graphs and tables (NC.M1.F-IF.4)
- Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: average rate of change

Mastering the Standard in this Unit

Comprehending the Standard

Students calculate the average rate of change of a function given a graph, table, and/or equation.

The average rate of change of a function $y = f(x)$ over an interval $a \le x \le b$ is $\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$.

This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.

It is an important connection for further courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature.

Assessing for Understanding

Students should be able to find the average rate of change over a specified interval.

Example: Find the average rate of change of each of the following functions over the interval $1 \le x \le 5$.

a) $f(x) = 3x - 7$
b) $g(x) = x^2 + 2x - 5$
c) $h(x) = 3(2)^x$

Example: The table below shows the average weight of a type of plankton after several weeks.

Time(weeks)	Weight (ounces)
8	0.04
9	0.07
10	0.14
11	0.25
12	0.49

(NCDPI Math 1 released EOC #21)

What is the average rate of change in weight of the plankton from week 8 to week 12?

a) 0.0265 ounce per week
b) 0.0375 ounce per week
c) 0.055 ounce per week
d) 0.1125 ounce per week

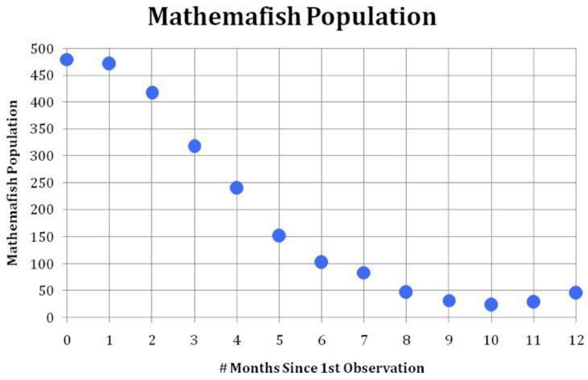
Example: The table below shows the temperature, T, in Tucson, Arizona t hours after midnight. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

t (hours after midnight)	0	3	4
T (temp. in ◦F)	85	76	70

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500)

Example: You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46



Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.

What to include in your summary report:

a) Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.
b) When was the population decreasing the fastest?
c) During what month did you notice the largest effects of the EPA intervention?
d) Explain the overall effects of the intervention.
e) Remember to justify all your conclusions using supporting evidence.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686)

The North Carolina High School Collaborative Instructional Framework

NC Math 1

Unit 2: Linear Functions

15 Days Block Schedule | **September 2017 Update** | 30 Days Traditional Schedule

RESEARCH BRIEF: **Unit 2: Linear Functions**

Essential Questions:

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables change?
- How can equations and inequalities be used to model real world situations?
- How can geometric properties in the coordinate plane be used to classify figures?

Learning Outcomes

- Students will describe the relationships among the graph, symbolic rule, table of values, and related situation for a linear function.
- Given a linear function, students will interpret the meaning of the slope and y-intercept of the function in context of the scenario.
- Students will interpret the meaning of the slope and y-intercept of the graph of a linear function in context.
- Given a graph, students will write the equation for a linear function.
- Given two points on a line, students will write the equation for the linear function passing through the points.
- Given a table of values, students will write the equation for the linear function associated with the table.
- Students will determine the domain and range of linear functions and interpret them in context of the scenario.

Student Objectives

- I will **interpret** the relationship of a linear function from various representations.
- I will **interpret** the meaning of the slope and y-intercept given a linear scenario. NC.M1.A-SSE.1a, NC.M1.A-SSE.1b
- I will **interpret** the meaning of the slope and y-intercept from a graph of a scenario. NC.M1.A-SSE.1a, NC.M1.A-SSE.1b
- I will **create** a graph of a linear function. NC.M1.A-CED.2
- I will **create** a linear equation given a graph, table of values, or verbal description. NC.M1.A-CED.2, NC.M1.F-BF.1a
- I will **identify** the domain and range of a linear function in context. NC.M1.F-IF.5, NC.M1.F-IF.7
- I will **predict** values based on a linear equation from a scenario.
- I will **use** technology, tables and graphs to approximate the solution to a linear equation. NC.M1.A-REI.11

- Students will use linear functions to answer questions about the situations that they describe.
- Students will use a linear model to predict the value of one variable given the value of the other and describe the rate of change in one variable as the other increases in a meaningful way.
- Students will use technology, graphs and table to approximate the solutions to linear equations.
- Students will use a graphing calculator or computer software to find the linear regression model for a set of data.
- Students will use technology to determine the correlation coefficient and interpret it as a measure of the strength and direction of a linear relationship .
- Students will write linear inequalities to represent scenarios.
- Students will estimate solutions to linear equations and inequalities by inspecting appropriate graphs and tables and interpret the meaning of the solutions in context.
- Students will solve and graph solutions of linear inequalities and interpret the meaning of the solution in context.
- Given an arithmetic sequence, students will write the recursive form for the sequence.
- Given an arithmetic sequence, students will write the explicit form for the sequence.
- Students will recognize that an arithmetic sequence is a linear function whose domain is a subset of the integers.
- Students will recognize that the terms of an arithmetic sequence are a subset of the range of a linear function.
- Students will use coordinates to find the midpoint or endpoint of a line segment.
- Students will use coordinates to prove the slope criteria for parallel and perpendicular lines.
- Students will find the equation of a line parallel or perpendicular to a given line and passing through a specified point.
- Students will use coordinates to compute the perimeter and area of triangles, rectangles, and polygons.
- Students will verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

- Given the value of one variable, I will **predict** the value of the other variable and **interpret** the rate of change in context.$^{\text{NC.M1.S-ID.6a, NC.M1.S-ID.7}}$
- I will **use** technology to find the linear regression from a set of data.$^{\text{NC.M1.S-ID.6a}}$
- I will **create and solve** linear inequalities from a scenario.
- I will **estimate** solutions from an equation or inequality by **analyzing** graphs and tables and **interpret** the meaning of the solution.
- I will **solve** and **graph** linear inequalities.
- I will **interpret** the meaning of the solutions to a linear inequality in context.
- I will **write** an arithmetic sequence in recursive form.$^{\text{NC.M1.F-BF.1a}}$
- I will **write** an arithmetic sequence in explicit form. $^{\text{NC.M1.F-BF.1a}}$
- I will **understand** the relationship between an arithmetic sequence and a linear function, including domain and range.
- I will use coordinates to **determine** the midpoint or endpoint of a line segment.$^{\text{NC.M1.G-GPE.6}}$
- I will **determine** lines are parallel, perpendicular, or neither as defined by the slope rules.$^{\text{NC.M1.G-GPE.5}}$
- I will **write** the equation of a line parallel or perpendicular to a given line through a given point.$^{\text{NC.M1.G-GPE.5}}$
- I will **compute** the perimeter and area of polygons using the Pythagorean Theorem.$^{\text{NC.M1.G-GPE.4}}$
- I will **verify/classify** the type of quadrilateral given four points.$^{\text{NC.M1.G-GPE.4}}$

Standards Addressed in this Unit

Identify, create, and graph linear equations and inequalities and interpret their key features.

- **NC.M1.A-SSE.1a**: Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.
- **NC.M1.A-SSE.1b**: Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.
- **NC.M1.A-CED.1**: Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.
- **NC.M1.A-REI.12**: Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.
- **NC.M1.A-CED.2**: Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.
- **NC.M1.F-BF.1a**: Build a function that models a relationship between two quantities. Write a function that describes a relationship between two quantities. Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).
- **NC.M1.A-REI.10**: Represent and solve equations and inequalities graphically. Understand that the graph of a two variable equation represents the set of all solutions to the equation.
- **NC.M1.A-REI.11**: Build an understanding of why the *x*-coordinates of the points where the graphs of two linear, exponential, or quadratic equations and intersect are the solutions of the equation and approximate solutions using a graphing technology or successive approximations with a table of values.
- **NC.M1.G-GPE.5**: Use coordinates to prove simple geometric theorems algebraically.Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems. Determine if two lines are parallel, perpendicular, or neither. Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

Determine the explicit and recursive formula for given arithmetic sequence.

- **NC.M1.F-IF.3**: Understand the concept of a function and use function notation. Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

- **NC.M1.F-BF.2:** Build a function that models a relationship between two quantities. Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.

- **NC.M1.A-REI.1:** Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

Understand and compare key features of linear functions.

- **NC.M1.F-LE.5:** Interpret expressions for functions in terms of the situation they model. Interpret the parameters a and b in a linear function f(x)=ax+b or an exponential function g(x)=abx in terms of a context.

- **NC.M1.F-IF.5:** Interpret functions that arise in applications in terms of the context. Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

- **NC.M1.S-ID.9:** Interpret linear models. Distinguish between association and causation.

- **NC.M1.F-IF.7:** Analyze functions using different representations. Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

- **NC.M1.F-IF.9:**Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Assess the line of best fit for a given set of data by using the correlation coefficient, residuals, and the least squares regression line.

- **NC.M1.S-ID.6a:** Summarize, represent, and interpret data on two categorical and quantitative variables. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a least squares regression line to linear data using technology. Use the fitted function to solve problems.

- **NC.M1.S-ID.6b:** Summarize, represent, and interpret data on two categorical and quantitative variables. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Assess the fit of a linear function by analyzing residuals.

- **NC.M1.S-ID.7:** Interpret linear models. Interpret in context the rate of change and the intercept of a linear model. Use the linear model to interpolate and extrapolate predicted values. Assess the validity of a predicted value.

- **NC.M1.S-ID.8**: Interpret linear models. Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.

Use geometric properties to classify & prove figures in the coordinate plane.

- **NC.M1.G-GPE.5**: Use coordinates to prove simple geometric theorems algebraically.Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems. Determine if two lines are parallel, perpendicular, or neither. Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

- **NC.M1.G-GPE.6**: Use coordinates to prove simple geometric theorems algebraically. Use coordinates to find the midpoint or endpoint of a line segment.

- **NC.M1.G-GPE.4**: Use coordinates to prove simple geometric theorems algebraically. Use coordinates to solve geometric problems involving polygons algebraically. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M1.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify parts of an expression using precise vocabulary (6.EE.2b)
- Interpret numerical expressions written in scientific notation (8.EE.4)
- For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)

Connections

- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)
- Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)
- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Quadratic term, exponential term

Mastering the Standard in this Unit

Comprehending the Standard

This set of standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

For this part of the standards, students recognize that the linear expression $mx + b$ has two terms, m is a coefficient, and b is a constant.

Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression.

Assessing for Understanding

Students should recognize that in the expression $2x + 1$, “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “$2x$” and “1” being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

> **Example**: The height *(in feet)* of a balloon filled with helium can be expressed by $5 + 6.3s$ where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.

NC.M1.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

Concepts and Skills

Pre-requisite

- Interpret a sum, difference, product, and quotient as a both a whole and as a composition of parts (6.EE.2b)
- Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)
- Interpret numerical expressions written in scientific notation (8.EE.4)

Connections

- Factor to reveal the zeros of functions and solutions to quadratic equations (NC.M1.A.SSE.3)
- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)
- Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)
- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential expression, quadratic expression

Mastering the Standard in this Unit

Comprehending the Standard

The set of A-SSE standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

Students identify parts of an expression as a single quantity and interpret the parts in terms of their context.

Assessing for Understanding

Students should understand that working with unsimplified expressions often reveals key information from a context.

Example: The expression $20(4x)+500$ represents the cost in dollars of the materials and labor needed to build a square fence with side length x feet around a playground. Interpret the constants and coefficients of the expression in context.

Example: Given that income from a concert is the price of a ticket times each person in attendance, consider the equation $I = 4000p - 250p^2$ that represents income from a concert where p is the price per ticket. What expression could represent the number of people in attendance?

Solution: *The equivalent factored form, p (4000-250p), shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize (4000-250p) as a single quantity for the number of people in attendance.*

NC.M1.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

Concepts and Skills

Pre-requisite

- Create two-step linear equations and inequalities from a context (7.EE.4)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)
- Solve linear and quadratic equations and linear inequalities (NC.M1.A-REI.3, NC.M1.A-REI.4)
- Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)
- Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard in this Unit

Comprehending the Standard

Students create equations and inequalities in one-variable and use them to solve the problems. In Math I, focus on linear contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can also be created from an associated function. After the students have created an equation, they can use other representations to solve problems, such as graphs and tables Students in Math I are not responsible for interval notation as a solution. They are to write answers to these inequalities using inequality notation.

Assessing for Understanding

Students should be able to create an equation from a function and use the equation to solve problems.

Example: A government buys x fighter planes at z dollars each, and y tons of wheat at w dollars each. It spends a total of B dollars, where $B = xz + yw$. In (a)–(c), write an equation whose solution is the given quantity.

a) The number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each.

b) The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 a ton, for a total of $50 million.

c) The price of a ton of wheat, given that a fighter plane costs 100,000times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million.

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/580)

Students should be able to create equations from various representations, such as verbal descriptions, and use them to solve problems.

Example: Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.

a) Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.

b) Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups,if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

(https://www.illustrativemathematics.org/content-standards/tasks/643)

NC.M1.A-REI.12

Represent and solve equations and inequalities graphically

Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Concepts and Skills

Pre-requisite

- Solve two-step linear inequalities (7.EE.4b)
- Solve linear inequalities in one variable (NC.M1.A-REI.3)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1)
- Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

5 – Use appropriate tools strategically

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.

Mastering the Standard

Comprehending the Standard

Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically. Similarly, we recognize linear inequalities to have infinitely many solutions.

It is an American tradition to shade the region that represent the solutions of the inequality. In other countries, they shade regions of the plane that do not contain solutions, marking that region out. This results in an unmarked solution region making it easier to identify and work with points in the solution region. This means that it is important for students to understand what the shaded region represents according to the context of the problem.

Assessing for Understanding

Students should be able to represent solutions to linear inequalities as a region of a plane.

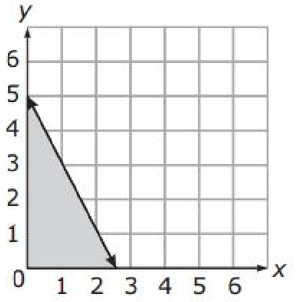
Example: What scenario could be modeled by the graph below? (multiple choice)

A) The number of pounds of apples, y, minus two times the number of pounds of oranges, x, is at most 5.

B) The number of pounds of apples, y, minus half the number of pounds of oranges, x, is at most 5.

C) The number of pounds of apples, y, plus two times the number of pounds of oranges, x, is at most 5.

D) The number of pounds of apples, y, plus half the number of pounds of oranges, x, is at most 5.



(NCDPI Math 1 released EOC #2)

NC.M1.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Concepts and Skills

Pre-requisite

- Construct a linear function that models the relationship between two quantities (8.F.4)
- Graph linear equations (8.EE.6)
- The graph of a function is the set of ordered pairs consisting of input and a corresponding output (8.F.1)
- Understand that the graph of a two variable equation represents the set of all solutions to the equation (NC.M1.A-REI.10)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Creating linear equations for a system (NC.M1.A-CED.3)
- Solving for a variable of interest in a formula (NC.M1.A-CED.4)
- The graph a function f is the graph of the equation $y = f(x)$ (NC.M1.F-IF.1)
- Interpret a function's domain and range in context (NC.M1.F-IF.5)
- Identify key features of linear, exponential and quadratic functions (NC.M1.F-IF.7)
- Building a function through patterns or by combining other functions (NC.M1.F-BF.1a , NC.M1.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and demonstrate its relation to the context.
New Vocabulary: exponential function, quadratic function

Mastering the Standard in this Unit

Comprehending the Standard

Students create equations in two variables.
Graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for the context and display adequate information about the relationship.
Students interpret the context and choose appropriate minimum and maximum values for a graph.
In Math I, focus on linear **contextual** situations for students to create equations in two variables.

Assessing for Understanding

Students should be able to create two variable equations from various representations, such as verbal descriptions, and use them to solve problems.

Students should be able to create a two variable equations, graph the relationship, and use graph to recognize key feature of the graph.

Example: The FFA had a fundraiser by selling hot dogs for \$1.50 and drinks for \$2.00. Their total sales were \$400.

a) Write an equation to calculate the total of \$400 based on the hot dog and drink sales.
b) Graph the relationship between hot dog sales and drink sales.
Note: This make a good connection to NC.M1.F-IF.5

NC.M1.F-BF.1a

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

a. Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Recognize arithmetic and geometric sequences as linear and exponential functions (NC.M1.F-IF.3)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

Connections

- Create and graph two variable equations (NC.M1.A-CED.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Translate between explicit and recursive forms (NC.M1.F-BF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify claims that a sequence defines a linear or exponential relationship.

New Vocabulary: arithmetic sequence, geometric sequence, exponential function

Mastering the Standard in this Unit

Comprehending the Standard

This standard is about building a function from different representations. In this part of the standard, the different representations include: sequences, graphs, verbal descriptions, tables, and ordered pairs.

This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

These functions can be written in function notation (linear or exponential) or as a sequence in explicit or recursive form.

Assessing for Understanding

Students should write functions from verbal descriptions as well as a table of values

Example: Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days.

Example: The table below shows the cost of a pizza based on the number of toppings.

Number of Toppings (n)	Cost (C)
1	$12.00
2	$13.50
3	$15.00
4	$16.50

Which function represents the cost of a pizza with n toppings?

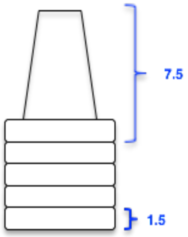
A) $C(n) = 12 + 1.5(n - 1)$

B) $C(n) = 1.5n + 12$

C) $C(n) = 12 + n$

D) $C(n) = 12n$

(NCDPI Math 1 released EOC #39)

Students should recognize explicit form of an arithmetic sequence as an equivalent structure to slope-intercept form of a linear function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the y-intercept, which gives meaning to the $n-1$ notation.	**Example:** The height of a stack of cups is a function of the number of cups in the stack. If a 7.5" cup with a 1.5" lip is stacked vertically, determine a function that would provide you with the height based on any number of cups. *Hint: Start with height of one cup and create a table, list, graph or description that describes the pattern of the stack as an additional cup is added.*  **Example:** There were originally 4 trees in an orchard. Each year the owner planted the same number of trees. In the 29th year, there were 178 trees in the orchard. Which function, $t(n)$, can be used to determine the number of trees in the orchard in any year, n? A) $t(n)=\frac{178}{29}n+4$ B) $t(n)=\frac{178}{29}n-4$ C) $t(n)=6n+4$ D) $t(n)=29n-4$ (NCDPI Math 1 released EOC #42) Students should write linear relationships as a sequence in explicit or recursive form. **Example:** Investigate the following sequence:  a) Create a recursive formula for the number of stars in the next pattern. b) Build an explicit formula for the number of stars in the nth pattern. c) How many stars are in the 43rd pattern? (www.visualpatterns.org)

NC.M1.A-REI.10

Represent and solve equations and inequalities graphically

Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Concepts and Skills

Pre-requisite

- Use substitution to determine if a number if a solution (6.EE.5)
- Graphing lines (8.EE.5, 8.EE.6, 8.F.3)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understanding functions as a rule that assigns each input with exactly one output (8.F.1)

Connections

- Creating and graphing two-variable equations (NC.M1.A-CED.2)
- Solutions to systems of equations (NC.M1.A-REI.5, NC.M1.A-REI.6)
- Understanding that the relationship between the solution of system of equations and the associated equation (NC.M1.A-REI.11)
- Representing the solutions to linear inequalities (NC.M1.A-REI.12)
- Relating a function to its graph, domain and range of a function (NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the solutions to a two variable equation and the link to a function.

Mastering the Standard in this Unit

Comprehending the Standard

Students understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement.

This standard contains no limitation and so applies to all function types, including those functions that a student cannot yet algebraically manipulate.

Students can explain and verify that every point (x, y) on the graph of an equation represents all values for x and y that make the equation true.

Assessing for Understanding

Students should be able to assess if a point is a solution to an equation.

Example: Consider three points in the plane, $P = (-4, 0)$, $Q = (-1, 12)$ *and* $R = (4, 32)$.

a) Find the equation of the line through P and Q.
b) Use your equation in (a) to show that R is on the same line as P and Q.

(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/10/tasks/1066)

Example: Which of the following points are on the graph of the equation $-5x + 2y = 20$? Which of the following points are of the graph of the equation? How do you know?

A) (4, 0)
B) (0, 10)
C) (-1, 7.5)
D) (2.3, 5)

NC.M1.A-REI.11

Represent and solve equations and inequalities graphically

Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)
- Solving systems of equations (NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

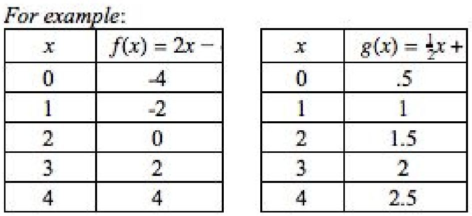
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

For a complete understanding, students will need exposure to both parts of this standard.



For example:

x	$f(x) = 2x -$
0	-4
1	-2
2	0
3	2
4	4

x	$g(x) = \frac{1}{2}x +$
0	.5
1	1
2	1.5
3	2
4	2.5

First, students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation.

Because $f(x) = g(x)$ when $x = 3$, 3 is the solution to the equation $2x - 4 = \frac{1}{2}x + \frac{1}{2}$ (As an extension, students could write an inequality to describe the relationship between the functions when $x < 3$ and when $x > 3$.)

In Math 1, students are expected to solve linear systems of equations algebraically. All other systems should be solved with technology, tables, and graphs.

Second, students should be able to use a system of equations to solve systems of equations.

For example:

Solve: $3x^2 - 2x + 1 = \frac{1}{2}x + 5$

Rewrite the equations as a system of equations: $f(x) = 3x^2 - 2x + 1$ $g(x) = \frac{1}{2}x + 5$

Using technology, graph the equations and look for points of intersection, where the same x produces $f(x) = g(x)$.

In Math 1, students are expected to solve linear equations using inverse operations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with

Assessing for Understanding

Students should be able to find approximate solutions to linear equations using technology, tables and graphs.

Example: The functions $f(m) = 18 + 0.4m$ and $g(m) = 11.2 + 0.54m$ give the lengths of two different springs in centimeters, as mass is added in grams, m, to each separately.

a) Graph each equation on the same set of axes.

b) What mass makes the springs the same length?

c) What is the length at that mass?

d) Write a sentence comparing the two springs.

technology, tables and graphs.	

NC.M1.G-GPE.5

Use coordinates to prove simple geometric theorems algebraically.

Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems.

- Determine if two lines are parallel, perpendicular, or neither.
- Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

Concepts and Skills

Pre-requisite

- Calculating rate of change given two points, a table or a graph (8.F.4)
- Derive the equation for a line in the coordinate plane (8.EE.6)

Connections

- Calculating and interpreting rate of change for a function (NC.M1.F-IF.6)
- Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)
- Analyze functions using different representations (NC.M1.F-IF.7, NC.M1.F-IF.9)
- Using concepts of points lines and planes to develop definitions of rigid motions in the plane (NC.M2.G-CO.2, NC.M2.G-CO.3, NC.M2.G-CO.4)
- Prove theorems about lines (NC.M2.G-CO.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others.

8 – Look for and express regularity in repeated reasoning.

- The slope formula is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

- Compare and contrast the equations of parallel and perpendicular lines. What similarities/differences must be present for parallel lines? Perpendicular lines? Intersecting lines?

Mastering the Standard in this Unit

Comprehending the Standard

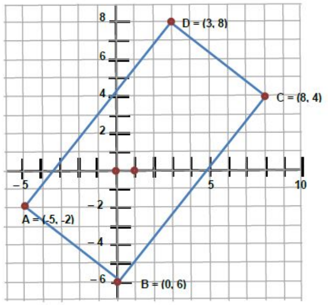
Students in 8th grade determine the slope and write the equation of non-vertical lines given two points, a table or graph. This standard is an extension and an application of this work as it asks students to compare two or more lines based on the characteristics of the lines presented.

- Parallelism – same slope
  $m_1 = m_2$, where $m = \frac{\Delta y}{\Delta x}$

Assessing for Understanding

Given coordinates, students can compare the characteristics, slopes and intercepts, of two or more lines. Student should be able to determine if two lines are parallel, perpendicular or intersecting based on the slopes of the two lines.

Example: Investigate the slopes of each of the sides of the rectangle ABCD (pictured on the right). What do you notice about the slopes of the sides that meet at a right angle? What do you notice about the slopes of the opposite sides that are parallel? Can you generalize what happens when you multiply the slopes of perpendicular lines?



● Perpendicularity – slopes are opposite reciprocals OR slopes have a product of (-1). $m_1 \cdot m_2 = -1$, where $m = \frac{\Delta y}{\Delta x}$ ● Intersecting – have completely different rates of change. It is useful to note that perpendicular lines are a subset of intersecting lines on coordinate plane. $m_1 \neq m_2$, where $m = \frac{\Delta y}{\Delta x}$ The slope formula ($m = \frac{y_2 - y_1}{x_2 - x_1}$) is an appropriate generalization and should be ***developed*** through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations. This is based on what students know about rate of change (slope) from MS mathematics.	Students should be able to find the slope and/or endpoint(s) of a line given the graph or coordinates of a line parallel or perpendicular to the given line. **Example:** Suppose a line k in a coordinate plane has slope $\frac{c}{d}$. a) What is the slope of a line parallel to k? Why must this be the case? b) What is the slope of a line perpendicular to k? Why does this seem reasonable? Students should be able to write the equation of line parallel or perpendicular to a given line. **Example:** Two points $A(0, -4)$, $B(2, -1)$ determine line AB . a) What is the equation of the line AB? b) What is the equation of the line perpendicular to line AB , passing through the point $(2, -1)$?

NC.M1.F-IF.3

Understand the concept of a function and use function notation.

Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

Concepts and Skills

Pre-requisite

- Interpret the equation $y = mx + b$ as being from a linear function and compare to nonlinear functions (8.F.3)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

Connections

- Relating the domain and range to a context (NC.M1.F-IF.5)
- Analyzing linear and exponential functions (NC.M1.F-IF.7)
- Build linear and exponential functions (NC.M1.F-BF.1)
- Translate between explicit and recursive forms (NC.M1.F-BF.2)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain a function written in recursive form using subset notation.
New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form, exponential function

Mastering the Standard in this Unit

Comprehending the Standard

Students should recognize that sequences are functions. A sequence can be described as a function, with the domain consisting of a subset of the integers, and the range being the terms of the sequence.
This standard should be taught with NC.M1.F-BF.2. Emphasize that arithmetic sequences are examples of linear functions.
It is important to note that sequences are not limited to arithmetic and geometric. It is expected that recursive form should be written in subset notation. Students should be familiar with writing and interpreting subset notation.
Now-Next can be used a tool for introduce the concepts of recursive form, but the expectation is that students will move to the more formal representations of recursive form.

Assessing for Understanding

Example: A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.

a) If the theater has 20 rows of seats, how many seats are in the twentieth row?
b) Explain why the sequence is considered a function.
c) What is the domain of the sequence? Explain what the domain represents in context.
d) What is the range of the sequence? Explain what the range represents in context.

NC.M1.F-BF.2

Build a function that models a relationship between two quantities.

Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3)
- Build functions from arithmetic and geometric sequences (NC.M1.F-BF.1a)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain their model in context.

New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form

Mastering the Standard in this Unit

Comprehending the Standard

Students should be able to use both the explicit and recursive forms of arithmetic sequences where the explicit form is a linear function.

Students are expected to use formal notation:

- a_n (NOW)
- a_{n-1} (PREVIOUS)
- a_{n+1} (NEXT)

(Students can use NEXT-NOW notation as they learn to recursive functions but will need to move to formal notation.)

This standard should be tied to NC.M1.F-IF.3, recognizing patterns and linking to function types.

Assessing for Understanding

Students should be able to build explicit and recursive forms of arithmetic sequences.

Example: A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats.

a) Write a recursive formula to find the number of seats in each row. How many seats are in row 5?
b) Write the explicit formula to determine which row has 94 seats?

Example: Given the sequence defined by the function $a_{n+1} = a_n + 12$ with $a_1 = 4$. Write an explicit function rule.

Note: Student may interpret 4 as the y-intercept since it is the first value; however, attending to the notation when $x = 1, y = 4$. Thus, the y-intercept for the explicit form is -8.

NC.M1.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.
- Solve multi-step equations (8.EE.7)

Connections

- Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)
- Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)
- Solve for a quantity of interest in a formula (NC.M1.A-CED.4)
- Solve linear and quadratic equations and systems of linear equations (NC.M1.A-REI.3, NC.M1.A-REI.4, NC.M1.A-REI.5, NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to defend their method of solving an equation and each step of the solving process.

New Vocabulary: quadratic equation

Mastering the Standard in this Unit

Comprehending the Standard

When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Properties of operations can be used to change expressions on either side of the equation to equivalent expressions.

In the properties of equality, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions.

Students do not have to name the property, but can describe the property using mathematical

Assessing for Understanding

Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency.

Example: Below are two methods to solve the same equation. Justify each step in the solving process. Which method do you prefer? Why?

Method 1:

$$5(x+3) - 3x = 55$$
$$5x + 15 - 3x = 55$$
$$2x + 15 = 55$$
$$2x + 15 - 15 = 55 - 15$$
$$2x = 40$$
$$\frac{2x}{2} = \frac{40}{2}$$
$$x = 20$$

Method 2:

$$5(x+3) - 3x = 55$$
$$\frac{5(x+3)}{5} - \frac{3x}{5} = \frac{55}{5}$$
$$x + 3 - \frac{3}{5}x = 11$$
$$\frac{2}{5}x + 3 = 11$$
$$\frac{2}{5}x + 3 - 3 = 11 - 3$$
$$\frac{2}{5}x = 8$$
$$\frac{5}{2}\left(\frac{2}{5}\right)x = \frac{5}{2}(8)$$

reasoning. *For example:* Transforming $2x-5=7$ to $2x=12$ is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. *Knowing this is the Addition Property of Equality is not the point of this standard.*	Students should be able to critique the solving process of others, recognize incorrect steps and provide corrective action to the process. **Example:** The following is a student solution to the inequality $\frac{5}{18}-\frac{x-2}{9}\leq\frac{x-4}{6}$. a) There are two mathematical errors in this work. Identify at what step each mathematical error occurred and explain why it is mathematically incorrect. b) How would you help the student understand his mistakes? c) Solve the inequality correctly. $\frac{5}{18}-\frac{x-2}{9}\leq\frac{x-4}{6}$ $\frac{5}{18}-\left(\frac{2}{2}\right)\frac{x-2}{9}\leq\left(\frac{3}{3}\right)\frac{x-4}{6}$ $\frac{5}{18}-\frac{2x-2}{18}\leq\frac{3x-4}{18}$ $5-(2x-2)\leq 3x-4$ $5-2x+2\leq 3x-4$ $7-2x\leq 3x-4$ $-5x\leq -11$ $x\leq\frac{11}{5}$ (https://www.illustrativemathematics.org/content-standards/HSA/REI/A/1/tasks/807) Note: While this standard does not cover inequalities, this could be a good extension.

NC.M1.F-LE.5

Interpret expressions for functions in terms of the situation they model.

Interpret the parameters a and b in a linear function $f(x) = ax + b$ or an exponential function $g(x) = ab^x$ in terms of a context.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship and interpret rate of change and initial value (8.F.4)
- Compare the coefficients and constants of linear equations in similar form (8.EEb)
- Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function

Mastering the Standard in this Unit

Comprehending the Standard

Students should know the meaning of the parameters in linear functions in the context of the situation.

Use real-world situations to help students understand how the parameters of linear functions depend on the context.

In a linear function $y = ax + b$ the value of "a" represents the slope (constant rate of change) while "b" represents the y intercept (initial value).

Assessing for Understanding

Students should be able to describe the effects of changes to the parameters of a linear and exponential functions.

Example: A plumber who charges \$50 for a house call and \$85 per hour can be expressed as the function $y = 85x + 50$. If the rate were raised to \$90 per hour, how would the function change?

Students should be able to interpret the parameters of a linear and exponential function.

Example: Lauren keeps records of the distances she travels in a taxi and what it costs:

Distance *d* in miles	**Fare *f* in dollars**
3	8.25
5	12.75
11	26.25

a) If you graph the ordered pairs *(d, f)* from the table, they lie on a line. How can this be determined without graphing them?
b) Show that the linear function in part a. has equation $f = 2.25d + 1.5$.
c) What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides

NC.M1.F-IF.5

Interpret functions that arise in applications in terms of the context.

Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

Concepts and Skills

Pre-requisite

- In middle school, students only informally considered restrictions to the domain and range based on context, such as understanding that measurements cannot be negative.
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)

Connections

- Recognize the domain of sequences (NC.M1.F-IF.3)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Analyze linear, quadratic, and exponential functions to identify key features (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard in this Unit

Comprehending the Standard

Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation.

Domain of graphs should be taught in the context of the situation they represent.

Graphs represented should be both discrete and continuous forms. Students do not need to know the terminology discrete and continuous, but they should be able to identify which is appropriate for each contextual situation.

Assessing for Understanding

Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation.

Example: Collin noticed that various combinations of nickels and dimes could add up to $0.65.

- Let x equal the number of nickels.
- Let y equal the number of dimes.

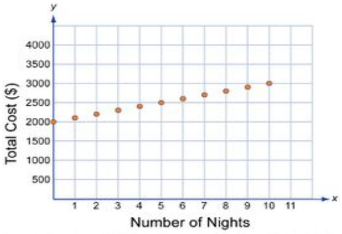
What is the domain where y is a function of x and the total value is $0.65? (NCDPI Math 1 released EOC #37)

a) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}
b) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}
c) {0, 1, 3, 5, 7, 9, 11, 13}
d) {1, 3, 5, 7, 9, 11, 13}

Example: Jennifer purchased a cell phone and the plan she decided upon charged her $50 for the phone and $0.10 for each minute she is on the phone. (The wireless carrier rounds up to the half minute.) She has budgeted $100 for her phone bill. What would be the appropriate domain for the cost as a function of the total minutes she used the phone? Describe what the point $(10, 51)$ represents in the problem.

Example: Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders' organization brings in as revenue is a function of the number of people, n, in attendance. If each ticket costs $30, find the domain of this function.
At a game, the Raiders has decided to honor fans who served in the military. For this event, the Raiders will be giving away 1,500 tickets to military families. How does this effect the domain and range of the function? What does this mean for the Raiders and their fans?

Example: An all-inclusive resort in Los Cabos, Mexico provides everything for their customers during their stay including food, lodging, and transportation. Use the graph at the right to describe the domain of the total cost function.



NC.M1.S-ID.9

Interpret linear models.

Distinguish between association and causation.

Concepts and Skills

Pre-requisite

- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)

Connections

- Fit a function to exponential data using technology (NC.M1.S-ID.6c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard in this Unit

Comprehending the Standard

In working with bivariate data in MS, students have previously investigated patterns of association between two quantities. Specifically, positive and negative associations and linear and non-linear associations.

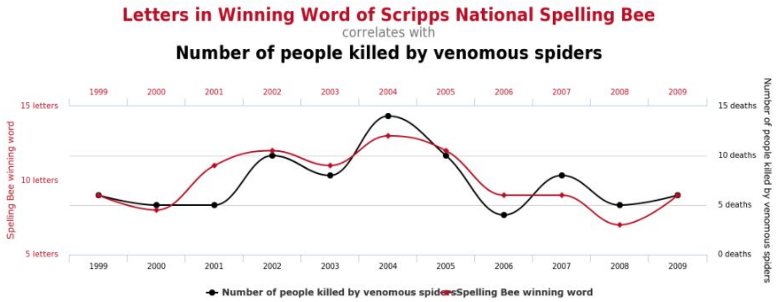
This standard addresses an often made misconception in regard to association, correlation and causation. Association indicates a relationship between two or more variables and correlation indicates the degree of association between two quantities. Causation, on the other hand, implies a cause and effect relationship when a strong relationship is observed.

Determining causation goes beyond the idea of mere association or a high degree of correlation and requires the design and analysis of a randomized experimental process.

Assessing for Understanding

Students will recognize that association does not imply causation.

Example: The following graph shows the correlation between *Letters in Winning Word of Scripps National Spelling Bee* and *Number of people killed by venomous spiders*. How does the graph support the phrase: association does not imply causation?



For more examples, explore the site http://tylervigen.com/.

Students will determine if statements of causation are reasonable or not and justify their opinion.

Example: A study found a strong, positive correlation between the number of cars owned and the length of one's life. Larry concludes that owning more cars means you will live longer. Does this seem reasonable? Explain your answer.

Example: Choose two variables that could be correlated because one is the cause of the other; defend and justify the selection of variables.

NC.M1.F-IF.9

Analyze functions using different representations.

Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Compare properties of two functions each represented in different ways (8.F.2)
- Formally define a function (NC.M1.F-IF.1)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Rewrite quadratic functions to identify key features (NC.M1.F-IF.8a)
- Interpret and explain growth and decay rates for an exponential function (NC.M1.F-IF.8b)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their use of a representation to make the comparison.

New Vocabulary: exponential function, quadratic function

Mastering the Standard in this Unit

Comprehending the Standard

Students should compare two functions in two different forms. The function types may be the same (linear & linear), but the representations should be different (e.g. numerical & graphical).

Assessing for Understanding

x	$g(x)$
−7	2
−5	3
−3	4
−1	5

Example: Dennis compared the y-intercept of the graph of the function $f(x) = 3x + 5$ to the y-intercept of the graph of the linear function that includes the points in the table below.

What is the difference when the y-intercept of $f(x)$ is subtracted from the y-intercept of $g(x)$?

A) -11.0

B) -9.3

C) 0.5

D) 5.5

(NCDPI Math 1 released EOC #22)

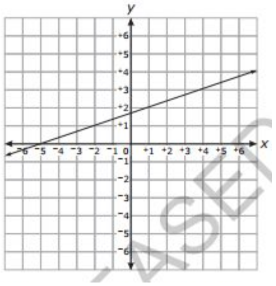
Example: Mario compared the slope of the function graphed to the right to the slope of the linear function that has an x-intercept of $\frac{4}{3}$ and a y-intercept of -2.

What is the slope of the function with the smaller slope?

A) $\frac{1}{5}$

B) $\frac{1}{3}$

C) 3



	D) 5 (NCDPI Math 1 EOC released #25)

NC.M1.F-IF.7

Analyze functions using different representations.

Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Concepts and Skills

Pre-requisite

- Interpret $y = mx + b$ as being linear (8.F.3)
- Determine rate of change and initial value of linear functions from tables and graphs (8.F.4)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)

Connections

- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Solving systems of equations (NC.M1.A-REI.6)
- Recognize the domain of sequences as integers (NC.M1.F-IF.3)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Build functions that describe a relationship between two quantities (NC.M1.F-BF.1a, NC.M1.F-BF.1b)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)
- Interpret the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their use of a representation.
New Vocabulary: exponential function, quadratic function

Mastering the Standard in this Unit	
Comprehending the Standard	**Assessing for Understanding**
Students should identify the key features of linear functions. Students should be aware of the key functions typically associated with each function type. Linear functions – domain & range, rate of change, intercepts, increasing/decreasing	Students should be able to identify key feature of linear, quadratic and exponential functions from the symbolic representation. **Example:** Describe the key features of the graph $f(x) = \frac{-2}{3}x + 8$ and use the key features to create a sketch of the function. Students should be able to identify key feature of linear functions from the graphical representation. **Example:** Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders' organization brings in as revenue is a function of the number of people, n, in attendance. If each ticket costs $30.00, find the appropriate domain and range of this function. (https://www.illustrativemathematics.org/content-standards/HSF/IF/B/5/tasks/631)

NC.M1.S-ID.6a

Summarize, represent, and interpret data on two categorical and quantitative variables.

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a least squares regression line to linear data using technology. Use the fitted function to solve problems.

Concepts and Skills

Pre-requisite

- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Informally fit a straight line assess the model fit judging the closeness of the data to line (8.SP.2)
- Analyze patterns and describe relationships between variables in context. (NC.M1.S-ID.8)

Connections

- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)
- Fit a function to exponential data using technology and use the model to solve problems (NC.M1.S-ID.6c)
- Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)
- Distinguish between association and causation (NC.M1.S-ID.9)
- Write a function that describes a relationship between two quantities (NC.M1.F-BF.1)
- Identify situations that can be modeled with linear and exponential functions and justify the appropriate model (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

Mastering the Standard in this Unit

Comprehending the Standard

In 8th grade, students created scatter plots and described patterns of association between two quantities. They also informally fit a straight line to data based on how closely the data points resembled a line. That knowledge is extended to fitting a linear regression equation to a set of data using technology. Technology includes graphing calculators, computer software/programs and web-based applets and tools.

Assessing for Understanding

Students can represent data on a scatter plot using an appropriate scale and describe the relationship between two quantitative variables.

HS Population	# of active band members
1200	150
1450	155
900	100
1500	125
1400	125
1005	120

Example: Represent the data from the table below in a scatter plot. Determine if and what the relationship is between the population of each high school and the number of active band members.

The initial exploration with technology should include a discussion of domain and range and their relationship to the graphing window. Most technology tools include an automatic feature that graphs data within a window representative of the data, however understanding of the graphing window can lead to further discussions about domain, range, interpolation and extrapolation.

Example: The data gives the number of miles driven and advertised price for 11 used models of a particular car.

Miles (thousands)	**Price**($)
22	17,998
29	16,450
35	14,998
39	13,998
45	14,599
49	14,988
55	13,599
56	14,599
69	11,998
70	14,450
86	10,998

a) Use a calculator or graphing technology to make a scatter plot of the data.
b) Find the correlation coefficient for the data above. Describe what the correlation coefficient means in regards to the data.
c) Fit a linear function to model the relationship between miles driven and the price of these cars.
d) How do you know that this is the best-fit model?
e) If a used car is driven 98,000 miles, what will the price be (to the nearest dollar)?
f) If the price of the car is $12,540, how many miles could have been driven (to the nearest thousand)?

NC.M1.S-ID.7

Interpret linear models.

Interpret in context the rate of change and the intercept of a linear model. Use the linear model to interpolate and extrapolate predicted values. Assess the validity of a predicted value.

Concepts and Skills

Pre-requisite

- Interpret the slope and y-intercept of a linear model in context (8.SP.3)

Connections

- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
- Interpret the parameters in linear or exponential functions in terms of a context (NC.M1.F-LE.5)
- Interpret key features in context to describe functions relating two quantities (NC.M1.F-IF.4)
- Calculate and interpret the avg. rate of change for a function (NC.M1.F-IF.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

Mastering the Standard in this Unit

Comprehending the Standard

Students have interpreted the slope and y-intercept of a linear model in 8th grade. This standard expands upon this notion to using the model to make **predictions**.

Interpolation is using the function to predict the value of the dependent variable for an independent variable that is in the midst of the data.

Extrapolation is using the function to predict the value of the dependent variable for an independent variable that is outside the range of our data.

Assessing for Understanding

Students can interpret the meaning of the rate of change and y-intercept in context and can interpolate and/or extrapolate predicted values using the linear model.

Example: Data was collected of the weight of a male white laboratory rat for the first 25 weeks after its birth. A scatterplot of the rat's weight (in grams) and the time since birth (in weeks) indicates a fairly strong, positive linear relationship. The linear regression equation $W = 100 + 40t$ (where W = weight in grams and t = number of weeks since birth) models the data fairly well.

a) Explain the meaning of the slope of the linear regression equation in context.
b) Explain the meaning of the y-intercept of the linear regression equation in context.
c) Based on the linear regression model, what will be the weight of the rat 10 weeks after birth?
d) Based on the linear regression model, at how many weeks will the rat be 760 grams?

NC.M1.S-ID.8

Interpret linear models.

Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.

Concepts and Skills

Pre-requisite

- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)

Connections

- Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

Mastering the Standard in this Unit

Comprehending the Standard

In working with bivariate data in MS, students have previously

The correlation coefficient, *r*, is a measure of the strength and direction of a linear relationship between two quantities in a set of data.

The magnitude (absolute value) of *r* indicates how closely the data points fit a linear pattern.

If $r = \pm 1$, all points fall exactly on a line. The sign of *r* indicates the direction of the relationship. The closer $|r|$ is to 1, the stronger the correlation and the closer $|r|$ is to zero, the weaker the correlation.

Assessing for Understanding

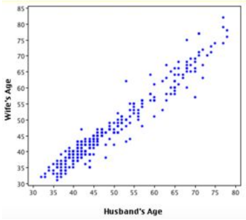
Students can interpret the correlation coefficient.

Example: The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data.

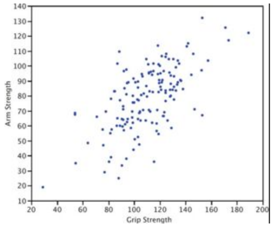
Students recognize the strength of the association of two quantities based on the scatter plot.

Example: Which correlation coefficient best matches each graph? Explain.

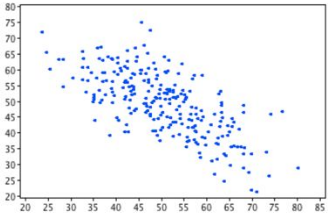
A $r =$ ________



B $r =$ ________



C $r =$ ________



r = –.48	*r* = .98	*r* = .88	*r* = –.17	*r* = 1	*r* = .31	*r* = –1

Instructions for TI-83 and TI-84 series calculators:

1: Go to the [catalog]. Click→ **2nd** then **0** .

2: Scroll down to → DiagnosticOn and press **enter** twice.

When '**Done**' appears on the screen the diagnostics are on and the calculator should now calculate the correlation coefficient (r) automatically when linear regression is performed.

Students will be able to analyze patterns in context between two variables and use graphing technology to determine whether a linear model is appropriate for the data.

Example: The following data set indicates the average weekly temperature and the number of sno-cones sold by Sno-Show Sno-cones each week in May for the temperatures noted.

Average weekly temperature	**# of Sno-cones sold**
68	500
74	600
74	700
80	800
82	1200

a) Using technology, sketch a scatter plot of the data.

b) Determine a linear regression model that could represent the data shown.

c) Determine the correlation coefficient.

d) Determine the strength and direction of the linear relationship.

e) Create a residual plot. Is a linear model appropriate for the data shown? Explain.

NOTE: Remind students to turn the Diagnostics on in the graphing calculator so that the correlation coefficient (r) appears when the regression equation is calculated.

NC.M1.G-GPE.6

Use coordinates to prove simple geometric theorems algebraically.

Use coordinates to find the midpoint or endpoint of a line segment.

Concepts and Skills

Pre-requisite

- Finding the distance between points in the coordinate plane (8.G.8)
- (7.RP.2d)

Connections

- Use coordinates to solve geometric problems involving polygons (NC.M1.G-GPE.4)
- Prove theorems about lines (NC.M2.G-CO.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others.

8 – Look for and express regularity in repeated reasoning.

- The midpoint formula is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

Mastering the Standard in this Unit

Comprehending the Standard

The midpoint partitions the ratio of two distinct points on the same line segment into 1:1; thus from either direction the point is the same.

The midpoint is always halfway between the two endpoints. The *x*-coordinate of the midpoint will be the mean of the *x*-coordinates of the endpoints and the *y*-coordinate will be the mean of the *y*-coordinates of the endpoints as indicated through the use of the midpoint formula.
This should derived from what students understand about distance.

The midpoint formula $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ is an appropriate generalization and should be ***developed*** through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations.

Assessing for Understanding

Given two points on a line, students can find the point that divides the segment into an equal number of parts.

Example: Jennifer and Jane are best friends. They placed a map of their town on a coordinate grid and found the point at which each of their house lies. If Jennifer's house lies at (9, 7) and Jane's house is at (15, 9) and they wanted to meet in the middle, what are the coordinates of the place they should meet?

Given the midpoint and an endpoint, students can use what they know about the midpoint to locate the other endpoint.

Example: If you are given the midpoint of a segment and one endpoint. Find the other endpoint.

a) Midpoint: (6, 2) endpoint: (1, 3)
b) Midpoint: (-1, -2) endpoint: (3.5, -7)

NC.M1.G-GPE.4

Use coordinates to prove simple geometric theorems algebraically.

Use coordinates to solve geometric problems involving polygons algebraically

- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
- Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

Concepts and Skills

Pre-requisite

- Finding the distance between points in the coordinate plane (8.G.8)
- Calculating rate of change from two points (8.F.4)
- Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)
- Finding midpoint/endpoint of a line segment, given either (NC.M1.G-GPE.6)

Connections

- Experiment with transformations in the plane (NC.M2.G-CO.2, NC.M2.G-CO.3, NC.M2.G-CO.3, NC.M2.G-CO.4)
- Geometric transformations as functions (NC.M2.F-IF.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others.

- Students must use algebraic reasoning as they solve geometric problems.

8 – Look for and express regularity in repeated reasoning

- The distance formula is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their claim that a set of points forms a particular shape using mathematical reasoning.

Mastering the Standard in this Unit

Comprehending the Standard

In upper elementary and middle grades, students calculated the area of triangles and special quadrilaterals using all four quadrants of the coordinate plane. Students also applied geometric measurement to real-world and mathematical problems and made use of properties of two-dimensional figures in order to calculate or estimate their lengths and areas.

This standard emphasizes the use of coordinates to solve geometric problems algebraically and continues with geometric measurement. Students will begin to demonstrate and analyze properties of geometric shapes using equations and graphs. This includes:

- Using previously learned formulas to find the perimeter of polygons and the area of triangles and rectangles.

Assessing for Understanding

Given coordinates of a polygon in the coordinate plane, students should be able to compute the lengths of segments and side lengths of polygons by finding the distance between points in the coordinate plane to:

- calculate the perimeter of polygons
- calculate the area of triangles and rectangles

Example: Find the perimeter and area of a polygon with vertices at $C(-1, 1)$, $D(3, 4)$, $E(6, 0)$, $F(2, -3)$ and $G(-4, -4)$. Round your answer to the nearest hundredth.

Given coordinates of a polygon in the coordinate plane, students should be able to verify the properties of any triangle or quadrilateral using the slopes of lines and lengths of segments that comprise the figure.

Example: Given ABC with altitude $\overline{CD}$, given $A(-4,-2)$, $B(8, 7)$, $C(1, 8)$ and $D(4, 4)$.

a) Calculate the area of ABC.

- Applying the slope to determine right angles in triangles and rectangles (perpendicular lines), to verify parallel sides in geometric figures; and to determine intersecting lines.

- Finding the perimeter of figures by computing the distance between points on the coordinate plane.

The distance formula ($d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$) is an appropriate generalization and should be ***developed*** through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations. This is based on what students know about finding the length of line segments in the coordinate plane (Pythagorean Theorem) from MS mathematics.

b) The altitude of a triangle is defined as is a line that extends from one vertex of a **triangle** perpendicular to the opposite side. Verify that $\overline{CD}$ is an altitude of ABC.

Example: The coordinates for the vertices of quadrilateral $MNPQ$ are $M\,(3,\ 0)$, $N\,(1,\ 3)$, $P\,(-2,\ 1)$, and $Q\,(0,\ -2)$.

a) Classify quadrilateral $MNPQ$.
b) Identify the properties used to determine your classification.

Given the properties of a rectangle or triangle, students can determine the missing coordinate(s).

Example*:* If quadrilateral $ABCD$ is a rectangle, where $A\,(1,\ 2)$, $B\,(6,\ 0)$, $C\,(10, 10)$ and $D\,(x,\ y)$ is unknown.

a) Find the coordinates of the fourth vertex Point D.
b) Verify that $ABCD$ is a rectangle providing evidence related to the sides and angles.

NC.M1.S-ID.6b

Summarize, represent, and interpret data on two categorical and quantitative variables.

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

b. Assess the fit of a linear function by analyzing residuals.

Concepts and Skills

Pre-requisite

- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)

Connections

- Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)
- Analyze patterns and describe relationships between variables in context. (NC.M1.S-ID.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others

Vocabulary

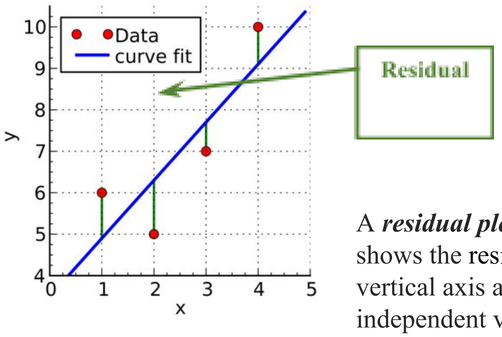
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

New Vocabulary: residual

Mastering the Standard in this Unit

Comprehending the Standard

A ***residual***, a measure of the error in prediction, is the difference between the actual y-value (y) and the predicted y-value ($\hat{y}$). Residuals are represented on the graph by the vertical distance between a data point and the graph of the function.



A ***residual plot*** is a graph that shows the residuals on the vertical axis and the independent variable on the horizontal axis. If the points in a residual plot are randomly dispersed around the horizontal axis, a linear regression model is appropriate for the data; otherwise, a non-linear model is more appropriate.

Assessing for Understanding

Students can create a residual plot from a given set of data and interpret the appropriateness of a linear model for the data set.

Year (0 = 1990)	Tuition Rate	Predicted Rate	Residuals
0	6546		
1	6996		
2	6996		
3	7350		
4	7500		
5	7978		
6	8377		
7	8710		
8	9110		
9	9411		
10	9800		

Students can determine the residual for any value in a data set.

Example: The table to the left displays the annual tuition rates of a state college in the U.S. between 1990 and 2000, inclusively. The linear function $R(t) = 326x + 6440$ has been suggested as a good fit for the data.

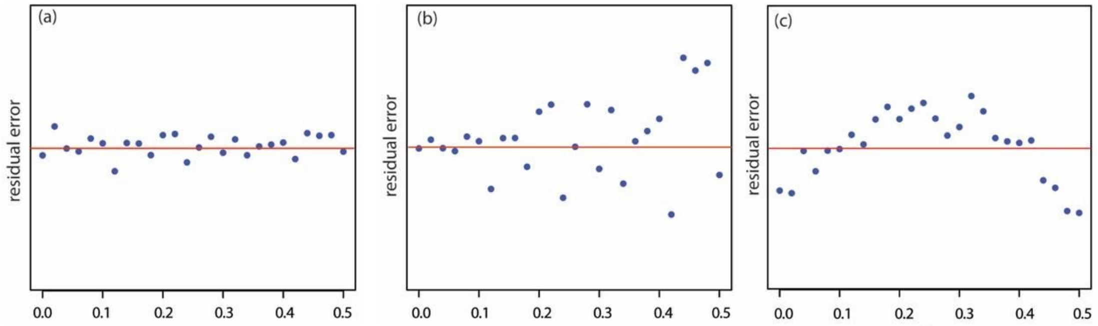
a) Extend the table to find the predicted rates based on the model and the residual values for each year.

b) Create the residual plot for the tuition rates.

c) Use the residual plot to determine the goodness of fit of the function for the data provided in the table.

Students can use a residual plot to determine the appropriateness of a linear model for a set of data.

Example: What do the following residual plots tell you about the appropriateness of a linear model for the functions they represent? Explain your responses.



The North Carolina High School Collaborative Instructional Framework

NC Math 1

Unit 3: Introduction to Exponential Functions

14 Days Block Schedule | **September 2017 Update** | 28 Days Traditional Schedule

RESEARCH BRIEF: **Unit 3: Exponential Functions**

Essential Questions:

- What are the basic patterns of exponential growth?
- How can the patterns of exponential growth be expressed with symbolic rules?
- What are the recursive and explicit forms for basic exponential functions?
- How are geometric sequences related to exponential functions?
- How can you represent and reason about functions involved in investments paying compound interest?
- What are some useful strategies for finding functions modeling patterns of change that are only approximately exponential?
- What mathematical patterns in tables, graphs, and symbolic rules are typical of exponential decay relations?
- How can you interpret parts of exponential growth and decay expressions in context of the scenario?
- How can logical analysis of an experiment be used as a check of a function model produced by your calculator or computer curve-fitting software?

Learning Outcomes	Student Objectives
● Students will recognize patterns of exponential growth and decay from multiple representations. ● Students will use the standard rules of exponents to write exponential expressions in equivalent forms. ● Students will model exponential functions using rules in the form $y = a(b^x)$ where $a > 0$ and $b > 1$ or $0 < b < 1$. ● Students should understand the key features of an exponential function's table, graph, and verbal representation.	● I will **explain** an exponential function using multiple representations as a model of growth or decay.[NC.M1.F-IF.2, NC.M1.F-IF.8b] ● I will **apply** the properties of exponents to write expressions in equivalent forms.[NC.M1.N-RN.2] ● I will **use** the standard form of an exponential function to graphically represent a translation between pairs of functions.[NC.M1.F-LE.5]

- Students should recognize the difference between *factor* and *rate.*
- Students can use the increasing rate of change to examine the difference between the end behavior of linear and exponential functions.
- Students will calculate the average rate of change over a specified interval and differentiate between linear and exponential.
- Students translate between explicit and recursive forms of geometric sequences and use both to model situations.
- Students can represent and reason about functions involved in exponential growth and decay situations.
- Students can use technology to solve and understand solutions of exponential equations and inequalities
- Students can use appropriate technology to calculate exponential regression for a scatterplot or a given set of data.
- Students will use exponential regression to predict outcomes from a given value.

- I will **explain** the key components of an exponential function given standard form and in context.[NC.M1.F-IF.4, NC.M1.A-SSE.1a, NC.M1.F-IF.5, NC.M1.F-IF.7, NC.M1.F-IF.9]
- I will **calculate** the average rate of change over a specified interval of a function. [NC.M1. F-IF.6]
- I will **calculate** the rate of change to **analyze** the end behavior of an exponential function.[NC.M1.F-IF.6, NC.M1.F-LE.3]
- I will **identify** situations as linear or exponential based on average rate of change. [NC.M1.F-LE.3]
- I will **recognize** the difference between factor and rate in an exponential functions.[NC.M1.F-LE.5]
- I will **develop** explicit and recursive forms of geometric sequences to **model** a scenario.[NC.M1.F-IF.3, NC.M1.F-BF.2]
- Given multiple representations, I will **create** a function for exponential growth and decay situations.[NC.M1.A-CED.1,NC.M1.A-CED.2, NC.M1.F-BF.1a, NC.M1.F-BF.1b]
- I will **explain** the solutions from exponential equations and inequalities in the context of a problem.[NC.M1.A.REI.10]
- I will **use** technology to **calculate** the exponential regression from a set of data and predict other values.[NC.M1.S-ID.6c]

Standards Addressed in this Unit

Understand and apply the properties of exponents.

- **NC.M1.N-RN.2**: Extend the properties of exponents. Rewrite algebraic expressions with integer exponents using the properties of exponents.

Determine the explicit and recursive formula for given geometric sequences.

- **NC.M1.F-IF.3**: Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

- **NC.M1.F-BF.2**: Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.

Evaluate, create, and interpret exponential functions in context.

- **NC.M1.F-IF.2:** Understand the concepts of a functions and use function notation. Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **NC.M1.F-IF.4:** Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.
- **NC.M1.A-CED.1:** Create equations that describe numbers or relationships. Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.
- **NC.M1.A-CED.2:** Create equations that describe numbers or relationships. Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.
- **NC.M1.A-REI.10:** Represent and solve equations and inequalities graphically. Understand that the graph of a two variable equation represents the set of all solutions to the equation.
- **NC.M1.F-LE.5:** Interpret expressions for functions in terms of the situation they model. Interpret the parameters a and b in a linear function f(x)=ax+b or an exponential function g(x)=abx in terms of a context.
- **NC.M1.F-IF.6:** Interpret functions that arise in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Identify situations and practical domains for exponential functions.

- **NC.M1.F-LE.1**: Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model for a situation based on the rate of change over equal intervals.
- **NC.M1.F-IF.5**: Interpret a function in terms of context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.
- **NC.M1.F-IF.7**: Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Compare, interpret, and explain key features of exponential functions.

- **NC.M1.F-IF.8b**: Analyze functions using different representations. Interpret and explain growth and decay rates for an exponential function.
- **NC.M1.F-LE.3**: Construct and compare linear and exponential models and solve problems. Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
- **NC.M1.A-SSE.1a**: Interpret the structure of expressions. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.
- **NC.M1.F-IF.9**: Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).
- **NC.M1.A-REI.11**: Build an understanding of why the *x*-coordinates of the points where the graphs of two linear, exponential, or quadratic equations and intersect are the solutions of the equation and approximate solutions using a graphing technology or successive approximations with a table of values.

Write and apply exponential functions given multiple representations.

- **NC.M1.F-BF.1a**: Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).
- **NC.M1.F-BF.1b**: Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.
- **NC.M1.S.ID.6c**: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to exponential data using technology. Use the fitted function to solve problems.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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The Math Resource for Instruction - Customized for the Content of this Unit

NC.M1.N-RN.2

Extend the properties of exponents.

Rewrite algebraic expressions with integer exponents using the properties of exponents.

Concepts and Skills

Pre-requisite

- Using the properties of exponents to create equivalent numerical expressions (8.EE.1)

Connections

- Use operations to rewrite polynomial expressions (NC.M1.A-APR.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Vocabulary – base, exponent, index

Students should be able to justify their steps in rewriting algebraic expressions.

Mastering the Standard

Comprehending the Standard

Students extend the properties of integer exponents learned in middle school with numerical expressions to algebraic expressions.

The process of “simplifying square roots” is not an expectation for Math 1 students In Math 2, students will extend the properties of exponents to rational exponents and rewrite, “simplify” all square roots.

Assessing for Understanding

Students should be able to use the properties of exponents to write expression into equivalent forms.

Example: Rewrite the following with positive exponents:

a) $(8x^{-4}y^{3})(-2x^{5}y^{-6})^{2}$

b) $\frac{(3m^{2}p^{-2}q)^{3}}{9m^{-3}q^{3}}$

Students should be able to use the new skills of applying the properties of exponents with skills learned in previous courses.

Example: Simplify: $\sqrt{25m^{14}p^{2}t^{4}}$

In 8th grade, students learned to evaluate the square roots of perfect squares and the cube root of perfect cubes. In Math 1, students can combine this previous skill with algebraic expressions. When addressing a problem like this in Math 1, students should be taught to rewrite the expression using the properties of exponents and then using inverse operations to rewrite. For example, $\sqrt{m^{14}} = \sqrt{(m^{7})^{2}} = m^{7}$.

In Math 1, the limitation from 8th grade of evaluating square roots of perfect squares and cube root of perfect cubes still applies.

NC.M1.F-IF.3

Understand the concept of a function and use function notation.

Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

Concepts and Skills

Pre-requisite

- Interpret the equation $y = mx + b$ as being from a linear function and compare to nonlinear functions (8.F.3)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

Connections

- Relating the domain and range to a context (NC.M1.F-IF.5)
- Analyzing linear and exponential functions (NC.M1.F-IF.7)
- Build linear and exponential functions (NC.M1.F-BF.1)
- Translate between explicit and recursive forms (NC.M1.F-BF.2)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain a function written in recursive form using subset notation.

New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form, exponential function

Mastering the Standard

Comprehending the Standard

Students should recognize that sequences are functions. A sequence can be described as a function, with the domain consisting of a subset of the integers, and the range being the terms of the sequence.

This standard connects to arithmetic and geometric sequences and should be taught with NC.M1.F-BF.2.

Emphasize that arithmetic and geometric sequences are examples of linear and exponential functions, respectively.

It is important to note that sequences are not limited to arithmetic and geometric. It is expected that recursive form should be written in subset notation. Students should be familiar with writing and interpreting subset notation.

Now-Next can be used a tool for introduce the concepts of recursive form, but the expectation is that students will move to the more formal representations of recursive form.

Assessing for Understanding

Example: A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.

a) If the theater has 20 rows of seats, how many seats are in the twentieth row?
b) Explain why the sequence is considered a function.
c) What is the domain of the sequence? Explain what the domain represents in context.
d) What is the range of the sequence? Explain what the range represents in context.

Example: A geometric sequence can be represented by the exponential function $f(x) = 400(\frac{1}{2})^x$. In terms of the geometric sequence, explain what $f(3) = 50$ represents.

Example: Represent the following sequence in explicit form: 1, 4, 9, 16, 25

Example: The Fibonacci numbers are sequence that are often found in nature. This sequence is defined by $a_n = a_{n-1} + a_{n-2}$ where $a_0 = 0$ and $a_1 = 1$. What are the first 10 terms of the Fibonacci sequence? Could you easily represent this pattern in explicit form?

NC.M1.F-BF.2

Build a function that models a relationship between two quantities.

Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3)
- Build functions from arithmetic and geometric sequences (NC.M1.F-BF.1a)

Connections

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The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain their model in context.

New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form

Mastering the Standard

Comprehending the Standard

Students should be able to use both the explicit and recursive forms of arithmetic and geometric sequences where the explicit form is a linear or exponential function, respectively.

Students are expected to use formal notation:

- a_n (NOW)
- a_{n-1} (PREVIOUS)
- a_{n+1} (NEXT)

(Students can use NEXT-NOW notation as they learn to create recursive functions but will need to move to formal notation.)

This standard should be tied to NC.M1.F-IF.3, recognizing patterns and linking to function types.

Students should recognize explicit form of an arithmetic sequence as an equivalent structure to

Assessing for Understanding

Students should be able to build explicit and recursive forms of arithmetic and geometric sequences.

Example: The sequence below shows the number of trees that a nursery plants each year.

$$2,\ 8,\ 32,\ 128\ldots$$

Let a_n represent the current term in the sequence and a_{n-1} represent the previous term in the sequence. Which formula could be used to determine the number of trees the nursery will plant in year n?

A) $a_n = 4a_{n-1}$

B) $a_n = \frac{1}{4}a_{n-1}$

C) $a_n = 2a_{n-1} + 4$

D) $a_n = a_{n-1} + 6$

Example: A single bacterium is placed in a test tube and splits in two after one minute. After two minutes, the resulting two bacteria split in two, creating four bacteria. This process continues.

a) How many bacteria are in the test tube after 5 minutes? 15 minutes?

b) Write a recursive rule to find the number of bacteria in the test tube from one minute to the next.

c) Convert this rule into explicit form. How many bacteria are in the test tube after one hour?

Example: A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats.

a) Write a recursive formula to find the number of seats in each row. How many seats are in row 5?

slope-intercept form of a linear function and explicit form of a geometric sequence as an equivalent structure to standard form of an exponential function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the y-intercept, which gives meaning to the $n-1$ notation.	b) Write the explicit formula to determine which row has 94 seats? **Example:** Given the sequence defined by the function $a_{n+1} = a_n + 12$ with $a_1 = 4$. Write an explicit function rule. Note: Student may interpret 4 as the y-intercept since it is the first value; however, attending to the notation when $x = 1, y = 4$. Thus, the y-intercept for the explicit form is -8. **Example:** Given the sequence defined by the function $a_{n+1} = \frac{3}{4}a_n$ with $a_1 = 424$. Write an explicit function rule.

NC.M1.F-IF.2

Understand the concept of a function and use function notation.

Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Concepts and Skills

Pre-requisite

- Use substitution to determine if a number if a solution (6.EE.5)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Define a function and use functions notation (NC.M1.F-IF.1)

Connections

- Creating and solving one variable equations (NC.M1.A-CED.1)
- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Function standards that relate domain and range (NC.M1.F-IF.3, NC.M1.F-IF.4, NC.M1.F-IF.5, NC.M1.F-IF.7)
- Comparing the end behavior of functions (NC.M1.F-LE.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the domain, range, input, output and the relationship between the variables of a function in context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function.

Students should be able to interpret statements in function notation in contextual situations.

Assessing for Understanding

Students should be able to use evaluate functions written in function notation.

Example: Evaluate $f(2)$ for the function $f(x) = 5(x-3)+17$.

Evaluate $f(2)$ for the function $f(x) = 1200(1+.04)^x$.

Evaluate $f(2)$ for the function $f(x) = 3x^2 + 2x - 5$.

Students should be able to evaluate functions and interpret the result in a context.

