North Carolina Community College System

Developmental Math Modular Curriculum

Module Outlines and Notes

BETA TEST VERSION – August 2011

Important Note: Beta tests of these modules during Fall 2011 may demonstrate the need to revise competencies/modules.

Redesign Principles

When the developmental math redesign began in North Carolina, a Math Redesign Task Force was appointed from nominations solicited from each of the NC Community Colleges. The task force was given principles to strive for in the redesign. These principles included the following:

- The new curriculum will be a modular approach.
- Developmental students will be able to complete the curriculum in an academic year.
- The new curriculum will be flexible to allow students to complete their required developmental math requirements at a pace that is appropriate to their needs and knowledge.
- Diagnostic testing will assure appropriate placement in modules.
- Each college will implement the new modular curriculum in a way that is appropriate for the needs and resources of the college.
- The modules will be rich in context and conceptual understanding.

Selection of Curriculum Objectives

The Developmental Math Redesign Task Force, made up of both developmental and curriculum math faculty, carefully considered what learning objectives to include in each module. Decisions centered on whether a given objective was truly a developmental topic or if the topic is part of the curriculum level course and taught there. Objectives were grouped into modules to facilitate conceptual understanding, allow for rich applications, and to eliminate repetition of topics. The modules are designed with each being a prerequisite of the next module. There are multiple exits points from the modules depending upon which curriculum math course the student is planning to take. (See Appendix.) The modules are also designed so that prerequisites for subsequent courses can be any consecutive sequence of modules, i.e. 1-4 or 1-5 or 1-8. This ensures that students are not required to complete more objectives than the subsequent course actually requires and, as a result, will allow students to reduce the amount of developmental math necessary to enroll and be successful in a curriculum course.

In the NCCCS Combined Course Library the modules' prefix will be DMA (for developmental math). Module 1 will be DMA 010, module 2 will be DMA 020, etc. The modules are referred to by their CCL prefix/number from this point forward.

Statement of Pedagogical Approach

The teaching approach that is recommended for the new developmental curriculum is contextual and conceptual. A module should begin, whenever possible, with a rich application with which students can connect and from which skills will emerge. In addition, conceptual understanding and connections to previously learned concepts will be emphasized throughout the module.

Students learn best when skills are placed into a meaningful context. This is most effective when the context is introduced before the procedural skills for problem solving. Deep understanding of mathematical concepts is as much the goal as is the ability of students to perform the required skills. While students are engaged in problem solving activities, effective questions to ask of students are: a) what is happening in the problem, b) why are you doing what you are doing, and c) how is your strategy working. Assessments to test for mastery of the student learning outcomes should be aligned with a conceptual understanding of the objectives.

Use of Calculators

The modules were developed with the understanding that students may have a calculator available to use throughout the sequence. The first 5 modules can be taught with a scientific calculator. However, there is no need to disallow the use of a graphing calculator in these modules. The last three modules require the use of a graphing calculator or comparable technology. Only students planning to transfer to a university (taking MAT 161 or MAT 171) will take these modules. Assessments must be carefully written to allow the use of the calculator and ensure the conceptual understanding of objectives.

What is Mastery?

An underlying principle of this math redesign is the concept of mastery. Students should master a module before they can move onto the next module or course. As a result, the task force has endorsed a pass/fail grading system since a module is either mastered or it is not. The details of this grade assignment will be forthcoming from NCCCS so that it is standard across all colleges.

The following items are strongly recommended:

1) Departments create and administer common assessments.
2) Each module has a rigorous final assessment that emphasizes conceptual understanding.
3) 90% of the overall grade for a module should consist of proctored assessments.
4) 10% of the overall grade for a module may consist of unproctored grades such as homework, attendance, etc.
5) Final module assessments can be taken twice with required remediation between attempts.
6) On minimum overall grade of 85% on a module indicates mastery.

Description of Module Components

This document outlines the content of each module. The following items are provided for each module:

Brief Description – a short paragraph describing the content of the module that will be used in the common course library.

Course Competencies – a list detailing what a student should be able to do after mastering the module.

Conceptual Student Learning Outcomes – a list of learning outcomes that are conceptually written. The expectation is that there will be assessment questions on these outcomes.

Suggested Timeline – a brief suggestion of how the module would be taught in four weeks.

Parameters and Teaching Tips – a list of items to consider as individual colleges set up the modules. Use of calculator, problems to include or avoid, suggested applications, methods to include or avoid, etc. are included.

Sample Introductory Application – it is recommended that the modules are introduced with an application to help students put the skills to be learned in context. Samples are provided to help instructors develop materials for their students.

Sample Conceptual Questions – these are questions that are written in a conceptual way whenever possible. A sample question is provided for each Conceptual Student Learning Outcome to help instructors develop questions for their students.

Student Success Tips – teaching developmental students involves much more than just teaching math. It is recommended that student success remain a part of the redesigned course and these suggestions are provided to help instructors incorporate student success in each module.

