

## Non-Rigid Registration and Atlases in Medical Image Analysis

Colin Studholme  
Associate Professor in Residence  
<http://radiology.ucsf.edu/bicg>

Department of Radiology  
University of California San Francisco

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## Non-Rigid Registration in Medical Image Analysis

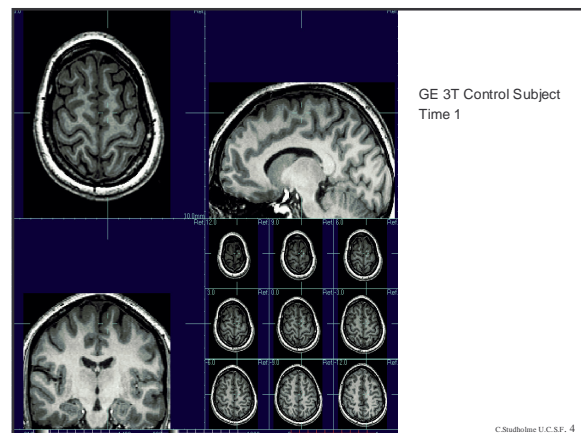
- Where is Non rigid registration needed in Medical Imaging?
- How do we describe image deformations?
  - Global Parametric Approaches
  - Dense Fields Approaches:
  - 'Physical' Models
  - Large Deformation Models:

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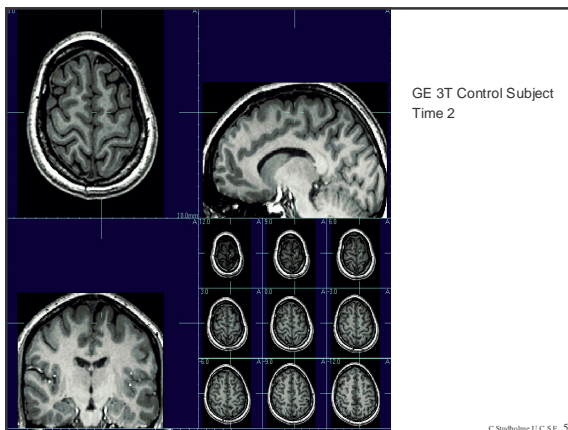
## Uses for Non-Rigid Registration

- Correcting/Accounting for Imaging Distortions
  - Scanner Induced Geometric changes
- Correcting Tissue Deformations
  - Subject Related Anatomical Changes
- Capturing Tissue Growth or Loss within a Subject
  - Studying Dementia or Tissue Growth: Deformation Based Morphometry
- Resolving Differences Between Subjects:
  - Spatial Normalization to Compare Image Data Across Populations

