Non-Holomorphic Cycles and Non-BPS Black Branes

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Supersymmetry vs. No Supersymmetry

Most of what we know about string theory: Supersymmetric backgrounds

Typically we consider compactifications on special holonomy manifolds

These are the only known stable solutions in string theory

Supersymmetry breaking backgrounds are all transient

However we live in a non-SUSY universe. We need to understand what happens if SUSY is broken.

Subject of a number of Swampland conjectures (related to the possible existence of dS or quasi-dS backgrounds).

We need analytic tools to address this class of string backgrounds.

Difficult task…

Intermediate approach:
Consider supersymmetric breaking states (non-BPS) in supersymmetric backgrounds

Supersymmetric BPS states in SUSY backgrounds have been well studied.

Prominent examples: Branes wrapped on calibrated cycles of manifolds of special holonomy;

For example: Branes on holomorphic cycles in Calabi-Yau

They lead to charged BPS branes ($T = |Q|$) where

$$Q \in H_*(M, \mathbf{Z})$$

For large Q (such as $Q \to NQ$ for $N \gg 1$)
They represent macroscopic BPS black branes

Have led to successful predictions of BPS black hole entropy:

$$S = \frac{A}{4}$$

The general approach (Strominger+V.)

1) Construct BPS strings in 1 higher dimension
2) Compactify on a circle and wrap the string on it
3) The high left-moving oscillatory BPS modes on the string can represent black holes
4) Use Cardy formula to get the entropy

$$S = 2\pi\sqrt{\frac{nc}{6}}$$

where c is the central charge on the string and n is oscillator mode

Do all charges states Q appear in the spectrum?

Yes! (One of the most basic Swampland conditions)

In a supersymmetric theory are all charge states Q stable?

No! Only the BPS ones are stable

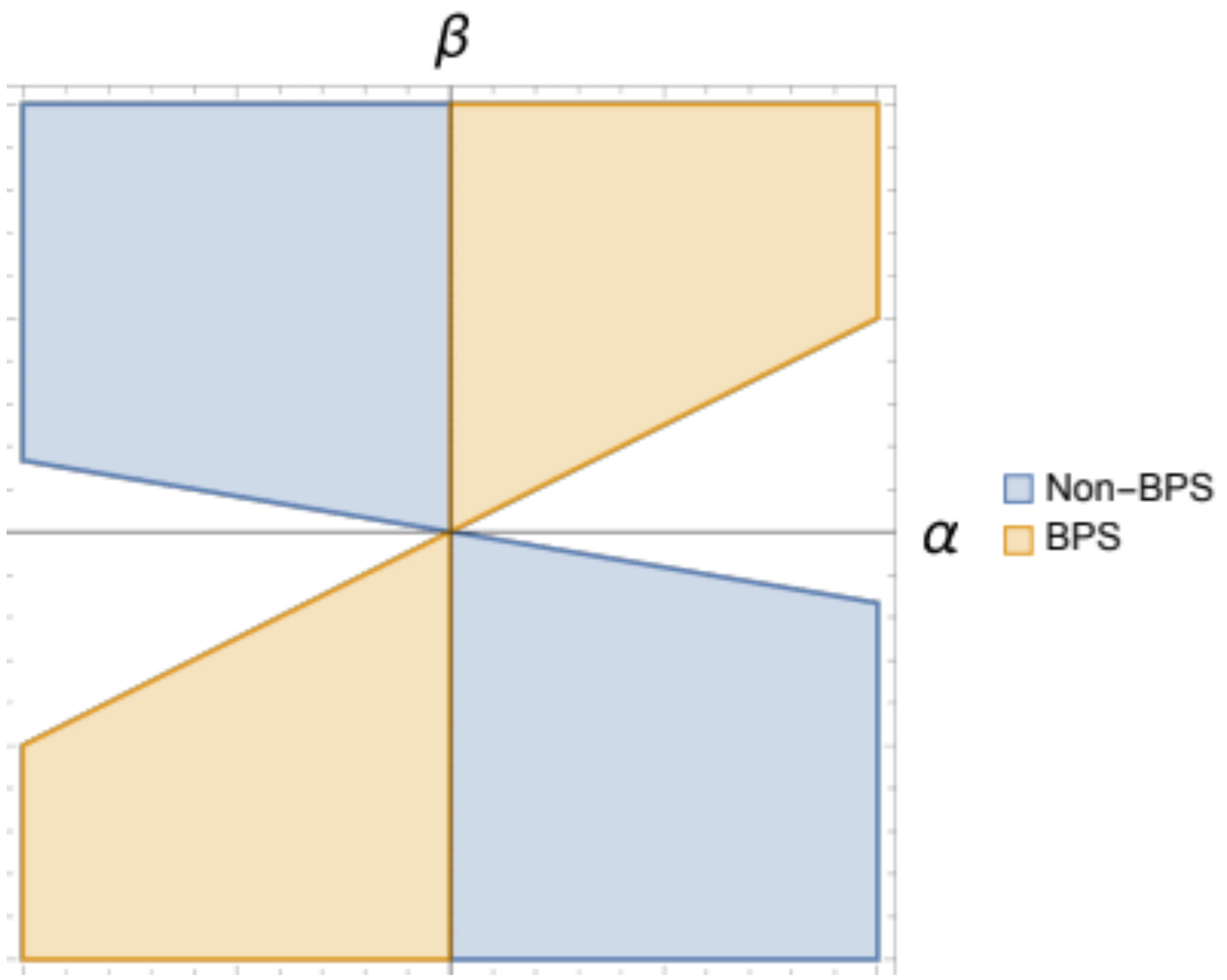
Are all charges represented by macroscopic branes?

No!

Are all BPS ones represented by macroscopic branes?

No!

Macroscopic black branes (blue and yellow regions):



White regions: Microscopic branes as well as unbound branes which should be viewed as combination of lower brane charges

Tensions of Branes vs Charge

For BPS states (microscopic or macroscopic)

$$T = |Q|$$

For macrscopic extremal non-BPS black branes (large Q)

$$T = |Q|$$

Big difference:

BPS ones are stable/Non-BPS ones are expected to decay

(One of the main motivations of Weak Gravity Conjecture (WGC))

What do extremal non-BPS black branes decay to?

A) Combination of BPS / anti-BPS states

$$T > T_1 + T_{\bar{2}}$$

B) non-BPS but stable mircroscopic states

For micrscopic non-BPS states

$$T < |Q|$$

Predicted by the Weak Gravity Conjecture
(and kinematics of decay)

Example: Heterotic String on Tori

Consider heterotic string wrapped on a circle with momentum n.

Charges are denoted by Narain Lattice vectors (Q_L, Q_R)

Physical states satisfy

$$\frac{1}{2}m^2 = \frac{1}{2}Q_R^2 + N_R = \frac{1}{2}Q_L^2 + N_L - 1$$

$$N_L - N_R = n$$

BPS states satisfy $N_R = 0 \rightarrow m^2 = Q_R^2$

non-BPS states extremal states satisfy $N_L = 0 \rightarrow m^2 = Q_L^2 - 2$

$$m < |Q_L|$$

In duality between M-theory on K3 and heterotic on T^3

BPS states are holomorphic curves

non-BPS states are non-holomorphic curves (unstable)

Class we study in this talk:

- 5d theory via M-theory on CY3 X.
- Non-BPS black holes from M2 branes on non-holomorphic curves.
- Non-BPS black strings from M5 branes on non-holomorphic divisors.
- Brane tension computed via the the attractor mechanism, which involves minimizing a rational function of the moduli.

$$T_p = \min\left(\sqrt{\frac{3}{2}V^p_{eff}}\right)$$

- These results are asymptotic and expected to hold for large charge.
- Also: compute the central charge of non-BPS strings.

Black Hole Results:

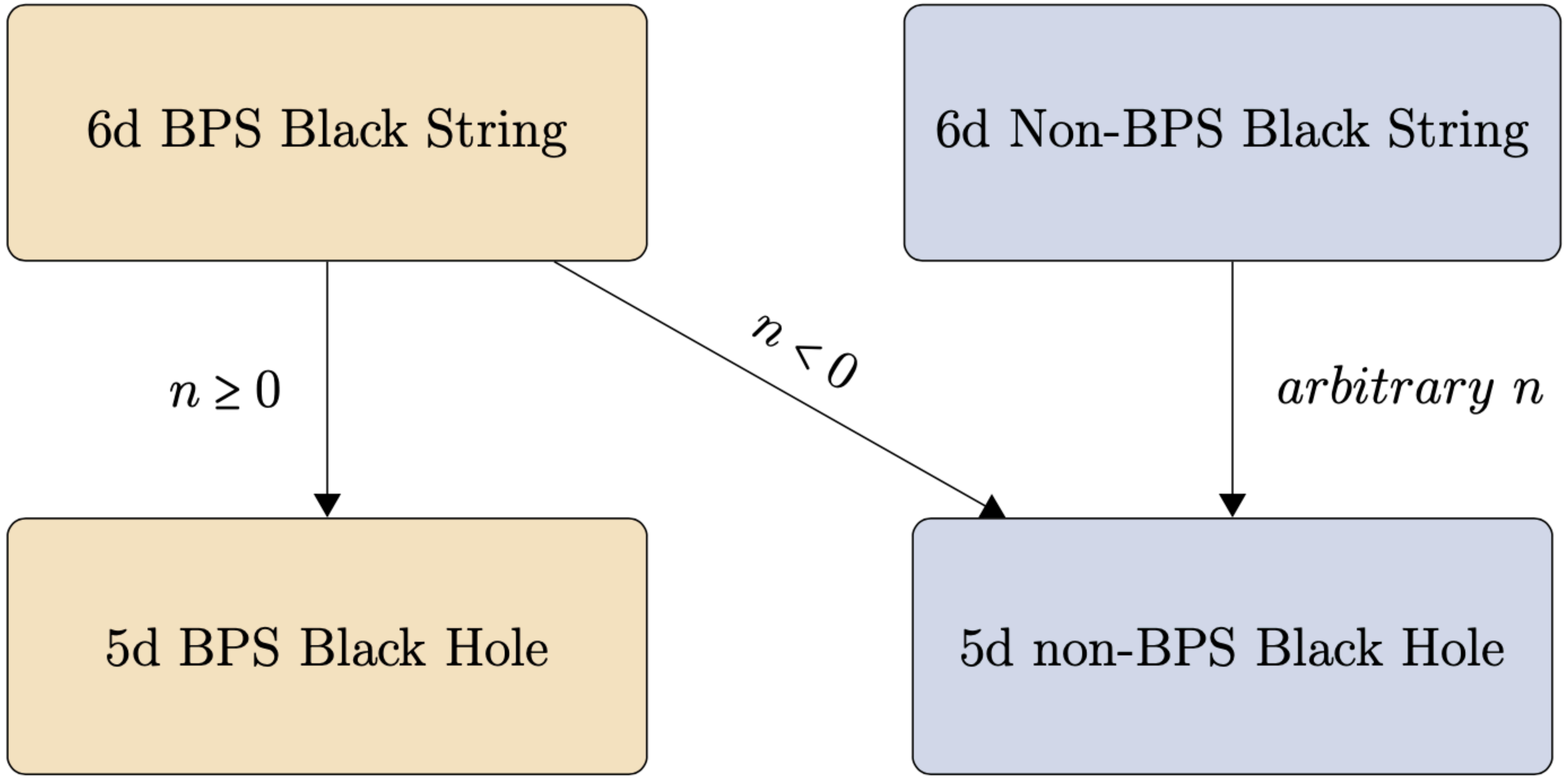
In the examples we consider we find that the black holes correspond to local, but not global, volume minimizers of the corresponding curve classes, as there is always a disconnected, piecewise-calibrated representative with smaller volume. Therefore the non-BPS extremal black hole can decay to BPS/anti-BPS constituents.

Black String Results:

In the examples we consider we find that
that sometimes the black strings correspond
to global volume minimizer of a given
class. In such cases we predict the existence of a stable
non-BPS microscopic string as the decay remnant.
We also find examples where this is not global volume
minimizers as there is a disconnected,
piecewise-calibrated representative with smaller volume.
Therefore in these other cases non-BPS extremal
black strings can decay to BPS/anti-BPS constituents.

