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Nonlinear Model Based Coordinated Adaptive Robust Control of Electro-hydraulic Robotic Arms via Overparametrizing Method

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Abstract

This paper studies the coordinated motion control of robotic manipulators driven by single-rod hydraulic actuators. Compared to conventional robot manipulators driven by electrical motors, hydraulic robot arms have a richer nonlinear dynamics and stronger couplings among various joints (or hydraulic cylinders). This paper presents a physical model based adaptive robust control (ARC) strategy to explicitly take into account the strong coupling among various hydraulic cylinders (or joints). To avoid the need of acceleration feedback for ARC backstepping design, the property that the adjoint matrix and the determinant of the inertial matrix could be linearly parametrized by certain suitably selected parameters is fully exploited and overparametrizing method is used. Theoretically, the resulting controller is able to take into account not only the effect of parametric uncertainties coming from the payload and various hydraulic parameters but also the effect of uncertain nonlinearities. Furthermore, the proposed ARC controller guarantees a prescribed output tracking transient performance and final tracking accuracy while achieving asymptotic output tracking in the presence of parametric uncertainties only. Simulation and experimental results on a three degree-of-freedom (DOF) hydraulic robot arm (a scaled down version of an industrial backhoe/excavator arm) are presented to illustrate the proposed control algorithm.

1 Introduction

Robotic manipulators driven by hydraulic actuators have been widely used in the industry for the tasks such as material handling and earth moving due to its high power density. These types of tasks typically require that the end-effectors of the manipulators follow certain prescribed desired trajectories in the working space. In order to meet the increasing requirement of productivity and performance of modern industry, the development of high speed and high accuracy trajectory tracking controllers for the coordinated motion of hydraulic robot manipulators is of practical importance.

The controller design for the hydraulic robotic manipulator is much more difficult both theoretically and experimentally than those for the conventional robotic manipulators driven by electrical motors, due to the following several reasons. First of all, unlike the electrical motors, the hydraulic cylinders are linear actuators and complicated mechanical mechanisms are needed to drive revolute joints. Such a configuration results in additional nonlinearites and stronger couplings among the dynamics of various joints. Secondly, in addition to the coupled MIMO nonlinear dynamics of the rigid robot arm, the dynamics of the hydraulic actuators must be considered in the control of a hydraulic arm, which substantially increases the controller design difficulties. It is well known that a robot arm including actuator dynamics [1] has a "relative degree" more than three. Synthesizing a controller for such a system usually requires joint acceleration feedback for a complete state feedback, which may not be a practical solution. Furthermore, the single-rod hydraulic actuator studied here has a much more complicated dynamics than electrical motors. The dynamics of a hydraulic cylinder is highly nonlinear [2] and may be subjected to nonsmooth and discontinuous nonlinearities due to directional change of valve opening and friction. The dynamic equations describing the pressure changes in the two chambers of a single-rod hydraulic actuator cannot be combined into a single load pressure equation, which not only increases the dimension of the system to be dealt with but also brings in the stability issue of the added internal dynamics. Finally, a hydraulic arm normally experiences large extent of model uncertainties including the large changes in load seen by the system in industrial use, the large variations in the hydraulic parameters (e.g., bulk modulus), leakages, friction, and external disturbances. Partly due to these difficulties, so far, the model-based coordinated robust control of a hydraulic arm has not been well studied and fewer results are available. In [3], the singular perturbation was used to synthesize a controller for a 6 axis hydraulic robot. In [4], a variable structure controller was developed to control a Caterpillar excavator without considering parametric uncertainties and uncertain nonlinearities associated with the system simultaneously. Theoretically, none of above schemes could address all the difficulties mentioned above well.

In [5, 6], the ARC approach proposed by Yao and Tomizuka in [7, 8, 9, 10] was generalized to provide a rigorous theoretical framework for the high performance robust motion control of a one DOF single-rod hydraulic actuator. The stability of zero output tracking error dynamics of single-rod hydraulic actuator was also addressed in [5, 6]. In [11], a physical model based ARC controller, which explicitly takes into account the strong coupling among various hydraulic cylinders (or joints), is proposed for a multi-DOF hydraulic arm. An observer motivated by the design in [1] is proposed to avoid the need of acceleration feedback for ARC backstepping design.

Experimental results show that the observer in [11] may be quite sensitive to measurement noises, which may limit the achievable performance in implementation. To by-pass this practical problem, a different method will be adopted in this paper to avoid the need for joint acceleration feedback. The method makes full use of the property that the adjoint matrix and the determinant of the inertial matrix can be linearly parametrized by certain suitably selected parameters, and employs certain overparametrizing technique to by-pass the need for acceleration feedback. The design is motivated by the researches in the electrical motor driven robot manipulators as summarized in [12] where two types of controllers are synthesized for a rigid-link flexible-joint robot. The first controller is a robust design which could ensure the Globally Uniform Ultimate Bounded (GUUB) stability in the presence of parametric uncertainties and uncertain nonlinearities. The second controller is an adaptive design which could guarantee Globally Asymptotic Stability (GAS) in the presence of parametric uncertainties only. The difference between the proposed ARC approach and the work done in [12] is that our approach effectively combines the design techniques of adaptive control (AC) and those of deterministic robust control (DRC). The basic idea is that: by using the robust feedback technique as in DRC [13, 14], the ARC will attenuate the effects of model uncertainties coming from both parametric uncertainties and uncertain nonlinearities as much as possible. In addition, certain parameter adaptation technique will be used to achieve a better model compensation for an improved performance. Theoretically, the proposed ARC approach achieves a guaranteed transient and final tracking accuracy for output trajectory tracking, which gets rid of the drawbacks of adaptive designs. At the same time, asymptotic output tracking is achieved in the presence of parametric uncertainties only, which overcomes the performance limitation of robust designs. The simulation and experimental results on a 3 DOF hydraulic arm will be presented to illustrate the proposed control algorithm.

