# New York State Mathematics <br> GLOSSARY - Grades PreK - 8 

This Glossary, intended for teacher use only, provides an understanding of the mathematical terms used in PreKgrade 8 level instruction as reflected in the New York State Mathematics Core Curriculum (Revised 2005). We encourage all teachers to become familiar with these terms and use them consistently throughout a student's educational program.

The following terms have been updated since the initial posting in June of 2005.
Guidance is provided on the exact language change and NYS teachers' rationale for the change in the document entitled, Updates to the PreK-Grade 8 Glossary (10-31-06).

| $\underline{\text { between }}$ | $\underline{\text { ones place }}$ |
| :--- | :--- |
| Cardinality Principle | $\underline{\text { pi }(\partial)}$ |
| $\underline{\text { fact family }}$ | $\underline{\text { right angle (change figure only) }}$ |
| *grid | *rule/function rule |
| $\underline{\text { hundreds place }}$ | $\underline{\text { substitute }}$ |
| $\underline{\text { hundredths place (replace hundredths) }}$ | $\underline{\text { tens place }}$ |
| $\underline{\text { linear equation }}$ | $\underline{\text { tenths place }}$ |
| $\underline{\text { linear inequality }}$ | $\underline{\text { thousands place }}$ |

* indicates a new addition to the glossary
absolute value $|n|$ The distance from 0 to a number, $n$, on a number line (e.g., $|-3|=3$, $|+3|=3$, and $|0|=0$ ).
accuracy How close a numerical measure is to its actual value.
acute angle An angle whose measure is greater than $0^{\circ}$ and less than $90^{\circ}$.
acute triangle $A$ triangle in which all three angles are acute.
Example:

add To combine two or more quantities to find one quantity called a total or sum.
addend One of the numbers in an indicated sum of two or more numbers (e.g., $3+5+1=9 ; 3,5$, and 1 are addends).
addition A mathematical operation of combining two or more numbers into a sum.
addition fact The addition of two single-digit addends producing sums to 18.
addition sentence An equation showing the sum of two or more numbers (e.g., $13+8=21$ ).
additive inverse A number that when added to a given number results in a sum of zero; the opposite of a number.
adjacent Next to each other.
adjacent side of an acute angle in a right triangle The leg of the right triangle that is a side of the acute angle.

Example: In right triangle $A B C$ below, $\overline{A C}$ is adjacent to angle $A$ and $\overline{B C}$ is adjacent to angle $B$.

adjacent angles Two angles in a plane that share a common side and share a common vertex but have no interior points in common (do not overlap).

Example: In the figure below, angles 1 and 2 are adjacent angles.

after Behind in place, subsequent to in time or order.
algebra The branch of mathematics that uses letters, symbols, and/or characters to represent numbers and express mathematical relationships.
algebraic Making use of or referring to concepts or methods of algebra.
algebraic expression A mathematical phrase that is written using one or more variables and constants, but which does not contain a relation symbol ( $<,>, \leq, \geq,=, \neq$ ) (e.g., $3 y+6$ ).
algebraic inequality An algebraic statement that is written using one or more variables and constants that shows a greater than or less than relationship (e.g., $3 x+7>22$ ).
algebraic pattern A set of numbers and/or variables in a specific order that form a pattern.
algebraic relationship To express the relationship between two or more numbers using an algebraic expression.

Example: The algebraic relationship which represents $2,4,6,8, \ldots$ is $2 n$
(where $n=1,2,3, \ldots$ ) and the algebraic relationship which represents $4,7,10,13, \ldots$ is $3 n+1$ (where $n=1,2,3, \ldots$ ).
algebraic solution The process of solving a mathematical problem using the principles of algebra.
algebraic term An algebraic expression, called a monomial, involving only multiplication between constants and at least one variable (e.g., 3xy).
algebraically Representing a mathematical situation using algebra.
algorithm An explicit step-by-step procedure for performing a mathematical computation or for solving a mathematical problem.
alternate exterior angles A pair of angles on the outer sides of two lines intersected by a transversal, but on opposite sides of the transversal.

Example:
alternate exterior angles:
$\angle 1$ and $\angle 8, \angle 2$ and $\angle 7$

alternate interior angles A pair of angles on the inner sides of two lines intersected by a transversal, but on opposite sides of the transversal.

## Example:

alternate interior angles:
$\angle 3$ and $\angle 6, \angle 4$ and $\angle 5$

altitude of a triangle or quadrilateral A line segment (or its length) drawn from a vertex perpendicular to the line containing the opposite side.
altitude of a cone or pyramid A line segment (or its length) drawn from the vertex of the cone perpendicular to the plane containing the base.
altitude of a cylinder or prism A line segment (or its length) drawn from any point on one base perpendicular to the plane containing the other base.
amount The sum, the whole, or aggregate of two or more quantities.
analog clock A clock with a minute hand and an hour hand.
angle A geometric figure formed by two non-collinear rays that have a common endpoint.

Example:

$\angle A B C$ has its vertex at point $B$.
angle bisector A segment or ray that divides an angle into two congruent angles.
angle pairs Pairs of angles with special relationships (e.g., supplementary angles, complementary angles, vertical angles, alternate interior angles, alternate exterior angles, corresponding angles, adjacent angles).
ante meridiem (a.m.) Before noon; the time between 12 midnight and 12 noon; 12 midnight is $12 \mathrm{a} . \mathrm{m}$.
apply To use a theorem or concept to solve an algebraic, numeric, or geometric problem.
approximation A mathematical quantity that estimates a desired quantity.
arc Part of a curve between any two of its points. (See major and minor arc)
Example: Minor arc $A B$ is an arc on circle $O$.

area The measure of the interior surface of a closed region or figure; area is measured in square units.

Example: The area of the rectangle is 33 square units.

| 11 |
| :--- |
|         |

argument The communication, in verbal or written form, of the reasoning process that leads to a valid conclusion; a valid argument is the result of the conjecture/reasoning process.
arithmetic expression (See numeric expression)
arithmetic sequence $A$ sequence of elements, $a_{1}, a_{2}, a_{3}, \ldots$, such that the difference of any two successive terms is a constant (e.g., $\{2,5,8,11,14, \ldots\}$ ).
arithmetically Computing using numbers.
array A set of objects or numbers arranged in an order, usually into rows and/or columns.
ascending order Arranged in order from least to greatest or smallest to largest.
associative property A property of real numbers that states that the sum or product of a set of numbers is the same, regardless of how the numbers are grouped.

Examples: Addition:

$$
2+(3.5+1.3)=(2+3.5)+1.3
$$

$$
\text { Multiplication: } \quad-6 \times(18 \times 7)=(-6 \times 18) \times 7
$$

attribute A characteristic that identifies an object or person as part of a group.
axis A horizontal or vertical line used in the Cartesian coordinate system used to locate a point on the coordinate graph.
axes The horizontal and vertical lines dividing a coordinate plane into four quadrants.
Example: The figure below illustrates the horizontal axis ( $x$ ) and the vertical axis (y).

bar graph A graph that uses horizontal or vertical bars to display data.

## Example:


base (1) A number that is raised to an exponent; (2) a particular side or face of a geometric figure; (3) a number that is multiplied by a rate or of which a percentage or fraction is calculated.

Examples: (1) $2^{3}$, where 2 is the base and 3 is the exponent
(2)

(3) Amount $=$ base $\times$ rate
base ten number system A place value number system in which ten digits, 0 through 9 , are used to represent a number and the value of each place is 10 times the value of the place to its right; the value of any digit in the number is the product of that digit and its place value.

|  | $\begin{aligned} & \infty \\ & \overline{0} \\ & \overline{\bar{O}} \\ & \stackrel{\bar{C}}{0} \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\bar{O}} \\ & \overline{\bar{\omega}} \end{aligned}$ |  |  | $\begin{aligned} & \text { の } \\ & \stackrel{\Gamma}{0} \\ & \hline \overline{\overline{=}} \end{aligned}$ |  |  | 0 0 0 0 0 0 0 | $\begin{aligned} & \frac{0}{0} \\ & \frac{0}{0} \\ & \frac{1}{3} \\ & \frac{1}{1} \end{aligned}$ | $\begin{gathered} \infty \\ \stackrel{\varrho}{0} \\ \hline \end{gathered}$ | $\begin{aligned} & \infty \\ & \underset{0}{\infty} \end{aligned}$ | and |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

10 ones $=1$ ten
10 tens $=1$ hundred
10 hundreds $=1$ thousand
10 thousands = 1 ten thousand
10 ten thousands = 1 hundred thousand 10 hundred thousands $=1$ million

10 tenths = 1 one
10 hundredths $=1$ tenth
10 thousandths = 1 hundredth
10 ten-thousandths = 1 thousandth
10 hundred-thousandths = 1 ten-thousandth
10 millionths $=1$ hundred-thousandth
basic whole number facts Addition, subtraction, multiplication, or division of all combinations of two numbers, using the numbers 0-9. (Sometimes the numbers 10-12 are included in the basic whole number facts.)
before In front of, earlier than, at an earlier time.
between Given two numbers, another number is said to be between those two numbers if it is greater than the first but less than the second.
binomial An algebraic expression consisting of two terms $(5 a+6)$.
bisect To divide a geometric figure into two congruent parts.
bisector A line segment, ray, line, or plane that divides a geometric figure into two congruent halves.
calculate To compute; to perform the indicated operation(s).
calendar A tabular arrangement of the days, weeks, and months of the year.
capacity The maximum amount a container can hold.
cardinal number $A$ number that denotes how many objects are in a set.
Cardinality Principle The concept that the last number counted represents the "manyness" or cardinality (the number of elements) of a set.

