Kirchhoff's Laws

Ohm's Law by itself is not sufficient to analyze circuits. Two of the important laws for analyzing circuits came from German physicist Gustav Robert Kirchhoff in 1847. These laws are formally known as Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL).

Kirchhoff's first law (KCL) is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically it can be written as

$$\sum_{n=1}^{N} i_n = 0 \qquad (2.1)$$

Here N is the number of branches connected to the node and i_n is the n^{th} current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative and current entering a node is considered as positive.

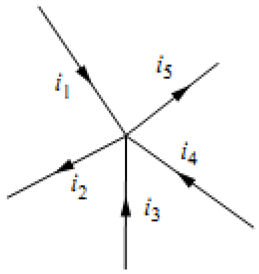

Figure 1 Current at a node illustrating KCL

Considering the node in figure 1, applying KCL gives

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

Or

$$i_1 + i_3 + i_4 = i_2 + i_5$$

So and alternative form of KCL can be stated as

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Note that KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point. In two dimensions, a closed boundary is the same as a closed path. As typically illustrated in the circuit of Figure 2, the total current entering the closed surface is equal to the total current leaving the surface.

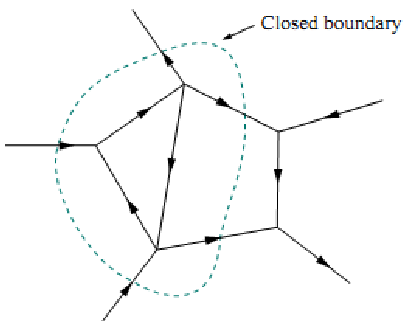

Figure 2 Applying KCL to a closed boundary

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A simple application of KCL is combining current sources in par-allel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in figure 3(a) can be combined as in figure 3(b). The combined or equivalent current source can be found by applying KCL to node a.

$$I_T + I_2 = I_1 + I_3$$

Or

$$I_T = I_1 - I_2 + I_3$$

A circuit cannot contain two different currents, I_1 and I_2, in series, unless $I_1 = I_2$; otherwise KCL will be violated.

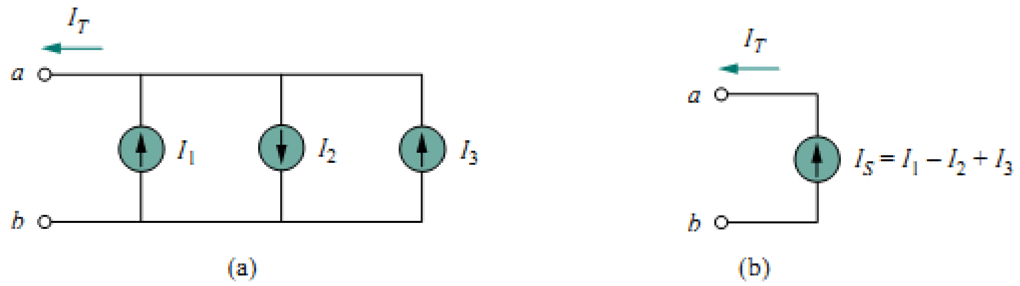


Figure 3 Current Sources in parallel (a) Original Circuit (b) equivalent Circuit

Kirchhoff's second law (KVL) is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

KVL can be mathematically expressed as

$$\sum_{m=0}^{M} v_m = 0 \qquad (2.2)$$

Here M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the m^{th} voltage.

To illustrate KVL, consider the circuit in figure 4. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1, +v_2, +v_3, -v_4, and\ +\ v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-\ v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Or

$$v_1 + v_4 = v_2 + v_3 + v_5$$

This can be interpreted as

$$Sum\ of\ voltage\ drops\ =\ Sum\ of\ voltage\ rises \qquad (2.3)$$

This is an alternative form of KVL. If we have travelled counterclockwise, we would have ended up with same equation.