DMA 010
Operations with Integers

Brief Description

In this module, the emphasis will be on a conceptual understanding of the problem events that require the use of integers and integer operations. Once students have a solid understanding of these concepts they will find the absolute value, evaluate and write some basic exponents, evaluate square roots, and use the correct order of operations. Geometry applications will include the perimeter and area of rectangles and triangles, angle facts, and the Pythagorean Theorem. Emphasis will be on solving contextual application problems.

Course Competencies

At the completion of this module, the student will be able to:

- Visually represent an integer and it's opposite on the number line
- Explain the concept of the absolute value of an integer
- Demonstrate the conceptual understanding of operations with integers to solve application problems
- Correctly apply commutative and associative properties to integer operations
- Apply the proper use of exponents and calculate the principal square root of perfect squares
- Simplify multi-step expressions using the rules for order of operations
- Solve geometric application problems involving area and perimeter of rectangles and triangles, angles, and correctly apply the Pythagorean Theorem

Conceptual Student Learning Outcomes

1.1 Demonstrate an understanding of the concept of integers within contextual application problems

1.2 Correctly represent integers on a number line

1.3 Demonstrate the correct use of additive inverses

1.4 Evaluate the absolute value of a number

1.5 Apply integer operations in solving contextual application problems

1.6 Correctly apply the associative and commutative properties

1.7 Demonstrate understanding of exponents by converting between exponential and expanded form

1.8 Evaluate exponents

1.9 Calculate the square root of numbers containing perfect squares

1.10 Evaluate integer expressions by using the correct order of operations

1.11 Distinguish between appropriate use of area and perimeter formulas to solve geometric application problems

1.12 Represent the events of a geometric application problem included in this module pictorially and evaluate the correct solution using the appropriate formula

Suggested Timeline

Week 1	Objectives 1.1 through 1.5
Week 2	Objectives 1.6 through 1.10
Weeks 3 and 4	Objectives 1.11 and 1.12, review and test

Parameters and Teaching Tips for DMA 010

- Introduce the module with a rich application. This will provide students an opportunity to discuss the skills that they will eventually use to solve the application as well as supply the student with a context from which the skills will be developed. Application problems should apply to curriculums represented in the class as much as possible.
- The use of a calculator is suggested in this module, beginning with SLO 1.7. Again, the emphasis is on conceptual understanding of the problem rather than on rote memorization. It is acknowledged that knowing multiplication tables is beneficial, but this lack of memorization should not prevent the student from solving the problem, hence the suggestion of allowing the use of calculators
- When evaluating expressions, students should not be expected to solve overly complex expressions. All radicands should be perfect squares.
- One of the key concepts of this module is that of additive inverse. It is helpful for students to connect the rules of adding signed numbers with a visual representation so that they can use the visualization strategy when they forget the rule.
- The underlying mathematics for multiplication and division of integers is beyond the scope of a developmental course, and those rules are best learned by memorization.
- While working with the geometry concepts in this unit, it is important to help students see the relationship between perimeter and area.
- Geometry vocabulary such as lines, rays, angles, ect… will be introduced and defined during the instructional component. However, direct assessment of vocabulary is not recommended as students will be assessed of this knowledge using contextual applications.
- Assist students in understanding the pace at which they must progress through this module in order to complete the requirements within the allotted timeframe. Additionally, provide guidance on test taking strategies and study skills.

Sample Introductory Application for DMA 010

You and your friend Patrick go on vacation. Patrick likes scuba diving and you like mountain climbing. You decide to separate for part of the day and do your individual activities. While you climb to an altitude of 2577 feet above sea level, Patrick is 49 feet below sea level. What is the vertical distance between you and Patrick?

- Draw a diagram to represent the events of the problem.
- Does your diagram accurately tell the story?
- Work with a partner and use integer tiles (labeled as 10s, 100s, 1000s) to determine the solution.
- What operation does the word difference indicate? How will you decide whether to use positive or negative tiles?

Sample Conceptual Questions for DMA 010

SLO	Sample question
1.1	Represent the following scenarios with a sketch and assign integer values: • 10º F and -50º F on a thermometer • Your checking account balance at $100 and your checking account balance overdrawn by $50 • Digging a hole 6 feet below ground level • Start with a hole 6 feet below ground level and fill the hole with three feet of mud. Are you above or below ground level? How far above or below ground level?
1.2	 Represent each quantity as an integer on the number line a) 10 degrees above zero b) a loss of 16 dollars c) a gain of 5 points d) 8 steps backward
1.3	Let a represent any number greater than zero and b represent any number less than zero. Are the following statements true or false? a) $a > -a$ b) $b > -b$ c) $a + (-a) = 0$ d) $-b + b = 0$ Give an example of parts c and d using numbers to support your answer.
1.4	On a number line how far from 0 is 4; how far from 0 is -4? If I travel four units in a negative direction and turn around and travel four units in a positive direction, where am I on the number line? Use number tiles to demonstrate this problem. What conclusion can you draw about numbers on a number line that have opposite signs?
1.5	a) The current temperature is -20°C. If it is forecast to be 9°warmer tomorrow, write a symbolic expression, and evaluate it to determine tomorrow's predicted temperature. b) On a cold day in April, the temperature rose from -6°C to 10°C. Write a symbolic expression, and evaluate it to determine the change in temperature. c) Yesterday, you wrote three checks, each for $15.75. How did these checks affect your balance? State your answer in words and as a signed number. d) During the last year, your friend lost $9876.34 in stocks. Express her average monthly loss as a signed number. e) The temperature of a chemical compound was -5ºC at 3:20 P.M. During a chemical reaction it increased 2ºC per minute until 3:52 P.M. What was the temperature at 3:52 P.M.? Create a chart to indicate the temperature at the following times: 3:21, 3:23, 3:25, 3:30, 3:40, 3:45, and 3:52. What operation can you use so that you don't have to do the problem repeatedly?

