Nonlinear dynamical system approach for state estimation using PMU/SCADA measurements

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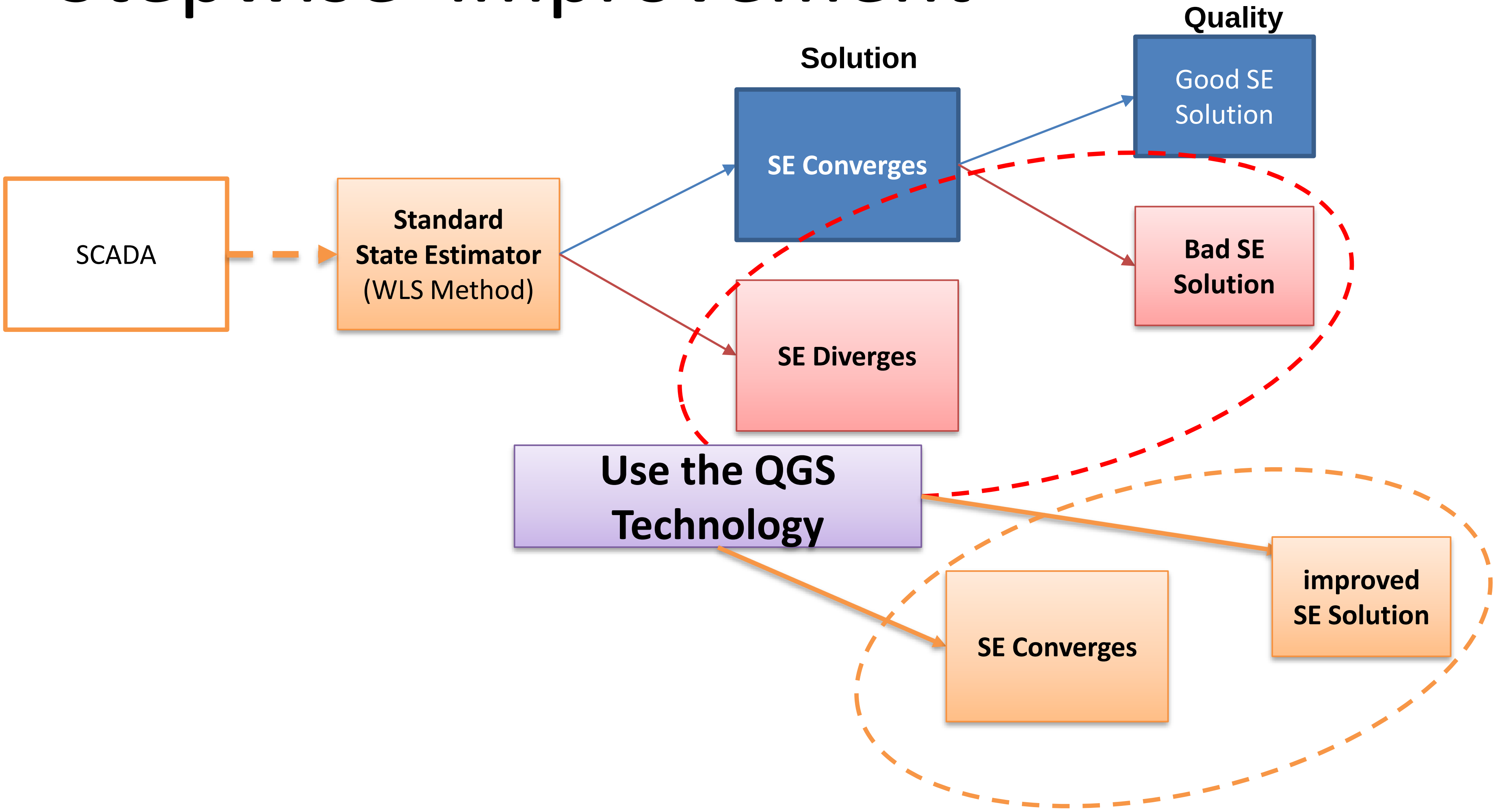
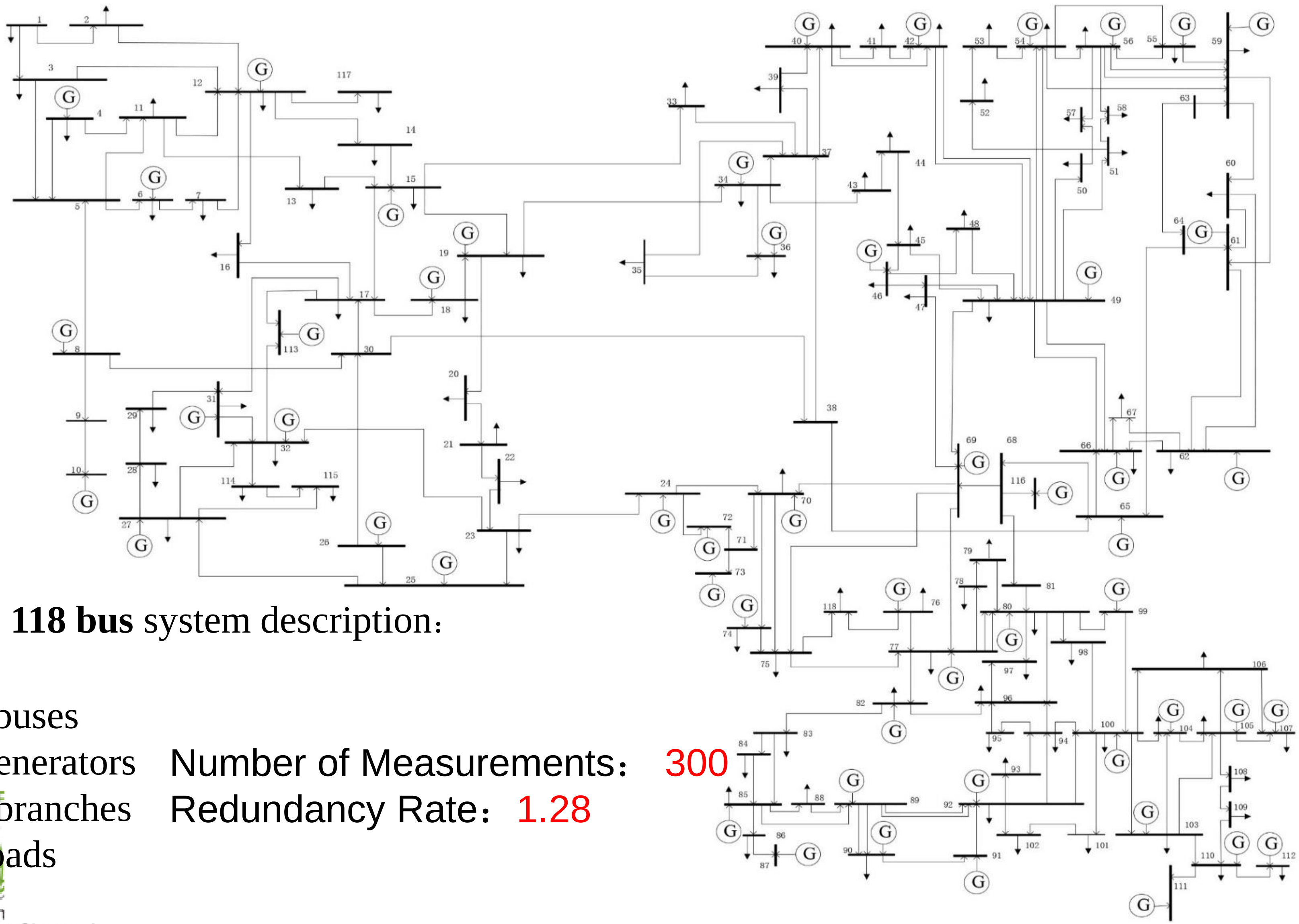
Stepwise Improvement



1. Provide satisfactory SE solution with less equipment installation (poor measurements redundancy so that the traditional WLS fails).



