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Leonardo da Vinci: Architecture and Mathematics

Sylvie Duvernoy, Guest Editor

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KIM WILLIAMS BOOKS

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CONTENTS

An Introduction to Leonardo's Lattices

Among the architectural and mathematical treatises that flourished during the Renaissance period, Leonardo's codices deserve special attention. They are not didactic treatises, arranged in several books that must be read from the first page to the last, but information about the scientific research in the Renaissance flows from their pages, full of sketches and notes as from an endless font. The reader always bumps into something new or unexpected when going through the drawings, whichever codex or whatever page he is exploring.

The sketches that can be seen on folio 899 of the *Codex Atlanticus* illustrate the design of a roof system assembled from simple elements, and describe the building process of this system based on the weaving of wooden logs that will generate a vaulted roof covering a wide space without intermediate supports (fig. 1). These drawings are quite unique in the pages of Leonardo and no repetition has been noticed in other codices, but they are not unique in the vast amount of architectural literature of Middle Ages and Renaissance.

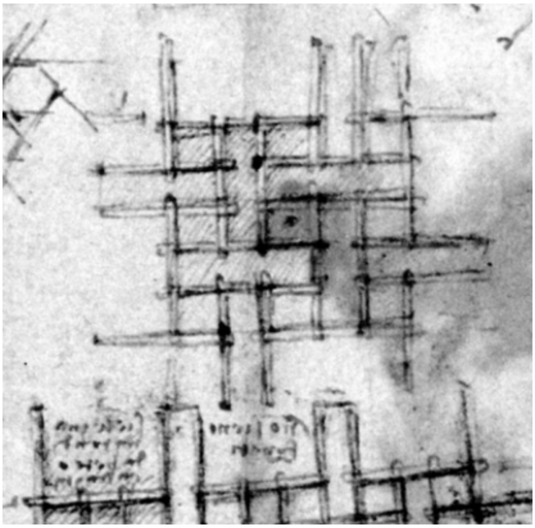

Fig. 1. Detail from Leonardo's *Codex Atlanticus*, folio 899

One of the drawings of Villard de Honnecourt's portfolio illustrates a building trick that can be considered to be an anticipation of Leonardo's research. The sketch by Villard shows a wooden floor built from beams that are all shorter than the dimensions of the room itself. The caption says: "in such a way you can work in a tower or a house with pieces of wood that are too short" (fig. 2). The beams are tied together according to a geometric pattern that makes it possible to cover of a span wider than the length of the beams themselves. We have no information whether Villard invented this trick together with some fellow builders, or if he inherited it from previous "know how", but the drawing is intended to transmit this technique to posterity and in fact it appears again, in drawing and in written description, in some treatises of the Renaissance.

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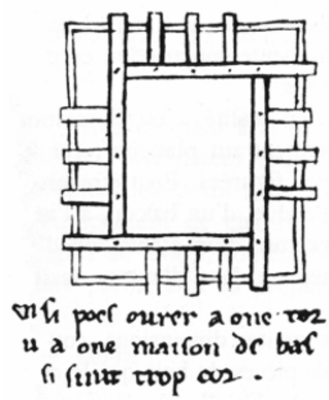


Fig. 2. How to build a floor with beams that are too short. From Villard de Honnecourt's portfolio (1225-1250)

In Book I of the *Seven Books on Architecture* by Sebastiano Serlio we can see a drawing that shows an application of the same technique: the system has been repeated and extended to build an even wider floor (fig. 3). Book I: *De Geometria* was first published in Paris in 1545, twenty-six years after Leonardo's death. Serlio's drawing looks like a combination of ancient problems and new solutions, but no explanation is given as far as design and science are concerned. Serlio only claims to present some possible solutions to inconveniences that often arise in the course of the architect's professional carreer. It is interesting to notice, however, that this practical trick of the trade appears in the more theoretical of the *Seven Books*, which deals with geometry.

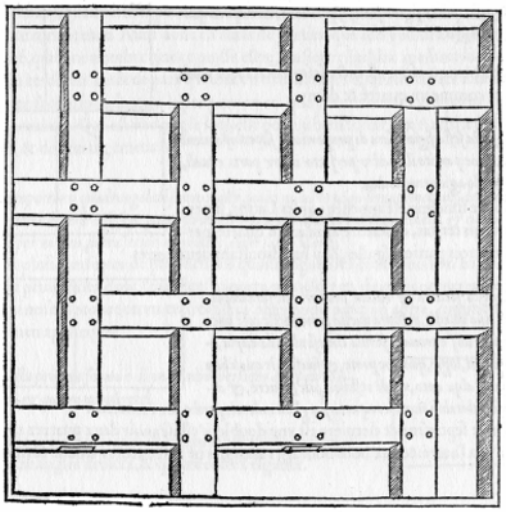


Fig. 3. How to build a floor of fifteen feet with beams that are one *braccio* short. From the *Seven Books on Architecture* by Sebastiano Serlio, Bk I: *De Geometria* (1545)

In the captions related to the drawings of folio 899 of the *Codex Atlanticus*, Leonardo does not claim to have discovered the construction technique that he illustrates. But while Villard and Serlio only deal with one single geometric pattern based on squares, Leonardo's investigates four different geometric patterns, not all of them based on orthogonality. In addition, since his beams are not abutted and nailed together they do not produce an horizontal surface, but rather a vaulted one. Because his notes are so specific in describing quantities and areas together with the details of the building process, it seems, however, quite certain that he actually took part in – or even directed – some building experiment of this kind.

