Neutrino Mass and Proton Decay in Renormalizable SO(10)

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Grand Unified Theories (GUTs): Elegant and ambitious

- Unifies all matter and forces

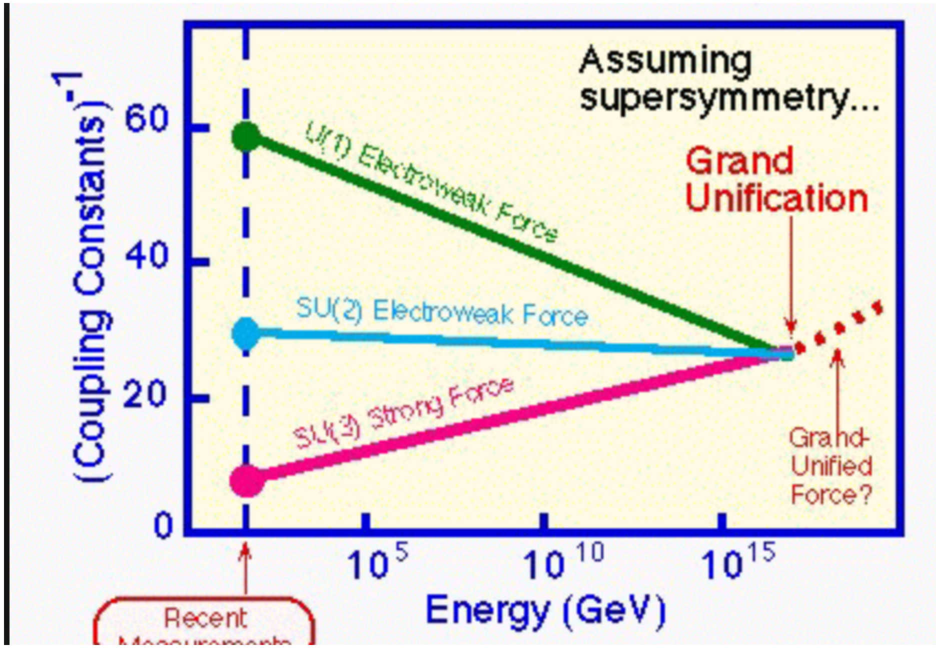
- Makes theory more predictive

  e.g. can predict $\boxed{\sin^2\theta_W}$ + more

- Also quantizes electric charges

Why SUSY GUTs

- Couplings unify at scale ~10^{16} GeV with susy breaking at TeV required to solve gauge hierarchy problem



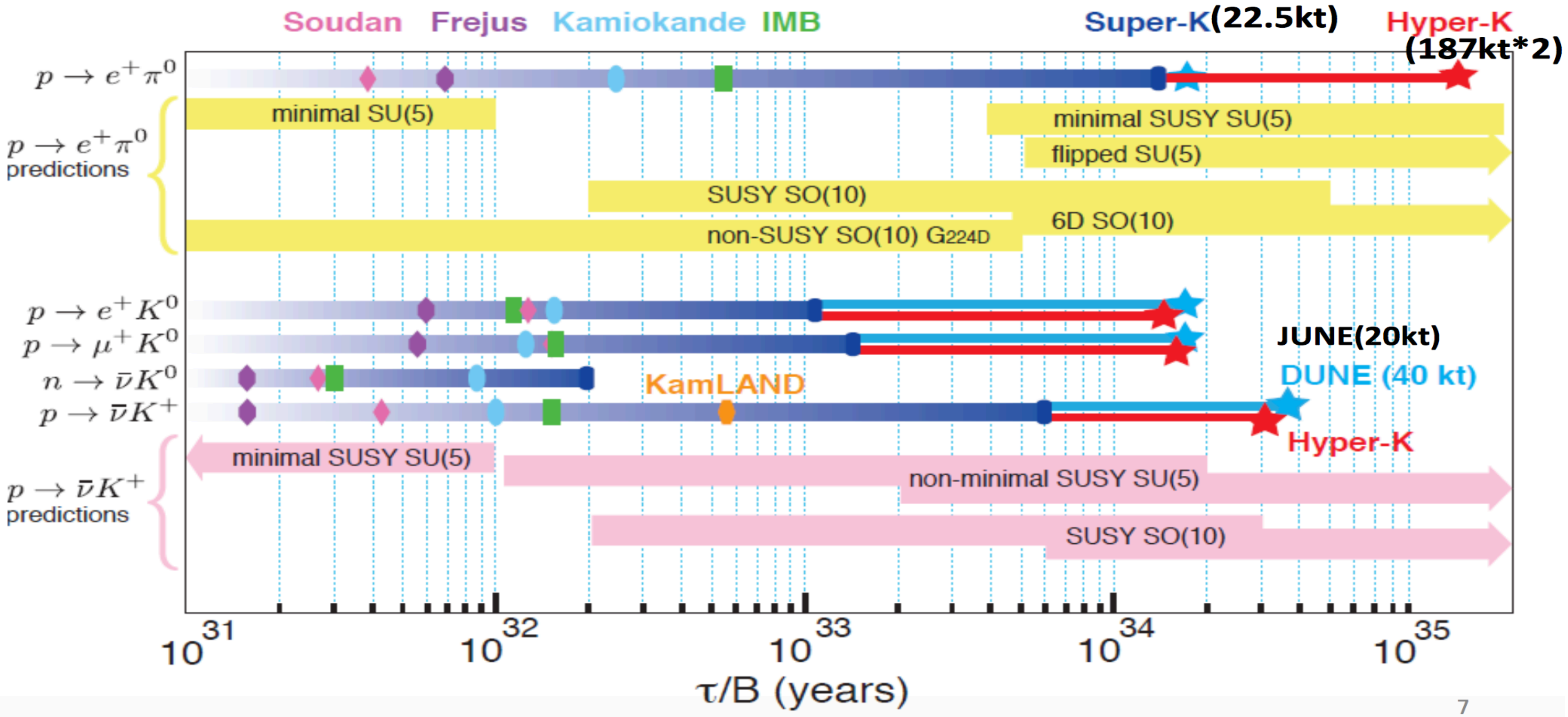
(Dimopoulos, Raby, Wilczek’81; Ibanez, Ross’81)

Where is SUSY?

Key prediction:proton decay

- GUT group→ Q-L unification→proton decay

$p \to e^+\pi^0\,;\, p \to K^+ + \bar{\nu}$ **no evidence yet**



No SUSY, No proton decay

why pursue GUTs?

Neutrino masses have provided new life to GUTs

Neutrino masses: 20 years before <1998

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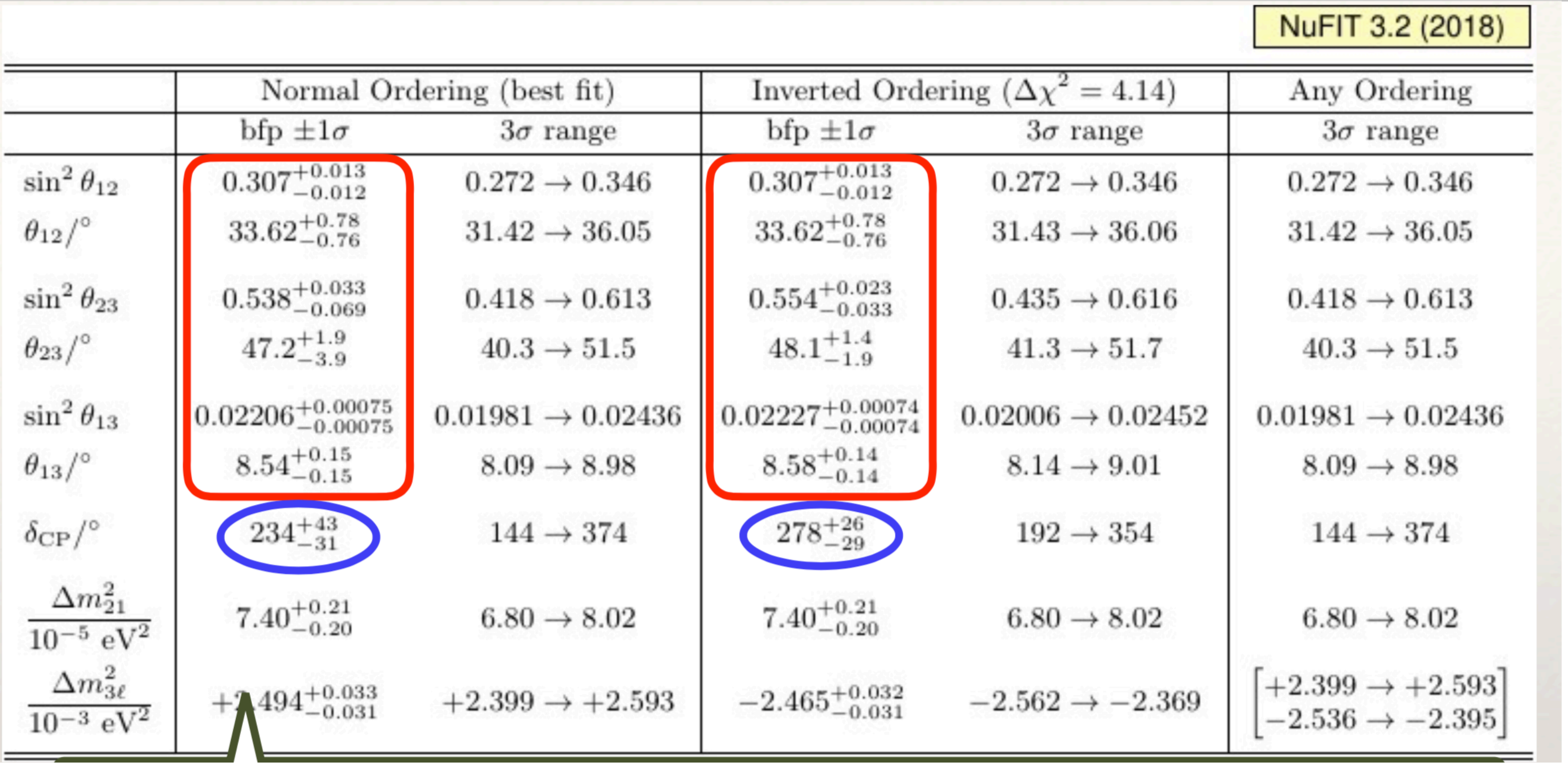
Neutrino masses and mixings now