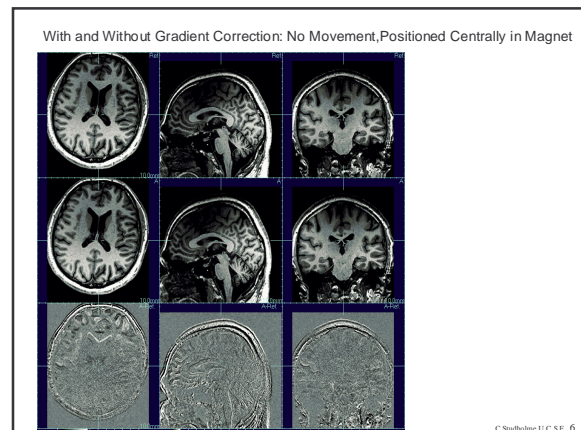
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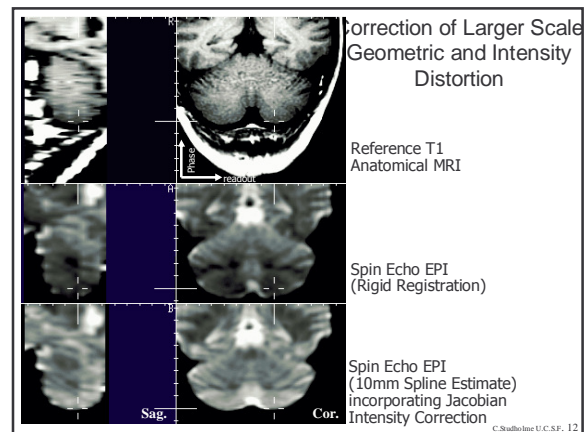
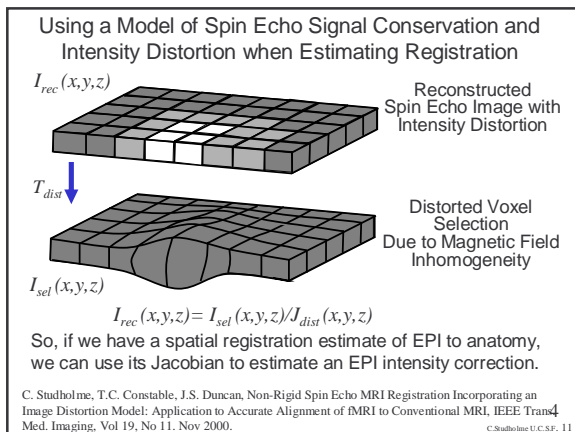
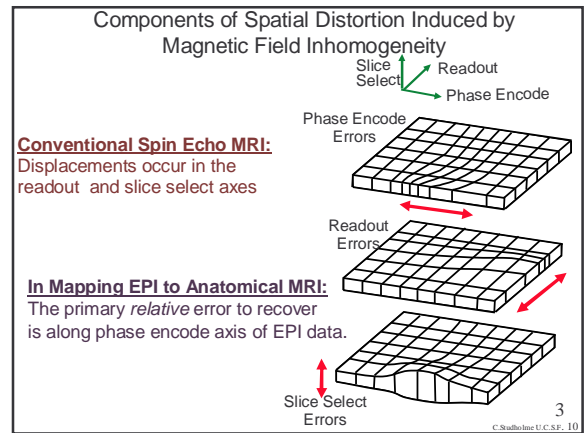
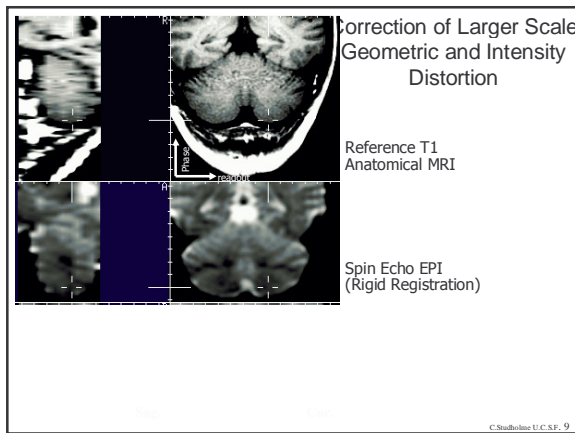
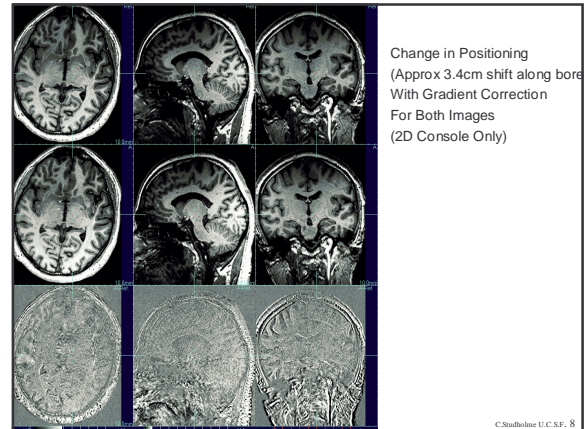
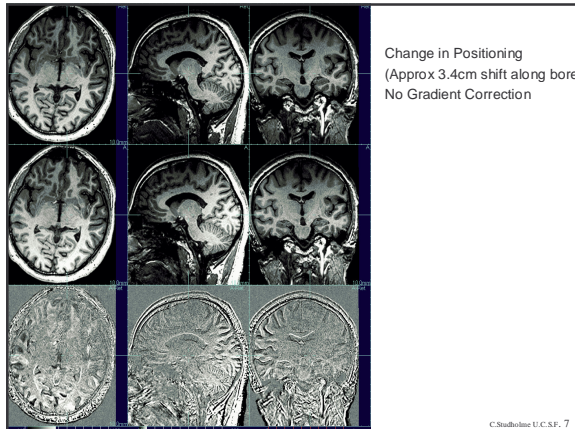
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### Spatial Normalisation: Bringing Image data into a Common Coordinate System

