

# Neutron Star and Superfluidity

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## Abstract

It is expected that under high density, nucleons in neutron star can form copper pairs and give rise to superfluidity. In this paper, the underlying principle will be briefly reviewed. Astrophysical implications such as explanation of pulsar glitches by the two components model and effect on cooling of neutron stars will be discussed.

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Emergent of superfluidity in neutron stars</b>	<b>2</b>
<b>3</b>	<b>Pulsar glitch and Two components model</b>	<b>4</b>
<b>4</b>	<b>Cooling of neutron stars</b>	<b>8</b>
<b>5</b>	<b>Conclusion</b>	<b>10</b>

# 1 Introduction

The idea of superfluidity exists inside neutron stars was first proposed by Migdal[1]. In analogy to electrons inside superconductor forming cooper pairs due to electron lattice interaction, it is expected that nucleons in neutron star at sufficiently high density and low temperature can also form copper pairs due to the long-range attractive nuclear force and lead to superfluidity and superconductivity[2, 3]. In this paper, I will only focus on nucleon superfluidity and left behind other possible interesting microphysics that may also take place inside neutron stars, including superconductivity due to protons and color-super-conductivity in color-flavor-locked phase of deconfined quark matters deep inside the interior of neutron stars. Interested readers may refer to [4] for more details. In section 2, I will briefly review the physical principles of how do superfluid phases emerge in nuclear matters of neutron stars. Similar to helium superfluid, superfluids in neutron stars also exhibit the properties of zero viscosity and quantized vortices. It is interesting that these microscopic physics do have observable effects on the macroscopic properties of neutron stars. Indeed, without superfluidity, some puzzling phenomena about dynamical and thermal evolution of neutron stars could not been understand. In section 3, I will talk about the two components model of weak coupling between crust layer of neutron star and superfluid core as a phenomenological model to explain post-glitch relaxation of neutron star and pinning and unpinning between nucleus and vortices in crust layer of neutron superfluid as mechanism of pulsar glitch. In section 4, I will talk about how does superfluidity affect the emission of neutrinos and hence affect the cooling curve of neutron star. Section 5 serves as conclusion.

## 2 Emergent of superfluidity in neutron stars

First of all, let us get a brief idea of what types of superfluids and where do these superfluids present in neutron star by looking at the typical interior structure of a neutron star in fig 1. In this "standard model" , there are 3 types of superfluid inside neutron star. We have neutron superfluid with configuration of  $^1S_0$  in the inner crust region and neutron superfluid with configuration  $^3P_2$  and  $^1S_0$  superconducting proton inside core (Since protons are charged superfluid, they are superconducting, however, we will not take about the consequences due to presence of such superconducting current, we

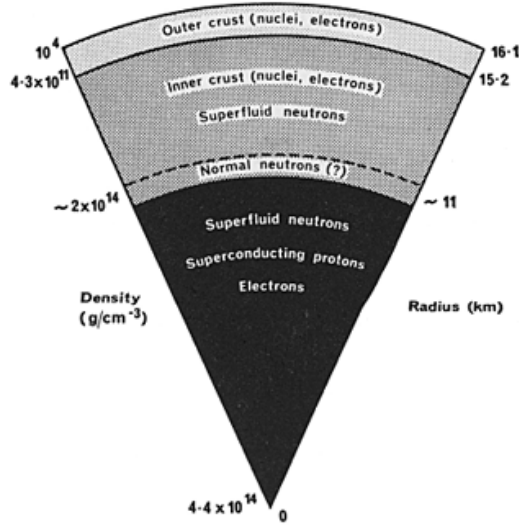


Figure 1: Typical structure of neutron star. Fig from [http://heasarc.gsfc.nasa.gov/docs/objects/binaries/neutron\\_star\\_structure.html](http://heasarc.gsfc.nasa.gov/docs/objects/binaries/neutron_star_structure.html)

will only focus on superfluidity only throughout this paper). The reason for the existence of three different types of superfluid is as follow. First, there is no superfluid in outer crust of neutron star because nucleon density is not high enough, nucleon are bounded inside nucleus and are not free to move. As density go up to about  $4 \times 10^{11} g/cm^3$  in inner crust region, a process called neutron drip where high momentum electrons are captured by proton to form neutron with emission of neutrino will occur. With more and more neutrons present, eventually extra neutrons must go to continuum states as all bound states are filled up. Degenerate neutron fermi sea will now form. Neutrons in neutron fermi sea interact through long-range attractive interaction and form cooper pairs, just as what happen to electrons in conventional s-wave superconductor. Typical temperature of neutron star is much smaller than the estimated critical temperature for superfluidity by many-body simulation except new-born neutron star [5]. Hence it is believed that s-wave neutron superfluid presents in inner crust of neutron star. However, protons are still locked inside nucleus and hence there is no proton superfluid in this region. As we continue to go inside neutron star to the core region, nucleus become so dense that they essentially merge together. At this point, all neutrons and protons are no longer in bound states, we will have both neutron