The papers that are presented in this issue of the *Nexus Network Journal* are the result of a workshop that took place in Vinci, birthplace of Leonardo, in June 2003, sponsored by the Leonardo Museum and Library of Vinci and Kim Williams Books, dedicated to the study of "Leonardo's lattices".

The workshop was composed of two parts: first, theoretical studies, and then the experimental phase. For the first part of the project, the seminar, a team of experts in various fields and from various nationalities discussed the use of geometry in the architecture of Leonardo as found in his sketchbooks. Presentations were made by Biagio di Carlo, Sylvie Duvernoy, Christopher Glass, Vesna Petresin, Mark Reynolds, Rinus Roelofs and João Pedro Xavier. The papers in this present issue all grew out of the research for and exchange of ideas during the workshop.

The second part of the project involved the actual construction of domes based upon Leonardo's system. During this second phase, which acted as the verification process of the theoretical research, our group decided to build four vaults following the instructions and drawings of Leonardo. It was important for us to take the discussion from the realm of theory into the realm of practice, since the construction of the dome allowed the theory to be tested. The construction was directed by Dutch artist Rinus Roelofs, who has worked with Leonardo's system of bar grids since 1989.

Leonardo himself gives some starting instructions:

> *Sien legnami tondi, d'abete o castagni. Non sieno forati*
>
> (Let them be round logs, of fir or chessnut. Let them not be drilled with holes).

Our beams where four meters long, and in order to arrange them very regularly we first made four notches in each, to mark the precise position where they had to intersect with each other. The notches also helped prevent the beams from sliding, since they were not fixed to one another by either nails or ropes, but were only "woven". For our first experiment we chose the orthogonal pattern based on the composition of square and rectangular shapes (fig. 4). This pattern may adequately cover a square space if regularly expanded from the center in both directions, or it can be developed along a single direction to cover a rectangular space, or even form the structure for a kind of bridge.

Fig. 4. First dome completed

Unlike masonry vaults or cupolas, which are usually built from the exterior towards the interior, that is, starting from the supporting walls and proceeding towards the center of the space, this kind of woven wooden structure starts from the center and expands outwards, the vaulted form rising in proportion to the width or diameter of the covered space and the thickness of the individual elements: the thicker the beams are, the higher the dome will rise.

The second vault that we built was the one based on a geometrical pattern in which hexagons and equilateral triangles alternate. We later tried another kind of orthogonal pattern composed only of squares of two different sizes (figs. 5 and 6).

Fig. 5. Second dome completed

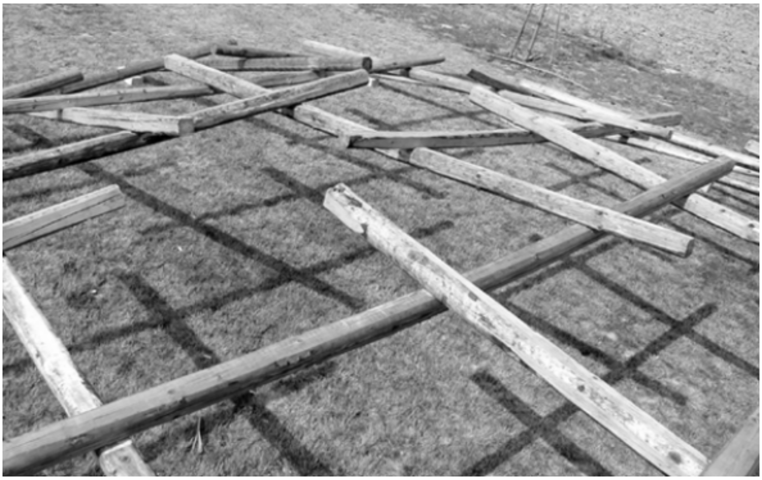


Fig. 6. Third dome completed

The rapidity of the building – and unbuilding – of the vaults allowed us to construct four of them in two days. The beams can be lifted and assembled by three or four men at most, no machinery being necessary, but the more the vault expands, the heavier it becomes and so the more difficult it is to lift and insert more beams at the edges to continue to enlarge the structure. The ultimate limit to the width of the vault is proportional to the strength and maximum resistance of the beams that touch the ground. Leonardo suggests doubling them in order to increase the stability of the structure:

> *ma con certezza si romperà li più deboli, li quali son li più carichi, e quelli che son piè carichi son quelli che toccan terra, che sostengano il tutto, li quali fien raddoppiati, che ne tocca a sostenere tre quarti di cantile*
>
> (but certainly the weakest ones will break, those which carry the greatest loads, and those carrying the greatest loads are those that touch the ground, which can be doubled, because it falls to them to carry three-quarters of the load).

Indeed, at one point one of our beams broke (one that was close to the edge but not actually resting on the ground). Part of the dome collapsed as a consequence, but not all of it, and we were able to repair the damage by removing the broken beam and inserting a new one, without having to dismantle it further.

Leonardo indicates that a vault covering a space of 45 *braccia* will have a height of 5 *braccia* (one Florentine *braccio* equals about 58.37 cm). The curvature of the vault is not very steep. The ratio of height to length is 1 to 9. The measurements we took of our own constructions confirmed this ratio. Some patterns produced a slightly lower curve than others, but overall the ratio of height to width varied between 1:8 and 1:10 (figs. 7 and 8).