2 Problem Formulation and Dynamic Models

The system under consideration is depicted in Fig.1, which represents a 3 DOF robot arm driven by three single-rod hydraulic cylinders. To make the results general, let us consider a n DOF robot arm driven by n hydraulic cylinders. The joint angles are represented by $q = [q_1, q_2, \ldots, q_n]^T$. $x = [x_1, x_2, \ldots, x_n]^T$ is the displacement vector of the hydraulic cylinders; each cylinder's displacement is uniquely related to the corresponding joint angle, i.e., $x_1(q_1)$, $x_2(q_2)$, and so on. The goal is to have joint angles q track any feasible desired motion trajectories as closely as possible for precision maneuver of the inertia load of the hydraulic robot arm. The rigid-body dynamics of the hydraulic arm can be described by:

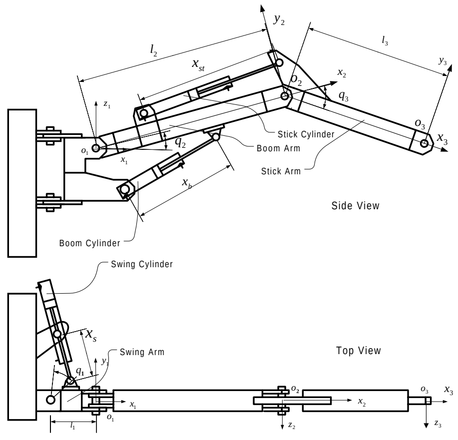

Figure 1: A Hydraulic Robot Arm

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \frac{\partial x}{\partial q}(A_1P_1 - A_2P_2) + T(t,q,\dot{q}) \quad (1)$$

where $P_1 = [P_{11}, P_{12}, \cdots, P_{1n}]^T$ and P_{1i} $(i = 1,2,\cdots,n)$ is the forward chamber pressure of the ith cylinder. $P_2 = [P_{21}, P_{22}, \cdots, P_{2n}]^T$ and P_{2i} $(i = 1,2,\cdots,n)$ is the return chamber pressure of the ith cylinder. $A_1 = diag[A_{11}, A_{12}, \cdots, A_{1n}]$ and $A_2 = diag[A_{21}, A_{22}, \cdots, A_{2n}]$ are the ram areas of the two chambers of the driving cylinders and $T(t,q,\dot{q}) \in R^n$ represents the lumped disturbance torque including external disturbances and terms like the joint friction torque.

Let m_L be the unknown payload mounted at the end of the nth arm, which is treated as a point mass for simplicity. Then, the inertial matrix $M(q)$, coriolis terms $C(q,\dot{q})$ and gravity terms $G(q)$ in (1) can be linearly parametrized with respect to the unknown mass m_L as

$$\begin{array}{l} M(q) = M_c(q) + M_L(q)m_L, \;\; G(q) = G_c(q) + G_L(q)m_L \\ C(q,\dot{q}) = C_c(q,\dot{q}) + C_L(q,\dot{q})m_L \end{array} \quad (2)$$

where $M_c(q)$, $M_L(q)$, $C_c(q,\dot{q})$, $C_L(q,\dot{q})$, $G_c(q)$, $G_L(q)$ are known nonlinear functions of q and $\dot{q}$. One of the properties of the inertia matrix $M(q)$ is that its inverse can be written as:

$$M^{-1}(q) = \bar{M}(q)/|M(q)| \quad (3)$$

where $|M(q)|$ represents the determinant of $M(q)$, $\bar{M}(q)$ represents the adjoint matrix of $M(q)$. Furthermore, both $\bar{M}(q)$ and $|M(q)|$ can be written as

$$|M(q)| = I = I_c + \sum_{i=1}^{n} I_{si} m_L^i, \quad \bar{M}(q) = \bar{M}_c + \sum_{i=1}^{n-1} \bar{M}_i m_L^i \quad (4)$$

where I_c, I_{si}, $\bar{M}_c$ and $\bar{M}_i$ are scalars and matrices of the known functions of joint position q with I_c, I_{si}, and I being scalars.