IEEE 118 bus system description：

118 buses
54 generators
186 branches
91 loads

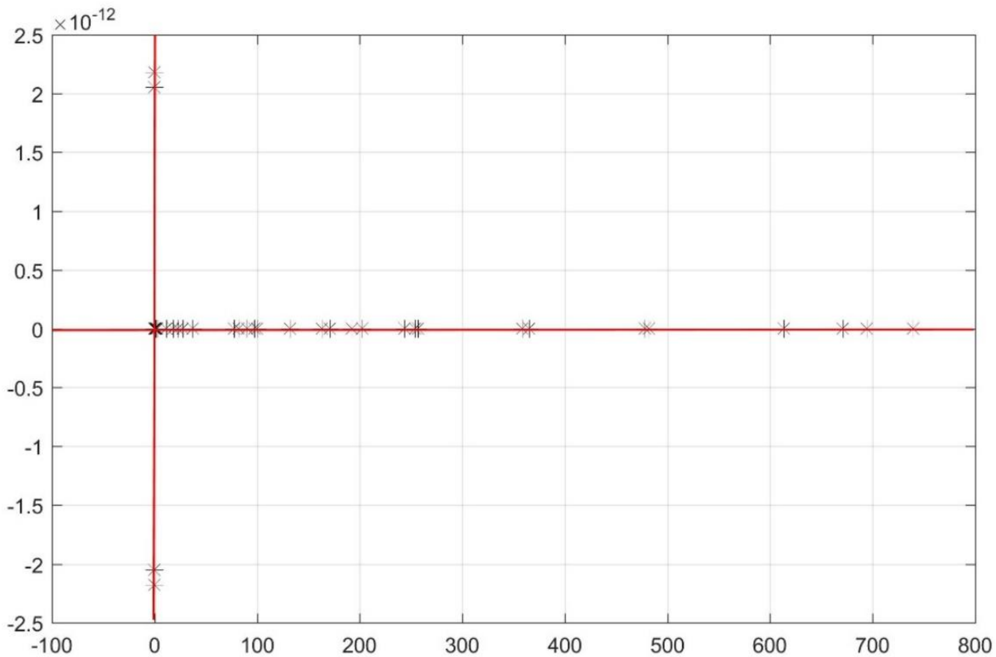
Number of Measurements： 300
Redundancy Rate： 1.28

1. Provide satisfactory SE solution with less equipment installation (poor measurements redundancy so that the traditional WLS fails).

redundancy Rate： 1.28

WLS-Gauss Newton

Fail to obtain a solution



cond(G)= 7.1439e+19

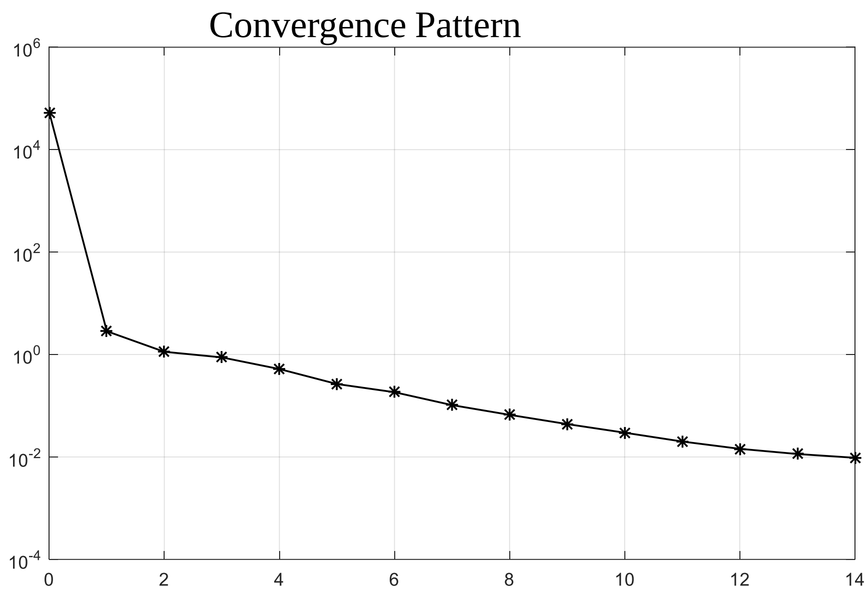
Unobservable state variables:

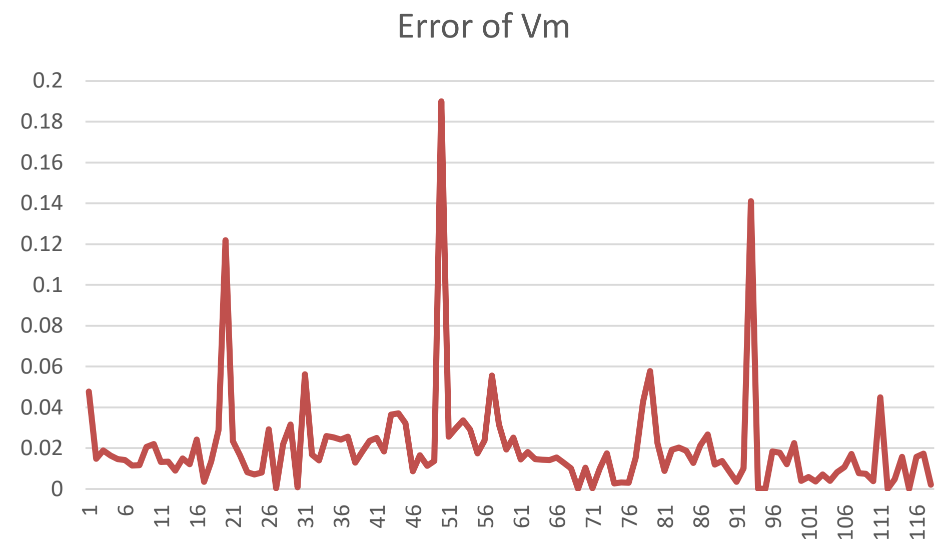
'Va112' 'Va20' 'Va38' 'Vm112'

Trivial Columns of Gain matrix :