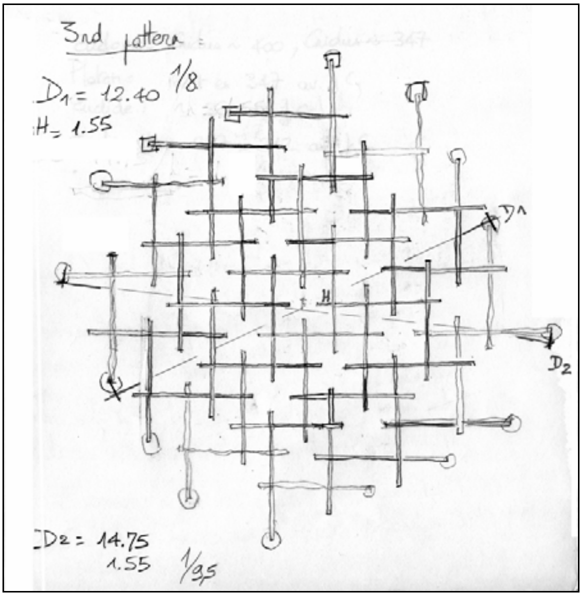


Fig. 7. Measurements of the third dome actually built: the space that has been covered can roughly been approximated to a square. The ratio between width and height of the dome varies from 1/8 to 1/9.5 according to the peripheral supports that are considered

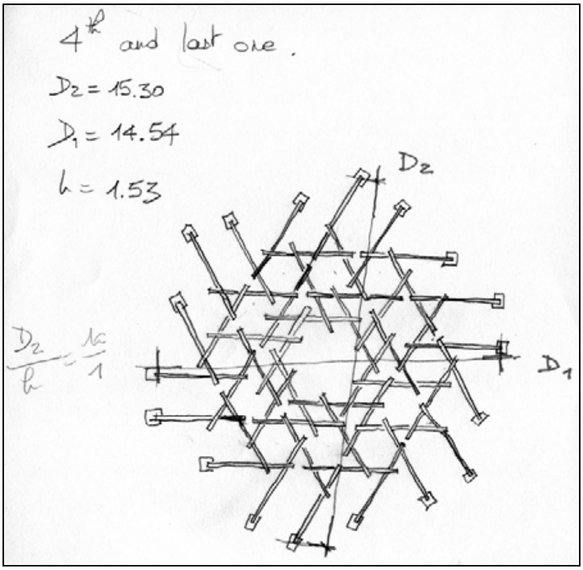


Fig. 8. Last but not least: geometric complexity increases in the pattern of the fourth built dome. Hexagons alternate with triangles and rhombuses. The dome is interrupted by a large central hexagonal oculus. The ratio between width and height – measured at the edge of the oculus – is close to 1/10

The fact that Leonardo speaks of beams that "touch the ground" shows that the experiment in which he participated was similar to ours. "His" vault too was erected on the ground and not on a peripheral masonry structure. But he also speaks of lifting the whole roof to raise it on some supports. For this operation machinery of ropes and levers appears to be required:

Debbon s'alzare tutti a un tratto colle lieve

(The whole should be lifted all at once with levers).

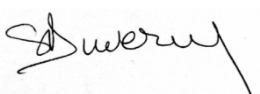
To complete the structure, this roof may be covered with fabric. Leonardo in his notes calculates the number of cloths necessary for a vault that covers a space of 45 *braccia*.

The whole building thus appears to be done with standard materials: small, identical timber beams, pieces of fabric *li quali ordinariamente si fan 30 braccia per ciascuno* (each of which is usually 30 *braccia*). The economy, ease and rapidity of this technology suggested to Carlo Pedretti, the major analyst of Leonardo's writings, that this kind of building was intended as some kind of temporary or emergency shelter. While this hypothesis is logical, nothing in Leonardo's own words of confirm the hypothesis, as he indicates nothing of the purpose and the function of such a structure.

We must consider this experiment as part of the general Renaissance research on relationships between mathematics and architecture. So far, those relationships have been investigated mainly with regards to the interaction between geometry and design, where geometrical shapes and patterns, together with their numerical proportions, guarantee the aesthetic result of the final architectural object. Here, in this experiment, geometry and mathematics are related to building technology, and Leonardo's concern obviously focuses on the role of geometrical shapes in structural stability.

Dome building is only one of the things that can be done with the Leonardo grids. The next step, by varying the basic building element, leads to building spheres, cylinders, and columns. Famous architects and engineers, such as Guastavino, Fuller and Snelson, would later study related structures.

We would like to take this opportunity to thank those who supported the 2003 Leonardo seminar and construction project: The Biblioteca and Museo Leonardiano of the city of Vinci, especially director Romano Nanni and librarian Monica Taddei, Laurent-Paul Robert for coordination of photography, and student helpers Lorenzo Matteoli and Alessio Mattu for help with the construction. And, of course, thank you very much to all those who participated for making the seminar and construction not only a learning experience, but fun as well.



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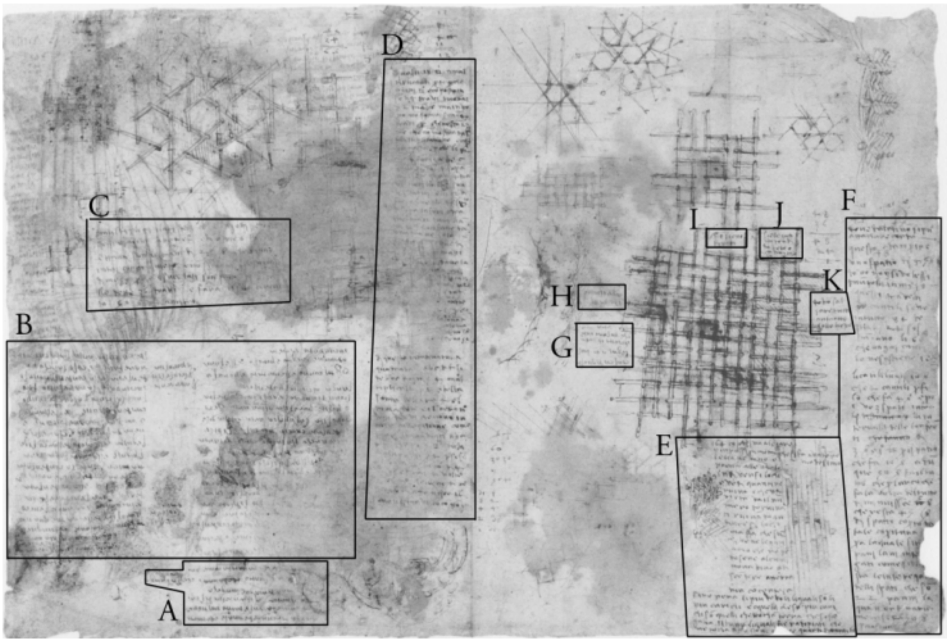
Research