Example: You placed a yam in the oven and, after 45 minutes, you take it out. Let f be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.

a) $f(0) = 65$
b) $f(5) < f(10)$
c) $f(40) = f(45)$
d) $f(45) > f(60)$

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/625)

	Example: The rule $f(x) = 50(0.85)^x$ represents the amount of a drug in milligrams , $f(x)$, which remains in the bloodstream after x hours. Evaluate and interpret each of the following: a) $f(0)$ b) $f(2) = k \cdot f(1)$. What is the value of k? c) $f(x) < 6$ ***Example:*** Suppose that the function $f(x) = 2x + 12$ represents the cost to rent x movies a month from an internet movie club. Makayla now has $10. How many more dollars does Makayla need to rent 7 movies next month? (NCDPI Math 1 released EOC #12) **Example:** Let $f(t)$ be the number of people, in millions, who own cell phones t years after 1990. Explain the meaning of the following statements. a) $f(10) = 100.3$ b) $f(a) = 20$ c) $f(20) = b$ d) $n = f(t)$ (https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/634)

NC.M1.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Concepts and Skills

Pre-requisite

- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

Connections

- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identification of key features and interpret those key features in context.

New Vocabulary: maximum, minimum

Mastering the Standard

Comprehending the Standard

Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that “tell a story” hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions.

Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and

Assessing for Understanding

Students should be able to identify and interpret key features of functions.

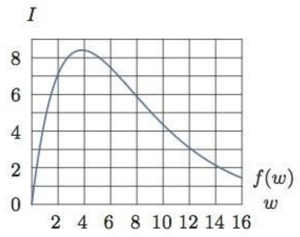
Example: An epidemic of influenza spreads through a city. The figure below is the graph of $I = f(w)$, where I is the number of individuals (in thousands) infected w weeks after the epidemic begins.

a) Estimate $f(2)$ and explain its meaning in terms of the epidemic.

b) Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form $f(a) = b$.

c) For approximately which w is $f(w) = 4.5$; explain what the estimates mean in terms of the epidemic.

d) An equation for the function used to plot the image above is $f(w) = 6w(1.3)^{-w}$. Use the graph to estimate the solution of the inequality $6w(1.3)^{-w} \geq 6$. Explain what the solution means in terms of the epidemic. *(This would make a great Honors level extension to this standard)*



(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/637)

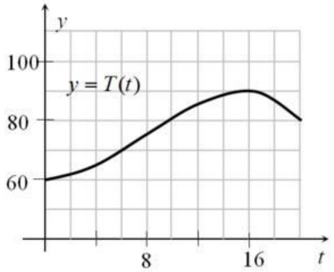
maximum/minimum) for any given graph, not just “function families”. This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulation algebraically.

By contrast, NC.M1.F-IF.7, has students work with specific functions in which students have the ability to use algebraic manipulation to identify additional key features.

Example: The figure shows the graph of T , the temperature (in degrees Fahrenheit) over one particular 20-hour period in Santa Elena as a function of time t .

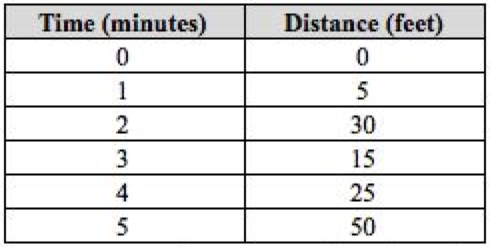
a) Estimate $T(14)$.
b) If $t = 0$ corresponds to midnight, interpret what we mean by $T(14)$ in words.
c) Estimate the highest temperature during this period from the graph.
d) When was the temperature decreasing?
e) If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/639)



Example: Eliana observed her dog, Lola, running around the yard and recorded the time and distance that Lola was away from her dog house in the table below.

a) Sketch a graph of Lola’s play time away from her dog house.
b) Describe what is happening between minutes 2 & 3.



Time (minutes)	Distance (feet)
0	0
1	5
2	30
3	15
4	25
5	50

NC.M1.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

Concepts and Skills

Pre-requisite

- Create two-step linear equations and inequalities from a context (7.EE.4)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)
- Solve linear and quadratic equations and linear inequalities (NC.M1.A-REI.3, NC.M1.A-REI.4)
- Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)
- Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students create equations and inequalities in one-variable and use them to solve the problems.

In Math I, focus on linear, quadratic, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can also be created from an associated function. After the students have created an equation, they can use other representations to solve problems, such as graphs and tables

For quadratic and exponential inequalities, the focus of this standard is to create the inequality and use that inequality to solve a problem. Solving these inequalities algebraically is NOT part of the standard. Once a student has the

Assessing for Understanding

Students should be able to create an equation from a function and use the equation to solve problems.

Students should be able to create equations from various representations, such as verbal descriptions, and use them to solve problems.

Example: Phil purchases a used truck for $11,500. The value of the truck is expected to decrease by 20% each year. When will the truck first be worth less than $1,000?

Example: Suppose a friend tells you she paid a total of $16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

a) Arizona, where the sales tax is 6.6%.
b) New York, where the sales tax is 8.25%.
c) A state where the sales tax is r.

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/582)

inequality, the student can use a table or graph to find a solution to the problem. Students in Math I are not responsible for interval notation as a solution. They are to write answers to these inequalities using inequality notation.	Students should be able to create inequalities and use those inequalities to solve problems. (Students are not expected to solve quadratic and exponential inequalities algebraically. Students should use technology, tables and graphs to solve problems.) **Example:** Susanna heard some exciting news about a well-known celebrity. Within a day she told 4 friends who hadn't heard the news yet. By the next day each of those friends told 4 other people who also hadn't yet heard the news. By the next day each of those people told four more, and so on. a) Assume the rumor continues to spread in this manner. Let N be the function that assigns to d the number of people who hear the rumor on the dth day. Write an expression for N(d). b) On which day will at least 100,000 people hear the rumor for the first time? c) How many people will hear the rumor for the first time on the 20th day? d) Is the answer to (c) realistic? Explain your reasoning. e) Create an inequality that could be used to determine when there will be greater than 200,000 people that have heard the rumor. https://www.illustrativemathematics.org/content-standards/HSF/LE/A/2/tasks/74

NC.M1.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Concepts and Skills

Pre-requisite

- Construct a linear function that models the relationship between two quantities (8.F.4)
- Graph linear equations (8.EE.6)
- The graph of a function is the set of ordered pairs consisting of input and a corresponding output (8.F.1)
- Understand that the graph of a two variable equation represents the set of all solutions to the equation (NC.M1.A-REI.10)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Creating linear equations for a system (NC.M1.A-CED.3)
- Solving for a variable of interest in a formula (NC.M1.A-CED.4)
- The graph a function f is the graph of the equation $y = f(x)$ (NC.M1.F-IF.1)
- Interpret a function’s domain and range in context (NC.M1.F-IF.5)
- Identify key features of linear, exponential and quadratic functions (NC.M1.F-IF.7)
- Building a function through patterns or by combining other functions (NC.M1.F-BF.1a , NC.M1.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students create equations in two variables. Graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for the context and display adequate information about the relationship.

Students interpret the context and choose appropriate minimum and maximum values for a graph.

In Math I, focus on linear, exponential and quadratic **contextual** situations for students to

Assessing for Understanding

Students should be able to create two variable equations from various representations, such as verbal descriptions, and use them to solve problems.

Example: The larger leg of a right triangle is 3 cm longer than its smaller leg. The hypotenuse is 6 cm longer than the smaller leg. How many centimeters long is the smaller leg?

(NCDPI Math 1 released EOC #13)

Example: The floor of a rectangular cage has a length 4 feet greater than its width, w. James will increase both dimensions of the floor by 2 feet. Which equation represents the new area, N, of the floor of the cage?

A) $N = w^2 + 4w$

B) $N = w^2 + 6w$

C) $N = w^2 + 6w + 8$

D) $N = w^2 + 8w + 12$

(NCDPI Math I released EOC #5)

create equations in two variables. While students will one be asked to rewrite expressions with integers exponents, in exponential functions, the domain is not restricted and students should use technology to see the continuity of exponential functions.	Students should be able to create a two variable equations, graph the relationship, and use graph to recognize key feature of the graph. **Example:** The FFA had a fundraiser by selling hot dogs for $1.50 and drinks for $2.00. Their total sales were $400. a) Write an equation to calculate the total of $400 based on the hot dog and drink sales. b) Graph the relationship between hot dog sales and drink sales. Note: This make a good connection to NC.M1.F-IF.5 **Example:** In a women's professional tennis tournament, the money a player wins depends on her finishing place in the standings. The first-place finisher wins half of $1,500,000 in total prize money. The second-place finisher wins half of what is left; then the third-place finisher wins half of that, and so on. a) Write a rule to calculate the actual prize money in dollars won by the player finishing in nth place, for any positive integer n. b) Graph the relationship between the first 10 finishers and the prize money in dollars. What pattern is indicated in the graph? What type of relationship exists between the two variables?

NC.M1.A-REI.10

Represent and solve equations and inequalities graphically

Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Concepts and Skills

Pre-requisite

- Use substitution to determine if a number if a solution (6.EE.5)
- Graphing lines (8.EE.5, 8.EE.6, 8.F.3)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understanding functions as a rule that assigns each input with exactly one output (8.F.1)

Connections

- Creating and graphing two-variable equations (NC.M1.A-CED.2)
- Solutions to systems of equations (NC.M1.A-REI.5, NC.M1.A-REI.6)
- Understanding that the relationship between the solution of system of equations and the associated equation (NC.M1.A-REI.11)
- Representing the solutions to linear inequalities (NC.M1.A-REI.12)
- Relating a function to its graph, domain and range of a function (NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the solutions to a two variable equation and the link to a function.

Mastering the Standard

Comprehending the Standard

Students understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement.

This standard contains no limitation and so applies to all function types, including those functions that a student cannot yet algebraically manipulate.

Students can explain and verify that every point (x, y) on the graph of an equation represents all values for x and y that make the equation true.

Assessing for Understanding

Students should be able to assess if a point is a solution to an equation.

Example: Consider three points in the plane, $P = (-4, 0)$, $Q = (-1, 12)$ *and* $R = (4, 32)$.

a) Find the equation of the line through P and Q.
b) Use your equation in (a) to show that R is on the same line as P and Q.

(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/10/tasks/1066)

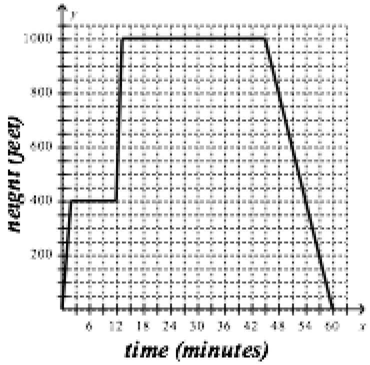
Example: Which of the following points are on the graph of the equation $-5x + 2y = 20$? Which of the following points are of the graph of the equation? How do you know?

a) (4, 0)
b) (0, 10)
c) (-1, 7.5)
d) (2.3, 5)

Example: Verify that $(-1,\ 60)$ is a solution to the equation $y = 15(\frac{1}{4})^x$. Explain what this means for the graph of the function.

Example: Without graphing, determine if the ordered pair $(2,\ -15)$ is on the graph of $y = 3x^2 + 2x - 1$. Explain.

Example: The graph to the right shows the height of a hot air balloon as a function of time.



Use the graph to answer the following:

a) What is the height of the hot air balloon 10 minutes after it has left the ground?
b) Approximately, when will the hot air balloon reach a height of 600 feet?
c) Explain what the point $(48,\ 800)$ on this graph represents.

NC.M1.F-LE.5

Interpret expressions for functions in terms of the situation they model.

Interpret the parameters a and b in a linear function $f(x) = ax + b$ or an exponential function $g(x) = ab^x$ in terms of a context.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship and interpret rate of change and initial value (8.F.4)
- Compare the coefficients and constants of linear equations in similar form (8.EEb)
- Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function

Mastering the Standard

Comprehending the Standard

Students should know the meaning of the parameters in both linear and exponential functions in the context of the situation.

Use real-world situations to help students understand how the parameters of linear and exponential functions depend on the context.

In a linear function $y = ax + b$ the value of "a" represents the slope (constant rate of change) while "b" represents the y intercept (initial value). In an exponential function $y = a(b)^x$ the value of "a" represents the y intercept (initial value) and "b" represents the growth or decay factor. When $b > 1$ the function models growth. When $0 < b < 1$ the function models decay.

Be cautious when interpreting the growth or decay rate. If the factor is 0.85 this means that it decreasing by 15%. If the factor is 1.05, this means that is increasing

Assessing for Understanding

Students should be able to describe the effects of changes to the parameters of a linear and exponential functions.

Example: A plumber who charges $50 for a house call and $85 per hour can be expressed as the function $y = 85x + 50$. If the rate were raised to $90 per hour, how would the function change?

Example: The equation $y = 8{,}000(1.04)x$ models the rising population of a city with 8,000 residents when the annual growth rate is 4%.

a) What would be the effect on the equation if the city's population were 12,000 instead of 8,000?
b) What would happen to the population over 25 years if the growth rate were 6% instead of 4%?

Students should be able to interpret the parameters of a linear and exponential function.

Example: A function of the form $f(n)=P(1+ r)n$ is used to model the amount of money in a savings account that earns 8% interest, compounded annually, where n is the number of years since the initial deposit.

a) What is the value of r? Interpret what r means in terms of the savings account?
b) What is the meaning of the constant P in terms of the savings account? Explain your reasoning.
c) Will n or $f(n)$ ever take on the value 0? Why or why not?

Example: Lauren keeps records of the distances and cost for her taxi rides.

a) If you graph the ordered pairs (d, f) from the table, they lie on a line. How can this be determined without graphing them?
b) Show that the linear function in part a. has equation $f = 2.25d + 1.5$.

Distance d in miles	Fare f in dollars
3	8.25
5	12.75
11	26.25

by 5	c) What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides

NC.M1.F-IF.6

Interpret functions that arise in applications in terms of the context.

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Concepts and Skills

Pre-requisite

- Determine and interpret the rate of change of a linear function (8.F.4)
- Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)

Connections

- Interpret key features of graphs and tables (NC.M1.F-IF.4)
- Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: average rate of change

Mastering the Standard

Comprehending the Standard

Students calculate the average rate of change of a function given a graph, table, and/or equation.

The average rate of change of a function $y = f(x)$ over an interval $a \le x \le b$ is $\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$.

This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.

It is an important connection for further courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and

Assessing for Understanding

Example: Find the average rate of change of each of the following functions over the interval $1 \le x \le 5$.

a) $f(x) = 3x - 7$

b) $g(x) = x^2 + 2x - 5$

c) $h(x) = 3(2)^x$

Example: The table below shows the average weight of a type of plankton after several weeks.

Time(weeks)	Weight (ounces)
8	0.04
9	0.07
10	0.14
11	0.25
12	0.49

(NCDPI Math 1 released EOC #21)

What is the average rate of change in weight of the plankton from week 8 to week 12?

A) 0.0265 ounce per week

B) 0.0375 ounce per week

C) 0.055 ounce per week

D) 0.1125 ounce per week

exponentials do not have constant rates of change due to their curvature.

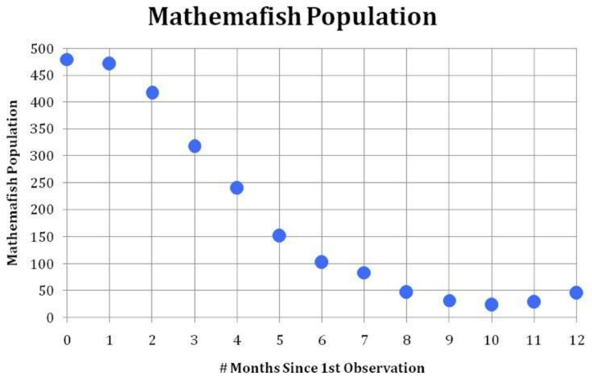
Example: The table below shows the temperature, T, in Tucson, Arizona t hours after midnight.
When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

t (hours after midnight)	0	3	4
T (temp. in ◦F)	85	76	70

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500)

Example: You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46



Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.
What to include in your summary report:

a) Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.
b) When was the population decreasing the fastest?
c) During what month did you notice the largest effects of the EPA intervention?
d) Explain the overall effects of the intervention.
e) Remember to justify all your conclusions using supporting evidence.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686)

NC.M1.F-LE.1

Construct and compare linear and exponential models and solve problems.

Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model for a situation based on the rate of change over equal intervals.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Formally define a function (NC.M1.F-IF.1)
- Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3)

Connections

- Build explicit and recursive forms of arithmetic and geometric sequences (NC.M1.F-BF.1a)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others
4 – Model with mathematics
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function

Mastering the Standard

Comprehending the Standard

Students should differentiate whether a situation (contextual, graphical, or numerical) can be represented best by a linear or exponential model.

Students should be able to identify whether a situation is linear or exponential based on the context in relation to the rate of change.

This standard can be taught with NC.MI.F-IF.3 and NC.MI.F-BF.2.

Assessing for Understanding

Students should be able to identify whether a situation is linear or exponential based on the context of the scenario and justify their decision.

Example: Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential. Explain.

Example: In (a)–(e), say whether the quantity is changing in a linear or exponential fashion.

a) A savings account, which earns no interest, receives a deposit of $723 per month.
b) The value of a machine depreciates by 17% per year.
c) Every week, 9/10 of a radioactive substance remains from the beginning of the week.
d) A liter of water evaporates from a swimming pool every day.
e) Every 124 minutes, ½ of a drug dosage remains in the body.

(https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/629)

Example: Monica did an experiment to compare two methods of warming an object. The results are shown in the table below. Which statement best describes her results?

a) The temperature using both methods changed at a constant rat
b) The temperature using both methods changed exponentially.
c) The temperature using Method 2 changed at a constant rate.
d) The temperature using Method 2 changed exponentially.

Time (hours)	**Temperature (℉) Method 1**	**Temperature (℉) Method 2**
0	0	1.5
1	5	3
2	11	6
3	15	12
4	19	24
5	25	48

(NCDPI Math 1 released EOC #24)

Example: According to Wikipedia, the International Basketball Federation (FIBA) requires that a basketball bounce to a height of 1300 mm when dropped from a height of 1800 mm.

Suppose you drop a basketball and the ratio of each rebound height to the previous rebound heig 1300:1800. Let h be the function that assigns to n the rebound height of the ball (in mm) on the nth bounce.

a) Complete the chart below, rounding to the nearest mm.
b) Write an expression for $h(n)$.
c) Solve an equation to determine on which bounce the basketball will first have a height of l than 100 mm. (Note: Students are not expected to solve part c algebraically but are expect take a table or graphical approach.)

(https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/

n	$h(n)$
0	1800
1	
2	
3	

Example: For each or the scenarios below, decide whether the situation can be modeled by a linear function, an exponential function, or neither. For those with a linear or exponential model, create a function which accurately describes the situation.

a) From 1910 until 2010 the growth rate of the United States has been steady at about 1.5% per year. The population in 1910 was about 92,000,000.
b) The circumference of a circle as a function of the radius.
c) According to an old legend, an Indian King played a game of chess with a traveling sage on a beautiful, hand-made chessboard. The sage requested, as reward for winning the game, one grain of rice for the first square, two grains for the second, four grains for the third, and so on for the whole chess board. How many grains of rice would the sage win for the nth square?
d) The volume of a cube as a function of its side length.

(https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/1910)

NC.M1.F-IF.5

Interpret functions that arise in applications in terms of the context.

Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

Concepts and Skills

Pre-requisite

- In middle school, students only informally considered restrictions to the domain and range based on context, such as understanding that measurements cannot be negative.
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)

Connections

- Recognize the domain of sequences (NC.M1.F-IF.3)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Analyze linear, quadratic, and exponential functions to identify key features (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students should be able to associate a reasonable domain and range to a graph as well as to a contextual situation.

The domain of a graph should be taught in the context of the situation it represents.

Graphs represented should be both discrete and continuous forms. Students do not need to know the terminology discrete and continuous, but they should be able to identify which is appropriate for each contextual situation.

Assessing for Understanding

Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation.

Example: Jacob is observing bacterial growth in a lab. He noted that the bacteria double in number every hour. There are 50 bacteria at the beginning of his experiment.

a) Build and graph a function to represent this scenario.

b) Determine the appropriate domain and range of the function if Jacob runs the experiment for 8 hours.

NC.M1.F-IF.7

Analyze functions using different representations.

Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Concepts and Skills

Pre-requisite

- Interpret $y = mx + b$ as being linear (8.F.3)
- Determine rate of change and initial value of linear functions from tables and graphs (8.F.4)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)

Connections

- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Solving systems of equations (NC.M1.A-REI.6)
- Recognize the domain of sequences as integers (NC.M1.F-IF.3)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Build functions that describe a relationship between two quantities (NC.M1.F-BF.1a, NC.M1.F-BF.1b)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)
- Interpret the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their use of a representation.
New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students should identify the key features of the three function families covered in Math 1: linear, quadratic, and exponential.

Students should be aware of the key functions typically associated with each function type.
Linear functions – domain & range, rate of change, intercepts, increasing/decreasing
Quadratic functions – domain & range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior
Exponential functions – domain & range, rate of change, increasing or decreasing (growth and decay), intervals of positive and negative values, and end behavior

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study end behavior of these function families, connect their mathematical thinking from “as we keep going out” or “as *x* gets really big” to “as *x* goes to infinity”.

At the Math 1 level, students should **<u>not</u>** be exposed to finding the line of symmetry of a quadratic function using the formula $x = \frac{-b}{2a}$, unless it is developed conceptually. This concept should be developed with a study of the quadratic formula, which will be done in Math 2.
If the students need to find the line of symmetry (not a requirement of Math 1), they can find the midpoint of the zeros of the function.

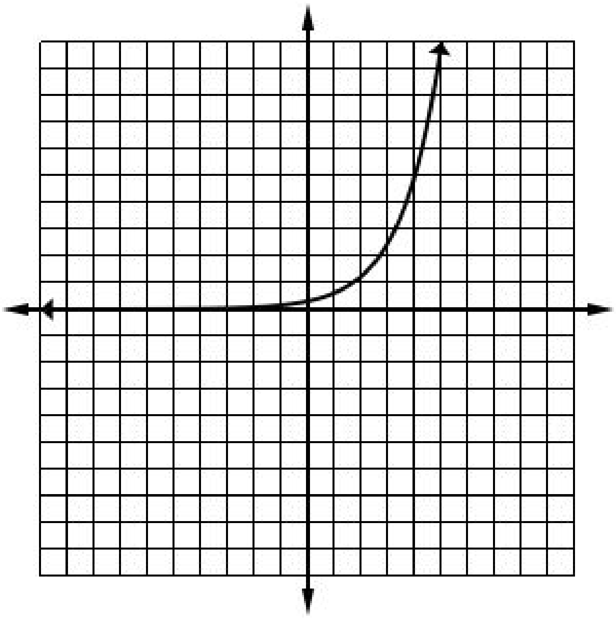
Assessing for Understanding

Students should be able to identify key feature of linear, quadratic and exponential functions from the symbolic representation.

Example: The function $f(x) = 300(0.70)^x - 25$ models the amount of aspirin left in the bloodstream after x hours. Graph the function showing the key features of the graph. Interpret the key features in context of the problem.

Students should be able to identify key feature of exponential functions from the graphical representation.

Example: Identify the key features of the graph below.



NC.M1.F-IF.8b

Analyze functions using different representations.

Use equivalent expressions to reveal and explain different properties of a function.

b. Interpret and explain growth and decay rates for an exponential function.

Concepts and Skills

Pre-requisite

- Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Formally define a function (NC.M1.F-IF.1)

Connections

- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, growth rate, decay rate

Mastering the Standard

Comprehending the Standard

This set of standards requires that students rewrite expressions of quadratic and exponential functions to reveal key features of their graphs.

This is the "why" behind rewriting an expression where NC.M1.A-SSE.1 interprets the rate in context. Therefore, these two standards should be taught together.

This standard should also tie to the key features of graphs in NC.M1.F.IF.7

Students should know the key features of an exponential function and how they relate to a contextual situation. Students should be able to find the initial value as well as the growth/decay rate for the interval based on the given context.

Assessing for Understanding

Students should know the key features of an exponential function and how they relate to a contextual situation.

Example: The expression $50(0.85)^x$ represents the amount of a drug in milligrams that remains in the bloodstream after x hours.

a) Describe how the amount of drug in milligrams changes over time.
b) What was the initial value of the drug in the bloodstream?
c) What would the expression $50(0.80)^x$ represent?
d) What new or different information is revealed by the changed expression?

Example: City Bank pays a simple interest rate of 3% per year, meaning that each year the balance increases by 3% of the initial deposit. National Bank pays an compound interest rate of 2.6% per year, compounded monthly, meaning that each month the balance increases by one twelfth of 2.6% of the previous month's balance.

a) Which bank will provide the largest balance if you plan to invest $10,000 for 10 years? For 15 years?
b) Write an expression for C(y), the City Bank balance, y years after a deposit is left in the account. Write an expression for N(m), the National Bank balance, m months after a deposit is left in the account.
c) Create a table of values indicating the balances in the two bank accounts from year 1 to year 15. For which years is City Bank a better investment, and for which years is National Bank a better investment?

(https://www.illustrativemathematics.org/content-standards/tasks/302)

NC.M1.F-LE.3

Construct and compare linear and exponential models and solve problems.

Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship and interpret rate of change (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Evaluate functions (NC.M1.F-IF.2)

Connections

- Calculate the average rate of change of an interval (NC.M1.F-IF.6)
- Identify and interpret key features, like rate of change, of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students experiment with the function types to build an understanding that the average rate of change over an interval for an exponential function will eventually surpass the rate of change of a linear or quadratic function over the same interval.

Students should be able to demonstrate this using various representations.

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study end behavior of these function families, connect their mathematical thinking from “as we keep going out” or “as x gets really big” to “as x goes to

Assessing for Understanding

Students should realize that an exponential function is eventually always bigger than a linear or quadratic function.

Example: Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month.

a) Whose account will have more money in it after two years?
b) After how many months will the accounts have the same amount of money in them?
c) Describe what happens as the money is left in the accounts for longer periods of time.

Example: Using technology, determine the average rate of change of the following functions for intervals of their domains in the table.

Functions	Average rate of change $0 \le x \le 10$	Average rate of change $10 \le x \le 20$	Average rate of change $20 \le x \le 30$	Average rate of change $30 \le x \le 40$	Average rate of change $40 \le x \le 50$
$f(x) = x^2$					
$f(x) = 1.17^x$					

a) When does the average rate of change of the exponential function exceed the average rate of change of the quadratic function?
b) Using a graphing technology, graph both functions. How do the average rates of change relate to what you see on the graph?

infinity”.	Note: You can use the information in your table to determine how to change the setting to see where the functions intersect. c) In your graphing technology, change the first function to $f(x) = 10x^2$ and adjust the settings to see where the functions intersect. What do you notice about the rates of change interpreted from the graph? d) Make a hypothesis about the rates of change about polynomial and exponential function. Try other values for the coefficient of the quadratic function to support your hypothesis.

NC.M1.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify parts of an expression using precise vocabulary (6.EE.2b)
- Interpret numerical expressions written in scientific notation (8.EE.4)
- For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)

Connections

- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)
- Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)
- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

- 2 – Reason abstractly and quantitatively.
- 4 – Model with mathematics
- 7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

- New Vocabulary: Quadratic term, exponential term

Mastering the Standard

Comprehending the Standard

The set of A-SSE standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

For this part of the standards, students recognize that the linear expression $mx + b$ has two terms, that m is a coefficient, and b is a constant.

Students are expected to recognize the parts of a quadratic expression, such as the quadratic, linear and constant term, or factors.

Assessing for Understanding

Students should recognize that in the expression $2x + 1$, “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “$2x$” and “1” being terms of the binomial expression. Also, a student recognizes that in the expression $4(3)^x$, 4 is the coefficient, 3 is the factor, and x is the exponent. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

Example: The height *(in feet)* of a balloon filled with helium can be expressed by $5 + 6.3s$ where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.

Example: The expression $-4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball t seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation.

For exponential expressions, students should recognize factors, the base, and exponent(s). Students extend beyond simplifying to interpret the components of an algebraic expression.	**Example:** The expression $35000(0.87)^t$ describes the cost of a new car t years after it has been purchased. Interpret the terms and coefficients of the expression in the context of this situation.

NC.M1.F-IF.9

Analyze functions using different representations.

Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Compare properties of two functions each represented in different ways (8.F.2)
- Formally define a function (NC.M1.F-IF.1)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Rewrite quadratic functions to identify key features (NC.M1.F-IF.8a)
- Interpret and explain growth and decay rates for an exponential function (NC.M1.F-IF.8b)

Connections

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The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

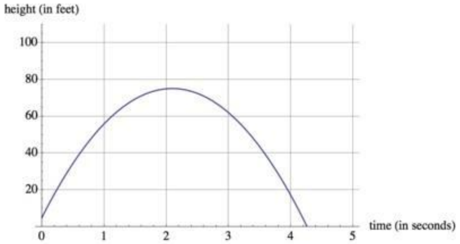
Students should be able to justify their use of a representation to make the comparison.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students should compare two functions in two different forms. The function types may be the same (linear & linear) or different (linear & exponential), but the representations should be different (e.g. numerical & graphical). It is important to note that the point of this standard is not to have students simply translate one function into the same form as the other function when given in different forms. Students should be able to use	**Example:** Suppose Brett and Andre each throws a baseball into the air. The height of Brett's baseball is given by $h(t) =- 16t^2 + 79t + 6$, where h is in feet and t is in seconds. The height of Andre's baseball is given by the graph below:

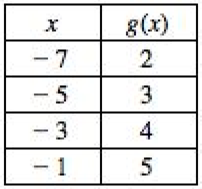
appropriate tools to compare the key features of functions.



Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher.

a) Who is right?
b) How long is each baseball airborne?
c) Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).

Example: Dennis compared the y-intercept of the graph of the function $f(x) = 3x + 5$ to the y-intercept of the graph of the linear function that includes the points in the table below.



x	$g(x)$
-7	2
-5	3
-3	4
-1	5

What is the difference when the y-intercept of $f(x)$ is subtracted from the y-intercept of $g(x)$?

A) -11.0
B) -9.3
C) 0.5
D) 5.5

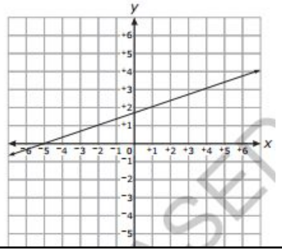
(NCDPI Math 1 released EOC #22)

Example: Joe is trying to decide which job would allow him to earn the most money after a few years.

- His first job offer agrees to pay him $500 per week. If he does a good job, they will give him a 2% raise each year.
- His other job offer agrees to pay him according to the following equation $f(x) = 20,800(1.03)^x$, where x represents the number of years and $f(x)$ his salary.

Which job would you suggest Joe take? Justify your reasoning.

Example: Mario compared the slope of the function graphed below to the slope of the linear function that has an x-intercept of $\frac{4}{3}$ and a y-intercept of -2.



What is the slope of the function with the smaller slope?

A) $\frac{1}{5}$
B) $\frac{1}{3}$
C) 3
D) 5

(NCDPI Math 1 EOC released #25)

NC.M1.A-REI.11

Represent and solve equations and inequalities graphically

Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)
- Solving systems of equations (NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

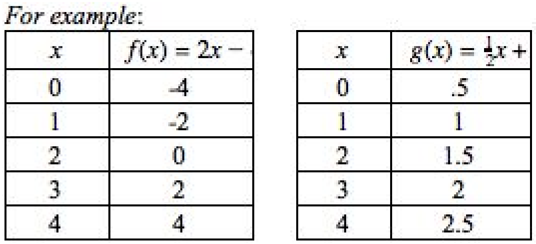
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

For a complete understanding, students will need exposure to both parts of this standard.



For example:

x	$f(x) = 2x -$
0	-4
1	-2
2	0
3	2
4	4

x	$g(x) = \frac{1}{2}x +$
0	.5
1	1
2	1.5
3	2
4	2.5

Students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation. Because $f(x) = g(x)$ when $x = 3$, 3 is the solution to the equation $2x - 4 = \frac{1}{2}x + \frac{1}{2}$

In Math 1, students are expected to solve linear equations using inverse operations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.

For example:

Solve: $3x^2 - 2x + 1 = \frac{1}{2}x + 5$

Rewrite the equations as a system of equations: $f(x) = 3x^2 - 2x + 1$ $g(x) = \frac{1}{2}x + 5$

Using technology, graph the equations and look for points of intersection, where the same x produces $f(x) = g(x)$.

Assessing for Understanding

Students should be able to find approximate solutions to exponential equations using technology, tables and graphs.

Example: Solve the following equations by graphing. Give your answer to the nearest tenth.

a) $3(2^x) = 6x - 7$

b) $10x + 5 =- x + 8$

Example: The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

a) Based on these assumptions, in approximately what year will this country first experience shortages of food?

b) If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

c) If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still

	occur? (https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/645

NC.M1.F-BF.1a

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

a. Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Recognize arithmetic and geometric sequences as linear and exponential functions (NC.M1.F-IF.3)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

Connections

- Create and graph two variable equations (NC.M1.A-CED.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Translate between explicit and recursive forms (NC.M1.F-BF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify claims that a sequence defines a linear or exponential relationship.

New Vocabulary: arithmetic sequence, geometric sequence, exponential function

Mastering the Standard

Comprehending the Standard

This standard is about building a function from different representations. In this part of the standard, the different representations include: sequences, graphs, verbal descriptions, tables, and ordered pairs.

This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

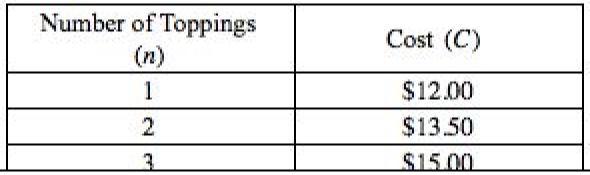
These functions can be written in function

Assessing for Understanding

Students should write functions from verbal descriptions as well as a table of values

Example: Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days.

Example: The table below shows the cost of a pizza based on the number of toppings.



Number of Toppings (n)	Cost (C)
1	$12.00
2	$13.50
3	$15.00

Which function represents the cost of a pizza with n toppings?

A) $C(n) = 12 + 1.5(n - 1)$

B) $C(n) = 1.5n + 12$

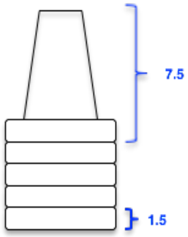
C) $C(n) = 12 + n$

notation (linear or exponential) or as a sequence in explicit or recursive form. Students should recognize explicit form of an arithmetic sequence as an equivalent structure to slope-intercept form of a linear function and explicit form of a geometric sequence as an equivalent structure to standard form of an exponential function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the y-intercept, which gives meaning to the $n-1$ notation.

D) $C(n) = 12n$

(NCDPI Math 1 released EOC #39)

Example: The height of a stack of cups is a function of the number of cups in the stack. If a 7.5” cup with a 1.5” lip is stacked vertically, determine a function that would provide you with the height based on any number of cups.
Hint: Start with height of one cup and create a table, list, graph or description that describes the pattern of the stack as an additional cup is added.



Example: There were originally 4 trees in an orchard. Each year the owner planted the same number of trees. In the 29th year, there were 178 trees in the orchard. Which function, $t(n)$, can be used to determine the number of trees in the orchard in any year, n?

A) $t(n) = \frac{178}{29}n + 4$
B) $t(n) = \frac{178}{29}n - 4$
C) $t(n) = 6n + 4$
D) $t(n) = 29n - 4$

(NCDPI Math 1 released EOC #42)

Students should write linear or exponential relationships as a sequence in explicit or recursive form.

Example: The price of a new computer decreases with age. Examine the table by analyzing the outputs.

a) Describe the recursive relationship.
b) Analyze the input and the output pairs to determine an explicit function that represents the value of the computer when the age is known.

Age	Value
1	$1575
2	$1200
3	$900
4	$650
5	$500
6	$400
7	$300

NC.M1.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Operations with polynomials (NC.M1.A-APR.1)
- Formally define a function (NC.M1.F-IF.1)

Connections

- Create and graph two variable equations (NC.M1.A-CED.2)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their process of building a new function.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

This standard is about building functions. In this part of the standard students should combine functions to represent a contextual situation.

This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

The algebraic skills behind this standard occur in NC.M1.A-APR.1. This standard should be taught throughout the year as each new function family is added to the course.

Assessing for Understanding

Students should combine functions to represent a contextual situation.

Example: Cell phone Company Y charges a $10 start-up fee plus $0.10 per minute, x. Cell phone Company Z charges $0.20 per minute, x, with no start-up fee. Which function represents the difference in cost between Company Y and Company Z?

A) $f(x) =- 0.10x - 10$

B) $f(x) =- 0.10x + 10$

C) $f(x) = 10x - 0.10$

D) $f(x) = 10x + 0.10$

(NCDPI Math 1 released EOC #23)

Example: A retail store has two options for discounting items to go on clearance.

- Option 1: Decrease the price of the item by 15% each week.
- Option 2: Decrease the price of the item by $5 each week.

If the cost of an item is $45, write a function rule for the difference in price between the two options.

Example: Blake has a monthly car payment of $225. He has estimated an average cost of $0.32 per mile for gas and maintenance. He plans to budget for the car payment the minimal he needs with an additional 3% of his total budget for incidentals that may occur. Build a function that gives the amount Blake needs to budget as a function of the number of miles driven.

NC.M1.S-ID.6c

Summarize, represent, and interpret data on two categorical and quantitative variables.

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

c. Fit a function to exponential data using technology. Use the fitted function to solve problems.

Concepts and Skills

Pre-requisite

- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)

Connections

- Create and graph equations that represent exponential relationships (NC.M1.A-CED.1)
- Recognize a geometric sequence as a subset of the range of an exponential function (NC.M1.F-IF.3)
- Exponential growth and decay (NC.M1.F-IF.8b)
- Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)
- Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)
- Interpret the parameters in linear or exponential functions in terms of a context (NC.M1.F-LE.5)
- Interpret key features in context to describe functions relating two quantities (NC.M1.F-IF.4)
- Interpret a function in terms of its domain and range in context (NC.M1.F-IF.5)
- Calculate and interpret the avg. rate of change for a function (NC.M1.F-IF.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

Mastering the Standard

Comprehending the Standard

Work with exponential functions is new to students. In 8th grade, students focused on identifying characteristics of linear functions and distinguishing them from non-linear functions. Students will use the same tools to explore exponential functions specifically.

This standard should be explored in context to help students make meaning of the behavior of exponential models. Technology can be used as a tool to make connections between symbolic, tabular and graphical representations of exponential functions. This will also help to build conceptual understanding of exponential growth and decay.

At this level, students should be able to support the use of an exponential model based on the graphical display and the understanding of the constant ratio between consecutive terms; a concept supported by the study of geometric sequences.

Students should be presented with exponential data and asked to fit the function to the data using technology. They should **NOT** have to *verify* the appropriateness of an exponential model; analysis at that level requires transformations for linearity, which is an advanced statistical concept.

Assessing for Understanding

Students can use graphing technology or a graphing calculator to determine the exponential model for a given data set or scatter plot.

Example: What is the exponential function that best models the number of gnats the scientists have gathered after the number of hours listed? How many hours will it take for 200 gnats to gather?

Hours	0	1	2	3	4
Number of gnats	12	20	35	60	80

Students can make connections between the graph, table, and symbolic representations of an exponential function.

Example: In an experiment, 300 pennies were shaken in a cup and poured onto a table. Any penny ‘heads up’ was removed. The remaining pennies were returned to the cup and the process was repeated. The results of the experiment are shown below. Write a function rule suggested by the context. Use the context to explain all values of the function. How are those values reflected in the table?

# of Rolls	0	1	2	3	4	5
# of Pennies	300	164	100	46	20	8

x

The North Carolina High School Collaborative Instructional Framework

NC Math 1

Unit 4: Introduction to Quadratic Functions and Equations

15 Days Block Schedule | **September 2017 Update** | 30 Days Traditional Schedule

RESEARCH BRIEF: Quadratic Functions

Essential Questions:

- What patterns of change are involved in projectile motion?
- What patterns of change appear in tables and graphs of values for quadratic functions?
- What functions model patterns of change that appear in tables and graphs of values for projectile motion?
- How can tables, graphs, and equations for quadratic functions be used to answer questions about the situations they represent?
- How are the values of a, b, and c related to patterns in the graphs and tables of values for quadratic functions in the form $f(x) = ax^2+bx+c$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2+c$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2+bx$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2+bx+c$?
- What strategies are useful in finding rules for quadratic functions?
- How are quadratic functions used in the real world?
- What strategies are useful in deciding whether two quadratic expressions are equivalent?
- What strategies are useful in deciding when one form of quadratic expression is more useful than another?
- What strategies can be used to transform quadratic expressions into useful equivalent forms?
- What are some effective methods for solving quadratic expressions algebraically?
- How do you determine when one method of solving quadratic is more efficient than another?

Learning Outcomes

- Students will use all arithmetic operations on polynomials.
- Students will determine patterns of change associated with quadratic functions.
- Students will use tables of values and graphs to estimate answers for questions about situations modeled by quadratic functions.
- Students will describe the effects of each parameter in the function rule $y=ax^2+bx+c$
- Students will determine whether two quadratic expressions are equivalent.
- Students will determine the most useful form of a quadratic function depending on s for different question types.
- Students will create equivalent quadratic expressions by expanding products of linear factors using the distributive property.
- Students will factor and solve quadratic equations using the greatest common factor.
- Students will solve quadratic equations using square roots.
- Students will factor quadratic equations into binomial factors and solve using the zero-product property.
- Students will write quadratic equations and inequalities to express questions about quadratic functions.
- Students will understand the relationship between factors and roots, solutions, x-intercepts, and zeros of a quadratic function.
- Students will solve a system of equations of a linear function and a quadratic function.
- Students will determine the vertex, x-intercept, and y-intercept from a graph, equation and in context with and without a graphing calculator.

Student Objectives

- I will **describe** the patterns of change in quadratic functions.
- I will **evaluate** tables and graphs to predict solutions of a quadratic model in context.
- I will **describe** how changing each part (*a, b, c*) of a quadratic equation alters the function.
- I will **calculate** the quadratic regression given a set of data.
- I will **justify** whether a pair of quadratic equations are equivalent.
- I will **determine** which quadratic form is best in the context of the problem.
- I will **use** the roots and/or factors of a polynomial to **write** a quadratic in standard form.
- I will **solve** a quadratic equation by factoring (GCF) or extracting the roots.
- I will **solve** a quadratic equation by factoring (binomial factors - difference of square, trinomials with and without leading coefficient).
- I will **create** a quadratic equation or inequality from a given scenario.
- I will **understand** the relationship between factors and roots, solutions, x-intercepts, and zeros.
- I will **locate** and **explain** the vertex, x-intercept, and y-intercept of a quadratic given a graph, an equation, and in context of a situation.
- I will **describe** key features of quadratic functions in context of the scenario they represent.

Standards Addressed in this Unit

Understand the terms and properties of polynomials.

- **NC.M1.A-APR.1:** Perform arithmetic operations on polynomials. Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.

- **NC.M1.A-SSE.1a**: Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

- **NC.M1.A-SSE.1b:** Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

Understand how changing the coefficients of a quadratic expression affect the key features of its related quadratic function.

- **NC.M1.F-IF.7:** Analyze functions using different representations. Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

- **NC.M1.F-IF.9:** Analyze functions using different representations. Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

- **NC.M1.F-LE.3:** Construct and compare linear, exponential, and quadratic models and solve problems. Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially exceeds a quantity increasing linearly or quadratically.

- **NC.M1.F-IF.6:** Interpret functions that arise in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

- **NC.M1.F-IF.8a:** Analyze functions using different representations. Use equivalent expressions to reveal and explain different properties of a function. Rewrite a quadratic function to reveal and explain different key features of the function.

- **NC.M1.A-REI.10**: Represent and solve equations and inequalities graphically. Understand that the graph of a two variable equation represents the set of all solutions to the equation.

- **NC.M1.F-IF2**: Understand the concept of a function and use function notation. Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **NC.M1.A-CED.2**: Create and graph equations in two variables to represent linear exponential, and quadratic relationships between quantities.

- **NC.M1.A-CED.1**: Create equations and inequalities in one variable that represent linear, exponential and quadratic relationships and use them to solve problems.

Understand the relationship between the factors of a quadratic expression and the solutions to its related quadratic equation.

- **NC.M1.A-SSE.3**: Write an equivalent form of a quadratic expression by factoring, where a is an integer of the quadratic expression, $ax^2 + bx + c$, to reveal the solutions of the equation or the zeros of the function the expression defines.

- **NC.M1.A-REI.4**: Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

- **NC.M1.A-APR.3**: Understand the relationship between zeros and factors of polynomials. Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.

- **NC.M1.A-REI.1**: Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

- **NC.M1.A-REI.11**: Build an understanding of why the *x*-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Build quadratic functions from other functions and interpret the key features of a quadratic function in context.

- **NC.M1.F-IF.4**: Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

- **NC.M1.A-APR.1:** Perform arithmetic operations on polynomials. Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.
- **NC.M1.A-SSE.1b:** Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.
- **NC.M1.F-BF.1b**: Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.
- **NC.M1.F-IF.5**: Interpret a function in terms of context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.
- **NC.M1.A-REI.11:** Build an understanding of why the *x*-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.
- **NC.M1.F-IF.7:** Analyze functions using different representations. Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.
- **NC.M1.S-ID.8:** Interpret linear models. Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M1.A-APR.1

Perform arithmetic operations on polynomials.

Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.

Concepts and Skills

Pre-requisite

- Add, subtract, factor and expand linear expressions (7.EE.1)
- Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)

Connections

- Rewrite expressions using the properties of exponents (NC.M1.N-RN.2)
- Understanding the process of elimination (NC.M1.A-REI.5)
- Rewrite a quadratic function to reveal key features (NC.M1.F-IF.8a)
- Building functions to model a relationship (NC.M1.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to compare operations with polynomials to operations with integers.

New Vocabulary: polynomial, quadratic expression

Mastering the Standard

Comprehending the Standard

Students connect their knowledge of integer operations to polynomial operations.

At the Math 1 level, students are only responsible for the following operations:

- adding and subtracting quadratic expressions
- adding, subtracting, and multiplying linear expressions

Assessing for Understanding

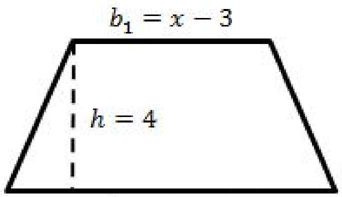
Students should be able to rewrite polynomial expressions using the properties of operations.

Example: Write at least two equivalent expressions for the area of the circle with a radius of $5x - 2$ kilometers.

Example: Simplify each of the following:

a) $(4x + 3) - (2x + 1)$
b) $(x^2 + 5x - 9) + 2x(4x - 3)$

Example: The area of a trapezoid is found using the formula $A = \frac{1}{2}h(b_1 + b_2)$, where A is the area, h is the height, and b_1 and b_2 are the lengths of the bases.

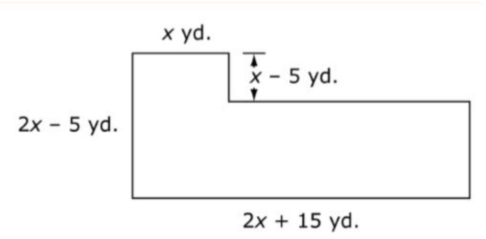


What is the area of the trapezoid?

A) $A = 4x + 2$
B) $A = 4x + 8$
C) $A = 2x^2 + 4x - 21$
D) $A = 2x^2 + 8x - 42$

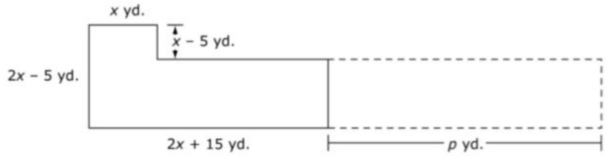
(*NCDPI Math I released EOC #33*)

Example: A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.



a) Write an expression for the area, in square feet, of this proposed parking lot. Explain the reasoning you used to find the expression.

b) The town council has plans to double the area of the parking lot in a few years. They plan to increase the length of the base of the parking lot by p yards, as shown in the diagram below.



c) Write an expression in terms of x to represent the value of p, in feet. Explain the reasoning you used to find the value of p.

Example: A cardboard box as a height of x, a width that is 3 units longer than the height, and a length that is 2 units longer than the width. Write an expression in terms of x to represent the volume of the box.

NC.M1.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify parts of an expression using precise vocabulary (6.EE.2b)
- Interpret numerical expressions written in scientific notation (8.EE.4)
- For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)

Connections

- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)
- Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)
- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Quadratic term, exponential term

Mastering the Standard

Comprehending the Standard

The set of A-SSE standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

For this part of the standards, students recognize that the linear expression $mx + b$ has two terms, that *m* is a coefficient, and *b* is a constant.

Students are expected to recognize the parts of a quadratic expression, such as the quadratic, linear and constant term, or factors.

For exponential expressions, students should recognize factors, the base, and exponent(s).

Students extend beyond simplifying to interpret the components of an algebraic expression.

Assessing for Understanding

Students should recognize that in the expression $2x + 1$, “2” is the coefficient, “2” and “*x*” are factors, and “1” is a constant, as well as “2*x*” and “1” being terms of the binomial expression. Also, a student recognizes that in the expression $4(3)^x$, 4 is the coefficient, 3 is the factor, and *x* is the exponent. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

> **Example:** The expression $-4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball *t* seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation.

NC.M1.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

Concepts and Skills

Pre-requisite

- Interpret a sum, difference, product, and quotient as a both a whole and as a composition of parts (6.EE.2b)
- Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)
- Interpret numerical expressions written in scientific notation (8.EE.4)

Connections

- Factor to reveal the zeros of functions and solutions to quadratic equations (NC.M1.A.SSE.3)
- Creating one and two variable equations (NC.M1.A-CED.1, NC.M1.A-CED.2, NC.M1.A-CED.3)
- Interpreting part of a function to a context (NC.M1.F-IF.2, NC.M1.F-IF.4, NC.M1.F-IF5, NC.M1.F-IF.7, NC.M1.F-IF.9)
- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential expression, quadratic expression

Mastering the Standard

Comprehending the Standard

The set of A-SSE standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

Students identify parts of an expression as a single quantity and interpret the parts in terms of their context.

Assessing for Understanding

Students should understand that working with unsimplified expressions often reveals key information from a context.

Example: A rectangle has a length that is 2 units longer than the width. If the width is increased by 4 units and the length increased by 3 units, write two equivalent expressions for the area of the rectangle.

Solution*: The area of the rectangle is $(x+5)(x+4) = x^2 + 9x + 20$. Students should recognize $(x+5)$ as the length of the modified rectangle and $(x+4)$ as the width. Students can also interpret $x^2 + 9x + 20$ as the sum of the three areas (a square with side length x, a rectangle with side lengths 9 and x, and another rectangle with area 20 that have the same total area as the modified rectangle.*

Example: Given that income from a concert is the price of a ticket times each person in attendance, consider the equation $I = 4000p - 250p^2$ that represents income from a concert where p is the price per ticket. What expression could represent the number of people in attendance?

Solution: *The equivalent factored form, $p\,(4000 - 250p)$, shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize $(4000 - 250p)$ as a single quantity for the number of people in attendance.*

NC.M1.F-IF.7

Analyze functions using different representations.

Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

Concepts and Skills

Pre-requisite

- Interpret $y = mx + b$ as being linear (8.F.3)
- Determine rate of change and initial value of linear functions from tables and graphs (8.F.4)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)

Connections

- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Solving systems of equations (NC.M1.A-REI.6)
- Recognize the domain of sequences as integers (NC.M1.F-IF.3)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Build functions that describe a relationship between two quantities (NC.M1.F-BF.1a, NC.M1.F-BF.1b)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)
- Interpret the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their use of a representation.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students should identify the key features of the three function families covered in Math 1: linear, quadratic, and exponential.

Students should be aware of the key functions typically associated with each function type. Quadratic functions – domain & range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$) . As students study end behavior of these function families, connect their mathematical thinking from "as we keep going out" or "as x gets really big" to "as x goes to infinity".

At the Math 1 level, students should **not** be exposed to finding the line of symmetry of a quadratic function using the formula $x = \frac{-b}{2a}$, unless it is developed conceptually.
This concept should be developed with a study of the quadratic formula, which will be done in Math 2.
If the students need to find the line of symmetry (not a requirement of Math 1), they can find the midpoint of the zeros of the function.

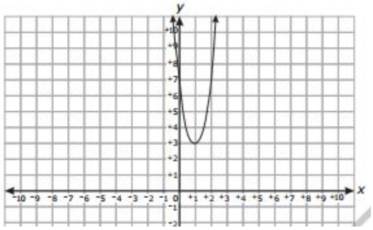
Assessing for Understanding

Students should be able to identify key feature of linear, quadratic and exponential functions from the symbolic representation.

Example: Without using the graphing capabilities of a calculator, sketch the graph of $f(x) = x^2 + 7x + 10$ and identify the x-intercepts, y-intercept, and the maximum or minimum point.

Students should be able to identify key feature of linear, quadratic and exponential functions from the graphical representation.

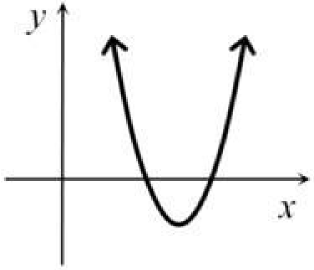
Example: Which of the following is the function graphed below?



A) $f(x) = 4x^2 - 8x + 7$
B) $f(x) = x^2 + 7x + 3$
C) $f(x) = 7x^2 - 4x + 3$
D) $f(x) = 3x^2 + x + 7$

(NCDPI Math 1 released EOC #4 modified)

Example: Which of the following could be the function of a real variable x whose graph is shown below? Explain.



$f_1(x) = (x+12)^2 + 4$	$f_5(x) =- 4(x+2)(x+3)$
$f_2(x) =- (x-2)^2 - 1$	$f_6(x) = (x+4)(x-6)$
$f_3(x) = (x+18)^2 - 40$	$f_7(x) = (x-12)(-x+18)$
$f_4(x) = (x-12)^2 - 9$	$f_8(x) = (24-x)(40-x)$

*This task could be modified for a Math 1 classroom to not use vertex form.
(https://www.illustrativemathematics.org/content-standards/HSF/IF/C/8/tasks/640)

NC.M1.F-IF.9

Analyze functions using different representations.

Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Compare properties of two functions each represented in different ways (8.F.2)
- Formally define a function (NC.M1.F-IF.1)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Rewrite quadratic functions to identify key features (NC.M1.F-IF.8a)
- Interpret and explain growth and decay rates for an exponential function (NC.M1.F-IF.8b)

Connections

-

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their use of a representation to make the comparison.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

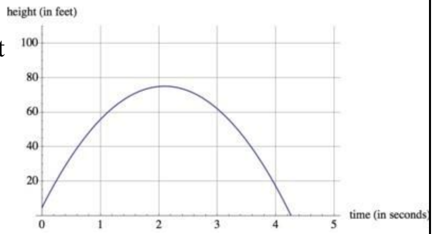
Students should compare two functions in two different forms. The function types may be the same (linear & linear) or different (linear & exponential), but the representations should be different (e.g. numerical & graphical).

It is important to note that the point of this standard is not to have students simply translate one function into the same form as the other function when given in different forms. Students should be able to use appropriate tools to compare the key features of functions.

Assessing for Understanding

Students should compare two functions in two different forms.

Example: Suppose Brett and Andre each throws a baseball into the air. The height of Brett's baseball is given by $h(t) = -16t^2 + 79t + 6$, where h is in feet and t is in seconds. The height of Andre's baseball is given by the graph below:



Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher.

a) Who is right?

b) How long is each baseball airborne?

c) Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).

Example: Kevin compared the y-intercept of the graph of the function $f(x) = 3x^2 + 5$ to the

x	$g(x)$
⁻7	2
⁻5	3

	y-intercept of the graph of the linear function that includes the points in the table to the right. What is the difference when the y-intercept of f(x) is subtracted from the y-intercept of g(x)?

NC.M1.F-LE.3

Construct and compare linear and exponential models and solve problems.

Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship and interpret rate of change (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Evaluate functions (NC.M1.F-IF.2)

Connections

- Calculate the average rate of change of an interval (NC.M1.F-IF.6)
- Identify and interpret key features, like rate of change, of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students experiment with the function types to build an understanding that the average rate of change over an interval for an exponential function will eventually surpass the rate of change of a linear or quadratic function over the same interval.

Students should be able to demonstrate this using various representations.

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study end behavior of these function families, connect their mathematical thinking from "as we keep going out" or "as x gets really big" to "as x goes to infinity".

Assessing for Understanding

Students should realize that an exponential function is eventually always bigger than a linear or quadratic function.

Example: Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month.

a) Whose account will have more money in it after two years?
b) After how many months will the accounts have the same amount of money in them?
c) Describe what happens as the money is left in the accounts for longer periods of time.

Example: Using technology, determine the average rate of change of the following functions for intervals of their domains in the table.

Functions	Average rate of change $0 \le x \le 10$	Average rate of change $10 \le x \le 20$	Average rate of change $20 \le x \le 30$	Average rate of change $30 \le x \le 40$	Average rate of change $40 \le x \le 50$
$f(x) = x^2$					
$f(x) = 1.17^x$					

a) When does the average rate of change of the exponential function exceed the average rate of change of the quadratic function?
b) Using a graphing technology, graph both functions. How do the average rates of change in your table relate to what you see on the graph?

	Note: You can use the information in your table to determine how to change the setting to see where the functions intersect. c) In your graphing technology, change the first function to $f(x) = 10x^2$ and adjust the settings to see where the functions intersect. What do you notice about the rates of change interpreted from the graph? d) Make a hypothesis about the rates of change about polynomial and exponential function. Try other values for the coefficient of the quadratic function to support your hypothesis.

NC.M1.F-IF.6

Interpret functions that arise in applications in terms of the context.

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

Concepts and Skills

Pre-requisite

- Determine and interpret the rate of change of a linear function (8.F.4)
- Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)

Connections

- Interpret key features of graphs and tables (NC.M1.F-IF.4)
- Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: average rate of change

Mastering the Standard

Comprehending the Standard

Students calculate the average rate of change of a function given a graph, table, and/or equation.

The average rate of change of a function $y = f(x)$ over an interval $a \leq x \leq b$ is $\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$.

This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.

It is an important connection for further courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature.

Assessing for Understanding

Example: Find the average rate of change of each of the following functions over the interval $1 \leq x \leq 5$.

- $f(x) = 3x - 7$
- $g(x) = x^2 + 2x - 5$
- $h(x) = 3(2)^x$

Example: The table below shows the average weight of a type of plankton after several weeks.

Time(weeks)	Weight (ounces)
8	0.04
9	0.07
10	0.14
11	0.25
12	0.49

What is the average rate of change in weight of the plankton from week 8 to week 12?

A) 0.0265 ounce per week
B) 0.0375 ounce per week
C) 0.055 ounce per week
D) 0.1125 ounce per week

(NCDPI Math 1 released EOC #21)

Example: The table below shows the temperature, T, in Tucson, Arizona t hours after midnight. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

t (hours after midnight)	0	3	4
T (temp. in ∘F)	85	76	70

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500)

NC.M1.F-IF.8a

Analyze functions using different representations.

Use equivalent expressions to reveal and explain different properties of a function.

a. Rewrite a quadratic function to reveal and explain different key features of the function

Concepts and Skills

Pre-requisite

- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Factor to reveal key features (NC.M1.A-SSE.3)
- Operations with polynomials (NC.M1.A-APR.1)
- Understand the relationship between linear factors and zeros (NC.M1.A-APR.3)
- Formally define a function (NC.M1.F-IF.1)

Connections

- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: quadratic function

Mastering the Standard

Comprehending the Standard

This set of standards requires that students rewrite expressions of quadratic and exponential functions to reveal key features of their graphs.

This is the "why" behind rewriting an expression where NC.M1.A-SSE.1 is the "how". Therefore, these two standards should be taught together.

This standard should also tie to the key features of graphs in NC.M1.F.IF.7

At the Math 1 level, students only know two forms of quadratics; standard and factored. Students SHOULD NOT complete the square or write a quadratic in vertex form. Therefore, other methods for finding the vertex should be used, such as calculating the midpoint between two zeros to find the x-value of the vertex and using function notation to determine the y-value of the vertex. Using a graphing utility to analyze key features of a quadratic function may be necessary.

At the Math 1 level, students should **not** be exposed to finding the line of symmetry of a quadratic function using the formula $x = \frac{-b}{2a}$, unless it is developed conceptually. This concept can be developed with a study of the quadratic formula in Math 2.

Assessing for Understanding

Students should be able to factor quadratic expressions to find key features of the quadratic function.

Example: Suppose $h(t) =-5t^2 + 10t + 15$ is the height of a diver above the water (in meters), t seconds after the diver leaves the springboard.

a) How high above the water is the springboard? Explain how you know.
b) When does the diver hit the water?
c) At what time on the diver's descent toward the water is the diver again at the same height as the springboard?
d) When does the diver reach the peak of the dive?

(https://www.illustrativemathematics.org/content-standards/HSF/IF/C/8/tasks/375)

Example: The function $f(t) =-5t^2 + 20t + 60$ models the approximate height of an object t seconds after it is launched. How many seconds does it take the object to hit the ground?

(NCDPI Math 1 released EOC #9)

If the students need to find the line of symmetry (not a requirement of Math 1), they can find the midpoint of the zeros of the function. The typical key features of a quadratic functions are: domain and range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior	**Example:** Suppose that the equation $V = 20.8x^2 - 458.3x + 3500$ represents the value of a car from 1964 to 2002. What year did the car have the least value? ($x = 0$ in 1964) A) 1965 B) 1970 C) 1975 D) 1980 (NCDPI Math 1 released EOC #19)

NC.M1.A-REI.10

Represent and solve equations and inequalities graphically

Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Concepts and Skills

Pre-requisite

- Use substitution to determine if a number if a solution (6.EE.5)
- Graphing lines (8.EE.5, 8.EE.6, 8.F.3)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understanding functions as a rule that assigns each input with exactly one output (8.F.1)

Connections

- Creating and graphing two-variable equations (NC.M1.A-CED.2)
- Solutions to systems of equations (NC.M1.A-REI.5, NC.M1.A-REI.6)
- Understanding that the relationship between the solution of system of equations and the associated equation (NC.M1.A-REI.11)
- Representing the solutions to linear inequalities (NC.M1.A-REI.12)
- Relating a function to its graph, domain and range of a function (NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the solutions to a two variable equation and the link to a function.

Mastering the Standard

Comprehending the Standard

Students understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement.

This standard contains no limitation and so applies to all function types, including those functions that a student cannot yet algebraically manipulate.

Students can explain and verify that every point (x, y) on the graph of an equation represents all values for x and y that make the equation true.

Assessing for Understanding

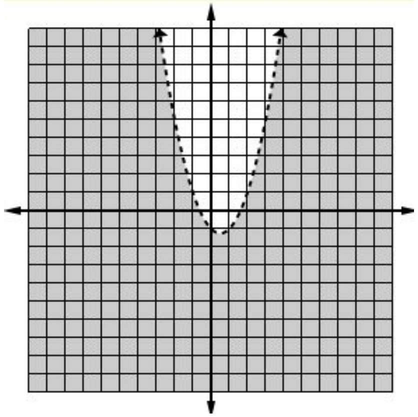
Students should be able to assess if a point is a solution to an equation.

Example: Without graphing, determine if the ordered pair $(2,-15)$ is on the graph of $y = 3x^2 + 2x - 1$. Explain.

Example: Given the function to the right, determine if the following points are solutions and explain each answer.

a) (2, 1)
b) (3, 8)
c) (-1, -4)

Note: ($y < x^2 - x - 1$ is the inequality of the graph)



NC.M1.F-IF.2

Understand the concept of a function and use function notation.

Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Concepts and Skills

Pre-requisite

- Use substitution to determine if a number if a solution (6.EE.5)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Define a function and use functions notation (NC.M1.F-IF.1)

Connections

- Creating and solving one variable equations (NC.M1.A-CED.1)
- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Function standards that relate domain and range (NC.M1.F-IF.3, NC.M1.F-IF.4, NC.M1.F-IF.5, NC.M1.F-IF.7)
- Comparing the end behavior of functions (NC.M1.F-LE.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the domain, range, input, output and the relationship between the variables of a function in context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function. Students should be able to interpret statements in function notation in contextual situations.

Assessing for Understanding

Students should be able to use evaluate functions written in function notation.

Example: Evaluate $f(2)$ for the function $f(x) = 3x^2 + 2x - 5$.

Students should be able to evaluate functions and interpret the result in a context.

Example: Suppose Matthew throws a baseball into the air. The height of the baseball at any given time, t, can be modeled by the function $h(t) = -16t^2 + 65t + 5$.

a) What is the height of the baseball after 2 seconds?

b) If $h(1) = 54$, what does this mean in context of the baseball scenario?

NC.M1.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Concepts and Skills

Pre-requisite

- Construct a linear function that models the relationship between two quantities (8.F.4)
- Graph linear equations (8.EE.6)
- The graph of a function is the set of ordered pairs consisting of input and a corresponding output (8.F.1)
- Understand that the graph of a two-variable equation represents the set of all solutions to the equation (NC.M1.A-REI.10)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Creating linear equations for a system (NC.M1.A-CED.3)
- Solving for a variable of interest in a formula (NC.M1.A-CED.4)
- The graph a function f is the graph of the equation $y = f(x)$ (NC.M1.F-IF.1)
- Interpret a function's domain and range in context (NC.M1.F-IF.5)
- Identify key features of linear, exponential and quadratic functions (NC.M1.F-IF.7)
- Building a function through patterns or by combining other functions (NC.M1.F-BF.1a , NC.M1.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and demonstrate its relation to the context.
New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students create equations in two variables. Students graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for the context and display adequate information about the relationship.Students interpret the context and choose appropriate minimum and maximum values for a graph.
In Math I, focus on linear, exponential and quadratic **contextual** situations for students to

Assessing for Understanding

Students should be able to create two variable equations from various representations, such as verbal descriptions, and use them to solve problems.

Example: The larger leg of a right triangle is 3 cm longer than its smaller leg. The hypotenuse is 6 cm longer than the smaller leg. How many centimeters long is the smaller leg?

Example: The floor of a rectangular cage has a length 4 feet greater than its width, w. James will increase both dimensions of the floor by 2 feet. Which equation represents the new area, N, of the floor of the cage?

a) $N = w^2 + 4w$
b) $N = w^2 + 6w$
c) $N = w^2 + 6w + 8$
d) $N = w^2 + 8w + 12$

create equations in two variables.	Students should be able to create two variable equations, graph the relationship, and use graph to recognize key feature of the graph. **Example:** Misha has a new rabbit that she named “Wascal.” She wants to build Wascal a pen, so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen. a) If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen? b) Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph. c) What are the dimensions of the pen that would allow Wascal the most area to run around? How do you know? (www.mathematicsvisionproject.org)

NC.M1.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

Concepts and Skills

Pre-requisite

- Create two-step linear equations and inequalities from a context (7.EE.4)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)
- Solve linear and quadratic equations and linear inequalities (NC.M1.A-REI.3, NC.M1.A-REI.4)
- Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)
- Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students create equations and inequalities in one-variable and use them to solve problems.

In Math I, focus on linear, quadratic, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can be created from an associated function when a given value is substituted in for either the independent or dependent variable.

After the students have created an equation, they can use other representations to assist in solving problems, such as graphs and tables.

For quadratic inequalities, the focus of this standard is to create the inequality and use that inequality to solve a problem. Solving these inequalities algebraically is *not* part of the standard. Once a student has the inequality, the student

Assessing for Understanding

Students should be able to create an equation from a function and use the equation to solve problems.