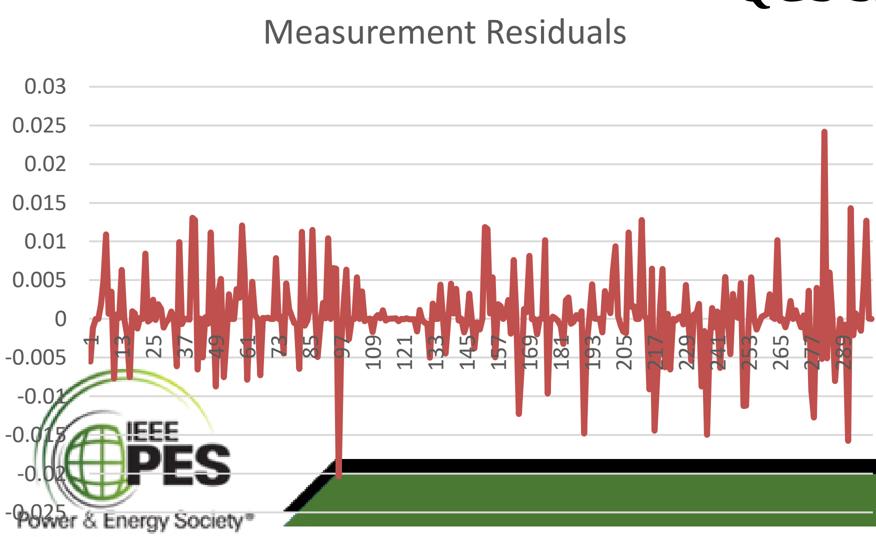
51 64 74 168

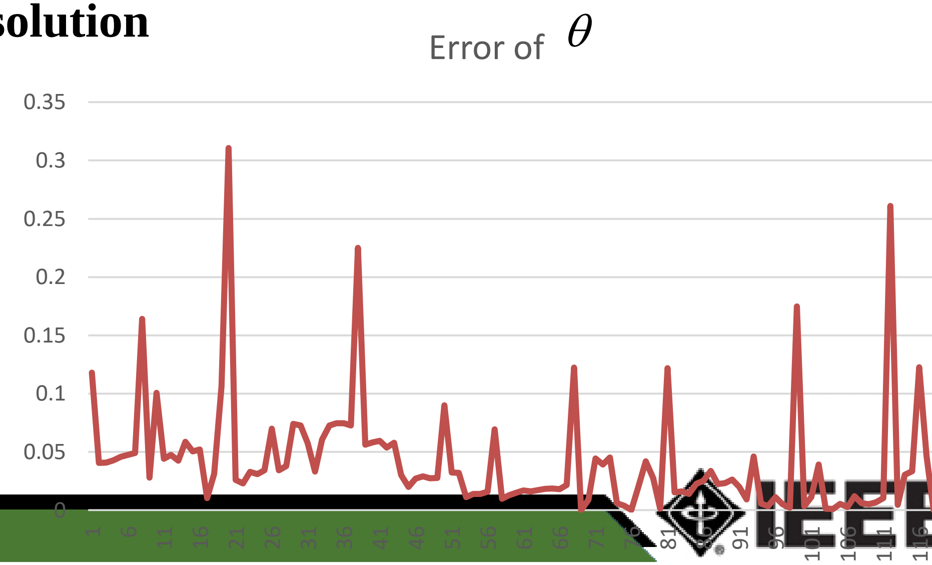
1. Provide satisfactory SE solution with less equipment installation (poor measurements redundancy so that the traditional WLS fails).