Transcription and Translation of Codex Atlanticus*, fol. 899 v*

Abstract. The basis for the 2003 seminar and construction project on Leonardo's roofing system was based on fol. 899v of the *Codex Atlanticus*. This paper is an transcription and translation to make that page more accessible.

Codex Atlanticus fol. 899v



Left face

A.

La linea circunferenziale conterrà in sé tanto minor vano, quant'ella dista … di maggior lunghezza…spazio che s'allunga … la sua capacità come hanno

The circumference line will itself measure less by that amount that it lies from … of a greater length … space that lengthens …its capacity as have

B.

…bile e condensabile … da il mezzo dell'arco …strigner si condensa, l'altra … parte che da esso arco … fori si dilata e ra … ne porosità … ella resta di spezie di vacuo … il quale é potentissimo e al continuo … disfarsi, il che non po disfare … ancora l'aria

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condensata distende, e se tale arco sta mezzo carico, l'aria condensata caccia per le insensibile porosità la soperchia aria e tira per le opposite porosità altra aria che ristora il vacuo, e cosi l'arco resta sanza alcuna potenzia.

…ble and condensable … from the middle of the arch … tightening, it pulls, the other … part of that arch …remains outside dilates and ra … porosity … there remains a kind of vacuum … which is most powerful and continuous …come undone, which cannot come undone … still the condensed air extends, and if such arch is half loaded, the condensed air pushes away, by means of the invisible porosity, the superfluous air and pulls, by means of the opposite porosity, the other air that restores the vacuum, and thus the arch stays without any force at all.

Ogni corpo elementato é poroso; adunque il legno é poroso e se sarà incurvato la metà…

Every body made of elements is porous; thus wood is porous and if it is curved by half…

Ogni corpo elementato é poroso e ogni porosità é piena in sé … e l'aria condensata e rarefatta é violente. Adunque la porosità del legname, quando é incurvato, una parte se ne condensa e una se ne rarefa. La rarefatta e la condensata ne farà ritornare nella prima natura … La condensata spigne e la rarefatta tira; seguita che 'l moto dell'arco si genera da soperchio a carestia, o voi dire da rarefazione e condensazione, e se l'arco sarà lasciato per alquanto tempo incurvato, la rarefazion si condenserà (attraendo per le insensibili porosità) e la condensazione si rarefarà, e cosi l'arco resterà colla acquisitata curvità.

Every body made of elements is porous and each porosity is complete in itself … and the condensed and rarefied air is violent. Thus the porosity of the wood, when it is curved, one part is condensed and the other is rarefied. The rarefied and the condensed [parts] will return to their original nature … the condensed pushes and the rarefied pulls; it follows that the motion of the arch generates an excess and a deficiency, that is to say, from rarefaction and condensation, and if the arch is left for some amount of time in a curved shape, the rarefaction will condense (attracting by means of the invisible porosity) and the condensation will rarefy, and thus the arch will remain with the curve acquired.

C.

Moltiplica i lati delli 6 esagoni per ordine delli esagoni *a b c d e f*, e di': '6 vie 6 fa 36,' e tante son le travi di 2 braccia che vanno in tal componimento e non si conta l'esagono di mezzo, perché i sua lati son fatti delle dette 36 travi, e sarà il suo circuito 50 braccia di diamitro.

Multiply the sides of the 6 hexagons by the order of the hexagons a b c d e f, that is, '6 by 6 makes 36,' and that is how many are the 2-braccia long beams in this arrangement, and the hexagon in the middle is not counted, because its sides are made of the 36 beams already counted, and its perimeter will be 50 braccia in diameter.

D.

Questi 18 quadrati a moltiplicarli per 4, perché ogni quadrato é composto in sé di 4 travi, tu arai 72 trave, ma in vero non sono se non 45 travi, che restan 27 men che non mostra tal moltiplicazione. E questo nasce perché li 2 quadrati di fori *a* e *c* e li 2 quadrati di sopra [c]ontengan 4 travi, tutti li altri son di 3 travi; e li quadrati di mezzo, cioè a b dal quadrato di sopra in fori che ha due travi, tutti li altri quadrati son fatti da una trave sola.

These 18 squares are multiplied by 4, because each square is itself composed of 4 beams, you will have 72 beams, but in fact there are only 45 beams, that is, 27 fewer than the product of the multiplication. This is because the 2 outside squares a *and* c *and the two upper squares contain 4 beams, all the others are of 3 beams; and the middle square, that is* a b *of the upper outside square has 2 beams, all the other squares are made with one beam only.*

Addunque di 72 travi che ti dava la predetta moltiplicazione, ne son diminuite 21, perché la somma di tale trave non son se non 51.

Thus from the 72 beams that are given by the multiplication described are taken away 21, because the sum of the beams is precisely 51.