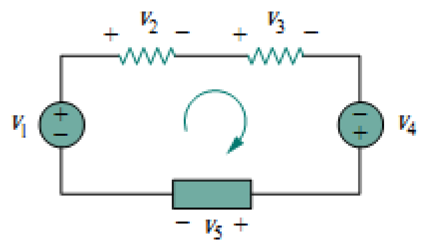


Figure 4 A single loop circuit illustrating KVL

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources. For example, for the voltage sources shown in Fig. 5(a), the combined or equivalent voltage source in Fig. 5(b) is obtained by applying KVL.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

Or

$$V_{ab} = V_1 + V_2 - V_3$$

To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.

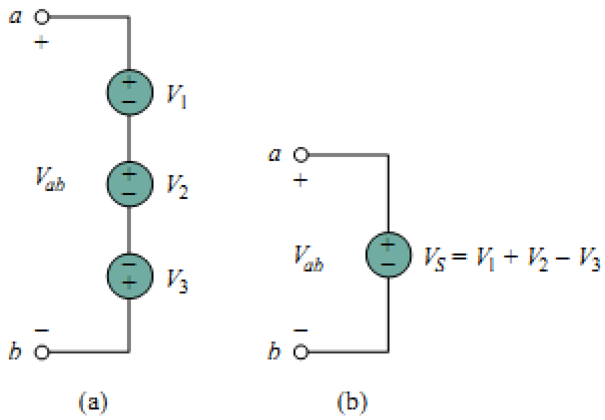


Figure 5 Voltage sources in series, (a) original circuit (b) equivalent circuit

EXAMPLE 2.5

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2.

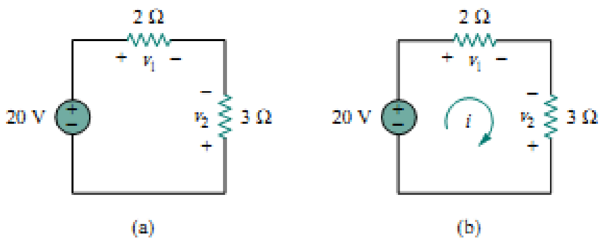


Figure 2.21 For Example 2.5.

Solution:

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \quad \Longrightarrow \quad i = 4\text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8\text{ V}, \qquad v_2 = -12\text{ V}$$

EXAMPLE 2.6

Determine v_o and i in the circuit shown in Fig. 2.23(a).

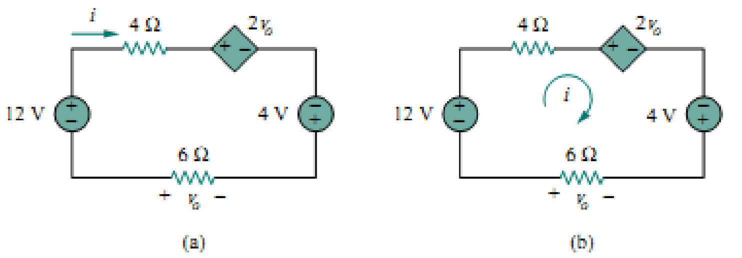


Figure 2.23 For Example 2.6.

Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad (2.6.1)$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad (2.6.2)$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Longrightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

EXAMPLE 2.7

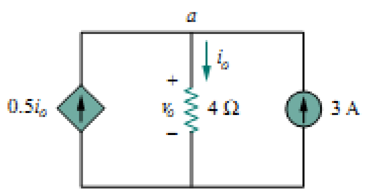


Figure 2.25 For Example 2.7.

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

Solution:

Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \quad \Longrightarrow \quad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

EXAMPLE 2.8

Find the currents and voltages in the circuit shown in Fig. 2.27(a).

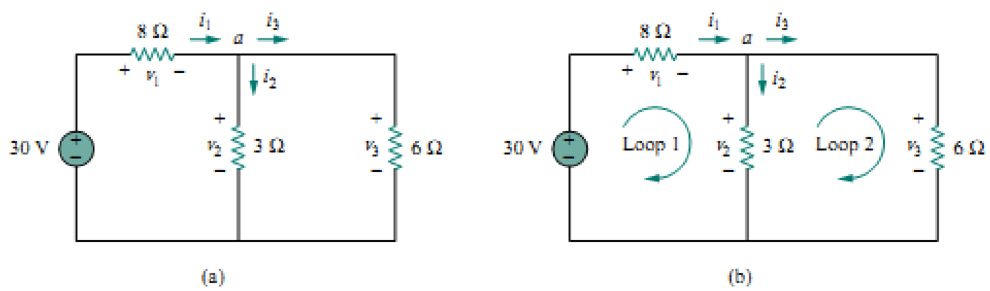


Figure 2.27 For Example 2.8.

Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \tag{2.8.1}$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Longrightarrow \qquad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \qquad \Longrightarrow \qquad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \qquad i_3 = 1 \text{ A}, \qquad v_1 = 24 \text{ V}, \qquad v_2 = 6 \text{ V}, \qquad v_3 = 6 \text{ V}$$

1. Alexander Practice problems 2.5-2.8

Series Resistors and Voltage Division

Combinations of resistors in series or parallel are so common, that it warrants some special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 6(a). The two resistors are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$V_1 = iR_1, \qquad V_2 = iR_2 \qquad (2.4)$$

If we apply KVL we get

$$-V + V_1 + V_2 = 0$$

Or

$$V = V_1 + V_2 = i(R_1 + R_2) \qquad (2.5)$$

Therefore

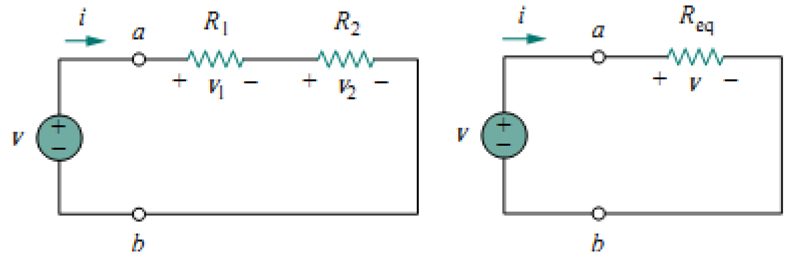
$$i = \frac{V}{R_1 + R_2} \qquad (2.6)$$

We can also find the current using the equivalent circuit of figure 6(b)

$$i = \frac{V}{R_{eq}}$$

So we can imply that

$$R_{eq} = R_1 + R_2 \qquad (2.7)$$



(a) (b)

Figure 6 (a)A series circuit with 2 resistors and (b) their equivalent circuit

In general it can be said as

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$\boldsymbol{R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n} \qquad \mathbf{(2.8)}$$

To determine voltage across each resistor, we substitute equation 2.6 in equation 2.4 to get

$$V_1 = \frac{R_1}{R_1 + R_2}V, \qquad V_2 = \frac{R_2}{R_1 + R_2}V \qquad (2.9)$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the principle of voltage division, and the circuit in Fig. 6 is called a voltage divider. In general, if a voltage divider has N resistors (R_1,R_2,...,R_N) in series with the source voltage v, the nth resistor (Rn) will have a voltage drop of

$$\boldsymbol{V_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N}V} \qquad \mathbf{(2.10)}$$

This rule is known as Voltage Division Rule (**VDR)**

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Parallel Resistors and Current Division

When two elements connect at a single node pair, they are said to be in parallel. Parallel-connected circuit elements have the same voltage across their terminals. Consider the circuit in figure 7(a) From Ohm's law,

$$V = i_1 R_1 = i_2 R_2$$

Or

$$i_1 = \frac{V}{R_1}, \qquad i_2 = \frac{V}{R_2}$$

Applying KCL at node a

$$i = i_1 + i_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}} \qquad (2.11)$$

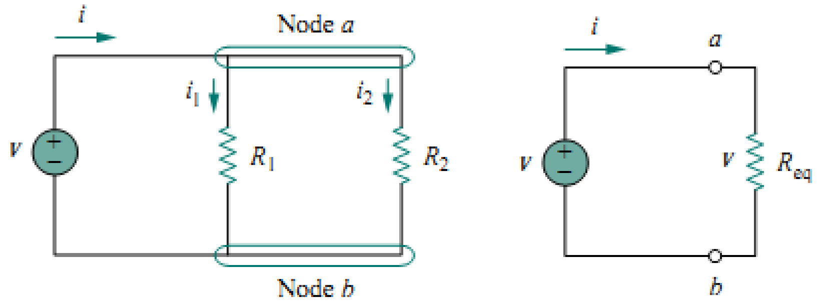


Figure 7 (a) Two Resistors in parallel and (b) their equivalent circuit

Where R_{eq} is the resistance in parallel (Figure 7(b))

