Problems

Задачи

# NEW VERY HARD PROBLEMS OF BALANCING CHEMICAL REACTIONS 

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#### Abstract

In this article are balanced two new very hard chemical reactions which possess atoms with fractional oxidation numbers. Considered chemical reactions belong to the class of chemical reactions which have unique coefficients. These reactions are balanced by the classical algebraic method. The decision to study these chemical reactions was made according to the fact that these reactions cannot be balanced by usage of a computer, because to date is not known any software package. A unique way to balance this class of chemical reactions is the application of mathematical methods. That was the cause that we chose the classical algebraic method for balancing chemical reactions.


Keywords: chemical reactions, balancing

## Introduction

Balancing chemical reactions is an amazing subject matter for mathematics and chemistry students who want to see the power of linear algebra as a scientific discipline. Mass balance of chemical reactions is one of the most highly studied topics in chemical education. Actually, balancing chemical reactions offers a superb didactic example of interconnection between chemistry and linear algebra (Risteski, 1990). In chemistry there are lots of ways to balance chemical reactions, but all of them have limited usage, because they hold only for some elementary chemical reactions. In fact, they are particular procedures founded by virtue of experience, but without any formal criteria. A survey of the references which treat problem of balancing chemical reactions through the prism of chemistry is given in the previous author's research works (Risteski, 2007a; 2007b; 2008a; 2008b; 2009).

Chemical ways for balancing chemical reactions are inconsistent, because they consider chemical reactions in an informal way, which produces only fallacies and paradoxes (Risteski, 2010; 2011).

Until the second half of the $20^{\text {th }}$ century there was no mathematical method for balancing chemical reactions in chemistry, other than the algebraic method. Chemists balanced simple particular chemical reactions using only change in oxidation number procedure, partial reactions procedure and other slightly different modifications derived
from them. Chemical ways were an assumption of traditional chemists, who thought that the solution of the general problem of balancing chemical reactions is possible by use of chemical procedures. But, practice showed that the solution of the century old problem is possible only by using contemporary mathematical method (Risteski, 2007a).

Also, in (Risteski, 2010) the author emphasized very clearly, that balancing chemical reactions is not chemistry; it is just linear algebra. From a scientific view point, a chemical reaction can be balanced if only if it generates a vector space. That is a necessary and sufficient condition for balancing a chemical reaction. This shows that chemical reaction must be considered as a formal whole, in a right sense of the word, if we like it to be balanced on a correct way. In opposite case, as it was done by the chemical ways, one obtains only absurd (Risteski, 2010; 2011).

Here, the considered chemical reactions belong to the class of chemical reactions with unique coefficients. Usually these chemical reactions are balanced by computer if they possess atoms with integer oxidation numbers. In an opposite case, the only way to balance these reactions is by the usage of mathematical methods. For that particular case we chose the algebraic method developed in (Risteski, 2012).

In the next section, two very hard problems of balancing chemical reactions will be solved.

## Main results

## Problem 1

First, will be balanced the following chemical reaction with atoms which possess fractional oxidation numbers

$$
\begin{gather*}
x_{1} \mathrm{C}_{2952} \mathrm{H}_{4664} \mathrm{~N}_{812} \mathrm{O}_{832} \mathrm{~S}_{8} \mathrm{Fe}_{4}+x_{2} \mathrm{Na}_{2} \mathrm{C}_{4} \mathrm{H}_{3} \mathrm{O}_{4} \mathrm{SAu}+x_{3} \mathrm{Fe}(\mathrm{SCN})_{2} \\
+x_{4} \mathrm{Fe}\left(\mathrm{NH}_{4}\right)_{2}\left(\mathrm{SO}_{4}\right)_{2} \cdot 6 \mathrm{H}_{2} \mathrm{O}+x_{5} \mathrm{C}_{4} \mathrm{H}_{8} \mathrm{Cl}_{2} \mathrm{~S}+x_{6} \mathrm{C}_{8} \mathrm{H}_{12} \mathrm{MgN}_{2} \mathrm{O}_{8} \rightarrow x_{7} \mathrm{C}_{55} \mathrm{H}_{72} \mathrm{MgN}_{4}  \tag{1}\\
+x_{8} \mathrm{Na}_{3.99} \mathrm{Fe}(\mathrm{CN})_{6}+x_{9} \mathrm{Au}_{0.987} \mathrm{SC}_{6} \mathrm{H}_{11} \mathrm{O}_{5}+x_{10} \mathrm{HClO}_{4}+x_{11} \mathrm{H}_{2} \mathrm{~S} .
\end{gather*}
$$