Di questa incatenatura quadrata *a b c d f* si debbe di ogni 5, 4 multiplicalli per 4 e della somma levare 4 e 'l rimanente é il vero numero delle travi che vanno in tale collegazione; come dire delli quadrati a b c d f, che son quadrati, tu li moltiplichi per 4 per sé … ogni quadrato fa 40 di 4 … e dirai: 4 volte fa 20; levane 4, resta 16; adunque 16 sono li travi che compongano …

For this square configuration a b c d f *for every 5, 4 multiply by 4 and from the product take away 4, and the remainder is the true number of beams that go in that configuration; that is, given squares* a b c d f*, which are squares, you multiply each by 4 … each square makes 40 of 4 … and I say: 4 times [five] makes 20; taking away 4, leaves 16; thus 16 are the beams that make it.*

Right face

E.

164 corde v'ha a legare a copresi in mentre che si fa, (a un tempo medesimo). Levane uno, e prova a levarlo in diversi lochi, e vedi quanti ne ruina e che varietà v'ha il numero de' ruinati, ruinando in diversi lochi; ma fa che sien bene legati a ciò che, rompendosene alcuni, non abbino a discendere a terra.

164 cords are to be tied and covered as it is made (at the same time). Take away one, and try taking it away in several places, and see how many fail, and how many different failures there are, failing in different places; but let them be well tied so that, when some break, they don't fall to the ground.

Ma con certezza si romperà li più deboli, li quali son li più carichi, e quelli che son piè carichi son quelli che toccan terra, che sostengano il tutto; li quali fien raddoppiati, che ne tocca a sostenere tre quarti di cantile.

But it is certain that the weakest will break, those that are the most loaded, and those that are most loaded are those that touch the ground, that support the whole; let them be doubled, since they have to support three quarters of the beams.

F.

Dove da' lati non si può attaccare corde.

On the sides you can't attach cords.

Questo é da coprire uno spazio di 45 braccia, dove non si volessi puntelli in mezzo.

This will cover a space of 45 braccia, where you don't want columns in the middle.

Questi travelli ovver cantili sono in tutto 84, de' quali 24 sostengano li 60, che ogni cantile ne sostiene 1 e ½.

These little beams, or cantili*, are 84 in all, of which 24 support the other 60, each of these supporting 1 and ½.*

Li cantili lunghi 10 braccia, che 4 cantili per filo, che fan 4(0) braccia; e poi v'é 3 spazi infra li intervalli delle lor fronti di 3 braccia e uno per ispazio, che fa 10 braccia. Adunque 50 braccia fa il tutto, che per l'arco che fa la volta del tutto, diminuisce 5 braccia, che resta 45 braccia di spazio coperto da tale copertura, sopra la quale si tira panni lani interrsegati, come si da … stan le intersegazion delli spazi, che son 22 panni, li quali ordinariamente si fan 30 braccia per ciascuno.

The cantili *are 10 braccia long, and there are 4 cantili per row, making 4(0) braccia; and then there are 3 spaces in the intervals between them that are 3 braccia and one per space, making 10 braccia. Thus 50 braccia is the total length, which due to arching height of the overall vault, is diminished by 5 braccia, so leaving 45 braccia of space covered by this covering, over which are pulled overlapping woollen cloths, as are used …are the overlaps of the spaces, which are 22 cloths, which are ordinarily 30 braccia each.*

G.

Ciascun di questi legni ha 4 *busi*, eccetto li 24 che posano in terra.

Each of these logs has 4 busi *(intersections?), except for the 24 that rest on the ground.*

H.

12 panni copre il tutto.

12 cloths will cover the whole.

I.

Non sieno forati.

Let them not be drilled (with holes).

J.

Sien legnami tondi, d'abete o castagni.

Let them be round logs, of fir or chestnut.

K.

Debbon s'alzare tutti a un tratto colle lieve.

It has to be lifted all at once with levers.

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Keywords: Leonardo da Vinci, grids, structural patterns, tilings

Research

Two- and Three-Dimensional Constructions Based on Leonardo Grids

Abstract. In 1989 I made a drawing of a net on a cube, consisting of 12 lines/elements. They were connected in a way that, a couple of months later, I recognised them in 899v in Leonardo's *Codex Atlanticus*. I don't know which moment impressed me the most: my own discovery of a very simple and powerful connecting system or the discovery of the Leonardo drawings, which implied that my own discovery was in fact a rediscovery. What we see in Leonardo's drawings are some examples of roof constructions built with a lot of straight elements. These drawings can be 'translated' into the following definition: On each element we define four points at some distance of each other – two points somewhere in the middle and two points closer to the ends. To make constructions with these elements we need only connect a middle point of one element to an end point of another one in a regular over-under pattern. Out of the simple definition of the elements, I designed many different patterns for my so-called "+ - - +" structures: domes, spheres, cylinders and other models were made.

Introduction

In 1989, I began constructing domes using notched bars assembled according to a simple rule. This led me to explore planar constructions based on this rule using fixed length "notched" linear segments. I was able to create a wide variety of patterns. From certain of these patterns I was able to construct spheres and cylinders with notched curved rods without the use of glue, rope, nails, or screws.

On folio 899v of the *Codex Atlanticus* we find, among others, three patterns with exactly the properties of these bar grids. In view of the way in which the patterns are drawn, oblong forms that seem to lie on each other, the most direct interpretation is that here we are dealing with stacking constructions, built from straight rods. Making a model leads to exactly the domes that I experimented myself. So the conjecture that Leonardo da Vinci is the first inventor of these constructions seems justified, although we cannot be sure about this.