Assuming no cylinder leakages, the actuator (or the cylinder) dynamics can be written as [2],

$$\begin{array}{l} \frac{V_1(x)}{\beta_e}\dot{P}_1 = -A_1\dot{x} + Q_1 = -A_1\frac{\partial x}{\partial q}\dot{q} + Q_1 \\ \frac{V_2(x)}{\beta_e}\dot{P}_2 = A_2\dot{x} - Q_2 = A_2\frac{\partial x}{\partial q}\dot{q} - Q_2 \end{array} \quad (5)$$

where $V_1(x) = V_{h1} + A_1 diag[x] \in R^{n\times n}$ and $V_2(x) = V_{h2} - A_2 diag[x]$ are the diagonal total control volume matrices of the two chambers of hydraulic cylinders respectively, which include the hose volume between the two chambers and the valves, $V_{h1} = diag[V_{h11}, V_{h12}, \cdots, V_{h1n}]$ and $V_{h2} = diag[V_{h21}, V_{h22}, \cdots, V_{h2n}]$ are the control volumes of the two chambers when $x = 0$, $diag[x] = diag[x_1, x_2, \cdots, x_n]$, $\beta_e \in R$ is the effective bulk modulus, $Q_1 = [Q_{11}, Q_{12}, \cdots, Q_{1n}]^T$ is the vector of the supplied flow rates to the forward chambers of the driving cylinders, and $Q_2 = [Q_{21}, Q_{22}, \cdots, Q_{2n}]^T$ is the vector of the return flow rates from the return chambers of the cylinders.

Let $x_v = [x_{v1}, x_{v2}, \cdots, x_{vn}]$ denotes the spool displacements of the valves in the hydraulic loops. Define the square roots of the pressure drops across the two ports of the first control valve as:

$$
\begin{aligned}
g_{31}(P_{11}, sign(x_{v1})) &= \begin{cases} \sqrt{P_s - P_{11}} & \text{for} \quad x_{v1} \geq 0 \\ \sqrt{P_{11} - P_r} & \qquad\; x_{v1} < 0 \end{cases} \\
g_{41}(P_{21}, sign(x_{v1})) &= \begin{cases} \sqrt{P_{21} - P_r} & \text{for} \quad x_{v1} \geq 0 \\ \sqrt{P_s - P_{21}} & \qquad\; x_{v1} < 0 \end{cases}
\end{aligned} \tag{6}
$$

where P_s is the supply pressure of the pump, and P_r is the tank reference pressure. Similarly, let g_{3i} and g_{4i} be the square roots of the pressure drops for the ith hydraulic loop. For simplicity of notation, define the diagonal square root matrices of the pressure drops as:

$$
\begin{aligned}
g_3(P_1, sign(x_v)) &= diag[g_{31}(P_{11}, sign(x_{v1})), \ldots, g_{3n}(P_{1n}, sign(x_{vn}))] \\
g_4(P_2, sign(x_v)) &= diag[g_{41}(P_{21}, sign(x_{v1})), \ldots, g_{4n}(P_{2n}, sign(x_{vn}))]
\end{aligned} \tag{7}
$$

Then, Q_1 and Q_2 in (5) are related to the spool displacements of the valves x_v by [2],

$$
Q_1 = k_{q1} g_3(P_1, sign(x_v)) x_v \quad , Q_2 = k_{q2} g_4(P_2, sign(x_v)) x_v \tag{8}
$$

where $k_{q1} = diag[k_{q11}, \ldots, k_{q1n}]$ and $k_{q2} = diag[k_{q21}, \ldots, k_{q2n}]$ are the *constant* flow gain coefficients matrices of the forward and return loops respectively. Thus, neglecting the valve dynamics, the control objective can be stated as:

Given the desired motion trajectory $q_d(t)$, the objective is to synthesize a control input $u = x_v$ such that the output $y = q$ tracks $q_d(t)$ as closely as possible in spite of various model uncertainties.

3 Adaptive Robust Controller Design

3.1 Design Model and Issues to be Addressed

In this paper, for simplicity, we consider the parametric uncertainties due to the unknown payload m_L, and the nominal value of the lumped disturbance T, T_n only. Other parametric ucnertainties can be dealt with in the same way if necessary. In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parametrize the system dynamics equation in terms of a set of unknown parameters. To achieve this, define the unknown parameter set as $\theta = [\theta_1, \theta_2^T]^T$ where $\theta_1 = m_L$ and $\theta_2 = T_n$. The system dynamic equations can thus be linearly parametrized in terms of θ as

$$
\begin{aligned}
&M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tfrac{\partial x}{\partial q}(A_1 P_1 - A_2 P_2) \\
&\quad + \theta_2 + \tilde{T}(t, q, \dot{q}), \quad \tilde{T} = T(t, q, \dot{q}) - T_n \\
&\dot{P}_1 = \beta_e V_1^{-1}(q) \left[-A_1 \tfrac{\partial x}{\partial q}\dot{q} + Q_1(u, g_3(P_1, sign(u))) \right] \\
&\dot{P}_2 = \beta_e V_2^{-1}(q) \left[A_2 \tfrac{\partial x}{\partial q}\dot{q} - Q_2(u, g_4(P_2, sign(u))) \right]
\end{aligned} \tag{9}
$$

Since the extent of the parametric uncertainties and uncertain nonlinearities are normally known, the following practical assumption is made. Parametric uncertainties and uncertain nonlinearities satisfy

$$
\begin{aligned}
\theta \quad &\in \quad \Omega_\theta \triangleq \{\theta : \ \theta_{min} < \theta < \theta_{max} \ \} \\
|\tilde{T}(t, q, \dot{q})| \quad &\leq \quad \delta_T(q, \dot{q}, t)
\end{aligned} \tag{10}
$$

where $\theta_{min} = [\theta_{1min}, , \theta_{2min}]^T$, $\theta_{max}=[\theta_{1max}, \theta_{2max}]^T$, and $\delta_T(t, q, \dot{q})$ are known.