## Solution

For balancing of this kind of reaction the computer is useless. From the mass balance of the above chemical reaction (1) one obtains this system of linear equations

$$
\begin{gathered}
2952 x_{1}+4 x_{2}+2 x_{3}+4 x_{5}+8 x_{6}=55 x_{7}+6 x_{8}+6 x_{9} \\
4664 x_{1}+3 x_{2}+20 x_{4}+8 x_{5}+12 x_{6}=72 x_{7}+11 x_{9}+x_{10}+2 x_{11} \\
812 x_{1}+2 x_{3}+2 x_{4}+2 x_{6}=4 x_{7}+6 x_{8} \\
832 x_{1}+4 x_{2}+14 x_{4}+8 x_{6}=5 x_{9}+4 x_{10}
\end{gathered}
$$

$$
\begin{gather*}
8 x_{1}+x_{2}+2 x_{3}+2 x_{4}+x_{5}=x_{9}+x_{11},  \tag{2}\\
4 x_{1}+x_{3}+x_{4}=x_{8}, \\
2 x_{2}=3.99 x_{8}, \\
x_{2}=0.987 x_{9}, \\
2 x_{5}=x_{10}, \\
x_{6}=x_{7} .
\end{gather*}
$$

By using of the method of the elemination of the variables, from the chemical reaction (1) and the system of linear equations (2) imediatelly follows the required coefficients

$$
\begin{align*}
& 30448582 \mathrm{C}_{2952} \mathrm{H}_{4664} \mathrm{~N}_{812} \mathrm{O}_{832} \mathrm{~S}_{8} \mathrm{Fe}_{4}+10833308052 \mathrm{Na}_{2} \mathrm{C}_{4} \mathrm{H}_{3} \mathrm{O}_{4} \mathrm{SAu} \\
& +  \tag{3}\\
& +3899586588 \mathrm{Fe}(\mathrm{SCN})_{2}+1408848684 \mathrm{Fe}_{2}\left(\mathrm{NH}_{4}\right)_{2}\left(\mathrm{SO}_{4}\right)_{2} \cdot 6 \mathrm{H}_{2} \mathrm{O}+5568665015 \mathrm{C}_{4} \mathrm{H}_{8} \mathrm{Cl}_{2} \mathrm{~S} \\
& +1379870764 \mathrm{C}_{8} \mathrm{H}_{12} \mathrm{MgN}_{2} \mathrm{O}_{8} \rightarrow 1379870764 \mathrm{C}_{55} \mathrm{H}_{72} \mathrm{MgN}_{4}+5430229600 \mathrm{Na}_{3.99} \mathrm{Fe}(\mathrm{CN})_{6} \\
& +10975996000 \mathrm{Au}_{0.987} \mathrm{SC}_{6} \mathrm{H}_{11} \mathrm{O}_{5}+11137330030 \mathrm{HClO}_{4}+16286436267 \mathrm{H}_{2} \mathrm{~S} .
\end{align*}
$$

Is it chemistry? No! It is a linear algebra. ${ }^{1)}$

## Problem 2

Next, will be balanced the following chemical reaction

$$
\begin{gathered}
x_{1} \mathrm{CaBeAsSAtCsF}{ }_{13}+x_{2}\left[\mathrm{Ru}\left(\mathrm{C}_{10} \mathrm{H}_{8} \mathrm{~N}_{2}\right)_{3}\right] \mathrm{Cl}_{2} \cdot 6 \mathrm{H}_{2} \mathrm{O}+x_{3} \mathrm{~W}_{2} \mathrm{Cl}_{8}\left(\mathrm{NSeInCl}_{3}\right)_{2} \\
+x_{4} \mathrm{Ca}\left(\mathrm{GaH}_{2} \mathrm{~S}_{4}\right)_{2}+x_{5}\left(\mathrm{NH}_{4}\right)_{2} \mathrm{MoO}_{4}+x_{6} \mathrm{~K}_{4} \mathrm{Fe}(\mathrm{CN})_{6}+x_{7} \mathrm{Na}_{2} \mathrm{Cr}_{2} \mathrm{O}_{7}+x_{8} \mathrm{MgS}_{2} \mathrm{O}_{3} \\
+x_{9} \mathrm{LaTlS}_{3}+x_{10} \mathrm{Na}_{3} \mathrm{PO}_{4}+x_{11} \mathrm{Ag}_{2} \mathrm{PbO}_{2}+x_{12} \mathrm{SnSO}_{4}+x_{13} \mathrm{HoHS}_{4}+x_{14} \mathrm{CeCl}_{3}+x_{15} \mathrm{ZrO}_{2} \\
+x_{16} \mathrm{Cu}_{2} \mathrm{O}+x_{17} \mathrm{Al}_{2} \mathrm{O}_{3}+x_{18}{\mathrm{Bi} 2 \mathrm{O}_{3}+x_{19} \mathrm{SiO}_{2}+x_{20} \mathrm{Au}_{2} \mathrm{O}+x_{21} \mathrm{TeO}_{3}+x_{22} \mathrm{CdO}}_{+}+x_{23} \mathrm{Hg}_{2} \mathrm{~S} \rightarrow x_{24}\left(\mathrm{NH}_{3}\right)_{3}\left[(\mathrm{PO})_{4} \cdot 12 \mathrm{MoO}_{3}\right]+x_{25} \mathrm{LaHgTlZrS} \\
6
\end{gathered}+x_{26} \mathrm{In}_{3} \mathrm{CdCeCl}_{12} .
$$