5d Black holes from BPS and non-BPS Strings

- M-theory on Elliptic CY threefold, can be obtained from F-theory on the same CY times a circle.
- 5d black holes in these cases can be viewed as wrapped 6d strings.
- Compute black hole entropy using Cardy formula. Results agree!



Black Holes and Strings from M-Theory on a Calabi-Yau Threefold

M-theory on Calabi-Yau Threefold X

- 5d field content from dimensional reduction on X:

  1. Gravity multiplet (that includes a vector).
  2. $h^{1,1}(X) - 1$ vector multiplets, each with a real scalar.
  3. $h^{2,1}(X) + 1$ hypermultiplets (which completely decouple!).

- Moduli of X are

$h^{1,1}(X)$ Kähler moduli t^I , $J = t^I \omega_I$, $\omega_I \in H^{1,1}(X)$.

$h^{2,1}(X)$ complex structure moduli.

M-theory on Calabi-Yau Threefold X

- Kähler moduli t^I are essentially the scalars in vector multiplets, **EXCEPT** the overall volume of X, which belongs to a hypermultiplet.

- Parametrize vector multiplet moduli space by $h^{1,1}$ fields t^I, with constraint

$$\mathrm{vol}(X) := \mathscr{V} = \frac{1}{6}\int_X J \wedge J \wedge J = \frac{1}{6} C_{IJK} t^I t^J t^K = 1\,.$$

- Relevant terms in the action:

$$S_5 = \frac{1}{2\kappa_5^2}\int\left(R * \mathbf{1} - G_{IJ} dt^I \wedge * dt^J - G_{IJ} F^I \wedge * F^J - \frac{1}{3!} C_{IJK} F^I \wedge F^J \wedge A^k\right),$$

- Gauge kinetic function and metric on moduli space:

$$G_{IJ} = -\frac{1}{2}\partial_I \partial_J \log(\mathscr{V})\,.$$

Charged particles and strings

- Introduced gauge and gravity fields, to construct charged black holes we need charged objects.
- In 5d, we have electrically charged particles, and magnetically charged strings.
- Wrap M2 brane around curve $C \subset X$ to get electrically charged particle.
- Charge under I-th $U(1)$ given by intersection of C with I-th divisor: $q_I = C \cdot D_I$.
- Central charge takes the form

$$Z_e = q_I t^I = \int_C J \, .$$

- For BPS particle central charge agrees with volume of wrapped curve, as measured by Kähler form.

Charged particles and strings

- Wrap M5 brane around divisor $D \subset X$ to get magnetically charged string.
- Magnetic charge given by wrapping number p^I of M5 brane around I-th divisor D_I .
- Central charge takes the form

$$Z_m = \frac{1}{2} \int_D J \wedge J = p^I \tau_I \, .$$

- For BPS string central charge agrees with volume of wrapped divisor, as measured by the Kähler form.

Black holes and strings

- Introduce enough charge, expect a black object to form.
- Presence of the charged objects also sources the scalars in the vector multiplets, forces scalars to flow to fixed values at the horizon (attractor mechanism).

Ferrara, Kallosh, Strominger

- Horizon moduli determined by minima of "effective potential":

$$V^e_{eff} = G^{IJ} q_I q_J \quad \text{or} \quad V^m_{eff} = 4 G_{IJ} p^I p^J$$

- Normalization is so that, in both cases

$$V^{e,m}_{eff} = \frac{2}{3} Z^2_{e,m} + G^{IJ} (\mathscr{D}_I Z_{e,m})(\mathscr{D}_J Z_{e,m})$$

BPS black holes and strings

- First, BPS solutions satisfy $\mathscr{D}_I Z = 0$. Horizon moduli t_0 fixed by solution. Some features:

1. BPS black holes then satisfy $q_I = \dfrac{1}{3\mathscr{V}}\tau_I Z \ \ \rightarrow C \cdot D_I \sim J^2 \cdot D_I$.

   It follows BPS black holes correspond to M2 branes wrapping curves given by the self-intersection of the Kähler divisor. These curves are examples of *strongly movable curves*, which are irreducible and foliate X.

2. Not all holomorphic curves correspond to large BPS black holes. For instance, rational curves cannot be written as the self-intersection of a Kähler divisor. Nor can any rigid curve.

- Similar for BPS black strings, for which an M5 brane must wrap an ample divisor.

BPS masses and tensions

- Horizon values of moduli t_0 fixed by attractor mechanism. Asymptotic values t_∞ are not, and can take any values inside Kähler cone.

- Mass of BPS black hole, and tension of BPS black string, given by central charge, with moduli evaluated at infinity:

$$M = Z_e|_{t=t_\infty} = \sqrt{\frac{3}{2}V^e_{eff}}|_{t=t_\infty}$$

$$T = Z_m|_{t=t_\infty} = \sqrt{\frac{3}{2}V^m_{eff}}|_{t=t_\infty}$$

Non-BPS black holes and strings

- Main case of interest in a brane wrapping a non-holomorphic cycle.
- These do not solve $\mathscr{D}_I Z = 0$ inside the Kähler cone. Instead, must solve $\mathscr{D}_I V_{eff} = 0$ inside Kähler cone.
- Non-BPS critical point is **not** automatically a minimum, and this must be checked case-by-case.
- Mass/tension still depends on asymptotic values of moduli t_∞, but is not given by central charge.

Non-BPS black holes and strings

- Simplified approach: set $t_\infty = t_0$, so-called “double extremal” black hole/string, in which the scalars do not flow.

- Setting $t_\infty = t_0$ essentially reduces to an ordinary Reissner-Nordström black hole/string, and mass/tension can be read off as

Meessen, Ortin, Perz, Shahbazi

$$M = \sqrt{\frac{3}{2} V^e_{eff}} \,\Big|_{t=t_0}, \quad T = \sqrt{\frac{3}{2} V^m_{eff}} \,\Big|_{t=t_0}$$

- If we fix $t_\infty = t_0$, solving the attractor equations is equivalent to minimizing the mass/tension, as a function of the moduli.

Minimal Cycles and Branes

Brane tension

- Consider n-cycle Σ_n, which is a local volume minimizer in its class $[\Sigma_n]$. An m-brane wrapping Σ_n gives a $(m-n)$-brane in the non-compact space, and has tension given by

$$T_{(m-n)} = \mathrm{vol}(\Sigma_n)$$

- For a BPS black hole/string, the brane must wrap holomorphic cycles, and then tension and volume are computed via calibration:

$$T = Z = \frac{1}{n!}\int_{\Sigma_n} J^n = \mathrm{vol}(\Sigma_n)$$

Brane tension and volumes

- However, for a non-BPS black hole/string, for which the brane wraps a non-holomorphic, locally volume-minimizing cycle, we have

$$T = \mathrm{vol}(\Sigma_n) \neq Z$$

- Therefore, computing T provides a **prediction** for the volume of the non-holomorphic, non-calibrated cycle Σ_n.

Computing the tension

- Objective: minimize V_{eff} with $\mathcal{V} = 1$.
- Constraint from effective field theory: minimum must be interior to Kähler cone, for control of EFT (some boundaries of Kähler cone are likely fine). This is what we mean by "large black object".
- Proposal: a minimum of V_{eff}, corresponding to a class $[\Sigma]$, whose moduli solution is interior to the Kähler cone, corresponds to a local volume minimizing representative $\Sigma \in [\Sigma]$. Moreover

$$\mathrm{vol}(\Sigma) = \int_\Sigma \sqrt{|g|}\, d^n x = \min\left(\sqrt{\frac{3}{2}V_{eff}}\right),$$

where one uses the appropriate V_{eff} depending on whether we consider black holes or black strings.

Definition

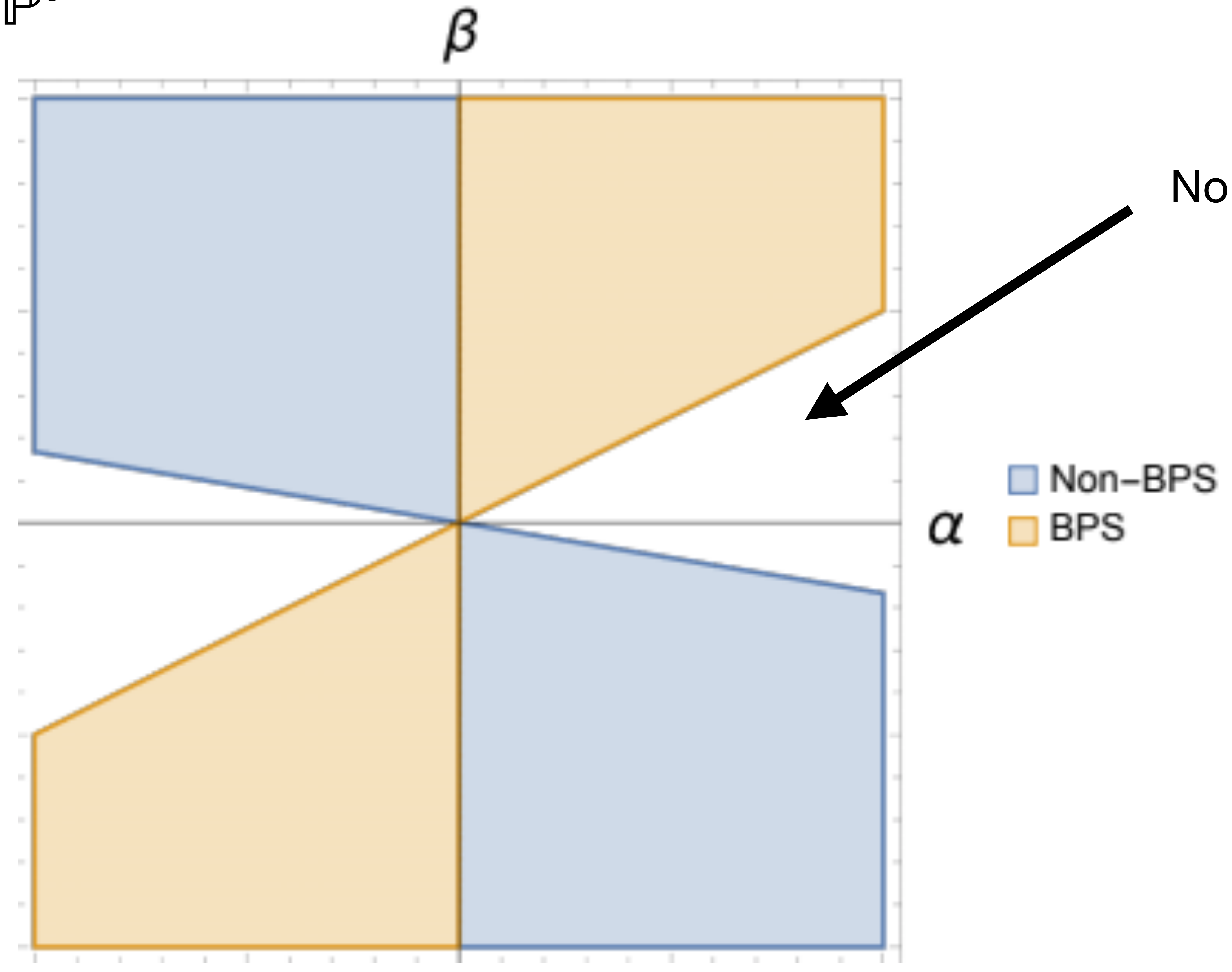
Consider an electric or magnetic charge, corresponding to a "large" even-dimensional homology class $[\Sigma]$ (e.g. $[\Sigma] = N[\Sigma_0], N \gg 1$) in a Calabi-Yau threefold X. If the corresponding black brane equations of motion are

1. solved in the strict interior of the Kähler cone, and

2. the solution is an attractor ,

we call the associated locally volume-minimizing representative Σ a ``*large black brane cycle*'' (LBBC).

Hypersurface in $\mathbb{P}^1 \times \mathbb{P}^3$



Not every “large” homology class has a large black brane!