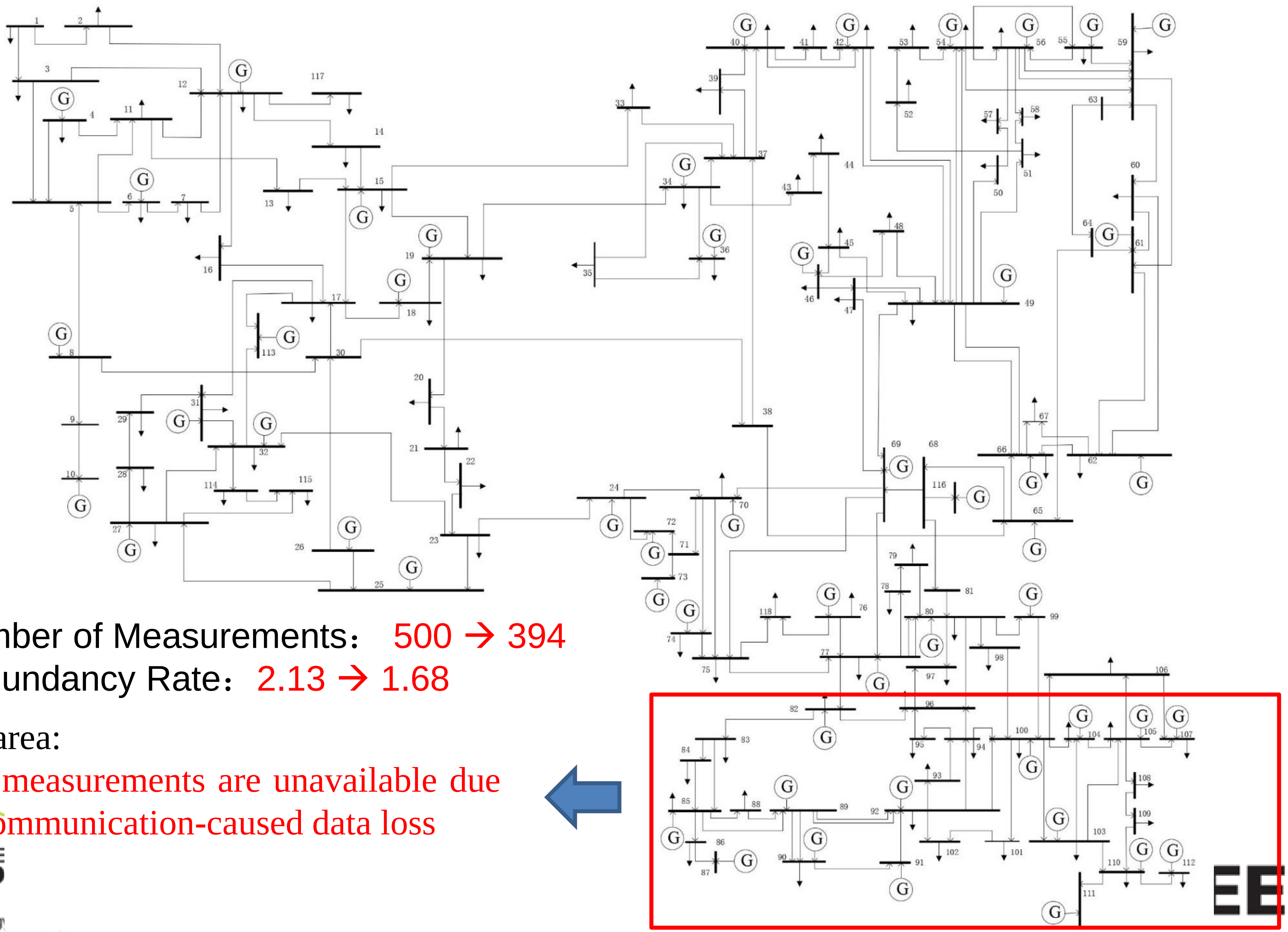


QGS SE solution





2. Robustness against losing observability due to communication-caused data loss



Number of Measurements：500 → 394

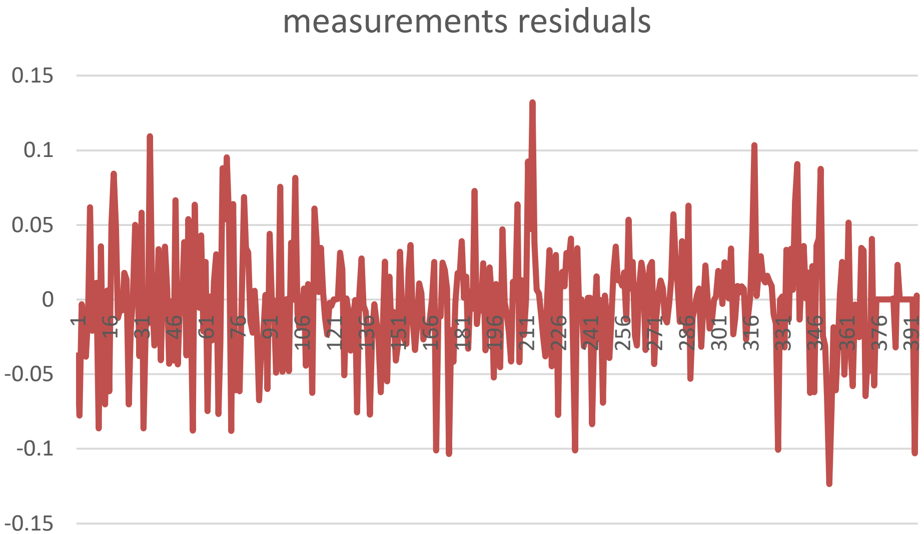
Redundancy Rate：2.13 → 1.68

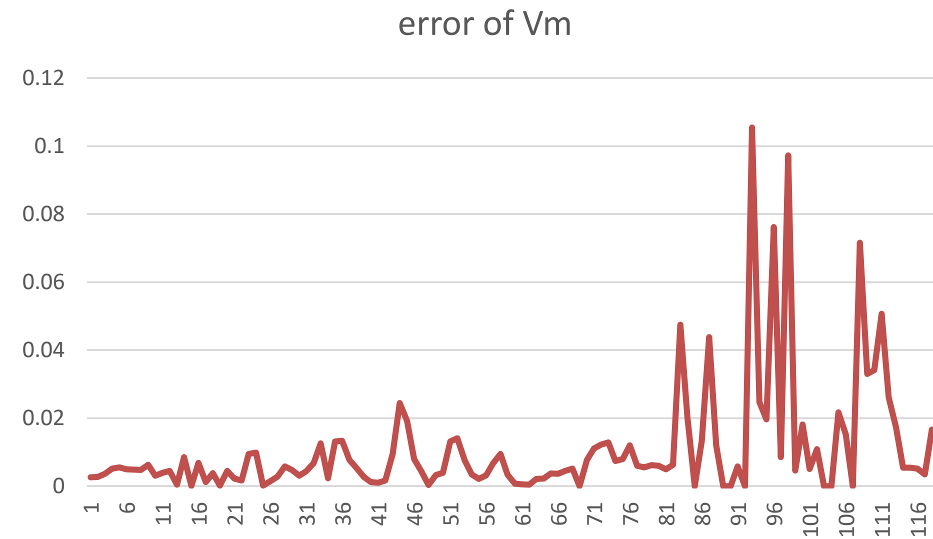
Subarea:

106 measurements are unavailable due to communication-caused data loss

2. Robustness against losing observability due to communication-caused data loss

QGS solution





Unobservable state variables:

‘Va85’ ‘Va87’ ‘Va89’ ‘Va90’ ‘Va91’
‘Va92’ ‘Va103’ ‘Va104’ ‘Va105’ ‘Va107’
‘Va110‘ ’Va111’ ‘Va112’ ‘Va83’ ‘Va84’
‘Va86’ ‘Va88’ ‘Va93’ ‘Va94’ ‘Va95’
‘Va101’ ‘Va102’ ‘Va106’ ‘Va108’ ‘Va109’
‘Vm85’ ‘Vm89’ ‘Vm90’ ‘Vm92’ ‘Vm103’
'Vm104' 'Vm107' 'Vm84' 'Vm86' 'Vm95'
'Vm102' 'Vm109'

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QGS Approach for State Estimation

Zero injections constrains + Zero error of every measurements +
Inequality constraints on state variables

$$\begin{cases} z_m - h_m(x) = 0 \\ C(x) = 0 \end{cases}$$

QGS dynamical formulation:

- always converge;
- Converge to an `optimal’ solution if unobservable

The convergence property of QGS approach will not be affected by the singularity of G matrix

Complete Characterization of Feasible region (Chiang & alberto, Stability Regions of Nonlinear Dynamical Systems, Cambridge Press, 2015, Chiang & Jiang, 2018 IEEE Trans. On Power Systems)

The feasible region defined by the following equality and inequality constraint functions:

$$\begin{cases} P_{Gi} - P_{Li} - V_i \sum_{j \in i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0 \\ Q_{Gi} - Q_{Li} - V_i \sum_{j \in i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0 \end{cases} \quad i \in \{1, \dots N_B\}$$

$$\begin{cases} P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \\ Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \end{cases} \quad i \in \{1, \dots N_G\}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \quad i \in \{1, \dots N_B\}$$

$$\begin{cases} \left|S_f\right| \le S_l^{\max} \\ \left|S_t\right| \le S_l^{\max} \end{cases} \quad l \in \{1, \dots N_L\}$$

Definition of Feasible Region

$$H(x) = \begin{bmatrix} C_E(u, y(u)) \\ C_I(u, y(u)) + \hat{S}^2 \end{bmatrix} = 0$$

$$x = (u, y(u), s(u, y)) \in \Re^{6N_G + 4N_B + 2N_L - 1}, \quad \hat{S} = (s_1, \ldots, s_{4N_G + 2N_B + 2N_L})$$

C_E

$$\begin{cases} V_i \sum_{j \in i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) + P_{Li} - P_{Gi} = 0 \\ V_i \sum_{j \in i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) + Q_{Li} - Q_{Gi} = 0 \end{cases} \quad i \in N_B$$

C_I

$$\begin{cases} P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \\ Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \end{cases} \quad i \in N_G$$

$$V_i^{\min} \le V_i \le V_i^{\max} \quad i \in N_B$$

$$|S_l| \le S_l^{\max} \quad l \in N_L$$

,
.

The complete constraint functions can be presented as the following equalities:

$$H(x) = 0, \quad x \in \Re^n$$

where $H = (h_1, \ldots, h_m)^T : \Re^n \to \Re^m$ $\quad n = 6N_G + 4N_B + 2N_L - 1$ $\quad m = 4N_G + 4N_B + 2N_L$

Definition 1: (Feasible region)

The feasible region is the set of control variables in which all the equality and inequality constraints of the problem are satisfied, i.e.,

$$FR = \{u \in \Re^{2N_G - 1} : H(x) = H(u, y(u), s(u, y)) = 0\}$$

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$\dot{x} = Q_H(x) = -DH(x)^T H(x)$

We seek to completely characterize the feasible region of a set of nonlinear constraint functions of OPF problems/State Estimation problems.