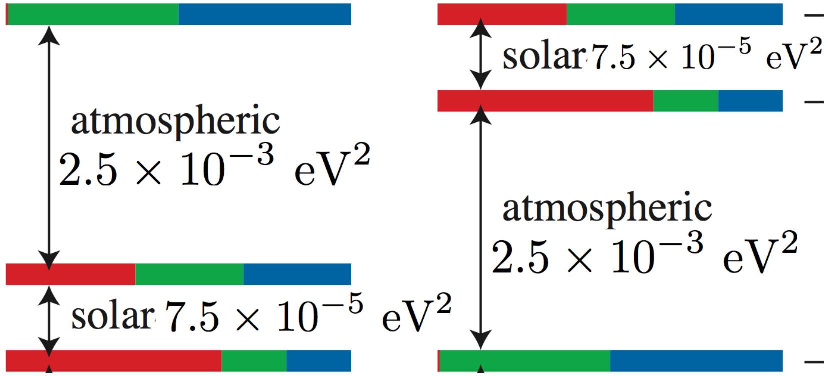
NuFIT 3.2 (2018)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.272 \to 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \to 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \to 36.06$	$31.42 \to 36.05$
$\sin^2\theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \to 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \to 0.616$	$0.418 \to 0.613$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \to 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \to 51.7$	$40.3 \to 51.5$
$\sin^2\theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \to 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \to 0.02452$	$0.01981 \to 0.02436$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \to 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \to 9.01$	$8.09 \to 8.98$
$\delta_{CP}/^\circ$	234^{+43}_{-31}	$144 \to 374$	278^{+26}_{-29}	$192 \to 354$	$144 \to 374$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$6.80 \to 8.02$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \to +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \to -2.369$	$\begin{bmatrix} +2.399 \to +2.593 \\ -2.536 \to -2.395 \end{bmatrix}$

Cosmology $\sum m_\nu [\text{eV}] < 0.3$

Things we do not know

- Mass ordering



- CP phase δ_{CP} (DUNE, T2K,..)
- Are their sterile neutrinos?
- Absolute scale $\sum m_\nu$
- Are they Majorana or Dirac?

(This talk assumes 3 neutrinos!)

Neutrino mass and revival of GUTs

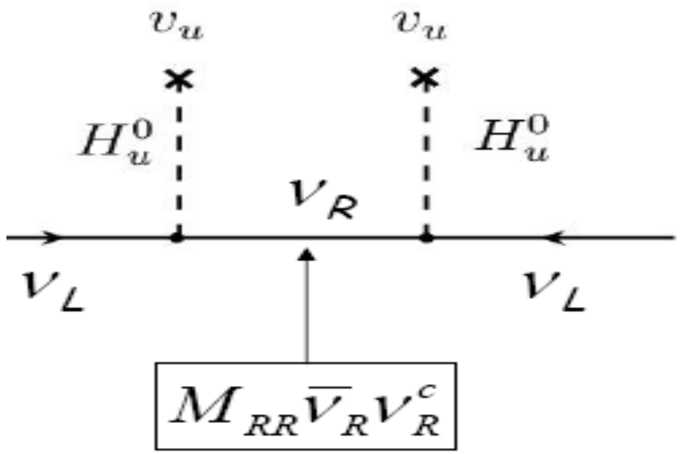
- $m_\nu \neq 0$ needs new physics beyond SM.
- Two new puzzles from neutrino mass discovery

(i) $m_\nu \ll m_q, m_\ell$

(ii) Lepton mixing patterns different from quarks

Seesaw paradigm for tiny neutrino mass fits well in GUTs

- SM+ RH neutrinos ν_R but with heavy Majorana mass



→ $m_\nu \simeq -\frac{m_D^2}{M_{\nu_R}}$

(Minkowski'77; Mohapatra,Senjanovic; Gell-Mann,Ramond, Slansky; Yanagida; Glashow'79)

- Q-L unification→ $m_{D33} \sim m_t$ → $M_{\nu_R} \sim 10^{14}$ GeV
- Fits well into GUT framework since $M_{\nu_R} \sim M_U$

GUT group SO(10): Just right for seesaw

- Two key ingredients of seesaw:

  (a) right handed neutrino

  (b) B-L symmetry
- Both are automatic in SO(10) unification:
- $SO(10) \supset B - L$
- Fundamental {16}- rep $\supset \mathrm{SM\ fermions} + \mathrm{N_R}$

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$$

GUT model building

(i) Coupling unification

(ii) GUT symmetry and fermions

(iii) Symmetry breaking (Higgs sector)

GUT model building

(i) Coupling unification (SO(10)→MSSM)

(ii) GUT symmetry and fermions ✔

(iii) Symmetry breaking (Higgs sector)

(Many ways)

Two ways to break B-L to get seesaw in SO(10) models

- Class I: Use $\{16_H\}+\{\overline{16_H}\}$

- Class II: $\{126\}+\{\overline{126}\}$

Two ways to break B-L and two classes of SO(10) models

- Class I: Use $\{16_H\}+\{\overline{16}_H\}$

  (a) Breaks R-Parity; so no natural DM;

  (b) Higher dim operators for fermion masses:

  > 63 parameters in the fermion sector

  (c) *String th. leads naturally to 16_H fields !*

- Class II: $\{126\}+\{\overline{126}\}$

  (a) R-parity automatic; Neutralino DM natural

  (b) with 10+126, # param=18 for fermions;

  very predictive; ideal model for neutrinos (Babu, Mohapatra'93)

Understanding Flavor: a challenge for GUTs

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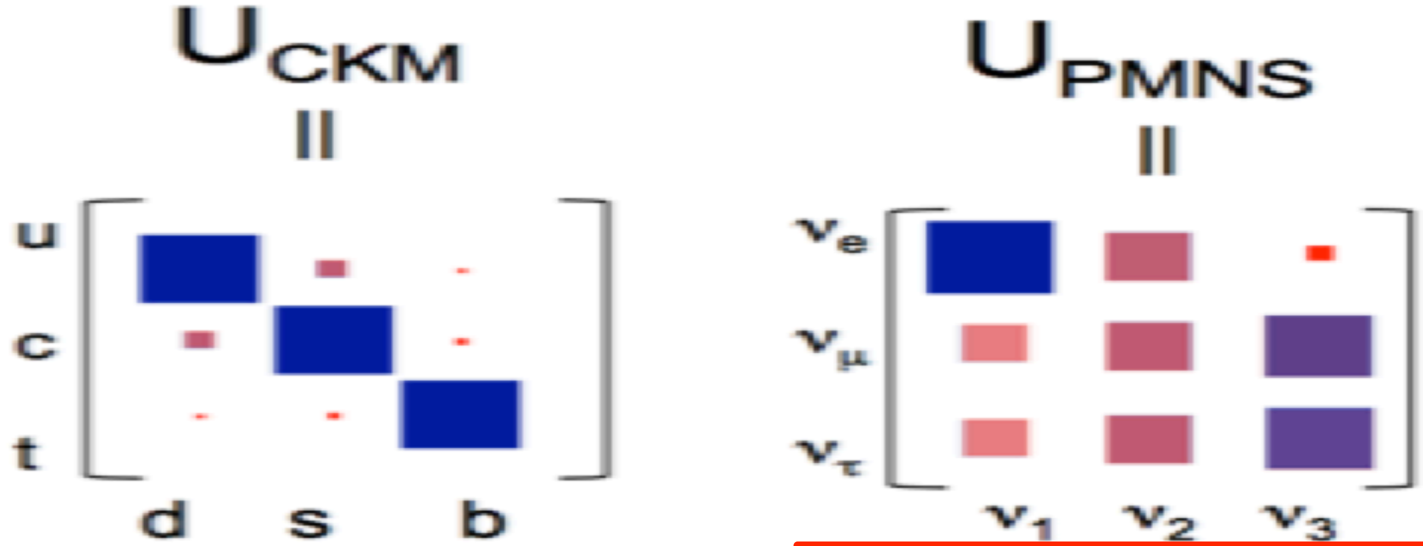