- Collecting data from different individual anatomies not trivial
- Need to locate corresponding location in atlas for a given measurement in the subject anatomy
- Need a Template (in atlas space) to match subject anatomy to Statistical Atlas
  - combine

- How do we derive a correspondence or mapping?
  - Estimate the warp that takes us from template to subject
- Need a [non-rigid] Registration algorithm for -> Spatial Normalisation

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### Example Group Spatial Normalization

(Studholme et al, proc. SPIE medical imaging 2001)

Global Normalized Mutual Information driven B-Spline Spatial Normalisation of Brain Anatomy

Compare Transformations

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### Non Rigid Registration

- Components of a non-rigid registration algorithm
  - Model or parameterization of the Transformation  $T$ 
    - What structural differences we can resolve
  - Registration (similarity) measure  $S(T)$ 
    - provide an absolute or relative measure of the quality of match
  - [Geometric Constraints]  $C(T)$ 
    - prevent unwanted or physically meaningless deformations
- so... need to vary  $T$  to find Optimum (here maximum) value for
 
$$F=S(T) - \alpha C(T)$$
- Optimization Method
  - Continuous refinement of many parameters
  - Often high dimensional search space
  - Constrained by corresponding spatial structures

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### Image Warping: How to model deformations between Images?

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### Mathematical Models for Spatial Transformations of Image Data

- Global Affine
- Non-Linear Global Parameterizations
- Spatially Local Parameterizations
- Dense Field Techniques

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### Deformation Models For Non-Rigid Registration

- Simplest Methods:
  - Use Global Linear or Affine model
  - Describing only global
    - Translations, Rotations,
    - Scaling and Skew

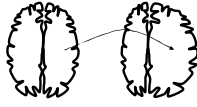
$$x' = a * x + t$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & tx \\ a_{21} & a_{22} & a_{23} & ty \\ a_{31} & a_{32} & a_{33} & tz \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

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### Deformation Models For Non-Rigid Registration

- More complex deformations: Globally Parameterize the Deformation Field: e.g. Polynomial function of location (here 1D)  
 $T(x) = a \cdot x^3 + b \cdot x^2 + cx + d$



- Modify Parameters  $a, b, c$  and  $d$  so global similarity  $F(T)$  maximized  
 [Woods et al, Automated Image registration, JCAT 1998.]

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### Deformation Models For Non-Rigid Registration

#### Cosine Basis Functions

$$T(x_i) = x_i - \sum_{j=1..J} t_{jd} b_{ij}(x_i)$$

$$T(y_i) = y_i - \sum_{j=1..J} t_{jd} b_{ij}(y_i)$$

$$T(z_i) = z_i - \sum_{j=1..J} t_{jd} b_{ij}(z_i)$$

where:

$$b_{m1}(x_i) = 1/\sqrt{M} \text{ for } m=1 \dots M$$

$$b_{mj}(x_i) = \sqrt{2}/\sqrt{M} \cos[\pi(2m-1)(j-1)/2M]$$

$$j=2 \dots J, m=1 \dots M$$

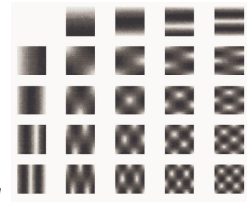


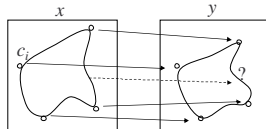
Figure 1. The lowest-frequency basis functions of a two-dimensional discrete cosine transform.

[Ashburner and Friston, Nonlinear Spatial Normalisation Using Basis Functions, Human Brain Mapping, 7:254-266, 1999]

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### Radial Basis Functions

- Given a set of corresponding landmarks, what happens between?



- An RBF estimates mapping for points not at landmarks
- For a given point  $x$ , it combines mappings from neighboring landmarks  $c_i$  weighted by a function of distance

$$y(x) = \sum_{i=1}^N w_i \phi(\|x - c_i\|)$$

- Where the basis function determines the form of the warp:  
 $\phi: \mathbb{R}^+ \rightarrow \mathbb{R}$

F. L. Bookstein, Principal Warps: Thin-Plate Splines and the Decomposition of Deformations. *IEEE Trans. on Pattern Anal. and Machine Intell.*, 11(6):567-585, 1989.

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### Different forms of Radial Basis Function:

- Thin Plate Spline:

$$\phi(r) = r^2 \log(r)$$

- Gaussian:

$$\phi(r) = \exp(-cr^2)$$

- Multiquadric

$$\phi(r) = \sqrt{r^2 + c^2}$$

D. Ruprecht and H. Muller. Free form deformation with scattered data interpolation methods. *Comp. Suppl.*, 8:267-281, 1993.