1.6	a) Describe the **associative property of multiplication** in your own words. Illustrate with a number example. b) Describe the **commutative property of multiplication** in your own words. Illustrate with a number example.
1.7	a) The temperature on the sun can be represented using scientific notation as $1.8 x 10^6$ degrees Fahrenheit. Express the temperature of the sun in expanded form. b) The energy generated per minute by the sun is 24,000 horsepower. Express the energy generated per minute in scientific notation.
1.8	Are -3^2 and $(-3)^2$ equal to the same value or are they different values? Consider what it means to raise a number to a power and explain your answer.
1.9	a) The opening in my wall units for a new television is 36" by 48". What is the largest size television I can put in that spot? Remember – when purchasing a television the size given represents the diagonal. b) A certain right triangle has legs that measure 3 ft. and 4 ft. The hypotenuse, or longest side, can be found by taking the square root of 25. What is the length of the hypotenuse of the right triangle?
1.10	Joyce solved the following problem using the order of operations. Is it correct or not? $2+3-5+7$ $5-12$ -7 Please explain why it is correct or why it is not correct. If the problem is not correct, please solve it correctly.
1.11	Draw the different rectangles (each with a different length and width) that are possible for a rectangle with an area of 24 square inches. Find the perimeters of rectangles that you drew.
1.12	A car is traveling on a road that is perpendicular to some train tracks. When the car is 30 yards from the crossing, the collision detector alarm sounds and the readout indicates that a train is just 50 yards away and heading towards the same crossing. Draw a diagram to model this situation, labeling all distances. What geometric figure will be used to help calculate the distance of the train from the crossing? How far does the train have to travel to the crossing?

Student Success Tips for DMA 010

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students.

- Discuss topics such as math anxiety, dyscalculia and myths about mathematics to elevate student's comfort.
- Introduce students to learning styles. Encourage them to determine their personal learning style. Visit any of the following sites for a free assessment of learning styles: http://vark-learn.com, http://www.howtolearn.com/start/learning-styles-inventory, http://www.engr.ncsu.edu/learningstyles/ilsweb.html, http://www.metamath.com/multiple/multiple_choice_questions.html.
  Offering learning opportunities for visual and kinesthetic learners is critical to helping all students learn mathematics.
- Assist students in understanding the pace at which they must progress through this module in order to complete the requirements within the allotted timeframe. Additionally, provide guidance on test taking strategies and study skills.
- Students should construct a timeline for the completion of the module. Have them be as detailed as possible; include all personal and professional obligations. Ideally, the timeline for completion would be designed on a daily basis. A daily log of student study habits can help student relate study time to grades earned.
- Help students learn to express the expectations of the module both verbally and in written form.
- If using a textbook, point out the distinct characteristics of the text. Important information such as definitions and examples are generally set apart on the page by boxes or vibrant coloring. If the text contains unit or cumulative reviews, encourage students to use them as study guides for assessments
- If using a textbook, students should be taught to use the index to assist in finding definitions, examples and explanations.
- Remind students how important it is to come to class prepared and to read any materials related to the day's content prior to coming to class.
- Students need to identify when they need help and how to get it. Quiz them on instructor office hours, e-mail address and phone number. Introduce students to the college a tutoring center.
- Encourage students to build study groups.
- Problem solving skills are crucial in mathematics. Help students develop a positive attitude toward problem solving.
- Adding pictures to the explanation of concepts and insisting that students draw diagrams will help students strengthen their visualization skills. When visualization is used as part of solving problems, students have an important problem solving strategy that can help them with future problems.

- Help students understand how to study effectively for tests. Remind students that they must work problems; mathematics is not a subject a person can stare at to study. Studying mathematics means working problems, practicing the mechanics.
- Developmental students bring with them to college many misconceptions about math. One of the biggest misconceptions is that math consists of rules to be memorized. Instructors can help students understand that as they advance in math courses, not only is this not true, but that it would be impossible for them to memorize how to do every problem. Teaching students to reason out the solutions to problems is a way that they can navigate simple and difficult mathematics problems without relying solely on memory of rules and procedures.
- Another misconception is that if you can't do a problem in two or three minutes, you just don't understand it and won't be able to do it. Giving students challenging, non-routine problems helps them practice having to persevere through problems when they don't immediately know how to work a problem. A scaffold approach should be used that guides the students through the problems with the use of targeted questions. As students' skills increase, students will be responsible for the problem solution with less assistance from the instructors' questions.
- When taking a test, scan the entire test first. Complete the problems that appear easy to you first. This will allow you more time for the problems you feel are more difficult. Use your full allotted time on tests; check your work whenever possible.