Cartesian (coordinate) plane The plane formed by a horizontal axis and a vertical axis, often labeled the $x$-axis and $y$-axis respectively.
categorical data Data that can be classified by type (e.g., colors, breeds of dogs); typically represented using a bar graph, circle graph, or pictograph.

Celsius A temperature measurement scale where $0^{\circ}$ represents the freezing point of water and $100^{\circ}$ represents the boiling point of water; formerly known as centigrade.
cent A unit of money equal to $\frac{1}{100}$ of one dollar.
centigrade (See Celsius)
centimeter (cm) A metric unit of length which is equal to one-hundredth of a meter.
central angle An angle whose vertex is at the center of a circle and whose sides contain radii of the circle.

Example: $\angle A O B$ is a central angle of circle $O$.

certainty An event that is certain to happen in a probability experiment.
Example: Drawing an odd number when selecting one number from a bag containing 6 slips of paper with the numbers $1,3,5,7,9$, and 11 on them.
change Money in the form of coins and/or dollars that is received when you purchase an item with more money than the item costs.
characteristic A distinguishing trait, quality, property, or attribute.
chart A diagram that illustrates information in the form of a table, graph, or picture.
chord A line segment connecting any two points on a circle. In the circle below, $\overline{A B}$ is a chord of circle $O$.

Example:

circle A plane closed curve consisting of all points a fixed distance from a fixed point called its center.
circle graph or pie graph A graph in which the data is represented by sectors of a circle; the total of all the sectors should be $100 \%$ of the data.

## Example: Per Capita US Consumption of Milk, 1995

$\square$ Whole ■ Lowfat $\square$ Skim


1995
circumference The distance around a circle, calculated by multiplying the length of the diameter of the circle by pi (i.e., $C=\partial d$ ).
classify To sort into categories or to arrange into groups by attribute(s).
classify triangles To categorize a triangle according to its angles (acute, obtuse, or right) or the lengths of its sides (equilateral, isosceles, or scalene).
closed figure A figure that starts and ends at the same point.
coefficient A constant that multiplies a variable (e.g., $3 x+4 y=14,3$ is the coefficient of $x$ and 4 is the coefficient of $y$ ).
coin A flat piece of metal issued by governmental authority as money (e.g., pennies, nickels, dimes, quarters).
collection An accumulation of objects gathered for study or comparison.
combination Any subset of a set considered without regard to order within the subset (e.g., the number of different committees of three that can be chosen from a group of twelve members).
combine like terms To simplify expressions by adding or subtracting like terms (e.g., $\left.4 x+5+3 x+9 x^{2}=7 x+5+9 x^{2}\right)$.
commission Earnings based on the amount of total sales.
common denominator A whole number greater than zero that is a common multiple of each denominator in two or more fractions (e.g., common denominators for $\frac{1}{6}$ and $\frac{3}{8}$ are $24,36,48, \ldots)$.
common factor A number, polynomial, or quantity that evenly divides into two or more numbers or algebraic expressions (e.g., 5 is a common factor of 15 and $30 ; 2 x$ is a common factor of $4 x y$ and $6 x^{2}$ ).
common multiple A whole number that is a multiple of two or more given numbers (e.g., common multiples of 2,3 , and 4 are $12,24,36,48, \ldots$ ).
commutative property of addition A property of real numbers that states that the sum of two terms is unaffected by the order in which the terms are added; i.e., the sum remains the same (e.g., $-2+3.5=3.5+-2$ ).
commutative property of multiplication A property of real numbers that states that the product of two factors is unaffected by the order in which they are multiplied; i.e., the product remains the same (e.g., $3 \times 5=5 \times 3$ and $5 \cdot x=x \cdot 5$ ).
compare To state the similarities or differences between two or more numbers, objects, or figures by considering size, shape, odd, even, or other attributes.
compass An instrument used to locate points at a given distance from a fixed point, and to describe circles and arcs.
compatible numbers Numbers that go together easily; a strategy often used to estimate sums, differences, products, and quotients.
compensation A strategy that can be used for addition which usually involves increasing one addend while decreasing the other by the same amount (e.g., When adding $46+38$, add 2 to 38 to make 40 and take two away from 46 , resulting in 44 ; then add $40+44$ to get 84 ).
complementary angles Two angles whose measures have a sum of $90^{\circ}$ (e.g., $50^{\circ}$ and $40^{\circ}$ are complementary angles).
compose Part of a process of grouping decomposed numbers into quantities that are easier to compute.
composite number A whole number greater than 1 that has more than two factors (e.g., 9 is a composite number because it has three factors: 1,3 , and 9 ).
compound event A combination of two or more simple events (e.g., the probability of rolling a 2 or a 3 when tossing a number cube).
computation The act or action of carrying out a series of operations.
compute To find the numerical result by applying arithmetic operations.
conclusion An answer or solution arrived at through logical or mathematical reasoning.
cone A solid bounded by a region called its base (usually a circle) in a plane and the surface formed by straight line segments which join points on the boundary of the base to a fixed point, called its vertex, not in the plane containing the base.
congruent Two or more figures having exactly the same shape and size; coinciding when superimposed.
conjecture (noun) A mathematical statement, thought to be true, which has neither been proven nor refuted by counterexample.
conjecture (verb) To make a prediction or a statement, based upon guesswork and thought to be true.
consecutive Following one right after the other in order (e.g., 1, 2, 3, ... are consecutive positive integers and $-2,0,2,4, \ldots$ are consecutive even integers).
conservation of number An understanding that rearranging a group of objects does not affect its number.
constant A quantity that does not change its value in a given expression or equation (e.g., In the expression $3 s+4,3$ and 4 are constants).
construction (compass and straightedge) A precise way of drawing that allows only two tools: the straightedge and compass; the compass establishes equidistance, and the straightedge establishes collinearity.
convert To change the form, but not the value of a particular number or quantity.
Examples: The improper fraction $\frac{8}{3}$ converts to $2 \frac{2}{3}$ as a mixed number.
32 inches converts to 2 feet 8 inches.

The unit rate \$0.08/oz. converts to $\$ 1.28 /$ pound
coordinate axes The two intersecting perpendicular lines in a plane that form the four quadrants for locating points, given the ordered pair of the points; the axes are referred to as the $x$-axis and the $y$-axis.
coordinate plane A plane containing a set of coordinate axes in which each point is located by a set of coordinates $(x, y)$; the point of intersection of the axes is called the origin and has coordinates $(0,0)$.

Example: On the coordinate plane below, point $A$ has coordinates $(-3,4)$.

coordinate geometry The study of geometry using a coordinate plane.
coordinate system (also called rectangular coordinate system) A method of representing points in the plane or in space by means of numbers; a point in a plane can be located by its distance from both a horizontal and a vertical line called the axes; the horizontal line is called the $x$-axis; the vertical line is called the $y$-axis; the pairs of numbers are called ordered pairs; the first number, called the $x$-coordinate, designates the distance along the horizontal axis; the second number, called the $y$-coordinate, designates the distance along the vertical axis; the point at which the two axes intersect has the coordinates $(0,0)$ and is called the origin.
coordinates An ordered pair of numbers that identifies a point on a coordinate plane, written as $(x, y)$.
corresponding angles Any pair of angles on the same side of the transversal, one interior and one exterior, formed when two parallel lines are intersected by a transversal.

Example: In the figure below, $\angle 1$ and $\angle 5, \angle 2$ and $\angle 6, \angle 3$ and $\angle 7, \angle 4$ and $\angle 8$ are corresponding angles.

corresponding sides Sides in the same relative position on two congruent or similar figures. (The corresponding sides of congruent figures are equal and the corresponding sides of similar figures are proportional.)
count To name the numbers in order up to and including a given number (e.g., count to ten); to determine the total number or amount, as in money.
count back A subtraction strategy of starting with the minuend and counting backward an amount equal to the amount of the subtrahend to arrive at the difference.
count on An addition strategy of starting with one addend (usually the larger) and counting forward an amount equal to the other addend to arrive at the sum.
counterexample An example to show that a rule is not true for all numbers.
Example: Show by counterexample that the commutative property does not work for subtraction. $4-5 \neq 5-4$
counting numbers All whole numbers greater than zero; also called natural numbers. cube A solid rectangular figure (prism) with 6 square faces.
cube of a number The third power of a number (e.g., $2^{3}=2 \times 2 \times 2=8$ ).
cubic unit $A$ unit for measuring volume.
cup A customary unit used to measure capacity; 1 cup $=8$ ounces.
currency The money of a country that circulates as a medium of exchange (e.g., coins, dollar bills, euros).
customary measurement system The system of measurement used mainly in the United States to measure length (e.g., inch, foot, yard, mile), mass (e.g., ounce, pound, ton), and capacity (fluid ounce, cup, pint, quart, gallon).
customary units The units of measure used in the customary measurement system.
cylinder A solid bounded by two parallel congruent closed curves (usually circles), called its bases, in a plane and the surface formed by straight line segments that join points on the each of the closed curves.

