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Negative Transverse Impedance

1 Introduction

In Ref. [1] we report an observation that the horizontal and the vertical loss factors have opposite signs for several types of geometries. Recently, measurements in the SPS show that the coherent tune shift in the horizontal direction has positive values whereas that in the vertical direction has negative ones. [2] Thus, the existence of negative transverse impedance gets confirmed in a real machine. This stimulates us to start a new round of systematic studies on this interesting phenomenon. The results obtained from our computer simulations are presented in this note.

It is known that, for a circularly symmetric geometry, the transverse wakefield has a positive first peak. This has been discussed in detail by A. W. Chao. [3] After having studied a couple of examples, Chao concludes, among others, "In general, one finds that the polarity of the transverse wake forces is such that it always hurts a short beam". However, the proof given in Ref. [3] is limited to the resistive wall case and the case where the boundary is infinitely periodic and has rotational symmetry. The generality of this conclusion is questionable. In fact, in the particular example that we will discuss below, the above conclusion is no longer valid.

Fig. 1 is reproduced from Ref. [3], which illustrates that the first peak of the transverse wake is positive and that the transverse wake force further deflects ("hurts") the test charge that closely follows the source charge. This is a correct picture insofar as the structure has rotational symmetry. However, when this symmetry is broken, the whole picture may change. The first peak may become negative and the transverse wake force may help the test charge stay closer to the beam tube axis, as we find in the SPS case.

2 Simulations of SPS Adaptors

Figure 2 shows the structures of two types of adaptors used in the SPS, which are likely to be the contributors to the negative impedance measured in the horizontal direction. In our simulations, these adaptors are approximated by a geometry that consists of a circularly symmetric cavity and two beam tubes of a rectangular cross-section, as shown in Fig. 3. The detailed structures of the adaptors are considered not to be essential for our studies and are thus ignored. When an off-axis Gaussian bunch of an r.m.s. length of 15 cm traverses this geometry, the transverse wake potentials and the loss factors are computed by the 3-D code MAFIA. (Ref. [4])

In order to see how a positive peak of the transverse wake potential could become a negative one when the rotational symmetry embedded in a structure is broken, the following procedure is taken. The middle part of Fig. 3, i.e. the circularly symmetric cavity, is kept unchanged, with a radius of 9 cm, while the cross-section of the beam tubes on both sides varies in the following way.

1. The cross-section is approximated by a circle of a radius of 3 cm. The results can then be compared with that calculated by the 2-D simulations.

2. The cross-section is deformed to a square, of which the half-width of the horizontal side, x, is equal to the half-height of the vertical side, y, and both are equal to 3 cm. The results should be close to that obtained from the first run above.

3. Keeping the vertical dimension y fixed at 3 cm, we vary the horizontal dimension x from 3 cm to 7 cm.

4. Decreasing the vertical dimension y to 2 cm, repeat step 3.

The results of these runs are summarized in Figs. 4(a) and (b). In Fig. 4(a) it is seen that, as the dimension x increases, the first peak of the horizontal wake decreases. When x equals 4.5 cm (3.5 cm) and y equals 3 cm (2 cm), this peak is almost zero. When x increases further, the peak becomes a negative one. To visualize this transition, Fig. 5(a) exhibits a positive peak of the horizontal wake potential when x and y are both equal to 3 cm, and Fig. 5(b) a negative one when x is increased to 7 cm.

Fig. 4(b) shows the behavior of the horizontal loss factor. It is similar to that seen in Fig. 4(a).

As a comparison, the results from 2-D TBCI calculations, in which the rectangular beam tubes are replaced by circular ones, are also included in Figs. 4(a) and (b). They are in good agreement with that obtained from MAFIA.

We have tried different mesh sizes, beam tube lengths and cavity lengths in the simulations and found that these are irrelevent to our results, just as expected.

In contrast to the horizontal direction, the first peak of the vertical wake, as well as the vertical loss, are all positive in these runs. This is obviously due to the specification of the ratios between the two transverse dimensions in our simulations.