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (2.12)$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad (2.13)$$

We can see that if $R_1 = R_2$, $\quad R_{eq} = \frac{R_1}{2}$

We can extend the equation 2.13 for any number of resistances in parallel. SO the equation for equivalent resistance become

$$\boldsymbol{R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}} \qquad \mathbf{(2.14)}$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N} \qquad (2.15)$$

Given the total current i entering node a in Fig. 7, how do we obtain current i_1 and i_2? We know that the equivalent resistor has the same voltage, or

$$V = iR_{eq} = \frac{i\, R_1 R_2}{R_1 + R_2} \qquad (2.16)$$

So

$$i_1 = \frac{R_2 \;\; i}{R_1 + R_2}, \quad i_2 = \frac{R_1\, i}{R_1 + R_2} \qquad (2.17)$$

Which shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit in Fig. 7 is known as a current divider. Notice that the larger current flows through the smaller resistance. This rule is known as Current Division Rule (**CDR)**

Another form of the CDR can be written as

$$i_j = \frac{V}{R_j} = \frac{R_{eq}}{R_j} i \qquad (2.18)$$

Here
R_{eq} = Equivalent resistance
R_j = J[th] Resistor
i_i = Current through J[th] Resistor

As an extreme case, suppose one of the resistor in figure 7 is zero, say $R_2 = 0$; ie $R_2$is a short circuit as shown in figure 8(a) since $R_2 = 0$, $i_1 = 0$, $i_1 = i$. This means that the entire current flows through R_2 and bypasses R_1. Thus when a circuit is short circuited, as shown in figure 8(a) two things should be kept in mind

1. The equivalent resistance is 0
2. The entire current flows through the short circuit

Another extreme case is when $R_2 = \infty$, that is, R_2 is an open circuit, as shown in figure 8(b) The current in this case flows through the least resistance path and flows through R_1 thus the equivalent resistance is $R_{eq} = R_1$

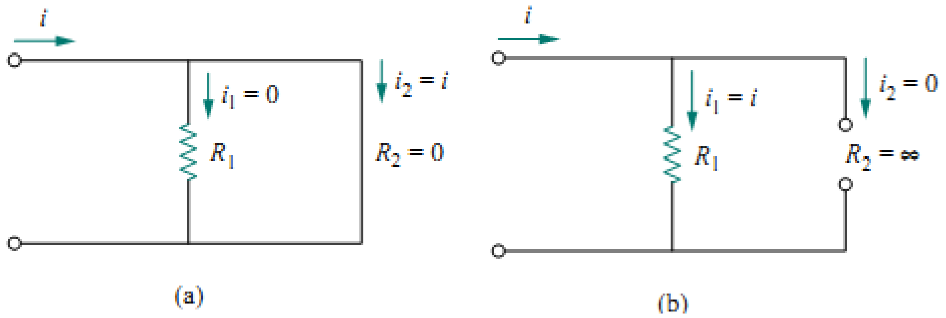


Figure 8 (a) Short Circuit and (b) Open Circuit

EXAMPLE 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6+3} = 2\ \Omega$$

(The symbol ∥ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4+6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

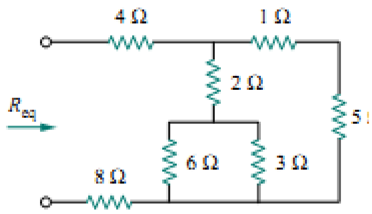
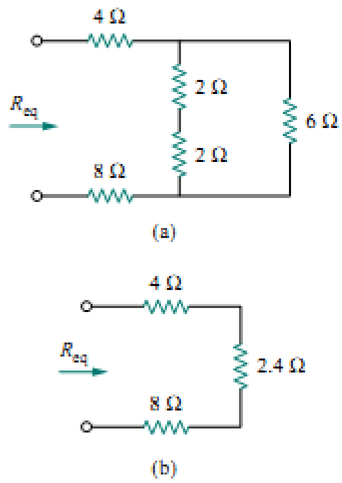


Figure 2.34 For Example 2.9.