At this stage, it can be seen that the main difficulties in controlling (9) are: (i) The system dynamics are highly nonlinear and coupled, due to either the nonlinear robot dynamics or the dependence of the effective driving torque on joint angle (terms like $\frac{\partial x(q)}{\partial q}$) and the nonlinearities in the hydraulic dynamics; (ii) The system has large extent of parametric uncertainties due to the large variations of inertial load m_L; (iii) The system may have large extent of lumped uncertain nonlinearities $\tilde{T}$ including external disturbances and unmodeled friction forces; (iv) The added nonlinear hydraulic dynamics are more complex than the electrical motor dynamics; (v) The model uncertainties are mismatched, i.e. both parametric uncertainties and uncertain nonlinearities appear in the dynamic equations which are not directly related to the control input $u = x_v$.

To address the challenges mentioned above, following general strategies will be adopted in the controller design. Firstly, the nonlinear physical model based analysis and synthesis will be employed to deal with the nonlinearities and coupling of the system dynamics. Secondly, the ARC approach [7, 10] will be used to handle the effect of both parametric uncertainties and uncertain nonlinearities; fast robust feedback will be used to attenuate the effect of various model uncertainties as much as possible while parameter adaptation will be introduced to reduce model uncertainties for high performance. Thirdly, backstepping design via ARC Lyapunov function will be used to overcome the design difficulties caused by the unmatched model uncertainties. Finally, the property that the adjoint matrix and the determinant of the inertial matrix could be linearly parametrized by certain suitably selected parameters is fully exploited so that certain overparametrizing method can be employed to avoid the need for joint acceleration feedback. The details are outlined below.

3.2 Controller Design using Overparametrizing

The design parallels the recursive backstepping design procedure via ARC Lyapunov functions in [10, 5] as follows.

Step 1

Define a switching-function-like quantity as $z_2 = \dot{z}_1 + k_1 z_1 = \dot{q} - \dot{q}_r$, where $\dot{q}_r = \dot{q}_d - k_1 z_1$, and $z_1 = q - q_d(t)$, in which $q_d(t)$ is the reference trajectory and k_1 is a positive feedback gain. The design in this step is to make z_2 as small as possible with a guaranteed transient performance. The design is the same as in [11] and is briefly outlined below.

Define a positive semi-definite (p.s.d) function as $V_2 =$

$\frac{1}{2}z_2^T M z_2$. From(9) and the property that $\dot{M}(q) - 2C(q,\dot{q})$ is a skew symmetric matrix [15], its derivative is given by

$$\begin{aligned}\dot{V}_2 &= z_2^T(M\dot{z}_2 + \tfrac{1}{2}\dot{M}z_2)\\ &= z_2^T[-M\ddot{q}_r - C_(\dot{q},q)\dot{q}_r - G(q) + \tfrac{\partial x}{\partial q}(A_1P_1 - A_2P_2) + \theta_2 + \tilde{T}]\end{aligned} \quad (11)$$

Define the load pressure as $P_L = A_1P_1 - A_2P_2$. If we treat P_L as the virtual control input to (11), a virtual control law P_{Ld} for P_L will be synthesized such that z_2 is as small as possible with a guaranteed transient performance. Since (11) has both parametric uncertainties θ_1 and θ_2 and uncertain nonlinearity $\tilde{T}$, the ARC approach proposed in [10] will be generalized to accomplish the objective. The control function P_{Ld} consists of two parts given by

$$\begin{aligned}P_{Ld}(q,\dot{q},\hat{\theta}_1,\hat{\theta}_2,t) &= P_{Lda} + P_{Lds}\\ P_{Lda} &= (\tfrac{\partial x}{\partial q})^{-1}[\hat{M}\ddot{q}_r + \hat{C}(\dot{q},q)\dot{q}_r + \hat{G}(q) - \hat{\theta}_2 - K_2(t)z_2]\end{aligned} \quad (12)$$

where $K_2(t)$ is a positive feedback gain matrix, $\hat{M}(q) = M_c + M_L\hat{\theta}_1$, $\hat{C}(\dot{q},q) = C_c + C_L\hat{\theta}_1$, and $\hat{G}(q) = G_c + G_L\hat{\theta}_1$. Substituting (12) into (11) and let $z_3 = P_L - P_{Ld}$ represent the input discrepancy, we will have

$$\dot{V}_2 = z_2^T[\tfrac{\partial x}{\partial q}P_{Lds} - \phi_2\tilde{\theta} + \tilde{T}] - z_2^T K_2(t)z_2 + z_2^T\tfrac{\partial x}{\partial q}z_3 \quad (13)$$

where $\phi_2 = [-M_L\ddot{q}_r - C_L(\dot{q},q)\dot{q}_r - G_L(q), I_{n\times n}]$. Then P_{Lds} can be chosen to satisfy:

$$\begin{aligned}&\text{condition i} && z_2^T[\tfrac{\partial x}{\partial q}P_{Lds} - \phi_2\tilde{\theta} + \tilde{T}] \le \varepsilon_2\\ &\text{condition ii} && z_2^T\tfrac{\partial x}{\partial q}P_{Lds} \le 0\end{aligned} \quad (14)$$

where ε_2 is a design parameter which can be arbitrarily small. Essentially, condition i of (14) shows that P_{Lds} is synthesized to dominate the model uncertainties coming from both parametric uncertainties $\tilde{\theta}$ and uncertain nonlinearities $\tilde{T}$, and condition ii is to make sure that P_{Lds} is dissipating in nature so that it does not interfere with the functionality of the adaptive control part P_{Lda}. How to choose P_{Lds} to satisfy constraints like (14) can be worked out in the same way as in [8, 9]. The adaptive function τ_θ and the adaptation law are given by

$$\tau_\theta = \phi_2 z_2 \quad \dot{\hat{\theta}} = Proj(\Gamma_\theta\tau_\theta) \quad (15)$$

where $Proj(\bullet)$ denote the discontinuous projection defined in [16, 17, 7], and Γ_θ denotes the adaptive gain matrix.