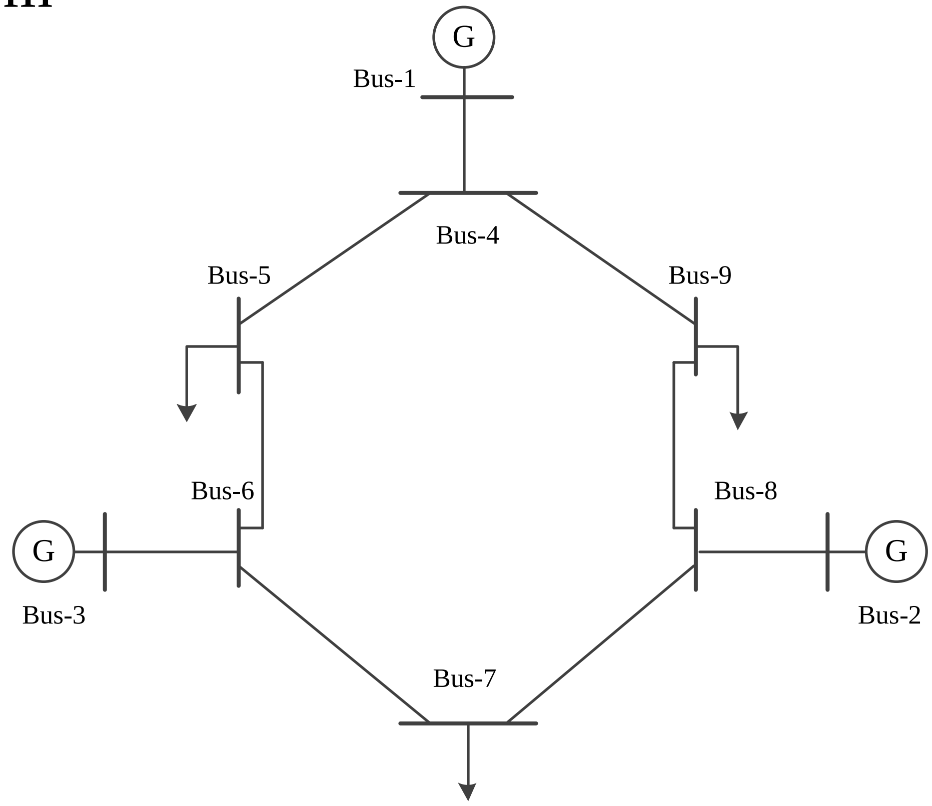
To this end, we consider the following nonlinear dynamical system, termed Quotient Gradient System (QGS) which is closely related to the nonlinear constraint functions:

$$\dot{x} = Q_H(x) = -DH(x)^T H(x)$$

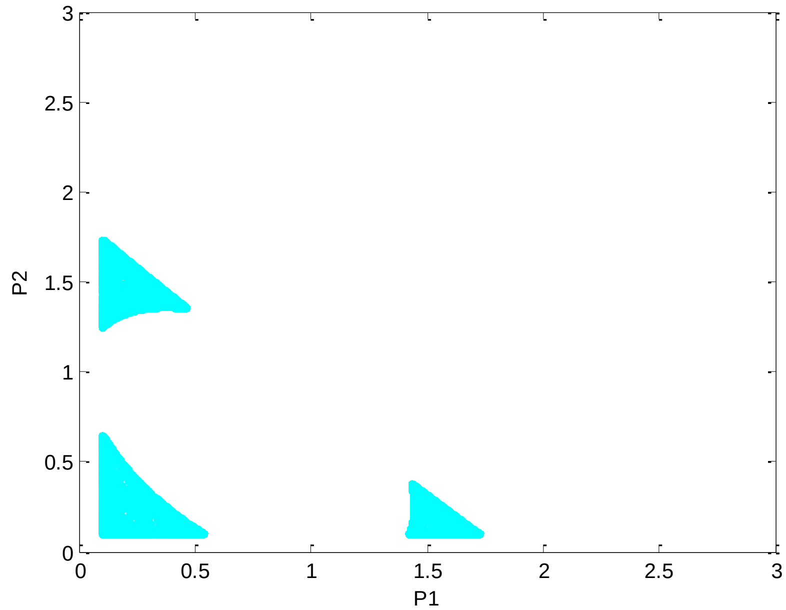
where $DH(x)$ is the Jacobian matrix of $H(x)$.

Feasible Regions & SEMs

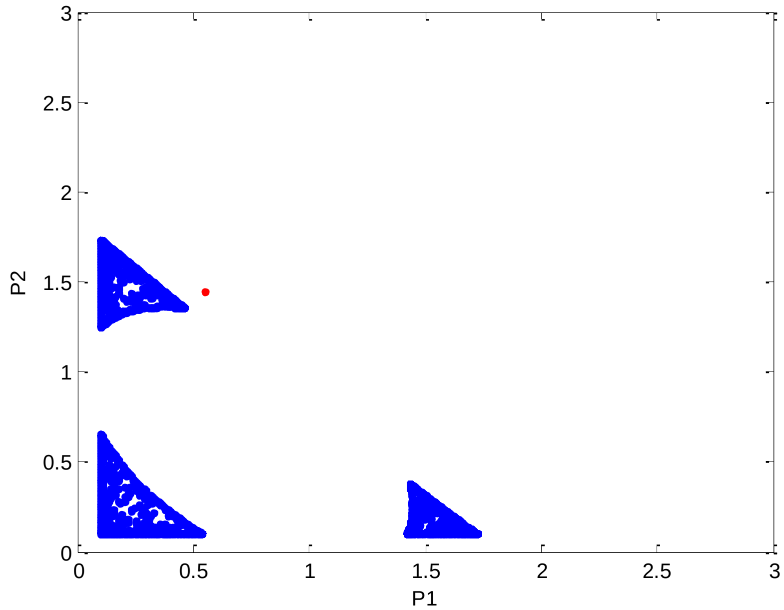
9-bus system



Feasible Regions & SEMs of 9-bus system



(a) Feasible regions on *P1-P2* projection plane



(b) SEMs on *P1-P2* projection plane

Theorem 5: (Complete characterization of the feasible region)

A path-connected set is a feasible component of constraint set <u>if and only if</u> it is a non-degenerate stable equilibrium manifold of QGS; i.e.,

$$FR = \bigcup_{j=1} \Sigma_j^r$$

where Σ_j^r is a regular stable equilibrium manifold of QGS.