## Examples:


data Information collected and used to analyze a particular concept or situation.
day $A$ unit used to measure time; 1 day $=24$ hours.
decagon A polygon with ten sides.
decimal fraction A fraction using a decimal point to represent the denominator of 10 or a power of 10.
decimal number A fractional number written using base ten notation; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number).
decimal point A period or dot separating the ones place from the tenths place in decimal numbers or dollars from cents in money.
decimeter One-tenth of a meter, a decimeter is approximately the distance from one's wrist to the end of one's thumb.
decompose To break a number into smaller units to simplify computation (e.g., $15=10+5)$.
decomposition The process of breaking a number into smaller units to simplify computation.
decrease To take away or become smaller.
deductive reasoning Using logic to arrive at a conclusion from a given premise.
degree The unit of measurement for angles; $1^{\circ}$ is the measure of an angle which forms $\frac{1}{360}$ of a circle.
degree of a monomial The sum of the exponents on the variables contained in a term (e.g., the degree of $3 x^{2} y^{3}$ is 5 ).
degree of a polynomial The largest degree among all its terms.
denominator The quantity below the line in a fraction. It represents the number of equal parts into which the whole is divided.
density The amount of mass per unit of volume.
dependent events Two events in which the outcome of the first event affects the outcome of the second event (e.g., drawing two marbles from a bag of red, green, and blue marbles without replacement).
diameter A chord of a circle which passes through the center of the circle.
Example: In the diagram below, $\overline{A B}$ is the diameter of circle $O$.

difference The amount remaining after one quantity is subtracted from another.
digit Any one of the ten numerical symbols: $0,1,2,3,4,5,6,7,8$, or 9 .
digital clock A clock on which the time is displayed numerically (e.g., The time is displayed as 12:22).
dilation A transformation that stretches or shrinks a function or graph both horizontally and vertically by the same scale factor.
dime A coin with a value of 10 cents or $\frac{1}{10}$ of one dollar.
dimensions Measurements of a figure (e.g., the length, width, and height of a prism).
discount A reduction made from the gross amount or value of something; a reduction made from the regular or list price.
discover To make note of possible patterns and generalizations that result from investigation/exploration.
distance The length of the line segment joining two points.
distributive property A property of real numbers that states that the product of the sum or difference of two numbers is the same as the sum or difference of their products.

Example: Multiplication over addition: $\quad 2(15+4)=2 \times 15+2 \times 4$
Multiplication over subtraction: $\quad 4(12-8)=4 \times 12-4 \times 8$
dividend A number to be divided by another number (divisor).
divisible If a number has a whole number answer when divided by a second number, the first number is divisible by the second number; $x$ is divisible by $y$ if and only if $x=q y$ where $y$ is a whole number.
division A mathematical operation involving two numbers that tells how many groups there are or how many are in each group.
divisor The number by which the dividend is divided.
dollar Currency that is worth 100 cents.
domain The set of values for which a function is defined.
double To make twice as great or as many; to increase by adding an equal amount; to amount to twice the number.
double-bar graph A graph that uses pairs of bars to compare and show the relationship between data.
double-line graph A graph that uses pairs of lines that show change over time to represent and compare data.
edge A line segment where two faces of a three-dimensional figure intersect.
elapsed time The difference between two times; the amount of time that has passed.
endpoint A point at either end of a line segment or the beginning point of a ray.
equal to ( = ) A symbol that means two things have the same amount, size, number, or value.
equation A mathematical sentence stating that two expressions are equal.
equidistant Having equal distances.
equilateral Having equal sides.
equivalent Equal in value.
Examples: $3+3$ is equivalent to $2 \times 3$ (equivalent numerical expressions)
2.9 is equivalent to 2.90

1 yard is equivalent to 3 feet (equivalent lengths)

$$
\frac{2}{3} \text { is equivalent to } \frac{8}{12} \quad \text { (equivalent fractions) }
$$

equivalent fractions Two or more fractions that have the same quotient or that name the same region, part of a set, or part of a segment (e.g., $\frac{1}{3}=\frac{3}{9}$ ).
equivalent ratios Two ratios that are equal. (See equivalent fractions)
estimate An answer that is an approximation.
evaluate To find the value of a mathematical expression.
Example: Evaluate the expression $2 y-7$ when $y=5$

$$
2(5)-7=10-7=3
$$

even number $A$ whole number that is a multiple of 2 .
event A set of one or more outcomes in a probability experiment (e.g., given a number cube with the numbers 1 to 6 on the faces, the rolling of an even number is an event).
exchange rate The ratio of the value of one currency in relation to the value of another.
exchange rate table A table displaying the ratios of the values of several currencies in relation to each other.
expanded form A way to write a number that shows the value of each digit (e.g., $4556=4000+500+50+6)$.
expectation The degree of probability of an occurrence based on probability, what would be expected to happen.
expected value An average value found by multiplying the value of each possible outcome by its probability, then summing all the products.
experiment An action or process carried out under controlled conditions in order to discover an unknown effect or law, to test or establish a hypothesis, or to illustrate a known law.
experimental probability Probability based on experimental data; the ratio of the total number of times the favorable outcomes happens to the total number of times the experiment is done found by repeating the experiment several times, given by the
formula $P(E)=\frac{\text { number of successful outcomes }}{\text { total number of outcomes }}$.
explain (See justify)
explore To look for patterns or relationships between elements within a given setting.
exponent A number that tells how many times the base is used as a factor; in an expression of the form $b^{\mathrm{a}}, a$ is called the exponent, $b$ is the base, and $b^{\mathrm{a}}$ is a power of $b$.
exponential form $A$ number written using exponents (e.g., $32=2^{5}$ ).
expression A mathematical representation containing numbers, variables, and operation symbols; an expression does not include an equality or inequality symbol.

Examples: $(5+2)-27 \div 3$ (arithmetic/numerical expression)

$$
2 a+3 b \quad \text { (algebraic/symbolic expression) }
$$

extend a pattern To continue a pattern through several cycles.
exterior angle of a polygon An angle formed by one side of a polygon and the extension of the adjacent side.

Example:

extremes The first and last terms in the ratios of a proportion.
Example: $\quad \frac{4}{10}=\frac{16}{40} \quad$ The numbers 4 and 40 are the extremes.
face Polygons which bound the surface of a geometric solid.

fact family A set of facts, each of which relates the same three numbers through addition and subtraction or through multiplication and division. (e.g., $3+4=7,4+3=7$, $7-4=3,7-3=4$ and /or $2 \times 5=10,5 \times 2=10,10-:-5=2$, and $10-:-2=5$ )
factor (noun) A number or expression that is multiplied by another to yield a product (e.g., a factor of 32 is 8 because $8 \times 4=32$ and a factor of $5 x^{3}$ is $5 x$ because $\left.5 x\left(x^{2}\right)=5 x^{3}\right)$.
factor (verb) To express as a product of two or more factors.
Fahrenheit A temperature scale based on $32^{\circ}$ as the freezing point of water and $212^{\circ}$ as the normal boiling point of water.
fair share The amount that each person receives when something is divided equally.
feet The plural form of foot.
fixed distance A distance is kept constant.
flip (See reflection)
foot $A$ customary unit used to measure length; 1 foot $=12$ inches.
formula A mathematical statement, equation, or rule that shows a relationship between two or more quantities.

$$
\begin{array}{lll}
\text { Examples: } & P=3 s & \text { (perimeter of an equilateral triangle) } \\
& d=r \times t & \text { (distance }=\text { rate } \times \text { time) }
\end{array}
$$

four-digit number A number that contains four digits.
fraction A number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$ which can be read as a divided by $b$.
frequency The number of times an event occurs.
frequency table A table that shows how often each item, number, or range of numbers occurs in a set of data.
front-end estimation Using the leading, or left-most, digits to make an estimate quickly and easily.

Example: Using front-end estimation to estimate the sum of 594, 32, and 221, an initial estimate would be $5+0+2$ hundreds or 700 .
function A mathematical relationship between two variables, an independent variable and a dependent variable, where every value of the independent variable corresponds to exactly one value of the dependent value.
function notation A notation in which a function is named with a letter and the input is shown in parentheses after the function name (e.g., $f(x)=x^{2}+1$ represents the function $y=x^{2}+1$, where the letter f is the name of the function, and $\mathrm{f}(x)$, read as f of $x$, stands for the output for the input $x$ ).
fundamental counting principle The principle that states that all possible outcomes in a sample space can be found by multiplying the number of ways each event can occur.
gallon A customary unit used to measure capacity; 1 gallon = 4 quarts.
geometric figure Any combination of points, lines, planes, or curves in two or three dimensions.
geometric pattern An arrangement of geometric figures that repeats.
geometric shape Any regular or irregular polygon, circle, or combination of geometric figures.

geometry The branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, planes, and two- and three-dimensional figures.
gram (g) A metric unit used to measure mass; 1000 grams = 1 kilogram.
graph A graphic representation used to show a numerical relationship.
graphically To solve a problem or demonstrate a relationship using a number line or coordinate graph.
greater than (>) A relationship showing that the first term or expression has a value larger than the second term or expression (e.g., $5+3>5-2$ and $2 x-4<18$ ).
greatest The largest amount.
greatest common divisor (GCD) The largest integer or the polynomial (monomial) of highest degree that is an exact divisor or each of two or more integers or polynomials.
greatest common factor (GCF) The greatest number or expression that is a factor of two or more numbers or expressions (e.g., the greatest common factor of 12 and 18 is 6 ).
grid - A network of uniformly spaced horizontal and vertical lines.
group A number of individuals or objects that are assembled together or that have some unifying relationship.
half hour A period of time lasting 30 minutes.
halving Dividing or separating into two equal parts; reducing to one half.
height (1) The perpendicular distance from a vertex to the line containing the opposite side of a plane figure; the length of a perpendicular from the vertex to the plane containing the base of a pyramid or cone; the length of a perpendicular between the planes containing the bases of a prism or cylinder. (See altitude)
heptagon A polygon with seven sides and seven angles.
hexagon A polygon with six sides and six angles.