Some characteristics of LBBCs

1. Locally-volume minimizing:
    - Correspond to black objects, which decay via classically disallowed trajectories.
2. Connected:
    - Bound states.
    - BPS case: strongly movable curves or very ample divisors.
    - Non-holomorphic curves: predicted volumes generally different from piecewise-calibrated, disconnected representatives.
3. Large in charge:
    - Correspond to black objects, and so obtained by e.g., $[\Sigma] = N\,[\Sigma_0],\, N \gg 1$.

Complex structure independence

- Volume given by effective potential, expect it to be independent of complex structure! Well-known for a holomorphic cycle, but perhaps surprising for a non-holomorphic one, even if even-dimensional.

- **Conjecture:** Consider an LBBC, Σ, in a Calabi-Yau threefold X, and let the moduli take the corresponding attractor values t_0. For these values, the volume of Σ is asymptotically independent of the complex structure moduli.

  More precisely $\lim_{N\to\infty} vol([\Sigma])/N$ is independent of the complex structure moduli (and N), where $[\Sigma] = N[\Sigma_0]$.

- Physics proof given by complex-structure independence of effective potential.

Recombination and Examples

Recombination

- Consider an even-dimensional cycle in a Calabi-Yau threefold that admit a *piecewise-calibrated* representative $\Sigma^{\cup}$, given by the union of holomorphic and anti-holomorphic cycles:

$$\Sigma^{\cup} = (\Sigma^{h}) \cup (\Sigma^{\bar{h}})$$

- Volume of $\Sigma^{\cup}$ is given by the sum of the volumes of its constituents, each of which is calibrated:

$$\mathrm{vol}(\Sigma^{\cup}) = \mathrm{vol}(\Sigma^{h}) + \mathrm{vol}(\Sigma^{\bar{h}}) = \int_{\Sigma^{h}} J^{n} + \Big| \int_{\Sigma^{\bar{h}}} J^{n} \Big|$$

Recombination

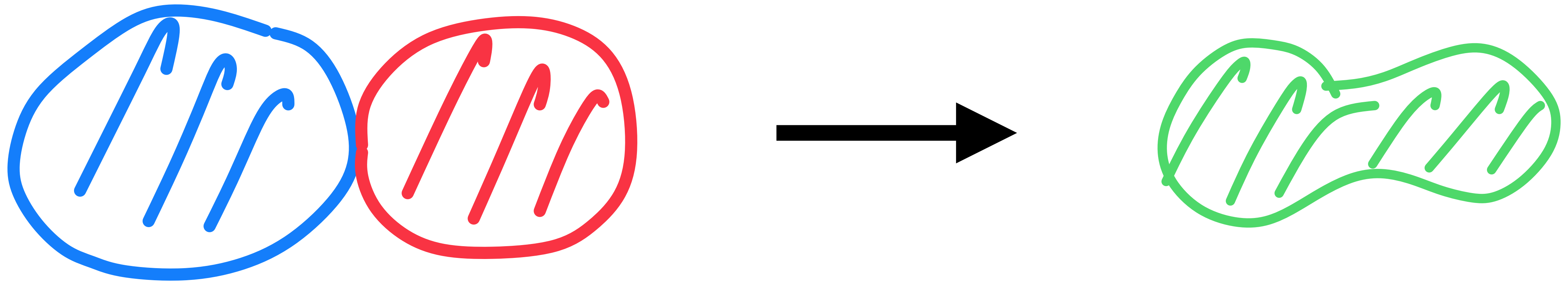
- Question: when is $\mathrm{vol}(\Sigma) < \mathrm{vol}(\Sigma^{\cup})$, for all $\Sigma^{\cup}$?

- When this happens, we say that $[\Sigma]$ undergoes *recombination*; that is, the holomorphic and anti-holomorphic constituents fuse to form a smaller cycle.

- Only explicit recombination results for CY manifolds are for K3, via Micallef-Wolfson and Sen. Prediction for recombination in CY3/CY4 using WGC.

  Demirtas, CL, McAllister, Stillman

- Can we use the attractor mechanism to identify examples of recombination?



Simple example: hypersurface in $\mathbb{P}^2 \times \mathbb{P}^2$

- Consider a generic anti-canonical hypersurface $X \subset \mathbb{P}^2 \times \mathbb{P}^2$.
- Basis of divisors $\{D_1, D_2\}$ given by restrictions of hyperplanes $\{\hat{D}_1, \hat{D}_2\}$.
- Expand Kähler form as $J = t_1 D_1 + t_2 D_2$, volume takes the form

$$\mathscr{V} = \frac{3}{2} t_1 t_2 (t_1 + t_2)$$

- Holomorphic curves in X and $\mathbb{P}^2 \times \mathbb{P}^2$ generated (over $\mathbb{Z}$) by the toric curves in $\mathbb{P}^2 \times \mathbb{P}^2$:

$$C_1 = \hat{D}_1^2 \cdot \hat{D}_2 \,,\, C_1 = \hat{D}_2^2 \cdot \hat{D}_1$$

Black holes

- Wrap M2 brane on $\alpha C_1 + \beta C_2$.
- Black hole effective potential:

$$V_{eff} = \frac{\alpha^2 t_2^2 \left(2t_1^2 + 2t_1 t_2 + t_2^2\right) - 2\alpha\beta t_1^2 t_2^2 + \beta^2 t_1^2 \left(t_1^2 + 2t_1 t_2 + 2t_2^2\right)}{t_1^2 + t_1 t_2 + t_2^2} ,$$

- Minimize while holding $\mathscr{V} = 1$.

BPS Black holes

- BPS equations of motion:

$$\beta x^2 + 2(\beta - \alpha)x - \alpha = 0\,, \qquad x = \frac{t_1}{t_2}\,.$$

- Subject to

$$\frac{3}{2}x(1+x)t_2^3 = 1$$

- For solution inside Kähler cone, need $t_2 > 0$, $x > 0$. Satisfied when

$\{\beta < 0 \;\text{ and }\; \alpha < 0\} \quad \text{or} \quad \{\beta > 0 \;\text{ and }\; \alpha > 0\}$, solutions are automatically attractor.

non-BPS Black holes

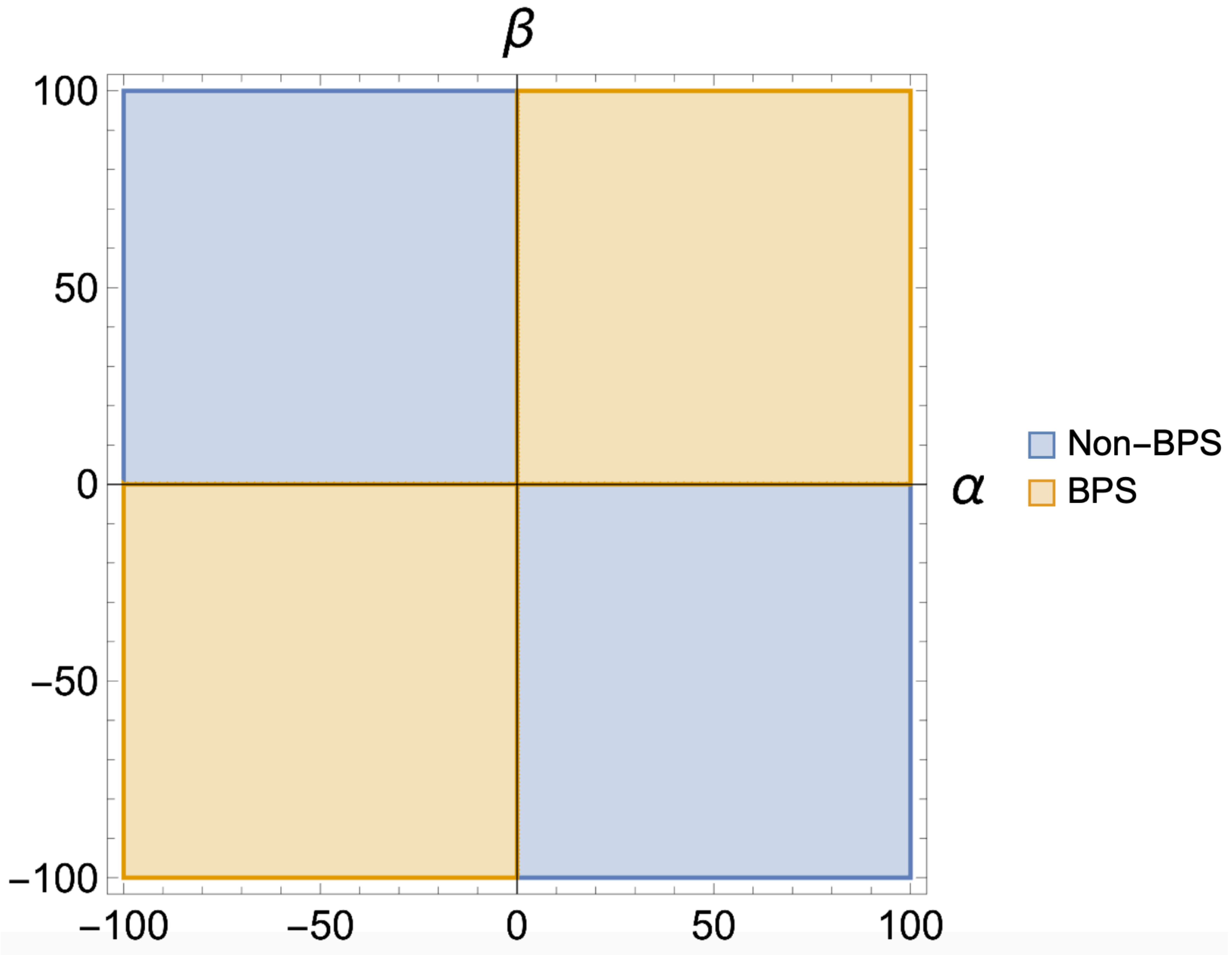
- non-BPS equations of motion:

$$2\beta x^5 + x^4(4\alpha + 3\beta) + x^3(8\alpha + 7\beta) + x^2(7\alpha + 8\beta) + x(3\alpha + 4\beta) + 2\alpha = 0$$

- Solution inside Kähler cone when

$$\{\beta < 0 \ \text{ and } \ \alpha > 0\} \quad \text{or} \quad \{\beta > 0 \ \text{ and } \ \alpha < 0\}$$

- Solutions can be checked to be attractors.



Non-holomorphic cycle volumes

- Simplest example: $\alpha = -\beta$: $[C] = \beta[C_2 - C_1]$. Moduli stabilized on symmetric locus

$$t_1 = t_2 = \left(\frac{1}{3}\right)^{1/3}$$

- Black hole mass given by

$$M = \mathrm{vol}(\Sigma) = \sqrt{2}3^{1/6}\,|\,\beta\,| \simeq 1.69\,|\,\beta\,|$$

- Volume of piecewise calibrated representative of $\beta[C_2 - C_1]$ given by

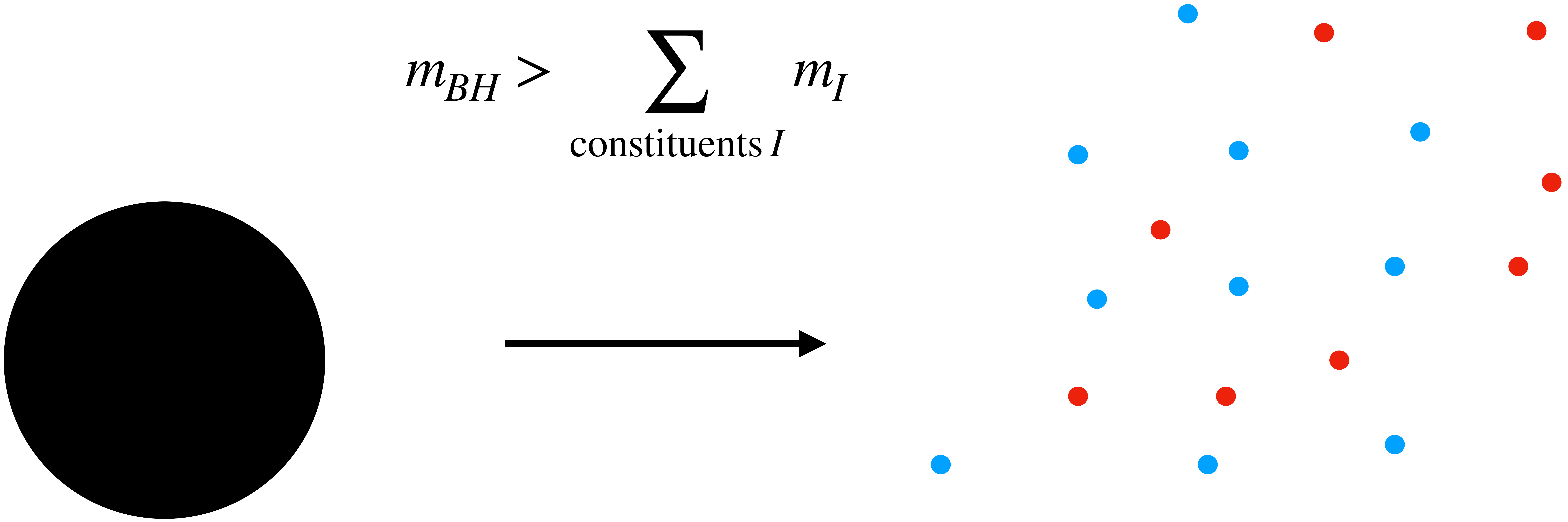
$$\mathrm{vol}(\Sigma^{\cup}) = \frac{2}{3^{1/3}}\,|\,\beta\,| \simeq 1.39\,|\,\beta\,|$$

- Therefore we find $\mathrm{vol}(\Sigma) > \mathrm{vol}(\Sigma^{\cup})$. Same result for all non-holomorphic curves!