DMA 020
Fractions and Decimals

Brief Description

In this module, the emphasis will be on a conceptual understanding of the relationship between fractions and decimals and the problem events that result in the use of fractions and decimals to find a solution. Once students have a solid understanding of these concepts they will effectively apply operations with fractions and decimals to solve contextual application problems. Geometric application problems will include circumference and area of circles and the concept of π. Students will be able to use appropriate technology to solve applications problems involving decimals.

Course Competencies

At the completion of this module, the student will be able to:

- Solve contextual application problems involving operations with fractions and decimals
- Visually represent fractions and their decimal equivalents
- Simplify fractions
- Find the LCD of two fractions
- Correctly perform arithmetic operations on fractions
- Explain the relationship between a number and its reciprocal
- Correctly order fractions and decimals on a number line
- Convert decimals between standard notation and word form
- Round decimals to a specific place value
- Estimate sums, differences, products, and quotients with decimals
- Demonstrate an understanding of the connection between fractions and decimals
- Convert between standard notation and scientific notation
- Solve geometric applications involving the circumference and area of circles

Conceptual Student Learning Outcomes

2.1 Solve conceptual problems involving fractions and decimals.

2.2 Visually represent fractions

2.3 Simplify fractions

2.4 Visually represent equivalent fractions and correctly place the values on the number line

2.5 Add and subtract fractions with like denominators

2.6 Write an equivalent fraction with a given denominator

2.7 Add and subtract fractions with unlike denominators using the correct LCD

2.8 Visually represent the sum and difference of two fractions with unlike denominators

2.9 Multiply fractions

2.10 Visually represent multiplication of fractions

2.11 Divide fractions using reciprocals

2.12 Visually represent decimals

2.13 Correctly round decimals to a specific place value

2.14 Estimate sums, differences, products, and quotients with decimals

2.15 Demonstrate an understanding of the connection between fractions and decimals

2.16 Distinguish between the appropriate use of circumference and area of a circle in solving geometric applications

2.17 Represent events in geometric problems pictorially and evaluate solution using correct formulas

2.18 Correlate negative exponents to fractions and decimals in base 10

2.19 Convert between standard notation and scientific notation

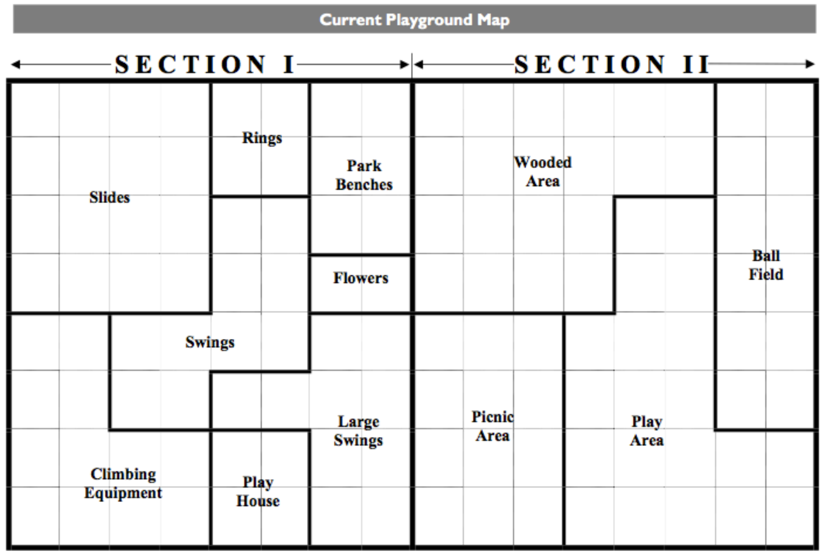
Suggested Timeline

Week 1	Objectives 2.1 through 2.5
Week 2	Objectives 2.6 through 2.13
Weeks 3 and 4	Objectives 2.14 through 2.19, review and test