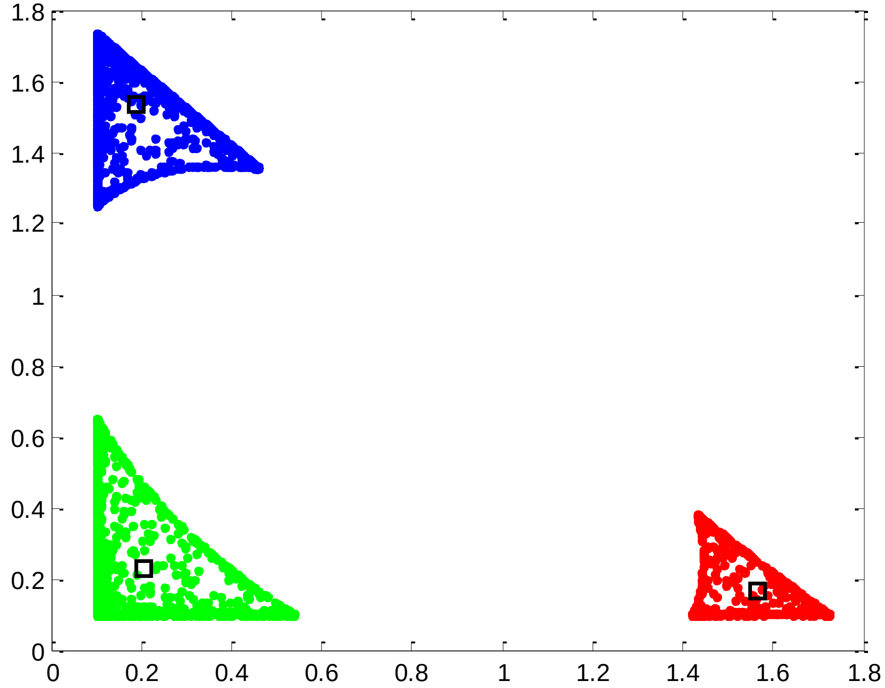
The proposed QGS nonlinear system does provide a novel computational mechanism to locate a path-connected feasible component. In particular, we can show that the QGS possesses another desirable property: every trajectory converges to an equilibrium manifold.

Theorem 6: (Complete stability)
Every trajectory of QGS converges to one of the equilibrium manifolds.

Convergence regions for feasible regions

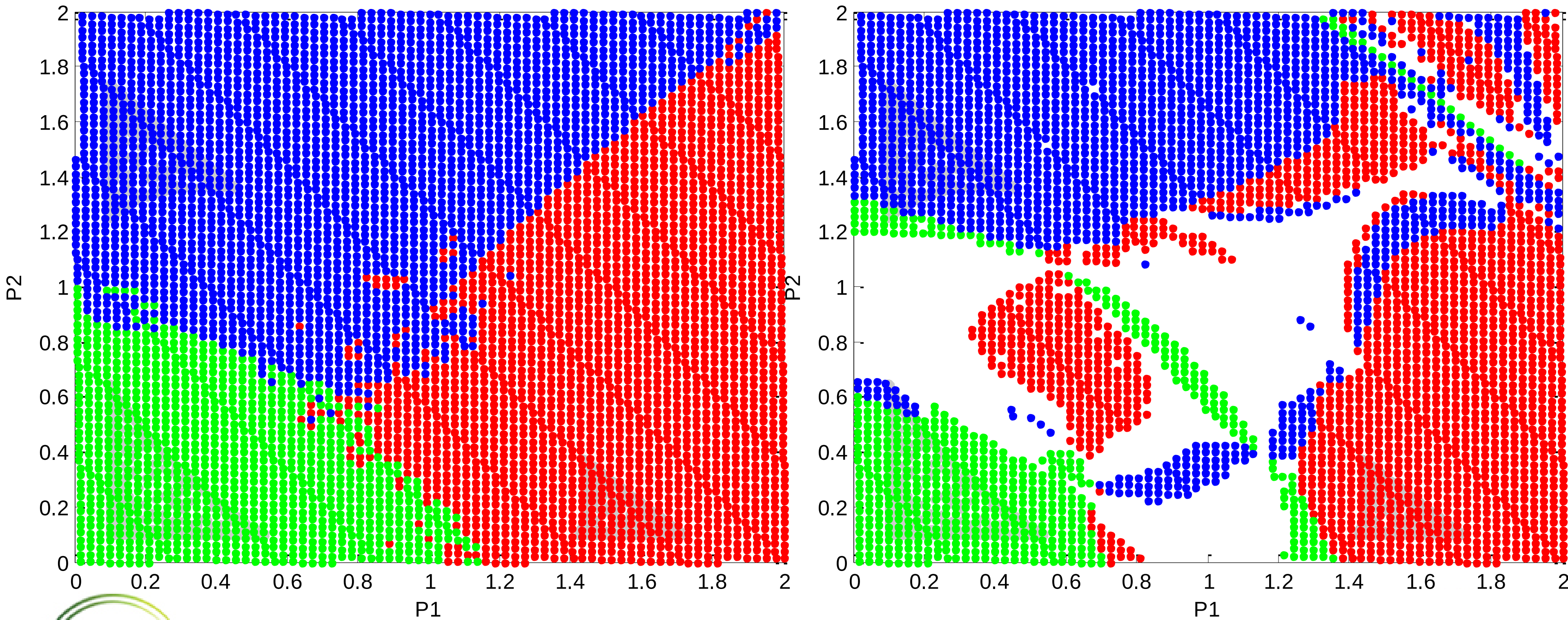
For the 9-bus system

- There are 3 feasible regions
- So the initial points on tangent plane are clustered into 3 groups



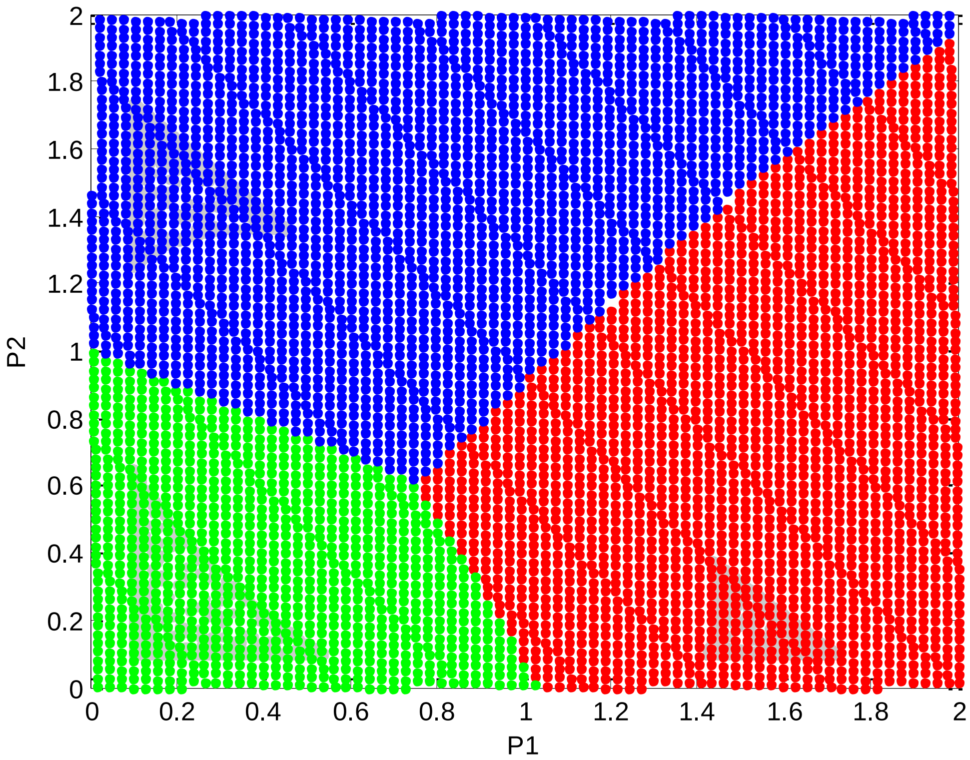
Convergence regions for feasible regions