DISCLAIMER

3 Simulations of Other Geometries

In addition to the rectangular-beam-tube/circular-cavity type of geometry, we have tested some other types of 3-D structures.

1. The beam tubes are circularly symmetric and the cavity is of a rectangular shape. In this case, we no longer see any negative transverse impedance for various width-to-height ratios of the rectangle.

2. Both the beam tubes and the cavity have rectangular cross-sections. The results of this geometry are more complicated. When we repeat steps 3 and 4 above, the horizontal loss factor decreases when x increases, but never becomes negative. The first peak of the horizontal wake potential, on the other hand, does cross the zero line and takes negative values when x is big enough.

4 Discussion

When a beam traverses a discontinuity in a beam chamber, it always loses energy. Negative transverse impedance would imply an energy gain of a beam in the transverse direction. Thus one might be concerned about energy conservation. But actually there is no violation of the conservation of energy in our case. The energy gain in one transverse (horizontal) direction comes from the energy loss in another transverse (vertical) direction and the longitudinal direction. The total energy change of the beam is still a loss.

The wake potentials that we compute are, of course, not the same thing as the wakefields that are discussed in Ref. [3]. The wake potential of a finite bunch is the convolution integral of the wakefield and the line density of the bunch. Therefore, one might argue that Fig. 1 is still a correct picture whereas there may be a quick turn over of the first wake peak in our case. This scenario is quite unlikely, although we cannot prove it impossible at this moment.

A qualitative understanding of negative transverse impedance can be given as follows. The transverse impedance is proportional to the difference of the coherent image coefficient, ξ, and the incoherent image coefficient, ε. As the horizontal dimension of a geometry increases, ξ_H would decrease. In the limit case that the horizontal dimension becomes infinity, ξ_H would be equal to zero, because a horizontal displacement of the beam would not change the forces acting on it. [5] Meanwhile, ε_H would remain finite. Therefore, when the horizontal dimension is large enough, the difference, $\xi_H - \varepsilon_H$, would change its sign. So would the horizontal impedance.

For the time being, our results are solely obtained from computer simulations. A further analytical study on this interesting subject is necessary in order to convince ourselves the truthfulness of the results. This work is under way.

In summary, our simulations demonstrate that the negative transverse impedance may appear when the rotational symmetry embedded in a discontinuity is broken, and that the geometries that we have studied may be the sources of the positive horizontal tune shift measured in the SPS.

We thank Dr. D. Brandt for bringing Ref. [2] to our attention and for helpful discussions. We also thank Dr. J. Cook for encouragement and for many enjoyable discussions we had with him.

References

[1] W. Chou and Y. Jin, *Impedance Studies - Part 3: Transverse-Loss Compensation*, ANL Light Source Note LS-114 (April 1988).

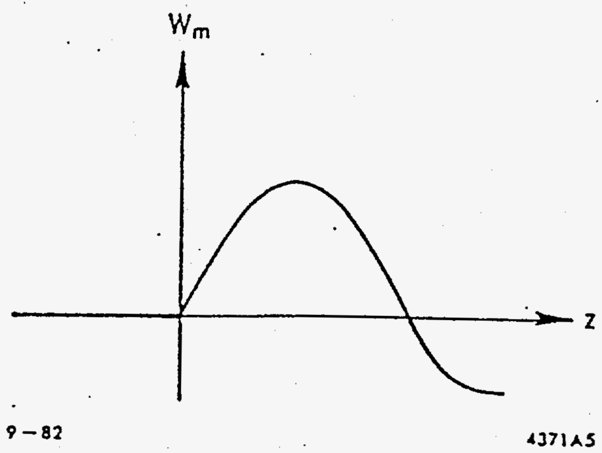
[2] D. Brandt et al., *Tune Shift Measurements in the SPS*, SPS/AMS/Note/88-14 (December 14, 1988).