## Examples:


histogram A special kind of bar graph that displays the frequency of data that has been organized into equal intervals; the intervals cover all possible values of data, therefore there are no spaces between the bars of the graph; the horizontal axis is divided into continuous equal intervals.

horizontal Parallel to or in the plane of the horizon.
hour (h) A unit used to measure time, $\frac{1}{24}$ of a day; 1 hour $=60$ minutes.
hour hand The shorter hand on an analog clock.
hundreds place The place value located three places to the left of the decimal point in a number; a digit in the hundreds place has a value of 100 times the value of the digit.
hundred chart A $10 \times 10$ grid representing the numbers from 1 to 100 in rows and columns of ten.
hundredths place The place value located two places to the right of the decimal point in a number; one out of a hundred equal parts of a whole.
hypotenuse The side of a right triangle opposite the right angle; the longest side of a right triangle.

Example:

identity element for addition The number in a set which when added to any number $n$ in the set yields the given number; in general, $e$ is the identity element of addition for a set if $n+e=n$ for all $n$ in the set; the identity element for addition is zero because $a+0=a$ and $0+a=a$.
identity element for multiplication The number in a set which when any number $n$ in the set is multiplied by, yields the given number; in general, $e$ is the identify element of multiplication for a set if $n \times e=n$ for all $n$ in the set; the identity element for multiplication is one because a $1=1 a=a$.
image The figure created when another figure, called the pre-image, undergoes a transformation.
impossible outcome or impossibility An event that cannot occur in a probability experiment (e.g., rolling the number 7 when tossing a six-sided number cube labeled 1 to 6).
improper fraction $A$ fraction whose numerator is greater than its denominator.
independent events Two or more events in which the outcome of one event has no effect on the outcome of the other event or events.
inch (in) A customary unit for measuring length or distance; 12 inches = 1 foot; roughly equivalent to the distance from the end of one's thumb to the first joint.
income The amount of money received for labor, for services, from the sale of goods or property, or from investments.
increase To become larger in size or quantity.
indirect proof The method of proof that assumes the negation of what is to be proved and deduces a contradiction.
inductive reasoning Making a generalization from specific cases; used to formulate a general rule after examining a pattern.
inequality A mathematical statement containing one of the symbols: $>,<, \geq, \leq$, or $\neq$ to indicate the relationship between two quantities (e.g., $6-2>4 \div 2,5 z \leq 25$, and $7 \neq 2+2$ ).
informally Not according to prescribed rules; casually without formal rules.
input value $A$ value assigned to a variable in a given formula or expression that allows the formula or expression to be evaluated.

Example: Evaluate the perimeter of a rectangle given the following input values: $L=12, W=5$ and the formula $P=2 L+2 W$

$$
\begin{aligned}
& P=2 L+2 W \\
& P=2(12)+2(5) \\
& P=24+10 \\
& P=34
\end{aligned}
$$

integers The set of numbers consisting of the counting numbers $(1,2,3,4, \ldots)$, their opposites ( $-1,-2,-3,-4, \ldots$ ), and zero.
integral Refers to an integer; an integral solution to a problem cannot be a decimal or fraction.
integral exponent An exponent that is an integer (e.g., in the expression $2^{-1},-1$ is the integral exponent of the base number 2).
interest The amount of money charged for borrowing money or the profit (usually money) that is made on invested capital.
interest rate The percent of interest charged on money borrowed or earned on money invested.
interior angle An angle on the inside of a polygon formed by two adjacent sides of the polygon.
intersecting lines Lines that share a common point.
intersection of sets The set of elements that belong to each of two or more sets (e.g., if Set $A=\{2,4,6,8,10\}$ and Set $B=\{1,2,3,4,5,6\}$, then the intersection of sets $A$ and $B$ is $\{2,4,6\}$ ).
interval A set containing all numbers between two given numbers (the endpoints) and one endpoint, both endpoints, or neither endpoint.
invalid An approach or example that is basically flawed and does not lead to the correct solution of the problem.

Example: Simplify the expression: $12+8 \div 2 \times 3^{2}$

An invalid approach would be to simplify the expression from left to right, disregarding the order of operations.

$$
\text { Invalid: } \quad 12+8 \div 2 \times 3^{2}
$$

$$
20 \div 2 \times 3^{2}
$$

$10 \times 3^{2}$
$30^{2}$
Incorrect answer: 900
Correct answer:
Valid: $12+8 \div 2 \times 3^{2}$
$12+8 \div 2 \times 9$
$12+4 \times 9$
$12+36$
48
inverse operation An operation that is the opposite of, or undoes, another operation; addition and subtraction are inverse operations; multiplication and division are inverse operations.
inverse property A property of real numbers that states that the result of two real numbers that when combined will result in the identity element; when a number is added to its additive inverse, the sum is always zero; (e.g., $8+-8=0$ ); when a number is multiplied by its multiplicative inverse, the product is always 1 . (See additive inverse and multiplicative inverse)
investigate (See explore)
irrational number A real number that cannot be represented as an exact ratio of two integers; the decimal form of the number never terminates and never repeats (e.g., $\pi$, $\sqrt{2}, \sqrt{10}, 0.010010001 \ldots$ ).
irregular polygon A polygon whose sides and angles are not all congruent.
irregular shape (See irregular polygon)
irrelevant information Extraneous information that has no bearing on the problem and cannot be used in its solution.

Example: A DVD player costs $\$ 339.50$. Bria has $\$ 550$ in her savings account. If she pays $\$ 35$ down and one monthly payment of $\$ 22.50$, how much more must she pay?

Relevant information: Cost: \$339.50
Down payment: \$35
Additional payment: \$22.50
Irrelevant information: Savings account balance: \$550.
isosceles trapezoid A trapezoid in which the two nonparallel sides are congruent.
isosceles triangle $A$ triangle with at least two congruent sides.
Example:

justify To provide an argument for a mathematical conjecture; it may be an intuitive argument or a set of examples that support the conjecture; the argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.
key A table for decoding or interpreting; a notation that explains something such as the value of each symbol or picture on a pictograph.
kilogram (kg) A metric unit to measure mass; 1 kilogram = 1000 grams.
kilometer (km) A metric unit of length roughly equivalent to $\frac{6}{10}$ of a mile;
1 kilometer = 1000 meters.
law of exponents for division The quotient of two numbers in exponential form with the same base is equal to that base with a power equal to the difference of the powers of each number; i.e., subtract their exponents: $\frac{a^{b}}{a^{c}}=a^{(b-c)}$.

Example: $\frac{a^{5}}{a^{3}}=a^{2}$
law of exponents for multiplication The product of two or more numbers in exponential form with the same base is equal to that base raised to the power equal to the sum of the powers of each number; i.e., add their exponents: $a^{b} a^{c}=a^{b+c}$.

Example: $a^{3} a^{4}=a^{7}$
least Smallest in quantity, size, or degree.
least common denominator (LCD) The smallest common multiple of two given denominators (e.g., the LCD of $\frac{1}{3}$ and $\frac{1}{8}$ is 24 ).
least common multiple (LCM) The smallest number, greater than zero, that is a multiple of two or more numbers (e.g., the LCM of 20 and 25 is 100 ; the LCM of $5 x^{2}$ and $3 x y$ is $15 x^{2} y$ ).
leg of a right triangle One of the two sides that form the right angle of a right triangle; the sides that are not the hypotenuse. (See hypotenuse)
length The distance from one end of an object to the other end.
less than (<) A relationship showing that the first term or expression has a value smaller than the second term or expression (e.g., $2<3$ or $-5<-1$ ).
like denominators A whole number, greater than zero, that is the denominator of two or more fractions. (See common denominator)

Example: The fractions $\frac{2}{7}$ and $\frac{3}{7}$ have like (the same) denominators.
like terms Terms that have the same literal part (e.g., in the expression $3 x^{2}+4 x+5+2 x+4 x^{3}+5 x^{2}, 3 x^{2}$ and $5 x^{2}$ are like terms, as are $4 x$ and $2 x$ ).
line An infinite set of points in opposite directions forming a straight path; it has only one dimension, length (e.g., $\overrightarrow{A B}$ is read "line $A B$ ").
linear equation An equation of the first degree that contains one or more variables; a linear equation with one or two variables yields a straight line when represented graphically (e.g., $3 x-1=-7$ or $x+2 y=12$ ).
linear inequality An inequality of the first degree that contains one or more variables (e.g., $3 x>2$ or $y<2 x-1$ ).
linear pair of angles A pair of adjacent angles formed by intersecting lines. Linear pairs of angles are supplementary.
line graph A graph that uses line segments to show changes in data; the data usually represents a quantity changing over time.