EXAMPLE 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

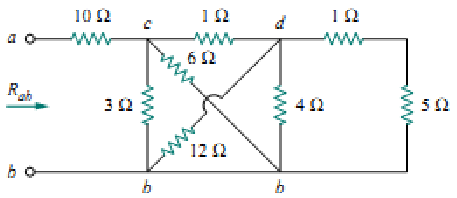


Figure 2.37 For Example 2.10.

Solution:

The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3+6} = 2\ \Omega \tag{2.10.1}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12+4} = 3\ \Omega \tag{2.10.2}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \tag{2.10.3}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of $1\ \Omega + 2\ \Omega = 3\ \Omega$. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

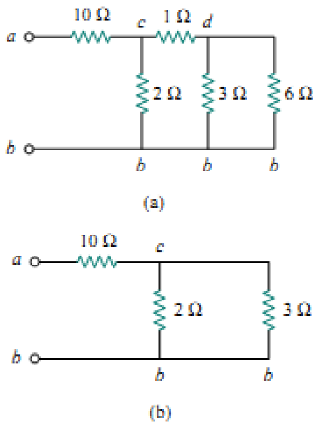


Figure 2.38 Equivalent circuits for Example 2.10.

EXAMPLE 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.

Solution:

The 6-Ω and 3-Ω resistors are in parallel, so their combined resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6+3} = 2\ \Omega$$

Thus our circuit reduces to that shown in Fig. 2.42(b). Notice that v_o is not affected by the combination of the resistors because the resistors are in parallel and therefore have the same voltage v_o. From Fig. 2.42(b), we can obtain v_o in two ways. One way is to apply Ohm's law to get

$$i = \frac{12}{4+2} = 2\ \text{A}$$

and hence, $v_o = 2i = 2 \times 2 = 4$ V. Another way is to apply voltage division, since the 12 V in Fig. 2.42(b) is divided between the 4-Ω and 2-Ω resistors. Hence,

$$v_o = \frac{2}{2+4}(12\ \text{V}) = 4\ \text{V}$$

Similarly, i_o can be obtained in two ways. One approach is to apply Ohm's law to the 3-Ω resistor in Fig. 2.42(a) now that we know v_o; thus,

$$v_o = 3i_o = 4 \quad \Longrightarrow \quad i_o = \frac{4}{3}\ \text{A}$$

Another approach is to apply current division to the circuit in Fig. 2.42(a) now that we know i, by writing

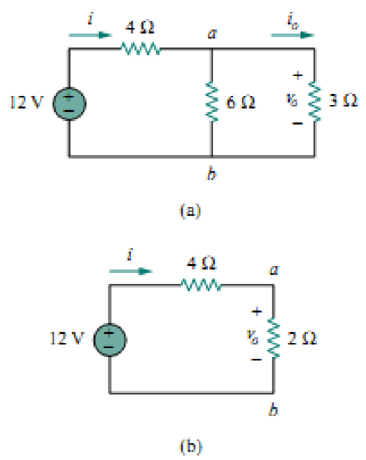


Figure 2.42 For Example 2.12: (a) original circuit, (b) its equivalent circuit.

$$i_o = \frac{6}{6+3}i = \frac{2}{3}(2\text{ A}) = \frac{4}{3}\text{ A}$$

The power dissipated in the 3-Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\text{ W}$$

EXAMPLE 2.13

For the circuit shown in Fig. 2.44(a), determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

Solution:

(a) The 6-kΩ and 12-kΩ resistors are in series so that their combined value is $6 + 12 = 18$ kΩ. Thus the circuit in Fig. 2.44(a) reduces to that shown in Fig. 2.44(b). We now apply the current division technique to find i_1 and i_2.