[3] A. W. Chao, *Coherent Instabilities of a Relativistic Bunched Beam*, SLAC-PUB-2946 (June, 1982).

[4] T. Weiland, IEEE Trans. Nucl. Sci., NS-32, 2738 (1985).

[5] B. Zotter, Nucl. Instrum. Methods, v. 129, 377 (1975).

(a)



(b)

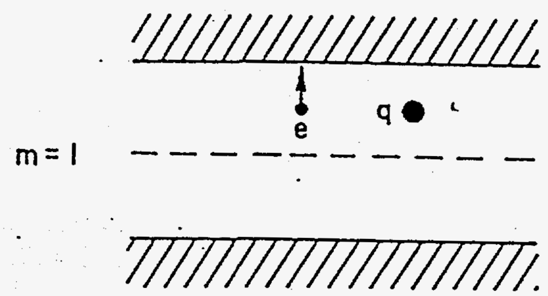


Fig. 1. (a) Sketch of the transverse wake function, which exhibits a positive peak.
(b) The transverse wake force further deflects the test charge.
Both figures are reproduced from Ref. 3.

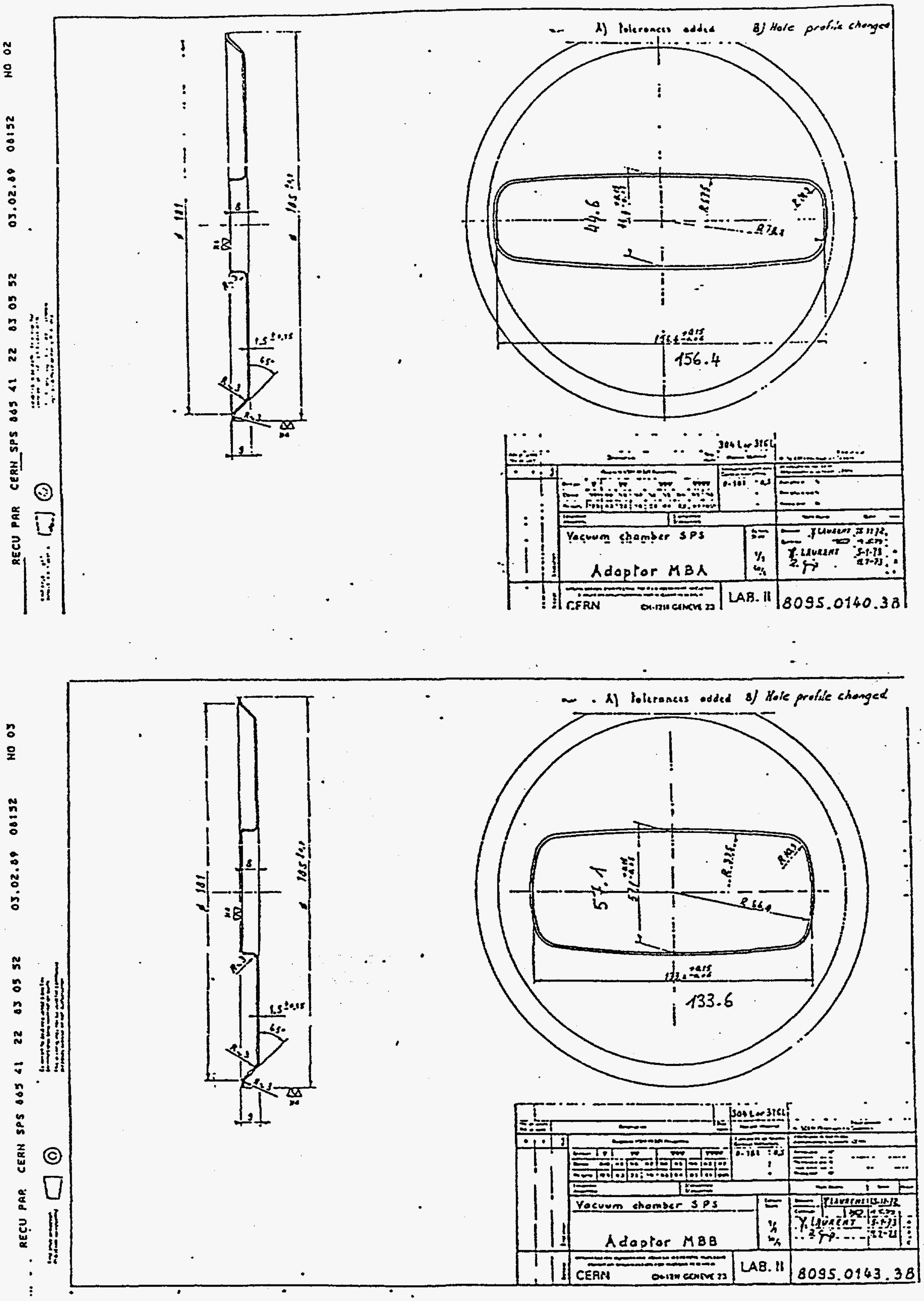


Fig. 2. Two types of adaptors in the SPS.