Example:

line segment The set of points on a line consisting of two fixed points (e.g., $P$ and $Q$ ) and all of the points between $P$ and $Q ; P$ and $Q$ are referred to as the endpoints of the segment.
line of symmetry A line that divides a figure into two congruent halves that are mirror images of each other; a simple test to determine if a figure has line symmetry is to fold the figure along the supposed line of symmetry and see if the two halves of the figure coincide.
liter (L) A metric unit used to measure capacity; 1 liter = 1000 milliliters.
logic The formal structure for reasoning.
logical According to the principles of logic.
logical reasoning The process of using a rational, systematic series of steps based on sound mathematical procedures to arrive at a conclusion; the drawing of conclusions from given facts and mathematical principles; often used as a problem solving strategy.
lowest terms The form of a fraction in which the numerator and denominator have no common factor except 1.
magnitude The numerical size or measure of an attribute (e.g., the area of a polygon, the volume of a solid).
major arc An arc of a circle that is longer than a semicircle.
manipulatives Physical objects that can be used to help solve mathematical problems (e.g., tangrams, base ten blocks, number cubes, cards, rulers, counters, pattern blocks, cubes).
map scale A key that provides an equivalence between a distance on a map and the associated real-world distance.
mass The amount of matter or substance in an object; commonly taken as a measure of the amount of material it contains and causes it to have weight in a gravitational field. [This should not be confused with weight, which is a measure of the force of gravity on an object. An apple weighs more on Jupiter than it does on Earth because Jupiter's gravity is stronger. However, the apple always has the same mass, no matter where it is.]
mathematical argument A chain of mathematical reasoning intended to convince the audience of the truth of a statement.
mathematical conjectures A reasoned guess stated in mathematical terms.
mathematical reasoning Applying mathematical techniques, concepts, and processes, either explicitly or implicitly.
mathematical relationship The connection between two quantities, properties, or concepts (e.g., the diameter of a circle is twice its radius).
mathematical sentence The expression of a complete thought or fact, it contains an equal sign ( $=$ ) or an inequality sign ( $>,<, \leq, \geq$, or $\neq$ ); a mathematical sentence is an open sentence if it contains an unknown quantity or a variable, and the value of that variable determines if the sentence is true or false; a mathematical sentence is a closed sentence if it has no variable or unknown and it can be determined to be conclusively true or false; a closed sentence is also known as a statement.
mathematical solution(s) The statement(s) or value(s) that correctly answers a mathematical question or completes an open mathematical sentence.
mathematical statement A mathematical sentence whose truth value can be determined to be either true or false.
mathematical symbol A character that is used to indicate a mathematical relation or operation; it has a precise mathematical meaning (e.g., $+, \div, \pm, \sqrt{ }, \neq$ ).
mean A measure of central tendency; the quotient obtained when the sum of the numbers in a set is divided by the number of addends.
means of a proportion The two middle terms in the ratios of a proportion (e.g., the means of the proportion $\frac{3}{4}=\frac{6}{8}$ are 4 and 6).
measure To find the dimensions or quantity (e.g., length, capacity) of an object or figure.

## Examples:

Find the perimeter of a regular hexagon whose side has a length of 6 cm .
Find the side of a square whose area is 81 square feet.
measure of angle The number of degrees or radians contained in the angle.
measure of central tendency A single number that represents a typical value for a set of numbers; the three most common measures of central tendency are the mean, median, and mode.
measurement The amount obtained by measuring.
median The middle number of a set of numbers arranged in increasing or decreasing order; if there is no middle number, the median is the average of the two middle numbers.

Examples: The median of the numbers $1,1,2,4,5,6$, and 7 is 4 .
The median of the numbers $1,1,2,4,5,6,7$, and 7 is 4.5 .
mental math Computations done by students "in their head" either in whole or in part.
method of proof A process used to prove or disprove a mathematical statement; it may contain direct statements with supporting reasons, examples, counterexamples or special cases.
meter A metric unit used to measure length; 1 meter $=100$ centimeters $=1000$ millimeters.
metric Of or using the metric system.
metric system A system of measurement based on the decimal system; the standard unit of length is a meter, of capacity is a liter, and of mass is a gram.
metric units Units used in the metric system:
length (meter): kilometer, hectometer, decameter, meter, decimeter, centimeter, millimeter
capacity (liter): kiloliter, hectoliter, decaliter, liter, deciliter, centiliter, milliliter
mass (gram): kilogram, hectogram, decagram, gram, decigram, centigram, milligram
mile A customary unit of length; 1 mile = 1760 yards $=5280$ feet.
milligram ( $\mathbf{m g}$ ) A metric unit used to measure mass; 1 milligram $=0.001$ gram.
milliliter (mL) A metric unit used to measure capacity; 1 milliliter $=0.001$ liter.
millimeter (mm) A metric unit used to measure length; 1 millimeter $=0.001$ meter. minor arc An arc of a circle that is less than a semicircle.
minuend In a subtraction problem, the number from which another number is to be subtracted.
minus A term that refers to subtraction or the symbol of subtraction.
minus sign (-) A mathematical symbol that means to subtract one amount from another amount.
minute $A$ unit used to measure time; 1 minute $=\frac{1}{60}$ of an hour.
minute hand The longer hand on an analog clock; it tells the minutes.
misleading graph A graph that leads the reader to make an incorrect conclusion or to form a false impression.
misleading statistics Statistics that lead the reader to make an incorrect conclusion or form a false impression.
missing value $A$ value omitted from an equation that is needed to make the equation true (e.g., $(2 \times 3) \times 5=2 \times(3 \times \square))$.
mixed number A number composed of an integer and a proper fraction (e.g., $3 \frac{2}{9}$ ). mode The number or members of a data set that occurs most frequently in the set.
model (noun) A visual representation that illustrates or further explains a mathematical principle or concept.
model (verb) To make or act out a representation of something, usually on a smaller scale or in a simpler way; to use pictures, diagrams, or physical objects to further demonstrate or clarify a problem.
money Something generally accepted as a medium of exchange, a measure of value, or a means of payment.
monomial A polynomial with one term; it is a number, a variable, or the product of a number (the coefficient) and one or more variables (e.g., $-\frac{1}{4}, x^{2}, 4 a^{2} b,-1.2, m^{2} n^{3} p^{4}$ ). month A unit used to measure time on a calendar; 12 months $=1$ year.
most Greatest in quantity, extent, or degree.
multiple The product of a given whole number and any other whole number.
multiple representations Various ways to present, interpret, communicate, and connect mathematical information and relationships.
multiplicand The number being multiplied in a multiplication problem.
multiplication A mathematical operation of combining groups of equal amounts; repeated addition; the inverse of division.
multiplicative inverse The reciprocal of a number; the number, which when multiplied by a given number, produces the multiplicative identity 1 ; in the set of real numbers the number a given number needs to be multiplied by to yield 1.; $n \times n$-inverse $=1$ for all $n$.

$$
\text { Examples: } \quad 7 \times \frac{1}{7}=1 \text { and } \frac{2}{3} \times \frac{3}{2}=1
$$

multiplier The number that one is multiplying by in a multiplication problem.
multiply (See multiplication)
multi-step equation or multi-step inequality An equation or inequality that requires two or more steps to solve.
natural numbers The set of counting numbers $\{1,2,3,4, \ldots\}$.
negative number A number that is less than zero; located to the left of zero on the number line.
nickel A coin with a value of 5 cents or $\frac{1}{20}$ of a dollar.
nonadjacent side of an angle in a triangle The side of the triangle that does not make up either side of the angle; the side opposite the specified angle; usually used in trigonometric ratios of the acute angles of a right triangle.
nonagon A polygon with nine sides and nine angles.
noncollinear Not lying on the same straight line.
nonlinear equation or inequality An equation or inequality that, when represented graphically, results in a graph that is not a straight line (e.g., circle, parabola, hyperbola).
nonrepeating decimal A decimal that does not repeat; it either terminates or continues in no discernible pattern.
nonstandard measurement The use of items as measurement tools that are not uniform in size (e.g., using fingers to measure something; one person's fingers are not necessarily the same size as another person's fingers).
nonstandard unit Any tangible item that can be used to measure something (e.g., paper clips, crayons).
nonterminating decimal A decimal that does not terminate; it either repeats or continues in no discernible pattern.
number The concept of an amount, quantity, or how many items there are in a collection.
number line A line on which each point represents a real number.
Example:

number sense Having a good conceptual understanding of numbers and number concepts.
number sentence A mathematical statement that has numbers, at least one operation sign, and an equal or inequality sign.
number system A system used to represent numbers. (See base ten number system)
numeral The written symbol that represents a number.
numeration The act or process of counting or numbering; a system of counting or numbering.
numerator The top number in a fraction; it tells the number of equal parts (numerator) out of the total number of parts (denominator) being described by the fraction.
numeric expression Any combination of words, variables, constants, and/or operators that result in a number; also known as an arithmetic expression.
numeric pattern An arrangement of numbers that repeat or that follow a specified rule.
numerically Expressed in or involving numbers or a number system.
obtuse angle An angle whose measure is greater than $90^{\circ}$ and less than $180^{\circ}$.
Example:

obtuse triangle A triangle containing one obtuse angle.
octagon A polygon with 8 sides and 8 angles.
Examples:

odd number An integer that when divided by 2 has a remainder of $\pm 1$; an integer that has $1,3,5,7$, or 9 in the ones place.
one-digit number A number consisting of just one digit; $0,1,2,3,4,5,6,7,8$, or 9.
ones place The place value located one place to the left of the decimal point in a number; it shows how many ones are in a number.
one-step equation or inequality An equation or inequality that can be solved in one step.
open figure A figure that is not closed; i.e., it does not start and end at the same point.
open sentence A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown (e.g., $3+n=5$ becomes true when $n=2$ ).
operations Procedures used to combine numbers, expressions, or polynomials into a single result (e.g., addition, subtraction, multiplication, division, exponents).
opposite The result of taking a number and changing its sign (e.g., the opposite of 5 is -5 , the opposite of -12 is $-(-12)$ or 12 ; a number and its opposite are equidistant from zero in a number line, but on opposite sides of zero. (See additive inverse)
order To place numbers or objects in a sequential arrangement (e.g., least to greatest or heaviest to lightest).
order of operations A specified sequence in which mathematical operations are expected to be performed; an arithmetic expression is evaluated by following these ordered steps: (1) simplify within grouping symbols such as parentheses or brackets, starting with the innermost; (2) apply exponents - powers and roots; (3) perform all multiplications and divisions in order from left to right; (4) perform all additions and subtractions in order from left to right.
ordered pair A set of two numbers named in an order that matters; represented by $(x, y)$ such that the first number, $x$, represents the $x$-coordinate and the second number, $y$, represents the $y$-coordinate when the ordered pair is graphed on the coordinate plane; each point on the coordinate plane has a unique ordered pair associated with it.
ordinal numbers Numbers used to specify position in a sequence (e.g., first, second, third, fourth).
origin The point on the coordinate plane where the $x$ - and $y$-axes intersect; has coordinates $(0,0)$.
ounce (oz) A customary unit used to measure mass; 1 ounce $=\frac{1}{16}$ pound; 16 ounces $=1$ pound .
outcome One of the possible events in a probability experiment (e.g., when tossing a fair coin there are two possible outcomes, heads or tails).
parallel lines Lines in the same plane that never intersect no matter how far they are extended; they are equidistant from each other.
parallelogram A quadrilateral with two pairs of parallel sides.

## Example:


$\overrightarrow{A B} \| \overrightarrow{C D}$
$\overrightarrow{A C} \| \overrightarrow{B D}$
pattern A design (geometric) or sequence (numeric or algebraic) that is predictable because some aspect of it repeats.

Examples: Geometric pattern:


Numeric pattern: $4,7,10,13, \ldots$
Algebraic pattern: $x, x^{2}, x^{3}, \ldots$
penny A coin with a value of one cent or $\frac{1}{100}$ of a dollar.
pentagon A polygon with 5 sides and 5 angles.

## Examples:



percent A number expressed in relation to 100; represented by the symbol \% (e.g., 40 parts out of 100 is $40 \%$ ).
percent decrease The magnitude of decrease expressed as a percent of the original quantity.
percent increase The magnitude of increase as a percent of the original quantity.
perfect square $A$ whole number resulting from multiplying an integer by itself; $a$ is a perfect square if $a=n \times n$ and $n$ is an integer (e.g., $16=4 \times 4$ and $121=(-11) \times(-11)$ ).
perimeter The distance around a closed figure.
perpendicular Two lines, segments, or rays that intersect to form right angles.
Example:

perpendicular bisector A line, segment, or ray that is perpendicular to and bisects a line segment.
personal reference Something that a person can refer to as a standard, for the purpose of comparison (e.g., knowing the width of your pinky finger is approximately 1 cm ).

Examples: The height of the room is about twice as tall as a student. The length of a fifth grader's arm is about 2 feet.
phenomena Something that is observable.
mathematical phenomena Problems related purely to mathematics.
Example: (negative number) $\times$ (negative number) $=$ positive number physical phenomena Problems in the physical world that involve math.

Example: Acceleration due to gravity.
social phenomena Problems in the real world relating mathematics and social concepts or events.

Example: Sharing 6 cookies among 3 friends.
physical model A representation of something using objects.
pi ( $\partial$ ) The ratio of the circumference of a circle to its diameter; pi is an irrational number whose approximate value is 3.141592654 .
pictograph A graph that uses pictures or symbols to represent data; an accompanying key indicates the value associated with each picture or symbol.

Example: $\quad$ Number of Cars Mr. Betz
Sold During One Week

|  | Pictograph |
| :---: | :---: |
| Mon | - a a |
| Tues | - a a |
| Wed | -a |
| Thurs | - a a Pa |
| Fri | -a |
| Key: | $a=5 \mathrm{cars}$ |

pint (pt) A customary unit used to measure capacity; 2 cups $=1$ pint; 2 pints $=1$ quart. place value The value of a digit in a number based on its position (e.g., in the number 28 , the 2 is in the tens place and the 8 is in the ones place).
plane A set of points forming a flat surface that extends without end in all directions.
plane figure A figure that lies on a flat surface; it has length, width, perimeter, and area.
plot To locate a point on a coordinate plane.
plus $A$ term that refers to addition or the symbol for addition.
plus sign The symbol (+) used to indicate addition.
point An exact location in space; a point has no dimension.
poll The results of a question or questions answered by a group of people.
polygon A closed plane figure formed by three or more line segments.
Examples:

polyhedron A three-dimensional figure that is bounded by four or more polygonal faces.
polynomial A monomial or the sum of two or more monomials whose exponents are positive.

Example: $5 a^{2}+b a-3$
population A group of people, objects, or events that fit a particular description; in statistics, the set from which a sample of data is selected.
positive number Any number greater than zero or to the right of zero on the number line.
post meridiem (p.m.) Afternoon; the times from 12 noon until 12 midnight; 12 noon is 12 p.m.
pound (lb) A customary unit used to measure mass; 1 pound $=16$ ounces.
power An exponent (e.g., in the expression $3^{8}, 8$ is the power and 3 is the base).
precise Exact in measuring; accurate.
precision A property of measurement that is related to the unit of measure used; the smaller the unit of measure used, the more precise the measurement is.

Example: 19 mm is more precise than 2 cm .
predict To be able to determine the next step or value (to make an educated guess), based on evidence or a pattern.
prediction An educated guess about an outcome.
pre-image In transformational geometry, the figure before a transformation is applied.
Example: If $R_{l}(A)=A^{\prime}, A$ is the pre-image of $A^{\prime}$.
preserved In transformational geometry, a property that is kept or maintained.
Example: In a translation the shape and size (property of congruence) is preserved.
prime factorization A method of writing a composite number as a product of its prime factors (e.g., $12=2 \times 2 \times 3=2^{2} \times 3$ ).
prime number A number greater than 1 that has exactly two different factors, 1 and itself.

## Examples:

| Prime Numbers |  | Non -Prime Numbers |  |
| :---: | :---: | :---: | :--- |
| Number | Factors | Number | Factors |
| 2 | 1,2 | 6 | $1,2,3,6$ |
| 7 | 1,7 | 8 | $1,2,4,8$ |
| 11 | 1,11 | 15 | $1,3,5,15$ |
| 17 | 1,17 | 25 | $1,5,25$ |

prism A three-dimensional figure (solid) that has two congruent and parallel faces that are polygons; these are the bases; the remaining faces are parallelograms.

Examples:

probability The chance of an event occurring; the ratio of the number of favorable outcomes to the total number of possible outcomes; the probability of an event must be greater than or equal to 0 and less than or equal to 1.

Example: $P($ rolling a 3$)=\frac{\text { the number of } 3 \text { 's on the faces }}{\text { the total number of faces }}=\frac{1}{6}$
problem solving strategies Various methods used to solve word problems; strategies may include, but are not limited to: acting it out, drawing a picture or graph, using logical reasoning, looking for a pattern, using a process of elimination, creating an organized chart or list, solving a simpler but related problem, using trial and error (guess and check), working backwards, writing an equation.
product The number that is obtained when two or more factors are multiplied.
profit The amount of money left after expenses have been subtracted from income.
proof A valid argument, expressed in written form, justified by axioms, definitions, and theorems.
proper fraction $A$ fraction whose numerator is less than its denominator.
properties Characteristics of a shape or object (e.g., size, shape, number of faces, or ability to be stacked or rolled).
properties of real numbers Rules that apply to the operations with real numbers.

## Examples:

Commutative Property

$$
\begin{array}{ll}
a+b=b+a & a b=b a \\
a+(b+c)=(a+b)+c & a(b c)=(a b) c \\
a(b+c)=a b+a c & \\
a+0=a & a \times 1=a \\
a+(-a)=0 & a \cdot 1=1 \\
& a \cdot 0=0
\end{array}
$$

Associative Property
Distributive Property Identity
Inverse
Zero Property
proportion An equation which states that two ratios are equivalent (e.g., $\frac{5}{10}=\frac{1}{2}$ or $5: 10=1: 2)$.
proportional reasoning Using the concept of proportions when analyzing and solving a mathematical situation.