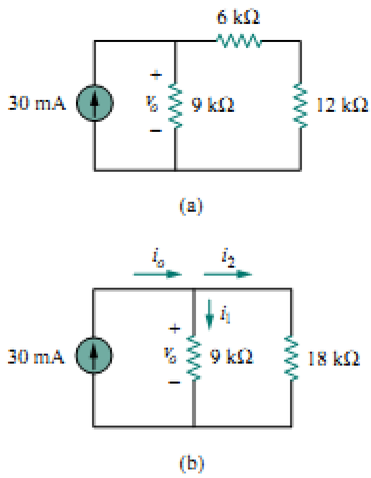


Figure 2.44 For Example 2.13: (a) original circuit, (b) its equivalent circuit.

$$i_1 = \frac{18{,}000}{9000 + 18{,}000}(30\text{ mA}) = 20\text{ mA}$$

$$i_2 = \frac{9000}{9000 + 18{,}000}(30\text{ A}) = 10\text{ mA}$$

Notice that the voltage across the 9-kΩ and 18-kΩ resistors is the same, and $v_o = 9{,}000 i_1 = 18{,}000 i_2 = 180$ V, as expected.

(b) Power supplied by the source is

$$p_o = v_o i_o = 180(30)\text{ mW} = 5.4\text{ W}$$

(c) Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2 R) = i_2^2 R = (10 \times 10^{-3})^2(12{,}000) = 1.2\text{ W}$$

Power absorbed by the 6-kΩ resistor is

$$p = i_2^2 R = (10 \times 10^{-3})^2(6000) = 0.6\text{ W}$$

Power absorbed by the 9-kΩ resistor is

$$p = \frac{v_o^2}{R} = \frac{(180)^2}{9000} = 3.6\text{ W}$$

or

$$p = v_o i_1 = 180(20)\text{ mW} = 3.6\text{ W}$$

Notice that the power supplied (5.4 W) equals the power absorbed ($1.2 + 0.6 + 3.6 = 5.4$ W). This is one way of checking results.

2. Alexander practice problem 2.9, 2.10, 2.12, 2.13

Wye-Delta Transformation (Y-Δ)

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 9. How do we combine resistors R1 through R6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 9 can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 10 and the delta (Δ) or pi (π) network shown in Fig. 11. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