Weak Gravity

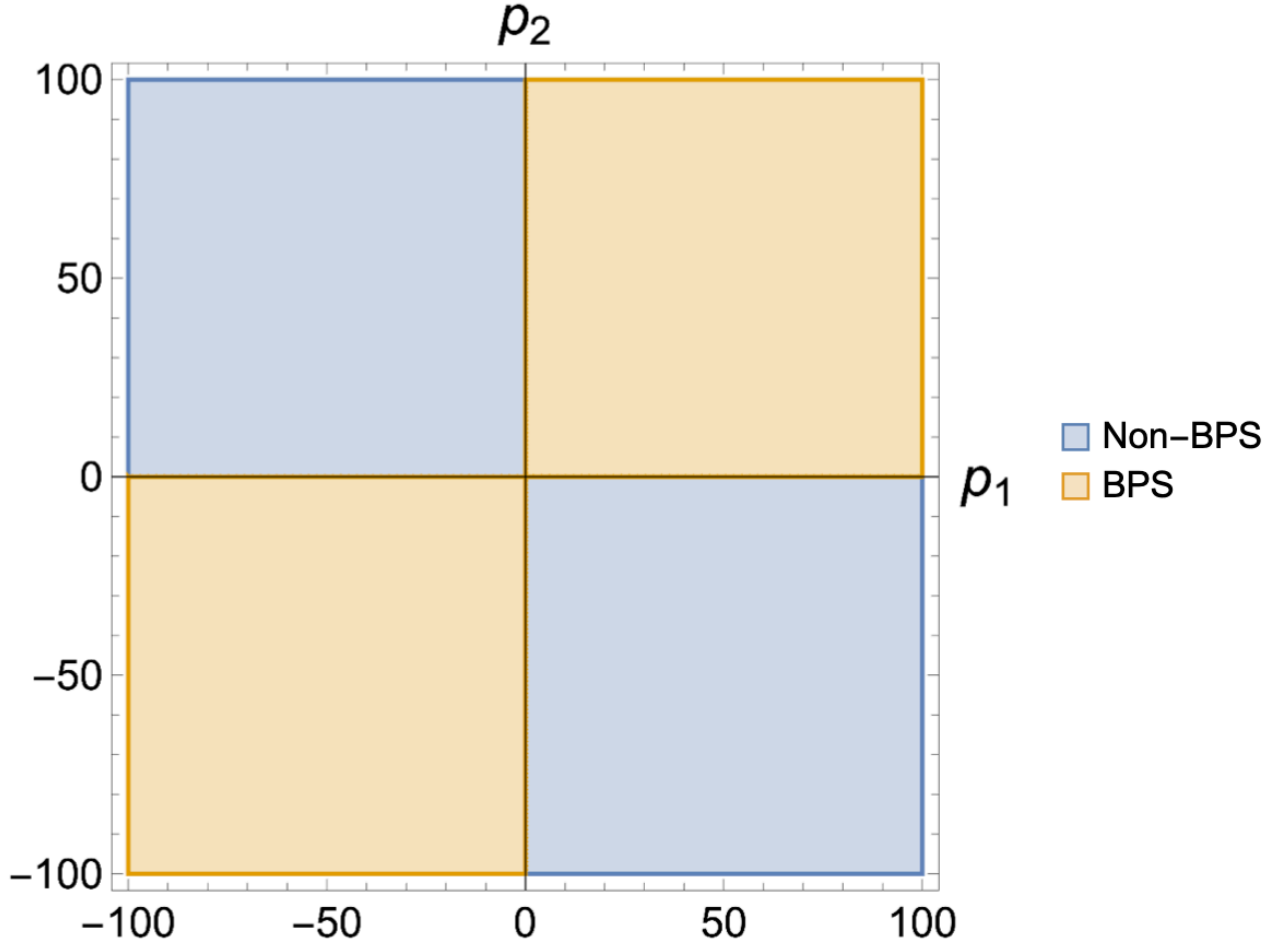
- Weak Gravity Conjecture: large non-BPS black holes can decay.
- In this example, we see WGC is satisfied: allowed decay channel is into widely separated BPS-anti-BPS constituents.

$$m_{BH} > \sum_{\text{constituents } I} m_I$$



Black strings in $X \subset \mathbb{P}^2 \times \mathbb{P}^2$

- $V_{eff} = \frac{9}{2}\left(p_1^2 t_2^2\left(2t_1^2 + 2t_1 t_2 + t_2^2\right) + 2p_1 p_2 t_1^2 t_2^2 + p_2^2 t_1^2\left(t_1^2 + 2t_1 t_2 + 2t_2^2\right)\right)$



Black strings in $X \subset \mathbb{P}^2 \times \mathbb{P}^2$

- Again consider symmetric locus $p_1 = -p_2$. Moduli again stabilized on symmetric locus $t_1 = t_2 = \left(\frac{1}{3}\right)^{1/3}$.

- Tension of non-BPS black string takes the form

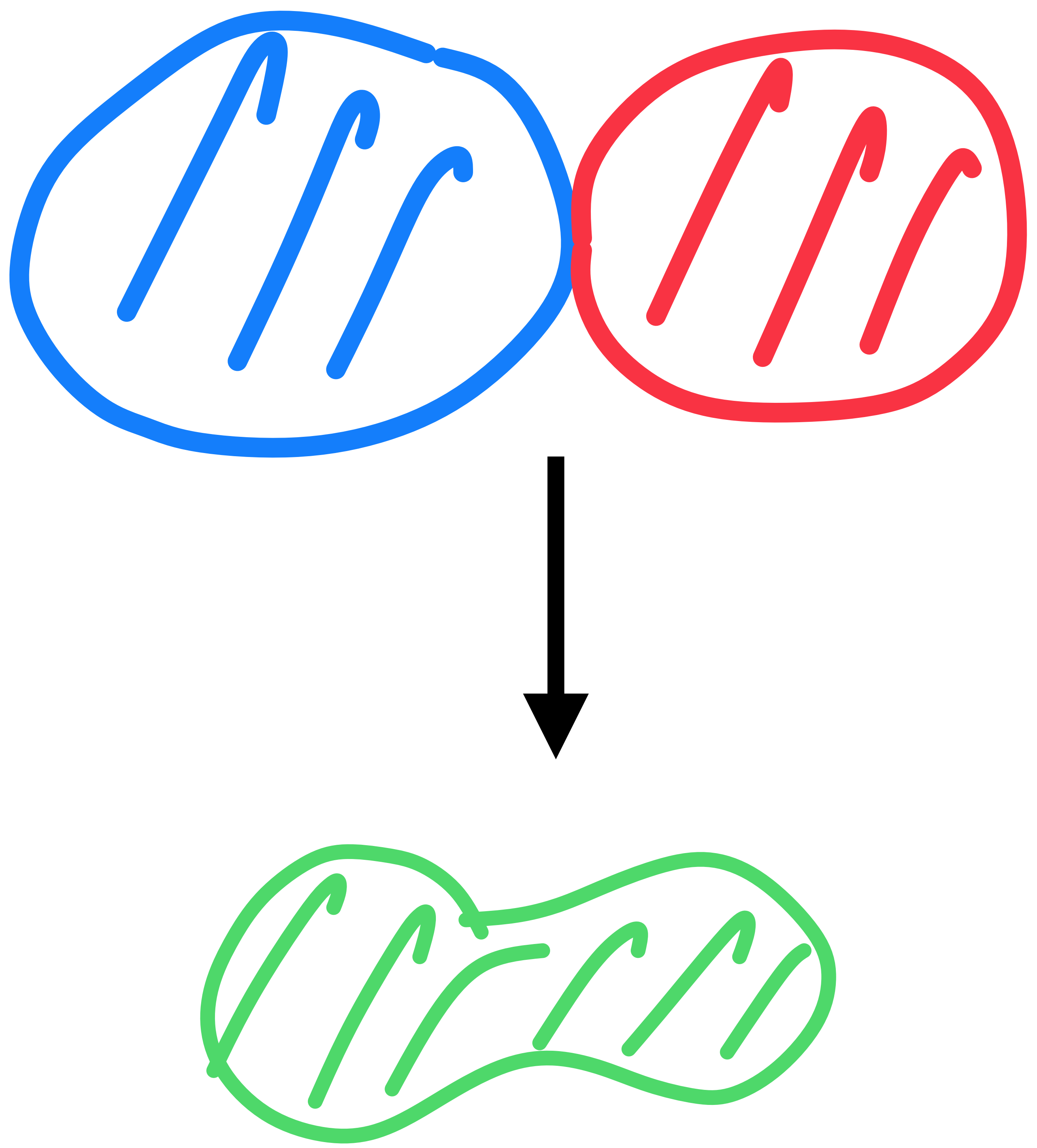
$$T = \mathrm{vol}(\Sigma) = \sqrt{2}3^{5/6}|p_2|.$$