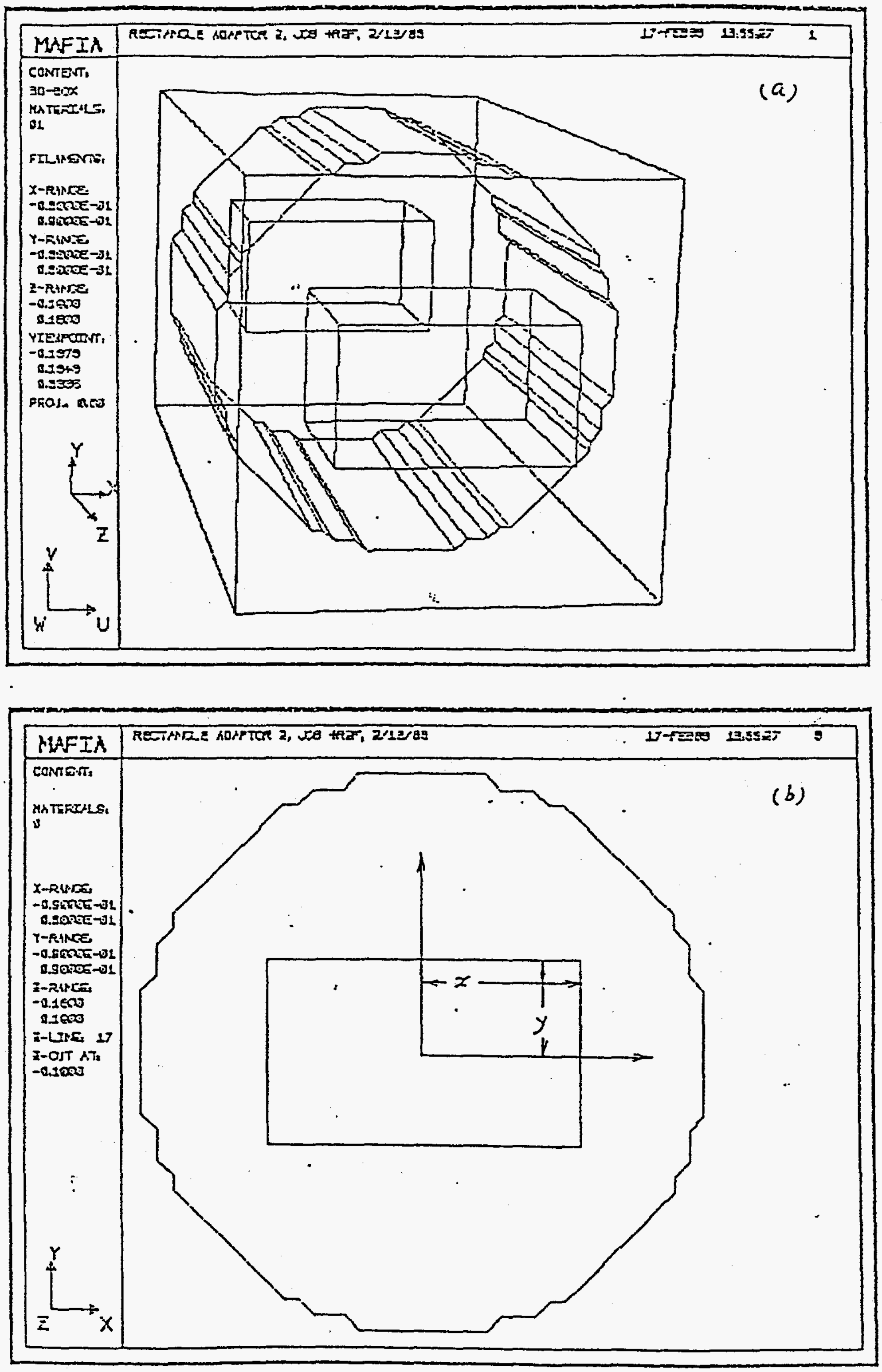
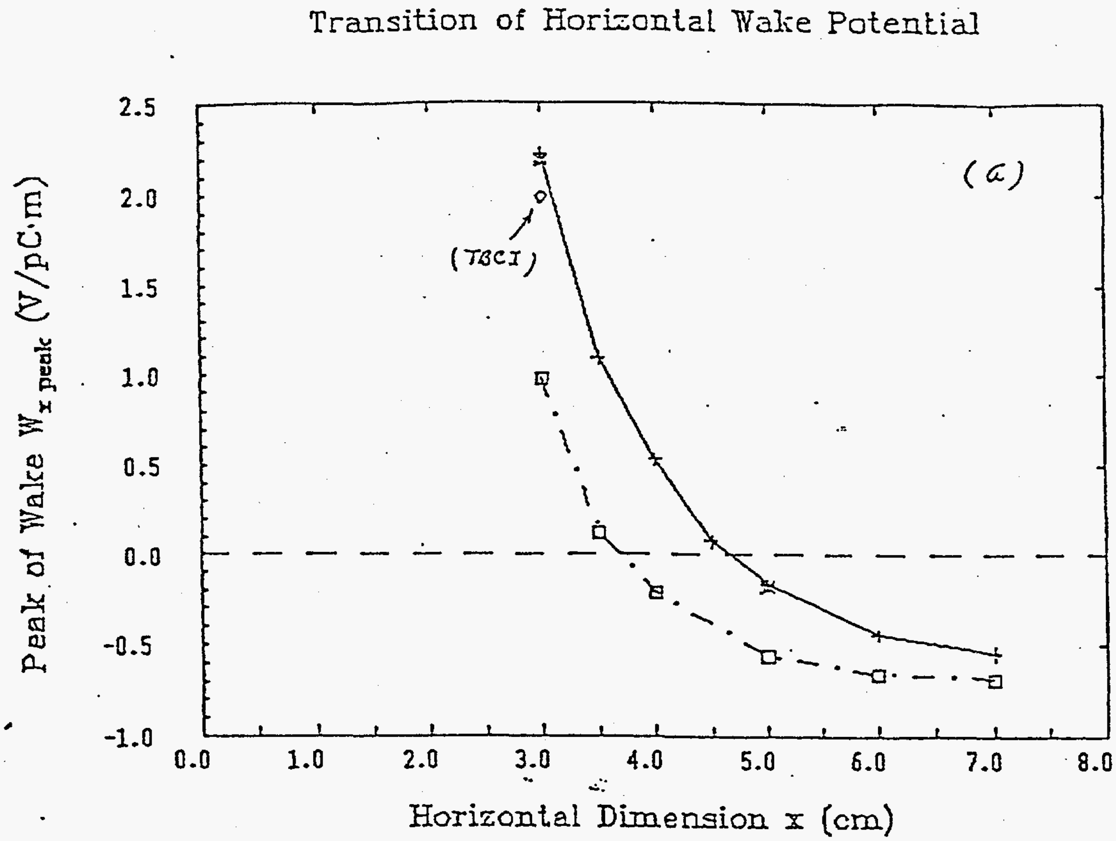


Fig. 3. The geometry used in the 3-D simulations. (a) 3-D view and (b) Front view.