## Solution

From (4) one can derive this system of linear equations

$$
\begin{gather*}
x_{1}+x_{4}=x_{35}, \\
x_{1}=x_{38}, \\
x_{1}=3 x_{31}, \\
x_{1}+8 x_{4}+2 x_{8}+3 x_{9}+x_{12}+4 x_{13}+x_{23}=6 x_{25}+7 x_{28}+4 x_{30}, \\
x_{1}=x_{32}, \\
13 x_{1}=5 x_{35}, \\
x_{2}=x_{27}, \\
30 x_{2}+6 x_{6}=4 x_{28}+2 x_{29}+x_{37}, \\
36 x_{2}+4 x_{4}+8 x_{5}+x_{13}=9 x_{24}+3 x_{28}+2 x_{33}+2 x_{37}+x_{39}, \\
6 x_{2}+2 x_{3}+2 x_{5}+6 x_{6}=3 x_{24}+2 x_{29}, \\
2 x_{2}+14 x_{3}+3 x_{14}=12 x_{26}+3 x_{32}+x_{39}, \\
6 x_{2}+4 x_{5}+7 x_{7}+3 x_{8}+4 x_{10}+2 x_{11}+4 x_{12}+2 x_{15}+x_{16}+3 x_{17}+3 x_{18}+2 x_{19}+x_{20} \\
+3 x_{21}+x_{22}=40 x_{24}+x_{28}+16 x_{30}+12 x_{31}+4 x_{36}+3 x_{37}+3 x_{38}+x_{39}+x_{40}, \\
2 x_{3}=2 x_{40}, \\
2 x_{3}=6 x_{34}, \\
2 x_{3}=3 x_{26}, \\
2 x_{4}=x_{33}, \\
x_{5}=12 x_{24}, \\
4 x_{6}=x_{29}, \\
x_{6}=2 x_{30},  \tag{5}\\
2 x_{7}+3 x_{10}=2 x_{28}, \\
2 x_{7}=x_{36}, \\
x_{8}=x_{30}, \\
x_{9}=x_{25}, \\
x_{9}=x_{25}, \\
x_{10}=4 x_{24}, \\
2 x_{11}=x_{27}, \\
x_{11}=x_{36}, \\
x_{12}=3 x_{31}, \\
x_{13}=x_{33}, \\
x_{14}=x_{26}, \\
x_{15}=x_{25}, \\
2 x_{16}=x_{32}, \\
2 x_{17}=0.97 x_{35}, \\
2 x_{18}=x_{31}, \\
3
\end{gather*}
$$

$$
\begin{gathered}
x_{19}=x_{34}+x_{38}, \\
2 x_{20}=x_{27}+x_{28}+x_{29}, \\
x_{21}=8 x_{27}, \\
x_{22}=x_{26}, \\
2 x_{23}=x_{25} .
\end{gathered}
$$

By using the same technique as in the previous problem, immediately follows required solution