Example: If triangle $A B C$ is similar to triangle $X Y Z$ and $A B=15$ when side $X Y=75$, find $B C$ when $Y Z=150$.


$$
\frac{15}{75}=\frac{a}{150}
$$

$$
a=30
$$

proportionality The quality, character, or fact of being proportional.
protractor An instrument used to find the degree measure of an angle.
pyramid A polyhedron whose base is a polygon and whose lateral faces are triangles that share a common vertex.

Example:


Pythagorean theorem The mathematical relationship stating that in any right triangle the sum of the squares of the two legs is equal to the square of the hypotenuse; if a and $b$ are the lengths of the legs and $c$ is the length of the hypotenuse, then $a^{2}+b^{2}=c^{2}$.
quadrant One of four sections of a coordinate grid separated by horizontal and vertical axes; they are numbered I, II, III, and IV, counterclockwise from the upper right.

quadratic Of the second degree.
quadratic equation A polynomial equation of degree 2; its general form is $a x^{2}+b x+c=0$, where $x$ is the variable and $a, b$, and $c$ are constants.
quadrilateral A polygon with 4 sides and 4 angles.
Examples:

square

rectangle

parallelogram trapezoid


irregular quadrilateral

irregular quadrilateral
quart (qt) A customary unit to measure capacity; 1 quart $=2$ pints.
quarter A coin with a value of 25 cents or $\frac{1}{4}$ of a dollar.
quotient The answer to the division of two numbers.
radii The plural form of radius.
radius $A$ line segment that extends from the center of a circle to any point on the circle.
Example: $\overline{O A}$ is a radius of circle $O$.

random sample A sample obtained by a selection from a population, in which element of the population has an equal chance of being selected.
range of a data set The difference between the greatest and the least values in a set of numbers.

Example: Given the data: 2, 7, 3, 14, -1, 6, 34, -3
The range is $34-(-3)=37$
range of a function The set of output values of a function.
rate A ratio that compares quantities of different units (e.g., miles per hour, price per pound, students per class, heartbeats per minute).
rate of change The amount the function's output increases or decreases for each unit of change in the input.
rate of interest The percent charged or paid for the use of money.
Example: $\$ 1000$ was borrowed at an interest rate of $8.5 \%$ per year for a period of 18 months. The rate of interest is $8.5 \%$.
ratio A comparison of two numbers or two like quantities by division (e.g., the ratio of girls to boys is two to three, $\frac{2}{3}$, 2 to 3 , or $2: 3$ )
rationale An explanation using logical reasoning, mathematical principles, or arguments.
rational number Any number that can be expressed as a ratio in the form $\frac{a}{b}$ where $a$ and $b$ are integers and $b \neq 0$.
raw data Collected information before the application of statistical procedures.
ray Part of a line that has one endpoint and extends infinitely in one direction.
real numbers The set of numbers that includes all rational and irrational numbers.

reasonable estimate An approximation of the result of a given problem or calculation using rational, logical procedures (e.g., rounding).
reasonableness The justification that a particular solution to a problem is within logical estimates.
reasoning Engaging in a process that leads to a conclusion or inference using known facts or assumptions.
reciprocal The number that is used to multiply a given number to obtain an answer of 1 (e.g., $\frac{2}{3} \times \frac{3}{2}=1$, therefore $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$ ). (See multiplicative inverse)
rectangle A quadrilateral with four right angles.

## Examples:


rectangular prism A prism whose six faces are rectangles.
Example:

reflection In transformational geometry, the figure formed by flipping a geometric figure (pre-image) over a line to obtain a mirror image; informally known as a "flip." (The line is the perpendicular bisector of each line segment connecting a point and its image.)
regroup (regrouping) A process used when subtracting numbers that contain two or more digits and where one of the digits in the subtrahend is greater than the corresponding digit in the minuend; a "trading process" that utilizes the equivalents of 1 hundred for 10 tens or 1 ten for 10 ones, etc.
regular polygon A polygon in which all sides and all angles are congruent.
Example:

regular shape (See regular polygon)
related facts (See fact family)
relative error The ratio of the absolute error in a measurement to the size of the measurement; often written as a percent and called the percent of error; the absolute error is the difference between an approximate number and the true number that it approximates.
relevant information Information applicable to the problem; information necessary for the solution of a problem; data that is pertinent, applicable, and essential in the solution of a problem.
remainder The amount left over when one number or polynomial is divided by another number or polynomial; if the remainder is 0 , it is usually said that there is no remainder.

Example:


4 is the remainder.
rename To express a number in a different form using regrouping or trading; used in performing arithmetic algorithms.
repeated addition Addition of equal groups; often used to model the concept of multiplication.
repeated subtraction Subtraction of equal groups from a number; a model or alternative algorithm for division.
repeating decimal A decimal in which one or more digits repeat infinitely (e.g., $0.3333 \ldots$ or $0 . \overline{3}, 5.272727 \ldots$, or $5 . \overline{27}$ ).
rhombus A parallelogram with two adjacent sides congruent (all four sides are congruent).

Example:

right angle An angle formed by two perpendicular lines, the measure of which is $90^{\circ}$.
Example:

right triangle A triangle with one right angle.

## Examples:


rotation A transformation, or movement, that results when a geometric figure is rotated about a fixed point; informally known as a "turn."
rotational symmetry A figure has rotational symmetry when it can be rotated around a central point, or point of rotation less than $360^{\circ}$ and still be identical to the original figure.
round a number To approximate the value of a whole number or decimal to a specific place value.

Example: Rounded to the nearest ten: Rounded to the nearest tenth:
125 rounds to 130
1.25 rounds to 1.3 (not 1.30 )

122 rounds to 120
1.22 rounds to 1.2 (not 1.20)
rule/function rule The horizontal equation that identifies the relationship between two variables or other valid rule including a verbal description which can be translated into an equation; an established standard pattern or behavior.
ruler A tool used to measure length.

sale price The price of a product after the discount has been subtracted from the original price.

Example:

| original | - | discount | $=$ | sale price |
| :---: | :---: | :---: | :---: | :---: |
| $\$ 24$ | - | $\$ 4$ | $=$ | $\$ 20$ |

sales The amount of money generated when goods are sold.
sample A representative part or a single item from a larger whole or group; a finite part of a statistical population whose properties are studied to gain information about the whole.
sample space A list of all possible outcomes in a given situation.
Example: The sample space for tossing two coins is:
$(H, H),(H, T),(T, H),(T, T)$.
sampling Selecting a small group which would be representative of the entire population; used in taking a survey.
scale (1) The ratio of the size of an object in a representation (drawing) of the object to the actual size of the object; the ratio of the distance on a map to the actual distance (e.g., the scale on a map is 1 inch:10 miles); (2) an instrument used to measure an object's mass.
scale drawing A proportionally correct drawing (enlargement or reduction) of an object or area.
scalene triangle A triangle with no congruent sides.
Example:

scientific notation A form of writing a number as the product of a power of 10 and a decimal number greater than or equal to 1 and less than 10 (e.g., $2,400,000=2.4 \times 10^{6}$, $240.2=2.402 \times 10^{2}, 0.0024=2.4 \times 10^{-3}$ ).
second $A$ unit to measure time; 1 second $=\frac{1}{60}$ of a minute.
sector of a circle The region of the circle formed by two radii and their intercepted arc.
Example: The shaded area in the circle below is a sector of circle $O$.

set $A$ well-defined collection of items.
sequence $A$ set of numbers arranged in a special order or pattern.
shape (See geometric shape)
side A line segment joining two adjacent vertices of a polygon.
Example: $\overline{A B}$ is a side of $\triangle A B C$.

similar triangles Triangles that have the same shape but not necessarily the same size; corresponding sides are in proportion and corresponding angles are congruent.

similar figures Figures that have the same shape but not necessarily the same size. (See similar triangles)
simple interest The amount obtained by multiplying the principal by the rate by the time; I = prt.

Example: Sue invested $\$ 100$ at a simple interest rate of $4 \%$. Find the interest she will earn in 3 years.

Given: $p=\$ 100, r=0.04, t=3$
Solution: $l=p r t$
$I=(\$ 100) \times(0.04) \times(3)$
I = \$12
simplify fractions To rename fractions to lowest terms by dividing the numerator and denominator by the greatest common factor of the numerator and denominator.
single-event experiment A probability experiment in which only one event can occur each time the experiment is performed (e.g., a number cube is rolled once, a coin is tossed once).
skip count To count by a given number (e.g., skip count by 2's: $2,4,6,8,10, \ldots$ ).
slide (See translation)
slope The measure of the steepness of a line; the ratio of vertical change to horizontal change.