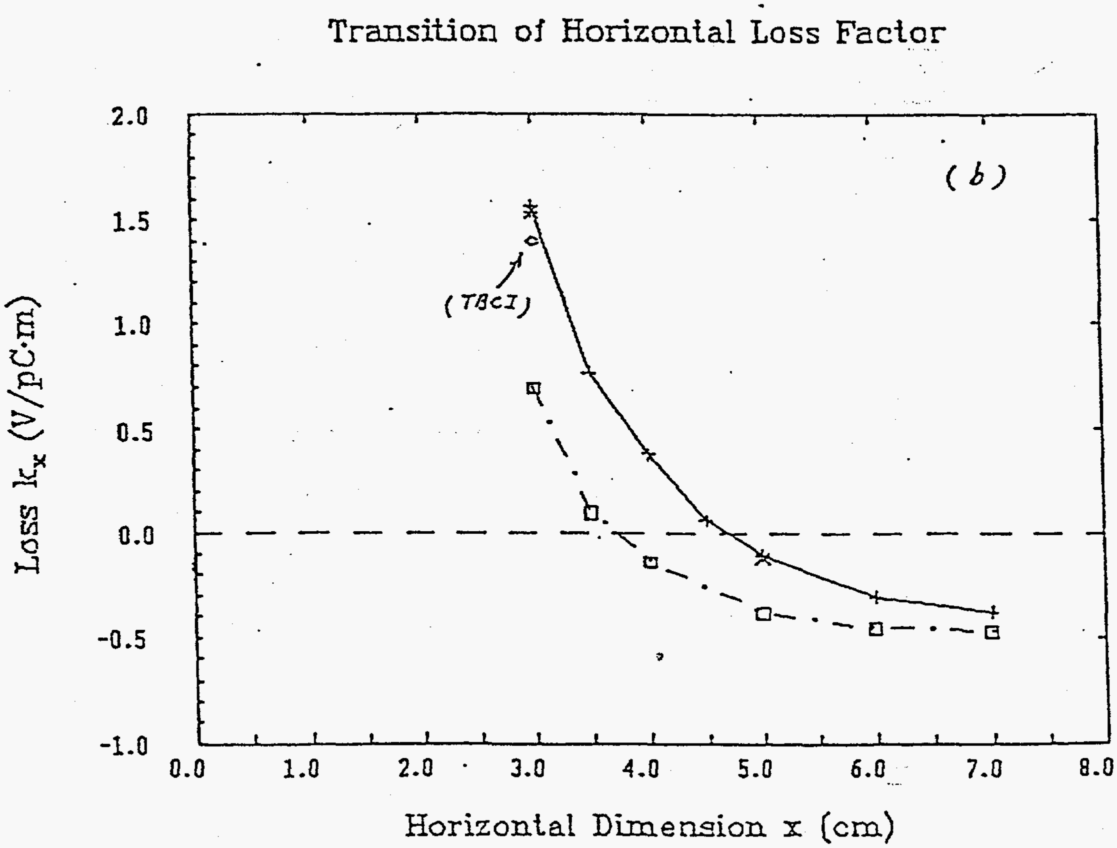


Fig. 4. (a) The first peak of the horizontal wake potential vs. x, the half-width of the horizontal side of the rectangular cross-section of beam tubes. (see Fig. 3)
(b) The horizontal loss factor vs. x. The solid lines correspond to a half-height of the vertical side, y, of 3 cm, and the dot-dashed ones to an y of 2 cm. Note that all the curves cross the zero line.