Convergence regions of IPOPT/MIPS on tangent plane



Convergence regions for feasible regions

Convergence regions of QGS on tangent plane



QGS formulation with residual magnitude constraints

Zero injections constraints + residual magnitude constraints

$$\begin{cases} \left(z_m - h_m(x)\right)^2 \le L_m \\ C(x) = 0 \end{cases}$$

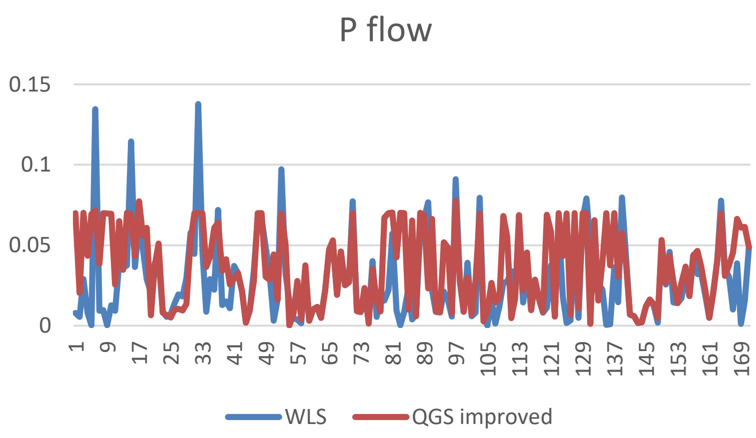
$$F(x) = \begin{bmatrix} \left(z_m - h_m(x)\right)^2 + \alpha_m{}^2 - L_m \\ C(x) \end{bmatrix} = 0$$

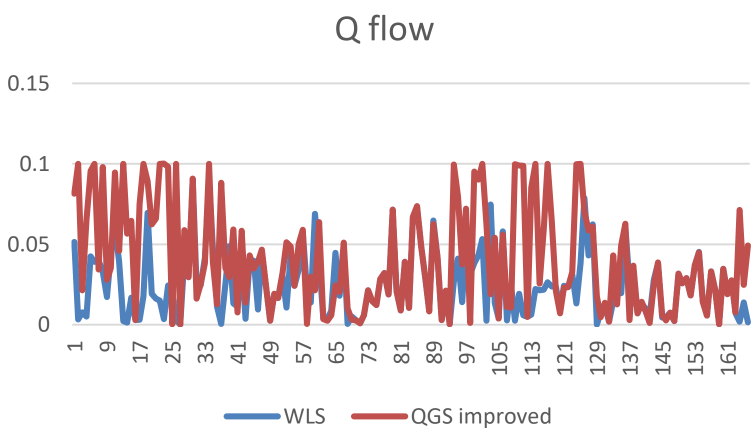
QGS formulation:

Number of Measurements: 500 with zero mean and standard deviation of 10 e-2 error
Redundancy Rate: 2.13

QGS improved

(**<u>residuals of P flow < 0.07 , the others <0.1</u>**)



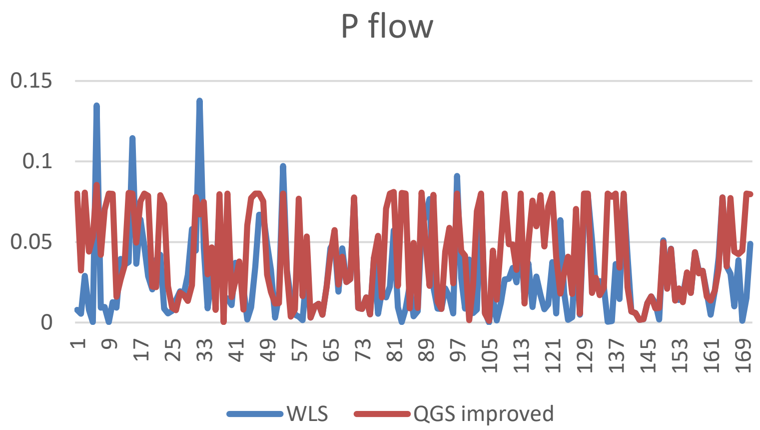


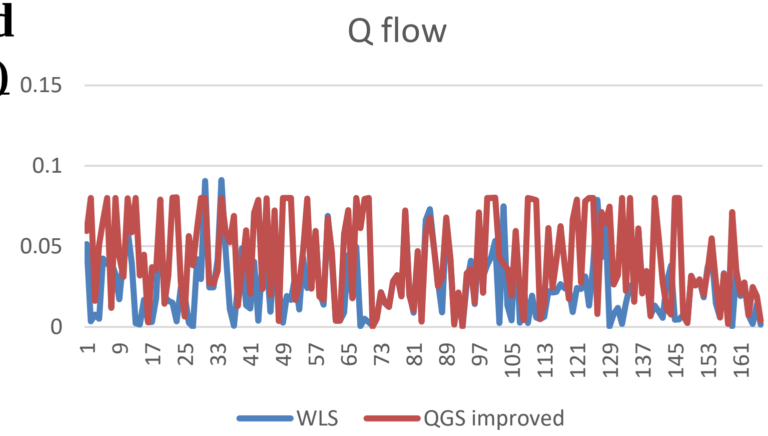
Number of Measurements： 500 with zero mean and standard deviation of 10 e-2 error