Step 2 In this step, an actual control law will be synthesized so that z_3 converges to zero or a small value with a guaranteed transient performance and accuracy. If we were to use the backstepping design strategy via ARC Lyapunov function as in [5, 10], then, the resulting ARC law would require the feedback of the joint acceleration $\ddot{q}$ since $\ddot{q}$ is needed in computing $\hat{\dot{P}}_{Ld}$, the calculable part of the derivative of the desired virtual control function P_{Ld}, for adaptive model compensation. In order to avoid the need for joint acceleration feedback, in the following, the property of the inertia matrix in (4) will be used as follows.

Multiply both side of first equation of (9) by $|M|M^{-1} = \bar{M}$, we will have

$$|M|\ddot{q} + \bar{M}C(\dot{q},q)\dot{q} + \bar{M}G = \bar{M}\tfrac{\partial x}{\partial q}P_L + \bar{M}\theta_2 + \bar{M}\tilde{T} \quad (16)$$

Define $C_t(\dot{q},q) = \bar{M}C(\dot{q},q)$, $G_t(q) = \bar{M}G$, $d_n = \bar{M}\theta_2$, $\tilde{d} = \bar{M}\tilde{T}$. Thus (16) could be expressed by

$$I\ddot{q} + C_t\dot{q} + G_t = \bar{M}\tfrac{\partial x}{\partial q}P_L + d_n + \tilde{d} \quad (17)$$

where I is a scalar. Similar to (2), C_t, G_t and d_n can be expressed by $C_t(\dot{q},q) = C_{tc} + \sum_{i=1}^n C_{ti}\theta_1^i$, $G_t(q) = G_{tc} + \sum_{i=1}^n G_{ti}\theta_1^i$, $d_n = \bar{M}_c\theta_2 + \sum_{i=1}^{n-1}\bar{M}_i\theta_1^i\theta_2$. Where C_{tc} and G_{tc} are of the known nonlinear functions of q and $\dot{q}$.

Redefine the unknown parameters as:$[\beta_1, \beta_2, \cdots, \beta_n, \beta_{n+1}^T, \beta_{n+2}^T, \cdots, \beta_{2n}^T] = [\theta_1, \theta_1^2, \cdots, \theta_1^n, \theta_2^T, \theta_1\theta_2^T, \cdots, \theta_1^{n-1}\theta_2^T]$. From (9), the derivative of z_3 is given by

$$\begin{aligned}\dot{z}_3 &= \dot{P}_L - \dot{P}_{Ld}\\ \dot{P}_L &= \beta_e[-(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} + (A_1V_1^{-1}Q_1 + A_2V_2^{-1}Q_2)]\\ \dot{P}_{Ld} &= \tfrac{\partial P_{Ld}}{\partial q}\dot{q} + \tfrac{\partial P_{Ld}}{\partial \dot{q}}\ddot{q} + \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \tfrac{\partial P_{Ld}}{\partial t}\end{aligned} \quad (18)$$

Define a p.s.d function as $V_3 = V_2 + \frac{1}{2}Iz_3^T z_3$. The derivative of V_3 is given by

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2|_{z_3=0} + z_2^T\tfrac{\partial x}{\partial q}z_3 + Iz_3^T\dot{z}_3 + \tfrac{1}{2}\dot{I}z_3^T z_3\\ &= \dot{V}_2|_{z_3=0} + z_3^T(\tfrac{\partial x}{\partial q}z_2 + I\dot{P}_L - I\dot{P}_{Ld} + \tfrac{1}{2}\dot{I}z_3)\end{aligned} \quad (19)$$

where $\dot{V}_2|_{z_3=0}$ represents the derivative of V_2 when $z_3 = 0$ and $I\dot{P}_{Ld}$ can be expressed by

$$I\dot{P}_{Ld} = \widehat{I\dot{P}_{Ld}} + \widetilde{I\dot{P}_{Ld}} \quad (20)$$