Thus, the name I gave to my bar grid construction system is "Leonardo grids", with which I was able to construct all kinds of structures out of simple elements using one single constructing rule. Most of the constructions I made where planar and static. However, the "Leonardo grid system" also makes possible the construction of non-planar and dynamic structures.

The system

The construction admits a simple description. We start with a number of rods, on each one determining four points as indicated in fig. 1. We call these points connecting points.

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Fig. 1

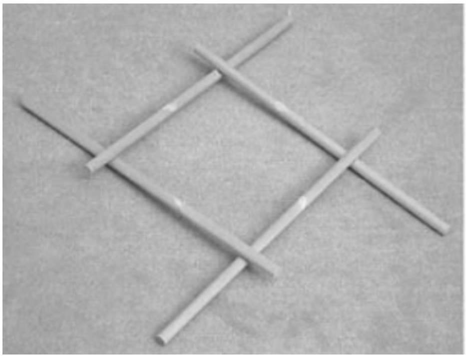

Fig. 2

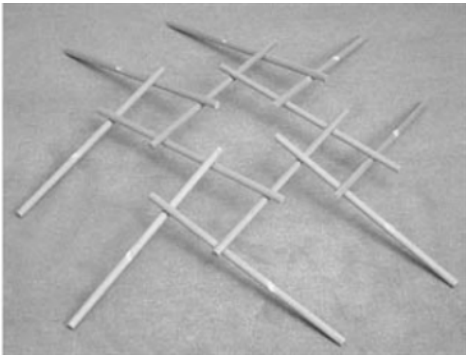

Fig. 3

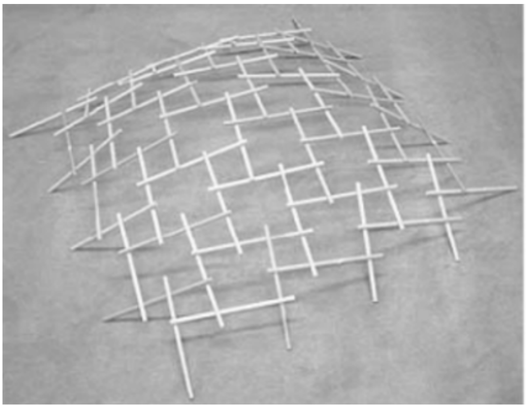

Fig. 4

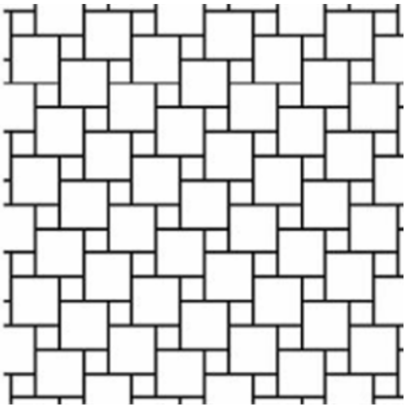

Fig. 5

We distinguish two types of connecting points: end points (closest to the ends of the rods) and interior points (the remaining points). Each rod has two end points and two interior points. In constructing the dome we now apply the following rules: one of the endpoints of a rod is placed on a free interior point of a different rod. At the end all connecting points of the rods have to be used as a connection between two rods, except near the border of the construction.

Now the actual construction of the dome turns out to be a simple task. Beginning with four rods, as in fig. 2, we extend the construction by continually adding rods at the bottom (fig. 3). Since we add one rod at the time, on the outer edge, the dome can be constructed by a single person. The four rods with which we have started will rise automatically during the building process and, at the end, the dome, consisting of 64 rods, will stand on the ground, resting on only 16 rods (fig. 4).

Following the above construction process various patterns can be formed, each leading to a dome-like construction. In the sequel we will call the patterns that can be formed with the above rules "bar grids". The bar grid of the dome of fig. 4 can be drawn simplified as in fig. 5. In this form the drawing looks like a tiling pattern. However, we are not interested in the tiles but in the joints between the tiles. So we have a grid consisting of straight lines representing the rods. A first investigation into the various possible bar grids soon resulted into dozens of patterns, some of which are shown in fig. 6.

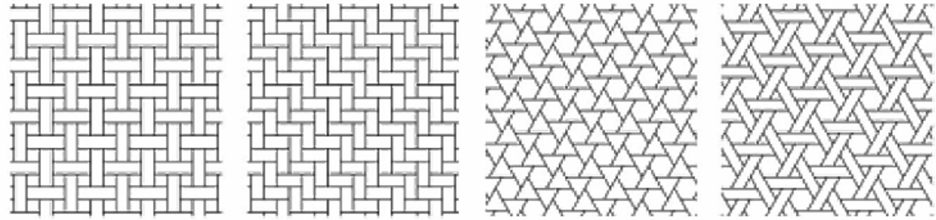

Fig. 6

From 2D to 3D

In the domes it is gravity that keeps the loose rods together. It follows that continuing the construction as far as a complete sphere is not possible. Yet it turns out that, using the above construction system, objects can be formed in which the elements themselves, instead of gravity, keep the construction together. For example, we can assemble a sphere from a number of rods, or more generally elements, without using connecting materials like wire or glue. The number of connecting points per elements and the connecting rules do not change. It is only the form of the elements that changes. For a sphere we use curved rods instead of straight ones.

A simple way to come to a design for such a sphere shaped construction is the following: in the bar grid of fig. 7 the midpoints of the hexagons are connected such that a pattern of triangles results (fig. 8). Eight of these triangles can be used to form an octahedron (fig. 9).