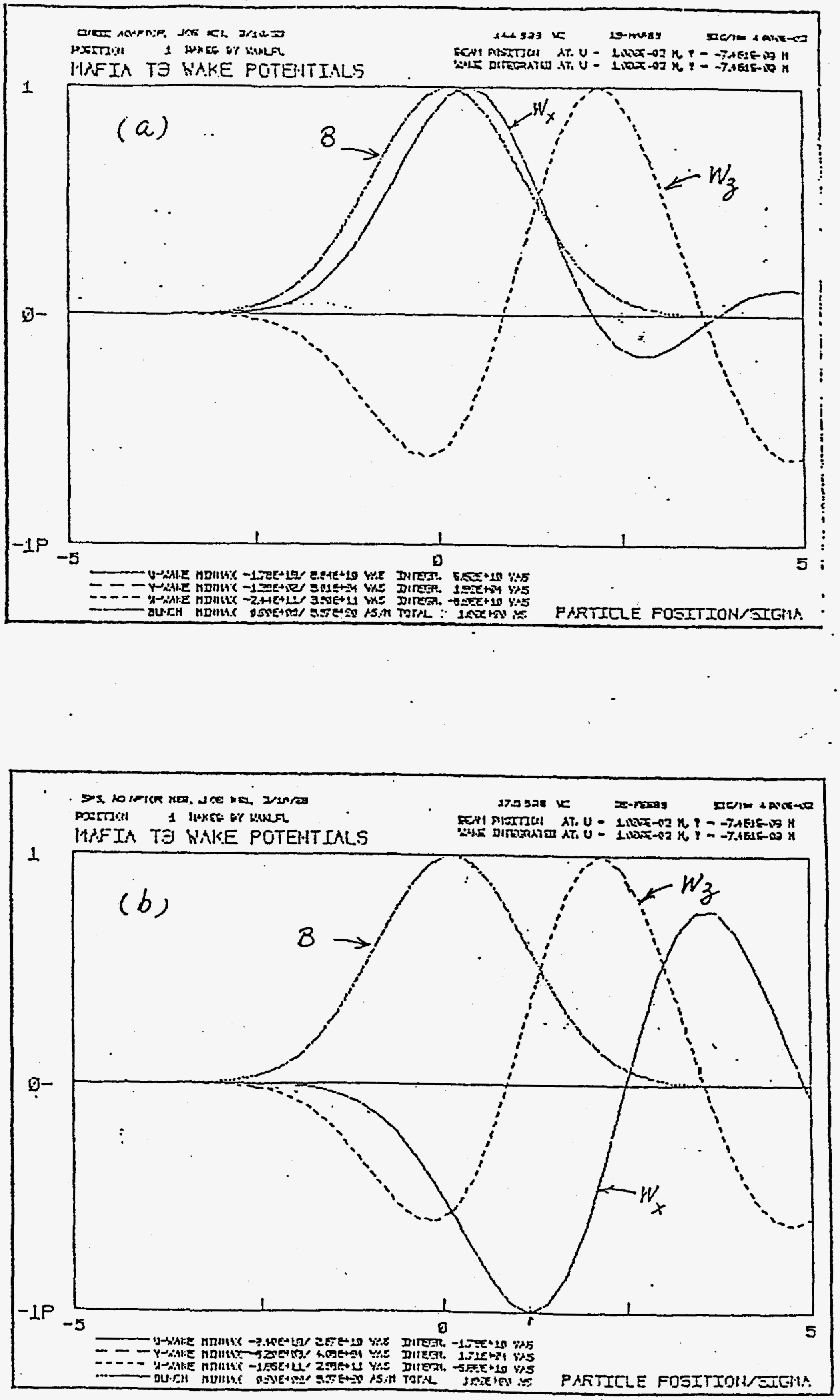


Fig. 5. The bunch (B), the horizontal wake (W_x) and the longitudinal wake (W_z) for an x of (a) 3 cm and (b) 7 cm. The scales on the vertical axis are normalized. The horizontal axis is in unit σ, the r.m.s. length of the bunch.