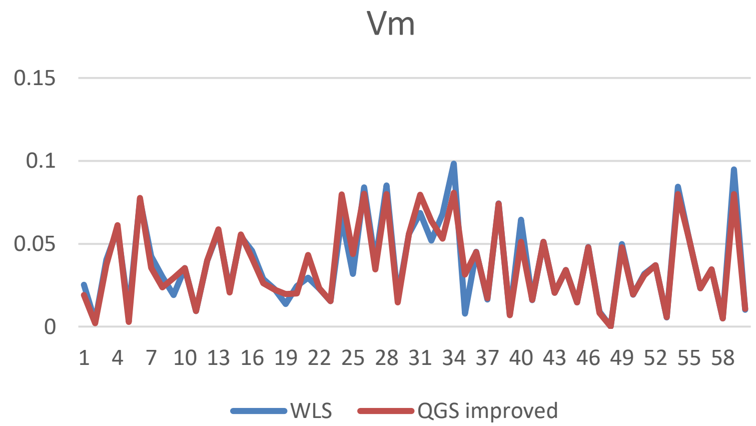
Redundancy Rate： 2.13

QGS improved

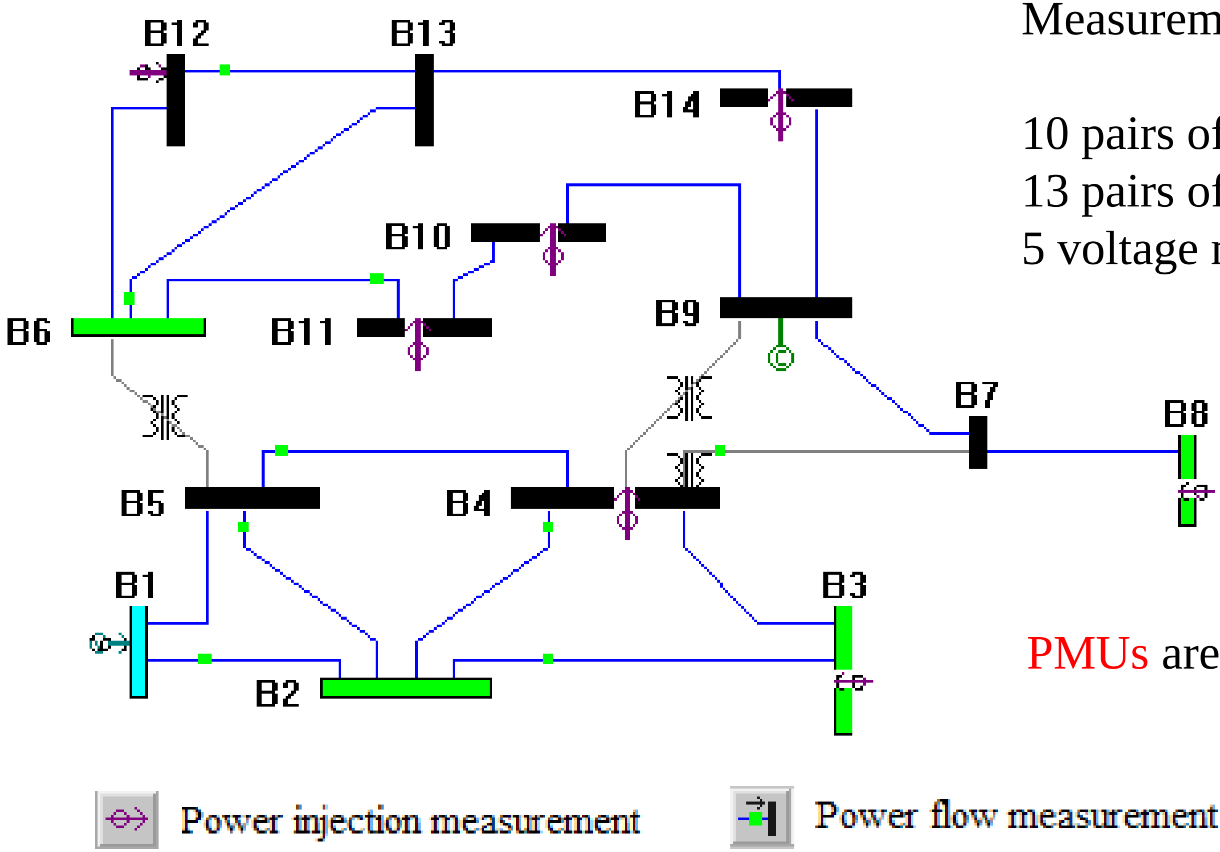
(min max 0.08)







6. Only PMU measurements are utilized between SCADA data.



Measurements:

10 pairs of injection measurements
13 pairs of flow measurements
5 voltage magnitude measurements

PMUs are deployed on buses:

6. . Only PMU measurements are utilized between SCADA data.

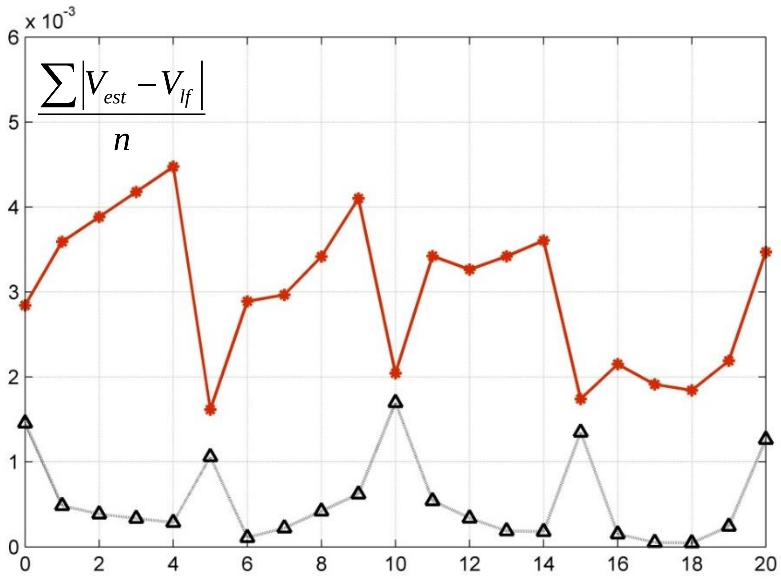
4 PMU data in every second

When SCADA data refreshed :

$$\begin{cases} z_{conv} - h_{conv}(x) \le L_{\mathrm{conv}} \\ z_{PMU} - h_{PMU}(x) \le L_{\mathrm{PMU}} \\ C(x) = 0 \end{cases}$$

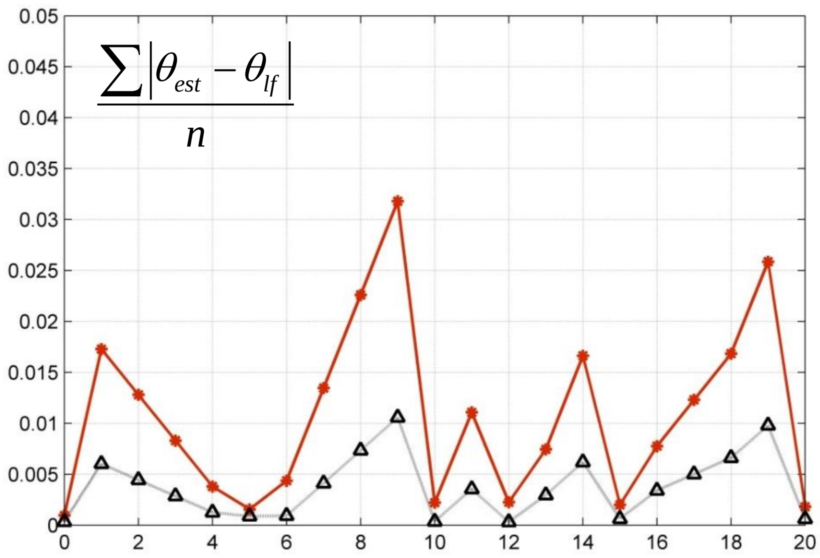
Between two refreshed SCADA data:

$$\begin{cases} z_{PMU} - h_{PMU}(x) \le L_{\mathrm{PMU}} \\ C(x) = 0 \end{cases}$$



$\frac{\sum |V_{est} - V_{lf}|}{n}$

t



$\frac{\sum |\theta_{est} - \theta_{lf}|}{n}$

t

Quasi-Gradient Smart (QGS) Methodology

- **Superior convergence property with less number of measurements (**for networks with poor measurement redundancy)
- **Convergence property with missing measurements**
- **Robust** against a set of measurements with a large quantity of current magnitudes
- **Flexible formulation** with residual and quality constraints for every measurement
- Novel approach to **utilize PMU measurements**
- **Easy integration of existing SE solvers and yet greatly improv**

Patent Pending

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Thank You !