- Volume of minimal piecewise-calibrated representative is

$$\mathrm{vol}(\Sigma^{\cup}) = 3^{4/3}|p_2|.$$

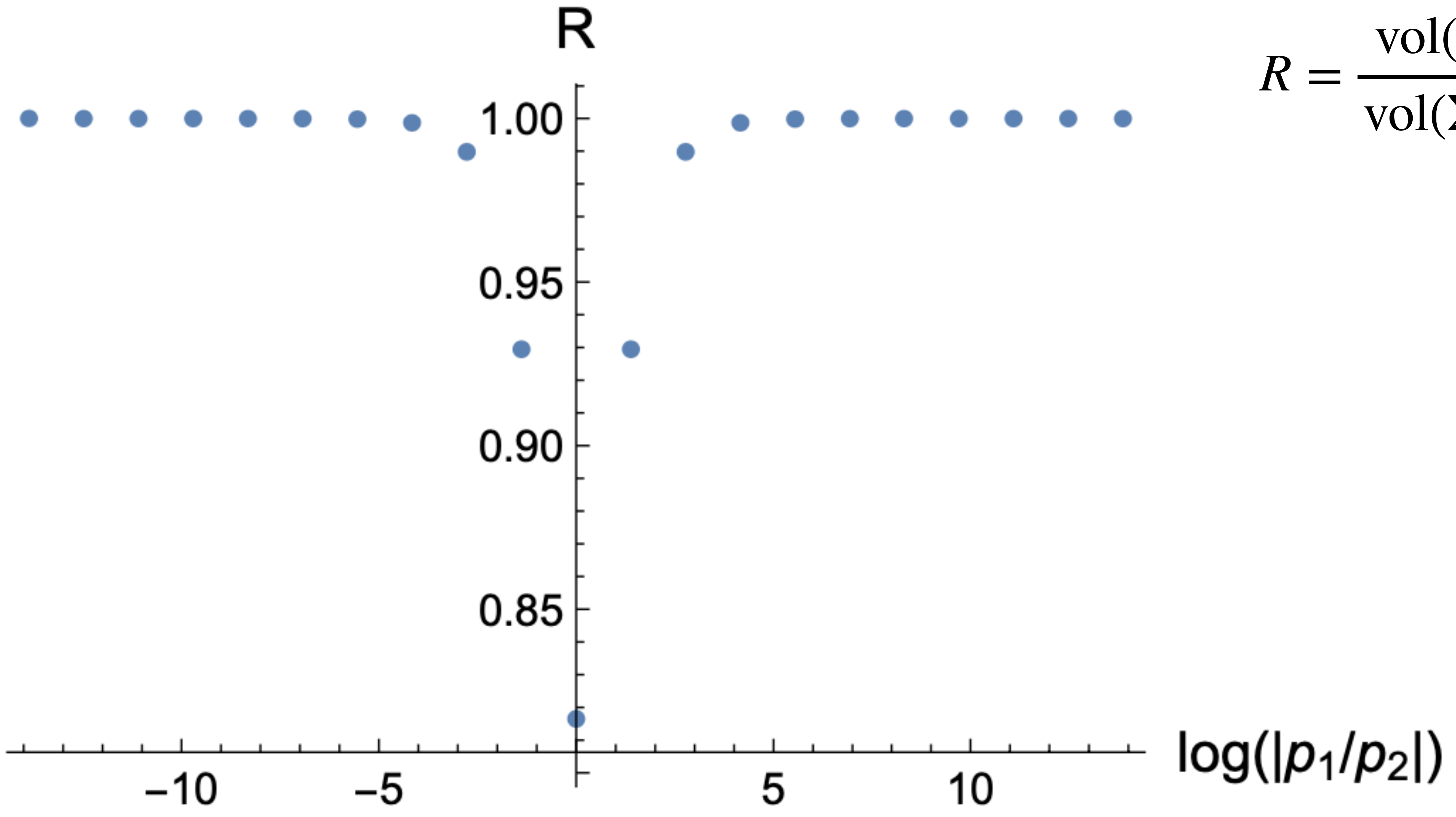
- Ratio is $R = \frac{\mathrm{vol}(\Sigma)}{\mathrm{vol}(\Sigma^{\cup})} = \sqrt{\frac{2}{3}}$, we find recombination

  in the class $p_2[D_2 - D_1]$!



Black strings in $X \subset \mathbb{P}^2 \times \mathbb{P}^2$

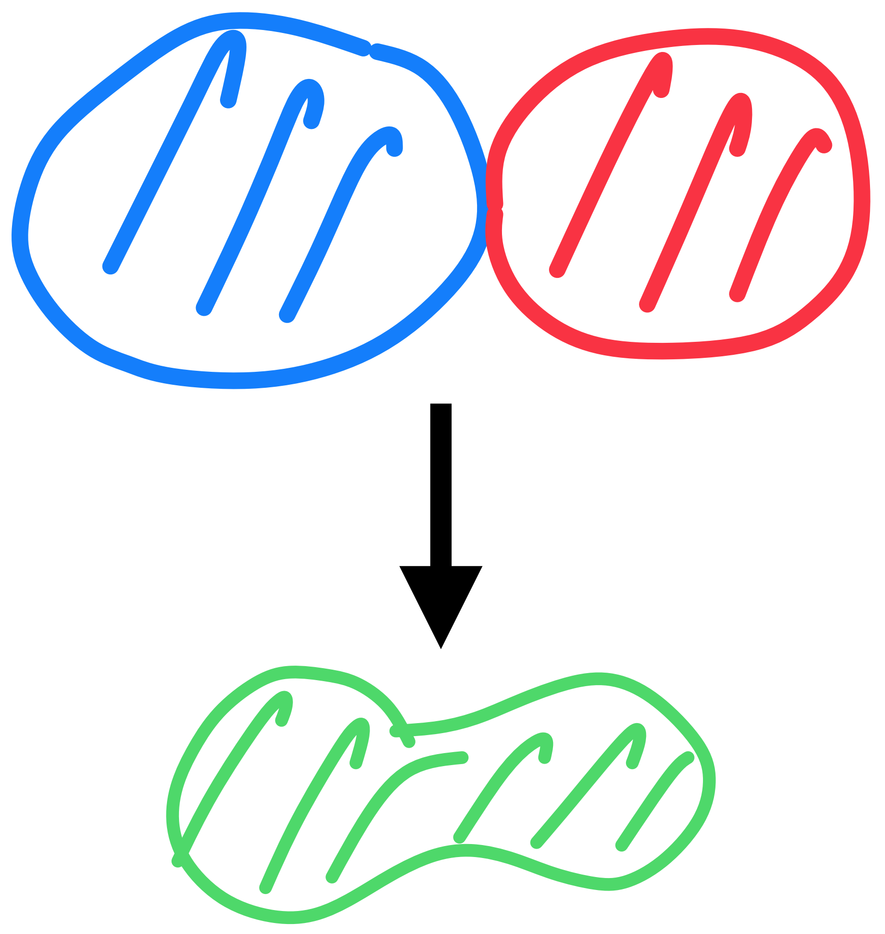
- Find that, on the symmetric locus, the divisor $n(D_1 - D_2)$, $n \gg 1$, recombines, via black string physics.
- Also find recombination for all non-holomorphic divisors, for their attractor moduli.



$$R = \frac{\mathrm{vol}(\Sigma)}{\mathrm{vol}(\Sigma^{\cup})}$$

Weak Gravity

- Black strings that exhibit recombination cannot decay completely to BPS-anti-BPS constituents, via conservation of energy.
- WGC then predicts the existence of a stable, microscopic non-BPS string.
- Two options: recombination of small cycles, or bound state.



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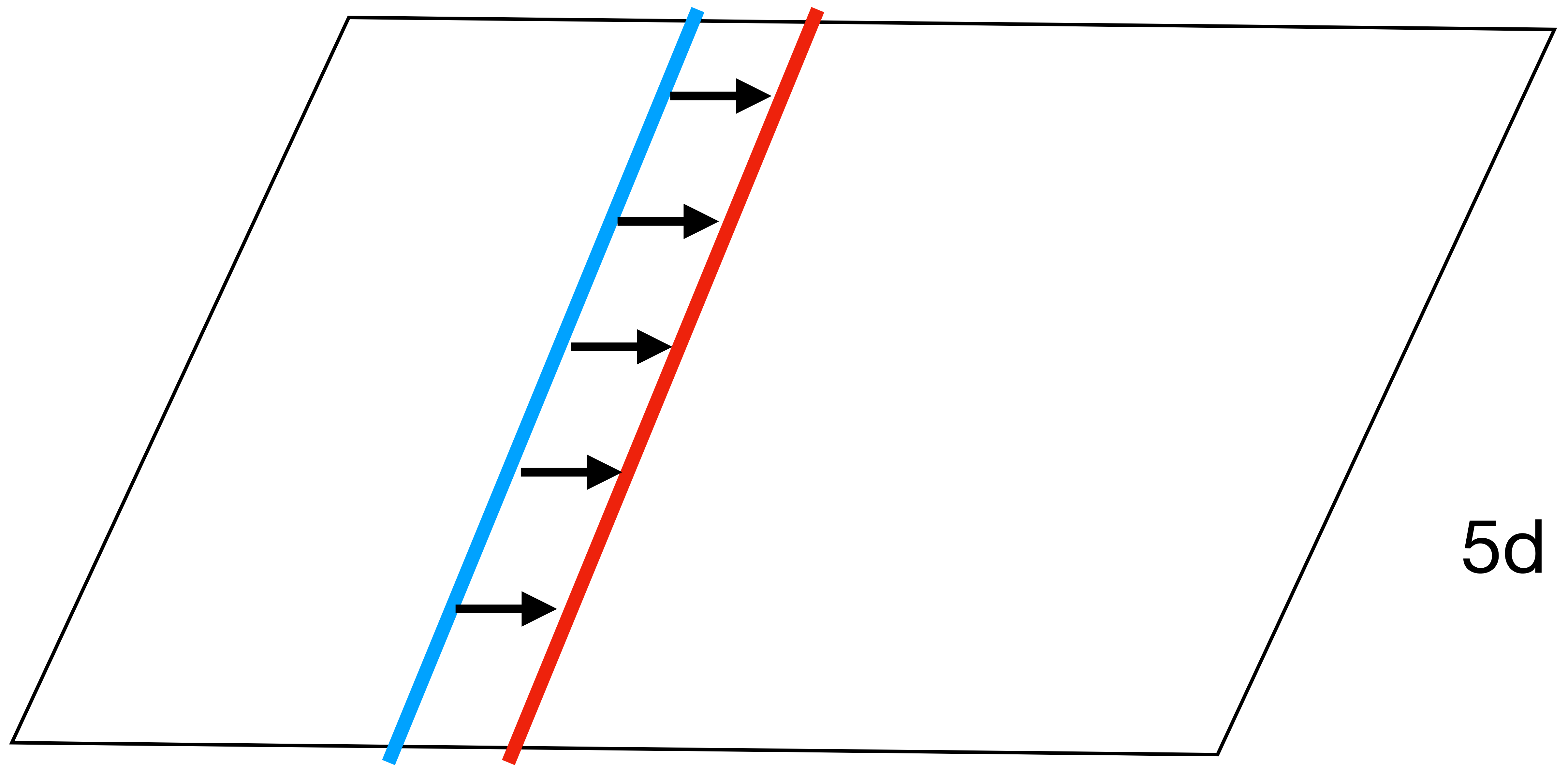
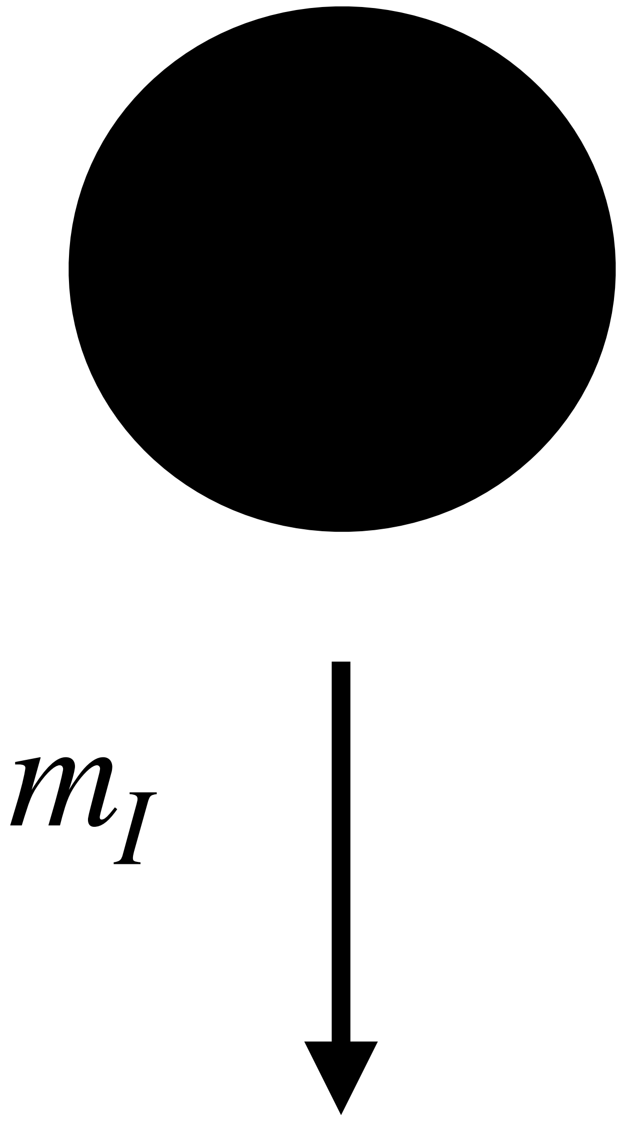