J. A. Little, D. L. G. Hill, and D. J. Hawkes. Deformations Incorporating Rigid Structures. *Computer Vision and Image Understanding*, 66(2):223-232, 1997.

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### Properties of RBF

- Many of the common forms (eg thin plate) provide optimally smooth deformations
- Generally stable to estimate weights for many different configurations of points.
- Change location of any landmark and whole deformation field changes:
  - Expensive to re-evaluate whole image match

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### Limitations of Global Parameterizations

$$T(x) = f(x, a, b, c)$$

- Each Parameter  $a, b, c, \dots$  Modifies entire image
  - Expensive to evaluate gradients of  $T$  wrt parameters
- Complex Brain shape differences requires a fine scale deformation
- Fine Scale deformation requires MANY parameters
  - High spatial frequencies for Cosine parameters
  - or: high order polynomial

So.. Need a way to simplify problem

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### Alternatives: Local Models

- Rather than have:
  - Many parameters
  - Where each influences the image deformation over the whole space:
- Need parameters that have localized influence on the deformation
  - Faster to Evaluate Image Match
- Forms of Spline can provide spatially localized deformation control

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### Spline Based Deformations with local support

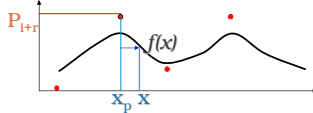
- Thin-Plate splines can be adapted to have local support:
- Mike Fornefett, Karl Rohr, and H. Siegfried Stiehl, Elastic Registration of Medical Images Using Radial Basis Functions with Compact Support, Computer Vision and Pattern Recognition, 1999
- Other forms using Specialized regular control knots can provide faster evaluation:

S. Lee, G. Wolberg, K.Y. Chwa and S.Y. Shin IEEE Trans. Vis. Comp. Graph., 1996 and 1997

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### B-Spline Models For Registration

- B-Spline Model:  $T(x)$  function of sparse knot values



$$f(x) = \sum_{r=0, N} P_{i+r} B_r(x-x_p)$$

Sum of contributions from local knots  $r=0..N$  only  
 The Basis functions  $B_x(\cdot)$  are specific polynomials eg Cubic  
 B-Spline with 4 controlling knots:

$$B_0(t) = (1-t)^3/6$$

$$B_1(t) = (3t^3 - 6t^2 + 4)/6$$

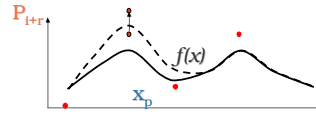
$$B_2(t) = (-3t^3 + 3t^2 + 3t + 1)/6$$

$$B_3(t) = t^3/6$$

S. Lee, G. Wolberg, K.Y. Chwa and S.Y. Shin IEEE Trans. Vis. Comp. Graph., 1996 and 1997

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### B-Spline Models For Registration



$$f(x) = \sum_{r=0, N} P_{i+r} B_r(x-x_p)$$

Move one knot and deformation changes only within A given range of knot locations.

A B-Spline Approximates: it does not interpolate!  
 Functions does not have to pass through knot values

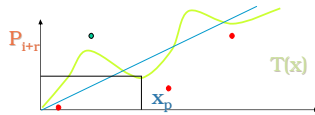
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### B-Spline Models For Registration

- B-Spline Transformation Model

$$T(x) = x + \sum_{r=0, N} P_{i+r} B_r(x-x_p)$$

B-Spline can Still Fold! (e.g. multiple x's map to the same value of  $T(x)$ )

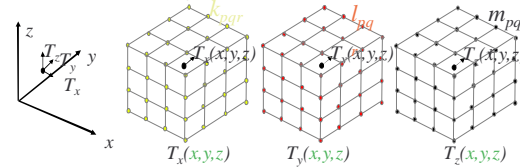


Can Test for Folding based on distance between knot values.