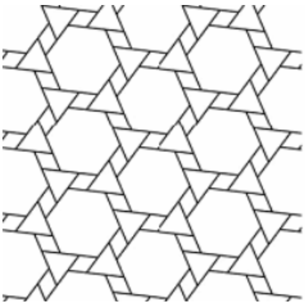

Fig. 7

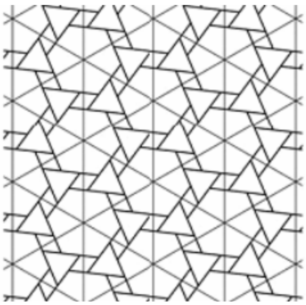

Fig. 8

Fig. 9

On this octahedron we now see a grid consisting of 24 bars and this can be used as a design for the sphere of fig. 10. The form of the elements has been determined such that no tension arises in the sphere. Only when closing the sphere some elasticity from the elements is required. The relative position of the elements causes the sphere to stay in one piece: each of the elements is prevented from falling by other elements. For the sphere of fig. 11, which consists of 90 elements, the icosahedron has been used as an intermediate step so that pentagons occur in the construction.

Beside domes and spheres other shapes have been realized, such as cylinders and ovoids (figs. 12, 13).

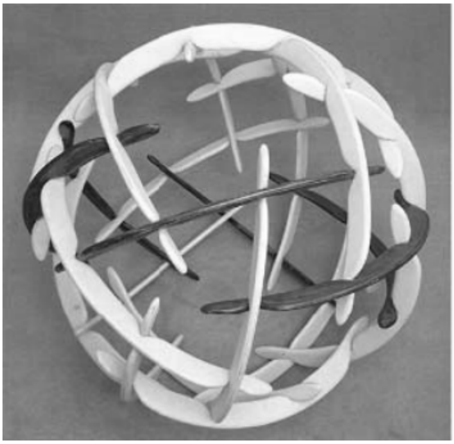

Fig. 10. Sphere, 24 elements sphere

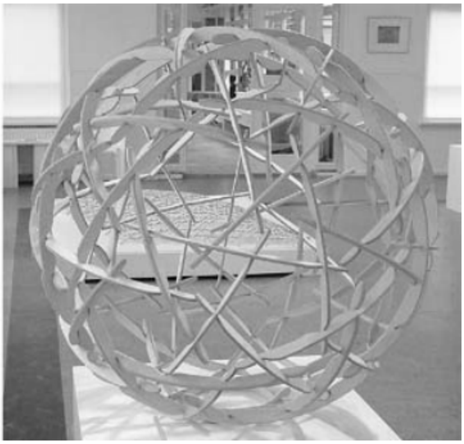

Fig. 11. Sphere, 90 elements

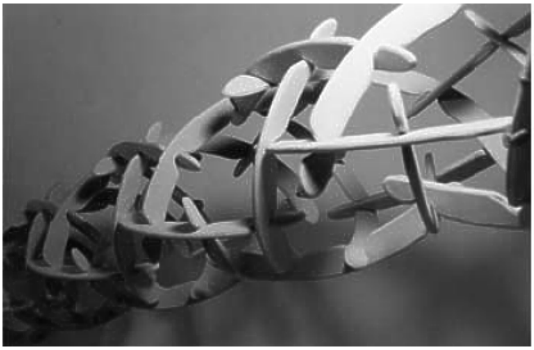

Fig. 12. Cylinder

Fig. 13. Ovoid

A real new step was made when designing some objects in which the inner space of the sphere is used too, as in fig. 17. This object has the form of two linked concentric spheres. The whole is a stable construction consisting of 24 elements. Each element is halfway (that is to say with two out of four connecting points) in the outer sphere and halfway in the inner sphere. The design was made by starting with two layers of Leonardo grids. The layers were placed above each other in such a way that after cutting each element in two parts, half elements from the upper layer could be connected to half elements of the under layer. The resulting structure again has all the properties needed for a Leonardo grid: each element is connected to 4 other elements in the right way: all endpoints connected to midpoints and all midpoints connected to endpoints. Half of the connecting points are on the surface of the inner sphere, the other half on the surface of the outer sphere. This is the first non-planar or real 3D result. And surprisingly enough, the total structure appeared to be stable.

Another approach was to start with interwoven patterns (fig. 14). A simple way to transform a flat pattern into a spherical construction is the use of polyhedron. When the pattern is hexagonal, the net of a icosahedron can be used. In the special case of the three interwoven patterns of fig. 15, the cut-off elements of pattern A will be connected to the cut-off elements of pattern B when folding the net into an icosahedron. And so with the cut-off elements of B and C and of C and A.

The result is real 3D Leonardo grid construction. And now all the connecting points are laying in the same spherical surface. In the pictures (figs. 16, 17, 18) you can see some variations.

Although this is a good approach it is hard to find a good set of interwoven patterns that can be used for this method.

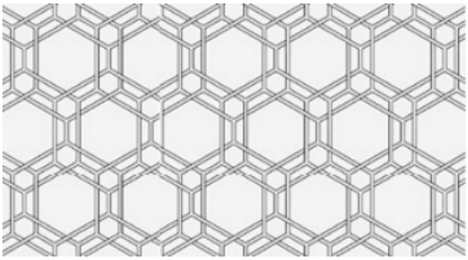

Fig. 14. Three interwoven patterns

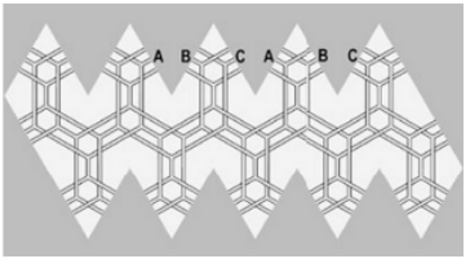

Fig. 15. Icosahedron, plan

Figs. 16-18. Interwoven spheres

Infinite double layer structures

To go one step further towards the Leonardo grid space frames, I first tried to find a way to construct infinite double layer structures. Space frames can be built by connecting polyhedra in systematic way. With cubes you can fill the space. And when you look at the graph that represents the cube you will notice that all vertices have degree 3, which was a condition for the Leonardo grids. A cubic frame can be made as a Leonardo grid construction in three different ways (figures 19, 20 and 21).