$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$

$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$

- *Is this diverse pattern even compatible with quark-lepton unification inherent in GUTs?*

Two approaches to quark-lepton flavor in SO(10) GUTs

- Minimal SO(10) models without flavor sym. Can meet the challenge

- Flavor symmetry augmented GUTs to understand needed Yukawa texture

$$SO(10) \times G_F$$

Two renormalizable SO(10) SUSY GUT models

- Scenario: SO(10) → MSSM → SM
- Note: $16_F \times 16_F = 10 + 126 + 120$
- Minimal renormalizable models with 10+126-Higgs (# param 18)(Babu, RNM'93)

$$W = h\psi\psi H + f\psi\psi\bar{\Delta}$$

- Next: 10+126+120+CP (# param=17+3)

$$W = h\psi\psi H + f\psi\psi\bar{\Delta} + g\psi\psi\Sigma$$

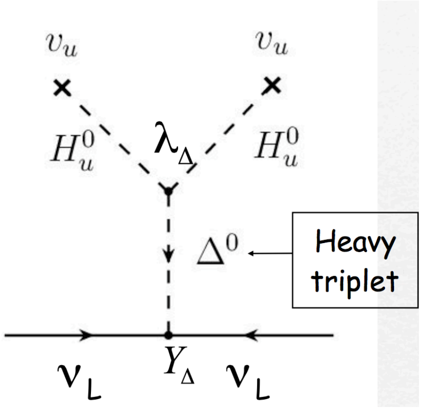
(Dutta, Mimura, RNM'04;
Bertolini, Frigerio,Malinsky'04)

Flavor in minimal SO(10): 10+126

- $$M_u = hv_u + f\kappa_u;\quad M_d = hv_d + f\kappa_d;$$ (Babu, RNM'92)
  $$M_\nu^D = hv_u - 3f\kappa_u;\quad M_l = hv_d - 3f\kappa_d;$$
- $$\mathcal{M}_\nu = fv_L - M_d \frac{1}{fv_R} M_d \;;\quad f = \frac{1}{4\kappa_d}(M_d - M_\ell)$$
- in GUTs $m_b(M_U) \simeq m_\tau(M_U)$ endows $\mathcal{M}_\nu$ with different flavor structure compared to $M_{u,e,d}$ and if f term dominates $\mathcal{M}_\nu$ (Type II), leads to maximal θ_{23} . (Bajc, Senjanovic,Vissani'2003)

Type II seesaw dominance

- What is type II seesaw: Add triplet to standard model; seesaw formula arises from



Lazaridis, Shafi, Wetterich; RNM, Senjanovic

Schechter, Valle'81

- Small nu mass from GUT scale triplet mass
- Triplet present in 126-Higgs field $\mathcal{M}_\nu = f \frac{v_{wk}^2}{M_{\Phi_3}}$

f is 126 Yukawa coupling

How does type II seesaw lead to maximal mixing?

- Consider 2 gen $\mathcal{M}_\nu$

$$\mathcal{M}_\nu = c(M_d - M_\ell)$$

$$M_d = m_b \begin{pmatrix} \sim\lambda^2 & \sim\lambda^2 \\ \sim\lambda^2 & 1 \end{pmatrix} \quad M_\ell = m_\tau \begin{pmatrix} a_1\lambda^2 & a_2\lambda^2 \\ a_2\lambda^2 & 1 \end{pmatrix}$$

- Suppose at GUT scale $m_b \simeq m_\tau(1+\lambda^2)$
- Then $\mathcal{M})_\nu = \begin{pmatrix} a'\lambda^2 & b'\lambda^2 \\ b'\lambda^2 & \lambda^2 \end{pmatrix}$

- $a_1, a_2, a', b' \sim 1$→ large θ_{23} natural (Bajc, Senjanovic,Vissani)

Moving on to 3 generations

- M_d and M_l typical:

$$M_{d,\ell} \approx m_{b,\tau} \begin{pmatrix} \sim\lambda^4 & \sim\lambda^5 & \sim\lambda^3 \\ \sim\lambda^5 & \sim\lambda^2 & \sim\lambda^2 \\ \sim\lambda^3 & \sim\lambda^2 & 1 \end{pmatrix}$$

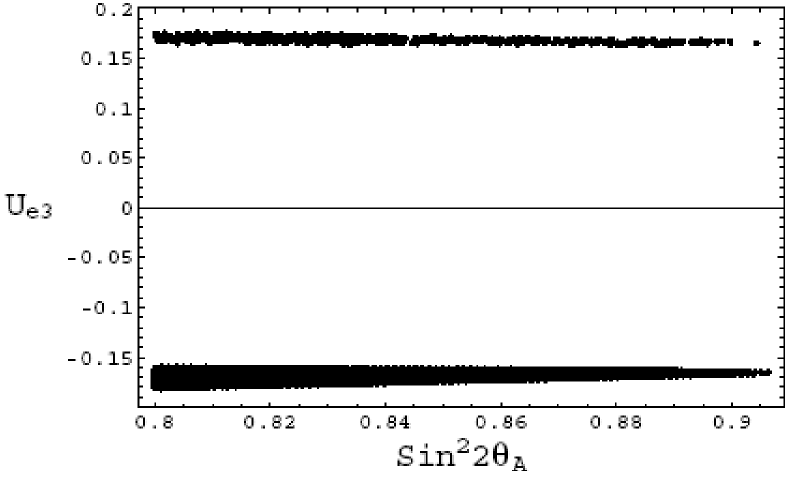
- B-tau unif at GUT scale→

$$M_\nu \;=\; c(M_d - M_\ell) \approx \; m_0 \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

- Atmospheric, solar, theta13, all large:(Goh, RNM, Ng'03)

Predictions of the model

- Works quantitatively:
  - ★ *normal hierarchy*:
  - ★ θ_{12}, θ_{23} large
  - ★ $\theta_{13} \approx \lambda$ "large" (Goh, RNM, Ng, 03 ; Babu, Macesanu'05)
  - ★ $\frac{m_{solar}}{m_{atmos}} \sim \lambda$



$$\theta_{13} \cong 0.15$$

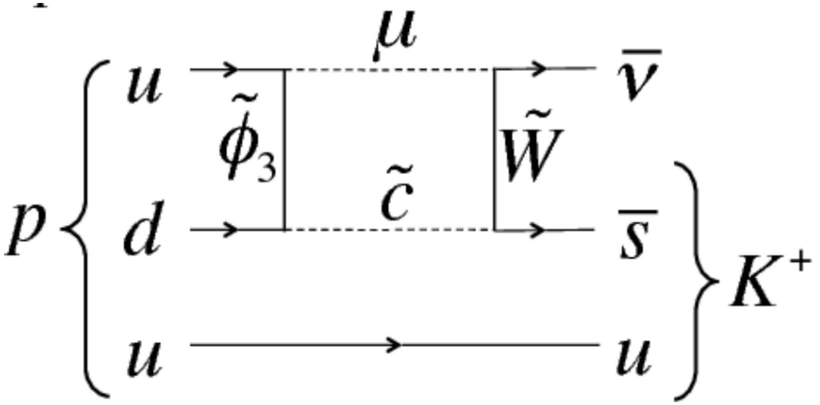
$$\text{Expt} : \theta_{13} = 0.14 - 0.156$$

Flavor in minimal SO(10): 10+126

- $\mathcal{M}_\nu = f v_L - M_d \frac{1}{f v_R} M_d$; $f = \frac{1}{4\kappa_d}(M_d - M_\ell)$
- Many detailed analysis with type II and type I + type II.
- Fukuyama, Okada’02; Babu, Macesanu’2005; Bertolini, Malinsky, Schwetz’06; Dutta, Mimura, RNM’07, Grimus, Kubock’07; Aulakh, Garg’05; Joshipura, Patel’11; Dueck, Rodejohann’13; Fukuyama, Ichikawa, Mimura’16; Babu,Bajc,Saad’16)