Can Prevent folding by adding a smoothness penalty term

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### Extend to 3D Displacement Along 3 Axes Using 3 3D lattices of control knots



- Describe Transformation  $T()$  in directions  $x$ ,  $y$  and  $z$  for each point in  $\{x,y,z\}$
- Parameterized by a Lattice of control parameters (knots)

$$Q_{pqr} = \{k_{pqr}, l_{pqr}, m_{pqr}\}$$

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Extend to 3D Displacement Along 3 Axes  
Using 3 3D lattices of control knots

Maximize Image Similarity  $Y()$  w.r.t.  $Q_{pqr}$

$$R(Q_{pqr}) = Y(Q_{pqr}) - \lambda \sum_x \frac{\partial^2 T(x, Q_{pqr})}{\partial x^2}$$

Registration Criteria    Regularization Penalty

Ruckert, Hayes, Studholme, Leach, Hawkes, Non-rigid registration of Breast MR images using Mutual Information Proc. MICCAI 1998 pp1144-1152, Springer, eds Wells, Colchester, Delp

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Rigid Estimate 65mm B-Spline, 40mm B-Spline, 25mm B-Spline, and 10mm

Studholme et al. 2001

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Example Group Spatial Normalization  
(Studholme et al. proc. SPIE medical imaging 2001)

Global Normalized Mutual Information driven B-Spline Spatial Normalisation of Brain Anatomy

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Capturing More Detail...

Dense Field Models

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Dense Field Methods

- Derive a voxel by voxel force field making images more similar
  - (local gradient of similarity measure with respect to individual voxel location)
- Move in the direction of the force field and re-evaluate

Need a model to describe how image responds to registration force

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Deformation Models for Registration

- Early approach applied to 3D brain images: Elastic Registration [Bajcsy, JCAT, 1983]
  - Applying a marked template to a new individual

$$T(\mathbf{x}) = \mathbf{x} + \mathbf{u}(\mathbf{x})$$

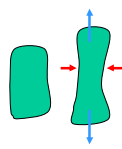
- Find a displacement field  $u(x)$  which balances the elastic energy of  $u(x)$  with the registration criteria  $S(x)$
- So the Elastic Deformation Model then is given by:
 
$$\mu \nabla^2 u(x) + (\lambda + \mu) \nabla(\nabla^T u(x)) = S(x)$$
 $\mu$  and  $\lambda$  are Lamé's elasticity constants

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### Deformation Models for Registration

- $\mu \nabla^2 u(x) + (\lambda + \mu) \nabla(\nabla^T u(x)) = S(x)$   
 $\mu$  and  $\lambda$  relate applied forces to the resulting strains, by the Poisson's Ratio:  
 $\sigma = \lambda / (\lambda + \mu)$

-> Ratio of Lateral Shrink to Extensional Strain.  
 Generally for registration  $\lambda = 0$   
 So registration force in one axis does not influence other axes

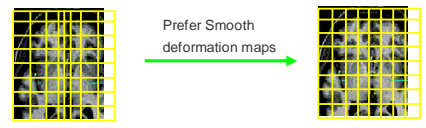


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### Elastic Deformation for Registration

$\mu \nabla^2 u(x) + (\lambda + \mu) \nabla(\nabla^T u(x)) = S(x)$

Key Idea: The Force balancing registration Criteria is a function of the derivatives of the deformation field [ $\nabla^T u(x)$  etc].  
 ..Rates of Change of displacements  $u(x)$  w.r.t. location  $x$ :



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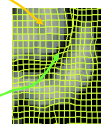
### Elastic Deformation for Registration

$\mu \nabla^2 u(x) + (\lambda + \mu) \nabla(\nabla^T u(x)) = S(x)$

If neighboring displacements are similar:  
 Local relative size is similar across image.

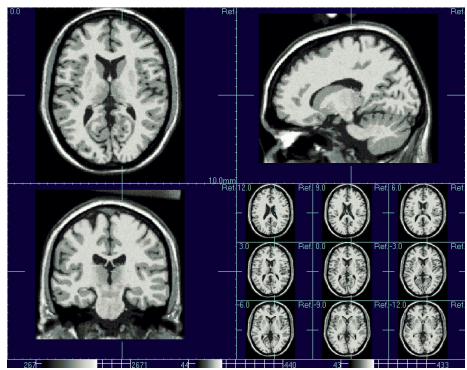
If neighboring displacements are different:  
 local relative size is changing.