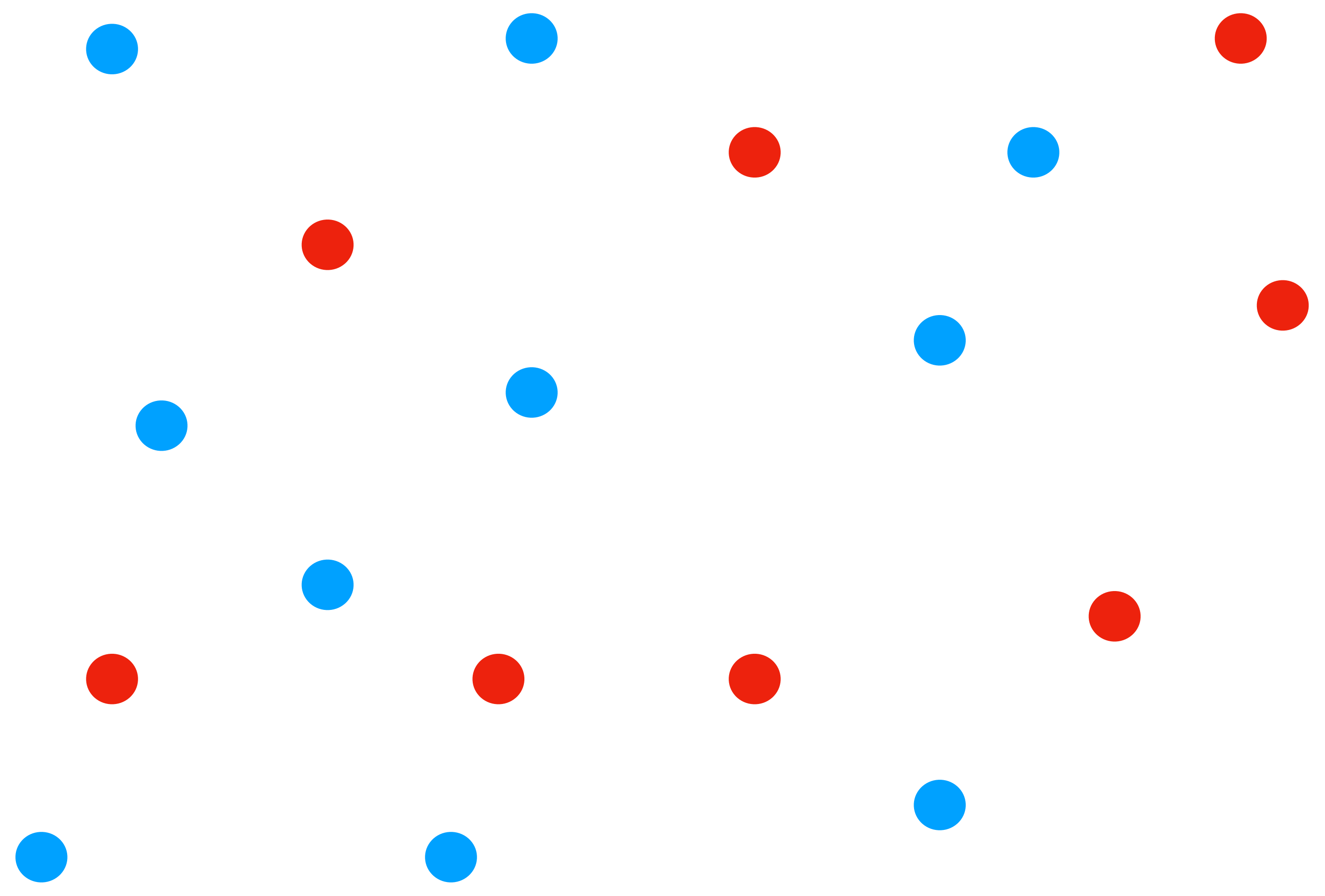
Black holes vs. Black Strings

- Find different behavior for black holes and black string, appears to be generic for examples studied
- For BHs studied CY hypersurfaces in 124 smooth Fano toric fourfolds, for which we know the generators of effective curves. Also looked at smooth elliptic fibrations. Found no black holes whose mass was SMALLER than the minimal piecewise-calibrated representative!
- Evidence for WGC for non-BPS black holes. Allowed decay channel into BPS and anti-BPS constituents.



$$m_{BH} > \sum_{\text{constituents } I} m_I$$

Skauli



Black holes vs. Black Strings

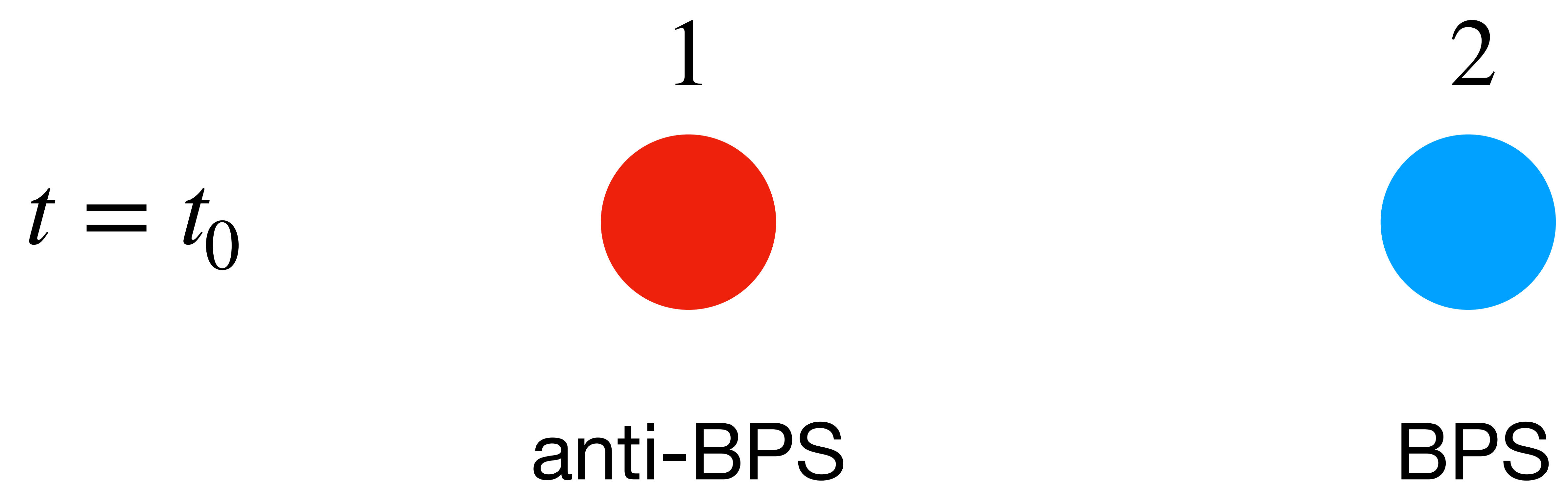
- Black strings are different. Found recombination in Calabi-Yau hypersurface in $\mathbb{P}^2 \times \mathbb{P}^2$ (and in $(\mathbb{P}^1)^4$).
- Also found examples where tension of black string was **GREATER** than minimal piecewise-calibrated, so black strings exhibit both types of behavior.
- Possible difference: divisors generically intersect, so expect localized modes on intersection of holomorphic and and anti-holomorphic branes. Condensation could lead to recombination. **See Sen's K3 analysis**
- Recombined strings predict the existence of stable non-BPS states, via the WGC.

Recombination and Forces

- So far have used black branes to search for recombination via direct computation, but can the physics explain why cycles might or might not recombine?

- Black hole formation process might. Consider separating the black hole charge into BPS and anti-BPS constituents, corresponding to M2 branes on holomorphic and anti-holomorphic curves.

- Set moduli to attractor values, bring a BPS particle and an anti-BPS particle close together, and compute the total (gravity, EM, scalar) force between them.



Electric: $F \sim G^{IJ} q^1_I q^2_J$

Magnetic: $F \sim G_{IJ} p^I_1 p^J_2$

Recombination and Forces

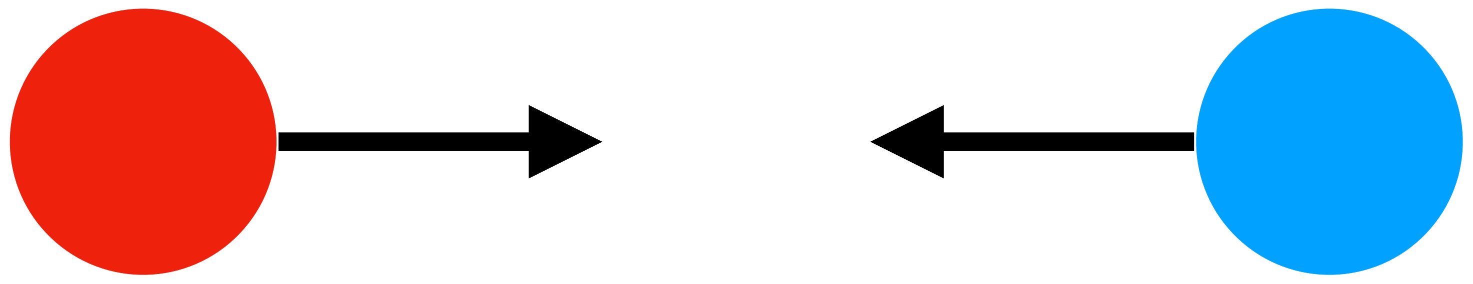
- If force is repulsive, need to put energy into the system to form black holes, so expect black hole mass to be greater than volume of piecewise-calibrated representative.

- On the other hand, if force is attractive, expect system to radiate energy as black hole forms, and so might find recombination.

- Find that all examples with recombination have attractive forces, all examples without recombination have repulsive forces, EXCEPT ONE. Consistency of this one example then predicts some interesting behavior.

Small curve recombination

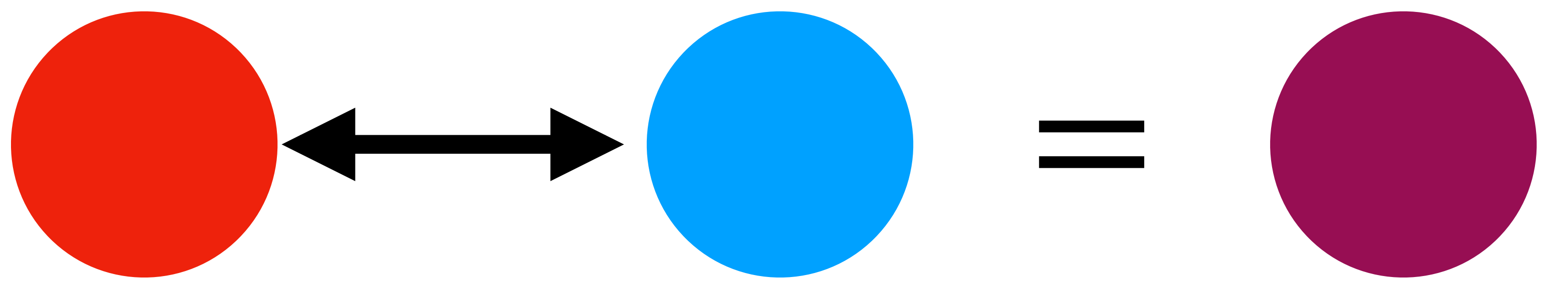
- Let X be a generic CY elliptic fibration over $\mathbb{P}^1 \times \mathbb{P}^1$. Consider the curve class $\Sigma = C_1 - C_2 + E$, where C_α are the respective $\mathbb{P}^1$ curves, and E is the fiber.
- Form a non-BPS black hole from wrapping $N \gg 1$ M2 branes on Σ. Large non-BPS solution exists.
- Mass of black hole greater than sum of masses of BPS-anti-BPS constituents.
- However, in the attractor background the force between M2 branes on $C_1 + E$ and M2 branes on $-C_2$ is attractive! This presents a puzzle.