Proton decay in SUSY GUTs

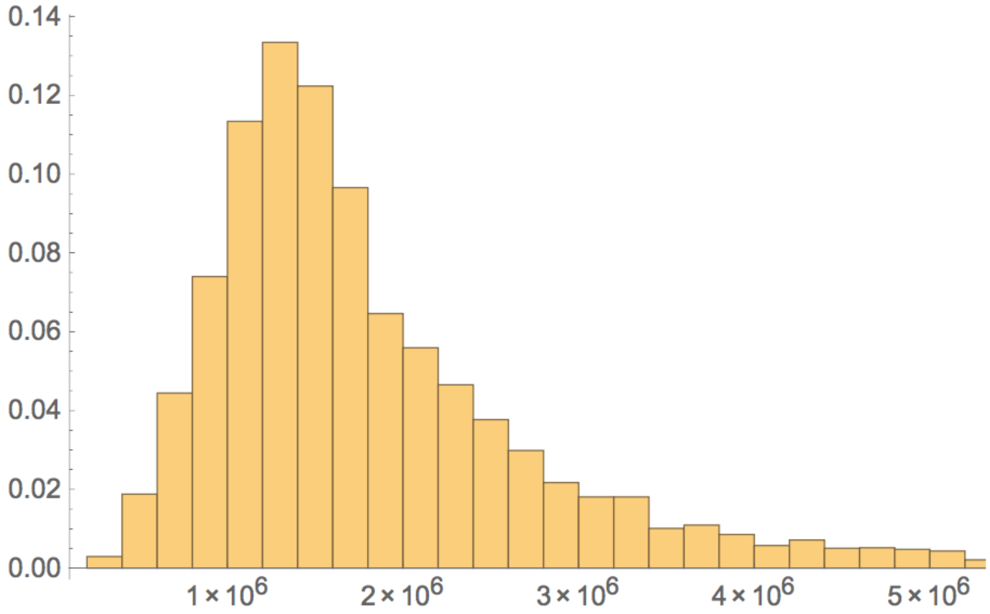
- In Non-SUSY models, $p \to e^+\pi^0$ dominates – does not test seesaw nor neutrino mass physics but is a key test of GUTs
- In SUSY SO(10)models, $p \to K^+\bar{\nu}$ connected to neutrino mixings: can test nu mass physics



Model with 10+126

- 10+126 model has tension with p-decay – specially RRRR operators troublesome:

$p \rightarrow K^+ + \bar{\nu}_\tau$ rate $\propto (\tan\beta)^2$



M_{susy} >232 TeV

(Babu, Bajc, Saad'18)

What if susy appears near a few TeV?

Next to minimal SO(10)
10+126+120+CP

- Fermion mass formula: $\tilde{g}$ imaginary; $\tilde{h}, \tilde{f}$ real

$$\mathcal{M}_u = \tilde{h} + r_2 \tilde{f} + r_3 \tilde{g}$$

$$\mathcal{M}_d = \frac{r_1}{\tan\beta}(\tilde{h} + \tilde{f} + \tilde{g})$$

$$\mathcal{M}_e = \frac{r_1}{\tan\beta}(\tilde{h} - 3\tilde{f} + c_e \tilde{g})$$

$$\mathcal{M}_{\nu_D} = \tilde{h} - 3 r_2 \tilde{f} + c_\nu \tilde{g},$$

- 17 parameters + 3 threshold effect parameters to fit 13+ 5 inputs (Dutta, Mimura, RNM'05)

CP and strong CP

- Note that $M_q = M_q^\dagger$ (due to antisym of $\tilde{g}$)
- → in the effective theory, $\theta = \mathrm{Arg\ Det\ M_q} = 0$
- Potential to solve the strong CP problem without the axion.
- Is this structure natural?

Naturalness with CPxZ$_2$

CP

Field	CP transform
$\Psi(16)$	$\Psi^*(16)$
$\mathrm{H}(10)$	$\mathrm{H}^*(10)$
$\bar\Delta(\overline{126})$	$\bar\Delta^*(\overline{126})$
$\Delta(126)$	$-\Delta^*(126)$
$\Sigma(120)$	$-\Sigma^*(120)$
$\mathrm{A}(45)$	$-\mathrm{A}^*(45)$
$\mathrm{S}(54)$	$\mathrm{S}^*(54)$
$\mathrm{X}(1)$	$\mathrm{X}^*(1)$

$$\theta \to \theta^*$$

→ $\tilde g$ imaginary

Z$_2$

$$\Delta(126) \to -\Delta(126)$$

$$W = \sum M_\phi \phi^2 + \lambda_1 X(A^2 - M^2)$$

$$\lambda_2 \Sigma A H + \lambda_3 \bar\Delta A \Sigma + \frac{\lambda_4}{\Lambda} \bar\Delta A^2 H$$

$$\lambda_5 S(HH + \Delta\Delta + \bar\Delta\bar\Delta) + \frac{1}{\Lambda}(\Delta\bar\Delta)^2$$

All other parameters in W real

all vevs real

(Preliminary)

(RNM, Severson'18)

Fermion mass fits and P-decay

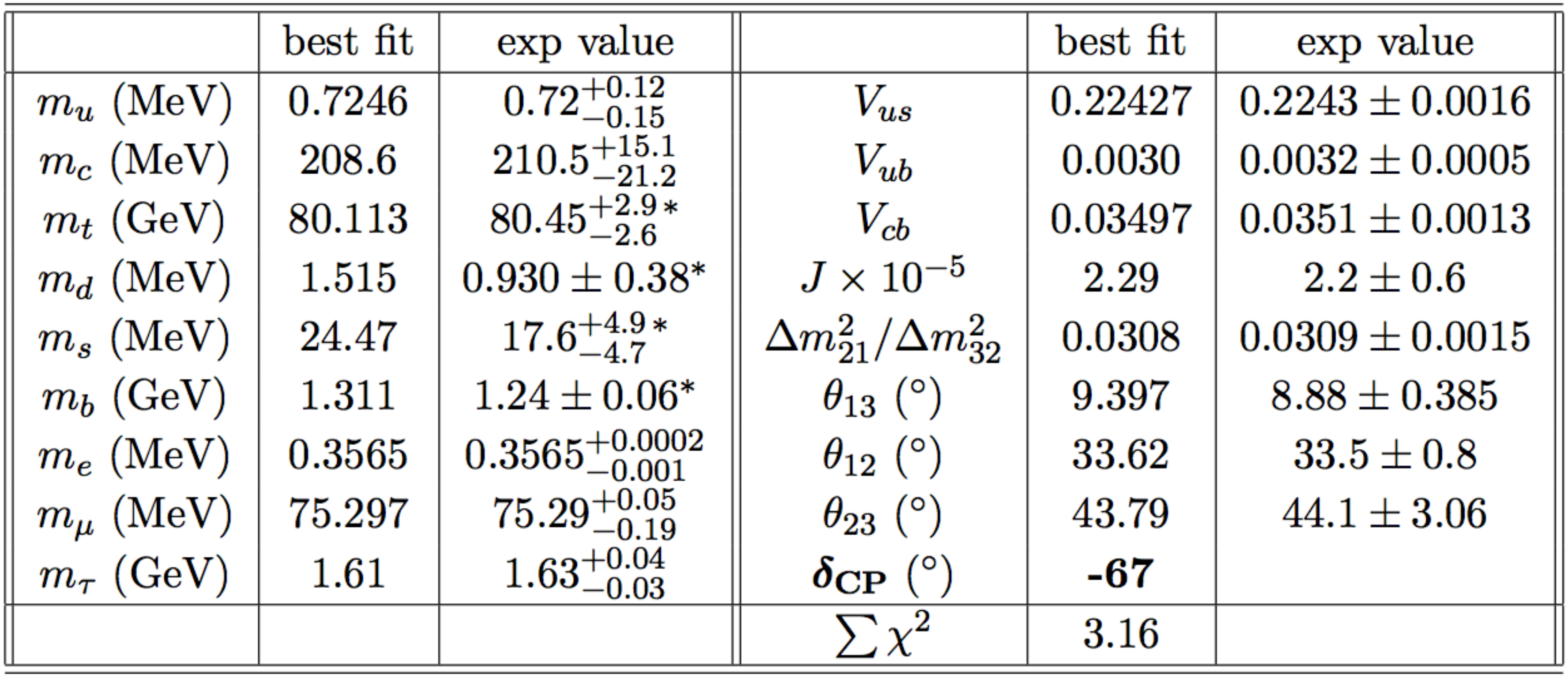
- In 10+126 models all Yukawas fixed by fermion mass fits and hence p-decay prediction firm:
- On the other hand in models with 120, one can choose textures to suppress proton decay: e.g.