Sample Introductory Application for DMA 020

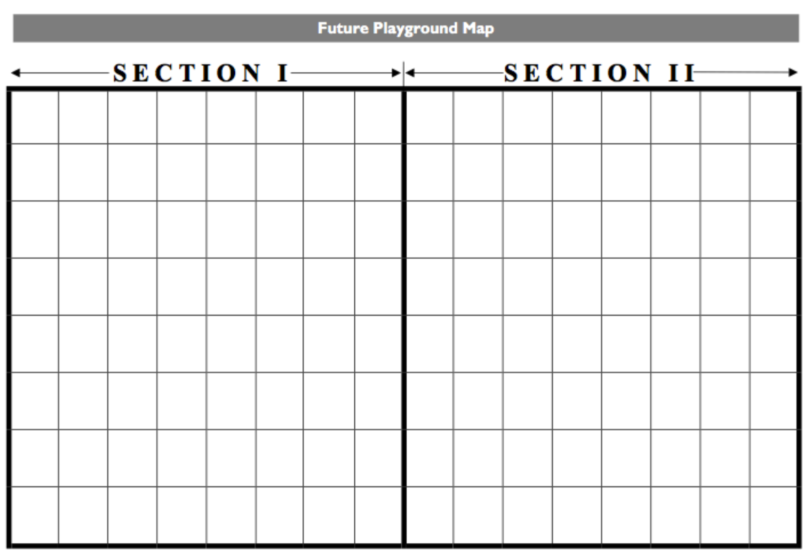
The playground at Central Elementary School is formed from two equal pieces of land that have been subdivided into 12 assorted areas with specific purposes. The P.T.A. has agreed to donate money to redesign and update the playground.

They want the existing 12 areas combined into four larger areas with the following specifications:



1. After the renovation, all 12 of the existing areas of the playground will be eliminated, and the whole playground will be divided into four new areas.
2. The ball field area will be joined to one other area, and together they will make up $\frac{1}{2}$ of one of the two equal parcels or sections of land. This will be the new Play Area.
3. The area containing the Play House will be combined with two other areas to form the new Primary Playground. This area will comprise $\frac{13}{32}$ of one of the two sections.
4. The area that holds the slides will be increased to equal $\frac{1}{2}$ of a section. This will be the new Playground Equipment Area.
5. The rest of the land in that section will be added to the park benches area. This will be the new Park & Picnic Area.
6. You will be able to walk through each of the four new areas of the playground without having to cross another area.

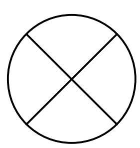
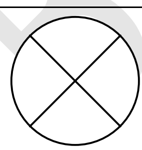
Use the clues above to find out which areas of the original playground were combined to make the four new areas. Draw a map of the new playground, and outline each of the four new areas. What fraction of the total playground area will be in each of the four new areas?

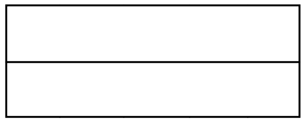


Parameters and Teaching Tips for DMA 020

- The key concept from this module is the relationship between fractions and decimal representations of numbers. The concept of a fraction as division leads into this discussion of the decimal form of a rational number
- It is assumed that operations with fractions include the following: signed numbers, proper fractions, improper fractions and mixed numbers.
- Instruct students on finding the reciprocal of a number and why zero has no reciprocal. These skills will not be measured directly but will be assessed in the context of division applications.
- Emphasis is placed on conceptual understanding of fractions and decimals.
- Introduce a variety of shapes that show fractions as part-whole relationships. The variety encourages students to use different strategies to solve the problems and develop their understanding about fractional representations. As students learn about fractions they come to understand that fractions have a size and can be compared, ordered, and represented as a point on the number line.
- When visually representing fractions assist the student in identifying the following: numerator, denominator, proper fraction, improper fraction, mixed numbers and shading parts of a figure to represent a given fraction. Direct assessment of these skills is not recommended, students abilities to use these skills will be assessed in application problems.
- When instructing on prime factorization you will define a factor, prime and composite numbers. Demonstrate the proper use of divisibility tests for 2, 3, 5 and 10. Students should not be directly assessed on these individual skills. It is recommended that the factorization should not have more than 5 factors nor should any one factor be greater than 13.
- Instruct the student on finding the LCD using multiples and prime factorization. These skills will be evaluated when assessing operations with fractions with unlike denominators.
- When instructing on decimals provide visual representation by placing at least four decimal numbers in the correct location on a number line. Also, order at least four numbers in both fraction and decimal form. Again, these skills will not be evaluated directly however they will assist students in determine reasonableness of solutions.
- It is recommended that rounding not exceed the ten-thousandths place.
- The approach to decimals should be similar to working with fractions, placing a strong and continued emphasis on models and oral language and then connecting this work with symbols.
- The use of a calculator for calculations with decimals is suggested for DMA 020. Care should be taken to write assessments that measure student's conceptual understanding and ability to appropriately use the calculator as a tool in problem solving. Application problems should include geometric applications from DMA 010.