A way to make a double layer structure is to connect these kinds of cubes as in fig. 22. This will result in non-planar infinite construction that has also dynamic properties. The elements can slide between certain boundaries and the total construction can be pressed together or stretched (figs. 23, 24).

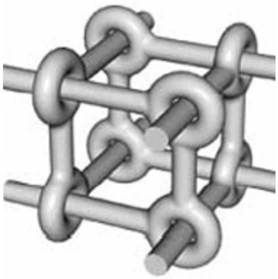

Fig. 19. Cubic frame A

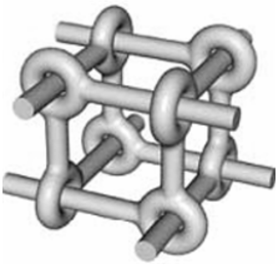

Fig. 20. Cubic frame B

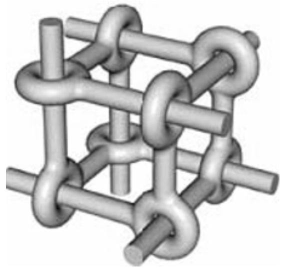

Fig. 21. Cubic frame C

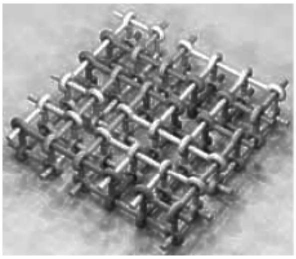

Fig. 22. Double layer structure

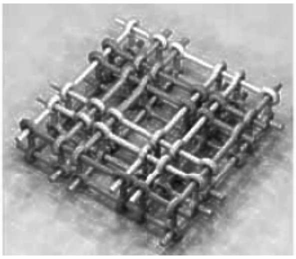

Fig. 23. Double layer structure, pressed together

Fig. 24. Double layer structure, stretched

Rings and strings

In the double layer structures the basic elements, the rods with the 4 connecting points, are linked together to form bigger units. We can distinguish two kinds of these bigger units: rings, a closed concatenation of a finite number of basic elements, and strings a (open) concatenation of an infinite number of basic elements. Some examples of both categories are shown in figs. 25, 26 and 27 (rings) and figs. 28, 29 and 30 (strings).

The constructions needn't limited to two layers, as can be seen in figs. 31, 32 and 33. Here an infinite 3D construction is build with one type of string, which is a concatenation of basic Leonardo grid elements.

The question still is whether it is possible to make space frame constructions out of basic elements, which are not linked.

Fig. 25. Double layer structure

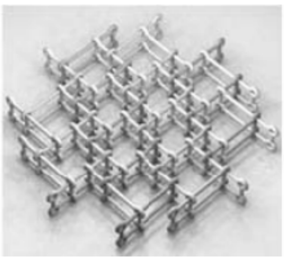

Fig. 26. Pressed together

Fig. 27. Stretched

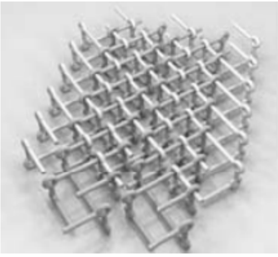

Fig. 28. Double layer structure

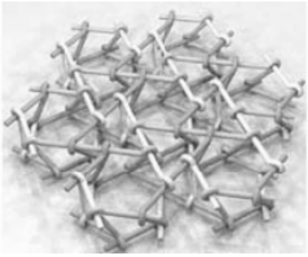

Fig. 29. Pressed together

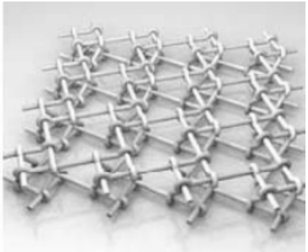

Fig. 30. Stretched

Fig. 31. 3D Strings

Fig. 32. 3D String structure

Fig. 33. 3D String structure

Grid transformation

Another, and maybe even better way to construct real 3D structures based on Leonardo grids appeared to be the use of transformation of the basic Leonardo grid from 2D to 3D (figs. 34, 35). The process can be described as follows: we can start with any pattern in which we have a hexagonal hole. We now keep the 6 sticks around this hole connected and change the hexagon from flat to skew. This change will cause a transformation of the sticks, which are connected to the first 6 sticks. The six parallelogram shaped holes in the pattern will also be parallelogram-shaped at the end of the process. But one of the connections around the triangular holes will get loose. The resulting structure now can be used as a layer with which we can create space frames.

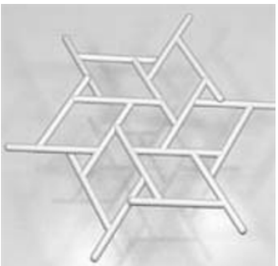

Fig. 34. Leonardo grid 2D

Fig. 35. Leonardo grid 3D

The discovery of this process lead to many designs of Leonardo grid space frames because the process could be applied on all the flat basic patterns. We can also start with the pattern with a square hole. In the same way the flat hexagon is transformed into a skew hexagon, a flat square can be transformed into a skew square. The over a hundred different