$$\tilde{h} = \begin{pmatrix} 0 & & \\ & 0 & \\ & & M \end{pmatrix}, \qquad \tilde{f} = \begin{pmatrix} \sim 0 & \sim 0 & f_{13} \\ \sim 0 & f_{22} & f_{23} \\ f_{13} & f_{23} & f_{33} \end{pmatrix}, \quad \tilde{g} = i \begin{pmatrix} 0 & g_{12} & g_{13} \\ -g_{12} & 0 & g_{23} \\ -g_{13} & -g_{23} & 0 \end{pmatrix}.$$

(Dutta, Mimura, RNM’05; Severson’15; RNM, Severson’18)

Fermion mass fit

- Best fit values for Type I with large CP phase



	best fit	exp value		best fit	exp value
m_u (MeV)	0.7246	$0.72^{+0.12}_{-0.15}$	V_{us}	0.22427	0.2243 ± 0.0016
m_c (MeV)	208.6	$210.5^{+15.1}_{-21.2}$	V_{ub}	0.0030	0.0032 ± 0.0005
m_t (GeV)	80.113	$80.45^{+2.9}_{-2.6}*$	V_{cb}	0.03497	0.0351 ± 0.0013
m_d (MeV)	1.515	$0.930 \pm 0.38^*$	$J \times 10^{-5}$	2.29	2.2 ± 0.6
m_s (MeV)	24.47	$17.6^{+4.9}_{-4.7}*$	$\Delta m^2_{21}/\Delta m^2_{32}$	0.0308	0.0309 ± 0.0015
m_b (GeV)	1.311	$1.24 \pm 0.06^*$	θ_{13} (°)	9.397	8.88 ± 0.385
m_e (MeV)	0.3565	$0.3565^{+0.0002}_{-0.001}$	θ_{12} (°)	33.62	33.5 ± 0.8
m_μ (MeV)	75.297	$75.29^{+0.05}_{-0.19}$	θ_{23} (°)	43.79	44.1 ± 3.06
m_τ (GeV)	1.61	$1.63^{+0.04}_{-0.03}$	$\boldsymbol{\delta_{\mathrm{CP}}}$ (°)	**-67**	
			$\sum \chi^2$	3.16	

$$\sum m_\nu = .074 \text{ eV}$$

RNM and Severson'18

New features in Proton decay

- This choice of textures suppresses 10 and 126 contributions except for those that arise from rotations to mass basis.
- There are new contributions (different from SU(5) and SO(10) with 10+126) due to 120:

→Usual kinds: (3, 1,-1/3) QQQL;
(3,1,-4/3) $U^cU^cD^cE$;

- New kind (3,3,-1/3) from 120 and 120-126

$$Q^T \vec{\tau} Q \cdot Q^T \vec{\tau} L$$

Chi-square values for CP phase (type I case)

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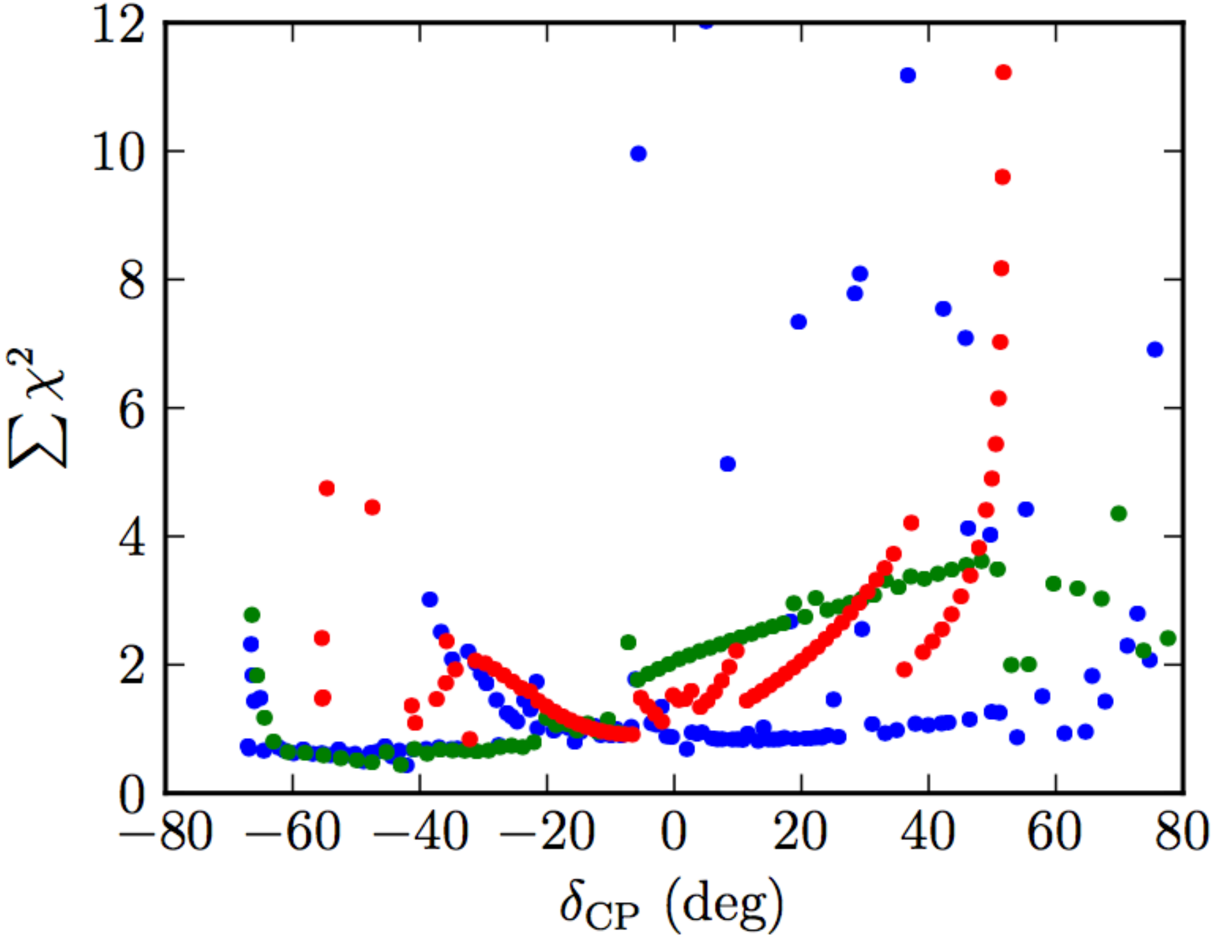
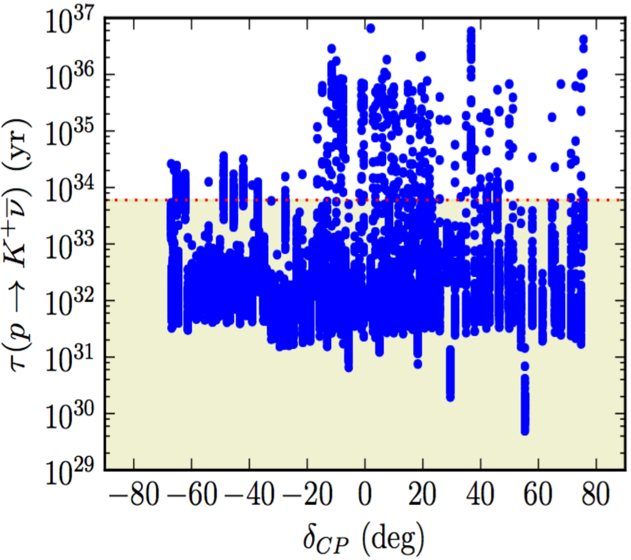
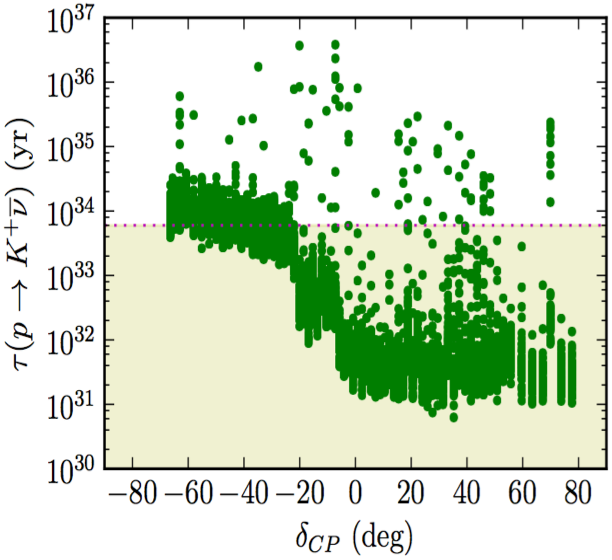
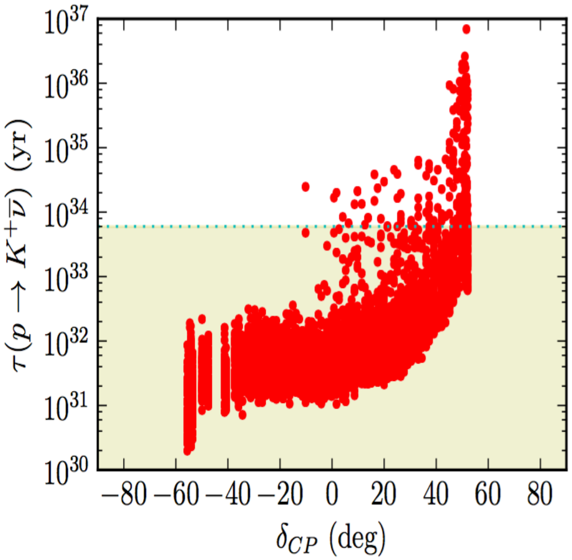