where

$$\begin{aligned}\widehat{I\dot{P}_{Ld}} &= \tfrac{\partial P_{Ld}}{\partial q}\hat{I}\dot{q} + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(-\hat{C}_t\dot{q} - \hat{G}_t + \hat{\bar{M}}\tfrac{\partial x}{\partial q}P_L + \hat{d}_n) + \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\hat{I}\dot{\hat{\theta}} + \tfrac{\partial P_{Ld}}{\partial t}\hat{I}\\ \widetilde{I\dot{P}_{Ld}} &= (\tfrac{\partial P_{Ld}}{\partial q}\dot{q} + \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \tfrac{\partial P_{Ld}}{\partial t})(-\textstyle\sum_{i=1}^n I_{si}\tilde{\beta}_i) + \tfrac{\partial P_{Ld}}{\partial \dot{q}}[\textstyle\sum_{i=1}^n(C_{ti}\dot{q} + G_{ti})\tilde{\beta}_i\\ &\quad - \textstyle\sum_{i=1}^{n-1}\bar{M}_i\tfrac{\partial x}{\partial q}P_L\tilde{\beta}_i - \bar{M}_c\tilde{\beta}_{n+1} - \textstyle\sum_{i=2}^n\bar{M}_{i-1}\tilde{\beta}_{n+i} + \tilde{d}]\\ \hat{I} &= I_c + \textstyle\sum_{i=1}^n I_{si}\hat{\beta}_i, \quad \hat{C}_t = C_{tc} + \textstyle\sum_{i=1}^n C_{ti}\hat{\beta}_i\\ \hat{G}_t &= G_{tc} + \textstyle\sum_{i=1}^n G_{ti}\hat{\beta}_i, \quad \hat{d}_n = \bar{M}_c\hat{\beta}_{n+1} + \textstyle\sum_{i=2}^n\bar{M}_{i-1}\hat{\beta}_{n+i}\\ \hat{\bar{M}} &= \bar{M}_c + \textstyle\sum_{i=1}^{n-1}\bar{M}_i\hat{\beta}_i\end{aligned} \quad (21)$$

$\widehat{I\dot{P}_{Ld}}$ represents the calculable part of $I\dot{P}_{Ld}$ and will be used in the model compensation part of the ARC control law in this step, $\widetilde{I\dot{P}_{Ld}}$ is the incalculable part of $I\dot{P}_{Ld}$ and will be attenuated by certain robust feedback.

Define $Q_L = A_1V_1^{-1}Q_1 + A_2V_2^{-1}Q_2$. From (19), Q_L can be treated as the virtual control input in this step and we will synthesize an ARC control function Q_{Ld} for Q_L such that P_L tracks P_{Ld}. Similar to the first step, Q_L is given by

$$\begin{aligned}Q_{Ld} &= Q_{Lda} + Q_{Lds}\\ Q_{Lda} &= (A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} + \tfrac{1}{\hat{I}\beta_e}(-\tfrac{\partial x}{\partial q}z_2 + \widehat{I\dot{P}_{Ld}} - \tfrac{1}{2}\hat{\dot{I}}z_3 - I_cK_3z_3)\end{aligned} \quad (22)$$

where $\hat{\dot{I}} = \dot{I}_c + \sum_{i=1}^n\dot{I}_{si}\hat{\beta}_i$ and K_3 is a positive feedback gain matrix. Substituting (22) in (19), we have

$$\dot{V}_3 = \dot{V}_2|_{z_3=0} + z_3^T(I\beta_eQ_{Lds} - \phi_3\tilde{\beta} - \tfrac{\partial P_{Ld}}{\partial \dot{q}}\tilde{d}) - z_3^T I_cK_3z_3 \quad (23)$$

where $\phi_3 = [\phi_{3(1)}, \cdots, \phi_{3(n-1)}, \phi_{3n}, \phi_{3(n+1)}, \phi_{3(n+2)}, \cdots, \phi_{3(2n)}]$.

$$
\begin{aligned}
\phi_{3(1)} &= I_{s1}[\beta_e Q_{Lda} - \beta_e(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} - \tfrac{\partial P_{Ld}}{\partial q}\dot{q} \\
&\quad - \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tfrac{\partial P_{Ld}}{\partial t}] + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(C_{t1}\dot{q} + G_{t1} - \bar{M}_1\tfrac{\partial x}{\partial q}P_L) + \tfrac{1}{2}\dot{I}_{s1}z_3 \\
\phi_{3(n-1)} &= I_{sn-1}[\beta_e Q_{Lda} - \beta_e(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} \\
&\quad - \tfrac{\partial P_{Ld}}{\partial g}\dot{q} - \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tfrac{\partial P_{Ld}}{\partial t}] + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(C_{tn-1}\dot{q} + G_{tn-1} \\
&\quad - \bar{M}_{n-1}\tfrac{\partial x}{\partial q}P_L) + \tfrac{1}{2}\dot{I}_{sn-1}z_3 \\
\phi_{3n} &= I_{sn}[\beta_e Q_{Lda} - \beta_e(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} \\
&\quad - \tfrac{\partial P_{Ld}}{\partial q}\dot{q} - \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tfrac{\partial P_{Ld}}{\partial t}] + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(C_{tn}\dot{q} + G_{tn}) + \tfrac{1}{2}\dot{I}_{sn}z_3 \\
\phi_{3(n+1)} &= -\tfrac{\partial P_{Ld}}{\partial \dot{q}}\bar{M}_c, \quad \phi_{3(n+2)} = -\tfrac{\partial P_{Ld}}{\partial \dot{q}}\bar{M}_1 \\
\phi_{3(2n)} &= -\tfrac{\partial P_{Ld}}{\partial \dot{q}}\bar{M}_{n-1}
\end{aligned}
\tag{24}
$$

Thus Q_{Lds} could be chosen to satisfy:

$$
\begin{aligned}
&\text{condition i} \quad z_3^T(I\beta_e Q_{Lds} - \phi_3\tilde{\beta} - \tfrac{\partial P_{Ld}}{\partial \dot{q}}\tilde{d}) \leq \varepsilon_3 \\
&\text{condition ii} \quad z_3^T I\beta_e Q_{Lds} \leq 0
\end{aligned}
\tag{25}
$$

where ε_3 is a positive design parameter.