Example: A ball thrown vertically upward at an initial velocity of v_0 ft/sec rises a distance d feet in t seconds, given by $d = 6 + v_0t - 16t^2$.

Write an equation whose solution is:

a) The time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet.

b) The speed with which the ball must be thrown to rise 20 feet in 2 seconds.

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/2/tasks/437)

Example: A ball thrown vertically upward at an initial velocity of 88 ft/sec rises a distance d feet in t seconds, given by $d = 6 + 88t - 16t^2$.

a) Write an inequality whose solution represents when the ball would be at least 78 ft above the ground.

b) Use the table to the right to find when the ball would be at least 78 ft. above the ground.

t (sec)	d (ft)
0	6
0.5	46
1	78
1.5	102
2	118
2.5	126
3	126
3.5	118
4	102
4.5	78
5	46
5.5	6

can use a table or graph to find a solution to the problem. Students in Math 1 are not responsible for using interval notation to represent a solution. They are to write answers to these inequalities using inequality notation.	Students should be able to create equations from various representations, such as verbal descriptions, and use them to solve problems. Students should be able to create inequalities and use those inequalities to solve problems. (Students are not expected to solve quadratic and exponential inequalities algebraically. Students should use technology, tables and graphs to solve problems.) **Example:** Stephen wants to create a landscaping feature in the shape of a parallelogram in his yard. Stephen has 200 square feet of mulch available for the project. To be most pleasing to the eye, he decides that he wants the length of the parallelogram to be 3 more than twice the width, measured in feet. If Stephen intends to cover the entire landscape feature in mulch, what can the width of the parallelogram be?

NC.M1.A-SSE.3

Write expressions in equivalent forms to solve problems.

Write an equivalent form of a quadratic expression by factoring, where a is an integer of the quadratic expression, $ax2 + bx + c$, to reveal the solutions of the equation or the zeros of the function the expression defines.

Concepts and Skills

Pre-requisite

- Factoring and expanding linear expressions with rational coefficients (7.EE.1)
- Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)

Connections

- Interpreting the factors in context (NC.M1.A-SSE.1b)
- Understanding the relationship between factors, solutions, and zeros (NC.M1.A-APR.3)
- Solving quadratic equations (NC.M1.A-REI.4)
- Rewriting quadratic functions into different forms to show key features of the function (NC.M1.F-IF.8a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to compare and contrast the zeros of a function and the solutions of a function.

New Vocabulary: quadratic expression, zeros, linear factors

Mastering the Standard

Comprehending the Standard

Students factor a quadratic in the form $ax^2 + bx + c$ where a is an integer in order to reveal the zeroes of the quadratic function.

Students use the linear factors of a quadratic function to explain the meaning of the zeros of quadratic functions and the solutions to quadratic equations in a real-world problem.

Assessing for Understanding

Students should understand that the reasoning behind rewriting quadratic expressions into factored form is to reveal different key features of a quadratic function, namely the zeros/x-intercepts.

Example: The expression $-4x^2 + 8x + 12$ represents the height of a coconut thrown from a person in a tree to a basket on the ground where x is the number of seconds.

a) Rewrite the expression to reveal the linear factors.
b) Identify the zeros and intercepts of the expression and interpret what they mean in regard to the context.
c) How long is the ball in the air?

Example: Part A: Three equivalent equations for $f(x)$ are shown. Select the form that reveals the zeros of $f(x)$ without changing the form of the equation.

$f(x) = -2x^2 + 24x - 54$ $\quad$ $f(x) =- 2(x-3)(x-9)$ $\quad$ $f(x) =- 2(x-6)^2 + 18$

Part B: Select all values of x for which $f(x) = 0$.

$-54, -18, -9, -6, -3, 0, 3, 6, 9, 18, 54$

(from the *Smarter Balanced Assessment Consortium*)

Students should understand that the reasoning behind rewriting quadratic expressions into factored form is to reveal the solutions to quadratic equations.

Example: A vacant rectangular lot is being turned into a community vegetable garden with a uniform path around it. The area of the lot

	is represented by $4x^2 + 40x - 44$ where x is the width of the path in meters. Find the width of the path surrounding the garden.

NC.M1.A-REI.4

Solve equations and inequalities in one variable.

Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

Concepts and Skills

Pre-requisite

- Factor linear expressions with rational coefficients (7.EE.1)
- Use square root to represent solutions to equations of the form $x^2 = p$, where p is a positive rational number; evaluate square roots of perfect squares (8.EE.2)
- Factor a quadratic expression to reveal the solution of a quadratic equation (NC.M1.A-SSE.3)
- Understand the relationship between linear factors and solutions (NC.M1.A-APR.3)

Connections

- Create one variable quadratic equations and inequalities and solve (NC.M1.A-CED.1)
- Justify a solution method and each step in the solution process (NC.M1.A-REI.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

7 – Look for a make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.

New Vocabulary: quadratic equation

Mastering the Standard

Comprehending the Standard

Students should focus on quadratics with one or two real solutions that can be solved by factoring or taking the square root.

This standard gives the algebraic reasoning of how to solve a quadratic equation to find the $x-$intercepts and zeroes that exist on the graph of the associated quadratic function.

This standard gives the "how" to solve a quadratic equation while NC.M1.A-APR.3 gives the "why". Therefore, these two standards should be taught together.

Students should be able to use the structure of the quadratic equation to determine whether to solve by using the square root as an inverse operation or by factoring.

When solving using the square root, students are only expected to evaluate perfect squares. All other square root solutions should either be left in square root form or estimated appropriately based on the context. Therefore, solving using the quadratic formula is not expected at this level.

Assessing for Understanding

Students should be able to solve quadratic equations using square root as the inverse operation.

Example: Solve:

a) $x^2 = 49$

b) $3x^2 + 9 = 72$

Students should be able to solve quadratic equations using factoring.

Example: Solve: $6x^2 + 13x = 5$

Students should be able to discuss their chosen solution method.

Example: Stephen and Brianna are solving the quadratic equation, $(x-4)^2 - 25 = 0$, in a classroom activity. Stephen believes that the equation can be solving using a square root. Brianna disagrees, saying that it can be solve using by factoring. Who is correct? Be prepared to defend your position.

NC.M1.A-APR.3

Understand the relationship between zeros and factors of polynomials.

Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.

Concepts and Skills

Pre-requisite

- Understand that is the product is zero, at least one of the factors is zero (3.OA.7)

Connections

- Factor quadratic expressions to reveal zeros of functions and solutions to equations (NC.M1.A-SSE.3)
- Justify the steps in solving a quadratic equation (NC.M1.A-REI.1)
- Solving quadratic equations (NC.M1.A-REI.4)
- Factor quadratic functions to reveal key features (NC.M1.F-IF.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to compare solutions functions to solutions of equations.

New Vocabulary: quadratic expression, quadratic equation, quadratic function, zeroes, linear factors, roots

Mastering the Standard

Comprehending the Standard

The focus of this standard is for students to use the multiplicative property of zero to create linear factors given a quadratic equation, and to solve those linear factors to find a zero of a function or a solution of an equation.

This standard should be taught with NC.M1.A-SSE.3 and NC.M1.A-REI.1.

Students can find the solutions of a factorable quadratic equation and use the roots to sketch its $x-$intercepts on the graph.

Assessing for Understanding

Students should be able to explain how they go from factored form to identifying the zeros of the function.

Example: Given the function $y = 2x^2 + 6x - 3$, list the zeroes of the function and sketch its graph.

Example: Sketch the graph of the function $f(x) = (x+5)^2$. How many zeros does this function have? Explain.

Note: It is a common error for students to assume that the solution or zero of linear factor, $(x-b)$, will always be the opposite of the constant term, b. If this is noticed, be sure to include examples in which $a \neq 1$.

Example: Which of the following are the solutions to the equation $x^2 - 13x = 30$?

A) $x =- 10$ & 3 B) $x = 10$ & -3 C) $x =- 15$ & 2 D) $x = 15$ & -2

Example: Which of the following has the largest x-intercept?

A) $x^2 + 4x - 12$ B) $(x+2)(x-5)$ C) $(x-1)^2 - 4$

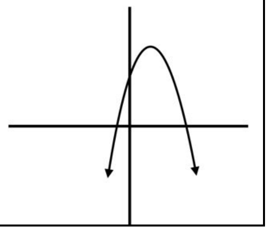
Students should understand the relationship between zeros/solutions and the quadratic expression.

Example: If the zeros of a function are $x = 2$ and $x = 7$, what was the function? Could there be more than one answer?

Example: Based on the graph to the right, which of the following functions could have produced the graph?

A) $f(x) = (x+2)(x+6)$

B) $f(x) = (x-2)(x+6)$



	C) $f(x) = (2 - x)(6 - x)$ D) $f(x) = (2 + x)(6 - x)$

NC.M1.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.
- Solve multi-step equations (8.EE.7)

Connections

- Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)
- Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)
- Solve for a quantity of interest in a formula (NC.M1.A-CED.4)
- Solve linear and quadratic equations and systems of linear equations (NC.M1.A-REI.3, NC.M1.A-REI.4, NC.M1.A-REI.5, NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to defend their method of solving an equation and each step of the solving process.

New Vocabulary: quadratic equation

Mastering the Standard

Comprehending the Standard

When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Properties of operations can be used to change expressions on either side of the equation to equivalent expressions.

In the properties of equality, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions.

Students do not have to name the property, but can describe the property using mathematical reasoning.

For example: Transforming $2x - 5 = 7$ to $2x = 12$ is possible because 5 = 5, so adding the same quantity to both sides of an

Assessing for Understanding

Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency.

Example: Below are two methods for solving the equation $5x^2 + 10 = 90$. Select one of the solution methods and construct a viable argument for the use of the method.

$$5x^2 + 10 = 90 \quad -10 = -10$$
$$5x^2 = 80 \quad \frac{5x^2}{5} = \frac{80}{5} \quad x^2 = 16$$
$$x = \pm\sqrt{16}$$
$$x = 4 \text{ or } x = -4$$

$$5x^2 + 10 = 90 \quad -90 = -90 \quad 5x^2 - 80 = 0$$
$$5(x^2 - 16) = 0 \quad 5(x+4)(x-4) = 0$$
$$x + 4 = 0 \text{ or } x - 4 = 0$$
$$x = 4 \text{ or } x = -4$$

equation makes the resulting equation true as well. *Knowing this is the Addition Property of Equality is not the point of this standard.*

NC.M1.A-REI.11

Represent and solve equations and inequalities graphically

Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)
- Solving systems of equations (NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

For a complete understanding, students will need exposure to both parts of this standard.

First, students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation.

For example:

x	$f(x) = 2x -$
0	-4
1	-2
2	0
3	2
4	4

x	$g(x) = \frac{1}{2}x +$
0	.5
1	1
2	1.5
3	2
4	2.5

Because $f(x) = g(x)$ when $x = 3$, 3 is the solution to the equation $2x - 4 = \frac{1}{2}x + \frac{1}{2}$

(As an extension, students could write an inequality to describe the relationship between the functions when $x < 3$ and when $x > 3$.)

Assessing for Understanding

Example: The functions $f(m) = 18 + 0.4m$ and $g(m) = 11.2 + 0.54m$ give the lengths of two different springs in centimeters, as mass is added in grams, m, to each separately.

a) Graph each equation on the same set of axes.
b) What mass makes the springs the same length?
c) What is the length at that mass?
d) Write a sentence comparing the two springs.

Example: Solve the following equations by graphing. Give your answer to the nearest tenth.

a) $3(2^x) = 6x - 7$
b) $10x + 5 =- x + 8$

Example: The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million

In Math 1, students are expected to solve linear systems of equations algebraically. All other systems should be solved with technology, tables, and graphs. Second, students should be able to use a system of equations to solve systems of equations. *For example:* Solve: $3x^2 - 2x + 1 = \frac{1}{2}x + 5$ Rewrite the equations as a system of equations $\{f(x) = 3x^2 - 2x + 1 \; g(x) = \frac{1}{2}x + 5$ Using technology, graph the equations and look for points of intersection, where the same x produces $f(x) = g(x)$. In Math 1, students are expected to solve linear equations using inverse operations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.	people per year. a) Based on these assumptions, in approximately what year will this country first experience shortages of food? b) If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year? c) If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur? (https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/645)

NC.M1.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

Concepts and Skills

Pre-requisite

- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

Connections

- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identification of key features and interpret those key features in context.

New Vocabulary: maximum, minimum

Mastering the Standard

Comprehending the Standard

Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that “tell a story” hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions.

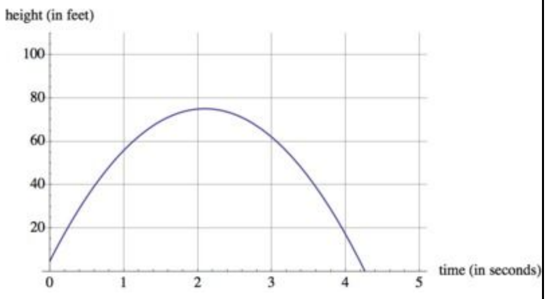
Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and maximum/minimum) for any given graph, not just “function families”. This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulation algebraically.

By contrast, NC.M1.F-IF.7, has students work with specific functions in which students have the ability to use algebraic manipulation to identify additional key

Assessing for Understanding

Students should be able to identify and interpret key features of quadratic functions from graphs, tables or verbal descriptions.

Example: Suppose Brett and Andre each throw a baseball into the air. The height of Brett's baseball is given by $h(t) =- 16t^2 + 79t + 6$, where h is in feet and t is in seconds. The height of Andre's baseball is given by the graph below: Interpret the x-intercept, y-intercept, and maximum in context of the baseball scenario.



Adapted from Illustrative Math

(https://www.illustrativemathematics.org/content-standards/tasks/1279)

features.	

NC.M1.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.

Concepts and Skills

Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Operations with polynomials (NC.M1.A-APR.1)
- Formally define a function (NC.M1.F-IF.1)

Connections

- Create and graph two variable equations (NC.M1.A-CED.2)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their process of building a new function.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

This standard is about building functions. In this part of the standard students should combine functions to represent a contextual situation.

This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

The algebraic skills behind this standard occur in NC.M1.A-APR.1. This standard should be taught throughout the year as each new function family is added to the course.

Assessing for Understanding

Students should combine functions to represent a contextual situation.

Example: The floor of a rectangular cage has a length 4 feet greater than its width, w. James will increase both dimensions of the floor by 2 feet. Which equation represents the new area, N, of the floor of the cage?

A) $N = w^2 + 4w$
B) $N = w^2 + 6w$
C) $N = w^2 + 6w + 8$
D) $N = w^2 + 8w + 12$

(NCDPI Math 1 released EOC #5)

NC.M1.F-IF.5

Interpret functions that arise in applications in terms of the context.

Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

Concepts and Skills

Pre-requisite

- In middle school, students only informally considered restrictions to the domain and range based on context, such as understanding that measurements cannot be negative.
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)

Connections

- Recognize the domain of sequences (NC.M1.F-IF.3)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Analyze linear, quadratic, and exponential functions to identify key features (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students should be able to associate a reasonable domain and range to a graph as well as to a contextual situation. The domain of a graph should be taught in the context of the situation it represents. Graphs represented should be both discrete and continuous forms. Students do not need to know the terminology discrete and continuous, but they should be able to identify which is appropriate for each contextual situation.	Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation. **Example:** Maggie tosses a coin off of a bridge into a stream below. The distance the coin is above the water is modeled by the equation $y = -16x^2 + 96x + 112$, where x represents time in seconds. What is a reasonable domain for the function?

NC.M1.S-ID.8

Interpret linear models.

Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.

Concepts and Skills

Pre-requisite

- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)

Connections

- Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

Mastering the Standard

Comprehending the Standard

In working with bivariate data in MS, students have previously

The correlation coefficient, r, is a measure of the strength and direction of a linear relationship between two quantities in a set of data.

The magnitude (absolute value) of r indicates how closely the data points fit a linear pattern.

If $r = \pm 1$, all points fall exactly on a line. The sign of r indicates the direction of the relationship. The closer $|r|$ is to 1, the stronger the correlation and the closer $|r|$ is to zero, the weaker the correlation.

Assessing for Understanding

Students can interpret the correlation coefficient.

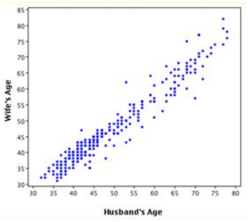
Example: The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data.

Students recognize the strength of the association of two quantities based on the scatter plot.

Example: Which correlation coefficient best matches each graph? Explain.

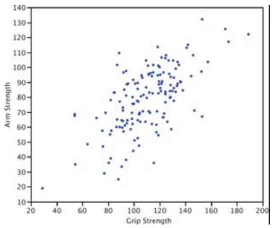
A $r =$ ________

-



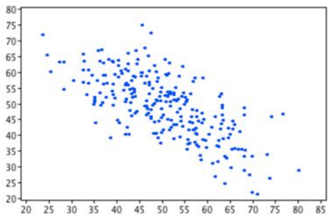
B $r =$ ________

-



C $r =$ __________

-



$r = -.48$	**$r = .98$**	**$r = .88$**	**$r = -.17$**	**$r = 1$**	**$r = .31$**	**$r = -1$**

Instructions for TI-83 and TI-84 series calculators:

1: Go to the [catalog]. Click⟶ **2nd** then **0** .

2: Scroll down to ⟶ DiagnosticOn and press **enter** twice.

When '**Done**' appears on the screen the diagnostics are on and the calculator should now calculate the correlation coefficient (r) automatically when linear regression is performed.

Students will be able to analyze patterns in context between two variables and use graphing technology to determine whether a linear model is appropriate for the data.

Example: The following data set indicates the average weekly temperature and the number of sno-cones sold by Sno-Show Sno-cones each week in May for the temperatures noted.

Average weekly temperature	**# of Sno-cones sold**
68	500
74	600
74	700
80	800
82	1200

a) Using technology, sketch a scatter plot of the data.

b) Determine a linear regression model that could represent the data shown.

c) Determine the correlation coefficient.

d) Determine the strength and direction of the linear relationship.

e) Create a residual plot. Is a linear model appropriate for the data shown? Explain.

NOTE: Remind students to turn the Diagnostics on in the graphing calculator so that the correlation coefficient (r) appears when the regression equation is calculated.

The North Carolina High School Collaborative Instructional Framework

NC Math 1

Unit 5: Systems of Equations and Inequalities

12 Days Block Schedule | **September 2017 Update** | 24 Days Traditional Schedule

RESEARCH BRIEF: Systems of Equations and Inequalities

Essential Questions:

- How is the solution of a system of equations represented on a graph, table, and in context of the scenario?
- How is the solution to a system of linear inequalities represented and what does it mean in context of the scenario?

Learning Outcomes	Student Objectives
• Given a scenario, students will create a system of linear equations to model the scenario. • Students will solve systems of linear equations using the table and graph on the graphing calculator. • Students will solve systems of linear equations algebraically with substitution and elimination. • Students will explain solutions to systems of linear equations in context. • Students will approximate solutions of systems of linear, quadratic, and exponential functions using technology. • Given a scenario, students will create a system of linear inequalities to model the scenario. • Students will solve systems of linear inequalities graphically without the use of a graphing calculator. • Students will solve systems of linear inequalities using the table and graph on the graphing calculator.	• I will **explore** systems of linear equations in different representations. • I will **explain** solutions to systems of linear equations in context. • I will **create** a system of equations to model a scenario. • I will be **challenged** to solve systems of linear equations algebraically. • I will **solve** systems of equations (linear, exponential, and/or quadratic) using technology. • I will **explore** systems of linear inequalities. • I will **solve** systems of linear inequalities graphically with and without a graphing calculator. • I will **understand** possible solutions to a system of linear inequalities in context.

- Students will explain solutions to systems of linear inequalities in context.

Standards Addressed in this Unit

Create, solve, and interpret systems of equations in context.

- **NC.M1.A-CED.3**: Create systems of linear equations and inequalities to model situations in context.
- **NC.M1.A-REI.5**: Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **NC.M1.A-REI.6**: Solve systems of equations using tables, graphs, or algebraic methods (substitution and elimination) to find the approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.
- **NC.M1.A-REI.11**: Build and understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) = g(x) and approximate solutions using a graphing technology or successive approximations with a table of values.

Create, solve, and interpret systems of inequalities in context.

- **NC.M1.A-REI.12**: Solve and represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M1.A-CED.3

Create equations that describe numbers or relationships.

Create systems of linear equations and inequalities to model situations in context.

Concepts and Skills

Pre-requisite

- Understanding a system of equations (8.EE.8)
- Creating linear equations in two variables (NC.M1.A-CED.2)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Use tables, graphs and algebraic methods to solve systems of linear equations (NC.M1.A-REI.6)
- Represent the solution to a system of linear inequalities as a region of the plane (NC.M1.A-REI.12)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and demonstrate its relation to the context.

Mastering the Standard

Comprehending the Standard

Students create a system of linear equations and inequalities that model real world situations.

The expectation for this standard is to create a system of linear equations or a system on linear inequalities that model a contextual situation. The system should include inequalities that limit the domain and range if necessary.

Connect this standard to NC.M1.A-REI.11 & 12 for solving the system of linear equations and NC.M1.A-REI.12 for representing the solutions to a system of linear inequalities.

Linear programming is not the intent of this standard. While it may be an extension of this standard and could be used as application, it is not the expectation that students be fluent in maximizing or minimizing based on constraints.

Assessing for Understanding

Students should be able to write inequalities that describe the limitations from a context for a system of inequalities.

Example: A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8.

a) Write a system of inequalities to represent the situation.
b) Graph the inequalities.
c) If the club buys 150 hats and 100 jackets, will the conditions be satisfied?
d) What is the maximum number of jackets they can buy and still meet the conditions?

Students should be able to write the system of equations based on context.

Example: The only coins that Alexis has are dimes and quarters.

- Her coins have a total value of $5.80.
- She has a total of 40 coins.

Which of the following systems of equations can be used to find the number of dimes, d, and the number of quarters, q, Alexis has?

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/3/tasks/220)

NC.M1.A-REI.5

Solve systems of equations.

Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions.

Concepts and Skills

Pre-requisite

- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Operations with polynomials (NC.M1.A-APR.1)
- Justify steps in a solving process (NC.M1.A-REI.1)

Connections

- Solving systems of equations and inequalities (NC.M1.A-REI.6)
- Understand that all points on the graph of an equation is a solution to that equation (NC.M1.A-REI.10)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why the process of elimination works.

New Vocabulary: elimination

Mastering the Standard

Comprehending the Standard

The focus of this standard is to explain a mathematical justification for the addition (elimination) method of solving systems of equations ultimately transforming a given system of two equations into a simpler equivalent system that has the same solutions as the original system.

Students should use the properties of equality to discuss why the process of elimination maintain the same solutions.

- When an equation is multiplied by a constant the set of solutions remains the same. Graphically it is the same line.
- When a two linear equations are added together, a third linear equation is formed that shares a common solution as the original equations. Graphically this means the three linear equations all intersect at the same point.
- The process of elimination is to obtain the value for one of the coordinates. Graphically, it is to get either a horizontal or vertical line that goes through the point of intersection.

Assessing for Understanding

Students should be able to understand the process of elimination through simple intuitive problems.

Example: Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that the two numbers, x and y, satisfy the equations $x + y = 10$ and $x - y = 4$.

Students should be able to identify systems composed of equivalent equations.

Example: Which of the following systems is equivalent to $x - 2y = 4 \textit{ and } 3x + y = 9$?

A) $x - 2y = 4 \textit{ and } 6x + 2y = 9$

B) $-3x + 6y = 4 \textit{ and } 3x + y = 9$

C) $x - 2y = 4 \textit{ and } 6x - 2y = 18$

D) $\frac{1}{2}x - y = 2 \textit{ and } 3x + y = 9$

NC.M1.A-REI.6

Solve systems of equations.

Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.

Concepts and Skills

Pre-requisite

- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Create equations for systems of equations (NC.M1.A-CED.3)
- Justify the steps in a solving process (NC.M1.A-REI.1)
- Solve linear equations in one variable (NC.M1.A-REI.3)
- Understand the mathematical reasoning behind the process of elimination (NC.M1.A-REI.5)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Understand the mathematical reasoning behind the methods of graphing, using tables and technology to solve systems and equations (NC.M1.A-REI.11)
- Analyze linear functions (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.
New Vocabulary: elimination

Mastering the Standard

Comprehending the Standard

Students solve a system of equations and then interpret its solution.
Students should be able to solve a system from a contextual situation. Therefore, this standard should be taught with NC.M1.A-CED.3

Students should not be required to use one method over another when solving a system of equations, but should be allowed to choose the best option for the given scenario. The focus of this standards should also not be limited to the algebraic methods.

Student were taught substitution and graphing methods in 8th grade.

Assessing for Understanding

Students should be able to create equations for system (NC.M1.A-CED.3), select an appropriate solution method, solve that system, and interpret the solution in context.

Example: José had 4 times as many trading cards as Philippe. After José gave away 50 cards to his little brother and Philippe gave 5 cards to his friend for his birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.

Example: A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special. What is the cost of each lunch special?

- On Thursday, the restaurant collected $467 selling 21 vegetarian specials and 40 chicken specials.
- On Friday, the restaurant collected $484 selling 28 vegetarian specials and 36 chicken specials.

Example: The math club sells candy bars and drinks during football games. How much does each candy bar sell for?

- 60 candy bars and 110 drinks will sell for $265.
- 120 candy bars and 90 drinks will sell for $270.

This is a capstone standard supported by several standards in this course. In order to have a complete understanding of this standard, these standards must be incorporated.

- The ability to create equations for a system from a contextual situation is addressed in NC.M1.A-CED.3.
- The understanding of the elimination method is addressed NC.M1.A-REI.5.
- The understanding for using methods graphing, and tables is taught in NC.M1.A-REI.11.

Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to NC.M1.G-GPE.5, which requires students to prove the slope criteria for parallel lines.

(NCDPI Math 1 released EOC #7)

Example: Two times Antonio’s age plus three times Sarah’s age equals 34. Sarah’s age is also five times Antonio’s age. How old is Sarah?

(NCDPI Math 1 released EOC #10)

Example: Lucy and Barbara began saving money the same week. The table below shows the models for the amount of money Lucy and Barbara had saved after x weeks.

Lucy’s Savings	$f(x) = 10x + 5$
Barbara’s Savings	$g(x) = 7.5x + 25$

After how many weeks will Lucy and Barbara have the same amount of money saved?

(NCDPI Math 1 released EOC #29)

Example: A streaming movie service has three monthly plans to rent movies online. Graph the equation of each plan and analyze the change as the number of rentals increase. When is it beneficial to enroll in each of the plans?

- Basic Plan: $3 per movie rental
- Watchers Plan: $7 fee + $2 per movie with the first two movies included with the fee
- Home Theater Plan: $12 fee + $1 per movie with the first four movies included with the fee

NC.M1.A-REI.11

Represent and solve equations and inequalities graphically

Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)
- Solving systems of equations (NC.M1.A-REI.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

For a complete understanding of the standard, students will need exposure to both parts of this standard.

First, students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation.

For example:

x	$f(x) = 2x -$
0	-4
1	-2
2	0
3	2
4	4

x	$g(x) = \frac{1}{2}x +$
0	.5
1	1
2	1.5
3	2
4	2.5

Because $f(x) = g(x)$ when $x = 3$, 3 is the solution to the equation $2x - 4 = \frac{1}{2}x + \frac{1}{2}$

(As an extension, students could write an inequality to describe the relationship between the functions when

Assessing for Understanding

Example: The functions $f(m) = 18 + 0.4m$ and $g(m) = 11.2 + 0.54m$ give the lengths of two different springs in centimeters, as mass is added in grams, m, to each separately.

a) Graph each equation on the same set of axes.
b) What mass makes the springs the same length?
c) What is the length at that mass?
d) Write a sentence comparing the two springs.

Example: Solve the following equations by graphing. Give your answer to the nearest tenth.

a) $3(2^x) = 6x - 7$
b) $10x + 5 =- x + 8$

$x < 3$ and when $x > 3$.) In Math 1, students are expected to solve linear systems of equations algebraically. All other systems should be solved with technology, tables, and graphs. Second, students should be able to use a system of equations to solve systems of equations. *For example:* Solve: $3x^2 - 2x + 1 = \frac{1}{2}x + 5$ Rewrite the equations as a system of equations $\{f(x) = 3x^2 - 2x + 1\ g(x) = \frac{1}{2}x + 5$ Using technology, graph the equations and look for point of intersection, where the same x produces $f(x) = g(x)$. In Math 1, students are expected to solve linear equations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.	**Example:** The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year. a) Based on these assumptions, in approximately what year will this country first experience shortages of food? b) If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year? c) If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur? (https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/645)

NC.M1.A-REI.12

Represent and solve equations and inequalities graphically

Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Concepts and Skills

Pre-requisite

- Solve two-step linear inequalities (7.EE.4b)
- Solve linear inequalities in one variable (NC.M1.A-REI.3)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections

- Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1)
- Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.

Mastering the Standard

Comprehending the Standard

Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically.

It is an American tradition to shade the region that represent the solutions of the inequality. In other countries, they shade regions of the plane that do <u>not</u> contain solutions, marking that region out. This results in an unmarked solution region making it easier to identify and work with points in the solution region. This means that it is important for students to understand what the shaded region represents according to the context of the problem.

Assessing for Understanding

Students should be able to represent solutions to linear inequalities and systems of linear inequalities as a region of a plane.

Example: Graph the solution set for the following system of inequalities:

$3x + 5y \le 10$

$y > -4$

Example: Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system.

$x - 3y > 0$

$x + y \le 2$

$x + 3y > -3$

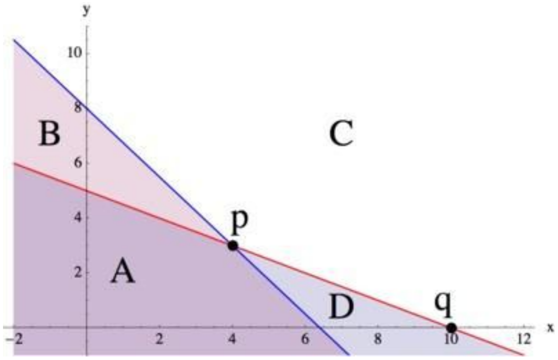
Example: Graph the following inequalities to determine the solution to the system:

$3x - 4y \le 7$

$y > -2x + 6$

$-9x + 4y \ge 1$

Example: Given below are the graphs of two lines, $y =- 0.5x + 5$ and $y =- 1.25x + 8$, and several regions and points are shown. Note that C is the region that appears completely white in the graph.



a) For each region and each point, write a system of equations or inequalities, using the given two lines, that has the region or point as its solution set and explain the choice of $\leq, \geq,$ *or* $=$ in each case. (You may assume that the line is part of each region.)

b) The coordinates of a point within a region have to satisfy the corresponding system of inequalities. Verify this by picking a specific point in each region and showing that the coordinates of this point satisfy the corresponding system of inequalities for that region.

c) In the previous part, we checked that specific coordinate points satisfied our inequalities for each region. Without picking any specific numbers, use the same idea to explain how you know that all points in the 3rd quadrant must satisfy the inequalities for region A.

(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/12/tasks/1205)

The North Carolina High School Collaborative Instructional Framework

NC Math 1

Unit 6: Descriptive Statistics

9 Days Block Schedule	**September 2017 Update**	18 Days Traditional Schedule

RESEARCH BRIEF: Univariate Statistics

Essential Questions:

- When is it most effective to use a box plot vs a histogram?
- How are shape, center, and spread used to interpret differences between data sets?
- How do extreme data points affect the shape, center, and spread of data sets?

Learning Outcomes	Student Objectives
• Students will interpret data using box plots and histograms. • Students will compare the shape, center, and spread of different data sets. • Students will determine the effects of outliers on the shape, center, and spread of data sets.	• I will use histograms and box plots to **interpret** data. NC.M1.S-ID.1 • I will **compare** the shape, center, and spread of different data sets. NC.M1.S-ID.2 • I will **explain** the effects of extreme data points (outliers) on shape, center, and/or spread. NC.M1.S-ID.3

Standards Addressed in this Unit

Understand how to summarize, represent, interpret and compare data on a single count or measurement variable.

- **NC.M1.S-ID.1**: Summarize, represent, and interpret data on a single count or measurement variable. Use technology to represent data with plots on the real number line (histograms and box plots).

- **NC.M1.S-ID.2**: Summarize, represent, and interpret data on a single count or measurement variable. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

- **NC.M1.S-ID.3**: Summarize, represent, and interpret data on a single count or measurement variable. Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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NC.M1.S-ID.1

Summarize, represent, and interpret data on a single count or measurement variable.

Use technology to represent data with plots on the real number line (histograms and box plots).

Concepts and Skills

Pre-requisite

- Displaying numerical data on line plots, dot plots, histograms and dot plots (6.SP.4)

Connections

- Comparing two or more data distributions using shape and summary statistics (NC.M1.S-ID.2)
- Examining the effects of outliers on the shape, center, and/or spread of data (NC.M1.S-ID.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

New Vocabulary: outlier, standard deviation

Mastering the Standard

Comprehending the Standard

This standard is an extension of 6th grade where students display numerical data using dot plots, histograms and box plots.

The standard involves representing data from contextual situations with histograms and box plots ***using technology***. Students should now be able to see that dot plots (line plots) are no longer appropriate for larger data sets. They should see that technology can quickly perform calculations and create graphs so that more emphasis can be placed on interpretation of the data.

Summary statistics include:

- 5-Number summary: minimum value (minX), maximum value (maxX), median (Med), lower quartile (Q_1) and upper quartile (Q_3)
- mean ($\bar{x}$)

Assessing for Understanding

Students can use appropriate technology to calculate summary statistics and graph a given set of data. Appropriate technology includes graphing calculators, software or online applications (e.g. http://technology.cpm.org/general/stats/).

Example: The table below shows the length of a class period for each of the schools listed in a NC school district. Choose and create an appropriate plot to represent the data. Explain your choice of plot.

School	**Class period** (minutes)	School	**Class period** (minutes)
Lincoln Middle	45	New Hope Middle	55
Central Middle	65	Sunnyside Middle	50
Oak Grove Middle	70	Pine Grove Middle	60
Fairview Middle	55	Green Middle	65
Jefferson Middle	60	Hope Middle	55
Roosevelt Middle	60		

● Sum ($\sum x$) ● standard deviation (Sx)* Graphs include: ● Histograms ● *Modified* Box plots – plots outliers as individual points. A point is determined to be an outlier if: o Lower outlier(s) $< 1.5 \cdot IQR$ o Upper outlier(s) $> 1.5 \cdot IQR$ *While technology gives values for the population standard deviation (σx), students will not use this measurement at this level.	**Example:** The following data set shows the number of songs downloaded in one week by each student in Mrs. Jones class: 10, 20, 12, 14, 12, 27, 88, 2, 7, 30, 16, 16, 32, 25, 15, 4, 0, 15, 6, 1, 0, 15, 12, 10, and 7. a) What are the summary statistics for the data? b) Construct two different graphs of the data. c) Describe the distribution of the data, citing both of the plots and the numerical summary statistics. d) What are the advantages to each data display? Explain.

NC.M1.S-ID.2

Summarize, represent, and interpret data on a single count or measurement variable.

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

Concepts and Skills

Pre-requisite

- Relating the choice of center and variability to shape of data (6.SP.5d)
- Informally compare graphical displays of two distributions to make inferences about two populations (7.SP.3)
- Informally compare numerical summaries of two distributions to make inferences about two populations (7.SP.4)
- Use technology to represent data (NC.M1.S-ID.1)

Connections

- Effects of outliers on shape, center, and/or spread (NC.M1.S-ID.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

New Vocabulary: standard deviation, outlier

Mastering the Standard

Comprehending the Standard

In middle school, students related the measure of center and variability to the shape and context of the data. Students know that symmetrical displays are more appropriate for the mean as a measure of center and mean absolute deviation (M.A.D) as a measure of variability. Likewise, they understand that skewed distributions or distributions with outliers are better described using median as a measure of center due to the fact that it is a *resistant* measure of center; and the interquartile range (IQR) as a measure of variability.

Context also plays an important role in the choice of summary statistic utilized. Students can examine the context to rationalize why particular measures are more appropriate than others.

Assessing for Understanding

Given two or more sets of data, students compare datasets and identify similarities and differences in shape, center and spread within the context of the data.

Example: Ms. Williams wants to analyze the scores for the first unit test of her 1st period and 4th period NC Math 1 classes. The scores for each class are below.

1st Period:	4th Period:
82, 100, 94, 68, 34, 72, 70, 96, 99, 92, 90, 85, 70, 46, 71, 71, 77, 78, 95, 82, 80, 100, 99, 72, 69, 74, 84, 87	100, 95, 72, 80, 97, 78, 89, 100, 93, 95, 66, 87, 85, 98, 89, 86, 80, 79, 94, 90, 92, 87, 88 81, 82

a) Calculate the mean, median, standard deviation, and interquartile range for each class.
b) Construct an appropriate graph to compare the two classes.
c) Write several sentences to compare the class grades in context.

The standard deviation is a new summary statistic for students. Its *development* should be based on the M.A.D (Mean Absolute Deviation) learned in the 6th grade. Essentially, students need to understand that SD like M.A.D is a measure of variability in the data. The larger SD, the more variable the data. Students should also know that standard deviation allows comparison of variability in multiple data sets regardless of the unit of measurement for the data sets.

An understanding of how the standard deviation is calculated can help students to conceptualize the value and why it's primarily used in association with mean as a measure of center.

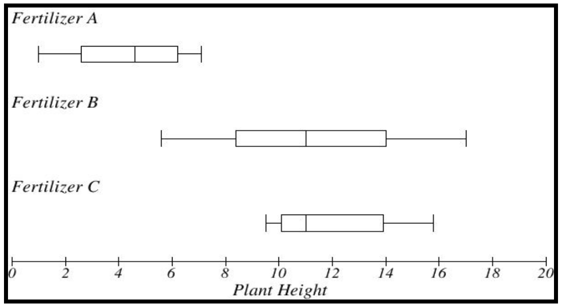
$$S_x = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2}$$

Using a relatively smaller data set and the list feature in the graphing calculator can make the calculations easier during development of the concept.

Given two or more graphs, students compare datasets and identify similarities and differences in shape, center and spread within the context of the data.

Example: Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 15 days, she measured the heights (in cm) of each set of seedlings. The data she collected and plots are shown below. Write a brief description comparing the three types of fertilizer. Which fertilizer do you recommend that Delia use? Explain your answer.

Fertilizer A			Fertilizer B			Fertilizer C		
7.1	6.3	1.0	11.0	9.2	5.6	10.5	11.8	15.5
5.0	4.5	5.2	8.4	7.2	12.1	14.7	11.0	10.8
3.2	4.6	2.4	10.5	14.0	15.3	13.9	12.7	9.9
5.5	3.8	1.5	6.3	8.7	11.3	10.3	10.1	15.8
6.2	6.9	2.6	17.0	13.5	14.2	9.5	13.2	9.7



Online Tools

Boxplot Grapher: http://www.imathas.com/stattools/boxplot.html

NC.M1.S-ID.3

Summarize, represent, and interpret data on a single count or measurement variable.

Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

Concepts and Skills

Pre-requisite

- Describing striking deviations from the overall pattern of a distribution (6.SP.5c)
- Use technology to create boxplots and histograms (NC.M1.S-ID.1)

Connections

- Comparing two or more data distributions using shape and summary statistics (NC.M1.S-ID.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others
4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision

Vocabulary

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

New Vocabulary: outlier, standard deviation

Mastering the Standard

Comprehending the Standard

An important part of data analysis includes examining data for values that represent abnormalities in the data. In MS, students *informally* addressed “striking deviations from the overall pattern” of a data distribution.

The identification of outliers is formalized in this standard. A value is mathematically determined to be an outlier if the value falls 1.5 IQRs below the 1st quartile or above the third quartile in a data set.

- Lower outlier(s) $< 1.5 \cdot IQR$
- Upper outlier(s) $> 1.5 \cdot IQR$

The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive

Assessing for Understanding

Students understand and use the context of the data to explain why its distribution takes on a particular shape (e.g. Why is the data skewed? Are there outliers?)

Example:
Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right?
Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left?
Why does the shape of the distribution of heights of the students at your school tend to be symmetrical?

Students should identify outliers of the data set and determine the effect outliers will have on the shape, center, and spread of a data set.

Example: The heights of players on the Washington High School’s Girls basketball team are recorded below:

5’ 10”	5’ 4”	5’ 7”	5’ 6”	5’ 5”	5’ 3”	5’ 7”	5’ 7”	5’ 8”

A student transfers to Washington High and joins the basketball team. Her height is 6’ 2”

a) What is the mean height of the team before the new player transfers in? What is the median height?
b) What is the mean height after the new player transfers? What is the median height?
c) What affect does her height have on the team’s height distribution and stats (center and spread)?
d) Which measure of center most accurately describes the team’s average height? Explain.

to outliers.

It is important to detect outliers within a distribution, because they can alter the results of the data analysis. The mean is more sensitive to the existence of outliers than other measures of center.

School	Length of class period (minutes)
Lincoln Middle	45
Central Middle	65
Oak Grove Middle	70
Fairview Middle	55
Jefferson Middle	60
Roosevelt Middle	60
New Hope Middle	55
Sunnyside Middle	50
Pine Grove Middle	60
Green Middle	65
Hope Middle	55

Example: The table to the left shows the length of a class period for each of the school's listed. If Cherry Lane Middle School's class period length of 100 minutes is added to the data above, what effect will it have on the mean, median, interquartile range, standard deviation and on the graph of the data?

The North Carolina High School Collaborative Instructional Framework

NC Math 2

Unit 1: Transformations of Functions and Geometric Objects

10 Days Block Schedule **September 2017 Update** 20 Days Traditional Schedule

RESEARCH BRIEF: **Unit 1: Transformations**

Essential Questions:

- How can coordinates be used to describe the properties of rigid and non-rigid transformations?
- How can a sequence of transformations be used to justify whether two figures are congruent or similar?
- How are transformations evident in real-world applications?
- How can geometric transformations be extended to the concept of functions?

Learning Outcomes	Student Objectives
• Given a figure (a preimage), students will create a new figure (an image) using a translation. • Given a figure (a preimage), students will create a new figure (an image) using a rotation. • Given a figure (a preimage), students will create a new figure (an image) using a reflection. • Given a figure (a preimage), students will create a new figure (an image) using a composition of rigid transformations. • Students will determine the order of rotational symmetry of a figure. • Students will determine the line symmetry of a figure. • Given a figure (a preimage), students will create a new figure (an image) using a dilation.	• I will use coordinates to **develop** function rules modeling transformations, line reflections, and rotations and size transformations centered at the origin. • I will use coordinates to **investigate** properties of figures under one or more rigid transformations or under similarity transformations. • I will **determine** which transformations produce congruent figures. • I will **determine** if a figure has rotational/line symmetry. • I will **determine** which transformations produce similar figures. • I will **explore** the concept of function composition using successive application of two transformations. • I will **generate** a two-variable function to describe each transformation.

- Given a figure (a preimage), students will create a new figure (an image) using a composition of rigid and similarity transformations.
- When performing transformations of figures, students will understand that the set of coordinates of the preimage is the domain and the set of coordinates of the image is the range.

- I will **recognize** that the domain of a transformation is the set of coordinates of the preimage.
- I will **recognize** that the range of a transformation is the set of coordinates of the image.

Standards Addressed in this Unit

Understanding and applying properties of transformations.

- NC.M2.F-IF.1: Extend the concept of a function to include geometric transformations in the plane by recognizing that:
    - the domain and range of a transformation function *f* are sets of points in the plane;
    - the image of a transformation is a function of its pre-image.

- NC.M2.F-IF.2: Extend the use of function notation to express the image of a geometric figure in the plane resulting from a translation, rotation by multiples of 90 degrees about the origin, reflection across an axis, or dilation as a function of its pre-image.

- NC.M2.G-CO.2: Experiment with transformations in the plane; represent transformations in the plane; compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations). Understand that rigid motions produce congruent figures while dilations produce similar figures.

- NC.M2.G-CO.3: Given a triangle, quadrilateral, or regular polygon, describe any reflection or rotation symmetry i.e., actions that carry the figure onto itself. Identify center and angle(s) of rotation symmetry. Identify line(s) of reflection symmetry. Represent transformations in the plane.

- NC.M2.G-CO.4: Verify experimentally properties of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

- NC.M2.G-CO.5: Given a geometric figure and a rigid motion, find the image of the figure. Given a geometric figure and its image, specify a rigid motion or sequence of rigid motions that will transform the pre-image to its image.

- NC.M2.G-CO.6: Determine whether two figures are congruent by specifying a rigid motion or sequence of rigid motions that will transform one figure onto the other.

- **NC.M2.G-SRT.1**: Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations with given center and scale factor:
    a. When a line segment passes through the center of dilation, the line segment and its image lie on the same line. When a line segment does not pass through the center of dilation, the line segment and its image are parallel.
    b. Verify experimentally the properties of dilations with given center and scale factor: The length of the image of a line segment is equal to the length of the line segment multiplied by the scale factor.
    c. The distance between the center of a dilation and any point on the image is equal to the scale factor multiplied by the distance between the dilation center and the corresponding point on the pre-image.
    d. Dilations preserve angle measure.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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The Math Resource for Instruction - Customized for the Content of this Unit

NC.M2.F-IF.1

Understand the concept of a function and use function notation.

Extend the concept of a function to include geometric transformations in the plane by recognizing that:

- the domain and range of a transformation function *f* are sets of points in the plane;
- the image of a transformation is a function of its pre-image.

Concepts and Skills

Pre-requisite

- Formally define a function (NC.M1.F-IF.1)

Connections

- Extend the use of a function to express transformed geometric figures (NC.M2.F-IF.2)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)
- Experiment with transformations on the plane (NC.M2.G-CO.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should discuss how an ordered pair can be the domain of a function.

New Vocabulary: preimage, image

Mastering the Standard for this Unit

Comprehending the Standard

Students need to understand that coordinate transformations are functions that have a domain and range that are points on the coordinate plane.

The domain consists of the points of the pre-image and the range consists of points from the transformed image.

This means that the transformed image is a function of its pre-image.

When listing the domain, the vertices of the geometric object are used. All points between the vertices are considered part of the domain. This means that when listing the domain and range of a function of a geometric transformation of a triangle, three points would be used for the domain and three points for the range.

Assessing for Understanding

Students should be able to find the domain and range of geometric transformations.

Example: If the domain of a function that is reflected over the x-axis is (3,4), (2,-1), (-1,2), what is the range?

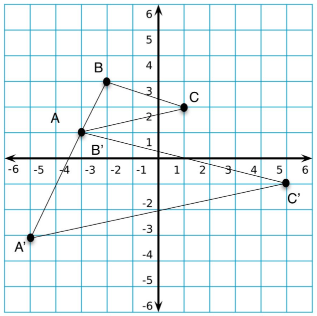
Example: If the domain of the coordinate transformation $f(x,y) = (y+1, -x-4)$ is $(1,4), (-3,2), (-1,-1)$, what is the range?

This transformation follow a rotation of 270 degree and a translation of right 1 and down 4.

Example: If the range of the coordinate transformation $f(x,y) = (-2x, -3y+1)$ is $(10,-2), (8,-5), (-2,4)$, what is the domain?

Example: Using the graph to the right, if this transformation was written as a function, identify the domain and range.

Note: While we often focus on the vertices for the transformation, the function for the transformation applies to all points on the geometric object.



NC.M2.F-IF.2

Understand the concept of a function and use function notation.

Extend the use of function notation to express the image of a geometric figure in the plane resulting from a translation, rotation by multiples of 90 degrees about the origin, reflection across an axis, or dilation as a function of its pre-image.

Concepts and Skills

Pre-requisite

- Describe the effects of dilations, translations, rotations, and reflections on geometric figure using coordinates (8.G.3)
- Interpret parts of a function as single entities in context (NC.M2.A-SSE.1b)
- Extend the concept of functions to include geometric transformations (NC.M2.F-IF.1)

Connections

- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Understand the effects of the transformation of functions on other representations (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should explain with mathematical reasoning how a dilation, rotation, reflection, and translation can be represented as a function.

Mastering the Standard for this Unit

Comprehending the Standard

Students use function notation to express a geometric transformation when performing the following operations:

- Translation $f(x,y)=(x+h,\ y+k)$, where h is a horizontal translation and k is a vertical translation.
- Rotation $90°$ counterclockwise or $270°$ clockwise $f(x,y)=(-y,x)$
- Rotation $180°$ $f(x,y)=(-x,-y)$
- Rotation $90°$ clockwise or $270°$ counterclockwise $f(x,y)=(y,-x)$
- Reflection over the x-axis $f(x,y)=(x,-y)$
- Reflection over the y-axis $f(x,y)=(-x,y)$
- Dilation $f(x,y)=(kx,ky)$ where k is the scale factor

Students should also continue to use function notation with all functions introduced in this course and Math 1.

Assessing for Understanding

Students should be able to identify the type of transformation through the function notation.

Example: Evaluate the function $f(x,y)=(-x,-y)$ for the coordinates (4,5), (3,1), and (-1,4). Graph the image of the transformation and describe the transformation with words.

Students should be able to use function notation to describe a geometric transformation.

Example: Write a function rule using function notation that will transform a geometric figure by rotating the figure $90°$ counterclockwise.

Example: Write a function rule using function notation that will translate a geometric figure 3 units to the right and 4 units down.

NC.M2.G-CO.2

Experiment with transformations in the plane.

Experiment with transformations in the plane.

- Represent transformations in the plane.
- Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations).
- Understand that rigid motions produce congruent figures while dilations produce similar figures.

Concepts and Skills

Pre-requisite

- Verify experimentally the properties of rotations, reflections and translations. (8.G.1)
- Understand congruence through rotations, reflections and translations (8.G.2)
- Use coordinates to describe the effects of transformations on 2-D figures (8.G.3)

Connections

- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and || lines and line segments (NC.M2.G-CO.4)
- Verify experimentally the properties of dilations given center and scale factor (NC.M2.G-SRT.1)
- Geometric transformations as functions (NC.M2.F-IF.1)
- Using function notation to express transformations (NC.M2.F-IF.2)
- Given a regular polygon, identify reflections/rotations that carry the image onto itself (NC.M2.G-CO.3)
- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

5 – Use appropriate tools strategically

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard for this Unit

Comprehending the Standard

In 8th grade, students understand transformations and their relationship to congruence and similarity through the use of physical models, transparencies, and geometry software.

In Math 2, students begin to formalize these ideas and connect transformations to the algebraic concept of function. A transformation is a new type of function that maps

Assessing for Understanding

Students describe and compare function transformations on a set of points as inputs to produce another set of points as outputs.

Example: A plane figure is translated 3 units right and 2 units down. The translated figure is then dilated with a scale factor of 4, centered at the origin.

a) Draw a plane figure and represent the described transformation of the figure in the plane.

two numbers (an ordered pair) to another pair of numbers. Transformations that are **rigid** (preserve distance and angle measure: reflections, rotations, translations, or combinations of these) and those that are not (stretches, dilations or rigid motions followed by stretches or dilations). Translations, rotations and reflections produce congruent figures while dilations produce similar figures. Note: It is not intended for students to memorize transformation rules and thus be able to identify the transformation from the rule. Students should understand the structure of the rule and how to use it as a function to generate outputs from the provided inputs.	b) Explain how the transformation is a function with inputs and outputs. c) Write a mapping rule for this function. d) Determine what type of relationship, if any, exists between the pre-image and the image after this series of transformations. Provide evidence to support your thinking. **Example:** Transform ΔABC with vertices $A\,(1,1)$, $B\,(6,3)$ and $C\,(2,13)$ using the function rule $(x,y) \rightarrow (-y,x)$ Describe the transformation as completely as possible. Note: As students work with transformations, many will begin to recall the transformations by recognizing the rule that was used. However, recognizing directly from the rules is not the expectation. Students can perform the transformation and then describe the transformation. In this case, a 90 degree counterclockwise rotation.

NC.M2.G-CO.3

Experiment with transformations in the plane.

Given a triangle, quadrilateral, or regular polygon, describe any reflection or rotation symmetry i.e., actions that carry the figure onto itself. Identify center and angle(s) of rotation symmetry. Identify line(s) of reflection symmetry. Represent transformations in the plane.

Concepts and Skills

Pre-requisite

- Understand congruence through rotations, reflections and translations (8.G.2)
- Use coordinates to describe the effects of transformations on 2-D figures (8.G.3)

Connections

- Geometric transformations as functions (NC.M2.F-IF.1)
- Using function notation to express transformations (NC.M2.F-IF.2)
- Understand that rigid motions produce congruent figures (NC.M2.G-CO.2)
- Verify experimentally properties of rigid motions in terms of angles, circles and lines (NC.M2.G-CO.4)
- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

- What kinds of figures have only rotational symmetry? What kinds of figures have only reflection symmetry? What kind have both? Why do you think this happens?

Mastering the Standard for this Unit

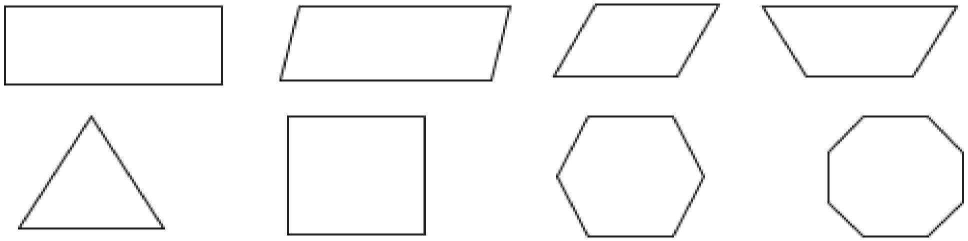
Comprehending the Standard

"The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are

Assessing for Understanding

Students describe and illustrate how figures such as an isosceles triangle, equilateral triangle, rectangle, parallelogram, kite, isosceles trapezoid or regular polygon are mapped onto themselves using transformations.

Example: For each of the following figures, describe and illustrate the rotations and/or reflections that carry the figure onto itself.



Students should make connections between the symmetries of a geometric figure and its properties. In addition to the example of an isosceles triangle noted above, figures with 180° rotation symmetry have opposite sides that are congruent.

Example: What connections can you make between a particular type of symmetry and the properties of a figure?

congruent.” (*Intro of HS Geometry strand of the CCSS-M*)	Students can describe and illustrate the center of rotation and angle(s) of rotation symmetry and line(s) of reflection symmetry.

NC.M2.G-CO.4

Experiment with transformations in the plane.

Verify experimentally properties of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Concepts and Skills

Pre-requisite

- Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)
- Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)
- Finding midpoint/endpoint of a line segment, given either (NC.M1.G-GPE.6)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard for this Unit

Comprehending the Standard

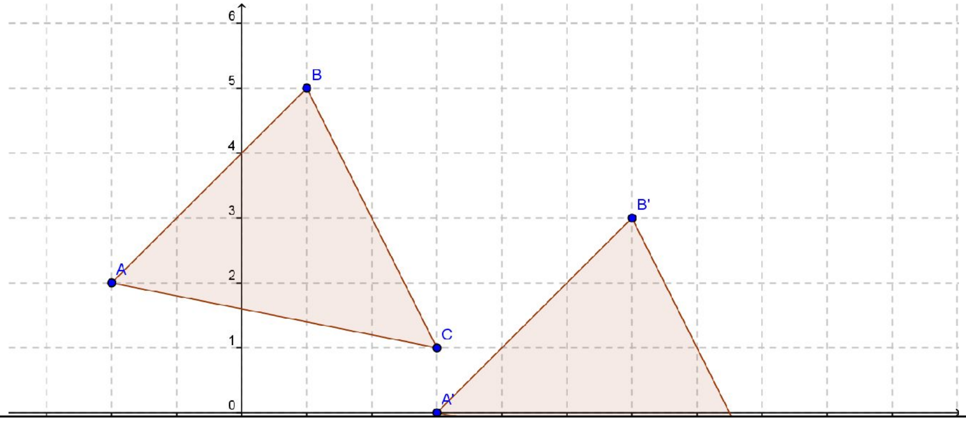
This standard is intended to help students develop the definition of each rigid motion in regards to the characteristics between pre-image and image points through experimentation.

- For *translations*: connecting points on the pre-image to corresponding points on the image produces line segments that are congruent and parallel.
- For *reflections*: the line of reflection is the perpendicular bisector of any line segment joining a point on the pre-image to the corresponding point on the image. Therefore, corresponding points on the pre-image and the image are equidistant from the line of reflection.

Assessing for Understanding

Students develop the definition of each transformation in regards to the characteristics between pre-image and image points.

Example: Triangle A’B’C’ is a translation of triangle ABC. Write the rule for the translation. Draw line segments connecting corresponding vertices. What do you notice?



Productive answers:
$(x, y) \rightarrow (x+5,\ y-2)$
$\overline{AA'} \parallel \overline{BB'} \parallel \overline{CC'}$
$\overline{AA'} \cong \overline{BB'} \cong \overline{CC'}$

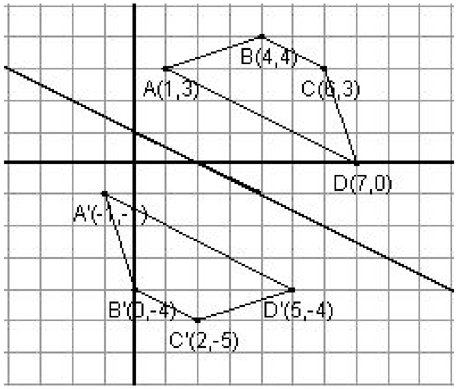
- For *rotations*: a point on the pre-image and its corresponding point on the image lie on a circle whose center is the center of rotation. Therefore, line segments connecting corresponding points on the pre-image and the image to the center of rotation are congruent and form an angle equal to the angle of rotation.

There are two approaches – both that should be used when teaching this standard. First, work with transformations on the coordinate plane. For this, students need to have some reasoning skills with figures on the coordinate plane. Calculating *distances* on the coordinate plane can help achieve this:

- show that the line of symmetry bisects the segment connecting image to preimage for a reflection;
- show that the segments connecting the image to center and preimage to center are the same length and represent the radius of the circle whose central angle is the angle of rotation
- show line segments are parallel for translations
- show line segments are perpendicular for reflection

The second approach is to work with the transformations on the Euclidean plane. Students should use tools (patty paper, mirrors, rulers, protractors, string, technology, etc) to measure and reason.

Example: Quadrilateral A'B'C'D' is a reflection of quadrilateral ABCD across the given line. Draw line segments connecting A to A' and C to C'. Label the points of intersection with the line of reflection as E and F. What do you notice?



Productive answers:

$\overline{AA'} \parallel \overline{CC'}$

$\overline{AE} \cong \overline{A'E}$

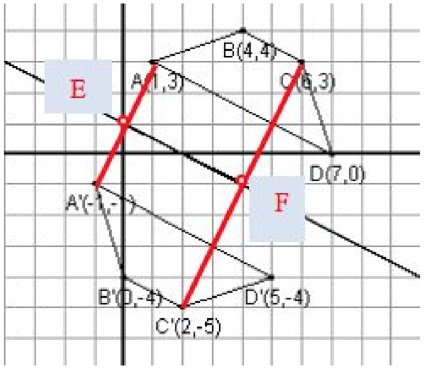
$\overline{CF} \cong \overline{C'F}$

$\overline{AA'} \perp \overline{EF}$

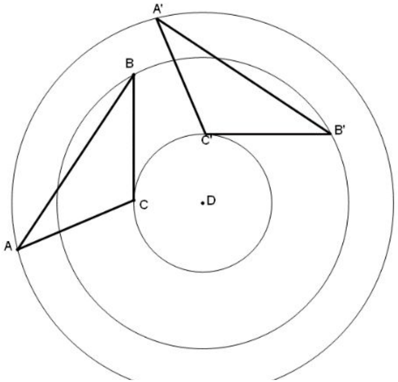
$\overline{CC'} \perp \overline{EF}$

A and A' are equidistant from the line of reflection.

C and C' are equidistant from the line of reflection.



Example: Triangle $A'B'C'$ is a rotation of triangle ABC. Describe the rotation, indicating center, angle, and direction. Draw line segments connecting corresponding vertices to the center. What do you notice?



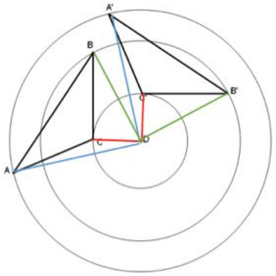
Triangle ABC is rotated 90° CW around point D.

Corresponding vertices lie on the same circle. The circles all have center D.

$\overline{CD} \cong \overline{C'D}$ and $m\angle CDC' = 90°$.

$\overline{AD} \cong \overline{A'D}$ and $m\angle ADA' = 90°$.

$\overline{BD} \cong \overline{B'D}$ and $m\angle BDB' = 90°$.



NC.M2.G-CO.5

Experiment with transformations in the plane.

Given a geometric figure and a rigid motion, find the image of the figure. Given a geometric figure and its image, specify a rigid motion or sequence of rigid motions that will transform the pre-image to its image.

Concepts and Skills

Pre-requisite

- Understand congruence through rotations, reflections and translations (8.G.2)

Connections

- Geometric transformations as functions (NC.M2.F-IF.1)
- Using function notation to express transformations (NC.M2.F-IF.2)
- Understand that rigid motions produce congruent figures (NC.M2.G-CO.2)
- Verify experimentally properties of rigid motions in terms of angles, circles and lines (NC.M2.G-CO.4)
- Given a regular polygon, identify reflections/rotations that carry the image onto itself (NC.M2.G-CO.3)
- Determining congruence through a sequence of rigid motions (NC.M2.G-CO.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard for this Unit

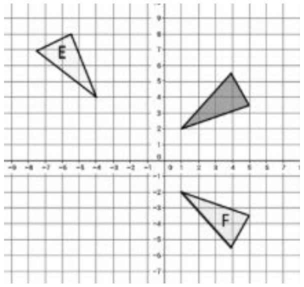
Comprehending the Standard

In 8th grade, students build an understanding of congruence through translations, reflections and rotation informally and in terms of coordinates. Students in MS verify that images transformed in the plane with rigid motions keep the same property as the preimage. They also note the effect of the rigid motion on the coordinates of the image and preimage. This standard extends the work in MS by requiring students to give precise descriptions of sequences of rigid motions where they specify exact points, lines and angles with coordinates and/or equations. Analytically, each rigid motion should be specified as follows:

- **For each rotation**, students should specify a point (x, y) and angle.
- **For each translation**, specific pairs of points (x, y) should be identified;
- **For each reflection**, the equation of the line ($y = mx + b$) should be identified.

Assessing for Understanding

Students transform a geometric figure given a rotation, reflection, or translation, using graph paper, tracing paper and/or geometry software.



Example: Using the figure on the right:

Part 1: Draw the shaded triangle after:

a) It has been translated -7 units horizontally and $+1$ units vertically. Label your answer ***A***.

b) It has been reflected over the x-axis. Label your answer ***B***.

c) It has been rotated 90° clockwise about the origin. Label your answer ***C***.

d) It was reflected over the line $y = 6$. Label the answer ***D***.

Part 2: Describe fully the transformation or sequence of transformations that:

a) Takes the shaded triangle onto the triangle labeled ***E***.

These specificities hold true whether working in the coordinate or Euclidean plane. Students must specify all points, lines of reflection/symmetry and angles of rotation.	b) Takes the shaded triangle onto the triangle labeled ***F***.

NC.M2.G-CO.6

Understand congruence in terms of rigid motions.

Determine whether two figures are congruent by specifying a rigid motion or sequence of rigid motions that will transform one figure onto the other.

Concepts and Skills

Pre-requisite

- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)

Connections

- Use the properties of rigid motions to show that two triangles are congruent if their corresponding sides and angles are congruent (NC.M2.G-CO.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard for this Unit

Comprehending the Standard

This standard connects to the 8th grade standard where students informally addressed congruency of figures through rigid motions to the formalized HS standard where students specifically defined points, lines, planes and angles of rigid motion transformations.

Students recognize rigid transformations preserve size and shape (or distance and angle) and develop the definition of congruence. This standard goes beyond the assumption of mere correspondence of points, lines and angles and thus establishing the properties of congruent figures.

Assessing for Understanding

Students use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.

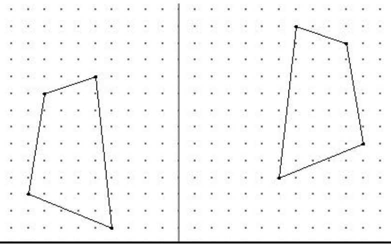
Example: Consider parallelogram ABCD with coordinates $A(2,-2)$, $B(4,4)$, $C(12,\ 4)$ and $D(10,-2)$. Consider the following transformations. Make predictions about how the lengths, perimeter, area and angle measures will change under each transformation below:

a) A reflection over the x-axis.
b) A rotation of 270° counterclockwise about the origin.
c) A dilation of scale factor 3 about the origin.
d) A translation to the right 5 and down 3.

Verify your predictions by performing the transformations. Compare and contrast which transformations preserved the size and/or shape with those that did not preserve size and/or shape. Generalize: which types of transformation(s) will produce congruent figures?

Students determine if two figures are congruent by determining if rigid motions will map one figure onto the other.

Example: Determine if the figures are congruent. If so, describe and demonstrate a sequence of rigid motions that maps one figure onto the other.



NC.M2.G-SRT.1

Understand similarity in terms of similarity transformations.

Verify experimentally the properties of dilations with given center and scale factor:

a. When a line segment passes through the center of dilation, the line segment and its image lie on the same line. When a line segment does not pass through the center of dilation, the line segment and its image are parallel.

b. Verify experimentally the properties of dilations with given center and scale factor: The length of the image of a line segment is equal to the length of the line segment multiplied by the scale factor.

c. The distance between the center of a dilation and any point on the image is equal to the scale factor multiplied by the distance between the dilation center and the corresponding point on the pre-image.

d. Dilations preserve angle measure.

Concepts and Skills

Pre-requisite

- Use coordinates to describe the effects of transformations on 2-D figures (8.G.3)
- Understand similarity through transformations (8.G.4)
- Finding the distance between points in the coordinate plane (8.G.8)
- Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)
- Understand that dilations produce similar figures (NC.M2.G-CO.2)

Connections

- Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)
- Determining similarity by a sequence of transformations; use the properties of dilations to show that two triangles are similar if their corresponding sides proportional and corresponding angles are congruent (NC.M2.G-SRT.2)
- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

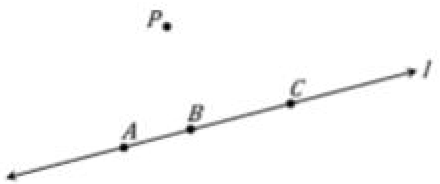
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard	Assessing for Understanding
Students use hands-on techniques (graph paper) and/or technology (geometry software) to experiment with dilations. This standard extends to the observance of the basic properties of dilations as they build a deeper	Students verify that a side length of the image is equal to the scale factor multiplied by the corresponding side length of the pre-image. **Example:** Given ΔABC with $A\,(-2,\ -4)$, $B\,(1,\ 2)$ and $C\,(4,-3)$. a) Perform a dilation from the origin using the following function rule $f(x,\ y)\rightarrow(3x,\ 3y)$. What is the scale factor of the dilation?

understanding of similarity. Students should understand that a dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.	b) Using ΔABC and its image $\Delta A'B'C'$, connect the corresponding pre-image and image points. Describe how the corresponding sides are related. c) Determine the length of each side of the triangle. How do the side lengths compare? How is this comparison related to the scale factor? d) Determine the distance between the origin and point A and the distance between the origin and point A'. Do the same for the other two vertices. What do you notice? e) Determine the angle measures for each angle of ΔABC and $\Delta A'B'C'$. What do you notice? Students perform a dilation with a given center and scale factor on a figure in the coordinate plane and verify that when a side passes through the center of dilation, the side and its image lie on the same line and the remaining corresponding sides of the pre-image and images are parallel. **Example:** Suppose we apply a dilation by a factor of 2, centered at the point P to the figure below.  a) In the picture, locate the images A', B', and C' of the points A, B, C under this dilation. b) What is the relationship between $lin\ AC$ and $line\ A'C'$? c) What is the relationship between the length of A'B' and the length of AB? Justify your thinking. (Teachers may add in coordinates into this problem initially to give students a concrete entrance to this concept.)