proton life time and δ_{CP} correlation in SO(10)

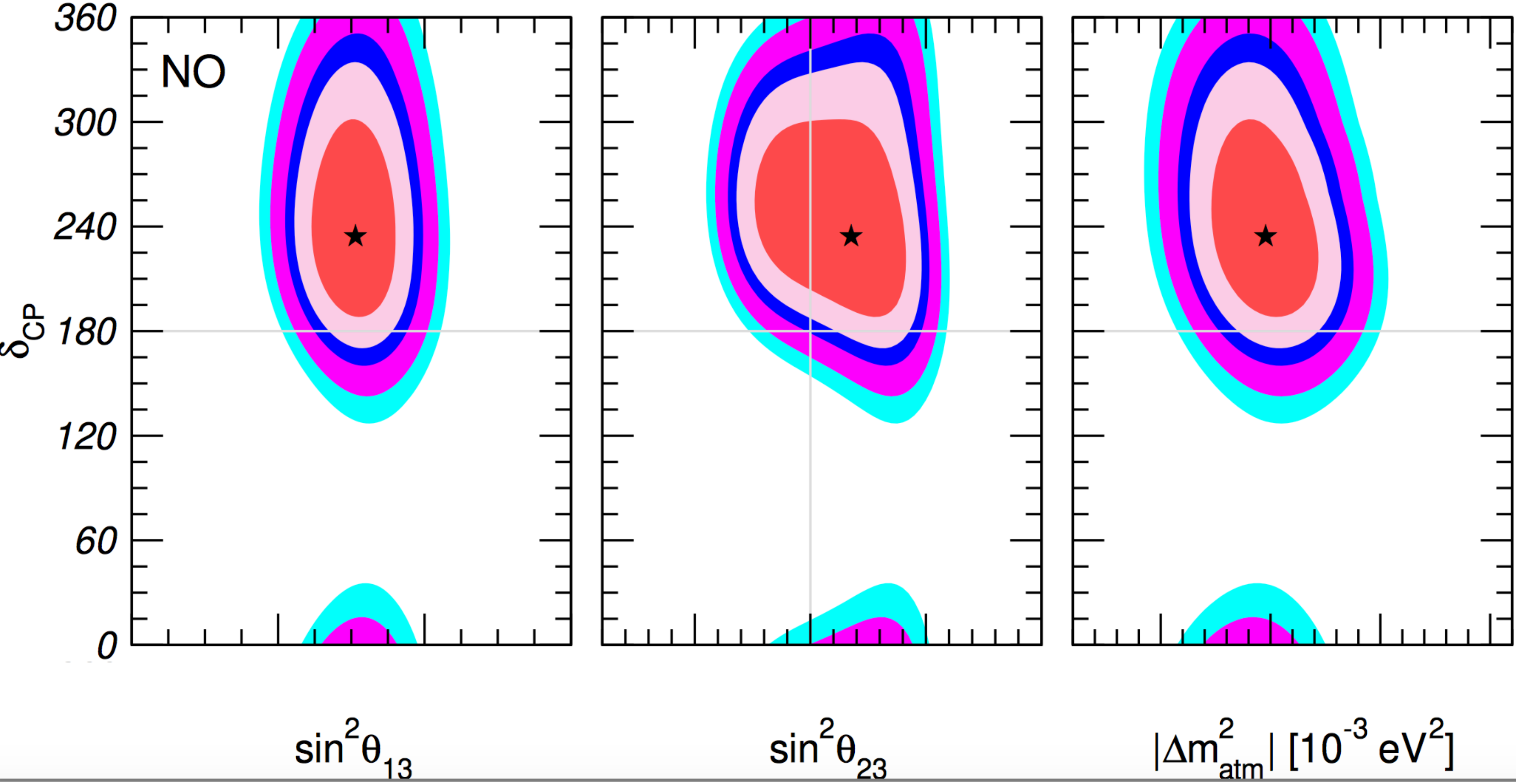




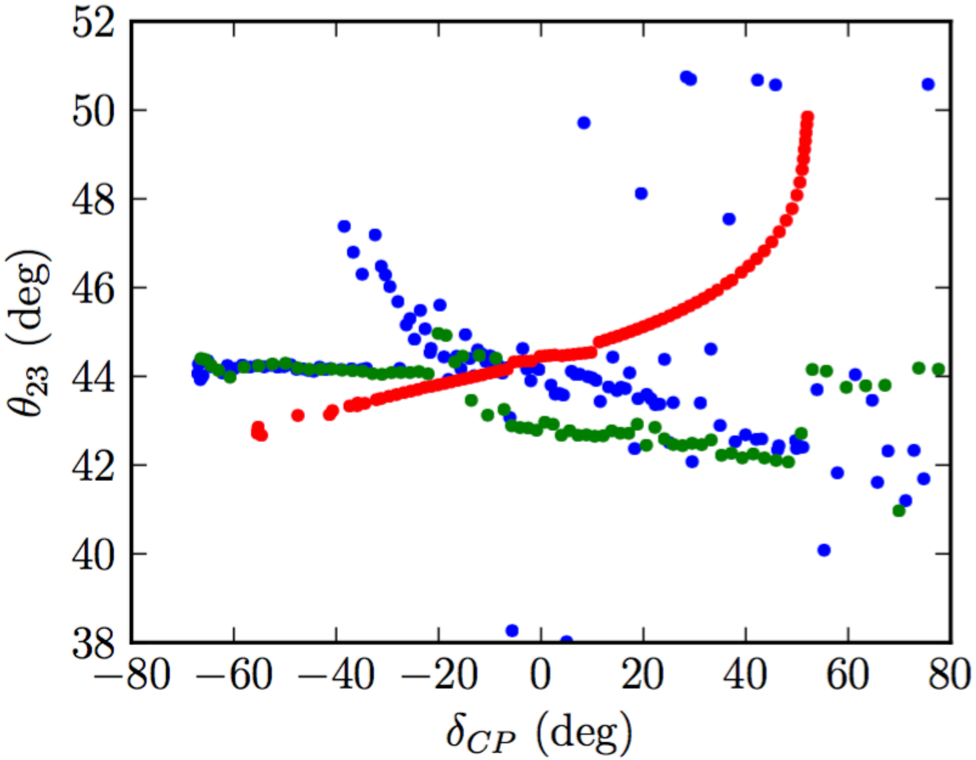


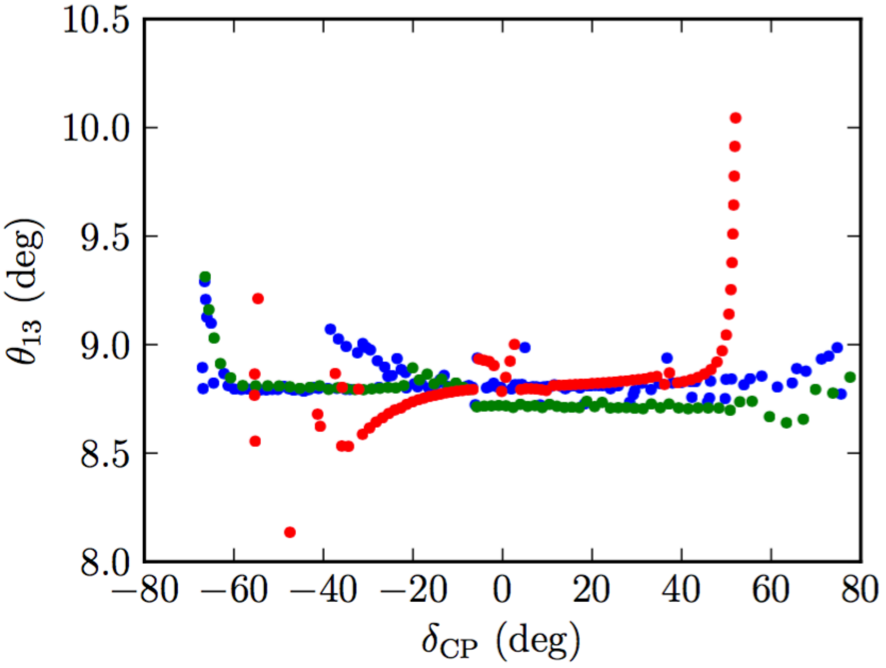
- M_{SUSY} =5 TeV
- Can help to test these models (RNM,Severson’18)