superfluid and proton superfluid. In neutron star core, due to extremely high density, short range repulsion of nuclear force will come into play, neutron superfluid will no longer be  $^1S_0$  superfluid. Instead, calculation showed that it is preferable for neutron to form  $^3P_2$  superfluid as  $^3P_2$  partial wave nucleon-nucleon interaction becomes attractive at such high density [6]. While in case of proton, since its number is much less than that of neutron, the short-range repulsion will be less dominant, calculation showed that they prefer to form  $^1S_0$  proton superfluid similar to that of conventional superconductor[7].

Due to the presence of superfluid, dynamical and thermal evolution of neutron stars will be affected. However, it should be noted that exact properties of superfluids, like energy gap and critical temperature are highly depend on nuclear strong interaction models and many-body theories at supranuclear density. Even though, we can still see how do neutron stars properties were affected without knowing the exact details of superfluid properties.

### 3 Pulsar glitch and Two components model

Although Pairing energy of neutrons inside neutron star only take up less than 1% of the total interaction energy and hence has negligible effect on its mass and radius [12], superfluidity can cause observable effects on properties of neutron star. In this and next section we will focus on consequences of superfluidity on macroscopic behavior of neutron stars. First, I will talk about pulsar glitch where properties of superfluid like quantized vortices are important for us to explain its origin and behaviors. In the next section, I will review how is thermal evolution of a neutron star being affected by superfluidity.

Pulsar glitch is the phenomenon that rotation frequency of pulsar exhibits a sudden increases, followed by a slow exponential relaxation of time scale varies from days to months. Fig 2 shows observation data of pulsar glitch of pulsar Vela.

Let us focus at the post glitch relaxation first. Assuming external torque to be uniform in time, the time-varying rate of change of rotation frequency suggests that neutron star cannot rotate as a rigid body. The simplest model beyond rigid body is to assume neutron star is composed of two components. The slow exponential relaxation indicates a underlying well-oil machinery, which is probably due to superfluidity [9, 10], this suggests simply taking neutron star to be composed of a rigid crust layer and core neutron super-

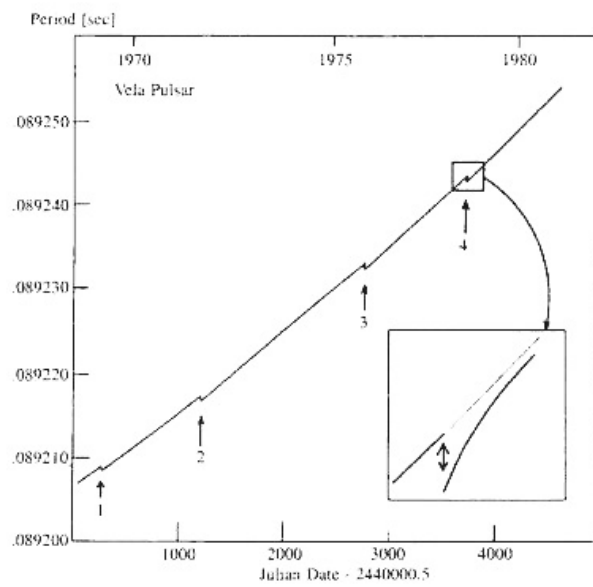


Figure 2: Observed Period of Pulsar Vela. Note the exponential relaxation after sudden decreases of rotation period with time scale of years. Fig from [8].

fluid may be able to explain the post glitch relaxation. This is indeed the so-called two components model [11]. It should be emphasized that the observed rotation frequency of a neutron star is the frequency of crust layer, we do not have direct observation on the frequency of core superfluid. Following [12], we can see how does two components model gives rise to temporal exponential decay mathematically, let us denote the moment of inertia and frequency of crust layer and superfluid core to be  $I_c$ ,  $\Omega_c$  and  $I_s$ ,  $\Omega_s$  respectively. The angular acceleration of crust is given by the total torque divided by its moment of inertia

$$\dot{\Omega}_c = \frac{N_{ext} - N_{int}}{I_c} \quad (1)$$

where I have written the torque in terms of external torque and internal torque explicitly. Internal torque is due to coupling between crust layer and superfluid core which may due to magnetic or viscous effect. We can write it as

$$N_{int} \equiv I_s \dot{\Omega}_s = I_c \frac{\Omega_c - \Omega_s}{\tau_c} \quad (2)$$