$$
\begin{gathered}
7731000 \mathrm{CaBeAsSAtCsF} 1_{3}+1502160\left[\mathrm{Ru}\left(\mathrm{C}_{10} \mathrm{H}_{8} \mathrm{~N}_{2}\right)_{3}\right] \mathrm{Cl}_{2} \cdot 6 \mathrm{H}_{2} \mathrm{O} \\
+9273600 \mathrm{~W}_{2} \mathrm{Cl}_{8}\left(\mathrm{NSeInCl}_{3}\right)_{2}+12369600 \mathrm{Ca}\left(\mathrm{GaH}_{2} \mathrm{~S}_{4}\right)_{2}+1560720\left(\mathrm{NH}_{4}\right)_{2} \mathrm{MoO}_{4} \\
+12054510 \mathrm{~K}_{4} \mathrm{Fe}(\mathrm{CN})_{6}+375540 \mathrm{Na}_{2} \mathrm{Cr}_{2} \mathrm{O}_{7}+6027255 \mathrm{MgSS}_{2} \mathrm{O}_{3}+37709196 \mathrm{LaTlS}_{3} \\
+520240 \mathrm{Na}_{3} \mathrm{PO}_{4}+751080 \mathrm{Ag}_{2} \mathrm{PbO}_{2}+7731000 \mathrm{SnSO}_{4}+24739200 \mathrm{HoHS}_{4} \\
+6182400 \mathrm{CeCl}_{3}+37709196 \mathrm{ZrO}_{2}+3865500 \mathrm{Cu}_{2} \mathrm{O}+9748791 \mathrm{Al}_{2} \mathrm{O}_{3}+1288500 \mathrm{Bi}_{2} \mathrm{O}_{3} \\
+10822200 \mathrm{SiO}_{2}+25438050 \mathrm{Au}_{2} \mathrm{O}+12017280 \mathrm{TeO}_{3}+6182400 \mathrm{CdO}+18854598 \mathrm{Hg}_{2} \mathrm{~S} \\
\rightarrow 130060\left(\mathrm{NH}_{3}\right)_{3}\left[(\mathrm{PO})_{4} \cdot 12 \mathrm{MoO}_{3}\right]+37709196 \mathrm{LaHgTlZrS}_{6}+6182400 \mathrm{In}_{3} \mathrm{CdCeCl}_{12} \\
+1502160 \mathrm{AgRuAuTe}_{8}+1155900 \mathrm{C}_{4} \mathrm{H}_{3} \mathrm{AuNa}_{2} \mathrm{OS}_{7}+48218040 \mathrm{KAu}(\mathrm{CN})_{2} \\
+6027255 \mathrm{MgFe}_{2}\left(\mathrm{SO}_{4}\right)_{4}+2577000 \mathrm{Sn}_{3}\left(\mathrm{AsO}_{4}\right)_{3} \mathrm{BiAt}_{3}+7731000 \mathrm{CuCsCl}_{3} \\
+24739200 \mathrm{GaHoH}_{2} \mathrm{~S}_{4}+3091200 \mathrm{~N}_{2} \mathrm{SiSe}_{6}+20100600 \mathrm{CaAl}_{0.97} \mathrm{~F}_{5}+751080 \mathrm{PbCrO}_{4} \\
+16332180 \mathrm{H}_{2} \mathrm{CO}_{3}+7731000 \mathrm{BeSiO}_{3}+54000120 \mathrm{HClO}+9273600 \mathrm{~W}_{2} \mathrm{O} .
\end{gathered}
$$

## Discussion

Well-known reactions with unique coefficients do not generate challenge for their future study, because right now they are balanced by a computer very easily if they possess atoms with integer oxidation numbers. Generally speaking, now it is a piece of cake. In an opposite case, when chemical reactions have atoms with fractional oxidation numbers, then their balancing is very hard and need special mathematical methods. Just these reactions were the subject matter in this article. Since this class of chemical reactions is not studied enough in the scientific literature, they were our main object for study.

What we did? For both considered chemical reactions, we determined their unique coefficients. From the obtained systems of linear equations, we determined required coefficients for every reaction. To date, chemists balanced only elementary particular reactions and they did not take into account reactions with fractional oxidation numbers. They did not pay attention to general cases and precise criteria for balancing chemical
reaction too. In this article we determined very accurately the unique coefficients of two chemical reactions with fractional oxidation numbers.

In other words, balancing chemical reactions with fractional oxidation numbers is possible only by using of mathematical methods, because there are not chemical methods and on top of all it is not possible such chemical methods to exist. Hopes that such chemical methods exist are pure illusion. Why? Reply to this question with a philosophical disposition is very easy and it looks like this. Foundation on chemistry is mathematical, but not chemical, because it uses basic mathematical entities. For instance, every chemical reaction can be reduced to a matrix equation, but opposite does not hold. That means that we have just one way of its foundation - mathematical. It is just one thing. Another thing is valence of elements in chemical reactions with fractional oxidation numbers we cannot determine. It happens not only with molecules with fractional oxidation numbers, the same appears in compound molecules with integer oxidation numbers. In this case we can speak only for algebra of the reaction, but not for change in oxidation number procedure that support chemists. These are facts that refute chemical reason.

## Conclusion

Balancing chemical reactions which possess atoms with fractional oxidation numbers is possible only by using of mathematical methods. In this article we considered two very hard chemical reactions which belong to the class of chemical reactions which have unique coefficients. These reactions are balanced by the classical algebraic method, because they cannot be balanced by usage of a computer. To date is not known any software package that can balance successfully this kind of reactions. That was the cause that we chose the classical algebraic method for balancing the above chemical reactions.

## NOTES

1. According to the above, balancing chemical equations independs of chemistry, but it is very important for chemistry. It is a linear algebra and it is a job for mathematicians.

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