Sample Conceptual Questions for DMA 020

SLO	Sample Question
2.1	Explain how to measure 1/6 cup of sugar given a bag of sugar and measuring cups with the following labels: 1 cup, 0.5 cup, 1/3 cup, 0.25 cup
2.2	 This fractional piece represents $\frac{1}{2}$ of a whole circle.  What fractional part of a whole circle does this shaded piece represent?  What fractional part of a whole circle does this shaded piece represent**?**
2.3	42/56 is a fraction that can be reduced, by more than one process. Explain how you would reduce this fraction.
2.4	Mike and three friends stopped at a pizza parlor Saturday night and shared a large pizza equally. The next day Mike and seven friends stopped at the same pizza parlor for a snack. This time the eight friends shared a large pizza equally among them and then ordered a second large pizza and shared it equally. Did Mike eat more pizza on Saturday or on Sunday? Or did he eat the same amount each day? Draw a diagram of the problem events before trying to solve the problem.
2.5	 Use the circle to show $\frac{3}{4} - \frac{1}{4}$.
2.6	Write the equivalent fraction of $\frac{7}{8}$ with a denominator of 40.
2.7	Erin made an apple pie. She used ½ of a tablespoon of cinnamon and ⅜ of a tablespoon of nutmeg. How much more cinnamon than nutmeg did Erin use?
2.8	 Use the circle to show $\frac{1}{2} + \frac{1}{4}$.
2.9	When ½ is multiplied by ⅔ why is the answer smaller than both of them?
2.10	 Cut this rectangle into ten equal pieces. Shade ⅖ of it. How many pieces are ½ of the ⅖ you just shaded?
2.11	Why is 5 divided by ½ larger than 5?

<table>
<tr><td>2.12</td><td> Shade 0.4 of the figure.</td></tr>
<tr><td>2.13</td><td>Tom went to the store to purchase a unique pen. However, the price for this particular pen was 3 pens for $11. How much did Tom have to pay for one pen? Round to the nearest cent.</td></tr>
<tr><td>2.14</td><td>If Alan earns $8.97 an hour, estimate his weekly pay (before taxes) if he works 40 hours per week.</td></tr>
<tr><td>2.15</td><td>An isosceles triangle measures 3 ½ inches on one side, and 5.2 inches on the congruent sides. What is the perimeter of the triangle?</td></tr>
<tr><td>2.16</td><td>Would you use the circumference or area of a circle to solve the following problems?
a) How much fencing is required to enclose a circular garden whose radius is 20 meters?
b) Which one of the following is a better buy: a large pizza with a 14-inch diameter for $13.00 or a medium pizza with a 7-inch diameter for $6.00?</td></tr>
<tr><td>2.17</td><td>Jason’s new pool needs a cover. The pool is round with a diameter of 20 feet. How many square feet of canvas will be needed to make the cover? If the canvas is available in 10 foot wide bolts, how many feet will he need to purchase? The cover needs to have an elastic border. How many feet of elastic will Jason need to purchase?</td></tr>
<tr><td>2.18</td><td>If $1000 = 10^3$, then $\frac{1}{1000} = 10^{\square}$.

Given the information below for exponents, fill in the remainder of the chart and explain why 2000 is equal to $2 * 10^3$ and why .002 is equal to $2 * 10^{-3}$.

<table>
<tr><td></td><td>Expanded form</td><td>Number or fraction equivalence</td><td>Decimal equivalence</td></tr>
<tr><td>10^4</td><td>1 * 10*10*10*10</td><td></td><td></td></tr>
<tr><td>10^3</td><td>1*10*10*10</td><td></td><td></td></tr>
<tr><td>10^2</td><td>1*10*10</td><td></td><td></td></tr>
<tr><td>10^1</td><td>1*10</td><td></td><td></td></tr>
<tr><td>10^0</td><td>1* no 10s</td><td></td><td></td></tr>
<tr><td>10^{-1}</td><td>1 divided by 10</td><td></td><td></td></tr>
<tr><td>10^{-2}</td><td>1 divided by 10*10</td><td></td><td></td></tr>
</table></td></tr>
<tr><td>2.19</td><td>a) The star nearest to Earth (excluding the Sun) is Proxima Centauri, which is 24,800,000,000,000 miles away. Write the number in scientific notation.
b) The thickness of human hair ranges from fine to coarse, with coarse hair about 0.0071 inch in diameter. Describe how you would convert this measurement into scientific notation.</td></tr>
</table>

Student Success Tips for DMA 020

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students.

- At the beginning of the module, students should create a time line in order to gain an understanding of the amount of time that the student will spend working on the course assignments. Each week, students should evaluate the time that they are spending on course assignments.
- Many students in developmental mathematics have math anxiety. It is important that students recognize the impact that math anxiety has on working memory especially during test taking. Students who experience math anxiety need to be aware that the anxiety may lead to an avoidance of math in the form of not coming to class, not doing homework, not studying for tests or not signing up for subsequent math courses. Some of the strategies for calming math anxiety are preparation for class, positive self-talk, and relaxation techniques.
- When faced with a problem that contains large numbers or fractions, replacing the more complex numbers with simpler numbers is an effective problem solving strategy. Instructors should model this strategy frequently until students become comfortable using it on their own.
- It is important that students who take developmental mathematics ask for help at an early stage when they do not understand a concept. One way in which instructors can help with students become comfortable asking for help is by having students work with in pairs and groups with many different students in the class. Instructors can also suggest peer tutoring if that is available on the college campus. Instructors, of course, will let students know when they are available for additional help outside of class time.