The North Carolina High School Collaborative Instructional Framework

NC Math 2
Unit 2: Similarity and Congruency

12 Days Block Schedule **September 2017 Update** 24 Days Traditional Schedule

RESEARCH BRIEF: **Unit 2: Similarity and Congruency**

Essential Questions:

- How do we use theorems and postulates to prove properties of triangles?
- How do we use transformations to justify similar and congruent triangles?
- How do we solve problems using properties of similarity?
- How do we use properties, postulates, and theorems to prove triangles congruent?
- How are properties of triangles used to solve application problems?

Learning Outcomes	Student Objectives
● Students will know and be able to use the angle relationship theorems involving two intersecting lines. ● Students will know and use the angle relationship involving two parallel lines cut by a transversal and their converses. ● Students will know and be able to use triangle congruence theorems to prove that triangles are congruent. ● Students will use congruent triangles to prove that corresponding parts of those triangles are congruent. ● Students will identify similar polygons and determine the scale factor of similar polygons. ● Students will know and use triangle similarity theorems to prove that triangles are similar.	● I will know and be able to use the angle relationship theorems involving two intersecting lines. ● I will know and be able to use the angle relationship involving two parallel lines cut by a transversal and their converses. ● I will know and be able to use triangle congruence theorems to prove that triangles are congruent. ● I will use congruent triangles to prove that corresponding parts of those triangles are congruent. ● I will identify similar polygons and determine the scale factor of similar polygons. ● I will know and use triangle similarity theorems to prove that triangles are similar.

● Students will know and use the Midsegment Theorem to solve problems. ● Students will recognize that rigid transformations preserve triangle congruence. ● Students will recognize that size transformations result in similar triangles.	● I will know and use the Midsegment Theorem to solve problems. ● I will recognize that rigid transformations preserve triangle congruence. ● I will recognize that size transformations result in similar triangles.

Standards Addressed in this Unit

Understand similarity through transformations, including dilations, and use the properties of similarity to solve problems.

- **NC.M2.G-CO.9**: (first three bullets) Prove theorems about lines and angles and use them to prove relationships in geometric figures including:
    - Vertical angles are congruent.
    - When a transversal crosses parallel lines, alternate interior angles are congruent.
    - When a transversal crosses parallel lines, corresponding angles are congruent.

- **NC.M2.G-SRT.1**: Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations with given center and scale factor:
    - b. Verify experimentally the properties of dilations with given center and scale factor: The length of the image of a line segment is equal to the length of the line segment multiplied by the scale factor.
    - c. The distance between the center of a dilation and any point on the image is equal to the scale factor multiplied by the distance between the dilation center and the corresponding point on the pre-image.
    - d. Dilations preserve angle measure.

- **NC.M2.G-SRT2a,b**: Understand similarity in terms of transformations.
    - a. Determine whether two figures are similar by specifying a sequence of transformations that will transform one figure into the other.
    - b. Use the properties of dilations to show that two triangles are similar when all corresponding pairs of sides are proportional and all corresponding pairs of angles are congruent.

- **NC.M2.G-SRT3**: Understand similarity in terms of transformations. Use transformations (rigid motions and dilations) to justify the AA criterion for triangles similarity.

- **NC.M2.G-SRT4**: (first bullet) Prove theorems involving similarity. Use similarity to solve problems and to prove theorems about triangles.
    - A line parallel to one side of a triangle divides the other two sides proportionally and its converse.

- **NC.M2.G-CO10**: (fourth bullet) Prove theorems about triangles and use them to prove relationships in geometric figures including:
    - The segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length.

Understand congruency through rigid motion transformations and use the properties of congruency to solve problems.

- **NC.M2.G-CO.6**: Determine whether two figures are congruent by specifying a rigid motion or sequence of rigid motions that will transform one figure onto the other.

- **NC.M2.G-CO7**: Use the properties of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

- **NC.M2.G-CO8**: Use congruence in terms of rigid motions. Justify the ASA, SAS, and SSS criteria for triangle congruence. Use criteria for triangle congruence (ASA, SAS, SSS, HL) to determine whether two triangles are congruent.

- **NC.M2.G-CO.9**: (fourth and fifth bullets)Prove theorems about lines and angles and use them to prove relationships in geometric figures including:
    - Points are on a perpendicular bisector of a line segment if and only if they are equidistant from the endpoints of the segment.
    - Use congruent triangles to justify why the bisector of an angle is equidistant from the sides of the angle.

- **NC.M2.G-CO10**: (third bullet) Prove theorems about triangles and use them to prove relationships in geometric figures including:
    - The base angles of an isosceles triangle are congruent.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M2.G-CO.9 (first three bullets)

Prove geometric theorems.

Prove theorems about lines and angles and use them to prove relationships in geometric figures including:

- Vertical angles are congruent.
- When a transversal crosses parallel lines, alternate interior angles are congruent.
- When a transversal crosses parallel lines, corresponding angles are congruent.

Concepts and Skills

Pre-requisite

- Use informal arguments to establish facts about angle sums and exterior angles in triangles and angles created by parallel lines cut by a transversal (8.G.5)
- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)
- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)

Connections

- Use triangle congruence to prove theorems about triangles (NC.M2.G-CO.10)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

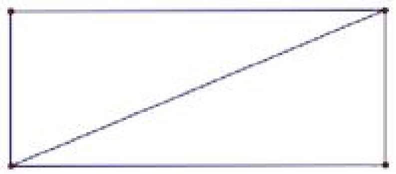
In 8th grade, students experimented with the properties of angles and lines. The focus in this standard is on *proving* the properties; not just knowing and applying them.

Students should use transformations and tactile experiences to gain an intuitive understanding of these theorems, before moving to a formal proof. *For example, vertical angles can be shown to be equal using a reflection across a line passing through the vertex or a 180° rotation around the vertex. Alternate interior angles can be matched up using a rotation around a point midway between the parallel lines on the*

Assessing for Understanding

Students can prove theorems about parallel lines cut by a transversal and the angles formed by the lines.

Example: A carpenter is framing a wall and wants to make sure the edges of his wall are parallel. He is using a cross-brace as show in the diagram.



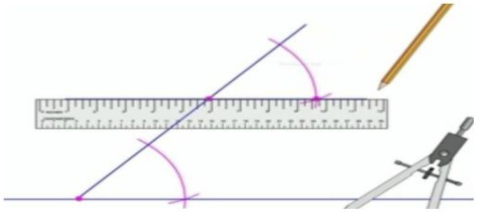
a) What are some different ways that he could verify that the edges are parallel?
b) Write a formal argument to show that the walls are parallel.
c) Pair up with another student who created a different argument than yours, and critique their reasoning. Did you modify your diagram as a result of the collaboration? How? Why?

transversal. Corresponding angles can be matched up using a translation.

Expose students to multiple formats for writing proofs, such as narrative paragraphs, bulleted lists of statements, flow diagrams, two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Students should not be required to master all formats, but to be able to read and analyze proofs in different formats, choosing a format (or formats) that best suit their learning style for writing proofs.

Example: The diagram below depicts the construction of a parallel line, above the ruler. The steps in the construction result in a line through the given point that is parallel to the given line. Which statement below justifies why the constructed line is parallel to the given line?

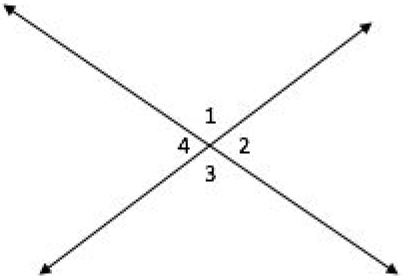
a) When two lines are each perpendicular to a third line, the lines are parallel.
b) When two lines are each parallel to a third line, the lines are parallel.
c) When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel.
d) When two lines are intersected by a transversal and corresponding angles are congruent, the lines are parallel.



Example: Using the image of the intersecting lines to the right:

a) Find the measure of the missing angles when the $m\angle 1 = 47$.
b) Explain how you found those angles. Will $m\angle 1$ and $m\angle 3$ always be the same? Can you think of any example when $m\angle 1$ and $m\angle 3$ could be different?

(Student explanations could include that because $m\angle 1$ and $m\angle 4$ are supplementary and $m\angle 4$ and $m\angle 3$ are supplementary so $m\angle 1$ and $m\angle 3$ must be equal by substitution).



NC.M2.G-SRT.1

Understand similarity in terms of similarity transformations.

Verify experimentally the properties of dilations with given center and scale factor:

b. Verify experimentally the properties of dilations with given center and scale factor: The length of the image of a line segment is equal to the length of the line segment multiplied by the scale factor.

c. The distance between the center of a dilation and any point on the image is equal to the scale factor multiplied by the distance between the dilation center and the corresponding point on the pre-image.

d. Dilations preserve angle measure.

Concepts and Skills

Pre-requisite

- Use coordinates to describe the effects of transformations on 2-D figures (8.G.3)
- Understand similarity through transformations (8.G.4)
- Finding the distance between points in the coordinate plane (8.G.8)
- Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)
- Understand that dilations produce similar figures (NC.M2.G-CO.2)

Connections

- Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)
- Determining similarity by a sequence of transformations; use the properties of dilations to show that two triangles are similar if their corresponding sides proportional and corresponding angles are congruent (NC.M2.G-SRT.2)
- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Students use hands-on techniques (graph paper) and/or technology (geometry software) to experiment with dilations. This standard extends to the observance of the basic properties of dilations as they build a deeper understanding of similarity.

Students should understand that a dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale

Assessing for Understanding

Students verify that a side length of the image is equal to the scale factor multiplied by the corresponding side length of the pre-image.

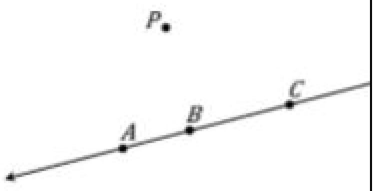
Example: Given ΔABC with $A\,(-2,-4)$, $B\,(1,\ 2)$ and $C\,(4,-3)$.

a) Perform a dilation from the origin using the following function rule $f(x,\ y)\rightarrow(3x,\ 3y)$. What is the scale factor of the dilation?

b) Using ΔABC and its image $\Delta A'B'C'$, connect the corresponding pre-image and image points. Describe how the corresponding sides are related.

c) Determine the length of each side of the triangle. How do the side lengths compare? How is this comparison related to the scale factor?

d) Determine the distance between the origin and point A and the distance between the origin and point

factor.	A'. Do the same for the other two vertices. What do you notice? e) Determine the angle measures for each angle of ΔABC and $\Delta A'B'C'$. What do you notice? Students perform a dilation with a given center and scale factor on a figure in the coordinate plane and verify that when a side passes through the center of dilation, the side and its image lie on the same line and the remaining corresponding sides of the pre-image and images are parallel. **Example:** Suppose we apply a dilation by a factor of 2, centered at the point P to the figure below.  a) In the picture, locate the images A', B', and C' of the points A, B, C under this dilation. b) What is the relationship between c) $lin\ AC$ and $line\ A'C'$? d) What is the relationship between the length of A'B' and the length of AB? Justify your thinking. (Teachers may add in coordinates into this problem initially to give students a concrete entrance to this concept.)

NC.M2.G-SRT.2

Understand similarity in terms of similarity transformations.

Understand similarity in terms of transformations.

a. Determine whether two figures are similar by specifying a sequence of transformations that will transform one figure into the other.
b. Use the properties of dilations to show that two triangles are similar when all corresponding pairs of sides are proportional and all corresponding pairs of angles are congruent

Concepts and Skills

Pre-requisite

- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)
- Verify experimentally properties of dilations with given center and scale factor (NC.M2.G-SRT.1)

Connections

- Use the properties of dilations to show that two triangles are similar if their corresponding sides proportional and corresponding angles are congruent Determining similarity by a sequence of transformations (NC.M2.G-SRT.2b)
- Use transformations for the AA criterion for triangle similarity (NC.M2.G-SRT.3)
- Verify experimentally that side ratios in similar right triangles are properties of the angle measures and use to define trig ratios (NC.M2.G-SRT.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
4 – Model with Mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Students use the idea of dilation transformations to develop the definition of similarity. They understand that a similarity transformation is a combination of a rigid motion and a dilation. Students demonstrate that in a pair of similar triangles, corresponding angles are congruent (angle measure is preserved) and corresponding sides are proportional. They determine that two figures are similar by verifying that angle measure is

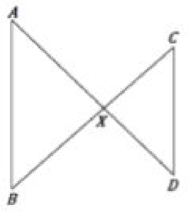
Assessing for Understanding

Students use the idea of dilation transformations to develop the definition of similarity.

Example: In the picture to the right, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles ΔAXB and ΔCXD. In each part a-d below, some additional *assumptions* about the picture are given. For each assumption:

- Determine whether the given assumptions are enough to prove that the two triangles are similar. If so, what is the correct correspondence of vertices. If not, explain why not.
- If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one variable to the other.

a) The lengths of AX and AD satisfy the equation $2AX = 3XD$.
b) The lengths AX, BX, CX, and DX satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$
c) Lines AB and CD are parallel.



(From Illustrative Mathematics)

preserved and corresponding sides are proportional.	d) $\angle XAB$ is congruent to angle $\angle XCD$.

NC.M2.G-SRT.3

Understand similarity in terms of similarity transformations.

Use transformations (rigid motions and dilations) to justify the AA criterion for triangle similarity.

Concepts and Skills

Pre-requisite

- Verify experimentally properties of dilations with given center and scale factor (NC.M2.G-SRT.1)
- Determining similarity by a sequence of transformations; use the properties of dilations to show that two triangles are similar if their corresponding sides proportional and corresponding angles are congruent (NC.M2.G-SRT.2)

Connections

- Use similarity to prove The Triangle Proportionality Theorem and the Pythagorean Theorem (NC.M2.G-SRT.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

5 – Use appropriate tools strategically

6 – Attend to precision

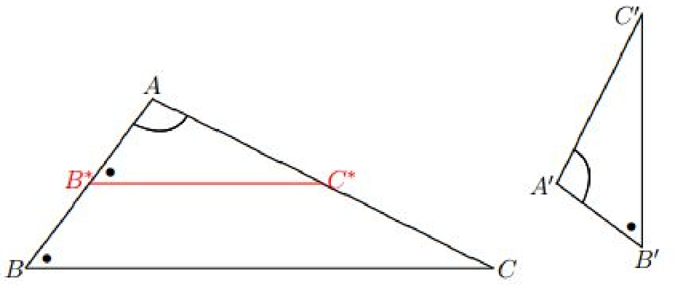
Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Given two triangles for which AA holds, students use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that the dilation will complete the mapping of one triangle onto the other. See p. 98 of Dr. Wu, *Teaching Geometry According to the Common Core Standards.*

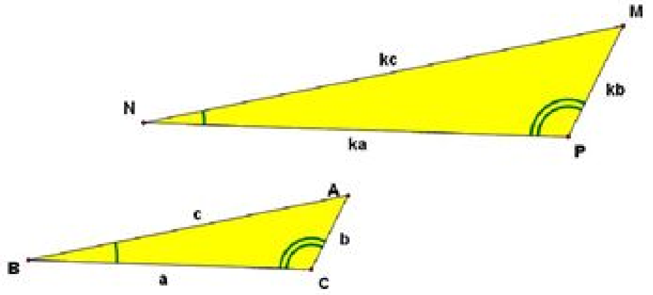


Dynamic geometry software visual of this process. (Geogebra.org)

Assessing for Understanding

Students can use the properties of dilations to show that two triangles are similar based on the AA criterion.

Example: Given that ΔMNP is a dilation of ΔABC with scale factor k, use properties of dilations to show that the AA criterion is sufficient to prove similarity.



NC.M2.G-SRT.4 (first bullet)

Prove theorems involving similarity.

Use similarity to solve problems and to prove theorems about triangles. Use theorems about triangles to prove relationships in geometric figures.

- A line parallel to one side of a triangle divides the other two sides proportionally and its converse.

Concepts and Skills

Pre-requisite

- Use transformations for the AA criterion for triangle similarity (NC.M2.G-SRT.3)

Connections

- Use trig ratios and the Pythagorean Theorem in right triangles (NC.M2.G-SRT.8)
- Derive the equation of a circle given center and radius using the Pythagorean Theorem (NC.M3.G-GPE.1)
- Prove theorems about parallelograms (NC.M3.G-CO.11)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)
- Understand apply theorems about circles (NC.M3.G-C.2)
- Use similarity to demonstrate that the length of the arc is proportional to the radius of the circle (NC.M3.G-C.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

2 – Reason abstractly and quantitatively

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

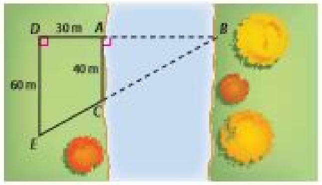
Students use the concept of similarity to solve problem situations (e.g., indirect measurement, missing side(s)/angle measure(s)). Students use the properties of dilations to prove that a line parallel to one side of a triangle divides the other two sides proportionally (often referred to as side-splitter theorem) and its converse.

The altitude from the right angle is drawn to the hypotenuse, which creates three similar triangles. The proportional relationships among the sides of these three triangles can be used to derive the Pythagorean relationship.

Assessing for Understanding

Students use similarity to prove the Pythagorean Theorem.

Example: Calculate the distance across the river, AB.

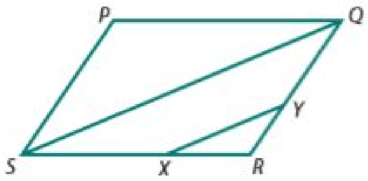


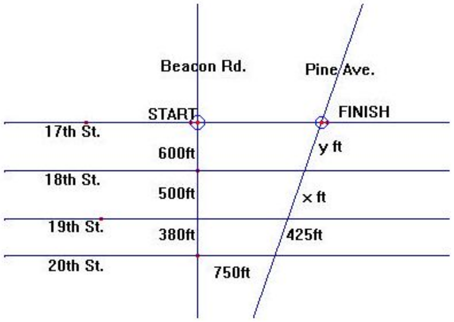
Students can use triangle theorems to prove relationships in geometric figures.

Example: In the diagram, quadrilateral PQRS is a parallelogram, SQ is a diagonal, and SQ || XY.

a. Prove that ΔXYR~ΔSQR.

b. Prove that ΔXYR~ΔQSP.





Example: Parade Route Problem
The parade committee has come up with the Beacon County Homecoming Parade route for next year. They want to start at the intersection of 17th Street and Beacon Road. The parade will proceed south on Beacon Road, turning left onto 20th Street. Then the parade will turn left onto Pine Avenue and finish back at 17th Street. For planning purposes, the committee needs to know approximately how long the parade will last. Can you help them? Justify your estimate. What assumptions did you make?

(adapted from http://www.math.uakron.edu/amc/Geometry/HSGeometryLessons/SideSplitterTheorem.pdf)

Example: Use similarity to prove the slope criteria for similar triangles.
(https://www.illustrativemathematics.org/content-standards/HSG/SRT/B/5/tasks/1876)

NC.M2.G-CO.10 (4th bullet)

Prove geometric theorems.

Prove theorems about triangles and use them to prove relationships in geometric figures including:

- The segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length.

Concepts and Skills

Pre-requisite

- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)
- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)
- Use triangle congruence to prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)

Connections

- Verify experimentally, properties of the centers of triangles (NC.M3.G-CO.10)
- Prove theorems about parallelograms (NC.M3.G-CO.11)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

5 – Use appropriate tools strategically

6 – Attend to precision

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

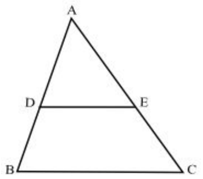
Mastering the Standard for this Unit

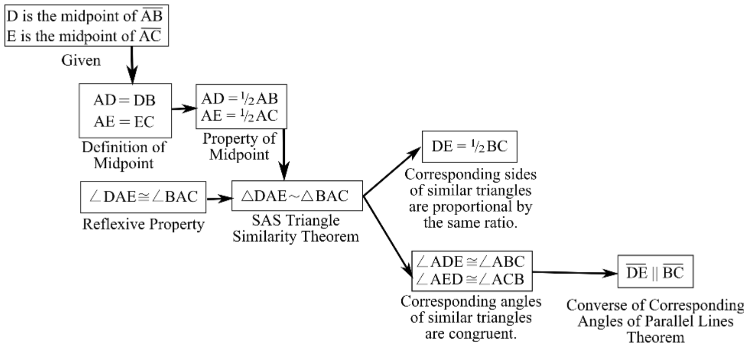
Comprehending the Standard

Encourage multiple ways of writing proofs, such as *narrative paragraphs* and *flow diagrams*. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of transparencies and dynamic geometry software can be important tools for helping students conceptually understand important geometric concepts.

Triangle Midsegment Theorem

Given that D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$, prove $\overline{DE} \| \overline{BC}$ and $DE = \frac{1}{2}BC$.





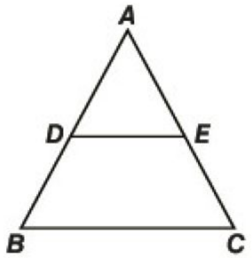
Assessing for Understanding

Students can prove theorems about triangles.

Example: In $\triangle ABC$, $DB = AD$ and $EC = AE$.

a) Given $DE = x$ and $BC = 3x - 6$, find DE and BC.

b) Given m∠ABC=69°, find the m∠ADE. Explain your reasoning.



NC.M2.G-CO.6

Understand congruence in terms of rigid motions.

Determine whether two figures are congruent by specifying a rigid motion or sequence of rigid motions that will transform one figure onto the other.

Concepts and Skills

Pre-requisite

- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)

Connections

- Use the properties of rigid motions to show that two triangles are congruent if their corresponding sides and angles are congruent (NC.M2.G-CO.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard for this Unit

Comprehending the Standard

This standard connects to the 8th grade standard where students informally addressed congruency of figures through rigid motions to the formalized HS standard where students specifically defined points, lines, planes and angles of rigid motion transformations.

Students recognize rigid transformations preserve size and shape (or distance and angle) and develop the definition of congruence. This standard goes beyond the assumption of mere correspondence of points, lines and angles and thus establishing the properties of congruent figures.

Assessing for Understanding

Students use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.

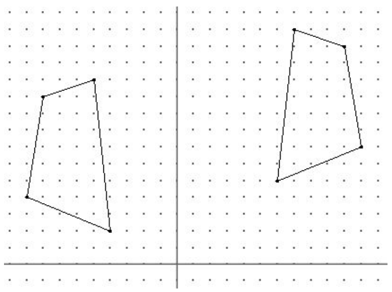
Example: Consider parallelogram ABCD with coordinates $A(2,-2)$, $B(4,4)$, $C(12,\ 4)$ and $D(10,-2)$. Consider the following transformations. Make predictions about how the lengths, perimeter, area and angle measures will change under each transformation below:

a) A reflection over the x-axis.
b) A rotation of 270° counterclockwise about the origin.
c) A dilation of scale factor 3 about the origin.
d) A translation to the right 5 and down 3.

Verify your predictions by performing the transformations. Compare and contrast which transformations preserved the size and/or shape with those that did not preserve size and/or shape. Generalize: which types of transformation(s) will produce congruent figures?

Students determine if two figures are congruent by determining if rigid motions will map one figure onto the other.

Example: Determine if the figures are congruent. If so, describe and demonstrate a sequence of rigid motions that maps one figure onto the other.



NC.M2.G-CO.7

Understand congruence in terms of rigid motions.

Use the properties of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Concepts and Skills

Pre-requisite

- Determining congruence through a sequence of rigid motions (NC.M2.G-CO.6)

Connections

- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

5 – Use appropriate tools strategically

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard for this Unit

Comprehending the Standard

A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed:

- to map lines to lines, rays to rays, and segments to segments and
- to preserve distances and angle measures.

Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

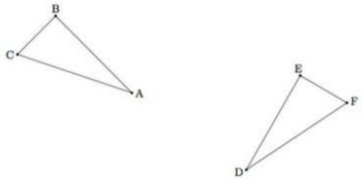
This standard connects the establishment of congruence to congruent triangle proofs based on corresponding sides and angles.

Assessing for Understanding

Students identify corresponding sides and corresponding angles of congruent triangles. Explain that in a pair of congruent triangles, corresponding sides are congruent (distance is preserved) and corresponding angles are congruent (angle measure is preserved). They demonstrate that when distance is preserved (corresponding sides are congruent) and angle measure is preserved (corresponding angles are congruent) the triangles must also be congruent.

Example: Illustrative Mathematics Task – Properties of Congruent Triangles

To the right is a picture of two triangles:



a) Suppose there is a sequence of rigid motions which maps ΔABC to ΔDEF . Explain why corresponding sides and angles of these triangles are congruent.

b) Suppose instead that corresponding sides and angles of ΔABC to ΔDEF are congruent. Show that there is a sequence of rigid motions which maps ΔABC to ΔDEF ’

NC.M2.G-CO.8

Understand congruence in terms of rigid motions.

Justify the ASA, SAS, and SSS criteria for triangle congruence. Use criteria for triangle congruence (ASA, SAS, SSS, HL) to determine whether two triangles are congruent.

Concepts and Skills

Pre-requisite

- Use the properties of rigid motions to show that two triangles are congruent if their corresponding sides and angles are congruent (NC.M2.G-CO.7)

Connections

- Use triangle congruence to prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)
- Use triangle congruence to prove theorems about triangles (NC.M2.G-CO.10)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Extending from the 7th grade standard where students examine the conditions required to determine a unique triangle, students come to understand the specific characteristics of congruent triangles which lays the groundwork for geometric proof. Proving these theorems helps students to then prove theorems about lines and angles in other geometric figures and other triangle proofs.

Videos of Transformation Proofs:

Animated Proof of SAS (YouTube)
Animated Proof of ASA (YouTube)

Assessing for Understanding

Students list the sufficient conditions to prove triangles are congruent: ASA, SAS, and SSS. They map a triangle with one of the sufficient conditions (e.g., SSS) onto the original triangle and show that corresponding sides and corresponding angles are congruent.

Example: Josh is told that two triangles ΔABC and ΔDEF share two sets of congruent sides and one set of congruent angles: $\overline{AB}$ is congruent to $\overline{DE}$, $\overline{BC}$ is congruent to $\overline{EF}$, and $\angle B$ is congruent to $\angle E$. He is asked if these two triangles must be congruent. Josh draws the two triangles marking congruent sides and angles. Then he says, "They are definitely congruent because two pairs of sides are congruent and the angle between them is congruent!"

a) Draw the two triangles. Explain whether Josh's reasoning is correct using triangle congruence criteria.

b) Given two triangles ΔABC and ΔDEF, give an example of three sets of congruent parts that will not always guarantee that the two triangles are congruent. Explain your thinking.

NC.M2.G-CO.9 (fourth and fifth bullets)

Prove geometric theorems.

Prove theorems about lines and angles and use them to prove relationships in geometric figures including:

- Points are on a perpendicular bisector of a line segment if and only if they are equidistant from the endpoints of the segment.
- Use congruent triangles to justify why the bisector of an angle is equidistant from the sides of the angle.

Concepts and Skills

Pre-requisite

- Use informal arguments to establish facts about angle sums and exterior angles in triangles and angles created by parallel lines cut by a transversal (8.G.5)
- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)
- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)

Connections

- Use triangle congruence to prove theorems about triangles (NC.M2.G-CO.10)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

In 8th grade, students experimented with the properties of angles and lines. The focus in this standard is on *proving* the properties; not just knowing and applying them.

Students should use transformations and tactile experiences to gain an intuitive understanding of these theorems, before moving to a formal proof. *For example, vertical angles can be shown to be equal using a reflection across a line passing through the vertex or a 180° rotation around the vertex. Alternate interior angles can be matched up using a rotation around a point midway between the parallel lines on the transversal. Corresponding angles can be matched up using a translation.*

Expose students to multiple formats for writing proofs, such as narrative paragraphs, bulleted lists of statements, flow diagrams, two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Students should not be required to master all formats, but to be able to read and analyze proofs in different formats, choosing a format (or formats) that best suit their learning style for writing proofs.

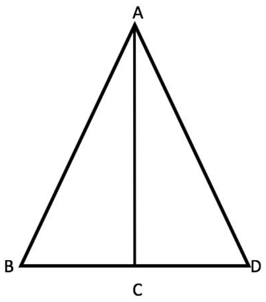
Assessing for Understanding

Students can prove theorems about intersecting lines and their angles.

Example: Prove that any point equidistant from the endpoints of a line segment lies on the perpendicular bisector of the line. [Example YouTube Proof: Point equidistant from segment end points is on perpendicular bisector]

Example: Given that ∠BAC≅∠DAC and that BC and DC are distances, prove that BC=DC.

(Students should be able to prove that there are right angles at C due to the definition of distance. Students should use CPCTC)



NC.M2.G-CO.10 (third bullet)

Prove geometric theorems.

Prove theorems about triangles and use them to prove relationships in geometric figures including:

- The base angles of an isosceles triangle are congruent.

Concepts and Skills

Pre-requisite

- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)
- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)
- Use triangle congruence to prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)

Connections

- Verify experimentally, properties of the centers of triangles (NC.M3.G-CO.10)
- Prove theorems about parallelograms (NC.M3.G-CO.11)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Encourage multiple ways of writing proofs, such as *narrative paragraphs* and *flow diagrams*. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of transparencies and dynamic geometry software can be important tools for helping students conceptually understand important geometric concepts.

Assessing for Understanding

Students can prove theorems about triangles.

Example: Prove the Converse of the Isosceles Triangle Theorem: If two angles of a triangle are congruent, then the sides opposite them are congruent.

Example: Prove that an equilateral triangle is also equiangular.

The North Carolina High School Collaborative Instructional Framework

NC Math 2

Unit 3: Quadratic Functions

20 Days Block Schedule

September 2017 Update

40 Days Traditional Schedule

RESEARCH BRIEF: **Unit 3: Quadratic Functions**

Essential Questions:

- What are the important features of each form of a quadratic function?
- When is it more appropriate to use one form of a quadratic function over another form?
- How can one form of a quadratic function be transformed to another form?
- Where does the Quadratic Formula come from and how is it derived?
- How does use of the quadratic formula suggest the need for complex numbers?
- What do the three forms of a quadratic function explain in context of a scenario?
- What do the solutions of a quadratic function explain in context of a scenario?

Learning Outcomes	Student Objectives
● Given a quadratic function in standard form, students will determine the effects of the a-value and c-value through the use of technology.	● Given a quadratic function in standard form, I will **determine** the effects of the a-value and c-value through the use of technology.
● Given an equation in standard form, students will transform the equation to factored form.	● Given an equation in standard form, I will **transform** the equation to factored form.
● Given an equation in factored form, students will transform the equation to standard form.	● Given an equation in factored form, I will **transform** the equation to standard form.
● Given the x-intercepts and a point on the parabola (y-intercept, vertex, etc.), students will generate the equation of the parabola.	● Given the x-intercepts and a point on the parabola (y-intercept, vertex, etc.), I will **generate** the equation of the parabola.
	● Given a quadratic function in standard form, I will **complete** the square to form the vertex form.

- Given a quadratic function in standard form, students will complete the square to form the vertex form.
- Given a quadratic function in vertex form, students will identify transformations that will map the quadratic parent function to the given quadratic function.
- Students will solve quadratic functions with real solutions by square roots, factoring, completing the square and the quadratic formula.
- Students will solve quadratic functions with complex solutions.
- Given a scenario, students will apply knowledge of quadratic functions to identify key components of the quadratic function in the scenario and explain these components in context.
- Given a scenario, students will apply knowledge of quadratic functions to solve the quadratic function in the scenario and explain the solutions in context.

- Given a quadratic function in vertex form, I will **identify** transformations that will map the quadratic parent function to the given quadratic function.
- I will **solve** quadratic functions with real solutions by square roots, factoring, completing the square and the quadratic formula.
- I will **solve** quadratic functions with complex solutions.
- Given a scenario, I will **apply** knowledge of quadratic functions to identify key components of the quadratic function in the scenario and explain these components in context.
- Given a scenario, I will **apply** knowledge of quadratic functions to solve the quadratic function in the scenario and explain the solutions in context.

Standards Addressed in this Unit

Interpret, compare, and analyze quadratic functions in different representations.

- **NC.M2.A.SS.E.1a:** Interpret expressions that represent a quantity in terms of its context.
    a. Identify and interpret parts of a quadratic, square root, inverse variation, or right triangle trigonometric expression, including terms, factors, coefficients, radicands, and exponents.

- **NC.M2.A.SS.E.1b:** Interpret expressions that represent a quantity in terms of its context.
    b. Interpret quadratic and square root expressions made of multiple parts as a combination of single entities to give meaning in terms of a context.

- **NC.M2.F.IF.4**: Interpret functions that arise in applications in terms of context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: domain and range, rate of change, symmetries, and end behavior.

- **NC.M2.F.IF.7**: Analyze functions using different representations. Analyze quadratic, square root, and inverse variation functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

- **NC.M2.F.IF.9**: Analyze functions using different representations. Compare key features of two functions (linear, quadratic, square root, or inverse variation functions) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

- **NC.M2.A.REI.7**: Use tables, graphs, and algebraic methods to approximate or find exact solutions of systems of linear and quadratic equations, and interpret the solutions in terms of a context.

Solve quadratic equations algebraically.

- **NC.M2.A.APR.1**: Perform operations on polynomials. Extend the understanding that operations with polynomials are comparable to operations with integers by adding, subtracting, and multiplying polynomials.

- **NC.M2.N-CN.1:** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ where a and b are real numbers.

- **NC.M2.A.SSE.3:** Interpret the structure of expressions. Write an equivalent form of a quadratic expression by completing the square, where is an integer of a quadratic expression, , to reveal the maximum or minimum value of the function the expression defines.

- **NC.M2.A.REI.4a:** Solve for all solutions of quadratic equations in one variable.
  a. Understand that the quadratic formula is the generalization of solving ax^2+bx+c by using the process of completing the square.

- **NC.M2.A.REI.4b:** Solve for all solutions of quadratic equations in one variable.
  b. Explain when quadratic equations will have non-real solutions and express complex solutions as $a+bi$ for real numbers a and b.

- **NC.M2.A.REI.1**: Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for quadratic, square root and inverse variation equations using mathematical reasoning.

- **NC.M2.F.IF.8**: Use equivalent expressions to reveal and explain different properties of a function by developing and using the process of completing the square to identify the zeros, extreme values, and symmetry in graphs and tables representing quadratic functions, and interpret these in terms of a context.

Transform and model quadratic functions.

- **NC.M2.F.BF.1**: Write a function that describes a relationship between two quantities by building quadratic functions with real solution(s) and inverse variation functions given a graph, a description of a relationship, or ordered pairs (include reading these from a table).
- **NC.M2.F.BF.3**: Understand the effects of the graphical and tabular representations of a linear, quadratic, square root, and inverse variation function f with $k \cdot f(x)$, $f(x) + k$, $f(x+k)$ for specific values of k (both positive and negative).
- **NC.M2.A.CED.1**: Create equations and inequalities in one variable that represent quadratic, square root, inverse variation, and right triangle trigonometric relationships and use them to solve problems.
- **NC.M2.A.CED.2**: Create and graph equations in two variables to represent quadratic, square root and inverse variation relationships between quantities.
- **NC.M2.A.CED.3**: Create systems of linear, quadratic, square root, and inverse variation equations to model situations in context.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M2.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a **quadratic**, square root, inverse variation, or right triangle trigonometric expression, including terms, factors, coefficients, radicands, and exponents.

Concepts and Skills

Pre-requisite

- Interpreting parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Creating equation to solve, graph, and make systems (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Solve and interpret one variable inverse variation and square root equations (NC.M2.A-REI.2)
- Interpreting functions (NC.M2.F-IF.4, NC.M2.F-IF.7, NC.M2.F-IF.9)
- Understand the effect of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.
4 – Model with mathematics
7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

When given an expression with a context, students should be able to explain how the parts of the expression relate to the context of the problem.

Students should be able to write equivalent forms of an expression to be able to identify parts of the expression that can relate to the context of the problem.

The parts of expressions that students should be able to interpret include any terms, factors, coefficients, radicands, and exponents.

Students should be given contexts that can be modeled with quadratic expressions.

Assessing for Understanding

Students should be able to identify and interpret parts of an expression in its context.

Example: The expression $-4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball t seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation.

Example: The area of a rectangle can be represented by the expression $x^2 + 8x + 12$. What do the factors of this expression represent in the context of this problem?

NC.M2.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret quadratic and square root expressions made of multiple parts as a combination of single entities to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpreting parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Creating equation to solve, graph, and make systems (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Solve and interpret one variable inverse variation and square root equations (NC.M2.A-REI.2)
- Interpreting functions (NC.M2.F-IF.4, NC.M2.F-IF.7, NC.M2.F-IF.9)
- Understand the effect of transformations on functions (NC.M2.F-IF.2, NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.
4 – Model with mathematics
7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe their interpretation of an expression.

Mastering the Standard for this Unit

Comprehending the Standard

When given an expression with a context that has multiple parts, students should be able to explain how combinations of those parts of the expression relate to the context of the problem.

Students should be able to write equivalent forms of an expression to be able to identify combinations of parts of the expression that can represent a quantity in the context of the problem.

Students should be given contexts that can be modeled with quadratic expressions.

Assessing for Understanding

Students should be able to see parts of an expression as a single quantity that has a meaning based on context.

Example: If the volume of a rectangular prism is represented by $x(x+3)(x+2)$, what can $(x+3)(x+2)$ represent?

Example: Sylvia is organizing a small concert as a charity event at her school. She has done a little research and found that the expression $-10x+180$ represents the number of tickets that will sell, given that x represents the price of a ticket. Explain why the income for this event can be represented by the expression $-10x^2+180x$. If all of the expenses will add up to $150, explain why the expression $-10x^2+180x-150$ represents the profit.

NC.M2.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: domain and range, rate of change, symmetries, and end behavior.

Concepts and Skills

Pre-requisite

- Interpret key features of graphs, tables and verbal descriptions (NC.M1.F-IF.4)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Extend the use of function notation to geometric transformations (NC.M2.F-IF.2)

Connections

- Analyze and compare functions (NC.M2.F-IF.7, 8, 9)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe how they identified key features of graph, table, or verbal description and interpret those key features in context.

Mastering the Standard for this Unit

Comprehending the Standard

When given a table, graph, or verbal description of a function that models a real-life situation, explain the meaning of the key features in the context of the problem.

Key features include: domain and range, rate of change, symmetries, and end behavior.

When interpreting rate of change students should be able to describe the rate at which the function is increasing or decreasing. For example, a linear function with a positive slope is increasing at a constant rate. A quadratic with a maximum point is increasing at a decreasing rate, reaching the maximum, and then decreasing at an increasing rate. An inverse variation function in the first quadrant is decreasing at a decreasing rate.

Connect this standard with NC.M2.F-IF.7. This standard focuses on interpretation from various representations whereas NC.M2.F-IF.7 focuses on generating different representations.

Also, this standard is not limited by function type and can include functions that students do not have the algebraic skills to manipulate. NC.M2.F-IF.7 lists specific function types for which students can use algebra to analyze key features of the function.

Assessing for Understanding

Students should be able to interpret key features of a function from a verbal description.

Example: Jason kicked a soccer ball that was laying on the ground. It was in the air for 3 seconds before it hit the ground again. While the soccer ball was in the air it reached a height of approximately 30ft. Assuming that the soccer ball's height (in feet) is a function of time (in seconds), interpret the domain, range, rate of change, line of symmetry, and end behavior in this context.

Students should be able to interpret key features of a function from a table.

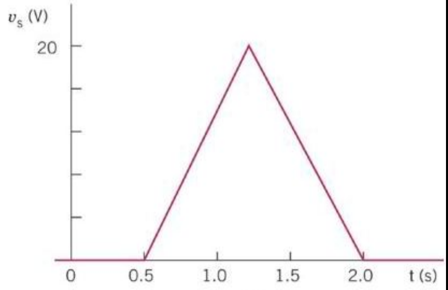
Example: Julia was experimenting with a toy car and 4ft ramp. She found that as she increased the height of one end of the ramp, the time that the car took to reach the end of the ramp decreased. She collected data to try to figure out the relationship between ramp height and time and came up with the following table.

Height (ft)	.25	.5	.75	1	1.25
Time (sec)	3.9	2.1	1.4	1.1	.9

Assuming that time is a function of height, interpret the domain, range, rate of change, and end behavior in terms of this context.

Students should be able to interpret key features of a function from a graph.

Example: The graph to the right is the voltage, *v*, in a given circuit as a function of the the time (in seconds). What was the maximum voltage and for how long did it take to complete the circuit?



NC.M2.F-IF.7

Analyze functions using different representations.

Analyze quadratic, square root, and inverse variation functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)

Connections

- Create and graph two variable equations (NC.M2.A-CED.2)
- Analyze quadratic functions rewritten into vertex form (NC.M2.F-IF.8)
- Compare functions (NC.M2.F-IF.8)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with mathematics
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Students need to be able to represent a function with an equation, table, graph, and verbal/written description.

When given one representation students need to be able to generate the other representations and use those representations to identify key features.

Key features include: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

Assessing for Understanding

Students should be able to find the appropriate key feature to solve problems by analyzing the given function.

Example: Represent the function $f(x) = 2(x+3)^2 - 2$ with a table and graph. Identify the following key features: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

NC.M2.F-IF.9

Analyze functions using different representations.

Compare key features of two functions (linear, quadratic, square root, or inverse variation functions) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Compare key features of two functions (NC.M1.F-IF.9)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.8)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

Students need to compare characteristics of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In this standard students are comparing any two of the following functions: Linear and Quadratic

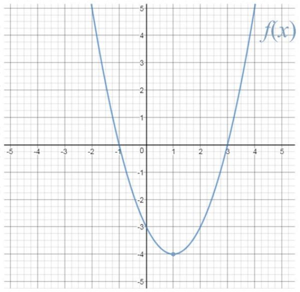
This means that students need to be able to compare functions that are in the same function family (for example quadratic vs quadratic) and functions that are in different function families The representations of the functions that are being compared needs to be different. For

Assessing for Understanding

Example: Chad was comparing two quadratic functions $f(x)$ and $g(x)$. The function $f(x)$ is given in the graph and $g(x)$ is given by the table.

a) What is the difference in the y-intercepts of each function?
b) Which function has the smallest minimum value and by how much?
c) What is the difference when the x-coordinate of the vertex of $g(x)$ is subtracted from the x-coordinate of the vertex of f(x)?

$g(x)$	x
-1	8
0	3
1	0
2	-1
3	0



Example: Eli and Jeb had a contest to see who could throw a football the highest. Eli released his football from an initial height of 5 feet and with an initial upward velocity of 40 ft/sec (the formula for projectile motion is $h(t) =- 16t^2 + v_0 t + h_0$ where v_0 represents the initial height and h_0 the initial height). The height of Jeb's ball can be modeled by the equation $j(t) =- 16t^2 + 35t + 6$.

a) Whose football went the highest and by how much?

example compare a graph of one function to an equation of another.	b) Whose football was in the air the longest

NC.M2.A-REI.7

Solve systems of equations.

Use tables, graphs, and algebraic methods to approximate or find exact solutions of systems of linear and quadratic equations, and interpret the solutions in terms of a context.

Concepts and Skills

Pre-requisite

- Use tables, graphs and algebraic methods to find solutions to systems of linear equations (NC.M1.A-REI.6)
- Operations with polynomials (NC.M2.A-APR.1)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)

Connections

- Create equations (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Analyze and compare functions (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to discuss the number of solutions possible in a system with a linear and quadratic function and a system with two quadratic functions.

Mastering the Standard for this Unit

Comprehending the Standard

Students solve a system containing a linear equation and a quadratic equation in two-variables. Students solve graphically and algebraically.

Students interpret solutions of a system of linear and quadratic equations in terms of a context.

Assessing for Understanding

Students should be able to efficiently solve systems of equations with various methods.

Example: In a gymnasium a support wire for the overhead score board slopes down to a point behind the basket. The function $w(x) = -\frac{1}{5}x + 38$ describes the height of the wire above the court, $w(x)$, and the distance in feet from the edge of the score board, x. During a game, a player must shoot a last second shot while standing under the edge of score board. The trajectory of the shot is $b(x) = -.08x^2 + 3x + 6$, where $b(x)$ is the height of the basketball and x is the distance from the player. Describe what could have happened to the shot. (All measurements are in feet.)

Example: The area of a square can be calculated with the formula $Area = s^2$ and the perimeter can be calculated with the formula $Perimeter = 4s$ where s is the length of a side of the square. If the area of the square is the same as its perimeter, what is the length of the side? Demonstrate how you can find the side length using algebraic methods, a table and with a graph.

Example: The student council is planning a dance for their high school. They did some research and found that the relationship between the ticket price and income that they will receive from the dance can be modeled by the function $f(x) = -100(x-4)^2 + 1500$. They also calculated their expenses and found that their expenses can be modeled by the function $g(x) = 300 + 10x$. What ticket price(s) could the student council charge for the dance if they wanted to break-even (the expenses are equal to the income)? Demonstrate how you can find the answer using algebraic methods, a table and with a graph.

NC.M2.A-APR.1

Perform arithmetic operations on polynomials.

Extend the understanding that operations with polynomials are comparable to operations with integers by adding, subtracting, and multiplying polynomials.

Concepts and Skills

Pre-requisite

- Operations with polynomials (NC.M1.A-APR.1)
- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)

Connections

- Solving systems of linear and quadratic equations (NC.M2.A-REI.7)
- Use equivalent expression to develop completing the square (NC.M2.F-IF.8)
- Understand the effect of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe their process to multiply polynomials.

Mastering the Standard for this Unit

Comprehending the Standard

The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, binomial, trinomial, polynomial, factor, and term.

Assessing for Understanding

Students should be able to rewrite polynomials into equivalent forms through addition, subtraction and multiplication.

Example: Simplify and explain the properties of operations apply.

a) $(x^3 + 3x^2 - 2x + 5)(x - 7)$

b) $4b(cb - zd)$

c) $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$

d) $(4x^2 - 3y^2 + 5xy) + (8xy + 3y^2)$

e) $(x + 4)(x - 2)(3x + 5)$

NC.M2.N-CN.1

Defining complex numbers.

Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ where a and b are real numbers.

Concepts and Skills

Pre-requisite

- The understanding of number systems is developed through middle school (8.NS.1)

Connections

- Solve quadratic equations in one variable (NC.M2.A-REI.4b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication
Complex Number, Imaginary
Students should be able to define a complex number and identify when they are likely to use them.

Mastering the Standard for this Unit

Comprehending the Standard

When students solve quadratic equations they should understand that there is a solution to an equation when a negative appears in the radicand. This solution does not produce x-intercepts for the function and is not included in the real number system. This means that it is now time to introduce students to a broader classification of numbers so that we have a way to express these solutions.

Students should know that every number can be written in the form $a + bi$,where *a* and *b* are real numbers and $i = \sqrt{-1}$,
are classified as complex numbers. If $a = 0$, then the number is a pure imaginary number. If $b = 0$ the number is a real number. This means that all real numbers are included in the complex number system and that the square root of a negative number is a complex number.

Students should connect what they have learned regarding properties of exponents to understand that $(\sqrt{-1})^2 = (-1)^{\frac{1}{2}*2} =-1$.

Students should be able to express solutions to a quadratic equation as a complex number.

Assessing for Understanding

Students should be able to rewrite expressions using what they know about complex numbers.

Example: Simplify.

	Problem	**Solution**
a)	i^2	$i^2 = (\sqrt{-1})^2 = (-1)^{\frac{1}{2}*2} =-1$
b)	$\sqrt{-36}$	$\sqrt{-36} = \sqrt{-1}\cdot\sqrt{36} = 6i$
c)	$2\sqrt{-49}$	$2\sqrt{-49} = 2\sqrt{-1}\cdot\sqrt{49} = 2\cdot 7i = 14i$
d)	$-3\sqrt{-10}$	$-3\sqrt{-10} =-3\sqrt{-1}\cdot\sqrt{10} =-3\cdot i\cdot\sqrt{10} =-3i\sqrt{10}$
e)	$5\sqrt{-7}$	$5\sqrt{-7} = 5\sqrt{-1}\cdot\sqrt{7} = 5\cdot i\cdot\sqrt{7} = 5i\sqrt{7}$
f)	$\frac{-3+\sqrt{9-4*2*5}}{4}$	$\frac{-3+\sqrt{9-4*2*5}}{4} = \frac{-3+\sqrt{-31}}{4} = \frac{-3+i\sqrt{31}}{4}$ Which can be written in the form $a + bi$ as $\frac{-3}{4} + \frac{\sqrt{31}}{4}i$

NC.M2.A-SSE.3

Interpret the structure of expressions.

Write an equivalent form of a quadratic expression by completing the square, where a is an integer of a quadratic expression, $ax^2 + bx + c$, to reveal the maximum or minimum value of the function the expression defines.

Concepts and Skills

Pre-requisite

- Rewrite quadratic expression to reveal zeros and solutions (NC.M1.A-SSE.3)
- Interpret parts of a function as single entities in context (NC.M2.A-SSE.1b)

Connections

- Understand the relationship between the quadratic formula and the process of completing the square (NC.M2.A-REI.4a)
- Find and compare key features of quadratic functions (NC.M2.F-IF.7, NC.M2.F-IF.8, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with mathematics
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to explain when the process of completing the square is necessary.

New Vocabulary: completing the square

Mastering the Standard for this Unit

Comprehending the Standard

When given an equation in the form $ax^2 + bx + c$ students should be able to complete the square to write a quadratic equation in vertex form: $a(x-h)^2 + k$.

Students should be able to determine that if $a > 0$ there is a minimum and if $a < 0$ there is a maximum.

Students should be able to identify the maximum or minimum point (h, k) from an equation in vertex form.

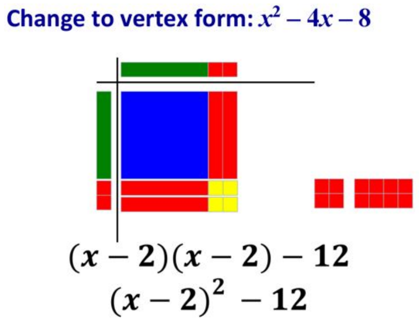
Algebra Tiles are a great way to demonstrate this process. You can demonstrate the reasoning for all of the steps in the process.
This process also links previous learning of the area model for multiplication.

Assessing for Understanding

Students should be able to reveal the vertex of a quadratic expression using the process of completing the square.

Example: Write each expression in vertex form and identify the minimum or maximum value of the function.

a) $x^2 - 4x + 5$
b) $x^2 + 5x + 8$
c) $2x^2 + 12x - 18$
d) $3x^2 - 12x - 1$
e) $2x^2 - 15x + 3$



Example: The picture at the right demonstrates the process of completing the square using algebra tiles. Looking at the picture, why might this process be called "completing the square"?

Note: There are at least two good answers to this question. First the product must form a square, so you must arrange and complete this missing parts using zero pairs to make the square. The second, completing the square is about finding the "new C" which in the process will be a square as seen in the yellow blocks in this picture.

NC.M2.A-REI.4a

Solve equations and inequalities in one variable.

Solve for all solutions of quadratic equations in one variable.

a. Understand that the quadratic formula is the generalization of solving $ax^2 + bx + c$ by using the process of completing the square.

Concepts and Skills

Pre-requisite

- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)

Connections

- Create and solve one variable equations (NC.M2.A-CED.1)
- Solve inverse variation and square root equations (NC.M2.A-REI.2)
- Explain that quadratic equations have complex solutions (NC.M2.A-REI.4b)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Analyze and compare functions (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
7 – Look for and make use of structure
8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to discuss the relationship between the quadratic formula and the process of completing the square.
New Vocabulary: completing the square, quadratic formula

Mastering the Standard for this Unit

Comprehending the Standard

Students have used the method of completing the square to rewrite a quadratic expression in standard NC.M2.A-SSE.3. In this standard students are extending the method to solve a quadratic equation.

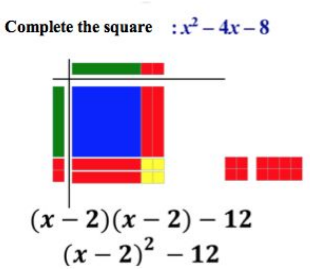
Some students may set the quadratic equal to zero, rewrite into vertex form $a(x-h)^2+k=0$, and then begin solving to get the equation into the form $(x-h)^2=q$ where $q=\frac{-k}{a}$. Other students may adapt the method (i.e. not having to start with the quadratic equal to 0) to get the equation into the same form.

Assessing for Understanding

Students should be able to explain the process of completing the square and be able to generalize it into the quadratic formula.

Example: by completing the square and the quadratic formula. How are the two methods related?

Example: We often see the need to create a formula when the same steps are repeated in the same type of problems. This is true for completing the square. Recall the steps for completing the square using a visual model, like algebra tiles. A completed example is provided to the right.
To make a formula, we need to generalize the process. To do this, we replace each coefficient with a variable and then solve with those variables in place and we treat those variables same as a numbers.



Students who write vertex form first

$$-2x^2-16x-20=0$$
$$-2(x^2+8x)-20=0$$
$$-2(x^2+8x+16)-20+32=0$$
$$-2(x+4)^2+12=0$$
$$-2(x+4)^2=-12$$
$$(x+4)^2=6$$
$$x+4=\pm\sqrt{6}$$
$$x=-4\pm\sqrt{6}$$

Students who adapts method

$$-2x^2-16x-20=0$$
$$-2x^2-16x=20$$
$$-2(x^2+8x)=20$$
$$-2(x^2+8x+16)=20-32$$
$$-2(x+4)^2=-12$$
$$(x+4)^2=6$$
$$x+4=\pm\sqrt{6}$$
$$x=-4\pm\sqrt{6}$$

This standard is about understanding that the quadratic formula is derived from the process of completing the square. Students should become very familiar with this process before introducing the quadratic formula. Students should understand completing the square both visually and symbolically. Algebra titles are a great way for students to understand the reasoning behind the process of completing the square.
It is not the expectation for students to memorize the steps in deriving the quadratic formula. (Remember that students have no experience with rational expressions which is required as part of completing the derivation on their own!)

Below are two columns. In the left is an example, similar to those you have been asked to solve. On the right is a generalized form of the problem. For the left column, provide a mathematical reason for each step as you have done before. (Refer back to a visual model as needed.) One the right side, identify how you can see that mathematical reasoning in the generalized form. When complete, try out the new formula with the example problem from the left column.

Completing the Square (*Example*)	**Completing the Square** (*Generalized*)
$3x^2+5x+4=0$	$ax^2+bx+c=0$
$x^2+\frac{5}{3}x+\frac{4}{3}=0$	$x^2+\frac{b}{a}x+\frac{c}{a}=0$
$x^2+\frac{5}{3}x+\frac{5^2}{2^2\cdot 3^2}=\frac{5^2}{2^2\cdot 3^2}-\frac{4}{3}$	$x^2+\frac{b}{a}x+\frac{b^2}{2^2\cdot a^2}=\frac{b^2}{2^2\cdot a^2}-\frac{c}{a}$
$x^2+\frac{5}{3}x+\frac{25}{36}=\frac{25}{36}-\frac{4}{3}$	$x^2+\frac{b}{a}x+\frac{b^2}{4\cdot a^2}=\frac{b^2}{4\cdot a^2}-\frac{c}{a}$
$x^2+\frac{5}{3}x+\frac{25}{36}=\frac{25}{36}-\frac{4}{3}\cdot\frac{12}{12}$	$x^2+\frac{b}{a}x+\frac{b^2}{4\cdot a^2}=\frac{b^2}{4\cdot a^2}-\frac{c}{a}\cdot\frac{4a}{4a}$
$x^2+\frac{5}{3}x+\frac{25}{36}=\frac{-23}{36}$	$x^2+\frac{b}{a}x+\frac{b^2}{4\cdot a^2}=\frac{b^2-4ac}{4\cdot a^2}$
$\left(x+\frac{5}{6}\right)^2=\frac{-23}{36}$	$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4\cdot a^2}$
$x+\frac{5}{6}=\pm\sqrt{\frac{-23}{36}}$	$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4\cdot a^2}}$
$x=\frac{-5}{6}\pm\frac{\sqrt{-23}}{6}$	$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$
$x=\frac{-5\pm i\sqrt{23}}{6}$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

NC.M2.A-REI.4b

Solve equations and inequalities in one variable.

Solve for all solutions of quadratic equations in one variable.

b. Explain when quadratic equations will have non-real solutions and express complex solutions as $a \pm bi$ for real numbers a and b.

Concepts and Skills

Pre-requisite

- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)
- Know there is a complex number and the form of complex numbers (NC.M2.N-NC.1)
- Solve quadratic equations (NC.M2.A-REI.4a)

Connections

- Create and solve one variable equations (NC.M2.A-CED.1)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)
- Solve inverse variation and square root equations (NC.M2.A-REI.2)
- Solve systems of equations (NC.M2.A-REI.7)
- Analyze and compare functions (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
5 – Use appropriate tools strategically
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to identify the number of real number solutions of a quadratic equation and justify their assertion.
New Vocabulary: complex solutions

Mastering the Standard for this Unit

Comprehending the Standard

Students recognize when the quadratic formula gives complex solutions and are able to write them as $a \pm bi$.

Students relate the value of the discriminant to the type of roots expected. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$. Students are not required to use the word discriminant, but should be familiar with the concepts of the discriminant.

Students should develop these concepts through experience and reasoning.

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2 - 4ac = 0$	1 real root	Intersects x-axis once
$b^2 - 4ac > 0$	2 real roots	Intersects x-axis twice
$b^2 - 4ac < 0$	2 complex solutions	Does not intersect x-axis

Assessing for Understanding

Students should be able to identify the number and type of solution(s) of a quadratic equation.

Example: How many real roots does $2x^2 + 5 = 2x$ have? Find all solutions of the equation.

Example: What is the nature of the roots of $x^2 + 6x + 10 = 0$? How do you know?

Examples: Solve each quadratic using the method indicated and explain when in the solving process you knew the nature of the roots.

a)	Square root	$3x^2 + 9 = 72$
b)	Quadratic formula	$4x^2 + 13x - 7 = 0$
c)	Factoring	$6x^2 + 13x = 5$
d)	Complete the square	$x^2 + 12x - 2 = 0$

Example: Ryan used the quadratic formula to solve an equation and his result was $x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-2)}}{2(1)}$.

a) Write the quadratic equation Ryan started with in standard form.
b) What is the nature of the roots?
c) What are the x-intercepts of the graph of the corresponding quadratic function?

Example: Solve $x^2 + 8x =- 17$ for x.

NC.M2.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a chosen solution method and each step of the solving process for **quadratic**, square root and inverse variation equations using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solving method and each step in the process (NC.M1.A-REI.1)
- Explain how expressions with rational exponents can be rewritten as radical expressions (NC.M2.N-RN.1)
- Use equivalent expressions to explain the process of completing the square (NC.M2.F-IF.8)

Connections

- Create and solve one variable equations (NC.M2.A-CED.1)
- Solve inverse variation, square root and quadratic equations (NC.M2.A-REI.2, NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Use trig ratios to solve problems (NC.M2.G-SRT.8)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Students need to be able to explain why they choose a specific method to solve an equation.
For example, with a quadratic equation, students could choose to factor, use the quadratic formula, take the square root, complete the square to take the square root, solve by graphing or with a table. Students should be able to look at the structure of the quadratic to make this decision. With a square root equation, students could choose to square both sides, solve by graphing or with a table.

Discussions on the solving processes and the benefits and drawbacks of each method should lead students to not rely on one solving process. Students should make determinations on the solving process based on the context of the problem, the nature and structure of the equation, and efficiency.

Assessing for Understanding

Students should be able to justify each step in a solving process.

Example: Explain why the equation $x^2 + 14 = 9x$ can be solved by determining values of x such that $x - 7 = 0$ and $x - 2 = 0$.

Example: Solve $3x^2 =- 4x + 8$. Did you chose to solve by factoring, taking the square root, completing the square, using the quadratic formula, or some other method? Why did you chose that method? Explain each step in your solving process.

Example: If a, b, c, and d are real numbers, explain how to solve how to solve $ax^2 + bx + c = d$ in 2 different methods. Discuss the pros and cons of each method.

While solving algebraically, students need to use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution. Students need to solve quadratic equations.	Students should be able to chose and justify solution methods. **Example:** To the right are two methods for solving the equation $5x^2 + 10 = 90$. Select one of the solution methods and construct a viable argument for the use of the method. ***Method A*** $5x^2 + 10 = 90$ $-10 = -10$ $5x^2 = 80$ $\frac{5x^2}{5} = \frac{80}{5}$ $x^2 = 16$ $x = \pm\sqrt{16}$ $x = 4 \; or \; x = -4$ ***Method B*** $5x^2 + 10 = 90$ $-90 = -90$ $5x^2 - 80 = 0$ $5(x^2 - 16) = 0$ $5(x+4)(x-4) = 0$ $x + 4 = 0 \; or \; x - 4 = 0$ $x = 4 \; or \; x = -4$ **Example:** To the right are two methods for solving the equation $2x^2 - 3x + 4 = 0$. Select one of the solution methods and construct a viable argument for the use of the method. ***Method A*** $2x^2 - 3x + 4 = 0$ $x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$ $x = \frac{3 \pm \sqrt{-23}}{4}$ $x = \frac{3 \pm i\sqrt{23}}{4}$ $x = \frac{3}{4} \pm \frac{i\sqrt{23}}{4}$ ***Method B*** $2x^2 - 3x + 4 = 0$ $x^2 - \frac{3}{2}x + 2 = 0$ $x^2 - \frac{3}{2}x + \frac{9}{16} = -2 + \frac{9}{16}$ $\left(x - \frac{3}{4}\right)^2 = \frac{-23}{16}$ $x - \frac{3}{4} = \pm\sqrt{\frac{-23}{16}}$ $x = \frac{3}{4} \pm \frac{i\sqrt{23}}{4}$

NC.M2.F-IF.8

Analyze functions using different representations.

Use equivalent expressions to reveal and explain different properties of a function by developing and using the process of completing the square to identify the zeros, extreme values, and symmetry in graphs and tables representing quadratic functions, and interpret these in terms of a context.

Concepts and Skills

Pre-requisite

- Rewrite a quadratic function to reveal key features (NC.M1.F-IF.8a)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)

Connections

- Creating and graphing equations in two variables (NC.M2.A-CED.2)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze and compare functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.9)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to explain which key features can be found from each form of a quadratic function.

New Vocabulary: completing the square

Mastering the Standard for this Unit

Comprehending the Standard

Students look at equivalent expressions of functions to identify key features on the graph and in a table of the function.

For example, students should factor quadratics to identify the zeros, complete the square to reveal extreme values and the line of symmetry, and look at the standard form of the equation to reveal the y-intercept.

Students could also argue that by factoring and finding the zeros they could easily find the line of symmetry by finding the midpoint between the zeros.

Once identifying the key features students should interpret them in terms of the context.

Assessing for Understanding

Students should be able use the process of completing the square to identify key features of the function.

Example: Coyote was chasing roadrunner, seeing no easy escape, Roadrunner jumped off a cliff towering above the roaring river below. Molly Mathematician was observing the chase and obtained a digital picture of this fall. Using her mathematical knowledge, Molly modeled the Road Runner's fall with the following quadratic functions:

$$h(t) = -16t^2 + 32t + 48$$

$$h(t) = -16(t+1)(t-3)$$

$$h(t) =- 16(t-1)^2 + 64$$

a) How can Molly have three equations?

b) Which of the rules would be most helpful in answering each of these questions? Explain.

   i. What is the maximum height the Road Runner reaches and when will it occur?

   ii. When would the Road Runner splash into the river?

   iii. At what height was the Road Runner when he jumped off the cliff?

Students should be able to identify the key features able to be found in each form of a quadratic function.

Example: Which of the following equations could describe the function of the given graph to the right? Explain.

$f_1(x) = (x + 12)^2 + 4$

$f_2(x) = -(x - 2)^2 - 1$

$f_3(x) = (x + 18)^2 - 40$

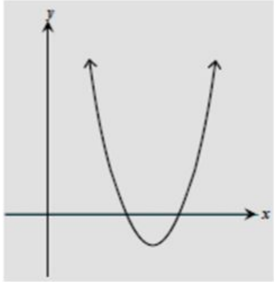
$f_4(x) = (x + 12)^2 + 4$

$f_5(x) = -4(x + 2)(x + 3)$

$f_6(x) = (x + 4)(x - 6)$

$f_7(x) = (x - 12)(-x + 18)$

$f_8(x) = (20 - x)(30 - x)$



NC.M2.F-BF.1

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities by **building quadratic functions with real solution**(s) and inverse variation functions given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

Concepts and Skills

Pre-requisite

- Build linear and exponential functions from tables, graphs, and descriptions (NC.M1.F-BF.1a)
- Creating and graphing equations in two variables (NC.M2.A-CED.2)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)

Connections

- Analyze and compare functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.8, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with mathematics
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to justify their chosen model with mathematical reasoning.

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

Given a graph, ordered pairs (including a table), or description of a relationship, students need to be able to write an equation of a function that describes a quadratic or inverse variation relationship.

Make sure that quadratic functions have real solutions. (Operations with complex numbers are **not** part of the standards.)

Student should realize that in an inverse variation relationship they can multiply the x and y coordinates of an ordered pair together to get the constant of proportionality.

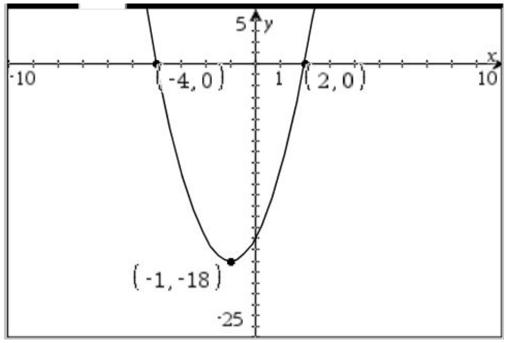
When given the x-intercepts and a point on a quadratic students can solve the equation $f(x) = a(x-m)(x-n)$ for a after substituting the x-intercepts for m and n, and the x and y coordinates from the point for x and $f(x)$. Once the student has solved for a they can plug a, m, and n into the equation so that their equation is written in factored form.

When given a maximum or minimum point on a quadratic and another point students can use the equation $f(x) = a(x-h)^2 + k$ to solve for a so that their function equation is written in vertex form.

Assessing for Understanding

Students should be able to build functions that model a given situation using the context and information available from various representations.

Example: Write an equation of the function given the graph.



NC.M2.F-BF.3

Build new functions from existing functions.

Understand the effects of the graphical and tabular representations of a linear, quadratic, square root, and inverse variation function *f* with $k \cdot f(x)$, $f(x)+k$, $f(x+k)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Operations with polynomials (NC.M2.A-APR.1)
- Extend the concept of functions to include geometric transformations (NC.M2.F-IF.1)

Connections

- Extend the use of function notation to express the transformation of geometric figures (NC.M2.F-IF.2)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze and compare functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make sense of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to compare and contrast the transformation of geometric figures and two variable equations expressed as functions.