When anatomical differences very localized (e.g. voxels in cortex) Registration Force balancing smoothness may under-estimate local contractions



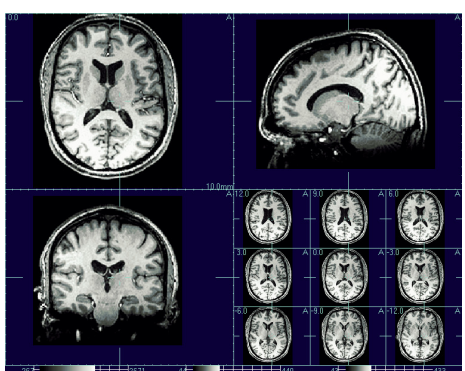
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### Example Elastic Warping of Brain Anatomy: Template (MNI)



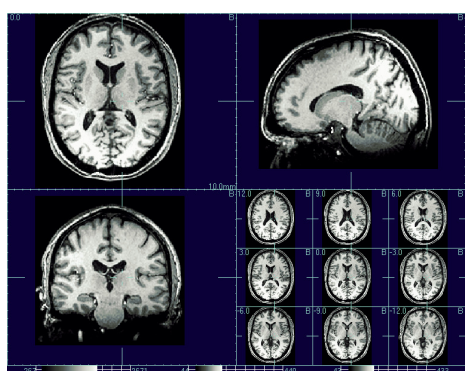
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### Example Elastic Warping of Brain Anatomy: Subject (affine)

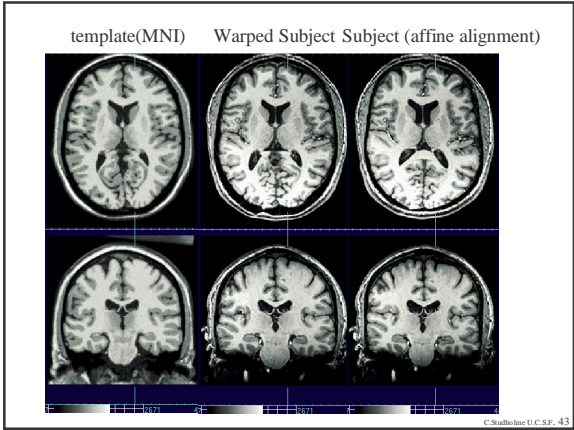


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### Example Elastic Warping of Brain Anatomy: Subject (Elastic Warp)



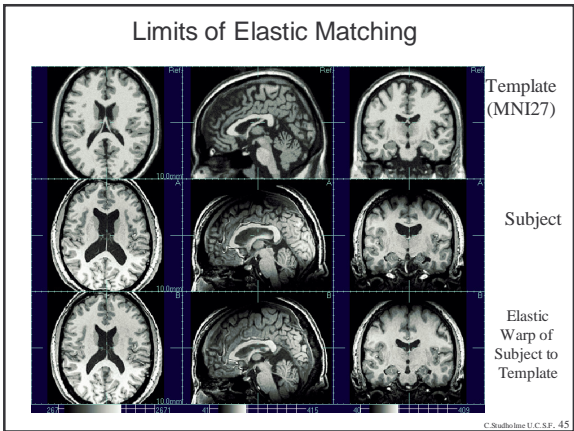
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### Elastic Deformation for Registration

- Can be used to prevent Singularities or Folding...
- But as displacement field evolves:
  - Deformation Energy builds up
- For extreme differences in anatomies:
  - deformation energy will prevent complete registration

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### Regularization and Large Deformations

$$\mu \nabla^2 u(x) + (\lambda + \mu) \nabla(\nabla^T u(x)) = S(x)$$

Simple Case      More Complex Case

Intermediate Anatomy

For "Large Deformations" When Evaluating "Distance" of deformation for regularization: linear distance can fail or limit our shape alignment

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### Regularization and Large Deformations