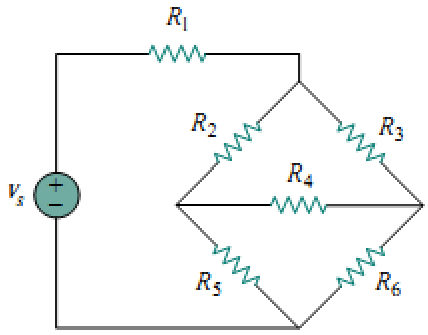


Figure 9 A bridge network

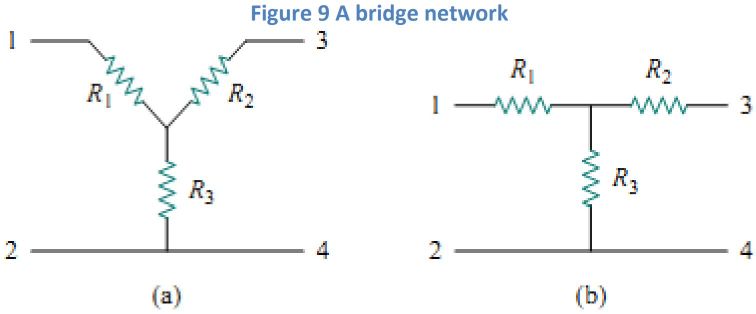


Figure 10 Two Forms of same network (a) Y and (b) T

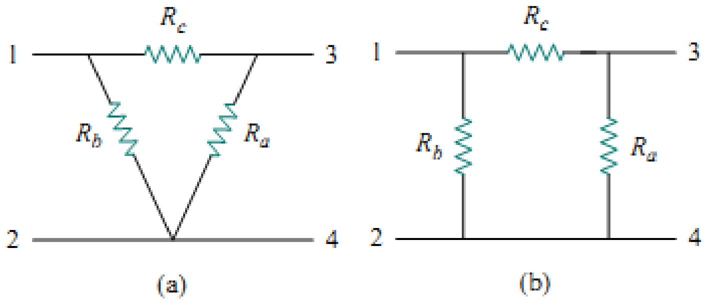


Figure 11 Two forms of same network (a) Δ and (b) π

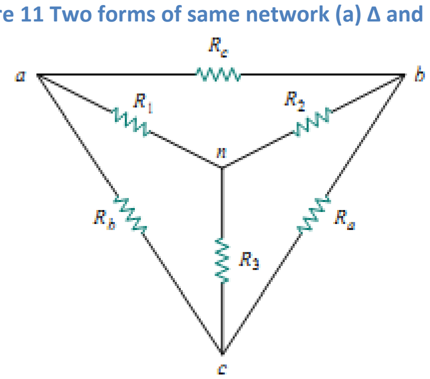


Figure 12 Superposition of Y and Δ networks as an aid in transforming one to another

Δ to Y conversion

The equations for Δ to Y conversion is as follow

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Y to Δ Conversion

The equations for Y to Δ conversion is as follows

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be balanced when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c$$

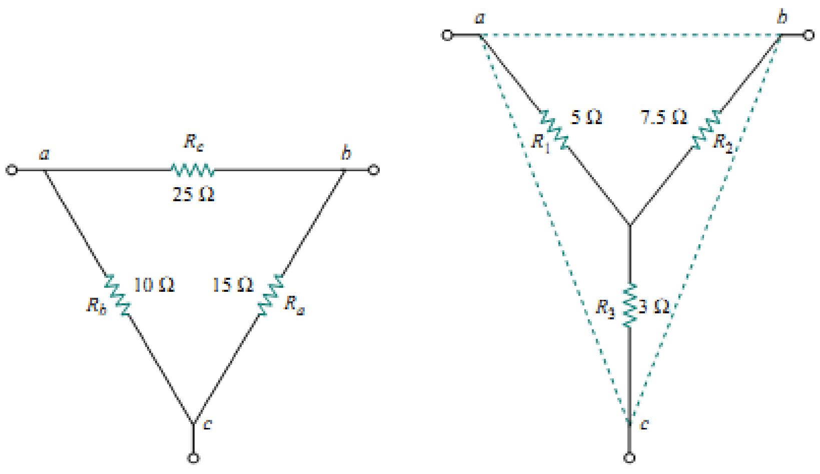
Under these conditions

$$R_Y = \frac{R_\Delta}{3} \qquad or\ R_\Delta = 3R_Y$$

Proof of Y-Δ transformations is available in Alexander book section 7.2

EXAMPLE 2.14

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.



(a) (b)

Solution:

Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{25 \times 10}{25 + 10 + 15} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

EXAMPLE 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

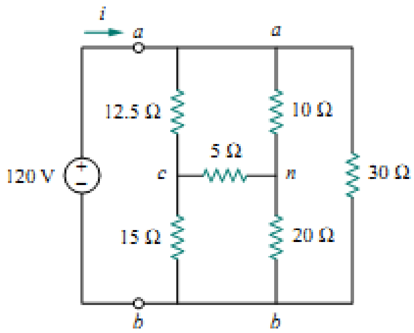


Figure 2.52 For Example 2.15.

Solution:

In this circuit, there are two Y networks and one Δ network. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

Thus from Eqs. (2.53) to (2.55) we have

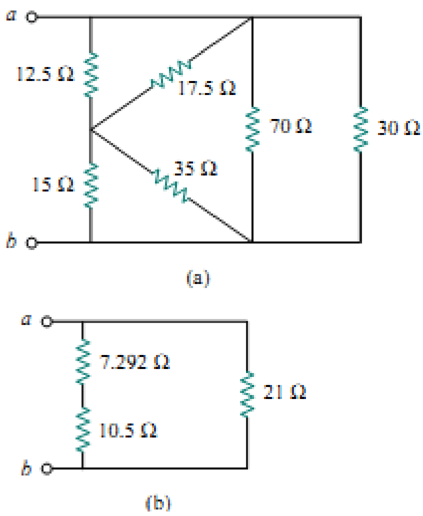
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

With the Y converted to Δ, the equivalent circuit (with the vo source removed for now) is shown in Fig. 2.53(a). Combining the pairs of resistors in parallel, we obtain



$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.2917\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = 9.632\ \Omega$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458\ \text{A}$$

3. Alexander practice problem 2.14, 2.15

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