New Vocabulary: inverse variation, constant of proportionality, vertical compression, vertical stretch

Mastering the Standard for this Unit

Comprehending the Standard

It is important to note that this standard is under the domain of building functions. The functions are being built for a purpose, to solve a problem or to offer insight.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions.

Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of *k*.

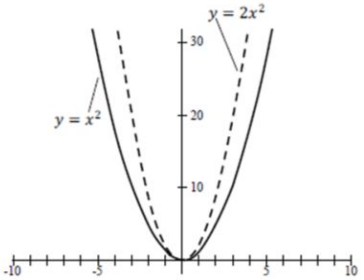
Students should understand how changes in the equation effect changes in graphs and tables of values.

- $k \cdot f(x)$ If $0 < k < 1$ there is a vertical compression meaning that the outputs of the function have been reduced since they were multiplied by a number between 0 and 1. If $k > 1$ there is a vertical stretch meaning that the outputs have all been multiplied by the same value. If k is negative, then all of the outputs will change signs and this will result in a reflection over the x-axis.
- $f(x)+k$ If k is positive all of the outputs are being increased by

Assessing for Understanding

Students should be able to describe the effect of transformations on algebraic functions.

Example: Compare the shape and position of the graphs of $f(x)=x^2$ and $g(x)=2x^2$ and explain the differences in terms of the algebraic expressions for the functions.



the same value and the graph of the function will move up. If k is negative, all of the outputs are being decreased by the same value and the graph of the function will move down.

- $f(x+k)$ If k is positive then all of the inputs are increasing by the same value. Since they are increasing before they are plugged into the operations of the function, the graph will move to the left. If k is negative, then all of the inputs are decreasing by the same value. Since they are decreasing before they are plugged into the operations of the function the graph will move to the right.

Students should focus on linear, quadratic, square root, and inverse variation functions in this course.

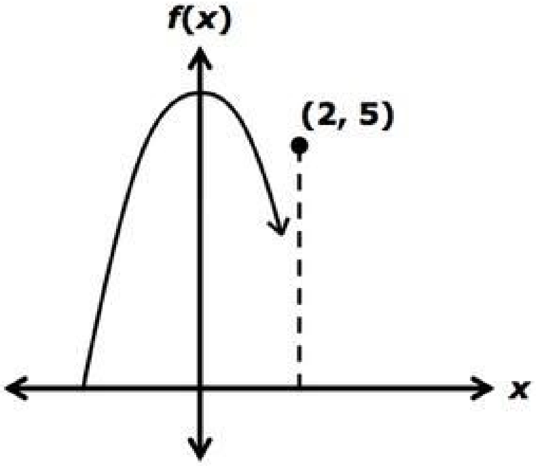
Example: A computer game uses functions to simulate the paths of an archer's arrows. The x-axis represents the level ground on which the archer stands, and the coordinate pair (2,5) represents the top of a castle wall over which he is trying to fire an arrow.

In response to user input, the first arrow followed a path defined by the function $f(x) = 6 - x^2$ failing to clear the castle wall.

The next arrow must be launched with the same force and trajectory, so the user must reposition the archer in order for his next arrow to have any chance of clearing the wall.

a) How much closer to the wall must the archer stand in order for the arrow to clear the wall by the greatest possible distance?
b) What function must the user enter in order to accomplish this?
c) If the user can only enter functions of the form $f(x+k)$, what are all the values of k that would result in the arrow clearing the castle wall?

https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/695



NC.M2.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent **quadratic**, square root, inverse variation, and right triangle trigonometric relationships and use them to solve problems.

Concepts and Skills

Pre-requisite

- Create and solve equations in one variable (NC.M1.A-CED.1)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Justify solving methods and each step (NC.M2.A-REI.1)

Connections

- Solve inverse variation, square root and quadratic equations (NC.M2.A-REI.2, NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Use trig ratios to solve problems (NC.M2.G-SRT.8)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equations as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

2 – Reason abstractly and quantitatively

4 – Model with mathematics

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Students should be able to determine a correct equation or inequality to model a given context and use the model to solve problems.

Focus on contexts that can be modeled with quadratic, square root, inverse variation, and right triangle trigonometric equations and inequalities.

Students need to be familiar with algebraic, tabular, and graphic methods of solving equations and inequalities.

Assessing for Understanding

Students should be able to create one variable equations from multiple representations, including from functions.

Example: Lava ejected from a caldera in a volcano during an eruption follows a parabolic path. The formula to find the height of the lava can be found by combining three terms that represent the different forces affecting the lava. The first term is the original height of the volcano. The second term concerns the speed at which the lava is ejected. The third term is the effect of gravity on the lava.

$$height(t) = original\ height + (initial\ speed\ of\ the\ lava)\cdot t + \tfrac{1}{2}(effects\ of\ gravity)\cdot t^2$$

The original height of the caldera is $936\,ft$. The lava was ejected at a speed of $64 ft/s$. The effect of gravity on any object on earth is approximately $-32ft/s^2$. Write and solve an equation that will find how long it will take for the lava to reach a height of 1000ft.

Example: The function $h(x) = 0.04x^2 - 3.5x + 100$ defines the height (in feet) of a major support cable on a suspension bridge from the bridge surface where x is the horizontal distance (in feet) from the left end of the bridge. Write a relation for each situation and find the solution.

a) Where is the cable less than 40 feet above the bridge surface?

b) Where is the cable at least 60 feet above the bridge surface?

	Example: Jamie is selling key chains that he has made to raise money for school trip. He has done a little research and found that the expression $-20x + 140$ represents the number of keychains that he will be able to sell, given that x represents the price of one keychain. Each key chain costs Jamie $.50 to make. Create a relationship to that shows his goal to make at least $150 profit.

NC.M2.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent **quadratic**, square root and inverse variation relationships between quantities.

Concepts and Skills

Pre-requisite

- Create and graph equations in two variables (NC.M1.A-CED.2)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)

Connections

- Write equations for a system (NC.M2.A-CED.3)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Analyze functions for key features (NC.M2.F-IF.7)
- Build quadratic and inverse variation functions (NC.M2.F-BF.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

In this standard students are creating equations and graphs in two variables.

Focus on contexts that can be modeled with **quadratic**.

This standard needs to be connected with other standards where students interpret functions, generate multiple representations, solve problems, and compare functions.

Assessing for Understanding

Students should be able to create an equation from a context or representation and graph the equation.

Example: The area of a rectangle is 40 in^2. Write an equation for the length of the rectangle related to the width. Graph the length as it relates to the width of the rectangle. Interpret the meaning of the graph.

Example: The formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r represents the radius of the circular cross-section of the cylinder and h represents the height. Given that $h = 10in$ …

a) Graph the volume as it relates to the radius.
b) Graph the radius as it relates to the volume.
c) Compare the graphs. Be sure to label your graphs and use an appropriate scale.

NC.M2.A-CED.3

Create equations that describe numbers or relationships.

Create systems of linear, **quadratic**, square root, and inverse variation equations to model situations in context.

Concepts and Skills

Pre-requisite

- Create equations for a system of equations in context (NC.M1.A-CED.3)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Create equations in two variables (NC.M2.A-CED.2)

Connections

- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equations as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
2 – Reason abstractly and quantitatively
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to justify their created equations through unit analysis.

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

Students create systems of equations to model situations in contexts.

Contexts should be limited to **linear, quadratic,** square root and inverse variation equations.

This standard should be connected with NC.M2.A-REI.7 where students solve and interpret systems and with NC.M2.A-REI.11 where students understand the representation of the solutions of systems graphically.

Assessing for Understanding

Students should be able to recognize when a context requires a system of equations and create the equations of that system.

> **Example:** In making a business plan for a pizza sale fundraiser, students determined that both the income and the expenses would depend on the number of pizzas sold. They predicted that $I(n) =- 0.05n^2 + 20n$ and $E(n) = 5n + 250$. Determine values for which $I(n) = E(n)$ and explain what the solution(s) reveal about the prospects of the pizza sale fundraiser.

The North Carolina High School Collaborative Instructional Framework

NC Math 2

Unit 4: Square Root & Inverse Variation Functions

15 Days Block Schedule | **September 2017 Update** | 30 Days Traditional Schedule

RESEARCH BRIEF: Square Root & Inverse Variation Functions

Essential Questions:

- What effects do r and k have on functions of the form $y = kx^r$ and $y = \frac{k}{x^r}$?
- What effects do a, h, and k have on functions of the form $y = a\sqrt{x-h}+k$?
- What strategies are useful in solving problems that involve links between two functions - one a linear function and one an inverse variation function?
- What strategies are effective in solving systems of equations that relate mixed functions (Linear, Quadratic, Inverse, and Square Root Functions)?

Learning Outcomes	**Student Objectives**
● Given data, students will recognize numeric and graphic patterns of change in direct and inverse variation relationships.	● Given data, I will **recognize** numeric and graphic patterns of change in direct and inverse variation relationships.
● Students will express direct and inverse variation relationships in symbolic forms.	● I will **express** direct and inverse variation relationships in symbolic forms.
● Students will recognize and represent relationships between variables that can be modeled by power functions $y = kx^r$ and $y = \frac{k}{x^r}$.	● I will **recognize** and **represent** relationships between variables that can be modeled by power functions $y = kx^r$ and $y = \frac{k}{x^r}$.
● Students will solve problems involving direct and inverse variation in context.	● I will **solve** problems involving direct and inverse variation in context.
● Students will analyze the effects of r and k on functions of the form $y = kx^r$ and $y = \frac{k}{x^r}$.	● I will **analyze** the effects of r and k on functions of the form $y = kx^r$ and $y = \frac{k}{x^r}$.
	● I will **investigate** functions of the form $y = a\sqrt{x-h}+k$.

- Students will investigate functions of the form $y = a\sqrt{x-h}+k$.
- Students will analyze the effects of a, h, and k on functions of the form $y = a\sqrt{x-h}+k$.
- Students will solve problems involving square root functions in context.
- Students will solve systems of equations which include any mixture of various functions.

- I will **analyze** the effects of a, h, and k on functions of the form $y = a\sqrt{x-h}+k$.
- I will **solve** problems involving square root functions in context.
- I will **solve** systems of equations involving linear, quadratic, inverse, and square root functions.

Standards Addressed in this Unit

Extend and apply the properties of rational exponents.

- NC.M2.N-RN.1: Explain how expressions with rational exponents can be rewritten as radical expressions.

- NC.M2.N-RN.2: Rewrite expressions with radicals and rational exponents into equivalent expressions using the properties of exponents.

- NC.M2.N-RN.3: Use the properties of rational and irrational numbers to explain why:
  - the sum or product of two rational numbers is rational;
  - the sum of a rational number and an irrational number is irrational;
  - the product of a nonzero rational number and an irrational number is irrational.

- NC.M2.A.SS.E.1a**:** Interpret expressions that represent a quantity in terms of its context.
  a. Identify and interpret parts of a quadratic, square root, inverse variation, or right triangle trigonometric expression, including terms, factors, coefficients, radicands, and exponents.

- NC.M2.A.SS.E.1b**:** Interpret expressions that represent a quantity in terms of its context.
  b. Interpret quadratic and square root expressions made of multiple parts as a combination of single entities to give meaning in terms of a context.

- NC.M2.A.CED.1: Create equations and inequalities in one variable that represent quadratic, square root, inverse variation, and right triangle trigonometric relationships and use them to solve problems.

- **NC.M2.A.CED.2**: Create and graph equations in two variables to represent quadratic, square root and inverse variation relationships between quantities.
- **NC.M2.F-BF.3**: Understand the effects of the graphical and tabular representations of a linear, quadratic, square root, and inverse variation function f with $k \cdot f(x)$, $f(x)+k$, $f(x+k)$) for specific values of k (both positive and negative).
- **NC.M2.A.CED.3:** Create systems of linear, quadratic, square root, and inverse variation equations to model situations in context.
- **NC.M2.A.REI.1**: Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for quadratic, square root and inverse variation equations using mathematical reasoning.
- **NC.M2.A-REI.2**: Solve and interpret one variable inverse variation and square root equations arising from a context, and explain how extraneous solutions may be produced.
- **NC.M2.A-REI.11**: Extend the understanding that the x -coordinates of the points where the graphs of two square root and/or inverse variation equations $y = f(x)$ *and* $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$, and approximate solutions using graphing technology or successive approximations with a table of values.
- **NC.M2.F.IF.4**: Interpret functions that arise in applications in terms of context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: domain and range, rate of change, symmetries, and end behavior.
- **NC.M2.F.IF.7**: Analyze functions using different representations. Analyze quadratic, square root, and inverse variation functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.
- **NC.M2.F.IF.9**: Analyze quadratic, square root, and inverse variation functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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The Math Resource for Instruction - Customized for the Content of this Unit

NC.M2.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a quadratic, **square root, inverse variation**, or right triangle trigonometric expression, including terms, factors, coefficients, radicands, and exponents.

Concepts and Skills

Pre-requisite

- Interpreting parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Creating equation to solve, graph, and make systems (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Solve and interpret one variable inverse variation and square root equations (NC.M2.A-REI.2)
- Interpreting functions (NC.M2.F-IF.4, NC.M2.F-IF.7, NC.M2.F-IF.9)
- Understand the effect of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, right triangle trigonometry

Mastering the Standard for this Unit

Comprehending the Standard

When given an expression with a context, students should be able to explain how the parts of the expression relate to the context of the problem.

Students should be able to write equivalent forms of an expression to be able to identify parts of the expression that can relate to the context of the problem.

The parts of expressions that students should be able to interpret include any terms, factors, coefficients, radicands, and exponents.

Students should be given contexts that can be modeled with quadratic, square root, inverse variation expressions.

Assessing for Understanding

Students should be able to identify and interpret parts of an expression in its context.

Example: The stopping distance in feet of a car is directly proportional to the square of its speed. The formula that relates the stopping distance and speed of the car is $D = k \cdot V^2$, where D represents the stopping distance in feet, k represents a constant that depends on the frictional force of the pavement on the wheels of a specific car, and V represents the speed the car was traveling in miles per hour.
When there is a car accident it is important to figure out how fast the cars involved were traveling. The expression $\sqrt{\frac{D}{k}}$ can be evaluated to find the speed that a car was traveling. What does the radicand represent in this expression?

Example: Ohm's Law explains the relationship between current, resistance, and voltage. To determine the current passing through a conductor you would need to evaluate the expression $\frac{V}{R}$, where V represents voltage and R represents resistance. If the resistance is increased, what must happen to the voltage so that the current passing through the conductor remains constant?

NC.M2.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret quadratic and **square root expressions** made of multiple parts as a combination of single entities to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpreting parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Creating equation to solve, graph, and make systems (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Solve and interpret one variable inverse variation and square root equations (NC.M2.A-REI.2)
- Interpreting functions (NC.M2.F-IF.4, NC.M2.F-IF.7, NC.M2.F-IF.9)
- Understand the effect of transformations on functions (NC.M2.F-IF.2, NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe their interpretation of an expression.

Mastering the Standard for this Unit

Comprehending the Standard

When given an expression with a context that has multiple parts, students should be able to explain how combinations of those parts of the expression relate to the context of the problem.

Students should be able to write equivalent forms of an expression to be able to identify combinations of parts of the expression that can represent a quantity in the context of the problem.

Students should be given contexts that can be modeled with quadratic and square root expressions.

Assessing for Understanding

Students should be able to see parts of an expression as a single quantity that has a meaning based on context.

Example: When calculating the standard deviation of a population you must first find the mean of the data, subtract the mean from each value in the data set, square each difference, add all of the squared differences together, divide by the number of terms in the data set and then take the square root. The expression used for calculating standard deviation of a population is $\sqrt{\frac{\sum(x-\mu)^2}{n}}$. Given the above description of the process of calculating standard deviation and what you have learned in a previous course about standard deviation being a measure of spread, answer the following questions.

a) Describe what you are finding when you calculate $x-\mu$.

b) Describe how the formula for standard deviation is similar to the formula for finding mean.

c) What part of the radicand would have to increase so that the value of the standard deviation would also increase: the numerator ($\sum(x-\mu)^2$) or the denominator (n)? Justify your answer.

NC.M2.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: domain and range, rate of change, symmetries, and end behavior.

Concepts and Skills

Pre-requisite

- Interpret key features of graphs, tables and verbal descriptions (NC.M1.F-IF.4)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Extend the use of function notation to geometric transformations (NC.M2.F-IF.2)

Connections

- Analyze and compare functions (NC.M2.F-IF.7, 8, 9)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe how they identified key features of graph, table, or verbal description and interpret those key features in context.

Mastering the Standard for this Unit

Comprehending the Standard

When given a table, graph, or verbal description of a function that models a real-life situation, explain the meaning of the key features in the context of the problem.

Key features include: domain and range, rate of change, symmetries, and end behavior.

When interpreting rate of change students should be able to describe the rate at which the function is increasing or decreasing. For example, a linear function with a positive slope is increasing at a constant rate. A quadratic with a maximum point is increasing at a decreasing rate, reaching the maximum, and then decreasing at an increasing rate. An inverse variation function in the first quadrant is decreasing at a decreasing rate.

Connect this standard with NC.M2.F-IF.7. This standard focuses on interpretation from various representations

Assessing for Understanding

Students should be able to interpret key features of a function from a verbal description.

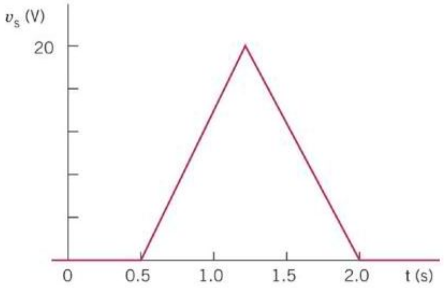
Example: Jason kicked a soccer ball that was laying on the ground. It was in the air for 3 seconds before it hit the ground again. While the soccer ball was in the air it reached a height of approximately 30ft. Assuming that the soccer ball's height (in feet) is a function of time (in seconds), interpret the domain, range, rate of change, line of symmetry, and end behavior in this context.

Students should be able to interpret key features of a function from a table.

Example: Julia was experimenting with a toy car and 4ft ramp. She found that as she increased the height of one end of the ramp, the time that the car took to reach the end of the ramp decreased. She collected data to try to figure out the relationship between ramp height and time and came up with the following table.

Height (ft)	.25	.5	.75	1	1.25
Time (sec)	3.9	2.1	1.4	1.1	.9

Assuming that time is a function of height, interpret the domain, range, rate of change, and end behavior in terms of this context.

whereas NC.M2.F-IF.7 focuses on generating different representations. Also, this standard is not limited by function type and can include functions that students do not have the algebraic skills to manipulate. NC.M2.F-IF.7 lists specific function types for which students can use algebra to analyze key features of the function.	Students should be able to interpret key features of a function from a graph. **Example:** The graph to the right is the voltage, *v*, in a given circuit as a function of the the time (in seconds). What was the maximum voltage and for how long did it take to complete the circuit? 

NC.M2.F-IF.7

Analyze functions using different representations.

Analyze quadratic, **square root, and inverse variation** functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)

Connections

- Create and graph two variable equations (NC.M2.A-CED.2)
- Analyze quadratic functions rewritten into vertex form (NC.M2.F-IF.8)
- Compare functions (NC.M2.F-IF.8)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with mathematics
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should explain which key features are necessary to find given the context of the problem.
New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

Students need to be able to represent a function with an equation, table, graph, and verbal/written description.

When given one representation students need to be able to generate the other representations and use those representations to identify key features.

Key features include: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

In Math 2 students should focus on quadratic, square root, and inverse variation functions.

Assessing for Understanding

Students should be able to find the appropriate key feature to solve problems by analyzing the given function.

Example: The distance a person can see to the horizon can be found using the function $d(h) = \sqrt{\frac{3h}{2}}$, where $d(h)$ represents the distance in miles and h represents the height the person is above sea level. Create a table and graph to represent this function. Use a table, graph, and the equation to find the domain and range, intercepts, end behavior and intervals where the function is increasing, decreasing, positive, or negative.

Example: Represent the function $f(x) = \frac{2}{x}$ with a table and graph. Identify the following key features: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

NC.M2.F-IF.9

Analyze functions using different representations.

Compare key features of two functions (linear, quadratic, square root, or inverse variation functions) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Compare key features of two functions (NC.M1.F-IF.9)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.8)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard

Comprehending the Standard

Students need to compare characteristics of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In this standard students are comparing any two of the following functions:

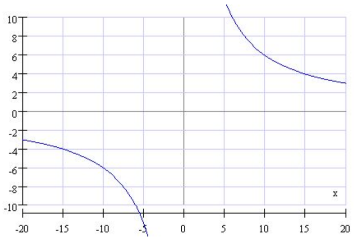
- Linear
- Quadratic
- Square root
- Inverse variation

Assessing for Understanding

Students should be able to compare key features of two functions in different representations.

Example: Compare the constant of proportionality for each of the following inverse variation models and list them in order from least to greatest.

x	y
5	36
10	18
15	12
20	9
25	7.2

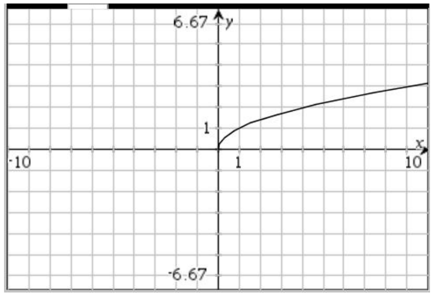


This means that students need to be able to compare functions that are in the same function family (for example quadratic vs quadratic) and functions that are in different function families (for example square root vs inverse variation).

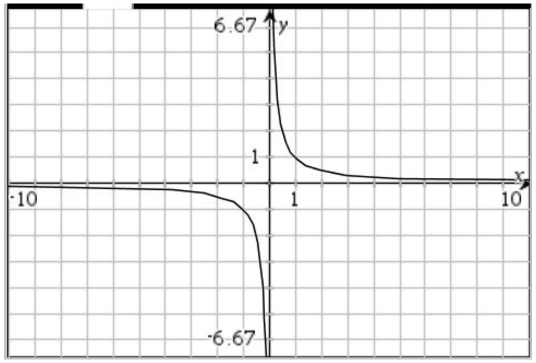
The representations of the functions that are being compared needs to be different. For example compare a graph of one function to an equation of another.

Example: Compare and contrast the domain and range, rate of change and intercepts of the two functions below represented below.

Meredith runs at a constant rate of 6 miles per hour when she runs on her treadmill. The distance that she runs on her treadmill is a function of the time that she is runs.



Example: Compare and contrast the end behavior and symmetries of the two functions represented below.



x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

NC.M2.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a chosen solution method and each step of the solving process for quadratic, **square root and inverse variation** equations using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solving method and each step in the process (NC.M1.A-REI.1)
- Explain how expressions with rational exponents can be rewritten as radical expressions (NC.M2.N-RN.1)
- Use equivalent expressions to explain the process of completing the square (NC.M2.F-IF.8)

Connections

- Create and solve one variable equations (NC.M2.A-CED.1)
- Solve inverse variation, square root and quadratic equations (NC.M2.A-REI.2, NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Use trig ratios to solve problems (NC.M2.G-SRT.8)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to predict the justifications of another student's solving process.
New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

Students need to be able to explain why they choose a specific method to solve an equation.
For example, with a square root equation, students could choose to square both sides, solve by graphing or with a table.

Discussions on the solving processes and the benefits and drawbacks of each method should lead students to not rely on one solving process. Students should make determinations on the solving process based on the context of the problem, the nature and structure of the equation, and efficiency.

While solving algebraically, students need to use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution.

Students need to solve quadratic, s**quare root and inverse variation equations.**

Assessing for Understanding

Students should be able to justify each step in a solving process.

Example: Solve $\frac{2}{x} = x + 1$. Did you chose to solve by factoring, taking the square root, completing the square, using the quadratic formula, or some other method? Why did you chose that method? Explain each step in your solving process.

Example: Solve $\sqrt{x+3} = 3x - 1$ using algebraic methods and justify your steps. Solve graphically and compare your solutions.

NC.M2.F-BF.3

Build new functions from existing functions.

Understand the effects of the graphical and tabular representations of a linear, quadratic, **square root, and inverse variation** function f with $k \cdot f(x)$, $f(x)+k$, $f(x+k)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Operations with polynomials (NC.M2.A-APR.1)
- Extend the concept of functions to include geometric transformations (NC.M2.F-IF.1)

Connections

- Extend the use of function notation to express the transformation of geometric figures (NC.M2.F-IF.2)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze and compare functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make sense of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to compare and contrast the transformation of geometric figures and two variable equations expressed as functions.

New Vocabulary: inverse variation, constant of proportionality, vertical compression, vertical stretch

Mastering the Standard for this Unit

Comprehending the Standard

It is important to note that this standard is under the domain of building functions. The functions are being built for a purpose, to solve a problem or to offer insight.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions. Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of k.

Students should understand how changes in the equation effect changes in graphs and tables of values.

- $k \cdot f(x)$ If $0 < k < 1$ there is a vertical compression meaning that the outputs of the function have been reduced since they were multiplied by a number between 0 and 1. If $k > 1$ there is a vertical stretch meaning that the outputs have all been multiplied by the same value. If k is negative, then all of the outputs will change signs and this will result in a reflection over the x-axis.
- $f(x)+k$ If k is positive all of the outputs are being increased by the same value and the graph of the function will move up. If k is negative, all of the outputs are being decreased by the same value and the graph of the function will move down.
- $f(x+k)$ If k is positive then all of the inputs are increasing by the same value. Since they are increasing before they are plugged into the operations of the function, the graph will move to the left. If k is negative, then all of the inputs are decreasing by the same value. Since they are decreasing before they are plugged into the

Assessing for Understanding

Example: Describe the effect of varying the parameters a, $\hbar$, and k on the shape and position of the graph of the equation$f(x) = a(x-\hbar)^2 + k$. Then compare that to the effect of varying the parameters a, h, and k on the shape and position of the graph of the equation $g(x) = a\sqrt{x-h} + k$.

Example: Describe the transformation that took place with the function transformation where $f(x) = \sqrt{x}$ is transformed to $g(x) = 2\sqrt{x+3} - 4$.

Example: Write an equation for the transformation of $f(x) = \frac{1}{x}$ after it has been translated 3 units to the right and reflected over the x-axis.

operations of the function the graph will move to the right.	

NC.M2.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent quadratic, **square root, inverse variation**, and right triangle trigonometric relationships and use them to solve problems.

Concepts and Skills

Pre-requisite

- Create and solve equations in one variable (NC.M1.A-CED.1)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Justify solving methods and each step (NC.M2.A-REI.1)

Connections

- Solve inverse variation, square root and quadratic equations (NC.M2.A-REI.2, NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Use trig ratios to solve problems (NC.M2.G-SRT.8)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equations as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
2 – Reason abstractly and quantitatively
4 – Model with mathematics
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to explain their reasoning behind their created equation.
New Vocabulary: inverse variation,

Mastering the Standard for this Unit

Comprehending the Standard	Assessing for Understanding
Students should be able to determine a correct equation or inequality to model a given context and use the model to solve problems. Focus on contexts that can be modeled with quadratic, square root, inverse variation, and right triangle trigonometric equations and inequalities. Students need to be familiar with algebraic, tabular, and graphic methods of solving equations and inequalities.	Students should be able to create one variable equations from multiple representations, including from functions. **Example:** In kickboxing, it is found that the force, *f*, needed to break a board, varies inversely with the length, *l*, of the board. Write and solve an equation to answer the following question: If it takes 5 lbs. of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long? **Example:** To be considered a 'fuel efficient' vehicle, a car must get more than 30 miles per gallon. Consider a test run of 200 miles. How many gallons of fuel can a car use and be considered 'fuel-efficient'? **Example:** The centripetal force F exerted on a passenger by a spinning amusement park ride is related to the number of seconds t the ride takes to complete one revolution by the equation $t = \sqrt{\frac{155\pi^2}{F}}$. Write and solve an equation to find the centripetal force exerted on a passenger when it takes 12 seconds for the ride to complete one revolution.

NC.M2.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent quadratic, **square root and inverse variation** relationships between quantities.

Concepts and Skills

Pre-requisite

- Create and graph equations in two variables (NC.M1.A-CED.2)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)

Connections

- Write equations for a system (NC.M2.A-CED.3)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Analyze functions for key features (NC.M2.F-IF.7)
- Build quadratic and inverse variation functions (NC.M2.F-BF.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

In this standard students are creating equations and graphs in two variables.

Focus on contexts that can be modeled with quadratic, square root and inverse variation relationships.

This standard needs to be connected with other standards where students interpret functions, generate multiple representations, solve problems, and compare functions.

Assessing for Understanding

Students should be able to create an equation from a context or representation and graph the equation.

Example: The formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r represents the radius of the circular cross-section of the cylinder and h represents the height. Given that $h = 10in$:

a) Graph the volume as it relates to the radius.
b) Graph the radius as it relates to the volume.
c) Compare the graphs. Be sure to label your graphs and use an appropriate scale.

Example: Justin and his parents are having a discussion about driving fast. Justin's parents argue that driving faster does not save as much time as he thinks. Justin lives 10 miles from school. Using the formula $r \cdot t = d$, where r is speed in miles per hour and d is the distance from school, rewrite the formula for t and graph. Do Justin's parents have a point?

NC.M2.A-CED.3

Create equations that describe numbers or relationships.

Create systems of linear, quadratic, **square root, and inverse variation** equations to model situations in context.

Concepts and Skills

Pre-requisite

- Create equations for a system of equations in context (NC.M1.A-CED.3)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Create equations in two variables (NC.M2.A-CED.2)

Connections

- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equations as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
2 – Reason abstractly and quantitatively
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to justify their created equations through unit analysis.

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

Students create systems of equations to model situations in contexts.

Contexts should be limited to linear, quadratic, square root and inverse variation equations.

This standard should be connected with NC.M2.A-REI.7 where students solve and interpret systems and with NC.M2.A-REI.11 where students understand the representation of the solutions of systems graphically.

Assessing for Understanding

Students should be able to recognize when a context requires a system of equations and create the equations of that system.

Example: The FFA has $2400 in a fund to raise money for a new tractor. They are selling trees and have determined that the number of trees they can buy to sell depends on the price of the tree p, according to the function $n(p) = \frac{2400}{p}$. Also, after allowing for profit, the number of trees that customers will purchase depends on the price which the group purchased the trees with function $c(p) = 300 - 6p$. For what price per tree will the number of trees that can be equal the number of trees that will be sold?

Example: Susan is designing wall paper that is made of several different sized squares. She is using a drawing tool for the square where she can adjust the area and the computer program automatically adjusts the side length by using the formula $s = \sqrt{A}$. The perimeter of the square can also be inputted into the computer so that the computer will automatically adjust the side length with the formula $s = \frac{p}{4}$. Susan wants to see what the design would look like if the perimeter and area of one of the squares was the same. Create a system of equations that Susan could solve so that she knows what to input into the computer to see her design. What is the side length that produces the same area and perimeter?

NC.M2.N-RN.1

Extend the properties of exponents to rational exponents.

Explain how expressions with rational exponents can be rewritten as radical expressions.

Concepts and Skills

Pre-requisite

- Rewrite algebraic expressions using the properties of exponents (NC.M1.N-RN.1)

Connections

- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)
- Justify the step in a solving process (NC.M2.A-REI.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision
7 – Look for and make use of structure
7 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to explain with mathematical reasoning how expressions with rational exponents can be rewritten as radical expressions.

Mastering the Standard for this Unit

Comprehending the Standard

The meaning of an exponent relates the frequency with which a number is used as a factor. So 5^3 indicates the product where 5 is a factor 3 times. Extend this meaning to a rational exponent, then $125^{\frac{1}{3}}$ indicates one of three equal factors whose product is 125.

Students recognize that a fractional exponent can be expressed as a radical or a root.
For example, an exponent of $\frac{1}{3}$ is equivalent to a cube root; an exponent of $\frac{1}{4}$ is equivalent to a fourth root.

Students extend the use of the power rule, $(b^n)^m = b^{nm}$ from whole number exponents i.e., $(7^2)^3 = 7^6$ to rational exponents.
They compare examples, such as $(7^{\frac{1}{2}})^2 = 7^{\frac{1}{2}*2} = 7^1 = 7$ to $(\sqrt{7})^2 = 7$ to establish a connection between radicals and rational exponents: $7^{\frac{1}{2}} = \sqrt{7}$ and, in general, $b^{\frac{1}{2}} = \sqrt{b}$.

Students can then extend their understanding to exponents where the numerator of the rational exponent is a number greater than 1. For example $7^{\frac{1}{2}*3} = 7^{\frac{3}{2}} = \sqrt{7^3} = (\sqrt{7})^3$.

Assessing for Understanding

Students should be able to use their understanding of rational exponents to solve problems.

Example: Determine the value of x

a) $64^{\frac{1}{2}} = 8^x$
b) $(12^5)^x = 12$

Students should be able to explain their reasoning when rewriting expressions with rational exponents.

Examples:

a) Write $x^{\frac{1}{5}}$ as a radical expression.
b) Write $(x^2y)^{\frac{1}{2}}$ as a radical expression.
c) Explain how the power rule of exponents, $(b^n)^m = b^{mn}$, can be used to justify why $(\sqrt[3]{b})^3 = b$.
d) Explain why $x^{\frac{2}{3}}$ is equivalent to $\sqrt[3]{x^2}$ and $(\sqrt[3]{x})^2$.

NC.M2.N-RN.2

Extend the properties of exponents to rational exponents.

Rewrite expressions with radicals and rational exponents into equivalent expressions using the properties of exponents.

Concepts and Skills

Pre-requisite

- Rewrite algebraic expressions using the properties of exponents (NC.M1.N-RN.1)
- Explain how expressions with rational expressions can be written as radical expressions (NC.M2.N-RN.1)

Connections

- Operations with polynomials (NC.M2.A-APR.1)
- Solve one variable square root equations (NC.M2.A-REI.2)
- Solve quadratic equations in one variable (NC.M2.A-REI.4a, NC.M2.A-REI.b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain their reasoning while simplifying expressions with rational exponents and radicals.

Mastering the Standard for this Unit

Comprehending the Standard

Students should be able to simplify expressions with radicals and with rational exponents.

Students should be able to rewrite expressions involving rational exponents as expressions involving radicals and simplify those expressions.

Students should be able to rewrite expressions involving radicals as expressions using rational exponents and use the properties of exponents to simplify the expressions.

Students should be able to explain their reasoning while simplifying expressions with rational exponents and radicals.

Assessing for Understanding

Students should be able to rewrite expressions with rational expression into forms that are more simple or useful.

Example: Using the properties of exponents, simplify:

a) $\left(\sqrt[4]{32^3}\right)^2$

b) $\dfrac{\sqrt[5]{b^3}}{b^{\frac{4}{3}}}$

Example: Write $\sqrt[3]{27x^2y^6z^3}$ as an expression with rational exponents.

Example: Write an equivalent exponential expression for $8^{\frac{2}{3}}$? Explain how they are equivalent.

Potential Solution: $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 2^2$

Example: Given $81^{\frac{3}{4}} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3$, which form would be easiest to calculate without using a calculator. Justify your answer?

Example: Determine whether each equation is true or false using the properties of exponents. If false, describe at least one way to make the math statement true.

a) $\sqrt{32} = 2^{\frac{5}{2}}$

b) $\sqrt{32} = 2^{\frac{5}{2}}$

c) $4^{\frac{1}{2}} = \sqrt[4]{64}$

d) $2^8 = \left(\sqrt[3]{16}\right)^6$

e) $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$

NC.M2.N-RN.3

Use properties of rational and irrational numbers.

Use the properties of rational and irrational numbers to explain why:

- the sum or product of two rational numbers is rational;
- the sum of a rational number and an irrational number is irrational;
- the product of a nonzero rational number and an irrational number is irrational.

Concepts and Skills

Pre-requisite

- Understand rational numbers (8.NS.1)

Connections

- These concepts close out the learning about the real number system.

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

Students know and justify that when

- adding or multiplying two rational numbers the result is a rational number.
- adding a rational number and an irrational number the result is irrational.
- multiplying of a nonzero rational number and an irrational number the result is irrational.

Note: Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction and the relationship between multiplication and addition.

Assessing for Understanding

Students should be able to explain the properties of rational and irrational numbers.

Example: Explain why the number 2π must be irrational.

Sample Response: *If 2π were rational, then half of 2π would also be rational, so π would have to be rational as well.*

Example: Explain why the sum of $3 + 2\pi$ must be irrational.

Example: Explain why the product of $3 \cdot \sqrt{2}$ must be irrational.

Example: Given one rational number $\frac{a}{b}$ and another rational number $\frac{r}{s}$, find the product of $\frac{a}{b} \cdot \frac{r}{s}$. Use this product to justify why the product of two rational numbers must be a rational number. Include in your justification why the number $\frac{a}{b}$ or $\frac{r}{s}$ could represent any rational number.

NC.M2.A-REI.2

Understand solving equations as a process of reasoning and explain the reasoning.

Solve and interpret one variable inverse variation and square root equations arising from a context, and explain how extraneous solutions may be produced.

Concepts and Skills

Pre-requisite

- Solve quadratic equations by taking square roots (NC.M1.A-REI.4)
- Interpret a function in context be relating it domain and range (NC.M1.F-IF.5)
- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)

Connections

- Know there is a complex number and the form of complex numbers (NC.M2.N-NC.1)
- Create and solve one variable equations (NC.M2.A-CED.1)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Use trig ratios and the Pythagorean Theorem to solve problems (NC.M2.G-SRT.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
7 – Look for and make use of structure
8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, extraneous solutions

Mastering the Standard for this Unit

Comprehending the Standard

Solve one variable inverse variations and square root equations that arise from a context.

Students should be familiar with direct variation, learned in 7th and 8th grades. Direct variations occur when two quantities are divided to produce a constant, $k = \frac{y}{x}$. This is why direct variation is linked to proportional reasoning. Indirect variations occur when two quantities are multiplied to produce a constant, $k = y \cdot x$.

Students should understand that the process of algebraically solving an equation can produce extraneous solutions. Students study this in Math 2 in connection mainly to square root functions. When teaching

Assessing for Understanding

Students should be able to solve inverse variation equations.

Example: Tamara is looking to purchase a new outdoor storage shed. She sees an advertisement for a custom built shed that fits into her budget. In this advertisement, the builder offers a 90 square foot shed with any dimensions. Tamara would like the shed to fit into her a corner of her backyard, but the width will be restricted by a tree. She remembers the formula for the area of a rectangle is $l \cdot w = a$ and solves for the width to get $w = \frac{a}{l}$. She then measures the restricted width to be 12 feet. What can be the dimensions of the shed?

Example: The relationship between rate, distance and time can be calculated with the equation $r = \frac{d}{t}$, where r is the rate (speed), d represents the distance traveled, and t represents the time. If the speed of a wave from a tsunami is 150 m/s and the distance from the disturbance in the ocean to the shore is 35 kilometers, how long will it take for the wave to reach the shore?

this standard, it will be important to link to the concept of having a limited domain, not only by the context of a problem, but also by the nature of the equation. Interpret solutions in terms of the context.	Students should be able to solve square root equations and identify extraneous solutions. **Example:** Solve algebraically: $\sqrt{x-1} = x-7$ a) Now solve by graphing. b) What do you notice? c) Check the solutions in the original equation. d) Why was an "extra" answer produced? **Example:** The speed of a wave during a tsunami can be calculated with the formula $s = \sqrt{9.81d}$ where s represents speed in meters per second, d represents the depth of the water in meters where the disturbance (for example earthquake) takes place, and 9.81 m/s^2 is the acceleration due to gravity. If the speed of the wave is 150 m/s, what is depth of the water where the disturbance took place?

NC.M2.A-REI.11

Represent and solve equations and inequalities graphically

Extend the understanding that the x-coordinates of the points where the graphs of two square root and/or inverse variation equations $y = f(x)$ *and* $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

- Understand the mathematical reasoning behind the methods of graphing, using tables and technology to solve systems and equations (NC.M1.A-REI.11)
- Create equations (NC.M2.A-CED.1, NC.M2.A-CED.3)

Connections

- Solve systems of equations (NC.M2.A-REI.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to discuss how technology impacts their ability to solve more complex equations or unfamiliar equation types.

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard for this Unit

Comprehending the Standard

Students understand that they can solve a system of equations by graphing and finding the point of intersection of the graphs. At this point of intersection the outputs $f(x)$ and $g(x)$ are the same when both graphs have the same input, x.

Students also understand why they can solve any equation by graphing both sides separately and looking for the point of intersection.

In addition to graphing, students can look at tables to find the value of x that makes $f(x) = g(x)$.

Assessing for Understanding

Students should be able to solve complex equations and systems of equations.

Example: Given the following equations determine the x-value that results in an equal output for both functions.

$$f(x) = \sqrt{3x-2}$$
$$g(x) = \sqrt{x+2}$$

Example: Solve for x by graphing or by using a table of values.

$$\frac{1}{x} = \sqrt{2x+3}$$

The North Carolina High School Collaborative Instructional Framework

NC Math 2

Unit 5: Relationships in Triangles

13 Days Block Schedule

September 2017 Update

26 Days Traditional Schedule

RESEARCH BRIEF: Trigonometry

Essential Questions:

- How can we use the definitions of trigonometric ratios to solve problems?
- How do angles of depression and elevation relate when solving problems involving right triangles?
- How can we use theorems about triangles to solve problems?

Learning Outcomes	Student Objectives
• Students will be able to prove the triangle angle sum theorem and the exterior angle theorem and use those relationships to solve problems. • Students will be able to explore basic properties of sine, cosine, and tangent functions with reference to their interrelationships and their patterns of change as the angle measure changes. • Students will be able to determine values of the sine, cosine, and tangent functions of an angle in standard position in a coordinate plane. • Students will be able to determine the sine, cosine, and tangent of an acute angle in a right triangle, and determine the angle given one of those ratios. • Students will be able to solve problems involving indirect measurement that can be modeled as parts of a right triangle.	• I will **prove** the triangle angle sum theorem and the exterior angle theorem. • I will **use** relationships in triangles, such as the triangle angle sum theorem and the exterior angle theorem, to solve problems. • I will **explore** basic properties of sine, cosine, and tangent functions with reference to their interrelationships and their patterns of change as the angle measure changes. • I will **determine** values of the sine, cosine, and tangent functions of an angle in standard position in a coordinate plane. • I will **determine** the sine, cosine, and tangent of an acute angle in a right triangle, and determine the angle given one of those ratios. • I will **solve** problems involving indirect measurement that can be modeled as parts of a right triangle.

- Students will be able to use relationships in right triangles, such as trigonometric ratios, angles of elevation and depression, and the Pythagorean Theorem to solve problems.
- Students will be able to use the relationships in the special right triangles (45-45-90 triangles and 30-60-90) to solve problems.

- I will **use** relationships in right triangles, such as trigonometric ratios, angles of elevation and depression, and the Pythagorean Theorem to solve problems.
- I will **apply** the relationships within 45-45-90 triangles and 30-60-90 triangles to solve problems.

Standards Addressed in this Unit

Understand, prove, and use properties of triangles to solve problems.

- **NC.M2.G.CO.10**: (first and second bullets) Prove theorems about triangles and use them to prove relationships in geometric figures including:
    - The sum of the measures of the interior angles of a triangle is 180 degrees.
    - An exterior angle of a triangle is equal to the sum of its remote interior angles.

- **NC.M2.G.SRT.4**: (second bullet) Use similarity to prove theorems about triangles. Use theorems about triangles to prove relationships in geometric figures.
    - Use the Pythagorean Theorem.

Use proportional reasoning to develop relationships between corresponding parts of similar triangles. Use these relationships to solve problems.

- **NC.M2.G-SRT.12**: Define trigonometric ratios and solve problems involving right triangles. Develop properties of special right triangles (45-45-90 and 30-60-90) and use them to solve problems.

- **NC.M2.G.SRT.6**: Verify experimentally that the side ratios in similar right triangles are properties of the angle measures in the triangle, due to the preservation of angle measure in similarity. Use this discovery to develop definitions of the trigonometric ratios for acute angles.

- **NC.M2.A.SSE.1a**: Identify and interpret parts of a quadratic, square root, inverse variation, or right triangle trigonometric expression, including terms, factors, coefficients, radicands, and exponents.

- **NC.M2.A.CED.1**: Create equations and inequalities in one variable that represent quadratic, square root, inverse variation, and right triangle trigonometric relationships and use them to solve problems.

- **NC.M2.G.SRT.8**: Use trigonometric ratios and the Pythagorean Theorem to solve problems involving right triangles in terms of a context.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M2.G-CO.10 (first and second bullets)

Prove geometric theorems.

Prove theorems about triangles and use them to prove relationships in geometric figures including:

- The sum of the measures of the interior angles of a triangle is 180º.
- An exterior angle of a triangle is equal to the sum of its remote interior angles.

Concepts and Skills

Pre-requisite

- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)
- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)
- Use triangle congruence to prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)

Connections

- Verify experimentally, properties of the centers of triangles (NC.M3.G-CO.10)
- Prove theorems about parallelograms (NC.M3.G-CO.11)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

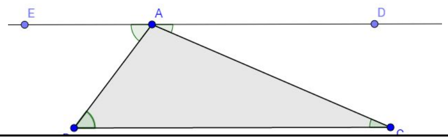
Comprehending the Standard

Encourage multiple ways of writing proofs, such as *narrative paragraphs* and *flow diagrams*. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of transparencies and dynamic geometry software can be important tools for helping students conceptually understand important geometric concepts.

Example Proofs:

Triangle Angle Sum Theorem



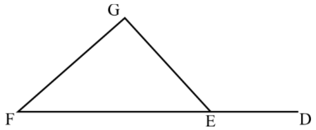
Given ΔABC, prove that the $m\angle A + m\angle B + m\angle C = 180°$.

Draw $line\ ED$ through point A, parallel to $line\ BC$. Since $line\ ED$ and $line\ BC$ are parallel, alternate interior angles

Assessing for Understanding

Students can prove theorems about triangles.

are congruent. Therefore, $\angle DAC \cong \angle ACB$ and $\angle EAB \cong \angle ABC$. By Angle Addition Postulate, $\angle EAB + \angle BAC + \angle DAC = \angle EAD$. Since $\angle EAD$ is a straight angle, its measure is 180°. Therefore $m\angle EAB + m\angle BAC + m\angle DAC = 180°$. Thus, the sum of the measures of the interior angles of a triangle is 180°.

<table>
<tr><td rowspan="5">Exterior Angle Theorem
Given the figure on the right, prove $m\angle EFG + m\angle FGE = m\angle DEG$
 </td><th>Statement</th><th>Reason</th></tr>
<tr><td>$m\angle DEG + m\angle GEF = 180^\circ$</td><td>Two angles that form a straight line are supplementary.</td></tr>
<tr><td>$m\angle EFG+m\angle FGE+m\angle GEF =180^\circ$</td><td>Sum of angles in a triangle is 180º</td></tr>
<tr><td>$m\angle EFG+m\angle FGE+m\angle GEF = m\angle DEG+m\angle GEF$</td><td>Substitution as both sums equal 180º.</td></tr>
<tr><td>$m\angle EFG+m\angle FGE=m\angle DEG$</td><td>Subtract $m\angle GEF$ from both sides of equation</td></tr>
</table>

NC.M2.G-SRT.4 (second bullet)

Prove theorems involving similarity.

Use similarity to solve problems and to prove theorems about triangles. Use theorems about triangles to prove relationships in geometric figures.

- The Pythagorean Theorem

Concepts and Skills

Pre-requisite

- Use transformations for the AA criterion for triangle similarity (NC.M2.G-SRT.3)

Connections

- Use trig ratios and the Pythagorean Theorem in right triangles (NC.M2.G-SRT.8)
- Derive the equation of a circle given center and radius using the Pythagorean Theorem (NC.M3.G-GPE.1)
- Prove theorems about parallelograms (NC.M3.G-CO.11)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)
- Understand apply theorems about circles (NC.M3.G-C.2)
- Use similarity to demonstrate that the length of the arc is proportional to the radius of the circle (NC.M3.G-C.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

2 – Reason abstractly and quantitatively

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard for this Unit

Comprehending the Standard

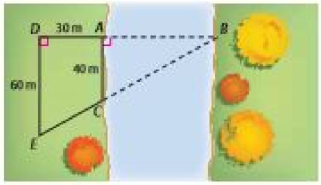
Students use the concept of similarity to solve problem situations (e.g., indirect measurement, missing side(s)/angle measure(s)). Students use the properties of dilations to prove that a line parallel to one side of a triangle divides the other two sides proportionally (often referred to as side-splitter theorem) and its converse.

The altitude from the right angle is drawn to the hypotenuse, which creates three similar triangles. The proportional relationships among the sides of these three triangles can be used to derive the Pythagorean relationship.

Assessing for Understanding

Students use similarity to prove the Pythagorean Theorem.

Example: Calculate the distance across the river, AB.



NC.M2.G-SRT.12

Define trigonometric ratios and solve problems involving right triangles.

Develop properties of special right triangles (45-45-90 and 30-60-90) and use them to solve problems.

Concepts and Skills

Pre-requisite

- Use similarity to prove The Triangle Proportionality Theorem and the Pythagorean Theorem (NC.M2.G-SRT.4)

Connections

- Verify experimentally that side ratios in similar right triangles are properties of the angle measures and use to define trig ratios (NC.M2.G-SRT.6)
- Use trig ratios and the Pythagorean Thm to solve problems (NC.M2.G-SRT.8)
- Understand apply theorems about circles (NC.M3.G-C.2)
- Build an understanding of trigonometric functions (NC.M3.F-TF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

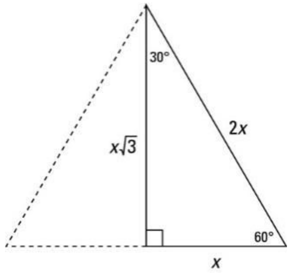
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

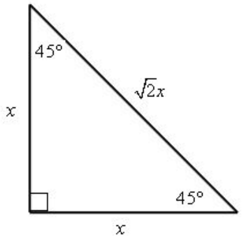
New Vocabulary: sine, cosine, tangent

Mastering the Standard for this Unit

Comprehending the Standard

By drawing the altitude to one side of an equilateral triangle, students form two congruent $30° - 60° - 90°$ triangles. Starting with an initial side length of $2x$, students use the Pythagorean Theorem to develop relationships between the sides of a $30° - 60° - 90°$ triangle.





Students begin by drawing an isosceles right triangle with leg length of x. Using the Isosceles Triangle Theorem, the Triangle Angle Sum Theorem, and the Pythagorean Theorem students develop and justify relationships between the sides of a $45° - 45° - 90°$ triangle.

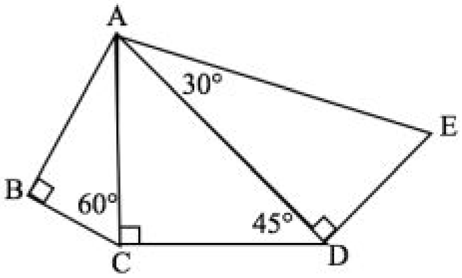
In Math 3, this relationship can be revisited with quadrilaterals by drawing the diagonal of a square to create two congruent $45° - 45° - 90°$ triangles. Using the properties of the diagonal and the Pythagorean Theorem, these relationships can be established in a different manner.

Assessing for Understanding

Students can solve problems involving special right triangles.

Example: The Garden Club at Heritage High wants to build a flower garden near the outdoor seating at the back of the school. The design is a square with diagonal walkways. The length of each side of the garden is 50 ft. How long is each walkway?

Example: If $AB = 8\sqrt{3}$, find AE.



NC.M2.G-SRT.6

Define trigonometric ratios and solve problems involving right triangles.

Verify experimentally that the side ratios in similar right triangles are properties of the angle measures in the triangle, due to the preservation of angle measure in similarity. Use this discovery to develop definitions of the trigonometric ratios for acute angles.

Concepts and Skills

Pre-requisite

- Determining similarity by a sequence of transformations; use the properties of dilations to show that two triangles are similar if their corresponding sides are proportional and their corresponding angles are congruent (NC.M2.G-SRT.2)

Connections

- Develop properties of special right triangles (NC.M2.G-SRT.12)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

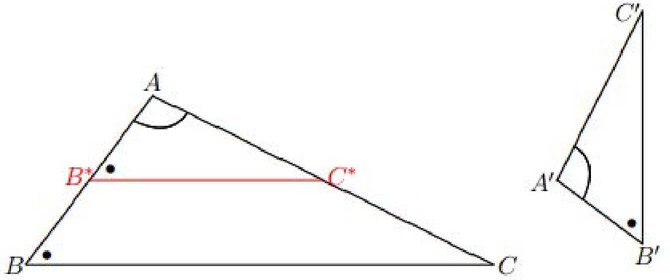
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: sine, cosine, tangent

Mastering the Standard for this Unit

Comprehending the Standard

Students establish that the side ratios of a right triangle are equivalent to the corresponding side ratios of *similar* right triangles and are a function of the acute angle(s). Because all right triangles have a common angle, the right angle, if two right triangles have an acute angle in common (i.e. of the same measure), then they are similar by the AA criterion. Therefore, their sides are proportional.

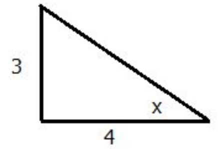


We define the ratio of the length of the side opposite the acute angle to the length of the side adjacent to the acute angle as the tangent ratio. Note that the tangent ratio corresponds to the slope of a line passing through the origin at an angle to the x-axis that equals the measure of the acute angle. For example, in the diagram below, students can see that

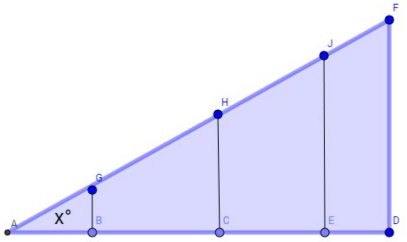
Assessing for Understanding

Students can use proportional reasoning to develop definitions of the trigonometric ratios of acute angles.

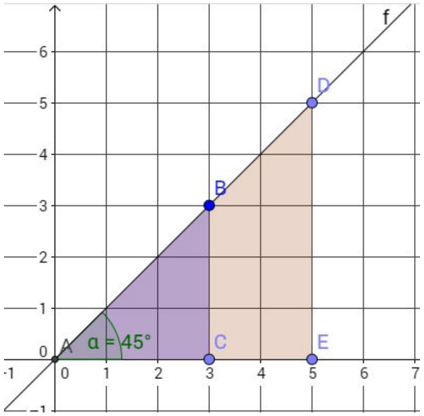
Example: Find the sine, cosine, and tangent of x.



Example: Explain why the sine of $x°$ is the same regardless of which triangle is used to find it in the figure below.



the tangent of 45° is 1, since the slope of a line passing through the origin at a 45° angle is 1. Using this visual, it is also easy to see that the slope of lines making an angle less than 45° will be less than 1; therefore the tangent ratio for angles between 0° and 45° is less than 1. Similarly, the slope of lines making an angle greater than 45° will be greater than 1; therefore, the tangent ratio for angles between 45° and 90° will be greater than 1.



Connect with 8.EE.6 “Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane.”

We define the ratio of the length of the side opposite the acute angle to the length of the hypotenuse as the sine ratio.

We define the ratio of the length of the side adjacent to the acute angle to the length of the hypotenuse as the cosine ratio.

NC.M2.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a quadratic, square root, inverse variation, or right triangle trigonometric expression, including terms, factors, coefficients, radicands, and exponents.

Concepts and Skills

Pre-requisite

- Interpreting parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Creating equation to solve, graph, and make systems (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Solve and interpret one variable inverse variation and square root equations (NC.M2.A-REI.2)
- Interpreting functions (NC.M2.F-IF.4, NC.M2.F-IF.7, NC.M2.F-IF.9)
- Understand the effect of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.
4 – Model with mathematics
7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, right triangle trigonometry

Mastering the Standard for this Unit

Comprehending the Standard

When given an expression with a context, students should be able to explain how the parts of the expression relate to the context of the problem.

Students should be able to write equivalent forms of an expression to be able to identify parts of the expression that can relate to the context of the problem.

The parts of expressions that students should be able to interpret include any terms, factors, coefficients, radicands, and exponents.

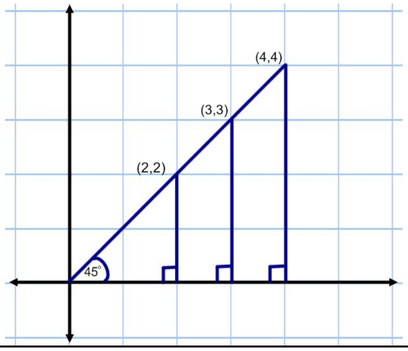
Students should be given contexts that can be modeled with quadratic, square root, inverse variation, or right triangle trigonometric expressions.

Assessing for Understanding

Students should be able to identify and interpret parts of an expression in its context.

Example: The tangent ratio is $\frac{y}{x}$ where (x, y) is a coordinate on the terminal side of the angle in standard position. Use the diagram to justify why the tangent of $45°$ is always 1. Then, expand that reasoning to justify why every individual angle measure has exactly one value for tangent.

Use similar reasoning to justify why every angle has exactly one value of sine and one value of cosine.



NC.M2.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent quadratic, square root, inverse variation, and **right triangle trigonometric** relationships and use them to solve problems.

Concepts and Skills

Pre-requisite

- Create and solve equations in one variable (NC.M1.A-CED.1)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Justify solving methods and each step (NC.M2.A-REI.1)

Connections

- Solve inverse variation, square root and quadratic equations (NC.M2.A-REI.2, NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Use trig ratios to solve problems (NC.M2.G-SRT.8)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equations as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
2 – Reason abstractly and quantitatively
4 – Model with mathematics
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to explain their reasoning behind their created equation.
New Vocabulary: inverse variation, right triangle trigonometry

Mastering the Standard for this Unit

Comprehending the Standard

Students should be able to determine a correct equation or inequality to model a given context and use the model to solve problems.

Focus on contexts that can be modeled with quadratic, square root, inverse variation, and right triangle trigonometric equations and inequalities.

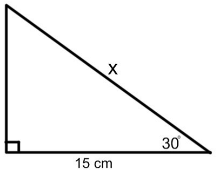
Students need to be familiar with algebraic, tabular, and graphic methods of solving equations and inequalities.

Assessing for Understanding

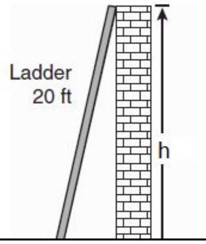
Students should be able to create one variable equations from multiple representations, including from functions

Students should be able to create equations using right triangle trigonometry.

Example*:* Write and solve an equation to find the hypotenuse of the following triangle.



Example: John has a 20-foot ladder leaning against a wall. If the height of the wall that the ladder needs to reach is at least 15ft, create and solve an inequality to find the angle the ladder needs to make with the ground.



NC.M2.G-SRT.8

Define trigonometric ratios and solve problems involving right triangles.

Use trigonometric ratios and the Pythagorean Theorem to solve problems involving right triangles in terms of a context.

Concepts and Skills

Pre-requisite

- Use similarity to prove The Triangle Proportionality Theorem and the Pythagorean Theorem (NC.M2.G-SRT.4)

Connections

- Develop properties of special right triangles (NC.M2.G-SRT.12)
- Understand apply theorems about circles (NC.M3.G-C.2)
- Build an understanding of trigonometric functions (NC.M3.F-TF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 - Model with mathematics (contextual situations are required)

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: sine, cosine, tangent

Mastering the Standard for this Unit

Comprehending the Standard	Assessing for Understanding
This standard is an application standard where students use the Pythagorean Theorem, learned in MS, and trigonometric ratios to solve application problems involving right triangles, including angle of elevation and depression, navigation, and surveying.	Students can use trig ratios and the Pythagorean theorem to find side lengths and angle measures in right triangles. **Example:** Find the height of a flagpole to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the flagpole is 50 feet. **Example:** A new house is 32 feet wide. The rafters will rise at a 36° angle and meet above the centerline of the house. Each rafter also needs to overhang the side of the house by 2 feet. How long should the carpenter make each rafter?

The North Carolina High School Collaborative Instructional Framework

NC Math 2
Unit 6: Probability

10 Days Block Schedule	**September 2017 Update**	20 Days Traditional Schedule

RESEARCH BRIEF: Probability

Essential Questions:
- How do I use the laws of probability to answer questions about real world phenomena?
- How can you find and organize the probabilities associated with random events?
- Under what conditions can you add individual probabilities to find the probability that a related event happens?
- How can you calculate the probability that two events both happen using the individual probabilities?
- How can you find probabilities in situations with conditions?
- How do you find probability of two events occurring when the events are not independent?

Learning Outcomes	**Student Objectives**
• Students will construct sample spaces for chance situations involving equally likely outcomes. • Students will construct probability distributions from sample spaces. • Students will determine whether events are mutually exclusive (disjoint). • Students will calculate P(A or B) using the Addition Rule or its special case for mutually exclusive events. • Students will determine whether events are independent. • Students will use an area model to find the probability that two independent events both occur.	• I will **construct** sample spaces for chance situations involving equally likely outcomes. • I will **construct** probability distributions from sample spaces. • I will determine whether events are mutually exclusive (disjoint). • I will **calculate** P(A or B) using the Addition Rule or its special case for mutually exclusive events. • I will **determine** whether events are independent. • I will **use** an area model to find the probability that two independent events both occur. • I will **use** the Multiplication Rule to calculate the probability that two independent events occur. • I will **find** conditional probabilities.

- Students will use the Multiplication Rule to calculate the probability that two independent events occur.
- Students will find conditional probabilities.
- Students will use the Multiplication Rule to calculate the probability that two events both occur when the events are not independent.

- I will **use** the Multiplication Rule to calculate the probability that two events both occur when the events are not independent.

Standards Addressed in this Unit

Understand, explain, and use conditional probabilities, the addition rule for probabilities, and the multiplication rules for probabilities.

- **NC.M2.S.IC.2**: Use simulation to determine whether the experimental probability generated by sample data is consistent with the theoretical probability based on known information about the population.

- **NC.M2.S.CP.1**: Describe events as subsets of the outcomes in a sample space using characteristics of the outcomes or as unions, intersections and complements of other events.

- **NC.M2.S.CP.3a**: Develop and understand independence and conditional probability.
    a. Use a two-way table to develop an understanding of the conditional probability of a given B (written P(A|B)) as the likelihood that A will occur given that B has occurred. That is, P(A|B) is the fraction of event B's outcomes that also belong to event A.

- **NC.M2.S.CP.3b**: Develop and understand independence and conditional probability.
    b. Understand that event A is independent from event B if the probability of event A does not change in response to the occurrence of event B. That is P(A|B) = P(A).

- **NC.M2.S.CP.4**: Represent data on two categorical variables by constructing a two-way frequency table of data. Interpret the two-way table as a sample space to calculate conditional, joint, and marginal probabilities. Use the table to decide if events are independent.

- **NC.M2.S.CP.5**: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

- **NC.M2.S.CP.6**: Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in context.

- **NC.M2.S.CP.7**: Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in context.
- **NC.M2.S.CP.8**: Apply the general Multiplication Rule P(A and B) = P(A)P(A|B), and interpret the answer in context. Include the case where A and B are independent: P(A and B) = P(A)P(B)

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

NC.M2.S-IC.2

Understand and evaluate random processes underlying statistical experiments

Use simulation to determine whether the experimental probability generated by sample data is consistent with the theoretical probability based on known information about the population.

Concepts and Skills

Pre-requisite

- Random sampling can be used to support valid inferences if the sample is representative of the population (7.SP.1)
- Approximate probabilities by collecting data and observing long-run frequencies (7.SP.6)

Connections

- Use simulation to understand how samples are used to estimate population means/proportions and how to determine margin of error (NC.M3.S-IC.4)
- Use simulation to determine whether observed differences between samples indicates actual differences in terms of the parameter of interest (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with Mathematics
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – simulation, experimental probability, theoretical probability

Mastering the Standard for this Unit

Comprehending the Standard

This standard is an expansion of MS (7th grade) where students approximate the probability of a chance event by collecting data and observing long-run relative frequencies of chance phenomenon. In the middle grades work, students understand that increasing the size of the trial yields results that are pretty consistent with the theoretical probability model. They also understand that randomization is an important element of sampling and that samples that reflect the population can be used to make inferences about the population.

This standard is extended to the idea of increasing the number of samples collected and examining the results of more samples opposed to larger sample sizes. This standard uses simulation to build an understanding of how taking more samples of the same size can be used to make predictions about the population of interest. Simulation can be used to mock real-world experiments. It is time saving and provides a way for students to conceptually understand and explain random phenomenon.

It is suggested at this level for students to conduct simulation using tactile tools and methods. Cards, number cubes, spinners, colored tiles and other common items are excellent tools for performing simulation. Technology can be used to compile and analyze the results, but should not be used to perform simulations at this level.

Assessing for Understanding

Students explain how well and why a sample represents the variable of interest from a population.

> **Example:** Multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability?

NC.M2.S-CP.1

Understand independence and conditional probability and use them to interpret data.

Describe events as subsets of the outcomes in a sample space using characteristics of the outcomes or as unions, intersections and complements of other events.

Concepts and Skills

Pre-requisite

- Find probabilities of compound events using lists, tables, tree diagrams and simulations (7.SP.8)

Connections

- Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b)
- Use the rules of probability to compute probabilities (NC.M2.S-CP.6, NC.M2.S-CP.7, NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – subset, union, intersections, complements

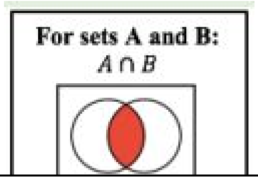
Mastering the Standard for this Unit

Comprehending the Standard

In MS (7th grade) students collect data to approximate relative frequencies of probable events. They use the information to understand theoretical probability models based on long-run relative frequency. This allows students to assign probability to simple events, therefore students develop the understanding for sample space as the collection of all possible outcomes. Additionally, MS students develop probability models for compound events using lists tables, tree diagrams and simulations.

This standard builds on the MS work by formalizing probability terminology associated with simple and compound events and using characteristics of the outcomes:

- The **intersection** of two sets A and B is the set of elements that *are common to both* set A and set B. It is denoted



Assessing for Understanding

Students define a sample space and events within the sample space.

Example: Describe the sample space for rolling two number cubes.
For the teacher: *This may be modeled well with a 6x6 table with the rows labeled for the first event and the columns labeled for the second event.*

Example: Describe the sample space for picking a colored marble from a bag with red and black marbles.
For the teacher*: This may be modeled with set notation.*

Example: Andrea is shopping for a new cellphone. She is either going to contract with Verizon (60% chance) or with Sprint (40% chance). She must choose between an Android phone (25% chance) or an IPhone (75% chance). Describe the sample space. ***For the teacher:*** *This may be modeled well with an area model.*

Example: The 4 aces are removed from a deck of cards. A coin is tossed and one of the aces is chosen. Describe the sample space. ***For the teacher:*** *This may be modeled well with a tree diagram.*

Students establish events as subsets of a sample space. An event is a subset of a sample space.

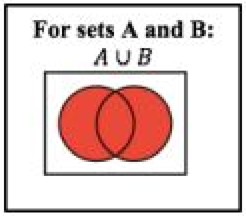
Example: Describe the event of rolling two number cubes and getting evens.

Example: Describe the event of pulling two marbles from a bag of red/black marbles.

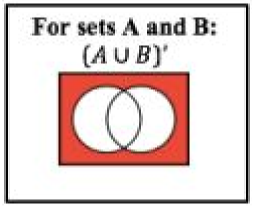
Example: Describe the event that the summing of two rolled number cubes is larger than 7 and even, and contrast it with the event that the sum is larger than 7 or even.

by $A \cap B$ and is read “A intersection B”

- The **union** of two sets A and B is the set of elements, which are *in A or in B, or in both*. It is denoted by $A \cup B$, and is read “A union B”



- The **complement** of the set is the set of elements that are members of the universal set but *are not in* It is denoted by ($A \cup B$)’



Example: If the subset of outcomes for choosing one card from a standard deck of cards is the intersection of two events: {queen of hearts, queen of diamonds}.

