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# Nonlinear Dirac Equation

## Spectral Stability of Solitary Waves

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*To Rasmey, Norinn, and Léa,*
N.B.

*To Mary and Natalie,*
A.C.

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# Index

# List of symbols

| Symbol | Meaning | Page |
|---|---|---|
| $1 \leq i,\, j \leq n$ | $1 \leq i \leq n$ and $1 \leq j \leq n$. | |
| $\mathbb{1}_S$ | A characteristic function of $S \subset T$: $\mathbb{1}_S(x) = \begin{cases} 1, & x \in S; \\ 0, & x \in T \setminus S. \end{cases}$ | |
| $\lvert\alpha\rvert$ | $\lvert\alpha\rvert = \alpha_1 + \cdots + \alpha_n, \quad \alpha \in \mathbb{N}_0^n$. | |
| $\big(a_i\big)_{i\in\mathbb{N}}$ | A sequence (ordered set) of elements $a_1,\, a_2,\, a_3,\, \dots$ . | |
| $\big\{a_i\big\}_{i\in\mathbb{N}}$ | A set of elements $a_1,\, a_2,\, a_3,\, \dots$ . | |
| $\boldsymbol{\alpha}^i,\, \boldsymbol{\beta},\, \mathbf{J}$ | $\boldsymbol{\alpha}^i = \begin{bmatrix} \operatorname{Re}\alpha^i & -\operatorname{Im}\alpha^i \\ \operatorname{Im}\alpha^i & \operatorname{Re}\alpha^i \end{bmatrix},\ \boldsymbol{\beta} = \begin{bmatrix} \operatorname{Re}\beta & -\operatorname{Im}\beta \\ \operatorname{Im}\beta & \operatorname{Re}\beta \end{bmatrix},\ \mathbf{J} = \begin{bmatrix} 0 & I_N \\ -I_N & 0 \end{bmatrix}$. | Page 178 |
| $\mathbb{B}_R(\mathbf{X})$ | $\{x \in \mathbf{X}\colon \quad \lvert x\rvert < R\}, \quad R > 0$. | |
| $\mathbb{B}_R^n,\ \mathbb{B}_R^n(x_0)$ | $\{x \in \mathbb{R}^n\colon\ \lvert x\rvert < R$ or $\lvert x - x_0\rvert < R$ if $x_0$ is provided$\}$, $R > 0$. | |
| $\mathscr{B}(\mathbf{X}),\, \mathscr{B}(\mathbf{X}, \mathbf{Y})$ | Vector space of bounded linear operators $\mathbf{X} \to \mathbf{X}$ and $\mathbf{X} \to \mathbf{Y}$. | Page 25 |
| $\mathscr{B}_0(\mathbf{X}),\, \mathscr{B}_0(\mathbf{X}, \mathbf{Y})$ | Vector space of compact linear operators $\mathbf{X} \to \mathbf{X}$ and $\mathbf{X} \to \mathbf{Y}$. | Page 26 |
| $\mathscr{C}(\mathbf{X}),\, \mathscr{C}(\mathbf{X}, \mathbf{Y})$ | Set of closed linear operators $\mathbf{X} \to \mathbf{X}$ and $\mathbf{X} \to \mathbf{Y}$. | Page 26 |
| $\mathbb{C}_+,\ \mathbb{C}_-$ | $\{z \in \mathbb{C}\colon \quad \operatorname{Im} z > 0\}, \quad \{z \in \mathbb{C}\colon \quad z < 0\}$. | |
| $C_{\mathrm{b}}^k(\mathbb{R}^n)$ | $\{f \in C(\mathbb{R}^n)\colon \quad \max\limits_{\alpha\in\mathbb{N}_0^n,\, \lvert\alpha\rvert\leq k} \sup_{x\in\mathbb{R}^n} \lvert\partial_x^\alpha f(x)\rvert < \infty\}$ | |
| $\mathscr{C}_{\lambda,\omega}(M, \mathcal{N}, \rho, \nu)$ | The set of admissible functions for the Carleman estimates for the linearization operator $J(D_m + V - \omega)$. | Page 131 |
| $\mathscr{C}_\lambda(M, \mathcal{N}, \rho, \nu)$ | The set of admissible functions for the Carleman estimates for the operator $D_m + V$. | Page 117 |
| $\mathbf{Cl}_\ell(\mathbb{C})$ | Clifford algebra over $\mathbb{C}$ formed by $\ell$ generators. | Page 143 |