Nu-Fit 3.2 result for CP phase



Correlations:Type I case

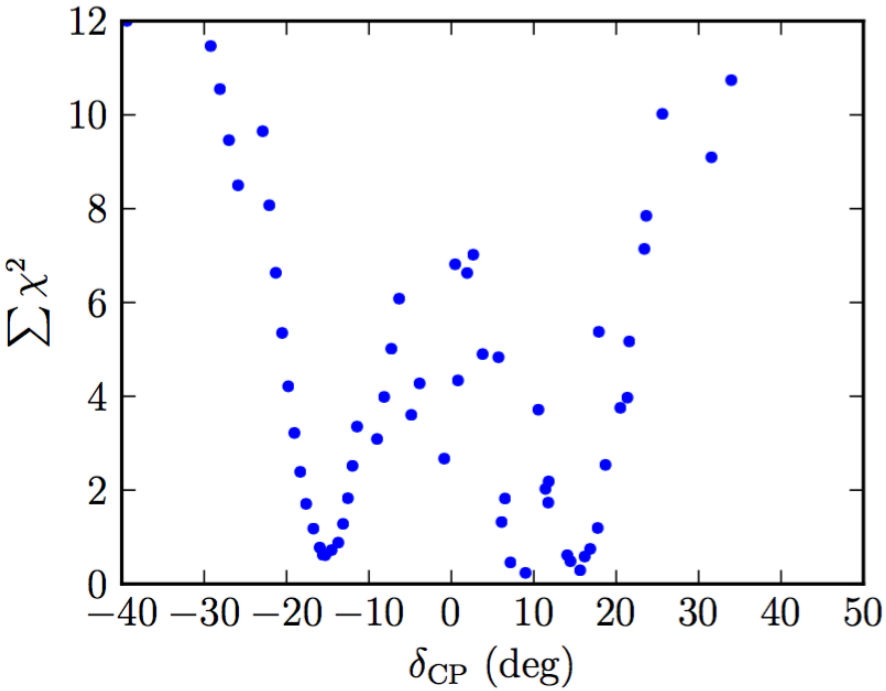


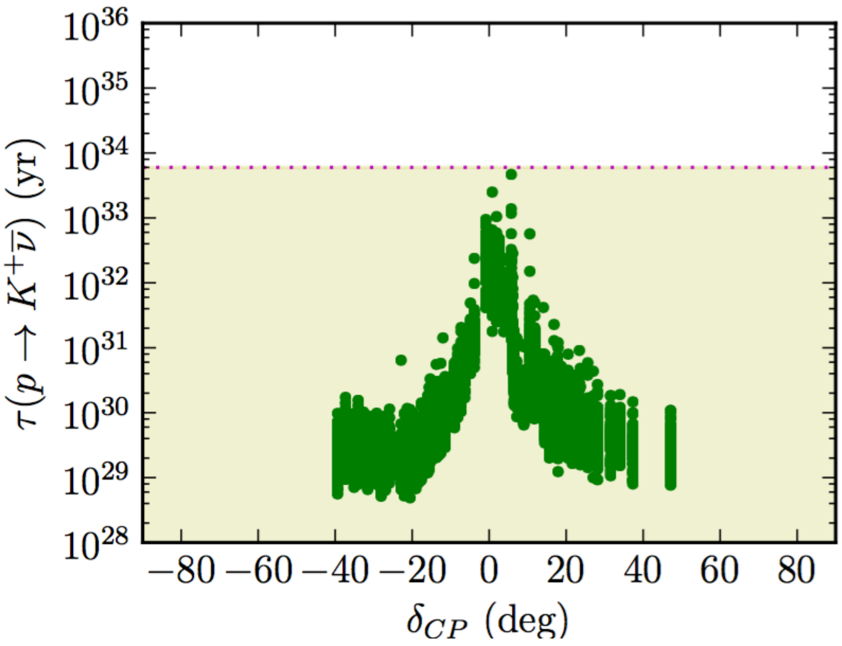


$$\delta_{CP} = +60^0 \text{ to} - 70^0$$

Predictions for type II case

- Mixing angles and p-decay





Measurement of leptonic Dirac phase can test type II version of the model !!

Beyond the fermion sector:

(i) Issue of coupling running beyond M_U

- Due to large reps, α_U blows up just after M_U

- The condition on Higgs for coupling not blowing up before M_{Pl}

$$b_H \leq 18 + \frac{2\pi}{\alpha_U \ell n(\frac{M_{Pl}}{M_U})} = 33$$

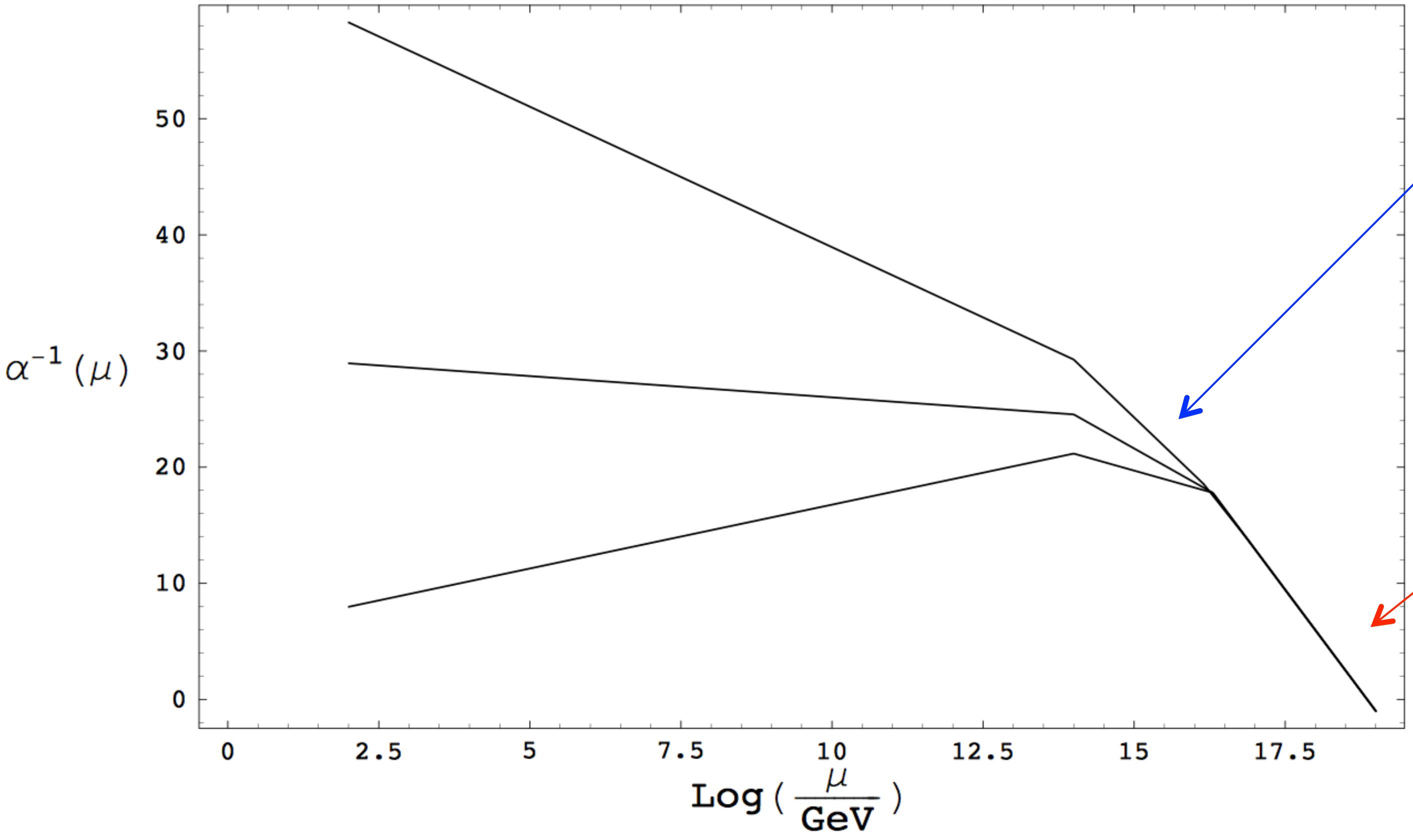
- For $b_{H(126)} = 35$; $b_{H(120)} = 28$; $b_{H(10)} = 1$

- What to do?