One Step      Multiple Steps: Regriding

Track Evolution over Curved Manifold

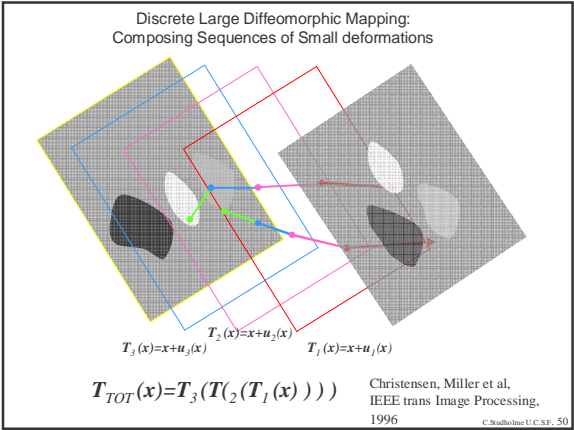
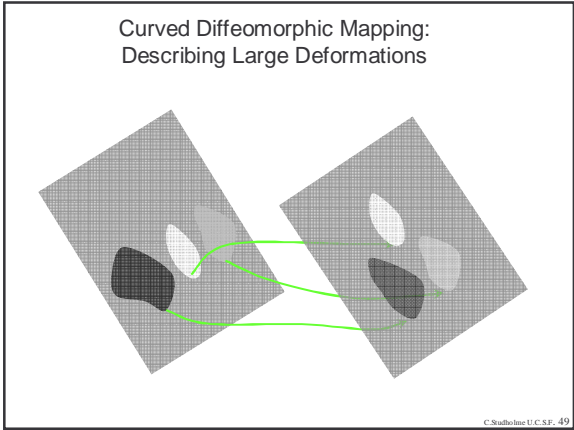
Christensen, Miller et al, IEEE trans Image Processing, 1996

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### Mapping between Anatomies: Describing Correspondence

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### Deformation Models For Registration

- Best known approach is a Viscous Fluid Deformation Model [Christensen, TIP, 1996] and (Freeborough&Fox98)
- For current deformation, evaluate Velocity Field:
 
$$\mu \nabla^2 v(x) + (\lambda + \mu) \nabla(\nabla^T v(x)) = S(u(t,x))$$

$$\mu \text{ is Shear Modulus, } \lambda \text{ is Lamé's Modulus}$$
- Evaluate a fractional update ( $\Delta t$  'seconds') of the displacement field along current velocity field:
 
$$u'(x) = u(x) + R \Delta t$$
 where
 
$$R = v(x) - v(x) \cdot [\partial u / \partial x]^T$$
- Then update the Force Field  $S(u(x,t))$  and iterate

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### Sparse Registration Force Field Driving Points into Better Alignment

In Regions of misaligned Tissue  
Force > 0  
 ↓  
 'Velocity' Field  
Smooth and Well Behaved  
i.e. no singularity points or folding  
 ↓  
 Then: Update displacement estimate  
u(x) along v(x)

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### Deformation Models For Registration

- Evaluate Velocity Field  $v(x)$  for Current Force
- Propagate mapping along Velocity Field (update  $u(x)$ )
- Update Force Field  $F(u(t,x))$  for Current Deformation

Key Idea: The fluid model ensures that the deformation preserves topology at each step:  
i.e. two points don't map to one point.

Can Resolve Complex Deformations:

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### Example Large Deformation

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Viscous Fluid Registration

Even Many Cortical Structures Deformed (intensities/tissue... look the same) BUT: not necessarily registered!

Derived from: Studholme et al, IEEE transactions on medical imaging, 2006

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Fluid registration can be dangerous..

Can be critically dependent on initialization: pre-registration and constraints on region applied to.

Key factor: Lots of engineering rather than mathematics

Fig. 5. The first 8 timesteps in a fluid registration of test image headB to headA showing misregistration of scalp to cortex.

H. Lester, S. Arridge, A survey of hierarchical nonlinear medical image registration., Pattern recognition, vol 32(1), 1999, page 129-149.

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## Atlases and Templates for Spatial Normalization of Anatomies

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## Overview

- What is an Atlas?
- Templates for Spatial Normalisation

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## An Atlas

In practice we might say an Atlas is:  
A map or spatial record of what we know about a region

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## Types of Atlas

Characteristics of an Atlas:

1. The type information we record in it
2. How we place that information within the atlas
3. How we display/project/extract that information

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### Atlases in Medical Imaging

1. An Atlas usually refers to an (often probabilistic) model of a population of spatial data (images).
2. Parameters determining the model are learned from a set of training data.
  - One or more subjects: eg atlas of brain regions
3. Simplest form is a template or average intensity.
  - Eg: Mean grey matter density, Mean PET tracer uptake
4. More complex forms capture
  - Higher order statistics: Variance, or other Model of Distribution
  - Complex parameterized models: eg Age

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### Example Atlases: Statistical Model of Brain Tissue Distribution in fetal brain

P. A. Habas et al, "Atlas-based segmentation of the germinal matrix from in utero clinical MRI of the fetal brain," in Medical Image Computing and Computer-Assisted Intervention, LNCS, vol. 5241, part I, pp. 351-358, September 2008.