DMA 030
Proportion/Ratios/Rates/Percents

Brief Description

In this module, the emphasis will be on a conceptual understanding of the problem events that are represented by ratios, rates, percents, and proportions. Once the students have a solid understanding of these concepts they will be able to use ratios, rates, proportions, and percents to solve conceptual application problems. Applications using U.S. customary and metric units of measurement and the geometry of similar triangles will be included.

Course Competencies

At the completion of this module, the student will be able to:

- Apply the concepts of ratio, rates, proportions, and percents to application problems
- Recognize and choose the correct units in application problems using ratios, rates, and proportions
- Calculate a unit rate
- Convert measurements within and between the U.S. customary and metric system using unit analysis
- Compare percents, decimals, and fractions
- Apply the concepts of part, whole, and percent to solve contextual applications

Conceptual Student Learning Outcomes

3.1 Demonstrate an understanding of the concepts of ratios, rates, proportions, and percents in the context of application problems

3.2 Write a ratio using a variety of notations.

3.3 Distinguish between events in a problem that should be represented by a ratio or a rate

3.4 Represent percent as "parts of 100"

3.5 Correctly convert between fractions, decimals, and percents

3.6 Calculate a unit rate

3.7 Solve application problems using ratios, rates, proportions, and percents

3.8 Recognize that two triangles are similar and solve for unknown sides using proportions in contextual applications

3.9 Convert measurements within the U.S. customary and metric system using unit analysis

3.10 Convert measurements between the U.S customary and metric Systems using unit analysis

Suggested Timeline

Week 1	Objectives 3.1 through 3.3
Week 2	Objectives 3.4 through 3.6
Weeks 3 and 4	Objectives 3.7 through 3.10, review and test

Sample Introductory Application for DMA 030

Scrumptious Scoops is a very popular ice cream parlor. To celebrate the Fourth of July, the store decided to serve free single scoops of its three most popular flavors to the audience at the Independence Day outdoor band concert. Mr. Scrumptious decided that he could determine how much ice cream he would need by using the data provided by the International Ice Cream Association. The town estimated that approximately 650 people would attend the band concert.

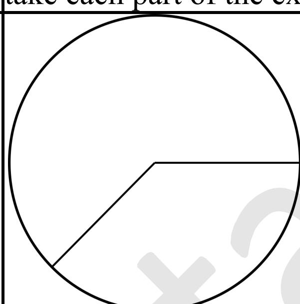
Favorite Flavor	Percent of Those Polled
Vanilla	55%
Chocolate	29%
Strawberry	16%

1. Assuming everyone will want a free scoop of ice cream, how many people would you expect to prefer chocolate?
2. How many half-gallons of chocolate ice cream should Mr. Scrumptious plan to have on hand to give to those people? How many pounds will that be? (*A gallon of ice cream weighs about 5 pounds and contains 4 quarts. One scoop of ice cream is ½ cup or about 68 grams. One gallon contains 16 cups, so one half-gallon contains 8 cups.)*

Parameters and Teaching Tips for DMA 030

- Introduce the module with a rich application. Students may not have all of the skills yet to solve the application but it will give them the context from which the skills can be developed.
- The key concepts from this module are the concept of part to whole and whole to part relationships, and the relationship between fractions, decimals, and percent. Instructors should help students understand that fractions, decimals and percent are three different representations of the same quantity, but in certain cases, one representation may be more commonly used than others. For example, we say $\frac{1}{2}$ inch but not 50% of an inch and sale prices may be 25% or $\frac{1}{4}$ off, but not 0.25 off.
- Applications of percents include but are not limited to sales tax, tips, discount, percent of increase, percent of decrease, simple interest.
- The percent equation is studied and used to answer the following types of questions: what is p% of a number, what number is p% of another number, and a number is what percent of another number? Students will work these types of problems using proportions, but tricks like "the number after of" should be avoided and replaced with the students' understanding of the significance of percentages.
- When students are learning about conversions, the fact that equivalent rates must have the same type of units (e.g. miles/hours can never equal gallons/hours) should be emphasized since this language/number connection will be important to students' success when they use algebraic equations in the next module to solve word problems.
- Ratio and proportion equations are very useful in solving application problems. Real world applications such as medical dosages, approximating wildlife in an area, and price per earnings ratios are some examples.
- When at all possible, bring measuring devices when working with application problems such as a cup, pint, quart, gallon jars, yard sticks, rulers.
- Provide conversion charts for converting with units.