Example: If point P is $\left(x_{1}, y_{1}\right)$ and point Q is $\left(x_{2}, y_{2}\right)$ the slope of $\overline{P Q}$ is

$$
\frac{\Delta y}{\Delta x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} .
$$

slope-intercept form The equation of a straight line in the form $y=m x+b$, where $m$ is the slope and $b$ is the $y$-coordinate of the point where the line intercepts the $y$-axis.
solid figure A three-dimensional geometric figure that has length, width, and height.
solution The value or values that make an equation, inequality, or open sentence true. solution set The set of values that make an equation or statement true.
solve To find the answer to an equation or a problem.
sort To separate objects into groups according to properties or characteristics.
spatial reasoning Drawing inferences or conclusions by using visual images.
sphere A three-dimensional figure with a set of points in space that are equidistant from a fixed point called the center.
square A rectangle with two adjacent sides congruent (all four sides will be congruent).
square a number To multiply a number by itself (e.g., $4 \times 4=16$ or $4^{2}=16$ ).
square root of a number A number (factor) that when multiplied by itself yields the original number (e.g., $\sqrt{16}=4$ or $\sqrt{16}=-4$ ).
standard form of a number A number is written in standard form when each digit is in a place value (e.g., twenty-nine thousand three hundred four is written as 29,304).
statistics The collection, organization, presentation, and analysis of data.
straight angle An angle that has a measure of $180^{\circ}$; an angle formed by two rays in opposite directions from their common endpoint.
straightedge A tool used to make a straight line; it can be thought of as a ruler without measurement marks.
strategy A method or system of steps used to solve problems. (See problem solving strategies)
subset $A$ set consisting of elements from a given set (e.g., if $B=\{1,2,3,4,5,6,7\}$ and $A=\{1,2,3\}$, then $A$ is a subset of $B$, written $A \subseteq B$ ).
substitute To replace variables in a given expression or equation with designated values in order to evaluate the expression or equation.

Example: To calculate the surface area of a rectangular prism with a width of 3 cm , length of 5 cm and a height of 4 cm , evaluate $S A=2 w l+2 w h+2 h l$ by substituting.

$$
\begin{aligned}
& S A=2 \mathrm{wl}+2 \mathrm{wh}+2 \mathrm{hl} \\
& S A=2(3)(5)+2(3)(4)+2(4)(5) \\
& S A=30+24+40 \\
& S A=94 \mathrm{~cm}^{2}
\end{aligned}
$$

subtraction A mathematical operation that finds the difference between two quantities or how much more one quantity is than a second quantity.
subtraction fact Number fact with minuends to 18 and single-digit subtrahends.
subtraction sentence An equation showing the difference of two numbers (e.g., $10-7=3$ ).
subtraction sign A symbol ( - ) that is read as "minus" or "take away" to represent subtraction.
subtrahend In subtraction, the number being subtracted from a given number.
sum The result when two or more quantities are added.
supplementary angles Two angles whose measures sum to $180^{\circ}$.
surface area The sum of the areas of the faces or curved surface of a threedimensional object.
survey To ask either written or verbal questions for the purpose of acquiring information/data.
symbol A notation used to represent an operation or abstract idea (e.g., +, -, >, $\infty$, or $\partial$ ).
symmetry The property of having the same size and shape across a dividing line or around a point.
system of equations A set of equations that may share a common solution or common solution(s).
system of inequalities A set of inequalities that may share a common solution or common solution(s).
table A systematic or orderly list of values, usually in rows and columns.
table of values An organized list of values from a function/relation.
tally mark A mark used to keep track of data being counted.
tangent A line segment, line, or ray that intersects a circle in only one point.
technical writing A genre of writing whose purpose is to describe, explain, or defend a mathematical idea, reasoning or process; the term "technical" refers to knowledge that is not widespread, that is more the territory of experts and specialists and whenever one attempts to write or say anything about a particular field of study, one is engaged in technical writing; technical writing is the delivery of technical information to readers (or listeners or viewers) in a manner that is adapted to their needs, level of understanding, and background; this ability to "translate" technical information to non-specialists is a key skill to any technical communicator; it requires the author to use specific mathematical language and typically the writing is precise and short; technical writing would be considered the opposite of narrative writing.
technology Tools such as calculator, computer, or personal data assistant (PDA) used to help represent/solve a problem.
temperature The extent of warmth or coldness of something; a thermometer is used to measure temperature in Fahrenheit or Celsius.
tens place A place value located two places to the left of the decimal point; a digit in the tens place has a value of 10 times the value of the digit.
tenths place The place value located one place to the right of the decimal point; one out of ten equal parts of a whole.
term The addends of an algebraic expression.
terminating decimal A decimal whose digits do not repeat; all terminating decimals are rational numbers (e.g., 0.7355).
thousands place The place value located four places to the left of the decimal point in a number; a digit in the thousands place has a value of 1000 times the value of the digit.
three-digit number A whole number greater than 99 and less than 1000.
three-dimensional figure An object that has length, width, and height; also called a solid figure (e.g., prism, pyramid, cylinder, cone).
time A system of measuring duration or a specific portion of duration (e.g., year, season, day, hour, minute, second).
ton $A$ unit of weight equal to 2,000 pounds.
tool Any instrument used to solve a problem, it does not necessarily involve technology.
transformation The result of a change made to an object; a collective term referring to translations, rotations, reflections, glide reflections, and dilations.
translation A transformation where every point moves the same distance in one or two directions within the plane; informally known as a "slide."
transversal A line that intersects two or more other lines.

Example: In the figure below, line $t$ is a transversal to lines / and $m$.

trapezoid A quadrilateral with exactly one pair of parallel sides.
Example: In the trapezoid below, $\overline{A B} \| \overline{C D}$.

trial and error (guess and check) A problem solving strategy whereby a reasonable estimate for an answer is made and checked in the problem. If the solution is not reached, the estimate is adjusted and checked again in the problem. This process continues until the correct answer is found.

Example: An archer shot 8 arrows at the target shown below. If each arrow hit the target and the archer's total score was 175, where did the arrows land?


By the use of trial and error, several combinations may be tried until the correct solution is found.
triangle A polygon with three sides and three angles.

## Examples:


trinomial A polynomial with exactly three terms.
turn (See rotation)
two-digit number A whole number greater than 9 and less than 100.
two-dimensional figure A figure that has length and width but no height (e.g., circle, square, triangle).
two-step algebraic equation An algebraic equation that requires two different steps such as multiplication/division and addition/subtraction to solve.
unit fraction $A$ fraction with a numerator of 1.
unit price The price of one item or one unit (e.g., \$0.15 per pound).
unit of capacity $A$ unit used to measure the amount that can be contained in an object (usually liquid); customary units include cup, pint, quart, and gallon; metric units include liter, kiloliter, and milliliter.
unlike denominators Two or more fractions with unequal denominators (e.g., $\frac{6}{17}$ and $\frac{3}{7}$ ).
unlikely In probability, an event that has a very small chance of occurring.
valid Based on proper procedures, a valid approach will lead to the correct solution of a problem.
value of a digit How much a digit is worth according to its place in a number.
variable A symbol used to represent a number or group of numbers in an expression or an equation.

Venn diagram A drawing showing relationships among sets.
Example: The Venn diagram below shows the students who play basketball, who run track, and who play basketball and run track.

verbal expression A phrase stating a relationship; can be translated into a mathematical/algebraic expression (e.g., twice a number can be represented as $2 n$ ).
verbal form A mathematical expression or relationship using words rather than symbols.
verbally Expressed in written form or spoken in words.
verify To ascertain or confirm that a mathematical property, concept, or statement is true.
vertex (1)The common endpoint of two sides of a polygon; (2) the common endpoint of two rays that form an angle; (3) the common point where two or more edges of a threedimensional solid meet.
vertical Straight up and down; perpendicular to the horizon.
vertical angles The non-adjacent angles formed by the intersection of two lines.
vertical line test $A$ test use to determine if a relation is a function; a relation is a function if there are no vertical lines that intersect the graph at more than one point.
vertices The plural form of vertex.
visual Any graphic representation such as a chart, picture, diagram, graph, etc.
volume The number of cubic units needed to fill a solid figure.
week $A$ unit used to measure time; 1 week $=7$ days.
whole numbers The set of counting numbers plus zero; $\{0,1,2,3, \ldots\}$.
whole unit $A$ whole standard quantity or amount (e.g., inch).
width One dimension of a two- or three-dimensional figure.
written symbol $A$ sign used to represent something such as an operation (,,$+- \times, \div$ ), a relationship ( $>,<, \geq, \leq$, or $\neq$ ), or a special quantity ( $\partial$ ).
$x$-axis The horizontal axis; the line whose equation is $y=0$.
$x$-intercept The point when a graph of an equation crosses the $x$-axis.
$\boldsymbol{y}$-axis The vertical axis; the line whose equation is $x=0$.
yard (yd) A customary unit used to measure length; 1 yard $=3$ feet.
year The time it takes the Earth to make a complete revolution around the sun; since it takes $365 \frac{1}{4}$ days, most years have 365 days, with an extra day added every four years (this year is referred to as a leap year).
$\boldsymbol{y}$-intercept The point when a graph of an equation crosses the $y$-axis.
zero The number which indicates no quantity, size, or magnitude; zero is neither negative nor positive; zero is the additive identity.
zero property of addition The property that states that the sum of a number and zero is that same number (i.e., $a+0=a$ for all a).
zero property of multiplication The property that states that the product of any number and zero is always zero (i.e., $a \times 0=0$ for all $a$ ).