Once the control function Q_{Ld} for Q_L is synthesized as given by (22), the actual control input u can be backed out from the continuous one-to-one nonlinear load flow mapping as follows. Noting that the elements of the diagonal matrices g_3, g_4, V_1, and V_2 are all positive functions, u_i, the control input for the ith hydraulic loop, should have the same sign as Q_{Ldi}. Thus

$$
u_i = [A_{1i}V_{1i}^{-1}k_{q1i}g_{3i}(P_{1i}, sign(Q_{Ldi})) + A_{2i}V_{2i}^{-1}k_{q2i}g_{4i}(P_{2i}, sign(Q_{Ldi}))]^{-1}Q_{Ldi} \tag{26}
$$

where $i = 1, 2, \ldots, n$. The adaptation law in this step is given by

$$
\dot{\hat{\beta}} = Proj(\Gamma_\beta \tau_\beta) \quad \tau_\beta = \phi_3 z_3 \tag{27}
$$

3.3 Main Theoretical Results

Theorem 1 *Let the parameter estimates $\hat{\theta}$ and $\hat{\beta}$ be updated by the adaptation law (15) and (27) respectively. Then, the following results hold if the control law (26) is applied:*

A. *In general, the tracking errors, z_1, z_2 and z_3, are bounded. Furthermore, V_3, an index for the bound of the tracking errors, is bound above by*

$$
V_3(t) \quad \leq exp(-\lambda_V t)V_3(0) + \tfrac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \tag{28}
$$

where $\lambda_V = \frac{2min\{k_2, k_3 I_c\}}{max\{k_M, I_M\}}$, $\varepsilon_V = \varepsilon_2 + \varepsilon_3$, k_M is the upper bound of the inertial matrix (i.e., $M(q) \leq k_M I_{3\times3}$), , I_M is the upper bound of the determinant of inertial matrix, and $k_2 = \inf_{t>0}\lambda_{min}(K_2(t)), k_3 = \lambda_{min}(K_3)$.

B *If after a finite time t_0, $\tilde{T} = 0$, i.e., in the presence of parametric uncertainties only, in addition to results in A, asymptotic output tracking is also obtained.* △

Proof : The details are outlined in Appendix A. □.

4 Simulation and Experimental Results

To study fundamental problems associated with the control of electro-hydraulic systems, a three-link robot arm (a scaled down version of industrial backhoe loader arm) driven by three single-rod hydraulic cylinders shown in Fig.1 has been set up. The three hydraulic cylinders are controlled by two proportional directional control valves and one servovalve. The detailed experimental set-up can be found in [5]. The exact model of the hydraulic arm shown in Fig.1 is quite messy and can be obtained from the authors, with parameters of the actual arm given in [11].

To illustrate the proposed algorithm, a simulation is first performed by neglecting the valve dynamics as assumed in the paper. The control gain and gain matrices in the simulation are: $k_1 = 150$, $K_2 = diag[150, 200, 220]$, $K_3 = diag[150,200,220]$. Adaptation gain matrices are $\Gamma_\theta = diag[30,\ 1\times10^{-8},\ 1\times10^{-7},\ 1\times10^{-7}\ ]$ and $\Gamma_\beta = diag[20, 2\times10^{-10},\ 2\times10^{-7}, 1\times10^{-8},\ 1\times10^{-7},\ 1\times10^{-7}, 1\times10^{-9},\ 1\times10^{-8},\ 1\times10^{-8}, 1\times10^{-10},\ 1\times10^{-9},\ 1\times10^{-9}]$. A typical point-to-point desired trajectory shown in Fig.2 is used for all three joints, which has a maximum velocity of $v_{max} = 0.1 rad/s$ and a maximum acceleration of $a_{max} = 0.2 rad/s^2$. The tracking errors of three joints are shown in Fig.3. As seen, the system has very small tracking errors during both transient periods and steady-state periods.

The experiments are carried out on the experimental setup described above. In the experiments, only swing and boom joints are actuated with stick joint fixed due to the hardware limitations at this stage. The controller parameters are $k_1 = 17$, $K_2 = diag[17, 23]$, $K_3 = diag[17,23]$. Adaptation gain matrices are $\Gamma_\theta = diag[0.1,\ 1\times10^{-8},\ 2\times10^{-7}]$ and $\Gamma_\beta = diag[0.01, 1\times10^{-10},\ 7\times10^{-9},\ 7\times10^{-8},\ 2\times10^{-10},\ 3\times10^{-9}]$. Same trajectory as in the simulation are used. The results shown in Fig.4 demonstrate that the proposed coordinated control algorithm achieves a satisfactory tracking performance. It is also noted that there is a significant discrepancy between the simulation and experiment tracking errors. Such a discrepancy is caused by the neglected valve dynamics in the simulation; the valves used in experiments are of normal industrial types and have a bandwidth not far higher that the hydraulic-mechanical natural frequency. As such, one of the future research will be devoted to the incorporation of valve dynamics in the controller design for an improved performance.