1. Describe the sample space for the experiment.
2. Describe the subset of outcomes for the union of two events.

NC.M2.S-CP.3a

Understand independence and conditional probability and use them to interpret data.

Develop and understand independence and conditional probability.

a. Use a 2-way table to develop understanding of the conditional probability of A given B (written P(A|B)) as the likelihood that A will occur given that B has occurred. That is, P(A|B) is the fraction of event B's outcomes that also belong to event A.

Concepts and Skills

Pre-requisite

- Understand patterns of association from two-way tables in bivariate categorical data (8.SP.4)

Connections

- Represent data on two categorical by constructing two-way frequency tables of data and use the table to determine independence (NC.M2.S-CP.4)
- Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5)
- Find conditional probabilities and interpret in context (NC.M2.S-CP.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
6 – Attend to precision

Disciplinary Literacy

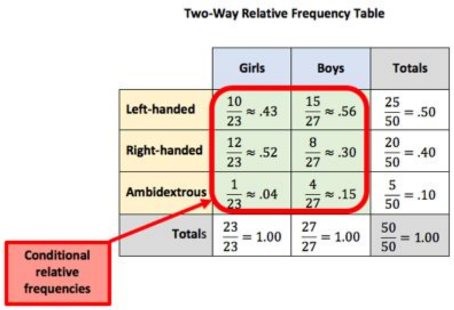
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – independence, conditional probability

Mastering the Standard for this Unit

Comprehending the Standard

Students created two-way tables of categorical data and used them to examine patterns of association in MS. They also displayed frequencies (counts) and relative frequencies (percentages) in two-way tables. This standard uses two-way tables to establish an understanding for conditional probability, that is given the occurrence of one event the probability of another event occurs.



Two-Way Relative Frequency Table

	Girls	Boys	Totals
Left-handed	$\frac{10}{23} \approx .43$	$\frac{15}{27} \approx .56$	$\frac{25}{50} = .50$
Right-handed	$\frac{12}{23} \approx .52$	$\frac{8}{27} \approx .30$	$\frac{20}{50} = .40$
Ambidextrous	$\frac{1}{23} \approx .04$	$\frac{4}{27} \approx .15$	$\frac{5}{50} = .10$
Totals	$\frac{23}{23} = 1.00$	$\frac{27}{27} = 1.00$	$\frac{50}{50} = 1.00$

a

The rows/columns determine the ***condition***. Using the example above, the probability that you select left-handed person, given

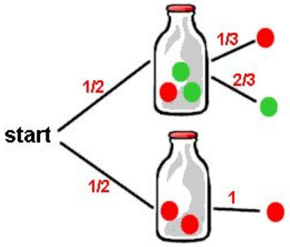
Assessing for Understanding

Students can use two-way tables to find conditional probabilities.

		Curfew		
		Yes	**No**	**Total**
Chores	**Yes**	51	24	75
	No	30	12	42
	Total	81	36	117

Example: Each student in the Junior class was asked if they had to complete chores at home and if they had a curfew. The table represents the data.

a. What is the probability that a student who has chores also has a curfew?
b. What is the probability that a student who has a curfew also has chores?
c. Are the two events have chores and have a curfew independent? Explain.

that it is a girl is the number of left-handed girls divided by the total number of girls → $P(Girl) = \frac{10}{23} \approx .43$. The ***condition*** in this problem is a ***girl*** therefore, the number of girls represents the total of the conditional probability.	Students understand conditional probability as the probability of A occurring given B has occurred. **Example:** What is the probability that the sum of two rolled number cubes is 6 given that you rolled doubles? **Example:** There are two identical bottles. A bottle is selected at random and a single ball is drawn. Use the tree diagram at the right to determine each of the following: a. $P\ (red \mid bottle\ 1)$ b. $P\ (red \mid bottle\ 2)$ 

NC.M2.S-CP.3b

Understand independence and conditional probability and use them to interpret data.

Develop and understand independence and conditional probability.

b. Understand that event A is independent from event B if the probability of event A does not change in response to the occurrence of event B. That is P(A|B) = P(A).

Concepts and Skills

Pre-requisite

- Understand patterns of association from two-way tables in bivariate categorical data (8.SP.4)

Connections

- Represent data on two categorical by constructing two-way frequency tables of data and use the table to determine independence (NC.M2.S-CP.4)
- Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5)
- Apply the general Multiplication Rule, including when *A* and *B* are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – independence, conditional probability

Mastering the Standard for this Unit

Comprehending the Standard

Assessing for Understanding

Students can use two-way tables to find conditional probabilities.

		Curfew		
		Yes	**No**	**Total**
Chores	**Yes**	51	24	75
	No	30	12	42
	Total	81	36	117

Example: Each student in the Junior class was asked if they had to complete chores at home and if they had a curfew. The table represents the data. Are the two events have chores and have a curfew independent? Explain.

NC.M2.S-CP.4

Understand independence and conditional probability and use them to interpret data.

Represent data on two categorical variables by constructing a two-way frequency table of data. Interpret the two-way table as a sample space to calculate conditional, joint and marginal probabilities. Use the table to decide if events are independent.

Concepts and Skills

Pre-requisite

- Understand patterns of association from two-way tables in bivariate categorical data (8.SP.4)

Connections

- Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b)
- Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5)
- Apply the general Multiplication Rule, including when *A* and *B* are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – joint probabilities, marginal probabilities

Mastering the Standard for this Unit

Comprehending the Standard

This standard builds upon the study of bivariate categorical data from MS. This standard supports data analysis from the statistical process.

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. **Analyze the data with appropriate methods.**
4. Interpret results and draw valid conclusions.

Students created two-way tables of categorical data and used them to examine patterns of association in 8th grade. They also displayed frequencies (counts) and relative frequencies (percentages) in two-way tables. Additionally, students have determined the sample space of simple and compound events in 7th grade. This standard expands on both of the 7th and 8th grade concepts to using the table to determine independence of two events.

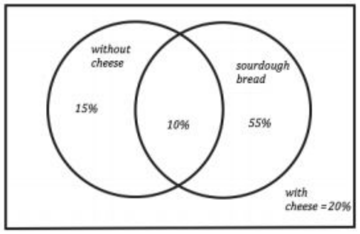
Assessing for Understanding

Students can create a two-way frequency table for data and calculate probabilities from the table.

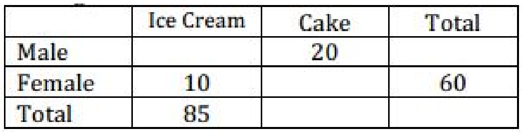
Example: Collect data from a random sample of students in your school on their favorite subject among math, science, history, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

Students can use a two-way table to evaluate independence of two variables.

Example: The Venn diagram to the right shows the data collected at a sandwich shop for the last six months with respect to the type of bread people ordered (sourdough or wheat) and whether or not they got cheese on their sandwich. Use the diagram to construct a two-way frequency table and then answer the following questions.



a. *P* (sourdough)
b. *P* (cheese | wheat)
c. *P* (without cheese or sourdough)
d. Are the events "sourdough" and "with cheese" independent events? Justify your reasoning.



	Ice Cream	Cake	Total
Male		20	
Female	10		60
Total	85		

	Example: Complete the two-way frequency table at the right and develop three conditional statements regarding the data. Determine if there are any set of events that independent. Justify your conclusion.

NC.M2.S-CP.5

Understand independence and conditional probability and use them to interpret data.

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Concepts and Skills

Pre-requisite

-

Connections

- Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b)
- Find conditional probabilities and interpret in context (NC.M2.S-CP.6)
- Apply the general Multiplication Rule, including when A and B are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard for this Unit

Comprehending the Standard

This standard is about helping students make meaning of data and statistical questions. It is about communicating in their own language what the data/graphs/information is "saying."

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. Analyze the data with appropriate methods.
4. **Interpret results and draw valid conclusions.**

This standard supports the idea of helping students to process the information around them presented in different formats or combination of formats (graphs, tables, narratives with percentages, etc.)

Assessing for Understanding

Students can use everyday language to determine if two events are dependent.

Example: Felix is a good chess player and a good math student. Do you think that the events "being good at playing chess" and "being a good math student" are independent or dependent? Justify your answer.

Example: Juanita flipped a coin 10 times and got the following results: T, H, T, T, H, H, H, H, H, H. Her math partner Harold thinks that the next flip is going to result in tails because there have been so many heads in a row. Do you agree? Explain why or why not.

Students can explain conditional probability using everyday language.

Example: A family that is known to have two children is selected at random from amongst all families with two children. Josh said that the probability of having two boys is $\frac{1}{3}$. Do you agree with Josh? Why or why not? Explain how you arrived at your answer?

NC.M2.S-CP.6

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in context.

Concepts and Skills

Pre-requisite

- Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b)

Connections

- Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5)
- Apply the general Multiplication Rule, including when A and B are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

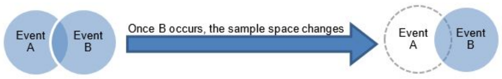
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard for this Unit

Comprehending the Standard

This standard should build on conditional probability and lead to the introduction of the addition and general multiplication rules of probability. Venn diagrams and/or tables of outcomes should serve as visual aids to build to the rules for computing probabilities of compound events.

The sample space of an experiment can be modeled with a Venn diagram such as:



So, the $P(B) = \frac{P(A \text{ and } B)}{P(B)}$

Assessing for Understanding

Students can find the conditional probability of compound events.

Example: If a balanced tetrahedron with faces 1, 2, 3, 4 is rolled twice.

(A): Sum is prime

(B): A 3 is rolled on at least one of the rolls.

a. Create a table showing all possible outcomes (sample space) for rolling the two tetrahedron.
b. What is the probability that the sum is prime (A) of those that show a 3 on at least one roll (B)?
c. Use the table to support the answer to part (b).

Example: Peter has a bag of marbles. In the bag are 4 white marbles, 2 blue marbles, and 6 green marbles. Peter randomly draws one marble, sets it aside, and then randomly draws another marble. What is the probability of Peter drawing out two green marbles? *Note: Students must recognize that this a conditional probability P(green | green).*

Example: A teacher gave her class two quizzes. 30% of the class passed both quizzes and 60% of the class passed the first quiz. What percent of those who passed the first quiz also passed the second quiz?

NC.M2.S-CP.7

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in context.

Concepts and Skills

Pre-requisite

- Describe events as subsets of the outcomes in a sample space based on characteristics of the outcomes or as unions, intersections or complements of other events (NC.M2.S-CP.1)

Connections

- Apply the general Multiplication Rule, including when A and B are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

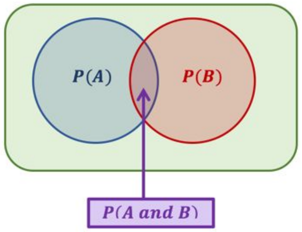
Mastering the Standard for this Unit

Comprehending the Standard

Students should apply the addition rule for computing probabilities of compound events and interpret them in context. Students should understand $P\ (A \text{ or } B)$ **OR** $P(A \cup B)$ to mean all elements of A and all elements of B excluding all elements shared by A and B.

The Venn diagram shows that when you include everything in both sets the middle region is included twice, therefore you must subtract the intersection region out once. The probability for calculating **joint** events is…

$$P\ (A \text{ or } B) = P(A) + P(B) - P\ (A \text{ and } B)$$



Students may recognize that if two events A and B are mutually exclusive, also called **disjoint,** the rule can be simplified to $P\ (A \text{ or } B) = P\ (A)\ +\ P\ (B)$ since for mutually exclusive events $P\ (A \text{ and } B) = 0$.

Assessing for Understanding

Students can apply the general addition rule for calculating conditional probabilities.

Example: Given the situation of drawing a card from a standard deck of cards, calculate the probability of the following:

a. Drawing a red card or a king
b. Drawing a ten or a spade
c. Drawing a four or a queen

Example: In a math class of 32 students, 18 boys and 14 are girls. On a unit test, 5 boys and 7 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

NC.M2.S-CP.8

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Apply the general Multiplication Rule P (A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in context. Include the case where A and B are independent: P (A and B) = P(A) P(B).

Concepts and Skills

Pre-requisite

- Describe events as subsets of the outcomes in a sample space based on characteristics of the outcomes or as unions, intersections or complements of other events (NC.M2.S-CP.1)

Connections

- Apply the Addition Rule and interpret in context (NC.M2.S-CP.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard for this Unit

Comprehending the Standard

Students should understand $P(A\ and\ B)$ **OR** $P(A \cap B)$ to mean all elements of A that are also elements of B excluding all elements shared by A and B. Two events must be ***independent*** to apply the general multiplication rule

$P(A\ and\ B) = P(A)P(A) = P(B)P(A|B)$

The general rule can be explained based on the definitions of independence and dependence. Events are either independent or dependent.

- Two events are said to be **independent** if the occurrence of one event does not affect the probability of the occurrence of the other event.
- Two events are **dependent** if the occurrence of one event does, in fact, affect the probability of the occurrence of the other event.

Sampling with and without replacement are opportunities to model independent and dependent events.

Assessing for Understanding

Students can apply the general multiplication rule for computing conditional probabilities.

Example: You have a box with 3 blue marbles, 2 red marbles, and 4 yellow marbles. You are going to pull out one marble, record its color, put it back in the box and draw another marble. What is the probability of pulling out a red marble followed by a blue marble?

Example: Consider the same box of marbles as in the previous example. However, in this case, we are going to pull out the first marble, leave it out, and then pull out another marble. What is the probability of pulling out a red marble followed by a blue marble?

Example: Suppose you are going to draw two cards from a standard deck. What is the probability that the first card is an ace and the second card is a jack (just one of several ways to get "blackjack" or 21)?

Students can use the general multiplication rule to determine whether two events are independent.

The North Carolina High School Collaborative Instructional Framework

NC Math 3
Unit 1: Functions and their Inverses

10 Days Block Schedule **September 2017 Updates** 20 Days Traditional Schedule

RESEARCH BRIEF: **Unit 1: Functions and Inverses**

Essential Questions:

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables increase?
- How can functions be used to model real world situations?

Learning Outcomes	Student Objectives
● Given a function students will determine key features of a graph, table, or context. ● Students should be able to compare features of two functions in different representations. ● Students should be able to understand and interpret a variety of functions as a correspondence between inputs and outputs. ● Create functions from a contextual situation. ● Understand relationships of functions and their inverses. ● Create inverse functions and restrict domain. ● Create an equation or inequality and interpret reasonable solutions in context. ● Given a function create an equation from various representations and use them to solve problems. ● Interpret structure of a function and relationship with graph, table, and/or context. ● Given two functions, solve and interpret equations graphically.	● I will **find** key **features** of a function from a graph, table, or context. ● I will **compare** features of two functions in different representations. ● I will **interpret** the relationship between input and output of a function. ● I will be able to **create** a function from various representations. ● I will **describe** the relationship between a function and its inverse. ● I will **create** an inverse function and be able to restrict the domain if needed. ● I will be able to read a word problem and **create** an equation or inequality. ● I will **interpret** parts of a function and their relationship with the graph, table, and context.

- I will be able to **solve** and **interpret** the solutions of two equations graphically.

Standards Addressed in this Unit

Compare functions using multiple representations and understand key features to interpret, analyze, and find solutions.

- **NC.M3.A-SSE.1b**: Interpret expressions that represent a quantity in terms of its context. b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

- **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

- **NC.M3.F-IF.9**: Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

- **NC.M3.F-IF.2**: Use function notation to evaluate piecewise defined functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **NC.M3.F-IF.7**: Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

- **NC.M3.F-BF.1b**: Write a function that describes a relationship between two quantities.
  b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

- **NC.M3.F-BF.3**: Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x) + k$, $f(x + k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

Understand inverse relationships, describe them in various representations, and use these relationships to solve, analyze and interpret.

- **NC.M3.F-BF.4a**: Find an inverse function.
    a. Understand the inverse relationship between exponential and logarithmic, quadratic and square root, and linear to linear functions and use this relationship to solve problems using tables, graphs, and equations.

- **NC.M3.F-BF.4b**: Find an inverse function.
    b. Determine if an inverse function exists by analyzing tables, graphs, and equations.

- **NC.M3.F-BF.4c**: Find an inverse function.
    c. If an inverse function exists for a linear, quadratic and/or exponential function, f, represent the inverse function, f^{-1}, with a table, graph, or equation and use it to solve problems in terms of a context

Understand absolute value and piecewise defined relationships, describe them in various representations, and use these relationships to solve, analyze and interpret.

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- **NC.M3.A-CED.1**: Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

- **NC.M3.A-CED.2**: Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

- **NC.M3.A-CED.3**: Create systems of equations and/or inequalities to model situations in context.

- **NC.M3.A-SSE.1a**: Interpret expressions that represent a quantity in terms of its context.
    a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

- **NC.M3.A-SSE.1b**: Interpret expressions that represent a quantity in terms of its context.
    b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

- **NC.M3.A-REI.1:** Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

- **NC.M3.A-REI.11**: Extend an understanding that the x-coordinates of the points where the graphs of two equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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The Math Resource for Instruction - Customized for the Content of this Unit

NC.M3.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Concepts and Skills

Pre-requisite

- Interpret key features from graph, tables, and descriptions (NC.M2.F-IF.4)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identified key features with mathematical reasoning.

Mastering the Standard

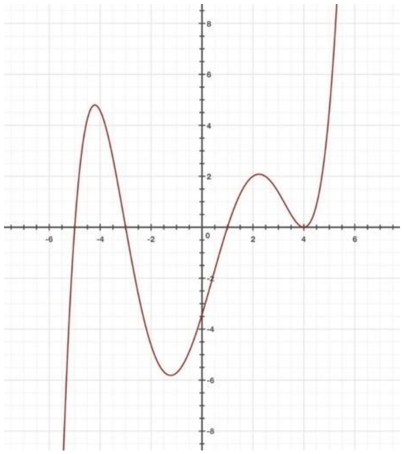
Comprehending the Standard	Assessing for Understanding
This standard is included in Math 1, 2 and 3. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math 3, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity as motion that is repeated in equal	This standard must be assessed using three important forms of displaying our functions: graphs, tables, and verbal descriptions/word problems. Students must be able to interpret each and how they apply to the key input-output values.

intervals of time and discontinuity as values that are not in the domain of a function, either as asymptotes or “holes” in the graph.

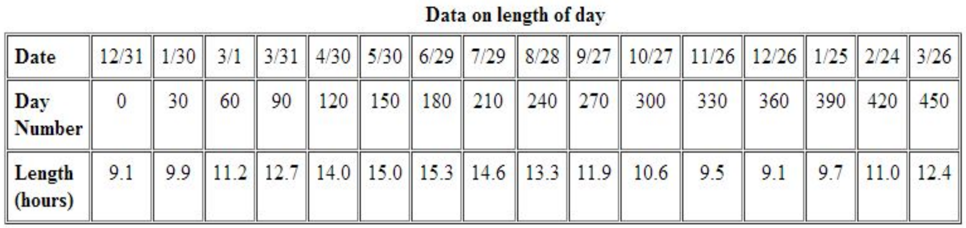
No limitations are listed with this standard. This means that all function types, even those found in more advanced courses. Students do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.
This is in contrast to NC.M3.F-IF.7, in which the specific function types are included. Students can work algebraically with those listed types and can analyze those functions in greater detail.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Example: For the function below, label and describe the key features. Include intercepts, relative max/min, intervals of increase/decrease, and end behavior.



Example*:* Over a year, the length of the day (the number of hours from sunrise to sunset) changes every day. The table below shows the length of day every 30 days from 12/31/97 to 3/26/99 for Boston Massachusetts.



Data on length of day

Date	12/31	1/30	3/1	3/31	4/30	5/30	6/29	7/29	8/28	9/27	10/27	11/26	12/26	1/25	2/24	3/26
Day Number	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450
Length (hours)	9.1	9.9	11.2	12.7	14.0	15.0	15.3	14.6	13.3	11.9	10.6	9.5	9.1	9.7	11.0	12.4

During what part of the year do the days get longer? Support your claim using information provided from the table.

NC.M3.F-IF.9

Analyze functions using different representations.

Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

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The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss how the comparison of a functions leads to a mathematical understanding, such as with transformations and choosing better models.

New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In Math 3, this standard can include two functions of any type students have learned in high school math in any representation. Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

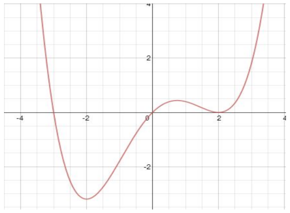
In assessing this standard, students must demonstrate that they can not only identify, but compare, the key features of two different functions. Appropriate question stems could include: Which is less/greater; Which will have a greater value at x = __; Which function has the higher maximum/lower minimum; etc.

Examples*:* If $f(x) =-(x+7)^2(x-2)$ and $g(x)$ is represented on the graph.

a) What is the difference between the zero with the least value of $f(x)$ and the zero with the least value of $g(x)$?

b) Which has the largest relative maximum?

c) Describe their end behaviors. Why are they different? What can be said about each function?



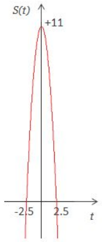
Example: Two objects dropped downward at the same time from a top of building. For both functions, t represents seconds and the height is represented in feet. One function is represented by the table and the other by the graph.

a) Which object was dropped from a greater height? Explain your answer.

b) Which object hit the ground first? Explain your answer.

c) Which object fell at a faster rate (in ft/sec)? Explain your answer.

t	$s(t)$
0	20
2.5	15
3.5	10
4.3	5
5	0



NC.M3.F-IF.2

Understand the concept of a function and use function notation.

Use function notation to evaluate piecewise defined functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Concepts and Skills

Pre-requisite

- Evaluate a function for inputs in their domain and interpret in context (NC.M1.F-IF.2)
- Interpret a function in terms of the context by relating its domain and range to its graph (NC.M1.F-IF.5)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Analyze and compare functions in various representations (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able how they know a point is a solution to piecewise defined function.

New Vocabulary: piecewise function

Mastering the Standard

Comprehending the Standard

The new concept students must understand from this standard is the notation of piecewise functions – mainly, that the function must be evaluated using different function rules for the different inputs in different domains. The function rules can include the new functions for this course (polynomial, rational, exponential) and functions from previous courses (linear, quadratic, root, etc.)

Additionally, students must recognize from word problems why certain domains apply to certain function rules.

A great introduction to piecewise functions could use absolute value as a piecewise function of two linear functions. Students take a function they are learning in this course and break it into two functions they have already learned in Math 1.

Assessing for Understanding

In assessing this standard, students must be able to evaluate all types of functions, and they must be able to determine the appropriate domain to use for each input value.

Example: For the following function: $h(x) = \{2^x,\ x < -3;\ \frac{3}{x},\ x \geq -3$

a) Evaluate $h(-4)$.

b) Evaluate $3\,h(0) + 2\,h(-3) - h(-6)$.

c) What is the domain of $h(x)$? Explain your answer.

Additionally, students must be able to explain the context of piecewise functions and how their domains apply.

Example: A cell phone company sells its monthly data plans according to the following function, with *f(x)* representing the total price and *x* representing the number of gigabytes of data used.

$$f(x) = \{19.95x + 60,\ for\ 0 \leq x \leq 3;\ 9.95x + 75, for\ 3 < x \leq 6;\ \ 125,\ for\ x > 6$$

a) If a customer uses 3 GB of data, how much will she pay?

b) How many GB of data are required so a subscriber does not pay any extra money per GB?

c) If you use 2.5 GB of data per month, what plan will be the cheapest?

d) How many GB of monthly data will make plan B's price equal to plan C?

NC.M3.F-IF.7

Analyze functions using different representations.

Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Concepts and Skills

Pre-requisite

- Analyze functions using different representations to show key features (NC.M2.F-IF.7)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Create and graph equations in two variables (NC.M3.A-CED.2)
- Analyze graphs and tables and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.9)
- Build functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Understand the effects of transformations on functions (NC.M3.F-BF.3)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)
- Compare the end behavior of functions using the rate of change (NC.M3.F-LE.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation best shows each of the key features.

Mastering the Standard

Comprehending the Standard

In previous math courses, students have identified the characteristic of graphs of other functions, including linear, quadratic, exponential, radical, and inverse variation functions. They should be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior.

In Math 3, these concepts are extended to piecewise, absolute value, polynomials, exponential, rational, and sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity are new features of functions that must be introduced. The intent of this standard is for students to find discontinuities in tables and graphs and to recognize their relationship to functions. Students are not expected to find an asymptote from a

Assessing for Understanding

In assessing this standard, students must demonstrate their ability to represent and determine the key features from algebraic and graphical representations of the functions.

Example: Graph $g(x) = x^3 + 5x^2 + 2x - 8$.

a) Identify zeroes.
b) Discuss the end behavior.
c) In what intervals is the function increasing? Decreasing?

function. (This could be an extension topic.) This standard will likely span multiple units, as most Math 3 courses teach polynomial, exponential, rational, and trigonometric functions in different units. These function characteristics will be repeated and reinforced throughout the course. Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.	**Example:** Given the following piecewise function: $h(x) = \begin{cases} x^2, & -3 \leq x < 3 \\ 2 - x, & 3 \leq x < 7 \end{cases}$ Discuss the key features, including domain and range, intercepts, relative maximum and minimums, end behavior and discontinuities.

NC.M3.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Concepts and Skills

Pre-requisite

- Build new function by combine linear, quadratic and exponential functions (NC.M1.F-BF.1b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify new function and discuss how the new function fits the context.

Mastering the Standard

Comprehending the Standard

This standard asks students to combine standard function types by addition, subtraction, and multiplication. In Math 3, we are NOT required to include composition, although it could be a valuable extension.

The key concept for teaching this standard is a review of adding and subtracting expressions (including combining like terms) and multiplying expressions (distributing polynomials and exponent rules).

Assessing for Understanding

In assessing this standard, students will need to perform the operations and determine from a context which operation is appropriate. The functions that students need to combine should be given in problems, but the operation can be determined from context if necessary.

Example: Last year, army engineers modeled the function of a bullet fired by a United States soldier from a certain weapon. The function $f(x) =- 16x^2 + 200x + 4$ modeled the path of the bullet. This year, the soldiers were supplied with more powerful guns that changed the path of the bullet from higher ground by adding the function $g(x) = 300x + 20$. What function models the path of the new bullet?

Example: Consider the functions: $f(x) = 4x + 9$ and $g(x) = -2x - 4$

a) Evaluate $f(-3)$.
b) Evaluate $g(-3)$.
c) Add $f(x) + g(x)$.
d) Evaluate $(f + g)(-3)$.
e) What do you notice? What properties have you learned that explain your answer?

NC.M3.F-BF.3

Build new functions from existing functions.

Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x)+k$, $f(x+k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Understand the effects of transformations on functions (NC.M2.F-BF.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build polynomial and exponential functions from a graph, description, or ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of k.

Mastering the Standard

Comprehending the Standard

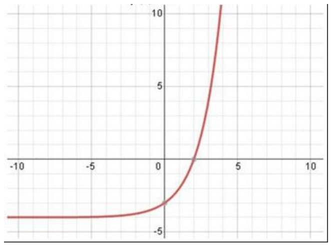
Students learned the translation and dilation rules in Math 2 with regard to linear, quadratic, square root, and inverse variation functions. In Math 3, we apply these rules to functions in general.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions. Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of k.

Assessing for Understanding

In demonstrating their understanding, students must be able to relate the algebraic equations, graphs, and tabular representations (ordered pairs) as functions are transformed. Appropriate questions will ask students to identify and explain these transformations.

Example: The graph of $f(x)$ and the equation of $g(x)$ are shown below. Which has a higher y-intercept? Explain your answer.



$g(x) = 2^x - 7$

Example: Use the table below to identify the transformations and write the equation of the absolute value function f(x).

x	-6	-5	-4	-3	-2
f(x)	3	1	-1	1	3

NC.M3.F-BF.4a

Build new functions from existing functions.

Find an inverse function.

a. Understand the inverse relationship between exponential and logarithmic, quadratic and square root, and linear to linear functions and use this relationship to solve problems using tables, graphs, and equations.

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- The existence of an inverse function and representing it (NC.M3.F-BF.4b, NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the relationship between inverse operations and inverse functions.

New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

Students have used inverse operations to solve equations in previous math courses, but this is the first time students are introduced to the concept of an inverse function. All of the F-BF.4 standards relate, but the progression of understanding the relationship, determining is an inverse exists, and solving for the inverse through the F-BF.4a, F-BF.4b, and F-BF.4c will enhance understanding.

For this part of the standard, the main concept students must understand is that an inverse function switches the input and output (x and y) for every point in the function. It is important to connect this concept to the reflection of one function, $f(x)$, across the line of symmetry $y = x$, to create the inverse function, $g(x)$. In Math 3, we are limiting the functions to linear, quadratic, square root, exponential, and

Assessing for Understanding

Students should first start by exploring the relationships between inverse functions.

Example: Complete the following tables for the given functions. Which are inverses? Explain your answer.

$f(x) = \frac{1}{10}x$

X	0	1	2	3	4
f(x)					

$g(x) = 10^x$

X	0	1	2	3	4
f(x)					

$h(x) = 10x$

X	0	1	2	3	4
f(x)					

$j(x) = log_{10} x$

X	1	100	1,000	10,000	100,000
f(x)					

logarithmic.

Students must also understand the common notation f^{-1} to represent inverse functions.

Students, while having worked with quadratic and square root functions, may not have explored all aspects of the inverse relationship.

Students started work with exponential functions in NC Math 1, and have not been exposed to logarithms before this course.

When speaking of inverse relationships, it is important for students to understand and communicate the reasoning for finding an inverse function. This can often be accomplished by considering the independent and dependent variables, the context of the problem, and a chosen solution pathway.

As students are solving problems using inverses, common formulas can help students understand this inverse relationship (Celsius/Fahrenheit conversions, geometry formulas, interest formulas). To understand the concept of an inverse function, students should be asked to explain the input as a function of the output and how this affects the values.

Example: The area of a square can be described as a function of the length of a side, $A(s) = s^2$.

a) What is the area of a square with side length 5 cm?
b) What is the length of a side of a square with an area 25 cm^2?
c) What relationship do a function of area given a side length and a function of side length given the area share? How do you know?
d) Use this relationship to solve for the length of a side of a square with an area of 200 cm^2.

Example: Complete the table to write the inverse for the following function. Is the inverse a function? Explain your answer.

x	1	2	3	4	5
f(x)	2	4	9	4	12

x					
f^{-1} (x)					

NC.M3.F-BF.4b

Build new functions from existing functions.

Find an inverse function.

b. Determine if an inverse function exists by analyzing tables, graphs, and equations.

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)
- Understand inverse relationships (NC.M3.F-BF.4a)

Connections

- Represent inverse functions (NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the reasoning in needing a restricted domain.

New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

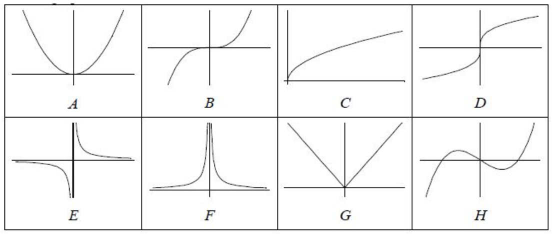
In Math 1, students learned to determine if a relation is a function by analyzing tables, equations, and graphs. In Math 3, students need to determine if a function is invertible and on what domain.

This part of the standard is not limited by function type. This means that students should be able to determine if any function or a portion of the function has an inverse function from different representations.

Assessing for Understanding

The standard states that students must determine if an inverse function exists, so presenting graphs, tables, and equations are all appropriate representations for students to analyze. Additionally, especially for quadratic functions, students must be able to determine the appropriate domain for a function to have an inverse.

Example: Which of the following functions have inverse functions? For those that are do not have inverse functions as a whole, divide the graph into sections that do have inverse functions.



Example: Use a table of $f(x) = 3x^2 - 18x + 5$ to determine possible domains on which $f^{-1}(x)$ is a function.

Example: Which of the following equations have an inverse function? How do you know, from the table and graph? For any that do not, how can we limit the domain of the function to ensure that it has an inverse?

a) $f(x) = 2x$ b) $f(x) = x^2$ c) $f(x) = 2^x$

Example: Determine which function(s) have an inverse function from the tables below. Provide a reason if an inverse function does not exist.

$g(x)$	x
-2	9
-1	3
0	1
1	3
2	9

$h(x)$	x
-2	-12
-1	-9
0	-6
1	-3
2	0

$j(x)$	x
-2	4
-1	2
0	1
1	0.5
2	0.25

$k(x)$	x
-2	5
-1	3
0	1
1	-1
2	1

NC.M3.F-BF.4c

Build new functions from existing functions.

Find an inverse function.

c. If an inverse function exists for a linear, quadratic and/or exponential function, f, represent the inverse function, f^{-1}, with a table, graph, or equation and use it to solve problems in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)
- Understand inverse relationships and determine if an inverse exist (NC.M3.F-BF.4a, NC.M3.F-BF.4b)

Connections

- Use logarithms to expression solutions to exponential functions (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation (tabular, graphical, or symbolic) is the most efficient to solve a particular problem.

New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

Once students understand the concept of a function that has an inverse, they can begin solving for the inverse functions. The idea of reversing the input and output (x and y) is central to solving for an inverse algebraically, and it should also be emphasized on the graph (reflection over the $y = x$ line) and table.

It is important to note; the algebraic approach can be complex in many cases. Often, tables and graphs can be used to solve problems in a more efficient and student friendly manner.

In Math 3, the functions are limited to linear, quadratic, and exponential. For quadratics, it must be emphasized that we have the equation in a form we can solve for the input variable, so this can be an appropriate concept in which to review completing the square and vertex form, from Math 2.

Assessing for Understanding

Most assessment items for this standard will ask students to solve for an inverse using a graph or equation. Real-world context exists with common conversion formulas, area/volume formulas, and interest formulas.

Example*:* Graph the inverse of $f(x) = -\frac{3}{2}x - 3$. How does $f^{-1}(x)$ relate to $f(x)$?

Example: Let $f(x) = x^2 + 7x + 9$. Does an inverse function exist for the entire domain of the function? Find the inverse of this function.

NC.M3.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

Concepts and Skills

Pre-requisite

- Create one variable equations and solve (NC.M2.A-CED.1)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Justify a solution method (NC.M3.A-REI.1)

Connections

- Justify a solution method (NC.M3.A-REI.1)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
1 – Make sense of problems and persevere in solving them
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
Student should be able to explain and defend the model they chose to represent the situation.
New Vocabulary: Absolute value equation

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematical equations to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.
Creating one variable equations and inequalities are included in Math 1, 2, and 3. In previous courses, students modeled with linear, exponential, quadratic, radical, and inverse variation equations. In Math 3, students will be expected to model with polynomial, rational, absolute value, and exponential equations. Students will need to analyze a problem, determine the type of equation, and set up and solve these problems. Students may need to create an equation from different representations found in the context. This makes it important for students to realize that equations can be derived as a specific instance of an associated function.

Students are expected to represent the solutions of an inequality using a number line and compound inequalities using inequality and interval notation.

Assessing for Understanding

Students should be able to create and solve problems algebraically and graphically. There should be a focus on using methods efficiently.

Example: Clara works for a marketing company and is designing packing for a new product. The product can come in various sizes. Clara has determined that the size of the packaging can be found using the function, $p(b) = (b)(2b+1)(b+5)$, where b is the shortest measurement of the product. After some research, Clara determined that packaging with $20,500\ cm^3$ will be the most appealing to customers. What are the dimensions of the package?

Example: A recent poll suggests that 47% of American citizens are going to vote for the Democratic candidate for president, with a margin of error of ±4.5%. Set up and solve an absolute value inequality to determine the range of possible percentages the candidate could earn. Based on your answer, can you determine if the Democratic candidate will win the election? Why or why not?

NC.M3.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

Concepts and Skills

Pre-requisite

- Create and graph two-variable equations (NC.M2.A-CED.2)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)

Connections

- Write the equations and inequalities of a system (NC.M3.A-CED.3)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Absolute value equation

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. In A-CED.1, writing and solving an equation is the essential skill required. In this standard, graphing the equation to determine key features is essential.

This standard is included in Math 1, 2, and 3. Throughout all three courses, students create equations in two variables and graph them on coordinate axes. In Math 3, absolute value, polynomial, and rational graphs are introduced, and exponential graphs are further developed to solve for the exponent.

Assessing for Understanding

Rate of growth and decay, work rate (and other rates), geometric, and other real-world examples provide the context for many of these problems.

Example: A biology student us studying bacterial growth. She was surprised to find that the population of the bacteria doubled every hour.

a) Complete the following table and plot the data.

Hours into study	0	1	2	3	4
Population (thousands)	4				

b) Write and equations for P, the population of the bacteria, as a function of time, t, and verify that it produces correct populations for t = 1, 2, 3, and 4.

https://www.illustrativemathematics.org/content-standards/tasks/385

NC.M3.A-CED.3

Create equations that describe numbers or relationships.

Create systems of equations and/or inequalities to model situations in context.

Concepts and Skills

Pre-requisite

- Write the equations for a system (NC.M2.A-CED.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Create and graph two variable equations (NC.M3.A-CED.2)

Connections

- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should justify the chosen models of each equation with mathematical reasoning.

Mastering the Standard

Comprehending the Standard

In Math 3, the systems of equations and inequalities that must be mastered include absolute value functions. In previous courses, students have worked with systems including linear and quadratic functions.

Function types are not limited in this standard as in previous courses. All function types are potential components of systems in Math 3. Students are not expected to solve complex systems algebraically, but should focus on more efficient method such as tables, graphs, and using technology. (Solving these systems algebraically can be an extension topic.)

Assessing for Understanding

In assessing this standard, graphical solutions can be highlighted using technology. Ideally, the functions and equations will come from a context.

Example: After receiving his business degree from UNC-Chapel Hill, John is offered positions with two companies. Company A offers him $80,000 per year, with a $1,000 increase every year. Company B offers him $60,000 per year with a 4% increase every year.

a) After how many years will the Company B salary be higher than Company A?
b) Which offer would you choose? Why?

NC.M3.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify and interpret parts of an expression in context (NC.M2.A-SSE.1a)

Connections

- Interpret parts of an expression as a single entity (NC.M3.A-SSE.1b)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret statements written in piecewise function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Absolute value, piecewise function

Mastering the Standard

Comprehending the Standard

Students need to be able to determine the meaning, algebraically and from a context, of the different parts of the expressions noted in the standard. At the basic level, this would refer to identifying the terms, factors, coefficients, and exponents in each expression.

Students must also be able to identify how these key features relate in context of word problems.

Assessing for Understanding

Students should be able to identify and **explain the meaning** of each part of these expressions.

Example: The Charlotte Shipping Company is needing to create an advertisement flyer for its new pricing for medium boxes shipped within Mecklenburg County. Based on the expressions of the function below, where c represents cost and p represent pounds, create an advertisement that discusses all the important details for the public.

$c(p) = \{11.45,\ \ p \le 12\frac{1}{3};\ .72\,p + 5.57,\ \ p > 12\frac{1}{3}$

Example: In a newspaper poll, 52% of respondents say they will vote for a certain presidential candidate. The range of the actual percentage can be expressed by the expression $|x - 4|$, where x is the actual percentage. What are the highest and lowest percentages that might support the candidate? Is the candidate guaranteed a victory? Why or why not?

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: piecewise function

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

As we add piecewise functions and expressions in Math 3, breaking down these expressions and functions into their parts are essential to ensure understanding.

For Example: Explain what operations are performed on the inputs -2, 0, and 2 for the following expression:

$$f(x) = \begin{cases} 3x, for\ x < 0 \\ \frac{1}{x}, for\ 0 \leq x < 2 \\ x^3, for\ x \geq 2 \end{cases}$$

Which input is not in the domain? Why not?

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: The expression, $0.0013x^3 - 0.0845x^2 + 1.6083x + 12.5$, represents the gas consumption by the United States in billions of gallons, where x is the years since 1960. Based on the expression, how many gallons of gas were consumed in 1960? How do you know?

NC.M3.A-REI.11

Represent and solve equations and inequalities graphically

Extend an understanding that the x-coordinates of the points where the graphs of two equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
Students should be able to explain how solutions obtained through algebraic methods and graphing can differ and understand the benefits and limitations of graphing.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2, and 3. In previous courses, students studied linear, exponential and quadratic functions. In Math 3, the type of function is not limited. Students are expected to find a solution to any equation or system using tables, graphs and technology.

Visual examples of rational equations explore the solution as the intersection of two functions and provide evidence to discuss how extraneous solutions do not fit the model.

Assessing for Understanding

Graphical solutions, often using technology, should be highlighted in assessing student mastery of this standard.

Example*:* Graph the following system and approximate solutions for $f(x) = g(x)$.

$$f(x) = \frac{x+4}{2-x} \text{ and } g(x) = x^3 - 6x^2 + 3x + 10$$

From the standard, we build that $f(x) = g(x)$ where $f(x) = y_1$ and $g(x) = y_2$

Example*:* Use technology to solve $e^{2x} + 3x = 15$, treating each side of the statement as two equations of a system.
Note: Algebraically solving equations with e is not an expectation of Math 3. Students should be able to solve any equations using a graphing technology.

Example: Solve the equation $5^{4x} = 2^{8x}$ graphically. Then, use the answer to show that the equation holds true for the x-value you find.

NC.M3.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solution method and the steps in the solving process (NC.M2.A-REI.1)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why it is necessary to write two equations to solve an absolute value equation.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. In Math 3, students should extend their knowledge of all equations they are asked to solve.

When solving equations, students will use mathematical reasoning to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Students do not have to use the proper names of the properties of operations and equality, but they should recognize and use the concepts associated with the properties.

Assessing for Understanding

Solving equations including justifications for each step, error analysis of solutions to equations, and comparing and analyzing different methods are all appropriate methods of assessing this standard.

Example: Julia is solving an absolute value inequality in class and has become stuck. Show Julia the next step and write down the explanation for that step so she can reference it on other problems.

Julia's steps:

$2|x+5|-3\leq 10$

$2|x+5|\leq 13$

$|x+5|\leq 6.5$

The North Carolina High School Collaborative Instructional Framework

NC Math 3

Unit 2: Exponential and Logarithmic Functions

9 Days Block Schedule | **September 2017 Update** | 18 Days Traditional Schedule

RESEARCH BRIEF: [Unit 2 Exponential and Logarithmic Functions]

Essential Questions:

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables increase?
- How can functions be used to model real world situations?

Learning Outcomes	Student Objectives
● Create exponential and logarithmic functions from a contextual situation. ● Create and solve exponential or logarithmic equations from a contextual situation. ● Given an exponential function students will determine key features of a graph, table, or context. ● Students should be able to compare features of two functions in different representations. ● Students should be able to understand and interpret domain and range of an exponential/logarithmic function. ● Understand inverse relationships of exponential and logarithmic functions. ● Create an equation or inequality and interpret reasonable solutions in context. ● Given a function create an equation from various	● I will **find** key **features** of an exponential and logarithmic function from a graph, table, or context. ● I will **compare** features of two functions in different representations. ● I will **interpret** the relationship between input and output of an exponential and logarithmic function. ● I will be able to **create** a exponential and logarithmic function from various representations. ● I will **describe** the inverse relationship between exponential and logarithmic functions. ● I will **create** an inverse function. ● I will be able to read a word problem and **create** an equation or inequality. ● I will **interpret** parts of a function and their relationship with the graph, table, and context.

representations and use them to solve problems.
- Interpret structure of an exponential/logarithmic function and relationship with graph, table, and/or context.
- Given two functions, solve and interpret equations graphically.

- I will be able to **solve** and **interpret** the solutions of two equations graphically.

Standards Addressed in this Unit

Understand how to create exponential equations and graphs with one or two variables, and be able to identify the different parts of an exponential equation and relate them to the real world.

- **NC.M3.A-CED.1**: Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

- **NC.M3.A-CED.2**: Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

- **NC.M3.A-SSE.1a**: Interpret expressions that represent a quantity in terms of its context.
    a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

- **NC.M3.A-SSE.1b**: Interpret expressions that represent a quantity in terms of its context.
    b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

- **NC.M3.A-SSE.2:** Use the structure of an expression to identify ways to write equivalent expressions.

- **NC.M3.A-SSE.3:** Write an equivalent form of an exponential expression by using the properties of exponents to transform expressions to reveal rates based on different intervals of the domain.

- **NC.M3.A-REI.1:** Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Recognize the relationship between exponential and logarithmic equations as inverses using multiple representations, interpret the key features of the graph, and use them to solve equations and model real world phenomena.

- **NC.M3.F-BF.3**: Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x) +$

k, $f(x + k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

- **NC.M1.F-IF.4:** Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

- **NC.M3.F-IF.7**: Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

- **NC.M3.F-IF.9**: Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

- **NC.M3.F-BF.1a**: Write a function that describes a relationship between two quantities.
    a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

- **NC.M3.F-BF.1b:** Write a function that describes a relationship between two quantities.
    b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

- **NC.M3.F-BF.4a**: Find an inverse function.
    a. Understand the inverse relationship between exponential and logarithmic, quadratic and square root, and linear to linear functions and use this relationship to solve problems using tables, graphs, and equations.

- **NC.M3.F-BF.4b**: Find an inverse function.
    b. Determine if an inverse function exists by analyzing tables, graphs, and equations.

- **NC.M3.F-BF.4c**: Find an inverse function.
    c. If an inverse function exists for a linear, quadratic and/or exponential function, f, represent the inverse function, f^{-1}, with a table, graph, or equation and use it to solve problems in terms of a context

- **NC.M3.F-LE.4**: Use logarithms to express the solution to $ab^{ct} = d$ where a, b, c, and d are numbers and evaluate the logarithm using technology.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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The Math Resource for Instruction - Customized for the Content of this Unit

NC.M3.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

Concepts and Skills

Pre-requisite

- Create one variable equations and solve (NC.M2.A-CED.1)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Justify a solution method (NC.M3.A-REI.1)

Connections

- Justify a solution method (NC.M3.A-REI.1)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Student should be able to explain and defend the model they chose to represent the situation.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematical equations to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Creating one variable equations and inequalities are included in Math 1, 2, and 3. In previous courses, students modeled with linear, exponential, quadratic, radical, and inverse variation equations. In Math 3, students will be expected to model with polynomial, rational, absolute value, and exponential equations. Students will need to analyze a problem, determine the type of equation, and set up and solve these problems. Students may need to create an equation from different representations found in the context. This makes it important for students to realize that equations can be derived as a specific instance of an associated function.

Students are expected to represent the solutions of an inequality using a number line and compound inequalities using inequality and interval notation.

Assessing for Understanding

Students should be able to create and solve problems algebraically and graphically. There should be a focus on using methods efficiently.

> **Example:** If the world population at the beginning of 2008 was 6.7 billion and growing at a rate of 1.16% each year, in what year will the population be double?

NC.M3.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

Concepts and Skills

Pre-requisite

- Create and graph two-variable equations (NC.M2.A-CED.2)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)

Connections

- Write the equations and inequalities of a system (NC.M3.A-CED.3)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. In A-CED.1, writing and solving an equation is the essential skill required. In this standard, graphing the equation to determine key features is essential.

This standard is included in Math 1, 2, and 3. Throughout all three courses, students create equations in two variables and graph them on coordinate axes. In Math 3, absolute value, polynomial, and rational graphs are introduced, and exponential graphs are further developed to solve for the exponent.

Assessing for Understanding

Rate of growth and decay, work rate (and other rates), geometric, and other real-world examples provide the context for many of these problems.

Example: A biology student is studying bacterial growth. She was surprised to find that the population of the bacteria doubled every hour.

a) Complete the following table and plot the data.

Hours into study	0	1	2	3	4
Population (thousands)	4				

b) Write an equation for P, the population of the bacteria, as a function of time, t, and verify that it produces correct populations for t = 1, 2, 3, and 4.

https://www.illustrativemathematics.org/content-standards/tasks/385

NC.M3.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify and interpret parts of an expression in context (NC.M2.A-SSE.1a)

Connections

- Interpret parts of an expression as a single entity (NC.M3.A-SSE.1b)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

Students need to be able to determine the meaning, algebraically and from a context, of the different parts of the expressions noted in the standard. At the basic level, this would refer to identifying the terms, factors, coefficients, and exponents in each expression.

Students must also be able to identify how these key features relate in context of word problems.

Assessing for Understanding

Students should be able to identify and **explain the meaning** of each part of expressions.

Example: A woman invests a specific amount of money which earns compounded interest at a particular rate. This situation is represented by the equation: $A=1000(1.023)^{2t}$. Determine the initial amount invested, the interest rate, and how often it is compounded.

(Remember: $A = P\left(1+\frac{r}{n}\right)^{nt}$)

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Piecewise function

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math1 and Math 2, so the new step is applying it to our Math 3 functions.

As we add piecewise functions and expressions in Math 3, breaking down these expressions and functions into their parts are essential to ensure understanding.

For Example: Explain what operations are performed on the inputs -2, 0, and 2 for the following expression:

$$f(x) = \{3x,\ for\ x < 0\ \frac{1}{x},\ for\ 0 \leq x < 2\ x^3,\ for\ x \geq 2$$

Which input is not in the domain? Why not?

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: Find the range using the appropriate expressions given the following domain values: -9, -6, -3, 0, 1.5, and 3.

$$g(x)\begin{cases} -3, for\ x < -6 \\ (x+1)^2, for -6 \leq x \leq 2 \\ 2^x, for\ x < 2 \end{cases}$$

NC.M3.A-SSE.3

Write expressions in equivalent forms to solve problems.

Write an equivalent form of an exponential expression by using the properties of exponents to transform expressions to reveal rates based on different intervals of the domain.

Concepts and Skills

Pre-requisite

- Use the properties of exponents to rewrite expressions with rational exponents (NC.M2.N-RN.2)

Connections

- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain their process of transforming an exponential expression using mathematical reasoning.

Mastering the Standard

Comprehending the Standard

Students have already learned about exponential expressions in Math 1. This standard expands on that knowledge to expect students to write equivalent expressions based on the properties of exponents.

Additionally, compound interest is included in this standard. In teaching students to fully mastery this concept, we must explain where the common compound interest formula originates. The relationship to the common $A = P(1+r)^t$ formula must be derived and explained.

Assessing for Understanding

For students to demonstrate mastery, they must be able to convert these expressions and explain why the conversions work mathematically based on the properties of exponents.

Example: Explain why the following expressions are equivalent.

$2(\frac{1}{2})^6$ $\quad$ $(\frac{1}{2})^5$ $\quad$ $2(\frac{1}{4})^3$

Students must be able to convert an exponential expression to different intervals of the domain.

Example: In 1966, a Miami boy smuggled three Giant African Land Snails into the country. His grandmother eventually released them into the garden, and in seven years there were approximately 18,000 of them. The snails are very destructive and need to be eradicated.

a) Assuming the snail population grows exponentially, write an expression for the population, p, in terms of the number, t, of years since their release.

b) You must present to the local city council about eradicating the snails. To make a point, you want to want to show the rate of increase per month. Convert your expression from being in terms of years to being in terms of months.

Modified from Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/tasks/638

NC.M3.F-BF.3

Build new functions from existing functions.

Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x)+k$, $f(x+k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Understand the effects of transformations on functions (NC.M2.F-BF.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build polynomial and exponential functions from a graph, description, or ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of *k*.

Mastering the Standard

Comprehending the Standard

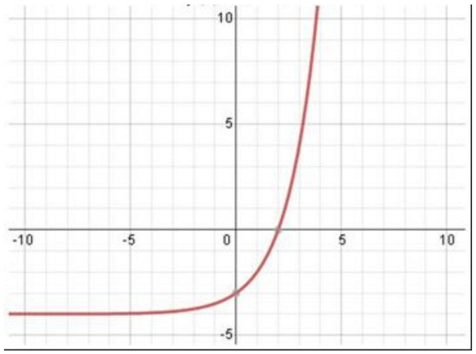
Students learned the translation and dilation rules in Math 2 with regard to linear, quadratic, square root, and inverse variation functions. In Math 3, we apply these rules to functions in general.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions. Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of *k*.

Assessing for Understanding

In demonstrating their understanding, students must be able to relate the algebraic equations, graphs, and tabular representations (ordered pairs) as functions are transformed. Appropriate questions will ask students to identify and explain these transformations.

Example: The graph of $f(x)$ and the equation of $g(x)$ are shown below. Which has a higher y-intercept? Explain your answer.



$g(x) = 2^x - 7$

NC.M3.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Concepts and Skills

Pre-requisite

- Interpret key features from graph, tables, and descriptions (NC.M2.F-IF.4)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identified key features with mathematical reasoning.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math 3, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity as motion that is repeated in equal intervals of time and discontinuity as values that are not in the domain of a function, either as asymptotes or “holes” in the graph.

No limitations are listed with this standard. This means that all function types, even those found in more advanced courses. Students do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.
This is in contrast to NC.M3.F-IF.7, in which the specific function types are included. Students can work algebraically with those listed types and can analyze those functions in greater detail.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

This standard must be assessed using three important forms of displaying our functions: graphs, tables, and verbal descriptions/word problems. Students must be able to interpret each and how they apply to the key input-output values.

Example: Peyton has a savings account at First National Bank. The amount of money in the account grows exponentially. The table below shows the amount of money in her account each year.

a) What is the y-intercept? What does it represent in the context of this problem?
b) Find the interest rate Peyton is earning on her money.

t	f(t)
0	$1200
1	$1254
2	$1310.40
3	$1369.40
4	$1431
5	$1495.40

NC.M3.F-IF.9

Analyze functions using different representations.

Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss how the comparison of a functions leads to a mathematical understanding, such as with transformations and choosing better models.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

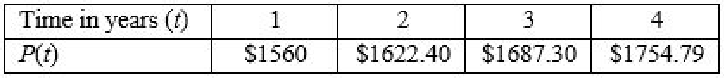
In Math 3, this standard can include two functions of any type students have learned in high school math in any representation. Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

In assessing this standard, students must demonstrate that they can not only identify, but compare, the key features of two different functions. Appropriate question stems could include: Which is less/greater; Which will have a greater value at x = __; Which function has the higher maximum/lower minimum; etc.

Example: Frank invested $2,000 into a savings account earning 2.5% interested annually. Paul invested money into a different account at the same time as Frank. The table below shows the amount of money in Paul's account after *t* years.



Time in years (t)	1	2	3	4
$P(t)$	$1560	$1622.40	$1687.30	$1754.79

a) Who had the larger initial investment?
b) Whose is earning a higher interest rate?
c) Over what interval of time will Frank have more money in his account? (show both inequality and interval notation)
d) Over what interval of time will Paul have more money in his account? (show both inequality and interval notation)

NC.M3.F-IF.7

Analyze functions using different representations.

Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Concepts and Skills

Pre-requisite

- Analyze functions using different representations to show key features (NC.M2.F-IF.7)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)

Connections

- Create and graph equations in two variables (NC.M3.A-CED.2)
- Analyze graphs and tables and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.9)
- Build functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Understand the effects of transformations on functions (NC.M3.F-BF.3)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)
- Compare the end behavior of functions using the rate of change (NC.M3.F-LE.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation best shows each of the key features.
New Vocabulary:

Mastering the Standard

Comprehending the Standard

In previous math courses, students have identified the characteristic of graphs of other functions, including linear, quadratic, exponential, radical, and inverse variation functions. They should be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior. In Math 3, these concepts are extended to piecewise, absolute value, polynomials, **exponential**, rational, and sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity are new features of functions that must be introduced. The intent of this standard is for students to find discontinuities in tables and graphs and to recognize their relationship to functions. Students are not expected to find an asymptote from a function.
Students are expected to use and interpret compound inequalities using inequality and

Assessing for Understanding

In assessing this standard, students must demonstrate their ability to represent and determine the key features from algebraic and graphical representations of the functions.

Example: If an adult takes 600 mg of ibuprofen, the amount remaining in their system can be modeled by the equation $I(t) = 600(0.72)^t$ where t represents the number of hours since taking the medicine.

a) What is the y-intercept of the graph and what does it represent?
b) At what rate if the body eliminating the drug?
c) Over what interval of time will there be at least 100 gm of ibuprofen in the person's body?

interval notation to describe key features when appropriate.	

NC.M3.F-BF.1a

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

Concepts and Skills

Pre-requisite

- Build quadratic functions given a graph, description, or ordered pair (NC.M2.F-BF.1)
- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Understand the effects of transforming functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss when multiple models can describe the information given, for example, when given the two roots, multiple models can contain those roots.

Mastering the Standard

Comprehending the Standard

This standard relates to building functions in two different contexts – polynomial (with real solutions) and exponential. In many Math 3 courses, it will be covered in two different units.

When building polynomial functions, only those with real solutions are considered. The relationship between solutions and factors, multiplicity and graphs, and the leading coefficient's sign relating to the end behaviors are all essential to build these functions.

When building exponential functions, students must be able to determine the initial value (a) and rate of growth (b) from the table, graph, or description presented. These problems can include those with compounding interest, doubling time and half-life.

Assessing for Understanding

For both functions, it is important that the assessment questions include algebraic "math" questions and questions in context. The answers to questions assessing this standard should be the actual function they are building, as other standards allow students to identify and interpret key features.

Example: The population of a certain animal being researched by environmentalists has been decreasing substantially. Biologists tracking the species have determined the following data set to represent the remaining animals:

Year	2010	2011	2012	2013	2014
Pop.	40,000	30,000	22,500	16.875	12,656

Assuming the population continues at the same rate, what function would represent the population $f(x)$ in year x, assuming x is the number of years after the year 2000?

NC.M3.F-BF.4a

Build new functions from existing functions.

Find an inverse function.

a. Understand the inverse relationship between exponential and logarithmic, quadratic and square root, and linear to linear functions and use this relationship to solve problems using tables, graphs, and equations.

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- The existence of an inverse function and representing it (NC.M3.F-BF.4b, NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the relationship between inverse operations and inverse functions.

New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

Students have used inverse operations to solve equations in previous math courses, but this is the first time students are introduced to the concept of an inverse function. All of the F-BF.4 standards relate, but the progression of understanding the relationship, determining is an inverse exists, and solving for the inverse through the F-BF.4a, F-BF.4b, and F-BF.4c will enhance understanding.

For this part of the standard, the main concept students must understand is that an inverse function switches the input and output (x and y) for every point in the function. It is important to connect this concept to the reflection of one function, $f(x)$, across the line of symmetry $y = x$, to create the inverse function, $g(x)$. In Math 3, we are limiting the functions to linear, quadratic, square root, exponential, and logarithmic.

Students must also understand the common notation f^{-1} to represent inverse functions. Students, while having worked with quadratic and square root functions, may not have explored all aspects of the inverse relationship.

Students started work with exponential functions in NC Math 1, and have not been exposed to logarithms before this course.

Assessing for Understanding

Students should first start by exploring the relationships between inverse functions.

Example: Complete the following tables for the given functions. Which are inverses? Explain.

$f(x) = \frac{1}{10}x$

X	0	1	2	3	4
f(x)					

$g(x) = 10^x$

X	0	1	2	3	4
f(x)					

$h(x) = 10x$

X	0	1	2	3	4
f(x)					

$j(x) = log_{10} x$

When speaking of inverse relationships, it is important for students to understand and communicate the reasoning for finding an inverse function. This can often be accomplished by considering the independent and dependent variables, the context of the problem, and a chosen solution pathway.

X	1	100	1,000	10,000	100,000
f(x)					

NC.M3.F-BF.4b

Build new functions from existing functions.

Find an inverse function.

b. Determine if an inverse function exists by analyzing tables, graphs, and equations.

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)
- Understand inverse relationships (NC.M3.F-BF.4a)

Connections

- Represent inverse functions (NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the reasoning in needing a restricted domain.

New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

In Math 1, students learned to determine if a relation is a function by analyzing tables, equations, and graphs. In Math 3, students need to determine if a function is invertible and on what domain.

This part of the standard is not limited by function type. This means that students should be able to determine if any function or a portion of the function has an inverse function from different representations.

Assessing for Understanding

The standard states that students must determine if an inverse function exists, so presenting graphs, tables, and equations are all appropriate representations for students to analyze. Additionally, especially for quadratic functions, students must be able to determine the appropriate domain for a function to have an inverse.

Example: Given the table below, tell if an inverse function exists and if it does, graph the inverse.

x	0	1	2	3	4
$f(x)$	0.25	0.5	1	2	4

Example: For the function represented in the table on the right, would an inverse function exist? Explain.

x	**y**
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

NC.M3.F-BF.4c

Build new functions from existing functions.

Find an inverse function.

c. If an inverse function exists for a linear, quadratic and/or exponential function, f, represent the inverse function, f^{-1}, with a table, graph, or equation and use it to solve problems in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)
- Understand inverse relationships and determine if an inverse exist (NC.M3.F-BF.4a, NC.M3.F-BF.4b)

Connections

- Use logarithms to expression solutions to exponential functions (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation (tabular, graphical, or symbolic) is the most efficient to solve a particular problem.

New Vocabulary: inverse function

Mastering the Standard

Comprehending the Standard

Once students understand the concept of a function that has an inverse, they can begin solving for the inverse functions. The idea of reversing the input and output (x and y) is central to solving for an inverse algebraically, and it should also be emphasized on the graph (reflection over the $y = x$ line) and table.

It is important to note; the algebraic approach can be complex in many cases. Often, tables and graphs can be used to solve problems in a more efficient and student friendly manner.

In Math 3, the functions are limited to linear, quadratic, and exponential. For

Assessing for Understanding

Most assessment items for this standard will ask students to solve for an inverse using a graph or equation. Real-world context exists with common conversion formulas, area/volume formulas, and interest formulas.

Example*:* Find the inverse of the function $g(x) = 2^x$ and demonstrate it as the inverse using input – output pairs.

quadratics, it must be emphasized that we have the equation in a form we can solve for the input variable, so this can be an appropriate concept in which to review completing the square and vertex form, from Math 2.

NC.M3.F-LE.4

Construct and compare linear and exponential models and solve problems.

Use logarithms to express the solution to $ab^{ct} = d$ where a, b, c, and d are numbers and evaluate the logarithm using technology.

Concepts and Skills

Pre-requisite

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method and each step in the solving process (NC.M3.A-REI.1)
- Understand the inverse relationship between functions (NC.M3.F-BF.4a)
- Represent inverse functions (NC.M3.F-BF.4c)

Connections

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The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss logarithms as the inverse function of an exponential function.

New Vocabulary: logarithm

Mastering the Standard

Comprehending the Standard

Building on the inverse relationship students conceptualized for exponents and logarithms in F-BF.4, students will rewrite exponents in logarithmic form and use it to solve equations, both algebraically and in the context of word problems.

Students will also need to be able to determine numerical approximations for the logarithms using technology.
For Example: Rewrite the following in logarithmic form. Then, evaluate the logarithms using technology.
a) $10^x = 1000$ b) $3^x = 1000$

Students should use the relationship between exponential and logarithmic functions to solve problems.

$$b^c = d \leftrightarrow log_b\ d = c$$

a. Students can use substitution to reveal another relationship that can be used to solve the original problem. For

Assessing for Understanding

Students must demonstrate the ability to solve exponential equations for an exponent variable using logarithms, and they should be able to express their answer in logarithmic form and using a decimal approximation.

Example: Consider the following investments.

a) A parent invests $2,000 at a 5% interest rate to help his daughter save for college. How long will it take his money to double? (Show your equation and the work.)
b) A banker invests $50,000 at a 5% interest rate to make money for Wells Fargo. How

example:

$$5^{x+3} = 372$$

The goal is to rewrite each expression so they both have the same base. In this case, we are using 10.
Starting with the expression on the left, $5 = 10^{m}$, rewrite using logarithmic form. We see that $m = log_{10} 5$. Using substitution, this means that $5 = 10^{log_{10} 5}$
Using the same procedure with the expression on the right we get, $372 = 10^{log_{10} 372}$.
We can now substitute these back into the original equation.

$$5^{x+3} = 372$$
$$(10^{log_{10} 5})^{x+3} = 10^{log_{10} 372}$$

Because this is an equation and both sides of the equation are base 10, the exponents must be equal. This reveals a new equation that can be used to solve for x.

$$(log_{10} 5)(x+3) = log_{10} 372$$
$$x = \frac{log_{10} 372}{log_{10} 5} - 3$$
$$x \approx .6776$$

b. Students are expected to rewrite an exponential equation into logarithmic form to find or approximate a solution. For example:

$$5^{x+3} = 372$$
$$log_5 372 = x + 3$$
$$log_5 372 - 3 = x$$
$$x \approx .6776$$

Students are **not** expected to know or use the properties of logarithms, *e*, or natural logs to solve problems. These can be extension topics, but are beyond the scope of the NC Math 3 standards.

long will it take the bank's money to double? (Show your equation and the work.)

c) What do you notice about the answers? Based on your work, why is that the case?

NC.M3.A-SSE.2

Interpret the structure of expressions.

Use the structure of an expression to identify ways to write equivalent expressions.

Concepts and Skills

Pre-requisite

- Justifying a solution method (NC.M2.A-REI.1)

Connections

- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

In Math 1 and 2, students factored quadratics. In Math 3, extend factoring to include strategies for rewriting more complicated expressions. Factoring a sum or difference of cubes, factoring a GCF out of a polynomial, and finding missing coefficients for expressions based on the factors can all be included.

For Example: When factoring a difference of cubes, is the trinomial factor always, sometimes or never factorable? How do you know?

Assessing for Understanding

This standard can be assessed mainly by performing the algebraic manipulation. Problems could include:

Example: Rewrite the following exponential equations to show the rate of growth or decay.

a) $A(t) = 500(1.035)^{t}$ answer: $A(t) = 500(1 + 0.035)^{t}$

b) $V(t) = 15{,}000(0.87)^{t}$ answer: $V(t) = 15{,}000(1 - 0.13)^{t}$

NC.M3.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solution method and the steps in the solving process (NC.M2.A-REI.1)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why it is necessary to write two equations to solve an absolute value equation.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. In Math 3, students should extend their knowledge of all equations they are asked to solve.

When solving equations, students will use mathematical reasoning to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Students do not have to use the proper names of the properties of operations and equality, but they should recognize and use the concepts associated with the properties.

Assessing for Understanding

Solving equations including justifications for each step, error analysis of solutions to equations, and comparing and analyzing different methods are all appropriate methods of assessing this standard.

Example: Solve the following three equations for x. Explain the rationale for the differences in your steps and solutions.

a) $2^6 = x$
b) $3(2)^6 = x$
c) $2^x = 6$
d) $3(2)^x = 6$

NC.M3.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Concepts and Skills

Pre-requisite

- Build new function by combine linear, quadratic and exponential functions (NC.M1.F-BF.1b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify new function and discuss how the new function fits the context.

Mastering the Standard

Comprehending the Standard

This standard asks students to combine standard function types by addition, subtraction, and multiplication. In Math 3, we are NOT required to include composition, although it could be a valuable extension.

The key concept for teaching this standard is a review of adding and subtracting expressions (including combining like terms) and multiplying expressions (distributing polynomials and exponent rules).

Assessing for Understanding

In assessing this standard, students will need to perform the operations and determine from a context which operation is appropriate. The functions that students need to combine should be given in problems, but the operation can be determined from context if necessary.

Example: A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.

The North Carolina High School Collaborative Instructional Framework

NC Math 3

Unit 3: Polynomial Functions

9 Days Block Schedule | **September 2017 Update** | 18 Days Traditional Schedule

RESEARCH BRIEF: Unit 3: Polynomial Functions

Essential Questions:

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables increase?
- How can functions be used to model real world situations?