| | |
|---|---|
| $\mathbf{coker}(A)$ | $\mathbf{Y}/\mathfrak{R}(A)$, the cokernel of a linear operator $A : \mathbf{X} \to \mathbf{Y}$. Page 38 |
| $\mathfrak{D}(A)$ | Domain of a linear operator $A$. Page 25 |
| $\partial_x^\alpha$ | $\partial_x^\alpha = \partial_{x^1}^{\alpha_1} \dots \partial_{x^n}^{\alpha_n} = \left(\frac{\partial}{\partial x^1}\right)^{\alpha_1} \dots \left(\frac{\partial}{\partial x^n}\right)^{\alpha_n}$ for $x = (x^1, \dots, x^n) \in \mathbb{R}^n$, $\alpha \in \mathbb{N}_0^n$. |
| $D_m$, $D_0$ | The Dirac operator: $D_m = -\mathrm{i}\boldsymbol{\alpha} \cdot \nabla + \beta m$, $D_0 = -\mathrm{i}\boldsymbol{\alpha} \cdot \nabla$. Page 62 |
| $\mathbb{D}_R$, $\mathbb{D}_R(z_0)$ | $\{z \in \mathbb{C}: \ \lvert z\rvert < R \ \text{ or } \ \lvert z - z_0\rvert < R \ \text{ if } z_0 \text{ is provided}\}$, $R > 0$. |
| $E^\perp$ | $\{\xi \in \mathbf{X}^*: \ \langle \xi, x\rangle = 0 \ \ \forall x \in E\}$, where $E$ is a subset of a Banach space $\mathbf{X}$. |
| $\mathrm{End}(\mathbb{C}^N)$ | The endomorphism ring of $\mathbb{C}^N$; $\mathrm{End}(\mathbb{C}^N) \cong M_N(\mathbb{C}) \cong \mathbb{C}^{N\times N}$. |
| $\Phi_A$, $\Phi_A^\pm$, $\Psi_A$ | Fredholm domains of a linear operator $A$. Page 38 |
| $\mathcal{G}(A)$ | Graph of a linear operator $A$. Page 26 |
| $\gtrapprox$ | $a \gtrapprox b$ if $a \in (b, b+\varepsilon)$ for $\varepsilon > 0$ small enough. |
| $H_r^1$, $L_r^2$, etc. | Corresponding subspaces of spherically symmetric functions. Page 17 |
| $I$, $I_{\mathbf{X}}$ | $I$ or $I_{\mathbf{X}} : \mathbf{X} \to \mathbf{X}$, $x \mapsto x \ \forall x \in \mathbf{X}$, where $\mathbf{X}$ is a Banach space. |
| $I_N$ | For $N \in \mathbb{N}$, $I_N = I_{\mathbb{C}^N} : \mathbb{C}^N \to \mathbb{C}^N$. |
| $\mathbf{ker}(A)$ | Kernel (null space) of a linear operator $A$. Page 27 |
| $\lessapprox$ | $a \lessapprox b$ if $a \in (b-\varepsilon, b)$ for $\varepsilon > 0$ small enough. |
| $\mathfrak{L}(A)$, $\mathfrak{L}_\lambda(A)$ | Root lineal of $A$ corresponding to eigenvalue zero ($\lambda$ if specified). Page 28 |
| $L^p_{\mathrm{loc}}$, $L^\infty_{\mathrm{loc}}$ | $\left\{ u \in \mathscr{D}': \ \ \varphi u \in L^p, \ \ \forall \varphi \in \mathscr{D} \right\}$ Page 12 |
| $\mathbb{N}$, $\mathbb{N}_0$ | $\mathbb{N} = \{1, 2, 3, \dots\}$, $\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$. |
| $\Omega_R^n$ | $\mathbb{R}^n \setminus \overline{\mathbb{B}_R^n} = \{x \in \mathbb{R}^n: \ \lvert x\rvert > R\}$, $R > 0$. |
| $\mathbf{O}(N, \mathbb{F})$ | Orthogonal matrices with coefficients in the field $\mathbb{F}$. Page 150 |
| $\bar\psi = \psi^*\beta$ | The Dirac adjoint of $\psi \in \mathbb{C}^N$. Page 165 |
| $\mathbb{R}_+$, $\mathbb{R}_-$ | $\{t \in \mathbb{R}: \ t > 0\}$, $\{t \in \mathbb{R}: \ t < 0\}$. |

| | |
|---|---|
| $r(A)$ | Spectral radius of a bounded linear operator $A$ <br> Page 35 |
| $r$, $\langle r\rangle$ | $r = \|x\|$ for $x \in \mathbb{R}^n$, $n \in \mathbb{N}$. The operators of multiplication by $\|x\|$ and $\langle x\rangle = (1+\|x\|^2)^{1/2}$ are denoted by by $r$ and $\langle r\rangle$, respectively. |
| $\mathfrak{R}(A)$ | Range of a linear operator $A$. <br> Page 25 |
| $\rho(A)$ | Resolvent set of a linear operator $A$. <br> Page 33 |
| $\mathbb{S}_R^{n-1}$ | The sphere in $\mathbb{R}^n$ of radius $R > 0$. |
| $\big(\sigma_i\big)_{1\le i\le n}$ | Generalized Pauli matrices $\sigma_i \in \mathrm{End}(\mathbb{C}^N)$ which satisfy the relations $\sigma_i\sigma_j^* + \sigma_j\sigma_i^* = 2\delta_{ij}I_N$, $1 \le i$, $j \le n$, with $n$, $N \in \mathbb{N}$. <br> Page 142 |
| $\sigma_{\mathrm{p}}$, $\sigma_{\mathrm{c}}$, $\sigma_{\mathrm{res}}$ | Point, continuous, and residual spectra of a linear operator. <br> Page 36 |
| $\sigma_{\mathrm{ess},k}$ | Different definitions of the essential spectra of a linear operator. <br> Page 50 |
| $\sigma_{\mathrm{ap}}$ | Approximate point spectrum of a linear operator. <br> Page 37 |
| $W^{s,p}(\mathbb{R}^n)$, $W_0^{s,p}(\Omega)$ | $L^p$-based Sobolev spaces of order $s$. <br> Page 12 |
| $\mathbb{Z}$ | $\{\dots, -2, -1, 0, 1, 2, 3, \dots\}$ |

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This monograph gives a comprehensive treatment of spectral (linear) stability of weakly relativistic solitary waves in the nonlinear Dirac equation. It turns out that the instability is not an intrinsic property of the Dirac equation that is only resolved in the framework of the second quantization with the Dirac sea hypothesis. Whereas general results about the Dirac-Maxwell and similar equations are not yet available, we can consider the Dirac equation with scalar self-interaction, the model first introduced in 1938. In this book we show that in particular cases solitary waves in this model may be spectrally stable (no linear instability). This result is the first step towards proving asymptotic stability of solitary waves.

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