where  $\tau_c$  is the crust-core coupling time, which is model dependence. Smaller  $\tau_c$  gives stronger coupling strength. Solving these coupled differential equations with the initial condition  $\Omega_o + \Delta\Omega_o$ , which represent pulsar glitch occur at  $t = 0$ . We can now get the following equation

$$\Omega_c(t) = \Omega_o(t) + \Delta\Omega_o [Qe^{-t/\tau} + (1 - Q)] \quad (3)$$

where  $\Omega_o t$  is the frequency without glitch,  $\tau = \tau_c(I_s/(I_s + I_c))$  and  $Q$  is called "healing parameter" which describe how does  $\Omega_c$  relax back to  $\Omega$ . When  $Q = 1$ , we have  $\Omega_c(t) \rightarrow \Omega_o(t)$  as  $t \rightarrow \infty$ . Figure 3 shows a plot of  $\Omega_c$  illustrating exponential time relaxation after glitch.

This model is elegant as we have only assumed the existence of superfluid in core of neutron star. It even does not matter whether it is proton superfluid or neutron superfluid for us to get the qualitative exponential time relaxation behavior. However, plenty of observation data show violations of the two-component model, for example [13]. More recent high-resolution data even show that up to four exponential decaying time scales are needed to fit observational data [14]. Nevertheless, Two components model do teach us that microphysics of superfluidity is necessary to understand properties of neutron stars. There are more recent models to explain the post-glitch relaxation [15] and superfluidity is the key ingredient in all these models.

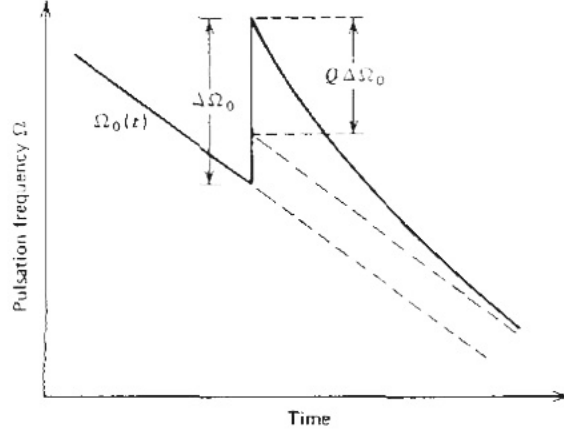


Figure 3: A plot showing how we can get the exponential relaxation after pulsar glitch from two-components model. Figure from [12].

Without superfluidity, we cannot have a comprehensive understanding on properties of neutron star.

Let's turn our focus to mechanism on pulsar glitch. Up to now, the exact mechanism of glitch is still uncertain, however, we can still get some insights from the properties of superfluid. Here, I will review one of the most recognized model of glitch mechanism [16]. The basic idea is pinning and unpinning between superfluid vortex (superfluid component) and nucleus (normal component) in inner crust layer of neutron star, transferring angular momentum from superfluid to normal component (This is different from the previous part in explaining the post-glitch relaxation where we are talking about superfluid in core of neutron star, now we are talking about on the superfluid in crust layer). Recall that in terrestrial laboratory, for rotating superfluid, we have quantized vorticity of  $\frac{N\hbar}{2m_N}n$  where  $m_N$  is the mass of neutron,  $N$  is the winding number and  $n$  is the number of vortices. Also recall that for the superfluid to rotate like a rigid body, we have  $\Omega_{eff} = \frac{\pi\hbar n}{2m_N}$ . Thus, for a fixed number of vortices, rotation frequency of superfluid is also fixed. Since it is energetically favourable for vortex to pin nucleus in crust layer [17], the numbers of vortices and hence rotation frequency of superfluid are fixed. As the crust layer are composed of charged particle like electrons and protons, it radiate and loss energy when it rotates, eventually rotation frequency of normal component in crust layer will decrease while that for

superfluid will remain the same. The increasing difference in rotation frequencies between the superfluid and normal component in crust will produce an outward Magnus force trying to unpin them [18]. Since the pinning force can sustain differential rotation up to  $10 \text{ rad s}^{-1}$ , it acts as a huge angular momentum reservoir, once the Magnus force exceed the pinning force between superfluid and nuclei, unpinning occur and huge angular momentum will be transferred to star surface, thus normal component in crust will spin-up and result in pulsar glitch. Once again, we see the importance of superfluidity for us to understand the dynamical properties of neutron star. So far, there is no theoretical model capable of explaining observation data of pulsar glitch and its exponential relaxation without considering superfluidity.

## 4 Cooling of neutron stars

Apart from affecting the dynamic of neutron star, superfluidity also cause significant contribution on thermal evolution of neutron star. Superfluidity affects the cooling history of neutron star mainly through two ways: 1) affecting neutrino emission process and 2) increasing heat capacity when temperature of neutron star is just below the critical temperature of superfluidity and suppress heat capacity by factor of  $e^{-\Delta/kT}$  when temperature is well below critical temperature [19]. In this paper, I will only focus on effect on neutrino emissivity by superfluidity.