Issue of coupling running beyond $M_{U:}$ one resolution

- Break symmetry by {210} and {54} and keep the SU(5) rep {15} at 10^{13} GeV. Unification works but SO(10) breaks at 10^{18} GeV
- SO(10)→SU(5)→MSSM (Goh, Nasri, RNM'05)

Coupling running



$\{15+\overline{15}\}$

$\{5,15\}+\{24\}$

- Can strong CP work with this?

(ii) Doublet-Triplet splitting: Orbifold embedding of SO(10)

- 5-D with $S_1/Z_2 x Z'_2$ for SO(10) (Dermisek, Mafi;H. Kim, Raby'03)
- Multiplets fragment and get pushed up reducing threshold effect (Fukuyama, Okada'16)
- Provides a way to solve D-T splitting problem
- Embedding of this model: open problem.

(iii) GUTs with Flavor symmetries:

- Quark lepton fits in GUTs require certain choices for the Yukawa couplings:
- Can we have a deeper understanding of the needed pattern of Yukawas?

- Perhaps symmetries can help! The vacuum of flavor GUTs may explain Yukawas! S_4 , A_4 ,

Summary and conclusion

- With no susy and no proton decay, neutrino mass provides only compelling theory motivation for GUTs.
- An example of predictive model:Renormalizable SO(10); natural DM;
- 10+126 most minimal model
- θ_{13} prediction confirmed!

- P-decay requires high susy Breaking scale.

Summary contd.

- 10+120+126 model
- Better for proton decay, if susy is discovered in the few-TeV range;

- Correlation between Dirac phase- p-decay rate,

  makes the model testable soon.

- Potential to provide a solution to strong CP problem without the axion.

Input parameters

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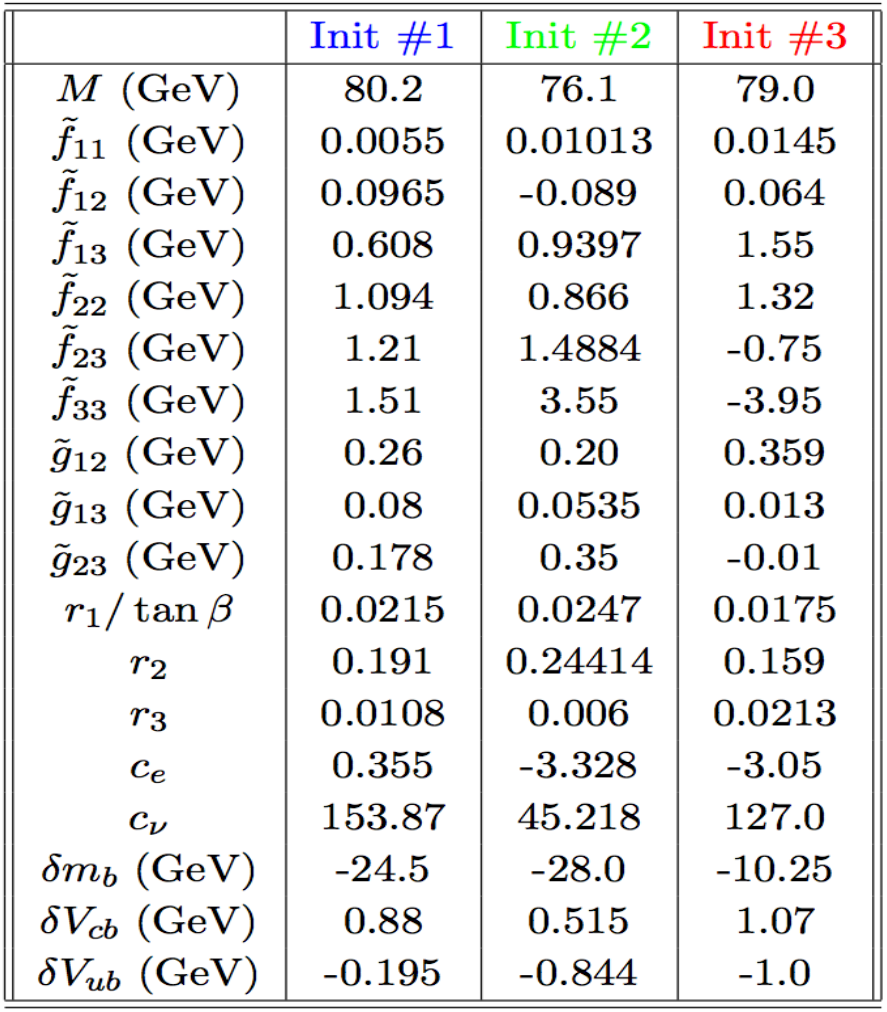

	Init #1	Init #2	Init #3
M (GeV)	80.2	76.1	79.0
$\tilde{f}_{11}$ (GeV)	0.0055	0.01013	0.0145
$\tilde{f}_{12}$ (GeV)	0.0965	-0.089	0.064
$\tilde{f}_{13}$ (GeV)	0.608	0.9397	1.55
$\tilde{f}_{22}$ (GeV)	1.094	0.866	1.32
$\tilde{f}_{23}$ (GeV)	1.21	1.4884	-0.75
$\tilde{f}_{33}$ (GeV)	1.51	3.55	-3.95
$\tilde{g}_{12}$ (GeV)	0.26	0.20	0.359
$\tilde{g}_{13}$ (GeV)	0.08	0.0535	0.013
$\tilde{g}_{23}$ (GeV)	0.178	0.35	-0.01
$r_1/\tan\beta$	0.0215	0.0247	0.0175
r_2	0.191	0.24414	0.159
r_3	0.0108	0.006	0.0213
c_e	0.355	-3.328	-3.05
c_ν	153.87	45.218	127.0
δm_b (GeV)	-24.5	-28.0	-10.25
δV_{cb} (GeV)	0.88	0.515	1.07
δV_{ub} (GeV)	-0.195	-0.844	-1.0

Embed strong CP solution: Example II

- Add singlets X with X→-X* and X→-X under CPxZ$_2$ and Y with Y→Y*; Y→ Y

$$W = M_X X^2 + \lambda X \Delta \bar{\Delta} + \bar{\Delta}\Sigma A + \Sigma A H$$

$$+\frac{\bar{\Delta} A^2 H}{M_P} + SAA + M_S S^2 + S^3 + M_A A^2 + \frac{(\Delta\bar{\Delta})^2}{M_P}$$

$$+Y(X^2 - M^{'2}) + M_Y Y^2 + Y^3 + Y S^2 + Y A^2$$

$$+\frac{i\kappa}{M_P} X \Delta A \bar{\Delta}$$

S$_4$ symmetry based model

- S$_4$ triplets $\phi_{1,2,3}$ and write superpotential for each: matter spinors in 3$_2$
- Minima for 3$_1$ ϕ_3 $W = \frac{1}{2}m\phi^2 - \lambda\phi^3 = \frac{1}{2}m(x^2+y^2+z^2) - \lambda xyz.$

$$\phi_3 = \frac{m}{\lambda}\{(1,1,1) \text{ or } (1,-1,-1) \text{ or } (-1,1,-1) \text{ or } (-1,-1,1)\}.$$

- For 3$_2$: $\phi_1 = (0,0,\pm 1)$ $\phi_2 = (0,\pm 1,\pm 1)$ (King,Ross,..)
- Choose W $W = (\phi_1\psi)(\phi_1\psi)H + (\phi_2\psi)(\phi_2\psi)\bar{\Delta} + (\phi_3\psi\psi)\bar{\Delta}$
- Leads to desired pattern→ TBM (DMM'10)

Proton decay in SO(10)

- 3 kinds: (3, 1,-1/3) mediated QQQL; (3,1,-4/3) $U^cU^cD^cE$; (3,3,-1/3) $Q^T\vec{\tau}Q\cdot Q^T\vec{\tau}L$
- Only the first two in minimal SU(5)
- Two kinds of operators: LLLL, RRRR
- In minimal SU(5), RRRR enhanced by $(\tan\beta)^4$
- Enhanced but not so much in SO(10)
- In SUSY SO(10), neutrino masses related to p-decay