$-C_2$ $\qquad\qquad$ $C_1 + E$

Small curve recombination

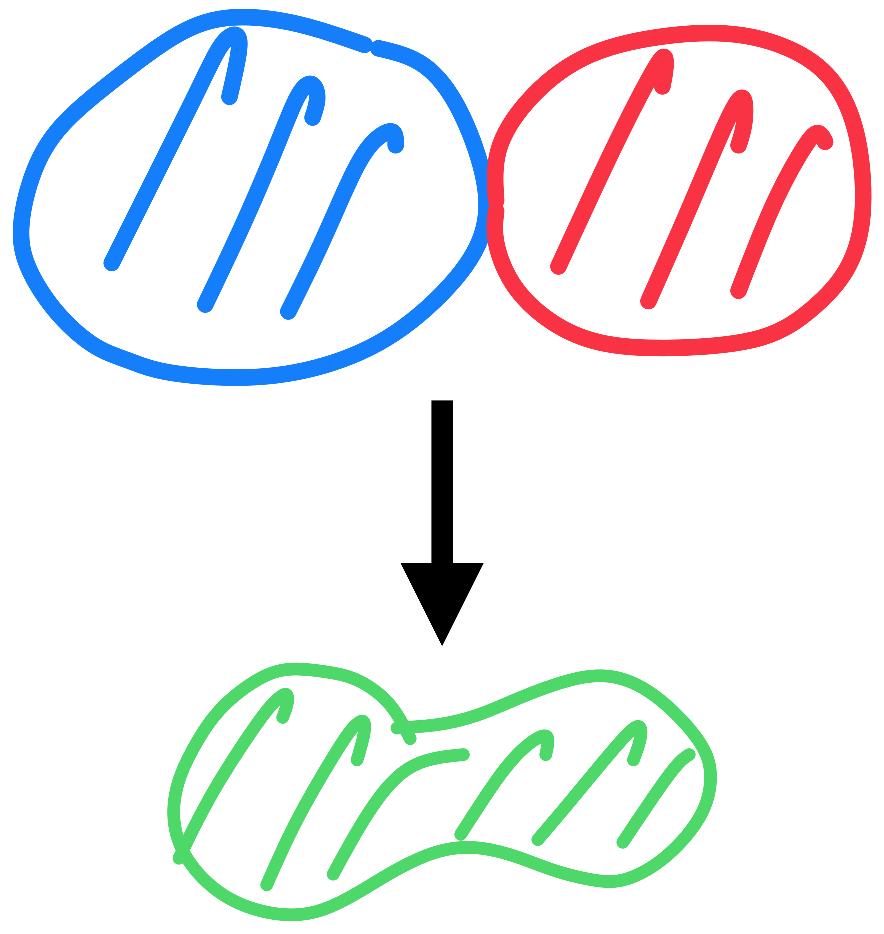
- Resolution: microscopic M2 branes on $C_1 + E$ and on $-C_2$ form a bound state B.



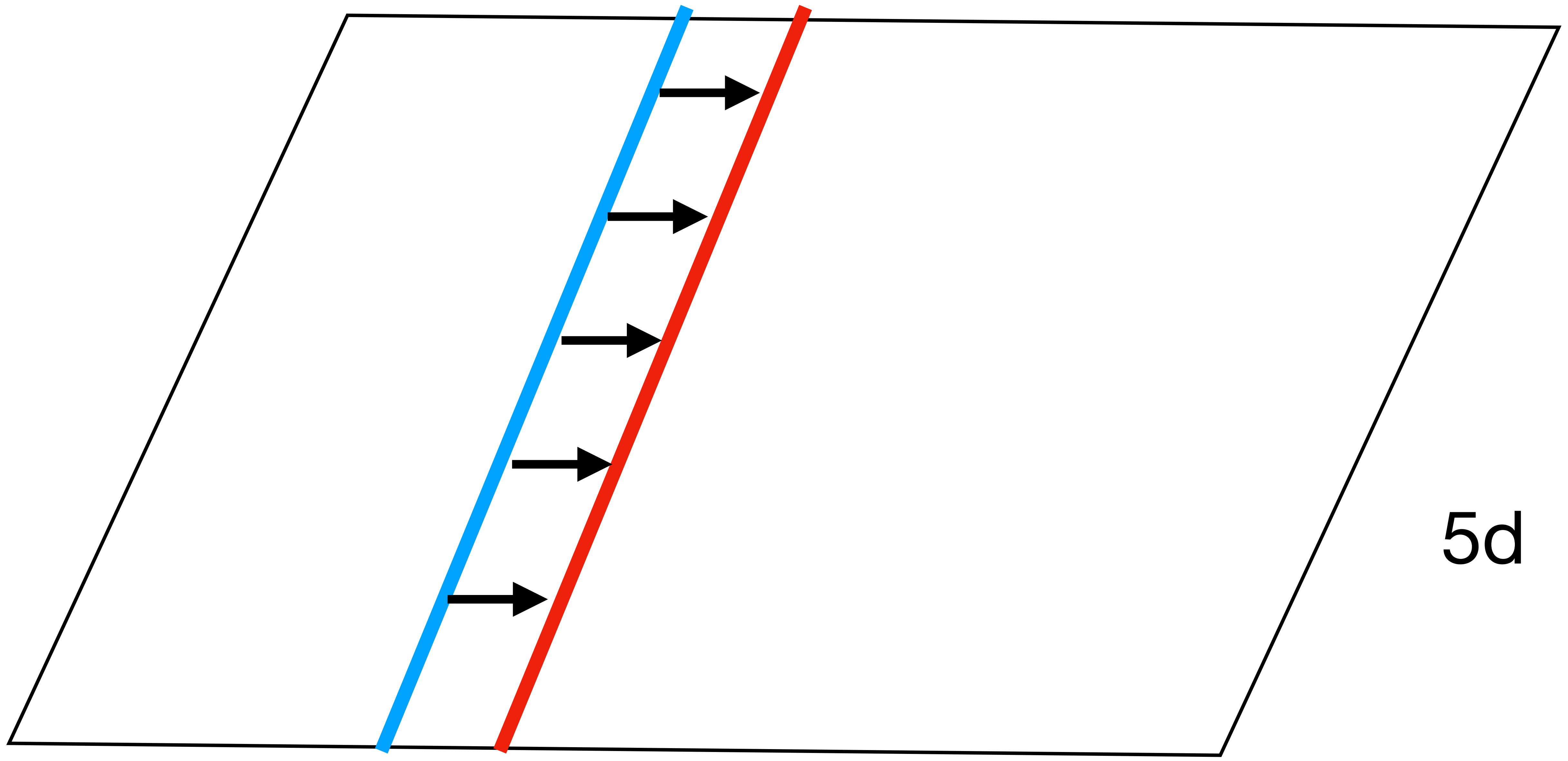
- Binding energy enough so that B is self-repulsive, and so to form a black hole out of many B's need to put energy into system.

Small curve recombination

- Bound state can either be from a 5d bound state, or from recombination of $C_1 + E$ and $-C_2$.



OR

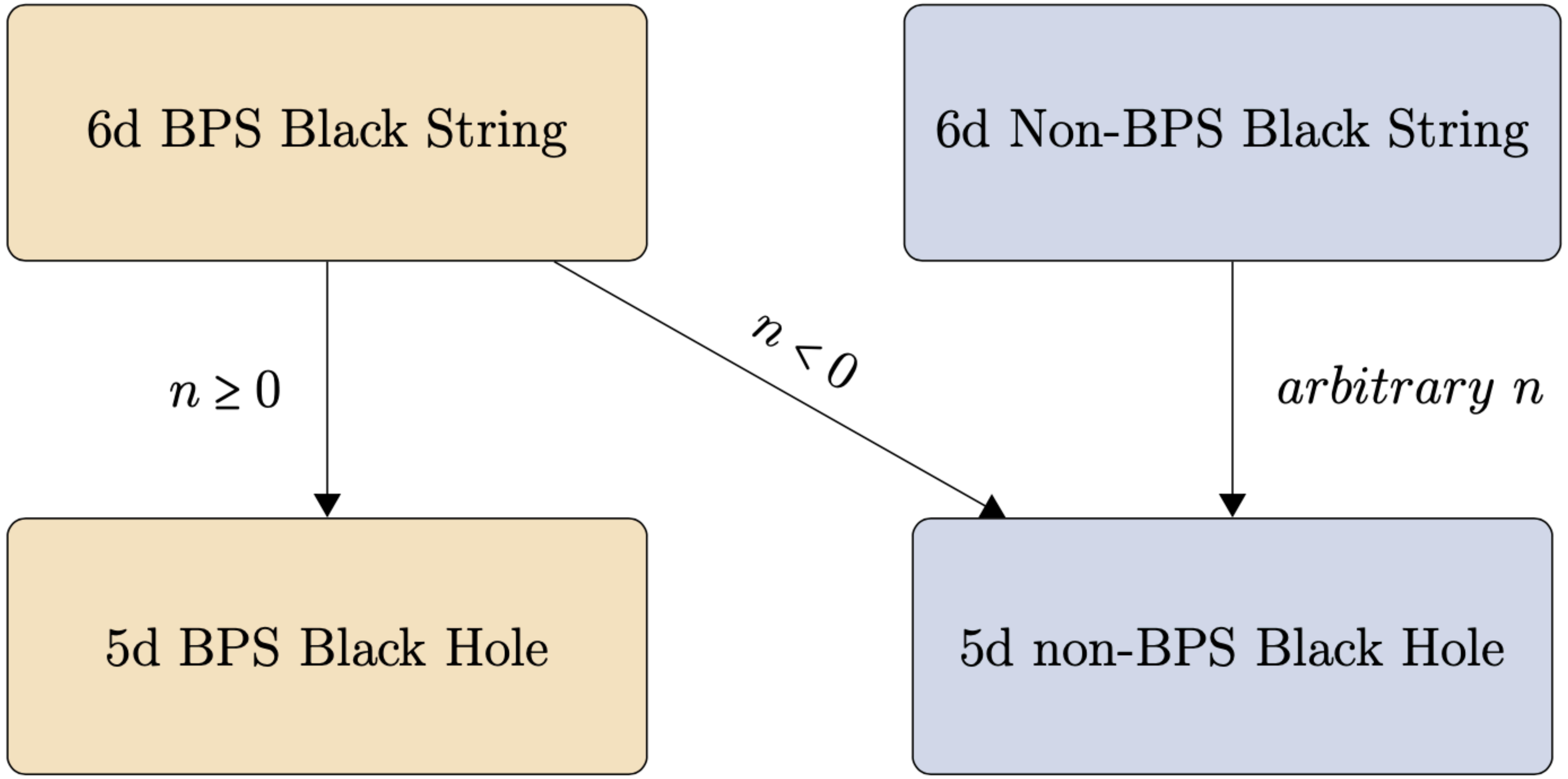


- Mathematical support for recombination: $[C_1 - C_2] \subset \mathbb{P}^1 \times \mathbb{P}^1$ recombines with FS metrics on $\mathbb{P}^1$ factors when ratio of Kähler classes is 3/2. Our ratio is instead 3, so a good example to check for recombination. Arezzo, La Nave

Central Charges of Non-BPS Strings

5d Black holes from BPS and non-BPS Strings

- So far focused on measure-theoretic aspects, but also can use these supergravity techniques to learn about the theory on the non-BPS branes.
- Compute central charge of BPS and non-BPS black strings via attractor mechanism and Brown-Henneaux formula in near-horizon (AdS3) limit.
- Circle reduction yields 5d black holes, compute central charge via entropy and Cardy formula. Results agree!



6d black strings

- $\mathcal{S}_6 = \int_{\mathcal{M}_6} \left[\frac{R}{2} * 1 - \frac{1}{4} g_{\alpha\beta} H^\alpha \wedge * H^\beta - \frac{1}{2} g_{\alpha\beta} dt^\alpha \wedge * dt^\beta \right]$ via IIB on 4d base B .
- $H^\alpha = dB^\alpha$.
- 6d black strings from D3 branes wrapping $C \subset B$.
- Near horizon geometry $AdS_3 \times S^3$.
- Effective potential for black strings (holding $\mathcal{V}_B = 1/2$):

$$V_{eff} = g^{\alpha\beta} q_\alpha q_\beta = \left(-\Omega^{\alpha\beta} + \frac{t^\alpha t^\beta}{\mathcal{V}_B} \right) q_\alpha q_\beta \quad .$$

Intersection matrix

6d BPS black strings

- BPS equation of motion:

$$t^{\alpha} = \frac{2\Sigma^{\alpha}\mathscr{V}_B}{(t^{\lambda}q_{\lambda})} \, ,$$

Wrapping numbers

which implies D3 branes wrap an ample curve in the base.