The main cooling channel of new born neutron star is through emission of neutrino. In a non-superfluid neutron star model, cooling mechanisms can be divided into two groups, the standard cooling and fast cooling processes [19]. The standard cooling includes modified Urca process [20] and nucleon-nucleon bremsstrahlung, while fast cooling is mainly due to direct Urca reaction [21]. In all cases, the presence of superfluid will reduce the emission of neutrino through these process [19]. Therefore, one expect that superfluidity will only decreases the cooling rate of neutron star. Indeed, this is not necessarily true. With presence of superfluid, a new neutrino generation mechanism is opened.

$$N \rightarrow N + \nu + \bar{\nu} \quad (4)$$

which is due to the energy gap in excitation spectrum. This new process can greatly increase the emission rate of neutrino. For detailed calculation of neutrino emissivity please refer to [19].



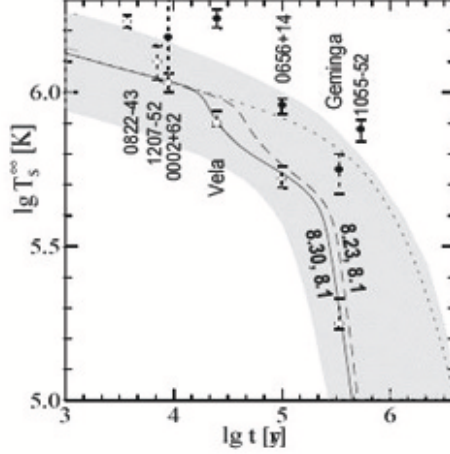


Figure 4: Cooling curves for neutron star, for detailed description, please refer to text. Fig from [19].

We will now see the importance of considering superfluidity in understanding cooling history of neutron star. Fig 4 shows cooling curves of neutron star based on difference theoretical models together with observation data. Dotted line corresponds to non-superfluid model, while for the solid and dashed curves, effects of superfluidity is considered (they correspond to different parameters). The filled circles are observed data points fitted by blackbody spectrum and open circles are data points fitted by Helium atmosphere model. Note that after taking into account superfluidity, theoretical solid curve can fit into all data points based on the Helium atmosphere model, while one is not able to match the observation data without considering superfluid.

The funny looking cooling curve for superfluid neutron star is because at temperature larger than critical temperature, which is expected for new born neutron star, there is no superfluid inside neutron star and cooling behavior is the same for superfluid and non-superfluid model. As neutron star cool, its temperature will eventually drop below critical temperature for onset of superfluid, hence, neutrino emission channel through eq 4 is opened up and cooling of neutron star is speeded up. As temperature continue to decrease, neutrino emissivity due to Cooper pair decreases as we can see from fig 4, hence the cooling rate will decrease and cooling behavior is now similar to

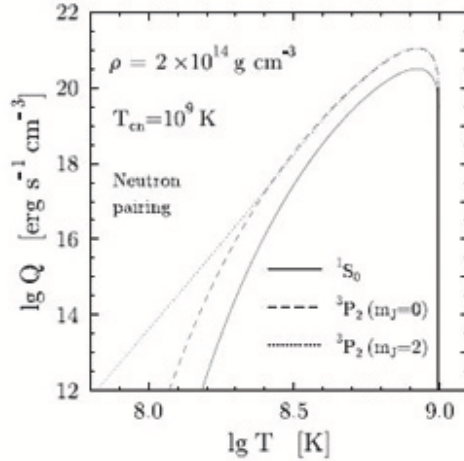


Figure 5: plot of log of emissivity against (surface) temperature. Just below critical temperature, emissivity is the highest, it drops with decreasing temperature afterward. Fig from [19].

that of non-superfluid neutron star.

Cooling behavior of neutron star is highly model dependence, different superfluid critical temperatures of neutron and proton give rise to different cooling curves. Hence, by precise measurement of cooling behaviors of neutron stars, one can put some constraints on models for superfluidity and get more knowledge about how does superfluidity emerge in neutron stars.

## 5 Conclusion

In this paper, emergence of superfluidity in neutron star is reviewed. Basically, we can divide the superfluids inside neutron star into three types,  $^1S_0$  neutron superfluid in inner neutron star crust,  $^3P_2$  neutron superfluid and  $^1S_0$  proton superfluid in neutron star core. Although exact detailed properties such as gap energy of these superfluids is still uncertain, superfluidity is still necessarily for us to have a complete understanding on dynamical and thermal evolution of neutron stars. Further studies on these behaviors may even help us to understand superfluidity in extreme condition inside neutron star.

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