Learning Outcomes	**Student Objectives**
• Model surface area and volume of geometric figures with polynomial functions. • Create and solve polynomial equations from a contextual situation. • Given a polynomial function students will determine key features of a graph, table, or context. • Students should be able to compare features of two functions in different representations. • Students should be able to understand and interpret domain and range of an polynomial. • Create an equation and interpret reasonable solutions in context. • Interpret the relationship between factors and zeros of a polynomials. • Given a function create an equation from various	• I will **find** key **features** of a polynomial function from a graph, table, or context. • I will **compare** features of two functions in different representations. • I will **interpret** the relationship between input and output of a polynomial function. • I will be able to **create** a polynomial function from various representations. • I will be able to read a word problem and **create** an equation or inequality. • I will **interpret** parts of a function and their relationship with the graph, table, and context. • I will be able to **solve** and **interpret** the solutions of two equations graphically.

representations and use them to solve problems.
- Interpret structure of a polynomial function and relationship with graph, table, and/or context.
- Given two functions, solve and interpret equations graphically.
- Apply Remainder Theorem, Factor Theorem, and the Division Algorithm.

- I will be able to **apply** Remainder Theorem, Factor Theorem, and Division Algorithm.

Standards Addressed in this Unit

Understand surface area and volume of geometric figures can be modeled by polynomial functions.

- **NC.M3.G-GMD.3**: Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

- **NC.M3.G-MG.1**: Apply geometric concepts in modeling situations Use geometric and algebraic concepts to solve problems in modeling situations:
    - Use geometric shapes, their measures, and their properties, to model real-life objects.
    - Use geometric formulas and algebraic functions to model relationships.
    - Apply concepts of density based on area and volume.
    - Apply geometric concepts to solve design and optimization problems.

Understand and apply the Fundamental Theorem of Algebra, the Remainder Theorem, the Factor Theorem, and the Division Algorithm. Create polynomial equations in one or two variables and use them to solve problems algebraically and graphically.

- **NC.M3.N-CN.9**: Use the Fundamental Theorem of Algebra to determine the number and potential types of solutions for polynomial functions.

- **NC.M3.A-SSE.1a**: Interpret expressions that represent a quantity in terms of its context.
    a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

- **NC.M3.A-SSE.1b**: Interpret expressions that represent a quantity in terms of its context.
    b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

- **NC.M3.A-SSE.2:** Use the structure of an expression to identify ways to write equivalent expressions.

- **NC.M3.A-APR.2**: Understand and apply the Remainder Theorem

- **NC.M3.A-APR.3**: Understand the relationship among factors of a polynomial expression, the solutions of a polynomial equation and the zeros of a polynomial function.

- **NC.M3.A-CED.1**: Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

- **NC.M3.A-CED.2**: Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

- **NC.M3.A-REI.1:** Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

- **NC.M3.F-BF.1a**: Write a function that describes a relationship between two quantities.
  a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

- **NC.M3.F-BF.1b:** Write a function that describes a relationship between two quantities.
  b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Recognize key features, zeros, and transformations of polynomial functions. Analyze a polynomial function and compare two or more functions by using their key features. Analyze and compare the relative rates of growth of exponential and polynomial functions.

- **NC.M1.F-IF.4:** Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

- **NC.M3.F-IF.7**: Analyze functions using different representations. Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

- **NC.M3.F-IF.9**: Analyze functions using different representations. Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

- **NC.M3.F-BF.1a**: Build a function that models a relationship between two quantities. Write a function that describes a relationship between two quantities. a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

- **NC.M3.F-BF.1b**: Build a function that models a relationship between two quantities. Write a function that describes a relationship between two quantities. b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.
- **NC.M3.F-BF.3**: Build new functions from existing functions. Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x) + k$, $f(x + k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).
- **NC.M3.F-LE.3**: Construct and compare linear and exponential models and solve problems. Compare the end behavior of functions using their rates of change over intervals of the same length to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a polynomial function.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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NC.M3.G-GMD.3

Explain volume formulas and use them to solve problems.

Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

Concepts and Skills
Pre-requisite
- Know and use formulas for volumes of cones, cylinders, and spheres (8.G.9)
Connections
- Solve for a quantity of interest in formulas (NC.M1.A-CED.4) - Apply geometric concepts in modeling situations (NC.M3.G-MG.1)

The Standards for Mathematical Practices
Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard. 1 – Make sense of problems and persevere in solving them
Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

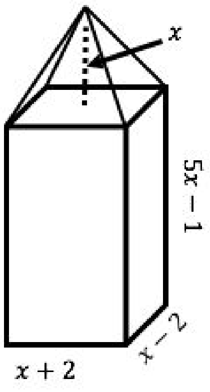
This standard focuses on volume and the use of volume formulas to solve problems. The figures may be a single shape or a composite of shapes.

Formulas for more complex figure should be provided.

Assessing for Understanding

Students should be able to identify the 3-D figures (prisms, cylinders, pyramids, cones and spheres) and the measurements needed to calculate the volume.

Example: Calculate the volume of the 3D figure below.



NC.M3.G-MG.1

Apply geometric concepts in modeling situations.

Apply geometric concepts in modeling situations

- Use geometric and algebraic concepts to solve problems in modeling situations:
- Use geometric shapes, their measures, and their properties, to model real-life objects.
- Use geometric formulas and algebraic functions to model relationships.
- Apply concepts of density based on area and volume.
- Apply geometric concepts to solve design and optimization problems.

Concepts and Skills

Pre-requisite

- Solve real world problems involving area, volume, and surface area (7.G.6)
- Use volume formulas to solve problems (NC.M3.G-GMD.3)

Connections

- Apply properties, definitions, and theorems of 2-D figures to solve problems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

For this standard, students should engage in problems that are more complex than those studied in previous grades. The standard combines geometric and algebraic concepts and focuses on four primary areas:

i. model real-world three-dimensional figures,
ii. model relationships,
iii. determine density based on area or volume

When students model real-world three dimensional figures they must recognize the plane shapes that comprise the figure. They must be flexible in constructing and deconstructing the shapes. Students also need to be able to identity the measures associated with the figure such as circumference, area, perimeter, and volume.

Students use formulas and algebraic functions when modeling relationships. This may include examining how the one measurement changes as another changes.

How does the volume of a cylinder change as the radius changes?
How does the surface area of a prism change as the height changes?

The concept of density based on area and volume is to calculate the mass per unit.

Assessing for Understanding

Students recognize situations that require relating two- and three- dimensional objects. They estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects. Students apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).

Use geometric and algebraic concepts to solve problems in modeling situations.

> **Example**: A gas company wants to determine what shape truck will hold the most gas to transport to the gas stations. The truck with a 58 foot bed can hold either a cylinder of diameter x ft. or a rectangular prism with a width and height of x ft. The have found out that a new, more advanced truck can increase the length of the diameter, width, and height by 4. Write a function to represent the volume of each container for the new truck. Which one can hold the most gas?

Examples for area density are:

Description	*Unit of Measure*
Data Storage	Gigabytes per square inch
Thickness of Paper	Grams per square meter
Bone density	Grams per square centimeter
Body Mass Index	Kilograms per square meter
Population	People per square mile

Examples for volume density are:

Description	*Unit of Measure*
Solids	Grams per cubic centimeter
Liquids	Grams per millliter (1 mL = 1 cubic cm)

NC.M3.N-CN.9

Use complex numbers in polynomial identities and equations.

Use the Fundamental Theorem of Algebra to determine the number and potential types of solutions for polynomial functions.

Concepts and Skills

Pre-requisite

- Understand the relationship between the factors and the zeros of a polynomial function (NC.M3.A-APR.3)

Connections

- Interpret parts of an expression (NC.M3.A-SSE.1a)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Creating equations to solve or graph (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method and the steps in the solving process (NC.M3.A-REI.1)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Finding and comparing key features of functions (NC.M3.F-IF.4, 7, 9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

3 – Construct viable arguments and critique the reasoning of others

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss how can you determine the number of real and imaginary solutions of a polynomial.

New Vocabulary: The Fundamental Theorem of Algebra

Mastering the Standard

Comprehending the Standard

Students know The Fundamental Theorem of Algebra, which states that *every polynomial function of positive degree n has exactly n complex zeros (counting multiplicities).* Thus a linear equation has 1 complex solution, a quadratic has two complex solutions, a cubic has three complex solutions, and so on. The zeroes do not have to be unique. For instance $(x-3)^2 = 0$ has zeroes at $x = 3$ and $x = 3$. This is considered to have a double root or a multiplicity of two.

Students also understand the graphical (x-intercepts as real solutions to functions) and

Assessing for Understanding

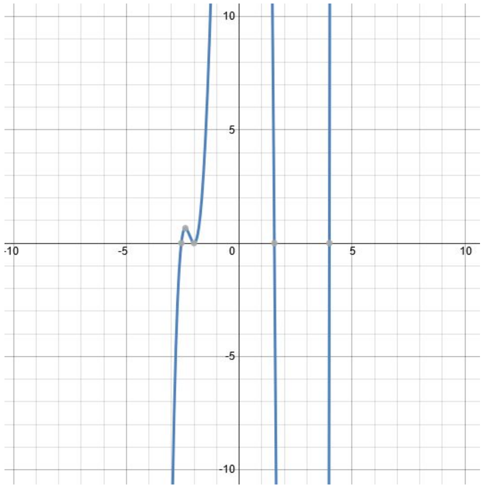
First, students need to be able to identify the number of solutions to a function by relating them to the degree.

Example: How many solutions exist for the function $(x) = x^4 - 10x + 3$?

Going deeper into the standard, students need to determine the types of solutions using graphical or algebraic methods, where appropriate.

Example (*real and imaginary solutions*): How many, and what type, of solutions exist for the function $(x) = x^4 - 10x^2 - 21x - 12$?

Example (*with multiplicity of 2*): How many, and what type, of solutions exist for the function $f(x) = x^5 - 3x^4 - 27x^3 + 19x^2 + 114x - 72$?

algebraic (solutions equal to zero by methods such as factoring, quadratic formula, the remainder theorem, etc.) processes to determine when solutions to polynomials are real, rational, irrational, or imaginary.	**Example**: What is the lowest possible degree of the function graphed below? How do you know? What is another possible degree for the function? 

NC.M3.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify and interpret parts of an expression in context (NC.M2.A-SSE.1a)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Interpret parts of an expression as a single entity (NC.M3.A-SSE.1b)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students need to be able to determine the meaning, algebraically and from a context, of the different parts of the expressions noted in the standard. At the basic level, this would refer to identifying the terms, factors, coefficients, and exponents in each expression. Students must also be able to identify how these key features relate in context of word problems.	Students should be able to identify and **explain the meaning** of each part of these expressions. **Example:** The expression $.0013x^3 - .0845x^2 + 1.6083x + 12.5$ represents the gas consumption by the United States in billions of gallons, where x is the years since 1960. Based on the expression, how many gallons of gas were consumed in 1960? How do you know?

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

As we add piecewise functions and expressions in Math 3, breaking down these expressions and functions into their parts are essential to ensure understanding.

For Example: Explain what operations are performed on the inputs -2, 0, and 2 for the following expression:

$f(x) = \quad \{3x,\ for\ x < 0\ \frac{1}{x},\ for\ 0 \leq x < 2\ x^3,\ for\ x \geq 2$

Which input is not in the domain? Why not?

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: If the expression $(x+2)(x-2)(5x-1)$ represents the measurements from a rectangular prism, what could entire expression and each of the factors represent?

NC.M3.A-APR.2

Understand the relationship between zeros and factors of polynomials.

Understand and apply the Remainder Theorem.

Concepts and Skills

Pre-requisite

- Evaluate functions (NC.M1.F-IF.2)
- Division of polynomials (NC.M3.A-APR.6)

Connections

- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method and the steps in the solving process (NC.M3.A-REI.1)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to accurately explain Remainder Theorem in their own words.

Recalled Vocabulary: Divisor, Dividend, Quotient, Remainder

Mastering the Standard

Comprehending the Standard

Students must understand that the Remainder Theorem states that if a polynomial $p(x)$ is divided by any binomial $(x-c)$, *which does not have to be a factor of the polynomial*, the remainder is the same as if you evaluate the polynomial for c (meaning to evaluate $p(c)$). If the remainder $p(c)=0$ then $(x-c)$ is a factor of $p(x)$ and c is a solution of the polynomial.

Students should be able to know and apply all of the Remainder Theorem. Teachers should not limit the focus to just finding roots.

Students can discover this relationship by completing the division and evaluating the function for the same value to see how the remainder and the function's value are the same.

Assessing for Understanding

Students should be able to apply the Remainder Theorem.

Example: Let $p(x) = x^5 - x^4 + 8x^2 - 9x + 30$. Evaluate $p(-2)$. What does the solution tell you about the factors of $p(x)$?

Solution: $p(-2) = 32$. This means that the remainder of $\frac{x^5 - x^4 + 8x^2 - 9x + 30}{x+2}$ is $\frac{32}{x+2}$. This also means that $x+2$ is not a factor of $x^5 - x^4 + 8x^2 - 9x + 30$.

Example: Consider the polynomial function: $P(x) = x^4 - 3x^3 + ax^2 - 6x + 14$, where a is an unknown real number. If $(x-2)$ is a factor of this polynomial, what is the value of a?

NC.M3.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

Concepts and Skills

Pre-requisite

- Create one variable equations and solve (NC.M2.A-CED.1)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Justify a solution method (NC.M3.A-REI.1)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Justify a solution method (NC.M3.A-REI.1)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Student should be able to explain and defend the model they chose to represent the situation.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematical equations to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Creating one variable equations and inequalities are included in Math 1, 2, and 3. In previous courses, students modeled with linear, exponential, quadratic, radical, and inverse variation equations. In Math 3, students will be expected to model with polynomial, rational, absolute value, and exponential equations. Students will need to analyze a problem, determine the type of equation, and set up and solve these problems. Students may need to create an equation from different representations found in the context. This makes it important for students to realize that equations can be derived as a specific instance of an associated function.

Students are expected to represent the solutions of an inequality using a number line and compound inequalities using inequality and interval notation.

Assessing for Understanding

Students should be able to create and solve problems algebraically and graphically. There should be a focus on using methods efficiently.

Example: Clara works for a marketing company and is designing packing for a new product. The product can come in various sizes. Clara has determined that the size of the packaging can be found using the function, $p(b) = (b)(2b+1)(b+5)$, where b is the shortest measurement of the product. After some research, Clara determined that packaging with $20,500\ cm^3$ will be the most appealing to customers. What are the dimensions of the package?

NC.M3.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

Concepts and Skills

Pre-requisite

- Create and graph two-variable equations (NC.M2.A-CED.2)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Write the equations and inequalities of a system (NC.M3.A-CED.3)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

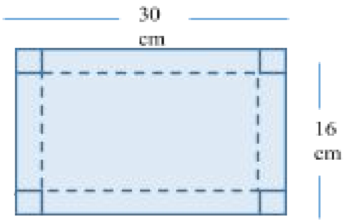
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. In A-CED.1, writing and solving an equation is the essential skill required. In this standard, graphing the equation to determine key features is essential.

This standard is included in Math 1, 2, and 3. Throughout all three courses, students create equations in two variables and graph them on coordinate axes. In Math 3, absolute value, polynomial, and rational graphs are introduced, and exponential graphs are further developed to solve for

Assessing for Understanding

Rate of growth and decay, work rate (and other rates), geometric, and other real-world examples provide the context for many of these problems.

Example: A company is manufacturing an open-top rectangular box. They have 30 cm by16 cm sheets of material. The bins are made by cutting squares the same size from each corner of a sheet, bending up the sides, and sealing the corners. Create an equation relating the volume V of the box to the length of the corner cut out x. Graph the equation and identify the dimensions of the box that will have the maximum volume. Explain.



the exponent.	

NC.M3.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Concepts and Skills

Pre-requisite

- Interpret key features from graph, tables, and descriptions (NC.M2.F-IF.4)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identified key features with mathematical reasoning.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math 3, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity as motion that is repeated in equal intervals of time and discontinuity as values that are not in the domain of a function, either as asymptotes or "holes" in the graph.

No limitations are listed with this standard. This means that all function types, even those found in more advanced courses. Students do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.

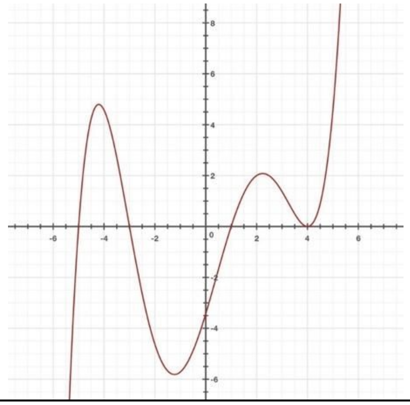
This is in contrast to NC.M3.F-IF.7, in which the specific function types are included. Students can work algebraically with those listed types and can analyze those functions in greater detail.

Students are expected to use and interpret compound inequalities using

Assessing for Understanding

This standard must be assessed using three important forms of displaying our functions: graphs, tables, and verbal descriptions/word problems. Students must be able to interpret each and how they apply to the key input-output values.

Example: For the function below, label and describe the key features. Include intercepts, relative max/min, intervals of increase/decrease, and end behavior.



inequality and interval notation to describe key features when appropriate.	

NC.M3.F-IF.9

Analyze functions using different representations.

Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

-

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss how the comparison of a functions leads to a mathematical understanding, such as with transformations and choosing better models.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

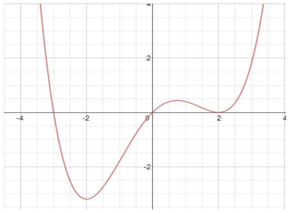
In Math 3, this standard can include two functions of any type students have learned in high school math in any representation. Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

In assessing this standard, students must demonstrate that they can not only identify, but compare, the key features of two different functions. Appropriate question stems could include: Which is less/greater; Which will have a greater value at x = __; Which function has the higher maximum/lower minimum; etc.

Examples*:* If $f(x) =- (x+7)^2(x-2)$ and $g(x)$ is represented on the graph.



a) What is the difference between the zero with the least value of $f(x)$ and the zero with the least value of $g(x)$?

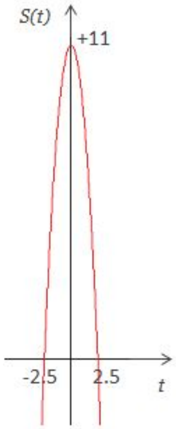
b) Which has the largest relative maximum?

c) Describe their end behaviors. Why are they different? What can be said about each function?

Example: Two objects dropped downward at the same time from a top of building. For both functions, *t* represents seconds and the height is represented in feet.

The function's data of the first object is given by this table:

t	$s(t)$
0	20
2.5	15
3.5	10
4.3	5
5	0



The function's graph of the second object is shown at the right:

a) Which object was dropped from a greater height? Explain your answer.
b) Which object hit the ground first? Explain your answer.
c) Which object fell at a faster rate (in ft/sec)? Explain your answer.

NC.M3.F-IF.7

Analyze functions using different representations.

Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Concepts and Skills

Pre-requisite

- Analyze functions using different representations to show key features (NC.M2.F-IF.7)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Create and graph equations in two variables (NC.M3.A-CED.2)
- Analyze graphs and tables and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.9)
- Build functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Understand the effects of transformations on functions (NC.M3.F-BF.3)
- Use graphs, tables and description to work with inverse functions (NC.M3.F-BF.4a, NC.M3.F-BF.4b, NC.M3.F-BF.4c)
- Compare the end behavior of functions using the rate of change (NC.M3.F-LE.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation best shows each of the key features.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

In previous math courses, students have identified the characteristic of graphs of other functions, including linear, quadratic, exponential, radical, and inverse variation functions. They should be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior.

In Math 3, these concepts are extended to piecewise, absolute value, polynomials, exponential, rational, and sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity are new features of functions that

Assessing for Understanding

In assessing this standard, students must demonstrate their ability to represent and determine the key features from algebraic and graphical representations of the functions.

must be introduced. The intent of this standard is for students to find discontinuities in tables and graphs and to recognize their relationship to functions. Students are not expected to find an asymptote from a function. (This could be an extension topic.) This standard will likely span multiple units, as most Math 3 courses teach polynomial, exponential, rational, and trigonometric functions in different units. These function characteristics will be repeated and reinforced throughout the course. Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.	**Example:** Graph $g(x) = x^3 + 5x^2 + 2x - 8$. a) Identify zeroes. b) Discuss the end behavior. c) In what intervals is the function increasing? Decreasing?

NC.M3.F-BF.1a

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

a. Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

Concepts and Skills

Pre-requisite

- Build quadratic functions given a graph, description, or ordered pair (NC.M2.F-BF.1)
- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the effects of transforming functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss when multiple models can describe the information given, for example, when given the two roots, multiple models can contain those roots.

Mastering the Standard

Comprehending the Standard

This standard relates to building functions in two different contexts – polynomial (with real solutions) and exponential. In many Math 3 courses, it will be covered in two different units.

When building polynomial functions, only those with real solutions are considered. The relationship between solutions and factors, multiplicity and graphs, and the leading coefficient's sign relating to the end behaviors are all essential to build these functions.

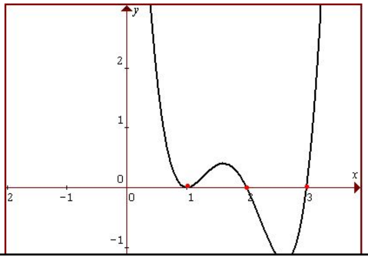
Assessing for Understanding

For both functions, it is important that the assessment questions include algebraic "math" questions and questions in context. The answers to questions assessing this standard should be the actual function they are building, as other standards allow students to identify and interpret key features.

Example: Build polynomial functions with a double root at -2 and another root at 5.

This example should be connected to NC.M3.F-BF.3, as students should understand which transformations functions do not change the zeros of the functions. This could also be connected to NC.M3.N-CN.9, as students should understand how to create multiple equations that could be solved with the same roots.

Example: Build a polynomial function that could represent the following graph, and explain how each characteristic you could see on the graph helped you build the function.



NC.M3.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Concepts and Skills

Pre-requisite

- Build new function by combine linear, quadratic and exponential functions (NC.M1.F-BF.1b)
- Operations with polynomials (NC.M1.A-APR.1)
- Operations with rational expressions (NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify new function and discuss how the new function fits the context.

Mastering the Standard

Comprehending the Standard

This standard asks students to combine standard function types by addition, subtraction, and multiplication. In Math 3, we are NOT required to include composition, although it could be a valuable extension.

The key concept for teaching this standard is a review of adding and subtracting expressions (including combining like terms) and multiplying expressions (distributing polynomials and exponent rules).

Assessing for Understanding

In assessing this standard, students will need to perform the operations and determine from a context which operation is appropriate. The functions that students need to combine should be given in problems, but the operation can be determined from context if necessary.

Example: Last year, army engineers modeled the function of a bullet fired by a United States soldier from a certain weapon. The function $f(x) =- 16x^2 + 200x + 4$ modeled the path of the bullet. This year, the soldiers were supplied with more powerful guns that changed the path of the bullet from higher ground by adding the function $g(x) = 300x + 20$. What function models the path of the new bullet?

Example: Consider the functions: $f(x) = 4x + 9$ and $g(x) = -2x - 4$

a) Evaluate $f(-3)$.

b) Evaluate $g(-3)$.

c) Add $f(x) + g(x)$.

d) Evaluate $(f + g)(-3)$.

e) What do you notice? What properties have you learned that explain your answer?

NC.M3.F-BF.3

Build new functions from existing functions.

Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k{\cdot}f(x)$, $f(x)+k$, $f(x+k)$ to include $f(k{\cdot}x)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Understand the effects of transformations on functions (NC.M2.F-BF.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build polynomial and exponential functions from a graph, description, or ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of *k*.

Mastering the Standard

Comprehending the Standard

Students learned the translation and dilation rules in Math 2 with regard to linear, quadratic, square root, and inverse variation functions. In Math 3, we apply these rules to functions in general.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions. Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of *k*.

Assessing for Understanding

In demonstrating their understanding, students must be able to relate the algebraic equations, graphs, and tabular representations (ordered pairs) as functions are transformed. Appropriate questions will ask students to identify and explain these transformations.

Example: Suppose $f(x)=x^2$ where *x* can be any real number.

a) Sketch a graph of the function *f*.
b) Sketch a graph of the function *g* given by $g(x)=f(x)+2$ g(x)=f(x).
c) How do the graphs of *f* and *g* compare? Why?
d) Sketch a graph of the function *h* given by $h(x) =-2\cdot f(x)$.
e) How do the graphs of *f* and *h* compare? Why?
f) Sketch a graph of the function *p* given by $p(x)=f(x+2)$.
g) How do the graphs of *f* and *p* compare? Why?

For commentary go to https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/741.

NC.M3.F-LE.3

Construct and compare linear and exponential models and solve problems.

Compare the end behavior of functions using their rates of change over intervals of the same length to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a polynomial function.

Concepts and Skills

Pre-requisite

- Calculate and interpret the average rate of change (NC.F-IF.6)
- Compare the end behavior of linear, exponential and quadratic functions (NC.M1.F-LE.3)
- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.7, NC.M3.F-IF.9)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the rate of change for each function type as the value of the domain increases.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1 and 3. In previous courses, students studied linear, exponential, and quadratic models. In Math 3, polynomial functions are included.

For Example: For the functions $f(x)=x^3$ and $g(x)=3^x$, which function has a greater value at:

a) $x = 0.5$ b) $x = 1$

c) $x = 1.5$ d) $x = 2$

Assessing for Understanding

Students must demonstrate that they understand how exponential functions ultimately increase at a greater rate than polynomial functions when considering the end behavior – namely, the rate of change is greater for an exponential function as the function increases to infinity.

Example: Using technology, determine the average rate of change of the following functions for intervals of their domains in the table.

Functions	Average rate of change $0\leq x\leq 10$	Average rate of change $10\leq x\leq 20$	Average rate of change $20\leq x\leq 30$	Average rate of change $30\leq x\leq 40$	Average rate of change $40\leq x\leq 50$
$f(x)=x^3$					
$f(x)=1.3^x$					

a) When does the average rate of change of the exponential function exceed the average rate of change of the polynomial function?

b) Using a graphing technology, graph both functions. How do the average rates of change in your table relate to what you see on the graph?

Note: You can use the information in your table to determine how to change the setting to see where the functions intersect.

c) In your graphing technology, change the first function to $f(x)=x^4$ and adjust the settings to see where the functions intersect. What do you notice about the rates of change interpreted from the graph?

e) $x = 2.5$ f) $x = 3$ g) $x = 3.5$ h) $x = 4$	d) Make a hypothesis about the rates of change about polynomial and exponential function. Try other values for the exponent of the polynomial function to support your hypothesis.

NC.M3.A-SSE.2

Interpret the structure of expressions.

Use the structure of an expression to identify ways to write equivalent expressions.

Concepts and Skills

Pre-requisite

- Justifying a solution method (NC.M2.A-REI.1)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Justify a solution method (NC.M3.A-REI.1)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Sum or Difference of Cubes

Mastering the Standard

Comprehending the Standard

In Math 1 and 2, students factored quadratics. In Math 3, extend factoring to include strategies for rewriting more complicated expressions. Factoring a sum or difference of cubes, factoring a GCF out of a polynomial, and finding missing coefficients for expressions based on the factors can all be included.

For Example: When factoring a difference of cubes, is the trinomial factor always, sometimes or never factorable? How do you know?

Assessing for Understanding

This standard can be assessed mainly by performing the algebraic manipulation. Problems could include:

Example: Factor $x^3 - 2x^2 - 35x$

Example: The expression $(x + 4)$ is a factor of $x^2 + kx - 20$. What is the value of k? How do you know?

Example: Factor $x^3 - 8$

NC.M3.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solution method and the steps in the solving process (NC.M2.A-REI.1)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why it is necessary to write two equations to solve an absolute value equation.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. In Math 3, students should extend their knowledge of all equations they are asked to solve.

When solving equations, students will use mathematical reasoning to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Students do not have to use the proper names of the properties of operations and equality, but they should recognize and use the concepts associated with the properties.

Assessing for Understanding

Solving equations including justifications for each step, error analysis of solutions to equations, and comparing and analyzing different methods are all appropriate methods of assessing this standard.

Example: Describe your process for solving the following polynomial and explain the mathematical reasoning for each step.

$$x^3 + 4x^2 + x = 6$$

The North Carolina High School Collaborative Instructional Framework

NC Math 3

Unit 4: Modeling with Geometry

7 Days Block Schedule	**September 2017 Update**	14 Days Traditional Schedule

RESEARCH BRIEF: Modeling with Geometry

Essential Questions:

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables increase?
- How can functions be used to model real world situations?
- How can functions be used to model geometric situations?

Learning Outcomes	**Student Objectives**
• Model surface area and volume of geometric figures with polynomial functions. • Create equations of a circle with completing the square. • Visualize cross-sections of 3D figures and the resulting figure from a rotation of 2D figure. • Apply geometric formulas to model real-life situations and solve optimization problems. • Prove geometric properties of triangles and quadrilaterals.	• I will **use** completing the square to **write** the equation of a circle. • I will **represent** the volume of a figure algebraically. • I will **interpret** the structure of a volume formula. • I will **identify** the resulting shape from cross-sections and rotations. • I will **use** geometric formulas to model real-life situations and **solve** for optimization problems. • I will **prove** properties of geometric figures, including triangles and quadrilaterals. • I will **solve** problems in properties of triangle points of concurrency.

Standards Addressed in this Unit

Derive the equation of a circle as well as distinguishing the center and radius of a circle from an equation.

- **NC.M3.G-GPE.1**: Translate between the geometric description and the equation for a conic section. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Implement surface area and volume of geometric figures and model using polynomial functions. Furthermore, relating cross sections with two-dimensional and three-dimensional figures.

- **NC.M3.G-GMD.3**: Explain volume formulas and use them to solve problems. Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

- **NC.M3.G-GMD.4**: Visualize relationships between two-dimensional and three-dimensional objects. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

- **NC.M3.G-MG.1**: Apply geometric concepts in modeling situations. Apply geometric concepts in modeling situations Use geometric and algebraic concepts to solve problems in modeling situations:
    - Use geometric shapes, their measures, and their properties, to model real-life objects.
    - Use geometric formulas and algebraic functions to model relationships.
    - Apply concepts of density based on area and volume.
    - Apply geometric concepts to solve design and optimization problems.

- **NC.M3.G-CO.14**: Prove geometric theorems. Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

- **NC.M3.A-SSE.1b**: Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

- **NC.M3.A-SSE.2:** Use the structure of an expression to identify ways to write equivalent expressions.

- **NC.M3.A-REI.1:** Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

- **NC.M3.F-BF.1b:** Write a function that describes a relationship between two quantities.
    b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)
- Interpret inverse functions in context (NC.M3.F-IF.4c)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

As we add piecewise functions and expressions in Math 3, breaking down these expressions and functions into their parts are essential to ensure understanding.

For Example: Explain what operations are performed on the inputs -2, 0, and 2 for the following expression:

$$f(x) = \{3x,\ for\ x < 0\ \frac{1}{x},\ for\ 0 \le x < 2\ x^3,\ for\ x \ge 2$$

Which input is not in the domain? Why not?

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: A progressive tax system increases the percentage of income tax as the income level increases. The following piecewise function describes a certain state's income tax. Write a paragraph explaining the tax system, and determine the amount of taxes paid by families with incomes of \$20,000, \$75,000, and \$160,000. Does this system seem fair? Why or why not?

$$\begin{cases} 0, for\ x \le 25{,}000 \\ 0.08x, for\ 25{,}000 < x \le 50{,}000 \\ 4000 + 0.15(x - 50{,}000), for\ 50{,}000 < x \le 125{,}000 \\ 15{,}250 + 0.3(x - 125{,}000) for\ x > 125{,}000 \end{cases}$$

NC.M3.A-SSE.2

Interpret the structure of expressions.

Use the structure of an expression to identify ways to write equivalent expressions.

Concepts and Skills

Pre-requisite

- Justifying a solution method (NC.M2.A-REI.1)

Connections

- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Justify a solution method (NC.M3.A-REI.1)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
7 – Look for and make use of structure
8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

In Math 1 and 2, students factored quadratics. In Math 3, extend factoring to include strategies for rewriting more complicated expressions. Factoring a sum or difference of cubes, factoring a GCF out of a polynomial, and finding missing coefficients for expressions based on the factors can all be included.
For Example: When factoring a difference of cubes, is the trinomial factor always, sometimes or never factorable? How do you know?

Assessing for Understanding

Example: The formula for the surface area of a cylinder is often written as $V = 2\pi rh + 2\pi r^2$.

a) Explain the meaning of each part of the formula.
b) Solve the formula for h, in terms of r and V. What might be the benefit of this new formula?

Note: In this example, part a) aligns with NC.M3.A-SSE.1b. For part b), students in Math 3 should should be able to look at the structure of the equation and use that structure it identity the best way forward. A more challenging extension would be to have students solve for r.

NC.M3.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solution method and the steps in the solving process (NC.M2.A-REI.1)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why it is necessary to write two equations to solve an absolute value equation.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. In Math 3, students should extend their knowledge of all equations they are asked to solve.

When solving equations, students will use mathematical reasoning to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Students do not have to use the proper names of the properties of operations and equality, but they should recognize and use the concepts associated with the properties.

Assessing for Understanding

Solving equations including justifications for each step, error analysis of solutions to equations, and comparing and analyzing different methods are all appropriate methods of assessing this standard.

> **Example:** The volume of a sphere is 523.6 in^3. Determine the radius of the sphere and justify each step of your algebraic reasoning.

NC.M3.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Concepts and Skills

Pre-requisite

- Build new function by combine linear, quadratic and exponential functions (NC.M1.F-BF.1b)
- Operations with polynomials (NC.M1.A-APR.1)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify new function and discuss how the new function fits the context.

Mastering the Standard

Comprehending the Standard

This standard asks students to combine standard function types by addition, subtraction, and multiplication. In Math 3, we are NOT required to include composition, although it could be a valuable extension.

The key concept for teaching this standard is a review of adding and subtracting expressions (including combining like terms) and multiplying expressions (distributing polynomials and exponent rules).

Assessing for Understanding

In assessing this standard, students will need to perform the operations and determine from a context which operation is appropriate. The functions that students need to combine should be given in problems, but the operation can be determined from context if necessary.

> **Example:** The length of the base of a rectangular prism is given as $x + 4$, and the width of the base is $x + 2$. The height of the rectangular prism is three more than two times the length. Build a function to model the volume of the rectangular prism.

NC.M3.G-GPE.1

Translate between the geometric description and the equation for a conic section.

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Concepts and Skills

Pre-requisite

- Apply the Pythagorean Theorem to find the distance between two points (8.G.8)
- Write an equivalent form of a quadratic expression by completing the square (NC.M2.A-SSE.3)

Connections

- Work with conic sections (4th level course)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

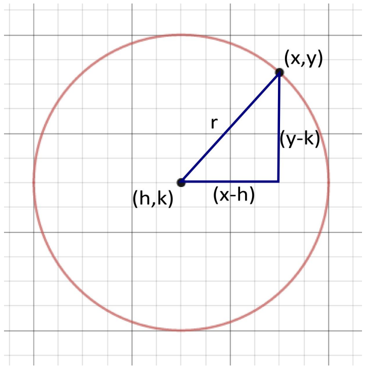
Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students derive the standard equation of a circle by reasoning with circles on the coordinate plane. Given a center (h, k) and a radius r, students determine that the horizontal distance from the center to a point (x, y) on the circle can be expressed by $(x - h)$. Likewise, the vertical distance from the center to the point can be expressed by $(y - k)$. These distances can be modeled by a vertical and horizontal line segment. The radius can be modeled by a line segment connecting the center to the point. A right triangle is formed and the Pythagorean Theorem can be applied to derive $(x-h)^2 + (y-k)^2 = r^2$.



For a circle equation in general form $x^2 + y^2 + cx + dx + e = 0$, students will use the process of completing the square to rewrite and identify the center and radius of the circle. (The process of completing the square is in Math 2 NC.M2.A-SSE.3.)

Assessing for Understanding

Students demonstrate an understanding of the equation of a circle by writing the equation using the center and radius.

Example: Write the equation of a circle that is centered at (-1,3) with a radius of 5 units.

Example: Using the whole numbers 1 – 9 as many times as you like, make the biggest circle by filling in the blanks below:

$$\square x^2 + \square y^2 = \square$$

Source: http://www.openmiddle.com/make-the-biggest-circle/

Example: Write an equation for a circle given that the endpoints of the diameter are (-2,7) and (4,-8)

Example: How many points with two integer coordinates are 5 units away from (-2, 3)?

Source: http://www.openmiddle.com/equidistant-points/

Students can rewrite the equation of a circle to identify the center and radius.

Example: Find the center and radius of the circle $4x^2 + 4y^2 - 4x + 2y - 1 = 0$.

NC.M3.G-GMD.3

Explain volume formulas and use them to solve problems.

Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

Concepts and Skills

Pre-requisite

- Know and use formulas for volumes of cones, cylinders, and spheres (8.G.9)

Connections

- Solve for a quantity of interest in formulas (NC.M1.A-CED.4)
- Apply geometric concepts in modeling situations (NC.M3.G-MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

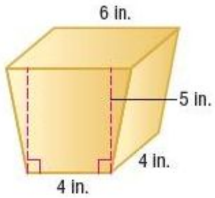
This standard focuses on volume and the use of volume formulas to solve problems. The figures may be a single shape or a composite of shapes.

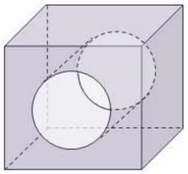
Formulas should be provided as the figures are more complex and the focus is on the modeling and solving problems.

Assessing for Understanding

Students should be able to identify the 3-D figures (prisms, cylinders, pyramids, cones and spheres) and the measurements needed to calculate the volume.

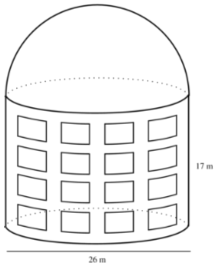
Example: A carryout container is shown. The bottom base is a 4-inch square and the top base is a 4-inch by 6-inch rectangle. The height of the container is 5 inches. Find the volume of food that it holds.





Example: A toy manufacturer has designed a new piece for use in building models. It is a cube with side length 7 mm and it has a 3 mm diameter circular hole cut through the middle. The manufacture wants 1,000,000 prototypes. If the plastic used to create the piece costs $270 per cubic meter, how much will the prototypes cost?

Example: The Southern African Large Telescope (SALT) is housed in a cylindrical building with a domed roof in the shape of a hemisphere. The height of the building wall is 17 m and the diameter is 26 m. To program the ventilation system for heat, air conditioning, and dehumidifying, the engineers need the amount of air in the building. What is the volume of air in the building?



NC.M3.G-GMD.4

Visualize relationships between two-dimensional and three-dimensional objects.

Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Concepts and Skills

Pre-requisite

- Describe 2-D cross-sections of rectangular prisms and pyramids (7.G.3)

Connections

- Apply geometric concepts in modeling situations (NC.M3.G-MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

This standard has two parts.

The first part is to identify the two-dimensional cross sections of three-dimensional objects.

Consider having students work with manipulatives such as play-dough and floss to make slices of three-dimensional shapes. Also, the *Cross Section Flyer* at http://www.shodor.org/interactivate/activities/CrossSectionFlyer/ can be used to allow students to predict and verify the cross section of different three-dimensional objects.

The second part is identifying three-dimensional objects generated by rotations of two-dimensional objects. There are a few interactive websites that students can use to explore.

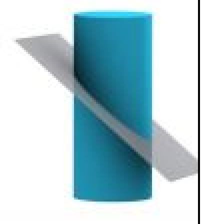
- 3D Transmographer
  http://www.shodor.org/interactivate/activities/3DTransmographer/
- Interactive Tool: Stacker
  http://www.scootle.edu.au/ec/viewing/L588/index.html
- Interactive Tool: Replicator
  http://www.scootle.edu.au/ec/viewing/L1059/index.html

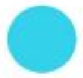
Assessing for Understanding

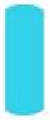
Students identify shapes of two-dimensional cross-sections of three-dimensional objects.

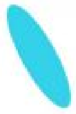
Example: Draw a figure that has the same cross section as a sphere.

Example: Which of the following is the cross section created by slicing the cylinder as shown in the figure to the right?



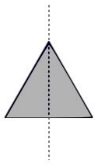


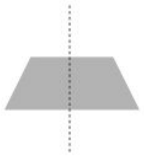


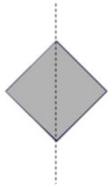


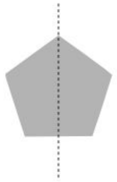
Students identify three-dimensional objects generated by rotations of two-dimensional objects.

Example: The shape at the right was created by rotating a two dimensional shape about an axis. Which of the following would create this shape?









NC.M3.G-MG.1

Apply geometric concepts in modeling situations.

Apply geometric concepts in modeling situations

- Use geometric and algebraic concepts to solve problems in modeling situations:
- Use geometric shapes, their measures, and their properties, to model real-life objects.
- Use geometric formulas and algebraic functions to model relationships.
- Apply concepts of density based on area and volume.
- Apply geometric concepts to solve design and optimization problems.

Concepts and Skills

Pre-requisite

- Solve real world problems involving area, volume, and surface area (7.G.6)
- Use volume formulas to solve problems (NC.M3.G-GMD.3)

Connections

- Apply properties, definitions, and theorems of 2-D figures to solve problems (NC.M3.G-CO.14)
- Identify 2-D cross sections; identify 3D objects (NC.M3.G-GMD.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

For this standard, students should engage in problems that are more complex than those studied in previous grades. The standard combines geometric and algebraic concepts and focuses on four primary areas:

i. model real-world three-dimensional figures,
ii. model relationships,
iii. determine density based on area or volume, and
iv. solve design and optimization problems.

When students model real-world three dimensional figures they must recognize the plane shapes that comprise the figure. They must be flexible in constructing and deconstructing the shapes. Students also need to be able to identity the measures associated with the figure such as circumference, area, perimeter, and volume.

Students use formulas and algebraic functions when modeling relationships. This may include examining how the one measurement changes as another changes.

Assessing for Understanding

Students recognize situations that require relating two- and three- dimensional objects. They estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects. Students apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).

Use geometric and algebraic concepts to solve problems in modeling situations.

Example: Janine is planning on creating a water-based centerpiece for each of the 30 tables at her wedding reception. She has already purchased a cylindrical vase for each table.

- The radius of the vases is 6 cm and the height is 28 cm.
- She intends to fill them half way with water and then add a variety of colored marbles until the waterline is approximately three-quarters of the way up the cylinder.
- She can buy bags of 100 marbles in 2 different sizes, with radii of 9mm or 12mm. A bag of 9mm marbles costs $3, and a bag of 12mm marbles costs $4.

a) If Janine only bought 9 mm marbles how much would she spend on marbles for the whole reception? What if Janine only bought 12 mm marbles? (Note: 1 cm^3 = 1 mL)

b) Janine wants to spend at most d dollars on marbles. Write a system of equalities and/or inequalities that she can use to determine how many marbles of each type she can buy.

How does the volume of a cylinder change as the radius changes?
How does the surface area of a prism change as the height changes?

The concept of density based on area and volume is to calculate the mass per unit.
Examples for area density are:

Description	*Unit of Measure*
Data Storage	Gigabytes per square inch
Thickness of Paper	Grams per square meter
Bone density	Grams per square centimeter
Body Mass Index	Kilograms per square meter
Population	People per square mile

Examples for volume density are:

Description	*Unit of Measure*
Solids	Grams per cubic centimeter
Liquids	Grams per millliter (1 mL = 1 cubic cm)

Design problems include designing an object to satisfy physical constraints. Optimization problems may maximize or minimize depending on the context.

c) Based on your answer to part b. How many bags of each size marble should Janine buy if she has $180 and wants to buy as many small marbles as possible?

Geometric shapes, their measures, and their properties to model real-life objects

Example: Describe each of the following as a simple geometric shape or combination of shapes. Illustrate with a sketch and label dimensions important to describing the shape.

a) Soup can label
b) A bale of hay
c) Paperclip
d) Strawberry

Use geometric formulas and algebraic functions to model relationships.

Example: A grain silo has the shape of a right circular cylinder topped by a hemisphere. If the silo is to have a capacity of 614π cubic feet, find the radius and height of the silo that requires the least amount of material to construct.

Density based problems

Example: A King Size waterbed has the following dimensions 72 in. x 84 in. x 9.5in. It takes 240.7 gallons of water to fill it, which would weigh 2071 pounds. What is the weight of a cubic foot of water?

Example: Wichita, Kansas has 344,234 people within 165.9 square miles. What is Wichita's population density?

NC.M3.G-CO.14

Prove geometric theorems.

Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

Concepts and Skills

Pre-requisite

- Prove theorems about parallelograms (NC.M3.G-CO.11)

Connections

- Use similarity to solve problems and to prove theorems about triangles (NC.M2.G-SRT.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically

Disciplinary Literacy

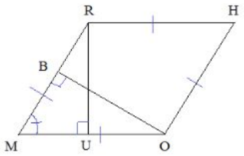
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

This standard is the application of the other two standards within this cluster NC.M3.G-CO.10 & 11. The other standards have students determine properties and prove theorems of figures. This standard is an application of those standards.
For this standard, instruction should provide students the opportunity to prove theorems for other two dimensional figures and to reason with figures to solve problems.
The geometric theorems may be for specific defined shapes. Consider including other quadrilaterals such as trapezoids and kites for students to explore. For example, prove the base angles of an isosceles trapezoid are congruent.

The geometric theorems may also be for a specific given figure. For example, given the rhombus RHOM, prove $\overline{RU} \cong \overline{OB}$.



Finally, this standard should be connected to NC.M3.G-C.2 where students are understanding and applying theorems about circles.

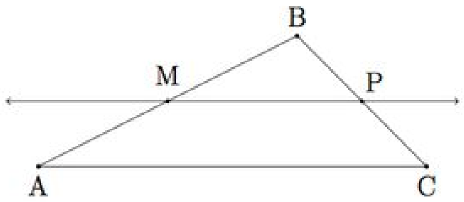
There is not a specific list of theorems for students to know and use. The focus is not on specific theorems but on construction of logical arguments and the ability of students to explain their reasoning with two-dimensional figures.

Assessing for Understanding

Students should demonstrate a solid understanding of lines and angles (Math 2), congruent triangles (Math 2), and properties of the centers of triangles (Math 3) and properties of parallelograms (Math 3). They should use their understanding of these properties, definitions and theorems to prove other geometric theorems and solve problems.

Example: Suppose ABC is a triangle. Let M be the midpoint of $\overline{AB}$ and P the midpoint of $\overline{BC}$ as pictured below:

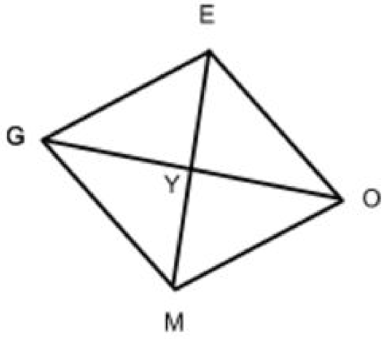
a) Prove that $\overline{MP}$ and $\overline{AC}$ are parallel.
b) Prove that $AC=2MP$.



Adapted from Illustrative Math

(https://www.illustrativemathematics.org/content-standards/tasks/1872)

Example: Given $\overline{EY} \cong \overline{YM}$, $\overline{GY} \cong \overline{YO}$, $\overline{EG} \cong \overline{EO}$. Prove GEOM is a rhombus.



The North Carolina High School Collaborative Instructional Framework

NC Math 3

Unit 5: Reasoning with Circles, Parallelograms and Triangles

15 Days Block Schedule **September 2017 Update** 30 Days Traditional Schedule

RESEARCH BRIEF: Reasoning with Geometry

Essential Questions:

- How can we use known properties of shapes to prove characteristics about triangles and quadrilaterals?
- What relationships exist between angles, segments, and circles?
- How can angles be measured in relation to the radius of a circle?

Learning Outcomes	**Student Objectives**
Students will extend knowledge of congruent triangles to proving theorems of parallelograms. Students will investigate properties of angles in circles. Students will investigate properties of segments in circles. Students will prove properties of the incenter, centroid, and circumcenter of triangles. Students will explore radian measure as the ratio of circumference to length of the radius and compare it to degree measure of angles. Students will discover the relationship of arc length to circumference and sector area to circle area given a central angle measure (in radians or degrees).	I will **prove** theorems of parallelograms I will **calculate** the arc length and area of a sector given an angle measure I will **calculate** measures of angles and segments with properties of circles. I will **convert** radian and degree angle measures. I will **use** points of concurrency to **find** segment lengths of lines intersecting within triangles

Standards Addressed in this Unit

Overarching Standards

- **NC.M3.A-SSE.1b**: Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

- **NC.M3.A-SSE.2:** Use the structure of an expression to identify ways to write equivalent expressions.

- **NC.M3.A-REI.1:** Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Construct logical arguments and explain reasoning with two-dimensional figures to prove geometric theorems about parallelograms and solve problems. Demonstrate an understanding of the properties of three of a triangle's points of concurrency.

- **NC.M3.G-CO.11:** Prove theorems about parallelograms.
  - Opposite sides of a parallelogram are congruent.
  - Opposite angles of a parallelogram are congruent.
  - Diagonals of a parallelogram bisect each other.
  - If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

- **NC.M3.G-CO.14**: Prove geometric theorems. Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

- **NC.M3.G-CO.10:** Verify experimentally properties of the centers of triangles (centroid, incenter, and circumcenter).

Understand properties of circles and how to apply them algebraically and geometrically. Demonstrate understanding that within circles, segments, lines, and angles create special relationships and use these to solve geometric problems.

- **NC.M3.G-C.2:** Understand and apply theorems about circles.
  - Understand and apply theorems about relationships with angles and circles, including central, inscribed and circumscribed angles.
  - Understand and apply theorems about relationships with line segments and circles including, radii, diameter, secants, tangents and chords.

- **NC.M3.G-C.5:** Using similarity, demonstrate that the length of an arc, *s,* for a given central angle is proportional to the radius, *r,* of the circle. Define radian measure of the central angle as the ratio of the length of the arc to the radius of the circle, *s/r*. Find arc lengths and areas of sectors of circles.

- **NC.M3.G-CO.14**: Prove geometric theorems. Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

- **NC.M3.G-MG.1**: Apply geometric concepts in modeling situations. Apply geometric concepts in modeling situations Use geometric and algebraic concepts to solve problems in modeling situations:
    - Use geometric shapes, their measures, and their properties, to model real-life objects.
    - Use geometric formulas and algebraic functions to model relationships.
    - Apply concepts of density based on area and volume.
    - Apply geometric concepts to solve design and optimization problems.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: In the equation of the circle $x^2 + (y-3)^2 = 16$, what does the $y-3$ represent?

NC.M3.A-SSE.2

Interpret the structure of expressions.

Use the structure of an expression to identify ways to write equivalent expressions.

Concepts and Skills

Pre-requisite

- Justifying a solution method (NC.M2.A-REI.1)
- Completing the Square (NC.M2.A-REI.4a)

Connections

- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

In Math 1 and 2, students factored quadratics. In Math 3, extend factoring to include strategies for rewriting more complicated expressions. Factoring a sum or difference of cubes, factoring a GCF out of a polynomial, and finding missing coefficients for expressions based on the factors can all be included. In Math 2, students completed the square to interpret and solve quadratic equations, and in this unit, students will complete the square to reveal properties of circles on the coordinate plane.

Assessing for Understanding

This standard can be assessed mainly by performing the algebraic manipulation. Problems could include:

Example: What are the center and radius of the circle given $x^2 + 8x - 13 + y^2 - 6y + 11 = 0$

NC.M3.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.

Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solution method and the steps in the solving process (NC.M2.A-REI.1)
- Use the Fundamental Theorem of Algebra (NC.M3.N-CN.9)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Use logarithms to express solutions to exponential equations (NC.M3.F-LE.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why it is necessary to write two equations to solve an absolute value equation.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. In Math 3, students should extend their knowledge of all equations they are asked to solve.

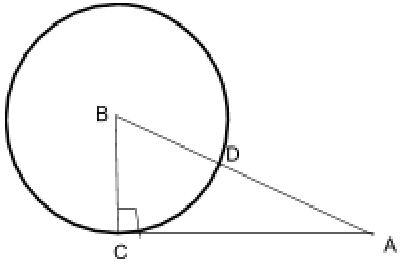
When solving equations, students will use mathematical reasoning to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Students do not have to use the proper names of the properties of operations and equality, but they should recognize and use the concepts associated with the properties.

Assessing for Understanding

Solving equations including justifications for each step, error analysis of solutions to equations, and comparing and analyzing different methods are all appropriate methods of assessing this standard.

Example: Triangle ABC is a right triangle, with AC tangent to circle B, AC = 8 and AD = 4. How would you calculate the radius of Circle B? Justify your reasoning.



NC.M3.G-CO.11

Prove geometric theorems.

Prove theorems about parallelograms.

- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.
- If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Concepts and Skills

Pre-requisite

- Prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)
- Use triangle congruence to prove theorems about lines, angles, and segments in triangles (NC.M2.G-CO.10)

Connections

- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)
- Apply geometric concepts in modeling situations (NC.M3.G.MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

5 – Use appropriate tools strategically

Disciplinary Literacy

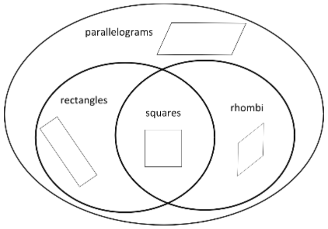
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

This standard is connected to the standards NC.M2.G-CO.8 & 9. Students use the triangle congruency theorems and theorems about lines and angles to prove theorems about parallelograms. The standard includes four specific theorems; however, student experience should not be limited to only these four.

Students should prove and apply the theorems listed. Application may include using the theorems to prove other theorems or to solve problems. (connect to NC.M3.G-CO.14 and NC.M3.G-MG.1).



Given the definition of a parallelogram (a quadrilateral with both pairs of opposite sides parallel) all other properties of a parallelogram can be proven.

Rectangles, rhombi, and squares are specific types of parallelograms. Consider including theorems that are specific to these such as:

- Diagonals of a rhombus are perpendicular bisectors.
- Diagonals of a square are congruent and perpendicular bisectors.
- Diagonals of a rhombus bisect the vertex angles.

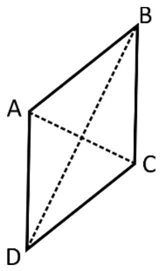
Proof is not solely about knowing the theorems. The goal of proof is to further develop the ability to construct logical arguments. Students should develop both *flow* and *paragraph* proofs. The

Assessing for Understanding

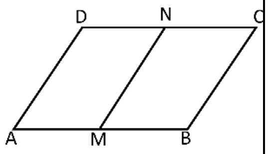
Students should prove theorems about parallelograms including:

a) Opposite sides of a parallelogram are congruent.
b) Opposite angles of a parallelogram are congruent
c) Diagonals of a parallelogram bisect each other
d) If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Students should apply proven theorems to prove additional theorems.



Example: Given ABCD is a rhombus prove the diagonals $\overline{BD}$ and $\overline{AC}$ are perpendicular bisectors.



Example: Suppose that ABCD is a parallelogram, and that M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. Prove that $\overline{MN} = \overline{AD}$ and that the line *line MN* is parallel to *line AD*.

construction of logical arguments and the ability to explain their reasoning is what will be expected from students.

NC.M3.G-CO.14

Prove geometric theorems.

Apply properties, definitions, and theorems of two-dimensional figures to prove geometric theorems and solve problems.

Concepts and Skills

Pre-requisite

- Prove theorems about parallelograms (NC.M3.G-CO.11)

Connections

- Use similarity to solve problems and to prove theorems about triangles (NC.M2.G-SRT.4)
- Understand and apply theorems about circles (NC.M3.G-C.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
1 – Make sense of problems and persevere in solving them
3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

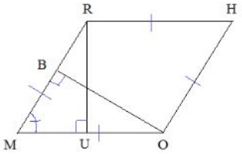
Mastering the Standard

Comprehending the Standard

This standard is the application of the other two standards within this cluster NC.M3.G-CO.10 & 11. The other standards have students determine properties and prove theorems of figures. This standard is an application of those standards.

For this standard, instruction should provide students the opportunity to prove theorems for other two dimensional figures and to reason with figures to solve problems.

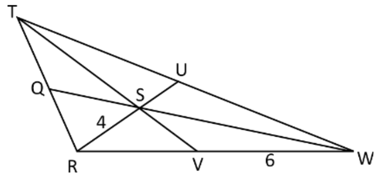
The geometric theorems may be for specific defined shapes. Consider including other quadrilaterals such as trapezoids and kites for students to explore. For example, prove the base angles of an isosceles trapezoid are congruent. The geometric theorems may also be for a specific given figure. For example,



Assessing for Understanding

Students should demonstrate a solid understanding of lines and angles (Math 2), congruent triangles (Math 2), and properties of the centers of triangles (Math 3) and properties of parallelograms (Math 3). They should use their understanding of these properties, definitions and theorems to prove other geometric theorems and solve problems.

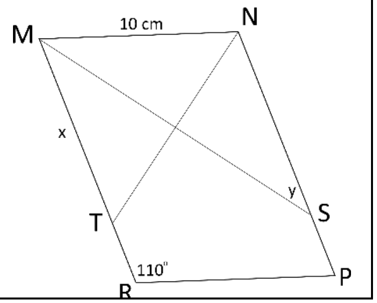
Students should use properties of the centers of triangles to solve problems.



Example: S is the centroid of ΔRTW; RS = 4, VW = 6 and TV = 9.
Find the length of each segment:

a) RV	b) SU	c) RU
d) RW	e) TS	f) SV

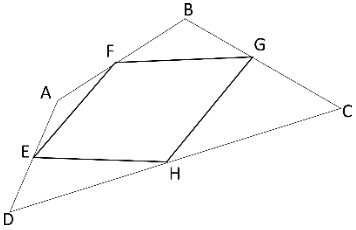
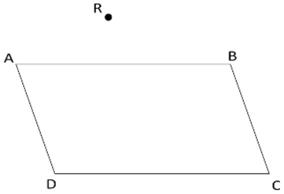
Students should use theorems about parallelograms to solve problems.



Example: Given MNPR is a parallelogram, $\overline{MS}$ bisects ∠RMN and $\overline{NT}$ bisects ∠MNP

a) Find the values of x and y.
b) Describe the relationship between $\overline{MS}$ and $\overline{NT}$

Example: In rectangle ABCD, AC = 3x + 15 and BD = 4x − 5. If AC and BD intersect at G, find the length of AG.

given the rhombus RHOM, prove $\overline{RU} \cong \overline{OB}$. Finally, this standard should be connected to NC.M3.G-C.2 where students are understanding and applying theorems about circles. There is not a specific list of theorems for students to know and use. The focus is not on specific theorems but on construction of logical arguments and the ability of students to explain their reasoning with two-dimensional figures.	Students should be able to prove geometric theorems. **Example:** Prove each of the following is true for an isosceles trapezoid. a) Base angles are congruent. b) Opposite angles are supplementary. c) Diagonals are congruent.  **Example:** For quadrilateral ABCD, points E, F, G and H are midpoints of their respective sides. Prove EFGH is a parallelogram. Students should be able to reason with two dimensional figures to solve problems. **Example:** In figure ABCD, AB||CD and AD||BC. Point R is in the same plane as ABCD. (Point R can be placed anywhere in the plane.)  Draw a straight line that passes through point R and divides ABCD into two congruent parts. Justify your reasoning that the two parts are congruent. Source: http://www.utdanacenter.org/k12mathbenchmarks/tasks/8_congruence.php

NC.M3.G-CO.10

Prove geometric theorems.

Verify experimentally properties of the centers of triangles (centroid, incenter, and circumcenter).

Concepts and Skills

Pre-requisite

- Use triangle congruence to prove theorems about lines, angles, and segments in triangles (NC.M2.G-CO.10)

Connections

- Understand and apply theorems about circles (NC.M3.G-C.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: centroid, incenter, circumcenter

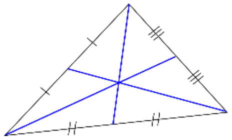
Mastering the Standard

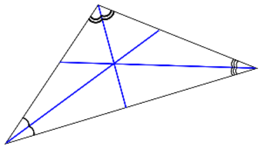
Comprehending the Standard

The goal is for students to be able to explore, make conjectures about the intersection of the different straight objects that produce the triangle centers, to justify why all three straight objects intersect at a common point, and why that point is an important feature of the triangle. The centers of triangles should be explored dynamically where students can discover them and their properties.

The centers of triangles are also known as points of concurrency for triangles. The three centers that are a focus for Math 3 are:

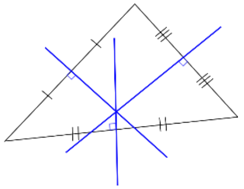
- Centroid – the point where the three medians of a triangle intersect





- Incenter – the point where the three angle bisectors of a triangle intersect

- Circumcenter – the point where the three perpendicular bisectors of the sides of a triangle intersect

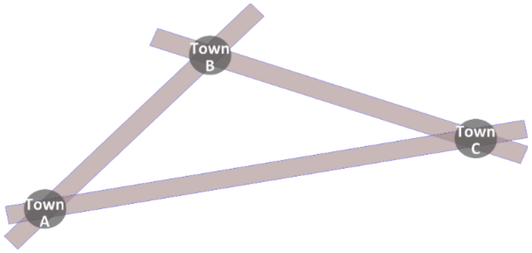


Assessing for Understanding

Students should demonstrate an understanding of the properties of the centers of triangles. The following task prompts students to consider the different centers, apply the properties to the context and make a decision about where to place the amphitheater.

Example:

A city plans to build an amphitheater and wants to locate it within easy access of the three largest towns in the area as shown on the map.



The developer must decide on the best location. The city will also have roads built for access directly to the towns or to the existing highways.

Describe how the developer might identify the location for the amphitheater. Choose one of the methods described and justify why this is the best location.

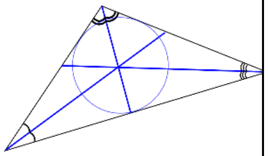
Possible student responses:

The circumcenter would place the amphitheater equidistant from the town. Roads would need to be built from the towns to the amphitheater. These roads would be the same distance.

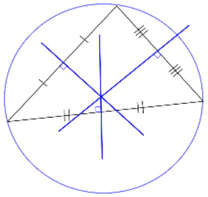
Once defined, students should experiment to verify the following properties:

- The centroid
  - always falls within the triangle
  - is located two-thirds of the way along each median or partitions the median into a ratio of 2:1 with the longest segment nearest the vertex
  - divides the triangle into six triangles of equal area
  - is the center of gravity for the triangle.

- The incenter
  - always falls within the triangle
  - equidistant from the sides of the triangle
  - is the center of the circle that is inscribed by the triangle; largest circle that will fit inside a circle and touch all three sides



- The circumcenter
  - falls inside when the triangle is acute; outside when it is obtuse, and on the hypotenuse when it is right.
  - equidistant from the vertices of the triangle
  - is the center of the circle that circumscribes the triangle; the circle that passes through all three vertices



The incenter would place the amphitheater from each road connecting the towns. Roads would need to be built from the existing roads to the amphitheater. These roads would be the same distance.

The centroid would place the amphitheater within the area surrounded by the three towns.

NC.M3.G-C.2

Understand and apply theorems about circles.

Understand and apply theorems about circles.

- Understand and apply theorems about relationships with angles and circles, including central, inscribed and circumscribed angles.
- Understand and apply theorems about relationships with line segments and circles including, radii, diameter, secants, tangents and chords.

Concepts and Skills

Pre-requisite

- Prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)
- Use similarity to solve problems and to prove theorems about triangles (NC.M2.G-SRT.4)

Connections

- Apply geometric concepts in modeling situations (NC.M3.G.MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

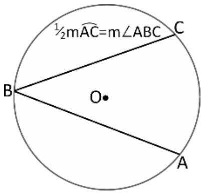
New Vocabulary: Circumscribe, inscribe, tangent

Mastering the Standard

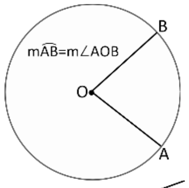
Comprehending the Standard

The following relationships with circles provide the foundation for reasoning with and applying theorems about circles:

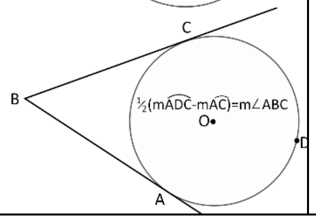
- Relationships with angles and circles
  - **Central angle** is an angle formed by two intersecting radii such that its vertex is at the center of the circle; the measure of the angle is equal to the measure of the intercepted arc



  - **Inscribed angle** is an angle with its vertex on the circle, formed by two intersecting chords; the measure of the angle is half the measure of the intercepted arc



  - **Circumscribed angle** is an angle formed by two tangents to a circle from the same point outside the circle; the measure of the angle is half the difference of the intercepted arcs



Assessing for Understanding

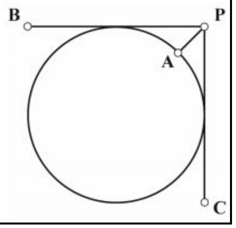
Students should have a strong command of the vocabulary: central angle, inscribed angle, circumscribed angle, tangent, arc (minor & major), secant, and chord.

Students demonstrate understanding when applying theorems about circles to explore other theorems.

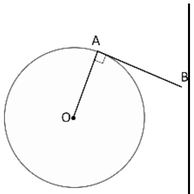
a) an angle inscribed in a semi-circle is a right angle.
b) the opposite angles in an inscribed quadrilateral are supplementary.
c) tangent lines drawn from a point outside a circle are equal in length.
d) when two chords intersect at a point interior to a circle, the chords are divided proportionally.
e) when two secants intersect at a point exterior to a circle, the lengths of the secants and the external parts are proportional.
f) if two chords are equivalent then their minor arcs are congruent and conversely
g) if two chords are equidistant from the center then they are congruent and conversely

Students demonstrate understanding when applying theorems about circles to solve problems with and without context.

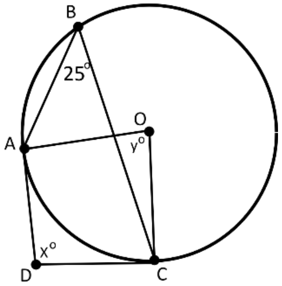
Example: A round table is pushed into a corner. The diameter of the table is 5 feet. Find the distance from the corner to the edge of the table.

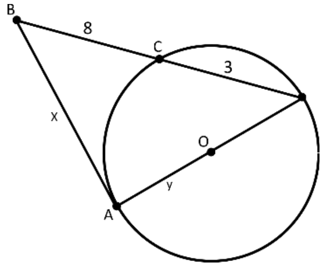


- Relationships with line segments and circles:
  - **Tangent line** intersects the circle exactly once at the point of tangency; the tangent line is perpendicular to the radius at the point of tangency



Example: Find the value of x and y.





NC.M3.G-C.5

Understand and apply theorems about circles.

Using similarity, demonstrate that the length of an arc, *s,* for a given central angle is proportional to the radius, *r,* of the circle. Define radian measure of the central angle as the ratio of the length of the arc to the radius of the circle, *s/r*. Find arc lengths and areas of sectors of circles.

Concepts and Skills

Pre-requisite

- Know the formulas for the area and circumference of a circle and use them to solve problems (7.G.4)
- Verify the properties of dilations with given center and scale factor (NC.M2.G-SRT.1)

Connections

- Understand radian measure as domain for trigonometric functions (NC.M3.G-TF.1)
- Apply geometric concepts in modeling situations (NC.M3.G-MG.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

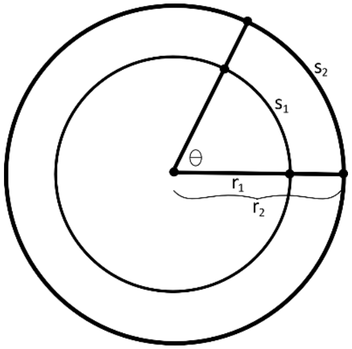
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Circles are similar figures; thus, any two arcs, subtended by the same central angle, will be proportional.