5 Conclusion

In this paper, a physical model based adaptive robust controller (ARC) is constructed for the coordinated motion control of a n degree-of-freedom(DOF) hydraulic arm driven by single-rod hydraulic actuators. The ARC controller explicitly takes into account the strong coupling among various hydraulic cylinders (or joints). In addition, the property that adjoint matrix and the determinant of the inertial matrix could be linearly parametrized by certain suitable selected parameters is fully exploited so that overparametrizing method can be employed to avoid acceleration feedback. The controller is able to take into account not only the effect of parametric uncertainties coming from the payload and various hydraulic parameters but also the effect of uncertain nonlinearities such as uncompensated friction forces and external disturbances. Theoretically, the proposed ARC controller guarantees a prescribed output tracking transient performance and final tracking accuracy while achieving asymptotic output tracking in the presence of parametric uncertainties. Simulation and experimental results for a three DOF hydraulic arm are presented to illustrate the proposed algorithm.

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Appendix A

Proof of Theorem 1: From (13) and (23), the derivative of V_3 could be expressed by

$$\begin{aligned}\dot{V}_3 = & z_2^T\left(\frac{\partial x}{\partial q}P_{Lds} - \phi_2\tilde{\theta} + \tilde{T}\right) + z_3^T\left(I\beta_e Q_{Lds} - \phi_3\tilde{\beta} - \frac{\partial P_{Ld}}{\partial \dot{q}}\tilde{d}\right)\\ & -z_2^T K_2 z_2 - z_3^T I_c K_3 z_3\end{aligned} \quad (29)$$

From the condition i of (14) and (25), (29) can be written as

$$\begin{aligned}\dot{V}_3 & \leq -z_2^T K_2 z_2 - z_3^T I_c K_3 z_3 + \varepsilon_2 + \varepsilon_3\\ & \leq -\lambda_V V_3 + \varepsilon_V\end{aligned} \quad (30)$$

which will lead to the part A of Theorem 1. For the part B of Theorem 1, define a Lyapnouv function V_a as

$$V_a = V_3 + \tfrac{1}{2}\tilde{\theta}^T\Gamma_\theta^{-1}\tilde{\theta} + \tfrac{1}{2}\tilde{\beta}^T\Gamma_\beta^{-1}\tilde{\beta} \quad (31)$$

From (29), condition ii of (14) and (25) and the property of projection mapping, the derivative of V_a satisfies

$$\begin{aligned}\dot{V}_a & = \dot{V}_3 + \tilde{\theta}^T\Gamma_\theta^{-1}\dot{\hat{\theta}} + \tilde{\beta}^T\Gamma_\beta^{-1}\dot{\hat{\beta}}\\ & \leq -\lambda_V V_3 + \tilde{\theta}^T\Gamma_\theta^{-1}(\dot{\hat{\theta}} - \Gamma_\theta\tau_\theta) + \tilde{\beta}^T\Gamma_\beta^{-1}(\dot{\hat{\beta}} - \Gamma_\beta\tau_\beta)\\ & \leq -\lambda_V V_3\end{aligned} \quad (32)$$

Thus, z_2, z_3 are in L_2. It is easy to check $\dot{z}_1$ is bounded, by Barbalat's Lemma $z_1 \to 0$ as $t \to \infty$, which leads to B of Theorem 1 $\square$.

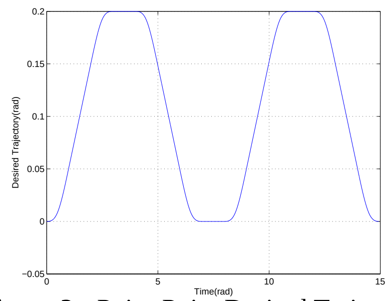

Figure 2: Point-Point Desired Trajectory

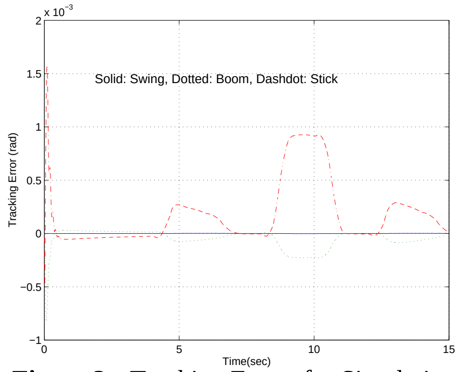

Figure 3: Tracking Errors for Simulation

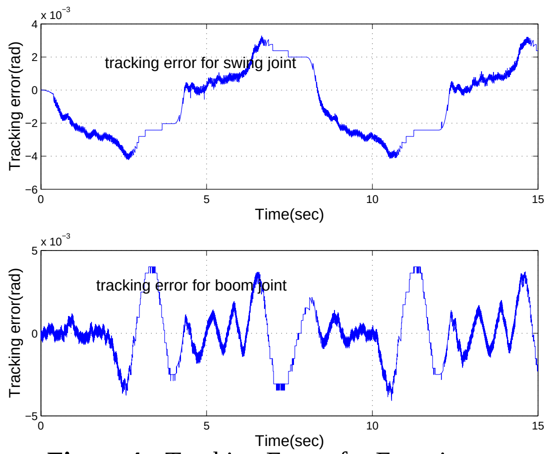

Figure 4: Tracking Errors for Experiments