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### Example Atlas: Complex Parameterized Models

21 weeks    22 weeks    23 weeks    24 weeks

Age-specific tissue probability maps generated in average space

P. A. Habas et al "A spatio-temporal atlas of the human fetal brain with application to tissue segmentation," in Medical Image Computing and Computer-Assisted Intervention, LNCS, vol. 5761, part I, pp. 289-296, September 2009.

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### How do we place information into an atlas? Independent Modalities

eg: Use 'structure' to place 'functional' measurements within the atlas

- Use MRI to normalize subject anatomy to a template anatomy
- Apply anatomical transformations to bring functional measurements in a subject into the atlas

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### How do we place information into an atlas? Same Modality

- Use neighboring structure to locate and place measurements within an atlas of the same type of measurements

- How accurately do we place that information?
- How does that neighboring information influence placement?

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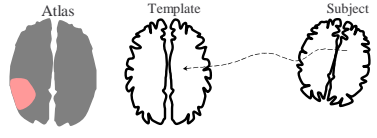
### How do we place information into an atlas? Shape Mapping: Morphometric Atlases

- Do not record imaging measurements in subject:  
BUT: record how subject was spatially re-arranged to fit the atlas
- Record how the shape of Subject and Atlas differ

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### Warping in Atlas Mapping

- Collecting data from different individual anatomies not trivial
- Need to locate corresponding points in atlas for a give measurement in the subject
- Need a Template (in atlas space) to match each subject to
  - Atlas
  - Template
  - Subject



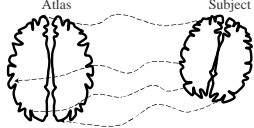
- How do we derive a correspondence or mapping?
  - Warp from template to subject?

Need a [non-rigid] Registration algorithm for -> Spatial Normalisation

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### Templates for Atlas Mapping

- What structure do we use as a target or template?
- Needs to contain information relevant to problem
  - Where are the ports along a coastline?
  - Where are the gyri delineating functional brain regions?
- Needs to Exclude irrelevant information:
  - Template should not contain a tumor if studying normal anatomy



- Need to have a representative shape:
  - Don't use a brain with rare sulcal patterns to study normal anatomy

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### Templates and Atlases

- Early Atlases for presentation/visualization:
  - Were often manually drawn
    - Broadmann[1]
  - 'Idealized' anatomies created by sketching features of interest
  - Difficult to compare results
- Modern Templates -> for Spatial normalisation:
  - Can be optimized for use with registration method

1. Broadmann, K: On the Comparative Localization of the Cortex 201-230.

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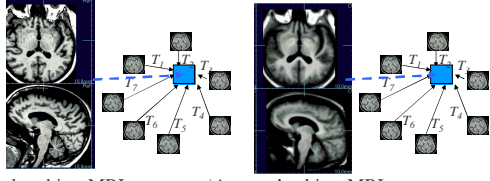
### Optimizing Templates

- Contrast/Intensity Properties
  - High signal to noise (average brain of MNI colin27?)
  - Show Imaging structures of interest:
    - T1W template for T1W matching -> structure
    - T2W template for T2W matching -> fMRI?
- Resolution
  - High isotropic resolution
    - Minimize loss of fine structure/tissue boundary
- Spatial Mathematical Properties:
  - Average 'Shape' of Anatomies studied
    - Aid in visualization of results
    - Improve registration algorithm?

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### Optimal Templates for Matching

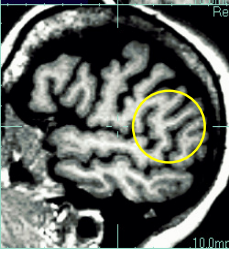
- Average Intensity? (older SPM/VBM analyses)
  - Register a set of MRI's to a single subject MRI:
    - and average intensities to form a new template:



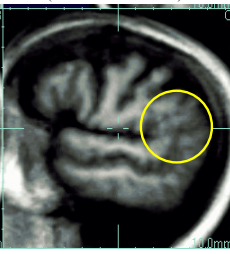
Single subject MRI      'Average' subject MRI

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Single Subject



Average of 14 subjects  
(Coarse deformation)



- Since images imperfectly aligned:
  - there is a Fundamental problem:
- Resulting 'average' image is not necessarily an example of a real anatomy which has been blurred.
- May even be topologically different: (eg sulci covered over)
- So... deforming an individual to it may be impossible!

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