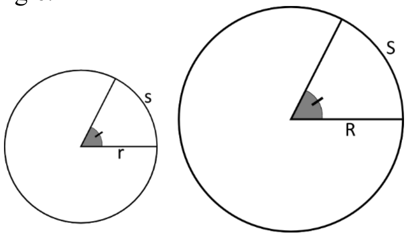
Since corresponding parts of similar figures are proportional then $\frac{r_1}{r_2} = \frac{s_1}{s_2}$ which can also be be written as $s_1 = (\frac{s_2}{r_2})r_1$. The structure of the equation reveals that the length of the arc is directly proportional to the radius and $\frac{s_2}{r_2}$ is the constant of proportionality.



Furthermore, a radian is defined as the ratio of the length of the arc to the radius of the circle, $\frac{s}{r}$, so the constant of proportionality is the radian measure of the angle.

Assessing for Understanding

Students demonstrate an understanding of the proportional relationship between the length of an arc and the radius of the circle by explaining how the following two diagrams could be used to prove that $s = kr$ where $k = \frac{S}{R}$ which is the radian measure of the central angle.

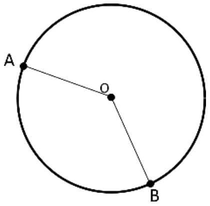


Students should use the definition of a radian to answer and solve problems.

Example: Explain why there are 2π radians in a circle. *Students explain that the radian measure is the ratio of the total length of the circle, 2πr, to the radius r. Thus* $\frac{2\pi r}{r} = 2\pi$ *radians.*

Example: The length of an arc is 18 cm and the radius of the circle is 6 cm. What is the radian measure of the central angle?

Example: A central angle measures 4.5 radians and has an arc length of 35 inches. What is the radius of the circle?

Using the reasoning presented, the arc length, s, can be calculated using the formula $s = \theta r$ where θ is the radian measure and r is the radius of the circle. The length of an arc subtended by a central angle can also be expressed as a fraction of the circumference. Given the central angle θ in degrees, the arc length is $s = \frac{\theta}{360°}(2\pi r)$. Given the central angle θ in radians, the arc length is $s = \frac{\theta}{2\pi}(2\pi r) = \theta r$. Similarly, the area of a sector can be expressed as a fraction of the area of the circle. Given the central angle in degrees and the radius r, the area of a sector is $\frac{\theta}{360°}(\pi r^2)$. Given the central angle in radians and the radius r, the area of the sector is $\frac{\theta}{2\pi}(\pi r^2) = \frac{\theta}{2}r^2 = \frac{sr}{2}$ where s is the arc length.	Students should be able to calculate arc lengths and areas of sectors of circles. **Example:** Given that $m\angle AOB = \frac{2\pi}{3} radians$ and the radius is 18 cm, what is the length of arc AB?  **Example:** Find the area of a sector with an arc length of 40 cm and a radius of 12 cm.

NC.M3.G-MG.1

Apply geometric concepts in modeling situations.

Apply geometric concepts in modeling situations

- Use geometric and algebraic concepts to solve problems in modeling situations:
- Use geometric shapes, their measures, and their properties, to model real-life objects.
- Use geometric formulas and algebraic functions to model relationships.
- Apply concepts of density based on area and volume.
- Apply geometric concepts to solve design and optimization problems.

Concepts and Skills

Pre-requisite

- Solve real world problems involving area, volume, and surface area (7.G.6)
- Use volume formulas to solve problems (NC.M3.G-GMD.3)

Connections

- Apply properties, definitions, and theorems of 2-D figures to solve problems (NC.M3.G-CO.14)
- Understand and apply theorems about circles (NC.M3.G-C.2)
- Find arc lengths and areas of sectors of circles (NC.M3.G-C.5)
- Identify 2-D cross sections; identify 3-D objects (NC.M3.G-GMD.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

For this standard, students should engage in problems that are more complex than those studied in previous grades. The standard combines geometric and algebraic concepts and focuses on four primary areas:

i. model real-world three-dimensional figures,
ii. model relationships,
iii. determine density based on area or volume, and
iv. solve design and optimization problems.

When students model real-world three dimensional figures they must recognize the plane shapes that comprise the figure. They must be flexible in constructing and deconstructing the shapes. Students also need to be able to identify the measures associated with the figure such as circumference, area, perimeter, and volume.

Assessing for Understanding

Students recognize situations that require relating two- and three- dimensional objects. They estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects. Students apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).

Use geometric and algebraic concepts to solve problems in modeling situations.

Example: Janine is planning on creating a water-based centerpiece for each of the 30 tables at her wedding reception. She has already purchased a cylindrical vase for each table.

- The radius of the vases is 6 cm and the height is 28 cm.
- She intends to fill them half way with water and then add a variety of colored marbles until the waterline is approximately three-quarters of the way up the cylinder.
- She can buy bags of 100 marbles in 2 different sizes, with radii of 9mm or 12mm. A bag of 9 mm marbles costs $3, and a bag of 12 mm marbles costs $4.

Students use formulas and algebraic functions when modeling relationships. This may include examining how the one measurement changes as another changes.

How does the volume of a cylinder change as the radius changes?

How does the surface area of a prism change as the height changes?

The concept of density based on area and volume is to calculate the mass per unit.

Examples for area density are:

Description	*Unit of Measure*
Data Storage	Gigabytes per square inch
Thickness of Paper	Grams per square meter
Bone density	Grams per square centimeter
Body Mass Index	Kilograms per square meter
Population	People per square mile

Examples for volume density are:

Description	*Unit of Measure*
Solids	Grams per cubic centimeter
Liquids	Grams per milliliter (1 mL = 1 cubic cm)

Design problems include designing an object to satisfy physical constraints. Optimization problems may maximize or minimize depending on the context.

a) If Janine only bought 9 mm marbles how much would she spend on marbles for the whole reception? What if Janine only bought 12 mm marbles? (Note: 1 cm^3 = 1 mL)

b) Janine wants to spend at most d dollars on marbles. Write a system of equalities and/or inequalities that she can use to determine how many marbles of each type she can buy.

c) Based on your answer to part b. How many bags of each size marble should Janine buy if she has $180 and wants to buy as many small marbles as possible?

Geometric shapes, their measures, and their properties to model real-life objects

Example: Describe each of the following as a simple geometric shape or combination of shapes. Illustrate with a sketch and label dimensions important to describing the shape.

a) Soup can label
b) A bale of hay
c) Paperclip
d) Strawberry

Density based problems

Example: A King Size waterbed has the following dimensions 72 in. x 84 in. x 9.5in. It takes 240.7 gallons of water to fill it, which would weigh 2071 pounds. What is the weight of a cubic foot of water?

Example: Wichita, Kansas has 344,234 people within 165.9 square miles. What is Wichita's population density?

The North Carolina High School Collaborative Instructional Framework

NC Math 3

Unit 6: Introduction to Rational Functions

10 Days Block Schedule
September 2017 Update
20 Days Traditional Schedule

RESEARCH BRIEF: Rational Functions

Essential Questions:

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables increase?
- How can functions be used to model real world situations?

Learning Outcomes	Student Objectives
• Create and solve rational equations from a contextual situation. • Given a rational function students will determine key features of a graph, table, or context. • Students should be able to compare features of two functions in different representations. • Students should be able to understand and interpret domain and range of a rational function. • Create an equation and interpret reasonable solutions in context. • Given a function create an equation from various representations and use them to solve problems. • Interpret structure of a rational function and relationship with graph, table, and/or context. • Given two functions, solve and interpret equations graphically. • Apply Remainder Theorem, Factor Theorem, and the Division Algorithm.	• I will **find** and **interpret** key features of a rational function from a graph, table, or context. • I will **compare** features of two functions in different representations. • I will **interpret** the relationship between input and output of a rational function. • I can **rewrite** and **simplify** a rational expression by factoring, long division, or synthetic division. • I will **explain** how operations on rational expressions are the same as simple fractions. • I will **multiply** and **divide** rational expressions. • I will **find** LCD in order to add and subtract rational expressions. • I will **recognize** the difference between adding rational expressions and solving rational expressions.

- I will **solve** a one variable rational equation algebraically or using a graph.
- I will **give examples** showing how extraneous solutions may arise when solving rational equation.
- I will **create** and **solve** a rational equation to solve an application.
- I will **interpret** the terms, factors, and coefficients of rational expressions.
- I will **apply** Remainder Theorem, Factor Theorem, and Division Algorithm.

Standards Addressed in this Unit

Recognize rational expressions as the division of two polynomials and use properties of simple fractions to analyze, perform arithmetic operations, create and solve equations that model real world phenomena.

- **NC.M3.A-SSE.1a**: Interpret expressions that represent a quantity in terms of its context. a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

- **NC.M3.A-SSE.1b**: Interpret expressions that represent a quantity in terms of its context. b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

- **NC.M3.A-SSE.2:** Use the structure of an expression to identify ways to write equivalent expressions.

- **NC.M3.A-APR.6:** Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.

- **NC.M3.A-APR.7a:** Understand the similarities between arithmetic with rational expressions and arithmetic with rational numbers.
  a. Add and subtract two rational expressions, $a(x)$ and $b(x)$, where the denominators of both $a(x)$ and $b(x)$ are linear expressions.

- **NC.M3.A-APR.7b:** Understand the similarities between arithmetic with rational expressions and arithmetic with rational numbers.
  b. Multiply and divide two rational expressions.

- **NC.M3.A-CED.1**: Create equations that describe numbers or relationships. Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

- **NC.M3.A-CED.2**: Create equations that describe numbers or relationships. Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

- **NC.M3.A-REI.1:** Justify a solution method for equations and explain each step of the solving process using mathematical reasoning.

- **NC.M3.A-REI.2:** Solve and interpret one variable rational equations arising from a context, and explain how extraneous solutions may be produced.

Understand and interpret the key features, uses and limitations of multiple representations of a rational function.

- **NC.M3.F-BF.1b:** Write a function that describes a relationship between two quantities.
    b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

- **NC.M1.F-IF.4:** Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

- **NC.M3.F-IF.7**: Analyze functions using different representations. Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

- **NC.M3.F-IF.9**: Analyze functions using different representations. Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M3.F-BF.1b

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.

b. Build a new function, in terms of a context, by combining standard function types using arithmetic operations.

Concepts and Skills

Pre-requisite

- Build new function by combine linear, quadratic and exponential functions (NC.M1.F-BF.1b)
- Operations with polynomials (NC.M1.A-APR.1)
- Operations with rational expressions (NC.M3.A-APR.7a, NC.M3.A-APR.7b)

Connections

- Create equation to graph and solve (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify new function and discuss how the new function fits the context.

Mastering the Standard

Comprehending the Standard

This standard asks students to combine standard function types by addition, subtraction, and multiplication. In Math 3, we are NOT required to include composition, although it could be a valuable extension.

The key concept for teaching this standard is a review of adding and subtracting expressions (including combining like terms) and multiplying expressions (distributing polynomials and exponent rules).

Assessing for Understanding

In assessing this standard, students will need to perform the operations and determine from a context which operation is appropriate. The functions that students need to combine should be given in problems, but the operation can be determined from context if necessary.

> **Example:** You are throwing a birthday party at a bowling alley for your little brother. It costs $75 to rent a room, plus an additional cost of $4.50 per child. Write a model that gives the average cost per child.
>
> **Example:** Information from an analysis of the past several years has allowed the owners of local pool to develop the following function rules for the number of customers *n(x)* and total profit *p(x)* based on the entrance fee to the pool *x*. Write an algebraic rule for the profit per customer in terms of the entrance fee *x*.

$$n(x) = 100 - 4x$$
$$p(x) = -3x^2 + 70x - 2$$

NC.M3.A-SSE.1a

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.

Concepts and Skills

Pre-requisite

- Identify and interpret parts of an expression in context (NC.M2.A-SSE.1a)

Connections

- Interpret parts of an expression as a single entity (NC.M3.A-SSE.1b)
- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Rational function

Mastering the Standard

Comprehending the Standard

Students need to be able to determine the meaning, algebraically and from a context, of the different parts of the expressions noted in the standard. At the basic level, this would refer to identifying the terms, factors, coefficients, and exponents in each expression.

Students must also be able to identify how these key features relate in context of word problems.

Assessing for Understanding

Students should be able to identify and **explain the meaning** of each part of these expressions.

Example: You were having a party and did not check to see how many slices each pizza was cut into at the beginning of the party. However, you assume that the pizza place would have cut all of the pizzas into equal slices. You still have 4 slices of one pizza and 3 of another. The following expression represents this situation. What does x represent in this expression?

$$\frac{4}{x}+\frac{3}{x}$$

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Create and graph equations and systems of equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret one variable rational equations (NC.M3.A-REI.2)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

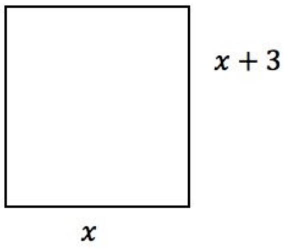
Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: Given the rectangle to the right, explain the meaning of the numerator of the following rational expression:

$$\frac{x^2+3x}{x+3}$$



NC.M3.A-APR.6

Rewrite rational expressions.

Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.

Concepts and Skills

Pre-requisite

- Long division of numerical expressions
- Operations with polynomial expressions (NC.M2.A-APR.1)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Operations with polynomial expressions (NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

If students learn synthetic division, students should be able to describe the limitations of the process.

Recalled Vocabulary: Divisor, Dividend, Quotient, Remainder

Mastering the Standard

Comprehending the Standard

In teaching this standard, students must be able to divide and simplify rational expressions by factoring and simplifying (inspection) and long division. It will be important for students to realize when each can and should be used.

Note: The use of synthetic division may be introduced as a method but students should recognize its limitations (division by a linear term). When students use methods that have not been developed conceptually, they often create misconceptions and make procedural mistakes due to a lack of understanding as to why the method is valid. They also lack the understanding to modify or adapt the method when faced with new and unfamiliar situations. Suggested viewing Synthetic Division: How to understand It by not doing it.

Assessing for Understanding

Students must not only be able to rewrite and divide the polynomials, but they will often need to determine the most appropriate method for performing the operation. Why questions, such as “Why did you choose inspection/long division/synthetic division to rewrite this expression?” can enhance the understanding.

Example: Express $\frac{-x^2+4x+87}{x+1}$ in the form $q(x) + \frac{r(x)}{b(x)}$.

Example: Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form.

Example: Determine the best method to simplify the following expressions, and explain why your chosen method is the most appropriate.

a) $\frac{6x^3+15x^2+12x}{3x}$ b) $\frac{x^2+9x+14}{x+7}$ c) $\frac{x^4+3x}{x^2-4}$ d) $\frac{x^3+7x^2+13x+6}{x+4}$

NC.M3.A-APR.7a

Rewrite rational expressions.

Understand the similarities between arithmetic with rational expressions and arithmetic with rational numbers.

a. Add and subtract two rational expressions, $a(x)$ and $b(x)$, where the denominators of both $a(x)$ and $b(x)$ are linear expressions.

Concepts and Skills

Pre-requisite

- Operations with fractions
- Operations with polynomial expressions (NC.M2.A-APR.1)

Connections

- Rewrite simple rational expressions (NC.M3.A-APR.6)
- Multiple and divide rational expressions (NC.M3.A-APR.7b)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students should understand that the same addition and subtraction properties that apply to fractions (adding and subtracting the numerators when they have a common denominator) also apply to rational expressions. With linear denominators, greatest common factors and multiply a rational expression times 1 (constant divided by a constant) will be important points to review. In previous math classes, many students might have learned to "cross multiply" to add or subtract fractions – we must fight this misconception so students truly understand why we use a common denominator.

Note: The revised standards only have students adding and subtracting rational expressions with linear denominators, so the concept of the common denominator can be stressed and understood, rather than more difficult algebraic manipulation.

For Example: $\frac{3x+7}{x-2} - \frac{3x+15}{2x-4}$

Assessing for Understanding

Students must be able to perform the operations and understand and explain the process (i.e. why they are factoring out a GCF, why they are finding a common denominator, why they are multiplying the numerator and denominator by the same factor, etc.)

Example: Simplify and explain your steps: $\frac{4x+13}{x-3} + \frac{x+2}{2x+6}$

Example: Why does multiplying a numerator and denominator by 2 NOT double the value of a rational expression?

NC.M3.A-APR.7b

Rewrite rational expressions.

Understand the similarities between arithmetic with rational expressions and arithmetic with rational numbers.

b. Multiply and divide two rational expressions.

Concepts and Skills

Pre-requisite

- Operations with fractions
- Operations with polynomial expressions (NC.M2.A-APR.1)
- Rewrite simple rational expressions (NC.M3.A-APR.6)

Connections

- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Building functions from graphs, descriptions and ordered pairs (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

Students should understand that the same multiplication and division properties that apply to fractions (multiplying the numerators and denominators for multiplication, multiplying times the reciprocal for division) also apply to rational expressions. In previous math classes, many students might have learned to "cross multiply" to divide fractions – we must fight this misconception so students truly understand why we multiply times a reciprocal.

Factoring will be a key review concept for teaching this standard.

Assessing for Understanding

Students must be able to perform the operations and understand and explain the process (i.e. why they are factoring each expression, why they can divide out common factors in the numerator and denominator, that a common denominator when dividing can be useful, etc.)

Example: Simplify and explain your steps.

a) $\left(\frac{2x+4}{x^2-6x}\right)\left(\frac{x^2-36}{4x+8}\right)$

b) $\left(\frac{x^2-4}{x^2+2x-5}\right)\div\left(\frac{x+2}{x^2+2x-5}\right)$

NC.M3.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.

Concepts and Skills

Pre-requisite

- Create one variable equations and solve (NC.M2.A-CED.1)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Justify a solution method (NC.M3.A-REI.1)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Student should be able to explain and defend the model they chose to represent the situation.

New Vocabulary: Rational equation

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematical equations to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Creating one variable equations and inequalities are included in Math 1, 2, and 3. In previous courses, students modeled with linear, exponential, quadratic, radical, and inverse variation equations. In Math 3, students will be expected to model with polynomial, rational, absolute value, and exponential equations. Students will need to analyze a problem, determine the type of equation, and set up and solve these problems. Students may need to create an equation from different representations found in the context. This makes it important for students to realize that equations can be derived as a specific instance of an associated function.

Students are expected to represent the solutions of an inequality using a number line and compound inequalities using inequality and interval notation.

Assessing for Understanding

Students should be able to create and solve problems algebraically and graphically. There should be a focus on using methods efficiently.

> **Example:** In a Math 3 class, the red group has four members. Brian can solve an equation in 5 minutes, Luis can solve one in 4 minutes, Sylvia can solve one in 6 minutes, and Tierra can solve one in 3 minutes. Set up and solve an equation to determine how long will it take the group to complete a 10 problem worksheet if they work together. Is this answer accurate, based on the context? Why or why not?

NC.M3.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.

Concepts and Skills

Pre-requisite

- Create and graph two-variable equations (NC.M2.A-CED.2)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Understand the relationship between the factors of a polynomial, solutions and zeros (NC.M3.A-APR.3)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Write the equations and inequalities of a system (NC.M3.A-CED.3)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M3.A-REI.11)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions from various representations and by combining functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Rational equation

Mastering the Standard

Comprehending the Standard

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. In A-CED.1, writing and solving an equation is the essential skill required. In this standard, graphing the equation to determine key features is essential.

This standard is included in Math 1, 2, and 3. Throughout all three courses, students create equations in two variables and graph them on coordinate axes. In Math 3, absolute value, polynomial, and rational graphs are introduced, and exponential graphs are further developed to solve for the exponent.

Assessing for Understanding

Rate of growth and decay, work rate (and other rates), geometric, and other real-world examples provide the context for many of these problems.

> **Example**: You are throwing a birthday party at a bowling alley for your little brother. It costs $75 to rent a room, plus an additional cost of $4.50 per child. Write and graph a model that gives the average cost per child.

NC.M3.A-REI.2

Understand solving equations as a process of reasoning and explain the reasoning.

Solve and interpret one variable rational equations arising from a context, and explain how extraneous solutions may be produced.

Concepts and Skills

Pre-requisite

- Solve and interpret one variable inverse variation and square root equations and explain extraneous solutions (NC.M2.A-REI.2)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Justify a solution method and each step in the solving process (NC.M3.A-REI.1)

Connections

- Creating one variable equations (NC.M3.A-CED.1)
- Analyze and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain when a rational equation will have an extraneous solution.

New Vocabulary: Rational equation, extraneous solution

Mastering the Standard

Comprehending the Standard

Students need to understand the process of solving rational equations, including finding the common denominator of all terms. It is important to keep in mind the limitations placed in NC.M3.A-APR.7.

Students also need to understand the relationship between rates and rational expressions, such as $speed = \frac{distance}{time}$.

Students should understand that the process of algebraically solving an equation can produce extraneous solutions. Students studied this in Math 2 in connection to square root functions.

Assessing for Understanding

To master this standard, students must be able to set up, solve, and evaluate the solutions to “real-world” rational equations.

Example: You are throwing a birthday party at a bowling alley for your little brother. It costs $75 to rent a room, plus an additional cost of $4.50 per child. If you only want to spend an average of $17 per child, how many children can you invite?

Example: Your Mom can clean your entire house in 3 hours. However, your dad takes 5 hours to clean the house. Determine how long it will take for them to clean the house if they work together.

When teaching this standard, it will be important to link to the concept of having a limited domain, not only by the context of a problem, but also by the nature of the equation.

Graphically, extraneous solution can be linked to discontinuities on the graph.

Additionally, students must be able to solve rational equations and understand how extraneous solutions can be produced. Graphic representations can often be used to find real solutions, but students must be able to identify when their algebraic solving process creates an extraneous solution.

Example: Consider the following equation.

$$\frac{x^2+x-2}{x+2} =- 2$$

Here are two algebraic methods that can be used to solve this equation.

Method 1:

$$\frac{x^2+x-2}{x+2} = -2$$

$$\frac{(x+2)(x-1)}{x+2} = -2$$

$$x - 1 = -2$$
$$x = -1$$

Method 2:

$$\frac{x^2+x-2}{x+2} = -2$$

$$x^2 + x - 2 = -2\,(x+2)$$

$$x^2 + x - 2 = -2x - 4$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2, -1$$

Verify that each step in the two methods is correct and answer the following questions.

a) Why does Method 2 produce two solutions?

b) Looking at original equation, how can you tell which of the solutions is extraneous?

Example: Graph the function $f(x) = \frac{x^2+x-2}{x+2}$ on a graphing calculator or app.

a) What do you notice about the graph?

b) Zoom into where the extraneous solution would be on the grid. What do you notice?

c) What are the implications of just looking at the graph for the solutions?

d) Now look at the table of the function. What do you notice?

NC.M3.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Concepts and Skills

Pre-requisite

- Interpret key features from graph, tables, and descriptions (NC.M2.F-IF.4)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identified key features with mathematical reasoning.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math 3, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity as motion that is repeated in equal intervals of time and discontinuity as values that are not in the domain of a function, either as asymptotes or “holes” in the graph.

No limitations are listed with this standard. This means that all function types, even those found in more advanced courses. Students do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.

This is in contrast to NC.M3.F-IF.7, in which the specific function types are included. Students can work algebraically with those listed types and can analyze those functions in greater detail.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

This standard must be assessed using three important forms of displaying our functions: graphs, tables, and verbal descriptions/word problems. Students must be able to interpret each and how they apply to the key input-output values.

> **Example:** The junior class is planning prom for this school year. The venue costs $1,200 to rent and there is an additional cost of $20 per person for food. Write a function to model the average cost per person at prom. Where is the vertical asymptote of this function and what does it represent in this problem?

NC.M3.F-IF.9

Analyze functions using different representations.

Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

-

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss how the comparison of a functions leads to a mathematical understanding, such as with transformations and choosing better models.

New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In Math 3, this standard can include two functions of any type students have learned in high school math in any representation. Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.

Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

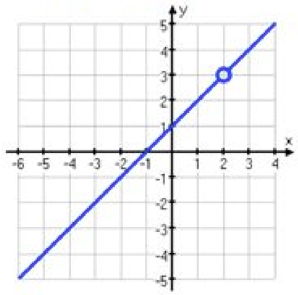
In assessing this standard, students must demonstrate that they can not only identify, but compare, the key features of two different functions. Appropriate question stems could include: Which is less/greater; Which will have a greater value at x = __; Which function has the higher maximum/lower minimum; etc.

Example: Find the difference between the x-values of the discontinuities for the two functions below:

Function 1:

$$\frac{x^2-5x+6}{x-3}$$

Function 2:



NC.M3.F-IF.7

Analyze functions using different representations.

Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Concepts and Skills

Pre-requisite

- Analyze functions using different representations to show key features (NC.M2.F-IF.7)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Understand and apply the Remainder Theorem (NC.M3.A-APR.2)
- Rewrite rational expressions (NC.M3.A-APR.6, NC.M3.A-APR.7a, NC.M3.A-APR.7b)
- Solve one variable rational equations (NC.M3.A-REI.2)

Connections

- Create and graph equations in two variables (NC.M3.A-CED.2)
- Analyze graphs and tables and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.9)
- Build functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Understand the effects of transformations on functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation best shows each of the key features.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

In previous math courses, students have identified the characteristic of graphs of other functions, including linear, quadratic, exponential, radical, and inverse variation functions. They should be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior. In Math 3, these concepts are extended to piecewise, absolute value, polynomials, exponential, rational, and sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity are new features of functions that must be introduced. The intent of this standard is for students to find discontinuities in tables and graphs and to recognize their relationship to functions. Students are not expected to find an asymptote from a function. (This could be an extension topic.)

This standard will likely span multiple units, as most Math 3 courses teach polynomial, exponential, rational, and trigonometric functions in different units. These function characteristics will be repeated and reinforced throughout the course.

Assessing for Understanding

In assessing this standard, students must demonstrate their ability to represent and determine the key features from algebraic and graphical representations of the functions.

Example: For $(x) = \frac{x+4}{2-x}$, discuss end behavior and any discontinuities.

The North Carolina High School Collaborative Instructional Framework

NC Math 3

Unit 7: Introduction to Trigonometric Functions

10 Days Block Schedule	**September 2017 Update**	20 Days Traditional Schedule

RESEARCH BRIEF: Trigonometric Functions

Essential Questions:

- How can the coordinates of any point on a circle be determined from the radius and angle of rotation?
- How can the trigonometric ratio be represented as a function of an angle measure?
- How do dependent variables change as independent variables increase for sine and cosine ratios represented on a circle and trigonometric curve?
- How can cosine and sine functions be used to model real world situations?

Learning Outcomes	**Student Objectives**
• Students should be able to convert between radian and degree measurements. • Students should explain the relationship between the domain and range represented as angle measurements and ratios respectively, of sine and cosine functions. • Students will understand the relationship between the cosine and sine values and the horizontal and vertical components of position. • Given a trigonometric function, students will determine key features of a graph, table, or context. • Students should be able to compare features of two functions in different representations. • Students will use prior knowledge of function transformations to build new sine and cosine functions.	• I will **calculate** an equivalent angle measurement given radians or degrees. • I will **interpret** the relationship between input and output of sine and cosine functions. • I will **evaluate** the value of sine and cosine functions given an angle measurement. • I will **determine** the coordinates of any point on a circle centered at the origin given the radius and angle measurement. • I can **sketch** and **recognize** angles in standard position to find coterminal and reference angles. • I will **compare** features of two functions in different representations.

- I will **interpret** parts of a function and their relationship with the graph, table, and context.
- I will be able to read a word problem or analyze a graph and **create** an equation or inequality.
- I will **describe** the transformations that have been applied to the sine and cosine functions given an equation or graph.

Standards Addressed in this Unit

Overarching Standards

- **NC.M3.A-SSE.1b**: Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Understand that trigonometric functions can be represented on the coordinate plane in different ways based on the quantities used for the axises. Understand the relationships between different representations of trigonometric functions on the coordinate plane. Identify and interpret key features of trigonometric functions.

- **NC.M3.F-IF.1:** Extend the concept of a function by recognizing that trigonometric ratios are functions of angle measure.

- **NC.M1.F-IF.4:** Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

- **NC.M3.F-IF.7**: Analyze functions using different representations. Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

- **NC.M3.F-IF.9**: Analyze functions using different representations. Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

- **NC.M3.F-BF.3**: Build new functions from existing functions. Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x) + k$, $f(x + k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

- **NC.M3.F-TF.1**: Understand radian measure of an angle as:
  - The ratio of the length of an arc on a circle subtended by the angle to its radius.
  - A dimensionless measure of length defined by the quotient of arc length and radius that is a real number.
  - The domain for trigonometric functions.

- **NC.M3.F-TF.2a**: Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.
  a. Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its y coordinate.

- **NC.M3.F-TF.2b**: Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.
  b. Interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its x coordinate.

- **NC.M3.F-TF.5**: Use technology to investigate the parameters, a, b, and h of a sine function, $f(x) = a{\cdot}sin(b{\cdot}x) + h$, to represent periodic phenomena and interpret key features in terms of a context.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

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The Math Resource for Instruction - Customized for the Content of this Unit

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret one variable rational equations (NC.M3.A-REI.2)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

As we add piecewise functions and expressions in Math 3, breaking down these expressions and functions into their parts are essential to ensure understanding.

For Example: Explain what operations are performed on the inputs -2, 0, and 2 for the following expression:

$$f(x) = \quad \{3x,\ for\ x < 0\ \frac{1}{x},\ for\ 0 \leq x < 2\ x^3,\ for\ x \geq 2$$

Which input is not in the domain? Why not?

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: Given the expression: $a \sin (bx) + c$

a) What do a, b, c, and x represent?

b) How would increasing each variable by a factor of 2 change the value of the expression?

Note: This example could also fit NC.M3.F-TF.5. For this standards, students must recognize that changing b and x have different impacts then a or c because they are “inputs” of a sine function. Teachers can give values for the variables to help students interpret.

	Students should notice the similarity of this expression as with function transformations (e.g., $a \cdot f(b \cdot x) + c$).

NC.M3.A-SSE.2

Interpret the structure of expressions.

Use the structure of an expression to identify ways to write equivalent expressions.

Concepts and Skills

Pre-requisite

- Justifying a solution method (NC.M2.A-REI.1)

Connections

- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

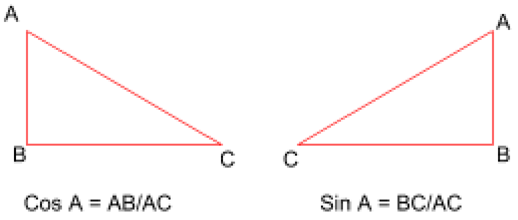
In Math 1 and 2, students factored quadratics. In Math 3, extend factoring to include strategies for rewriting more complicated expressions. Factoring a sum or difference of cubes, factoring a GCF out of a polynomial, and finding missing coefficients for expressions based on the factors can all be included.

For Example: When factoring a difference of cubes, is the trinomial factor always, sometimes or never factorable? How do you know?

Assessing for Understanding

This standard can be assessed mainly by performing the algebraic manipulation. Problems could include:

Example: Prove that sin $(x - \pi/2)$ is the same as cos (x). Use the triangles below if you need.



NC.M3.F-IF.1

Understand the concept of a function and use function notation.

Extend the concept of a function by recognizing that trigonometric ratios are functions of angle measure.

Concepts and Skills

Pre-requisite

- Define a function (NC.M1.F-IF.1)
- Verify experimentally that the side ratios in similar triangles are properties of the angle measures in the triangle (NC.M2.G-SRT.6)
- Understand radian measure of an angle (NC.M3.F-TF.1)

Connections

- Analyze and compare functions in various representations (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build an understanding of trig functions in relation to its radian measure (NC.M3.F-TF.2a, NC.M3.F-TF.2b)
- Investigate the parameters of the sine function (NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the output of trig functions as unit rates.

Mastering the Standard

Comprehending the Standard

This is an extension of previous learning. Students should already understand function notation, the correspondence of inputs and outputs, and evaluating functions. In Math 3, students should build an understanding of the unique relationship between the measure of the angle and the value of the particular trig ratio.

Also in Math 3, students build an understanding of radian measure.
See NC.M3.F-TF.1 for more information.

Students should also begin to see the graphical representations of trig functions, both on a unit circle and on a graph in which the domain is the measure of the angle and the range is the value of the associated trig ratio.
On the unit circle, the input is the measure of the angle and the output of the sine function is the y-coordinate of the vertex of the formed triangle and the output of the cosine function is the x-coordinate of the vertex of the formed triangle.
See NC.M3.F-TF.2a and NC.M3.F-TF.2b for more information.

Assessing for Understanding

Students should be able to create trig functions in various representations, recognizing that the domain of a trig function is the measure of the angle.

Example: Complete the function table for $f(\theta) = \sin\theta$ and $f(\theta) = \cos\theta$ and complete the following.

θ	$\sin\theta$	$\cos\theta$	θ	$\sin\theta$	$\cos\theta$
0			π		
$\frac{\pi}{6}$			$\frac{7\pi}{6}$		
$\frac{\pi}{4}$			$\frac{5\pi}{4}$		
$\frac{\pi}{3}$			$\frac{4\pi}{3}$		
$\frac{\pi}{2}$			$\frac{3\pi}{2}$		
$\frac{2\pi}{3}$			$\frac{5\pi}{3}$		
$\frac{3\pi}{4}$			$\frac{7\pi}{4}$		
$\frac{5\pi}{6}$			$\frac{11\pi}{6}$		

<u>Based on the table:</u>

a) Describe in your own words the relationship you see between the measure of the angle and the sine function.

b) If you were to graph $f(\theta) = \sin\theta$, what would it look like? What would be some of the key features?

c) Describe in your own words the relationship between the measure of the angle and the cosine function.

d) If you were to graph $f(\theta) = \cos\theta$, what would it look like? What would be some of the key feature?

e) How does $\sin\theta$ and $\cos\theta$ relate to each other?

NC.M3.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Concepts and Skills

Pre-requisite

- Interpret key features from graph, tables, and descriptions (NC.M2.F-IF.4)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)

Connections

- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identified key features with mathematical reasoning.

New Vocabulary: periodicity, discontinuity, amplitude, period, radian.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math 3, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity as motion that is repeated in equal intervals of time and discontinuity as values that are not in the domain of a function, either as asymptotes or “holes” in the graph.

No limitations are listed with this standard. This means that all function types, even those found in more advanced courses. Students do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.

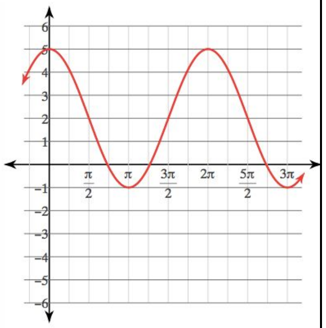
This is in contrast to NC.M3.F-IF.7, in which the specific function types are included. Students can

Assessing for Understanding

This standard must be assessed using three important forms of displaying our functions: graphs, tables, and verbal descriptions/word problems. Students must be able to interpret each and how they apply to the key input-output values.

Example*:* Jumper horses on carousels move up and down as the carousel spins. Suppose that the back hooves of such a horse are six inches above the floor at their lowest point and two-and-one-half feet above the floor at their highest point. Draw a graph that could represent the height of the back hooves of this carousel horse during a half-minute portion of a carousel ride.

Example: For the function to the right, label and describe the key features. Include intercepts, relative max/min, amplitude, period, midline, and frequency.



Example*:* Over a year, the length of the day (the number of hours from sunrise to sunset) changes every day. The table below shows the length of day every 30 days from 12/31/97 to 3/26/99 for Boston Massachusetts.

Data on length of day

Date	12/31	1/30	3/1	3/31	4/30	5/30	6/29	7/29	8/28	9/27	10/27	11/26	12/26	1/25	2/24	3/26
Day Number	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450
Length (hours)	9.1	9.9	11.2	12.7	14.0	15.0	15.3	14.6	13.3	11.9	10.6	9.5	9.1	9.7	11.0	12.4

work algebraically with those listed types and can analyze those functions in greater detail.	During what part of the year do the days get longer? Support your claim using information provided from the table.

NC.M3.F-IF.9

Analyze functions using different representations.

Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss how the comparison of a functions leads to a mathematical understanding, such as with transformations and choosing better models.

New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In Math 3, this standard can include two functions of any type students have learned in high school math in any representation. Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.

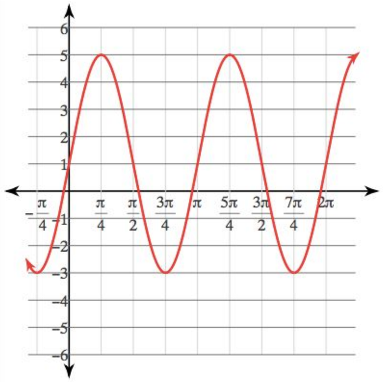
Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

In assessing this standard, students must demonstrate that they can not only identify, but compare, the key features of two different functions. Appropriate question stems could include: Which is less/greater; Which will have a greater value at x = __; Which function has the higher maximum/lower minimum; etc.

Examples: If $f(x) = -2sin(x) - 3$ and $g(x)$ is represented on the graph.

a) Which function has the greatest amplitude? Explain your reasoning.
b) Which has the largest relative maximum?
c) Describe each function's period. Why are they different? What can be said about each function?



NC.M3.F-IF.7

Analyze functions using different representations.

Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Concepts and Skills

Pre-requisite

- Analyze functions using different representations to show key features (NC.M2.F-IF.7)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Create and graph equations in two variables (NC.M3.A-CED.2)
- Analyze graphs and tables and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.9)
- Build functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Understand the effects of transformations on functions (NC.M3.F-BF.3)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation best shows each of the key features.
New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

In previous math courses, students have identified the characteristic of graphs of other functions, including linear, quadratic, exponential, radical, and inverse variation functions. They should be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior.

In Math 3, these concepts are extended to piecewise, absolute value, polynomials, exponential, rational, and sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity are new features of functions that must be introduced. The intent of this standard is for students to find discontinuities in tables and graphs and to recognize their relationship to functions. Students are not expected to find an asymptote from a function. (This could be an

Assessing for Understanding

In assessing this standard, students must demonstrate their ability to represent and determine the key features from algebraic and graphical representations of the functions.

Example: Graph $y = 3 \sin(x) - 5$ and answer the following questions:

a) What is the period?

extension topic.) This standard will likely span multiple units, as most Math 3 courses teach polynomial, exponential, rational, and trigonometric functions in different units. These function characteristics will be repeated and reinforced throughout the course. Students should be able to use interval notation as appropriate.	b) For the domain of $-2\pi < x < 2\pi$, identify any relative maxima and minima, intervals of increasing and decreasing, and lines of symmetry.

NC.M3.F-BF.3

Build new functions from existing functions.

Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k \cdot f(x)$, $f(x) + k$, $f(x + k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Understand the effects of transformations on functions (NC.M2.F-BF.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

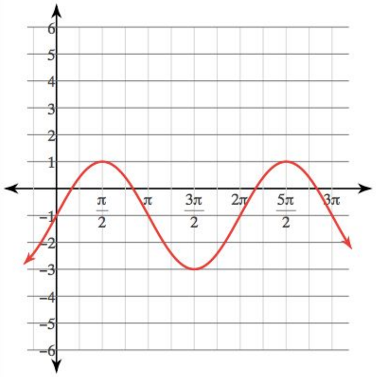
3 – Construct a viable argument and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of *k*.

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students learned the translation and dilation rules in Math 2 with regard to linear, quadratic, square root, and inverse variation functions. In Math 3, we apply these rules to functions in general. Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions. Students should be able to explain why $f(x+k)$ moves the graph of the function left or right depending on the value of *k*. • Note: Phase shifts and transformations of trigonometric functions are NOT required in Math 3. Those will be covered in the fourth math course.	In demonstrating their understanding, students must be able to relate the algebraic equations, graphs, and tabular representations (ordered pairs) as functions are transformed. Appropriate questions will ask students to identify and explain these transformations. **Example**: If $f(x) = sin(x)$ and the given graph is $g(x)$, write the equation for $g(x)$.  **Example**: Explain how the graph of $f(x) =- 1/2sin(3x) + 2$ is transformed from the parent sine function.

NC.M3.F-TF.1

Extend the domain of trigonometric functions using the unit circle.

Understand radian measure of an angle as:

- The ratio of the length of an arc on a circle subtended by the angle to its radius.
- A dimensionless measure of length defined by the quotient of arc length and radius that is a real number.
- The domain for trigonometric functions.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)

Connections

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Define radian measure (NC.M3.G-C.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the relationship between degrees and radians.

New Vocabulary: arc length

Mastering the Standard

Comprehending the Standard

To build the understanding of radian measure, students should first become familiar with degree measure.

In ancient times, when discussing angle measure, it was realized that the best way to describe angle measure was through a ratio. It was decided based on a different numbering system that they would divide a circle into 360 sectors and each of the sectors would measure 1 degree. The division of the circle into 360 sectors not only divided the angle, but also divided the arc of the circle as well. (Hence the measure of the central angle is the same as the measure of the intercepted arc.)

This means that a measure of $42°$ is $42(\frac{1}{360})$ of a circle or 42 divisions of the 360 divisions.

In modern times, as science and mathematics knowledge increased, the decision to divide a circle into 360 parts is arbitrary and less precise. This lead to the development of radian measures.

In this process, a ratio is still used, however the circle is not divided into parts but is described in the ratio of the circumference to the radius.

Here is a good resource to understand radian measure: Find radian measure by dividing arc length by radius (Learn Zillion)

By discovery (using string, rolling a can, etc.), students can determine that it takes just over 6 radii to create the circumference of a circle, and the teacher can relate that to 2π .

Assessing for Understanding

In mastering this standard, students will need to demonstrate an understanding of radian angle measure and applying the arc length formula (Arc Length = Radius • Radian Measure) to solve for any missing measure, both using basic measures and in the context of word problems. The following examples are from NC.M3.G-C.5 but require the understanding of this standard.

Example: An angle with a measure of 4 radians intercepts an arc with a length of 18 ft. What is the length of the radius of the circle?

Example*:* The minute hand on the clock at the City Hall clock in Stratford measures 2.2 meters from the tip to the axle.

a) Through what radian angle measure does the minute hand pass between 7:07 a.m. and 7:43 a.m.?
b) What distance does the tip of the minute hand travel during this period?

NC.M3.F-TF.2a

Extend the domain of trigonometric functions using the unit circle.

Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.

a. Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *y* coordinate.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should describe the relationship between sine represented on a unit circle and graphical representation of the sine function.

Mastering the Standard

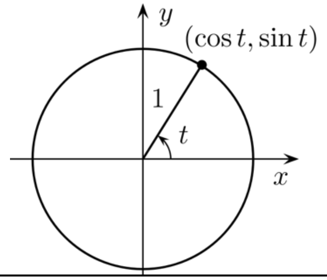
Comprehending the Standard

Students will be introduced to the unit circle and angle measures on the coordinate plane in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane.

A unit circle is used to develop the concepts of this standard to simplify the picture for students. In Math 3, students are only introduced to the trigonometric functions.

This standard builds upon previous understanding of the trig ratios in right triangles. *Sin* θ is the unit rate produced by the ratio of the length of the opposite side to the length of the hypotenuse.

$$sin\ \theta = \frac{length\ of\ opposite\ side}{length\ of\ hypotenuse}$$



Since we are working within a unit circle, and the hypotenuse is the radius of the unit circle, so the length of the hypotenuse is 1 unit. This means that $sin\ \theta = \frac{length\ of\ opposite\ side}{1}$, so with the unit circle, $sin\ \theta$ is the length of the opposite side.

Assessing for Understanding

Students apply reasoning to their knowledge of the relationship between angles and the sides of right triangles.

Example: A stink bug has crawled into a box fan and sits on the tip of the blade of the fan as seen below. The fan starts to turn slowly due to a breeze in the room.

a) Create a function and a graph that describes its change in height from its original position based on the angle of the blade from its original position.

b) What is the height of the stink bug when the blade has rotated 2 radians? $\frac{11\pi}{6}$ radians?

c) How much has the blade rotated when the stink bug's height is $-\frac{3}{4}$ feet? Can there be more than one answer?

This means that the height of the triangle, which is the y-coordinate of the vertex on the circle, is $sin\ \theta$.

The focus of this standard is on the relationship between the changing angle of the sine function and the value of the sine ratio. This should allow students to move from the unit circle to graphing the relationship on a coordinate plane in which the independent variable is the angle measure and the dependent variable is the value of the sine ratio (the y-coordinate from the unit circle). This is a strong connection to NC.M3.F-IF.1.

In general, from the unit circle, students should see that as the angle is near zero, the ratio of the length of the opposite side to the length of the hypotenuse is also near zero. As the angle starts to increase and approaches $90°$ or $\frac{\pi}{2}$, the value of the sine ratio approaches 1. This pattern continues around the unit circle and eventually demonstrates the periodicity of the sine function.

An in depth teaching of the unit circle, tangent and reciprocal ratios, coterminal angles, specific coordinates and the Pythagorean Identity are NOT appropriate for Math 3, as they will be covered in depth in the fourth math course.

Students should understand these relationships in degree and radian angle measure.

NC.M3.F-TF.2b

Extend the domain of trigonometric functions using the unit circle.

Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.

b. Interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its x coordinate.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
2 – Reason abstractly and quantitatively

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should describe the relationship between cosine represented on a unit circle and graphical representation of the cosine function.

Mastering the Standard

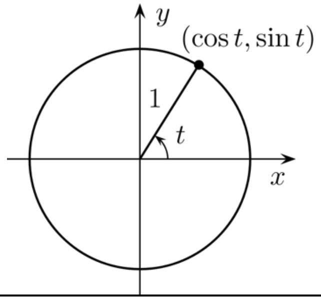
Comprehending the Standard

Students will be introduced to the unit circle and angle measures on the coordinate plane in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane.

A unit circle is used to develop the concepts of this standard to simplify the picture for students. In Math 3, students are only introduced to the trigonometric functions.

This standard builds upon previous understanding of the trig relationship in right triangle. $Cos\ \theta$ is the unit rate produced by the ratio of the length of the adjacent side to the length of the hypotenuse.

$$cos\ \theta = \frac{length\ of\ adjacent\ side}{length\ of\ hypotenuse}$$



Since we are working within a unit circle, and the hypotenuse is the radius of the unit circle, so the length of the hypotenuse is 1 unit. This means that $cos\ \theta = \frac{length\ of\ adjacent\ side}{1}$, so with the unit circle, $cos\ \theta$ is the length of the adjacent side.

This means that the base of the triangle, which is the x-coordinate of the vertex on the circle, is $cos\ \theta$.

Assessing for Understanding

Students apply reasoning to their knowledge of the relationship between angles and the sides of right triangles.

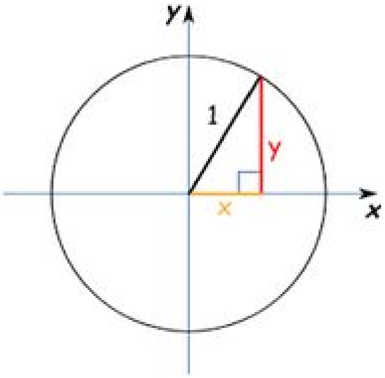
Example: Using the unit circle and segments below:

a) Why is the cosine value of the reference angle θ equal to x?

b) For $90^0 < \theta < 270^0$, why is the cosine value negative?

c) Why is the range of the cosine function $-1 \leq y \leq 1$?

d) Will the cosine value ever equal the sine value? Why or why not?



The focus of this standard is on the relationship between the changing angle of the cosine function and the value of the cosine ratio. This should allow students to move from the unit circle to graphing the relationship on a coordinate plane in which the independent variable is the angle measure and the dependent variable is the value of the cosine ratio (the *x*-coordinate from the unit circle). This is a strong connection to NC.M3.F-IF.1.

From the unit circle, students should see that as the angle is near zero, the ratio of the length of the opposite side to the length of the hypotenuse is also near 1. As the angle starts to increase and approaches $90°$ or $\frac{\pi}{2}$, the value of the cosine ratio approaches 0. This pattern continues around the unit circle and eventually demonstrates the periodicity of the cosine function.

As the angle changes, sine represents the change in the y-coordinate (height of the triangle) on the unit circle, cosine represents the change in the x-coordinate (length of the base of the unit circle).

Students should be able to not only see the relationship between the functions represented on a unit circle and the graphical representation on the coordinate plane, but should understand the relationship between the sine and cosine functions.

An in depth teaching of the unit circle, tangent and reciprocal ratios, coterminal angles, specific coordinates and the Pythagorean Identity are NOT appropriate for Math 3, as they will be covered in depth in the fourth math course.

Students should understand these relationships in degree and radian angle measure.

NC.M3.F-TF.5

Model periodic phenomena with trigonometric functions.

Use technology to investigate the parameters, a, b, and h of a sine function, $f(x) = a \cdot sin(b \cdot x) + h$, to represent periodic phenomena and interpret key features in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M3.A-SSE.1a)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)
- Build an understanding of trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss how changing the parameters effects the different representations.

New Vocabulary: period, amplitude

Mastering the Standard

Comprehending the Standard

It is important to not overreach with this standard. In Math 3, students are just being introduced to the concepts of the sine function and the effects of the various representations by changing parameters. As the phrase at the beginning of the standards states, students should use technology to investigate these changes.

There are several excellent online resources to investigate the change in parameters of trig functions. For some of these resources, you may need to create an account. Some of these resources are listed below. Some of the resources explore horizontal phase shift, which is not part of this standard.

Phase shifts and complicated trigonometric functions are not part of the standards for Math 3, as they will be covered in depth in the fourth math course. This is an introduction to the concept of a periodic graph through learning the sine function.

Assessing for Understanding

Students should be able to explain how the change in parameters affects the various representations and interpret them in a context.

Example: The following function describes the stock price for Facebook where m stands for the number of months since May 2012. Use technology to graph and create tables as needed.

$$f(m) =- 11 \cdot sin\,(\frac{2\pi}{4}m)\; + 38$$

a) Interpret the 38 in the context of the problem.
b) What does -11 mean in context of the problem and what is the significance of 11 being negative?
c) How long does it take for the pattern to start repeating?
d) During which months would you want to buy and sell stock in Facebook?

The North Carolina High School Collaborative Instructional Framework

NC Math 3
Unit 8: Statistics

5 Days Block Schedule

September 2017 Update

10 Days Traditional Schedule

RESEARCH BRIEF: Statistics

Essential Questions:

- When is it appropriate to use different statistical methods?
- What is the most appropriate statistical method given a situation?
- How can I use and evaluate statistical data to make decisions?

Learning Outcomes	Student Objectives
● Understand statistics as a process of making inferences about a population (parameter) based on results from a random sample (statistic). ● Acknowledge the role of randomization in using sample surveys, experiments, and observational studies to collect data and understand the limitations of generalizing results to populations (related to randomization) ● Understand simulation is useful for using data to make decisions. ● Know how to carry out a simulation with data for the purposes of: estimating population means or proportions, determining the margin of error for those estimates, and determining statistical significance. ● Understand that samples can differ by chance. ● Understand not all data that is reported is valid. Reports should be evaluated based on source, design of the study, and data displays.	● I will **distinguish** between a sample (statistic) and a population (parameter). ● I will **describe** how to select a random sample from a given population. ● I will **explain** the purposes and the differences of sample surveys, observational studies, and experiments, including how randomization applies to each. ● I will **distinguish** between sample surveys, observational studies, and experiments. ● I will **determine** how results of a statistical study can be generalized to make conclusions about a population based on the sample. ● I will **use** data from a sample survey to estimate a population mean or proportion with a margin of error. ● I will **determine** and **justify** if results from an experiment are statistically significant. ○ I will **identify** the parameter of interest in an experiment.

    - I will **select** and **calculate** sample statistics.
    - I will **calculate** the difference between the sample statistics.
    - I will **set up** and **complete** a simulation re-randomizing the groups.
    - I will **compare** the actual difference to the simulated differences to determine statistical significance.
- I will **state** a conclusion about the effectiveness or accuracy of a claim based on a sample.
- I will **evaluate** and make sense of a statistical article or website.

Standards Addressed in this Unit

Understand statistics as a process of making inferences about a population (parameter) based on results from a random sample (statistic).

- **NC.M3.S-IC.1**: Understand the process of making inferences about a population based on a random sample from that population.
- **NC.M3.S-IC.3**: Recognize the purposes of and differences between sample surveys, experiments, and observational studies and understand how randomization should be used in each.

Understand simulation is useful for using data to make decisions. Understand that samples can differ by chance.

- **NC.M3.S-IC.4**: Use simulation to understand how samples can be used to estimate a population mean or proportion and how to determine a margin of error for the estimate.
- **NC.M3.S-IC.5**: Use simulation to determine whether observed differences between samples from two distinct populations indicate that the two populations are actually different in terms of a parameter of interest.

Understand not all data that is reported is valid. Reports should be evaluated based on source, design of the study, and data displays.

- **NC.M3.S-IC.6:** Evaluate articles and websites that report data by identifying the source of the data, the design of the study, and the way the data are graphically displayed.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M3.S-IC.1

Understand and evaluate random processes underlying statistical experiments.

Understand the process of making inferences about a population based on a random sample from that population.

Concepts and Skills

Pre-requisite

- Use data from a random sample to draw inferences about a population (7.SP.2)

Connections

- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)
- Use simulation estimate a population mean or proportion (NC.M3.S-IC.4)
- Use simulation to determine whether observed differences between samples indicate the two populations are distinct (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: sample, population, random sample, inferential statistics

Mastering the Standard

Comprehending the Standard

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. Analyze the data with appropriate methods.
4. Interpret results and draw valid conclusions.

An essential understanding about the data collection step is that random selection can produce samples that represent the overall population. This allows for the generalization from the sample to the larger population in the last step of the process.

A *population* consists of everything or everyone being studied in an inference procedure. It is rare to be able to perform a census of every individual member of the population. Due to constraints of resources it is nearly

Assessing for Understanding

Students demonstrate an understanding of the different kinds of sampling methods.

Example: From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school advisory panel. Here are four different methods of making the selection. Which is the best sampling method, among these four, if you want the school panel to represent a fair and representative view of the opinions of your class? Explain the weaknesses of the three you did not select as the best.

a) Select the first three names on the class roll.
b) Select the first three students who volunteer.
c) Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix.
d) Select the first three students who show up for class tomorrow.

Students should recognize the need for random selection, describe a method for selecting a random sample from a given population, and explain why random assignment to treatments is important in the design of a statistical experiment.

Example: A department store manager wants to know which of two advertisements is more effective in increasing sales among people who have a credit card with the store. A sample of 100 people will be selected from the 5,300 people who have a credit card with the store. Each person in the sample will be

impossible to perform a measurement on every subject in a population. A *random sample* is a sample composed of selecting from the population using a chance mechanism. Often referred to as a simple random sample. *Inferential statistics* considers a subset of the population. This subset is called a statistical sample often including members of a population selected in a random process. The measurements of the individuals in the sample tell us about corresponding measurements in the population.	called and read one of the two advertisements. It will then be determined if the credit card holder makes a purchase at the department store within two weeks of receiving the call. a) Describe the method you would use to determine which credit card holders should be included in the sample. Provide enough detail so that someone else would be able to carry out your method. b) For each person in the sample, the department store manager will flip a coin. If it lands heads up, advertisement A will be read. If it lands tails up, advertisement B will be read. Why would the manager use this method to decide which advertisement is read to each person? Source: https://locus.statisticseducation.org/

NC.M3.S-IC.3

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Recognize the purposes of and differences between sample surveys, experiments, and observational studies and understand how randomization should be used in each.

Concepts and Skills

Pre-requisite

- Understand the process of making inferences (NC.M3.S-IC.1)

Connections

- Use simulation estimate a population mean or proportion (NC.M3.S-IC.4)
- Use simulation to determine whether observed differences between samples indicate the two populations are distinct (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Observational study, simulation, sample, population, random sample, inferential statistics

Mastering the Standard

Comprehending the Standard

Students understand the different methods of data collection, specifically the difference between an observational study and a controlled experiment, and know the appropriate use for each.

- *Observational study* – a researcher collects information about a population by measuring a variable of interest, but does not impose a treatment on the subjects. (i.e. examining the health effects of smoking)

- *Experiment* – an investigator imposes a change or treatments on one or more group(s), often called treatment group(s). A comparative experiment is where a control group is given a placebo to compare the reaction(s) between the treatment group(s) and the control group.

Assessing for Understanding

Students should be able to distinguish between the different methods.

Example: A student wants to determine the most liked professor at her college. Which type of study would be the most practical to obtain this information?

A) simulation
B) experiment
C) survey
D) Observation

Source: NC Measure of Student Learning CC Math III Spring 2013

Students understand the role that randomization plays in eliminating bias from collected data.

Example: Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students.

a) Describe the parameter of interest and a statistic the students could use to estimate the parameter.
b) Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
c) The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness.
d) Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above.

NC.M3.S-IC.4

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Use simulation to understand how samples can be used to estimate a population mean or proportion and how to determine a margin of error for the estimate.

Concepts and Skills

Pre-requisite

- Design and use simulation to generate frequencies for compound events (7.SP.8c)
- Understand the process of making inferences (NC.M3.S-IC.1)

Connections

- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)
- Use simulation to determine whether observed differences between samples indicate the two populations are distinct (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: simulation, sample, population, margin of error, parameter

Mastering the Standard

Comprehending the Standard

This standard has two parts:

1. Use simulation to understand how samples can be used to estimate a population mean or proportion
2. Use simulation to determine a margin of error for the estimate

Simulations may use physical manipulatives: dice, cards, beads, decks of playing cards. If available, simulations can be completed using technology. In either situation, students should have a clear understanding of how the simulation models the situation.

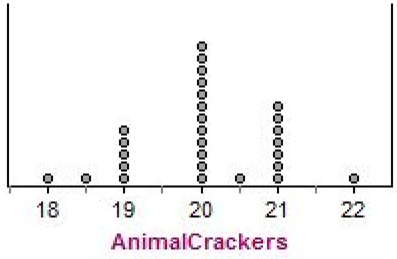
For the first part, students understand that a sample only provides an estimate of the population parameter. With repeated sampling, the estimates vary and a sampling distribution can be created to model the variation.

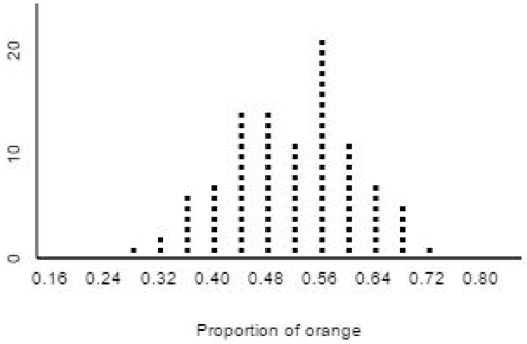
> Consider trying to determine the proportion of orange candies in Reese's Pieces. After taking a sample of 25 pieces, the proportion of orange is 0.40. Another sample has a proportion of orange as 0.60. By taking 100 random samples and computing the proportion of orange for each one a sampling distribution can be made.

Assessing for Understanding

Students should use a simulation to estimate a population mean or proportion and determine a margin of error for that estimate.

Example: The label on a Barnum's Animal Cracker box claims that there are 2 servings per box and a serving size is 8 crackers. The graph displays the number of animal crackers found in a sample of 28 boxes. Use the data from the 28 samples to estimate the average number of crackers in a box with a margin of error. Explain your reasoning or show your work.





Using the sampling distribution, students can estimate a population proportion using the mean of the distribution (0.51).

Simulation for Reese’s Pieces at http://www.rossmanchance.com/applets/OneProp/OneProp.htm?candy=1

For the second part, students understand that the margin of error is the maximum likely mistake in prediction. In other words, it is the most that a value of a sample statistic is likely to differ from the actual value of the population parameter.

One informal way of developing a margin of error from a simulation is to compute the value that is the range of the simulation’s results divided by 2 ($margin\ of\ error = \frac{range}{2}$).

In the Reese’s Pieces simulation, the margin of error would be 0.22 meaning the actual population proportion of orange candies is between 0.29 and 0.73.

Taking larger sample sizes would decrease the margin of error.

Changing the sample size to 50 gives a margin of error of 0.15.

Margin of error can be computed by formula; however, this standard is intended to engage students in using simulations to estimate. Note that confidence intervals are beyond what is intended in the standard. Students should have an idea of what margin of error is and how it is interpreted, which can lead informally to the idea of an interval estimate.

NC.M3.S-IC.5

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Use simulation to determine whether observed differences between samples from two distinct populations indicate that the two populations are actually different in terms of a parameter of interest.

Concepts and Skills

Pre-requisite

- Design and use simulation to generate frequencies for compound events (7.SP.8c)
- Understand the process of making inferences (NC.M3.S-IC.1)

Connections

- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)
- Use simulation estimate a population mean or proportion (NC.M3.S-IC.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: simulation, sample, population, parameter

Mastering the Standard

Comprehending the Standard

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. Analyze the data with appropriate methods.
4. Interpret results and draw valid conclusions.

This standard addresses parts 3 and 4 of this process. Once data is collected from an experiment, it is necessary to determine if there are differences between the two treatment groups. If so, are the differences due to the treatment or due to variation within the population?

Select a sample statistic to compare. For example, the mean of each sample.

> Consider the experiment where twenty male students were randomly assigned to one of two treatment groups of 10 students each, one group receiving 200 milligrams of caffeine and the other group no caffeine.
>
> The parameter of interest is the number of finger taps per minute. The sample statistics showed that the mean of the

Assessing for Understanding

Students should demonstrate an understanding of the process by

- identifying the parameter of interest,
- select and calculate sample statistics,
- calculate the difference between the sample statistic,
- set up and complete a simulation re-randomizing the groups,
- and compare the actual difference to the simulated differences

Example: Sal purchased two types of plant fertilizer and conducted an experiment to see which fertilizer would be best to use in his greenhouse. He planted 20 seedlings and used Fertilizer A on ten of them and Fertilizer B on the other ten. He measured the height of each plant after two weeks. Use the data below to determine which fertilizer Sal should use.

Fertilizer A	23.4	30.1	28.5	26.3	32.0	29.6	26.8	25.2	27.5	30.8
Fertilizer B	19.8	25.7	29.0	23.2	27.8	31.1	26.5	24.7	21.3	25.6

a) Use the data to generate simulated treatment results by randomly selecting ten plant heights from the twenty plant heights listed.

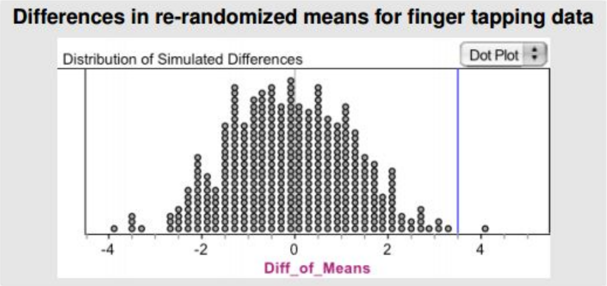
b) Calculate the average plant height for each treatment of ten plants.

c) Find the difference between consecutive pairs of treatment averages and compare. Does your simulated data provide evidence that the average plant heights using Fertilizer A and Fertilizer B is significant?

200 mg group was 3.5 taps more than the 0 mg group. Thus, an observed difference.

Use simulation to determine if the observed difference is due to the caffeine.

Is it possible that the 3.5 taps was due to randomization and not caffeine? In order to find out, re-randomize the participants and calculate the difference in means. Simulate this and create a distribution of the results.

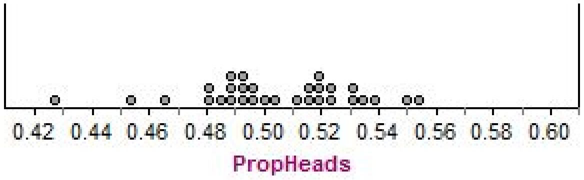


The results of the simulation shows that the difference of 3.5 is equaled or exceeded only once out of 400 trials this providing strong evidence that the caffeine is the cause of the increased tapping.

Source: http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf

Example: “Are Starbucks customers more likely to be female?” To answer the question, students decide to randomly select 30-minute increments of time throughout the week and have an observer record the gender of every tenth customer who enters the Starbucks store. At the end of the week, they had collected data on 260 customers, 154 females and 106 males. This data seems to suggest more females visited Starbucks during this time than males.

To determine if these results are statistically significant, students investigated if they could get this proportion of females just by chance if the population of customers is truly 50% females and 50% males. Students simulated samples of 260 customers that are 50-50 females to males by flipping a coin 260 then recording the proportion of heads to represent the number of women in a random sample of 260 customers (e.g., 0.50 means that 130 of the 260 flips were heads). Their results are displayed in the graph at the right.



Use the distribution to determine if the class’s data is statistically significant enough to conclude that Starbucks customers are more likely to be female.

NC.M3.S-IC.6

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Evaluate articles and websites that report data by identifying the source of the data, the design of the study, and the way the data are graphically displayed.

Concepts and Skills

Pre-requisite

- Use appropriate statistics to compare center and spread of two or more data sets and interpret differences in context (NC.M1.S-ID.2)
- Recognize the purpose and differences between samples and studies and how randomization is used (NC.M3.S-IC.3)

Connections

The Standards for Mathematical Practices

Connections

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Comprehending the Standard

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3. Analyze the data with appropriate methods.
4. Interpret results and draw valid conclusions.

When students are presented with information supported by data, they should critically examine the source of the data, the design of the study and the graphs to determine the validity of the article or website.

Students should recognize how graphs and data can be distorted to support different points of view. Students should use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays.

This standard connects to NC.M3.S-IC.1, 3, 4, & 5.

Assessing for Understanding

Students critically evaluate the source of the data, the design of the study, and the graphical displays.

Example: Read the article below from NPR.org then answer the following questions.

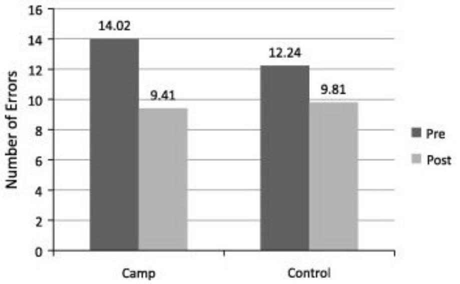
Kids and Screen Time: What Does the Research Say?
By Juana Summers
August 28, 2014

Kids are spending more time than ever in front of screens, and it may be inhibiting their ability to recognize emotions, according to new research out of the University of California, Los Angeles.

The study, published in the journal Computers in Human Behavior, found that sixth-graders who went five days without exposure to technology were significantly better at reading human emotions than kids who had regular access to phones, televisions and computers.

The UCLA researchers studied two groups of sixth-graders from a Southern California public school. One group was sent to the Pali Institute, an outdoor education camp in Running Springs, Calif., where the kids had no access to electronic devices. For the other group, it was life as usual.

At the beginning and end of the five-day study period, both groups of kids were shown images of nearly 50 faces and asked to identify the feelings being modeled. Researchers found that the students who went to camp scored significantly higher when it came to reading facial emotions or other nonverbal cues than the students who continued to have access to their media devices.



"We were pleased to get an effect after five days," says Patricia Greenfield, a senior author of the study and a distinguished professor of psychology at UCLA. "We found that the kids who had been to camp without any screens but with lots of those opportunities and necessities for interacting with other people in person improved significantly more."

If the study were to be expanded, Greenfield says, she'd like to test the students at camp a third time — when they've been back at home with smartphones and tablets in their hands for five days.

"It might mean they would lose those skills if they weren't maintaining continual face-to-face interaction," she says.

a) What is the source of the data?
b) Describe the design of the study.
c) After analyzing the graph, evaluate the claim that the "kids who had been to camp … improved significantly more."