- Gives minimum of effective potential (positive-definite Hessian).

- Find $\boxed{V_{eff} = (C \cdot C)}$

(self-intersection of curve wrapped by D3 branes), valid for curves with positive self-intersection, but not all such curves give valid solutions!

6d non-BPS black strings

- non-BPS equation of motion:

$$t^\lambda q_\lambda = 0\,,\ \ \mathscr{V}_B = \frac{1}{2}\,.$$

- Gives a minimum with $h^{1,1}(B) - 2$ flat directions.
- Find $\boxed{V_{eff} = -(C \cdot C)}$

  (negative self-intersection of curve wrapped by D3 branes), only valid for negative-self intersection curve, including many non-holomorphic curves in B.

- Again, not all negative self-intersection curves work.

The central charge

- For both the BPS and non-BPS case, horizon value of effective potential is

$$V_{eff} = |C \cdot C|$$

- Brown-Henneaux formula: $c = \dfrac{3l_{AdS}}{2G_3}$ (or central charge function extremization)

Kraus, Larsen

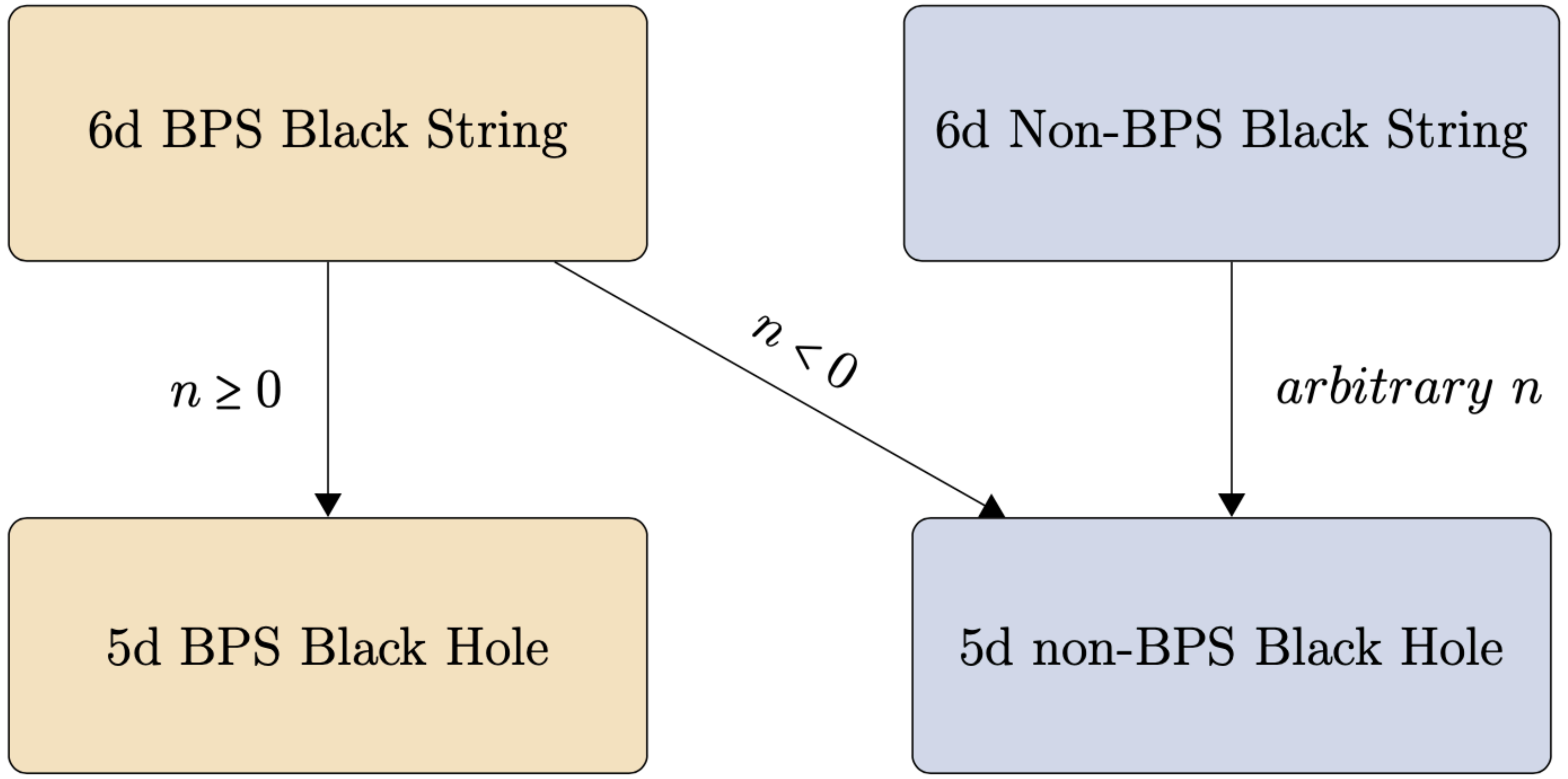
- Find

$$\boxed{c = 3|C \cdot C|,}$$

which agrees with the known BPS case, and gives a simple extension to the non-BPS case.

6d to 5d

- Compactify on circle to arrive at a 5d black hole, with n units of momentum around the circle.
- Arrive at 5d black holes with same charges by considering an M2 brane wrapping $C \subset B$, and elliptic fiber n times.
- Whether or not black hole is BPS depend on 1) if the 6d string was BPS and 2) relative momentum around the circle: $n \geq 0$ BPS, $n < 0$ non-BPS.



5d black holes from 6d BPS string

- First consider ample curve $C \subset B$, wrap fiber n times. Equations of motion for black hole:

$$t^\alpha = \frac{2V_B \Sigma^\alpha}{Z_B}, \; t^\alpha \text{ base Kähler moduli.}$$

$$V_B = \left(\frac{2n^2}{(C \cdot C)} \right)^{1/3}$$

- In agreement with 6d EOM for relative values of moduli.

- Doesn't depend on sign of n: $n \geq 0$ is BPS, $n < 0$ is non-BPS, interpreted as a non-BPS excitation of a BPS string (right-moving momentum around the circle instead of left).

5d black holes from 6d BPS string

- Entropy takes the form

$$S = \sqrt{2}\pi\sqrt{|n|(C \cdot C)}$$

- Matching with the Cardy formula $S = 2\pi\sqrt{\frac{c|n|}{6}}$, find

$$\boxed{c = 3C \cdot C,}$$

in agreement with 6d calculation.

- Via D3 brane on C, leading central charges is

$$c_L = 3C \cdot C\,,\, c_R = 3C \cdot C\,,$$

agrees with 6d calculation for both signs of n!

5d black holes from 6d non-BPS strings

- Now wrap a non-holomorphic curve $C \subset B$, EOM give

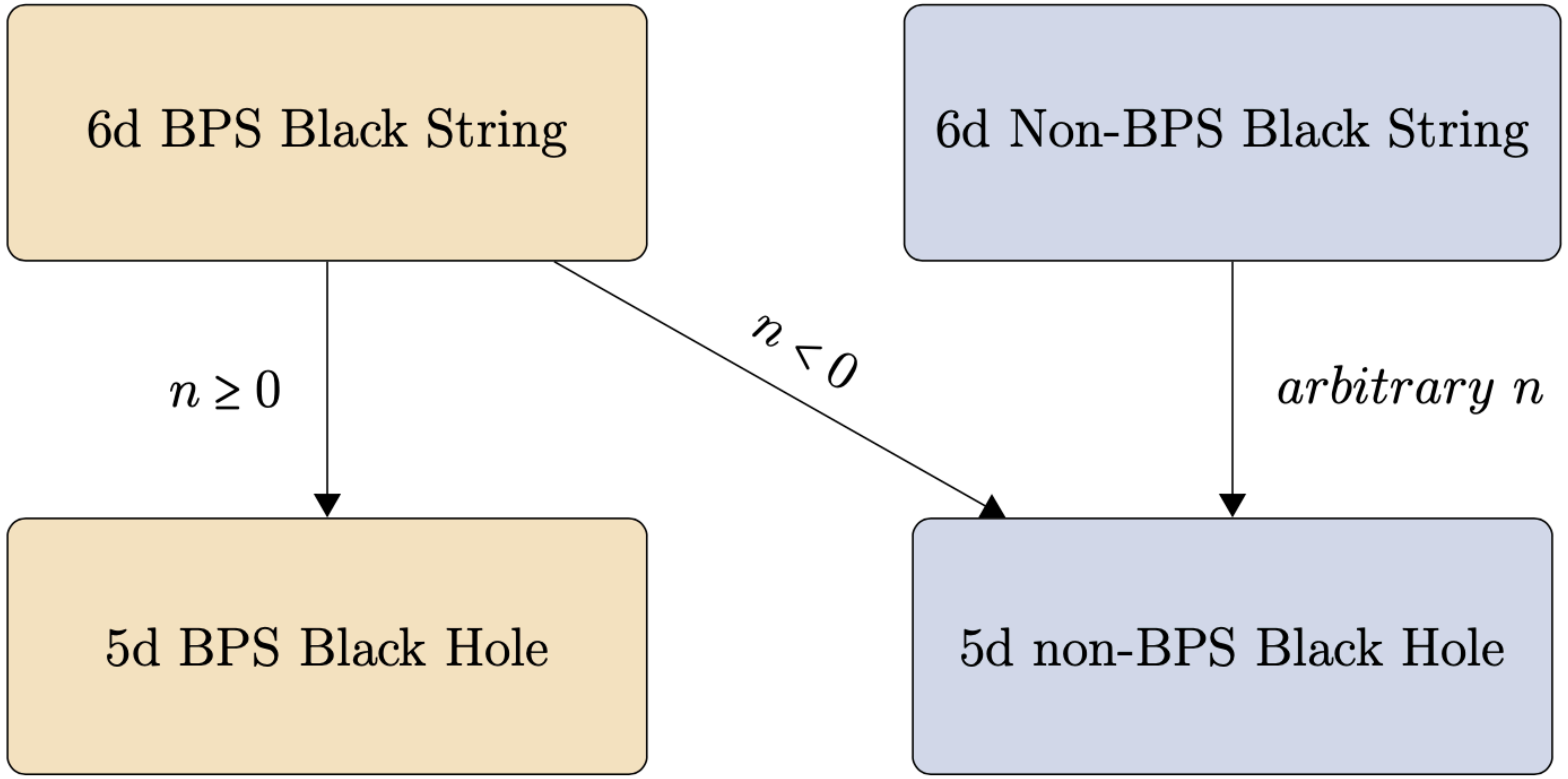
$$t^\alpha q_\alpha = 0\,,\; V_B = \left(-\frac{2n^2}{C \cdot C}\right)^{1/3}\,,$$

- Again $h^{1,1}(B) - 2$ flat directions, in agreement with 6d calculation!
- Valid for $C \cdot C < 0$.
- Via entropy and Cardy, find $\boxed{c = 3\,|\,C \cdot C\,|\,,}$ in agreement with 6d.

Summary: Central Charges

- Compute central charge of non-BPS string from 6d and 5d perspectives.
- Results agree, simple extension of BPS case:

$$c = 3\,|C \cdot C|$$



Summary: Minimal Cycles

- Used the attractor mechanism to compute tension of non-BPS black holes and black strings, corresponding to M2 and M5 branes wrapping non-holomorphic curves and divisors, respectively.
- This in turn provides a conjectural formula for the volumes of locally volume-minimizing, connected representatives of their corresponding homology classes.
- Black hole/curve case: found a great deal of evidence for WGC, since mass of black hole was always greater than sum of masses of BPS-anti-BPS constituents.
- Black string/divisor case: found examples that exhibit recombination, holomorphic-anti-holomorphic constituents fuse to make a smaller cycle. WGC predicts the existence of stable microscopic non-BPS string!

Thanks!