Sample Conceptual Questions for DMA 030

SLO	Sample question
3.1	a) The Fall semester enrollment at the college was 5200 students and for the Spring semester, it was 4800 students. What was the percent of decrease? b) A family spends $400 every month on food. If the family's income each month is $2400, what percent of the family's income is spent on food?
3.2	a) Two lengths are in the ratio of 8:5. Write this ratio in two other ways. Use an illustration to show what this means. b) An alloy consists of copper, zinc and tin in the ratio 2:3:5, representing the three amounts, respectively. Find the amount of each metal in 75 kilograms of the alloy.
3.3	Two friends were shooting basketball. John made 4 out of 6 free throws and Avery made 8 out of 12 free throws. Is this situation represented by a ratio or a rate? Are the two friends basketball shooting equivalent? If so, how did you determine it?
3.4	Using a 10-by-10-squares grid divided into 100 equal-sized parts, demonstrate 37.5%. Write the decimal representation that is equivalent to this. Write the fraction representation that is equivalent to this. Write the fraction in expanded base-ten fraction form (tenths, hundredths and thousandths). Using the definition of percent, take each part of the expanded form and explain why the fraction equals 37.5%.
3.5	 Consider the larger area of the circle. Write the area as a fractional part of the circle. Next, write the area as a decimal. Finally, write the area as a percent of the whole circle. Explain how the fraction, decimal, and percent values represent the same area.
3.6	A Toyota Corolla can travel 420 highway miles on 12 gallons of gas. A Chevrolet Cobalt requires 8 gallons of gas to travel 296 highway miles. Explain the calculations you do to compare the fuel efficiency for each model.
3.7	a) Susan decided she needs to lose a few pounds. There is a new diet that claims you can lose 10 pounds in 7 days. If she wants to lose 14 pounds, how long should it take her? b) Shoes Galore" is obviously a shoe store with a big SALE sign in the window that says "BUY AND SAVE." You want to buy two pairs of shoes. Each pair of shoes cost $29.00. One offer states "Buy one pair of shoes, get the second pair for free" and another states "Buy two pairs of shoes, 40% off of each pair." Decide which offer is the best deal for you. Explain your choice.
3.8	Marcus and Steve stood side-by-side one sunny summer day and notice that their shadows were different lengths. Marcus measured Steve's shadow and determined

	it was $9\frac{1}{2}$ feet. Steve measured Marcus' shadow and determined it was $10\frac{1}{4}$ feet. If Steve is $5'10''$ tall, how tall is Marcus?
3.9	A property in Italy is advertised at $645,000 for 6.8 hectares. a) Find the area of the property in acres. b) What is the price per acre?
3.10	You are mixing some concrete for a home project and you've calculated according to the directions that you need six gallons of water for your mix. But your bucket isn't calibrated, so you don't know how much it holds. You just finished a two-liter bottle of soda so you can use it to measure your water. How many times will you need to fill the soda bottle?

Student Success Tips for DMA 030

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students.

- At the beginning of the module, students should create a time line in order to gain an understanding of the amount of time that the student will spend working on the course assignments. Each week, students should evaluate the time that they are spending on course assignments.
- Students do not automatically know how to effectively study for a mathematics test and may in fact hold the misconception that you can't study for math – you either know it or you don't.
    - Emphasize that studying for a test begins with attending all classes, taking complete notes, and asking questions in class. Students need to ask questions as they arise and not wait until the day or two before a test.
    - Homework should be done when it is assigned. If a student can't get the answer to a homework problem remind them to get help.
    - Students should form study groups. Explain that they should take turns explaining out loud, in their own words, how each solution strategy is used. Discussing the solutions aloud helps knowledge get into long-term memory.
    - Remind students to start studying for a test several days to a week before the test by going over each section, reviewing notes and reworking the homework problems.
    - On the test problems from different sections may be all together. Encourage students to put themselves in a test-like situation: work problems from review sections at the end of chapters and work old tests.
    - Even though students are now adults, it is important to encourage them to get lots of sleep the night before the test. Math tests are easier when a student is mentally sharp.
- In math, one of the main problems that students face is converting information from working memory to long-term memory and/or abstract reasoning. Since the goal of mathematical instruction is not to simply gain mathematical knowledge but to use mathematical knowledge, it is important that as soon as a concept is introduced to students, they also see the function that the math concept plays in some aspect of their lives. To place information into long-term memory students must understand the math vocabulary and practice problems, but to place information into abstract reasoning, students must understand the concept. For most problems, students must use both long-term memory and abstract reasoning.

DMA 040
Expressions, Linear Equations, Linear Inequalities

Brief Description

In this module, the emphasis will be on a conceptual understanding of the problem events that result in graphic and algebraic representations of linear expressions, equations, and inequalities. Once the students have a solid understanding of these concepts they will be able to distinguish between, simplify, and solve in relation to linear expressions, equations, and inequalities. Emphasis will be on solving contextualized application problems using linear equations and inequalities.

Course Competencies

At the completion of this module, the student will be able to:

- Differentiate between expressions, equations, and inequalities
- Simplify and evaluate, when appropriate, expressions, equations, and inequalities
- Effectively apply algebraic properties of equality
- Correctly represent the solution to an inequality on the number line
- Represent the structure of application problems pictorially and algebraically
- Apply effective problem solving strategies to contextual application problems
- Demonstrate conceptual knowledge by modeling and solving applications using linear equations and inequalities

Conceptual Student Learning Outcomes

4.1 Demonstrate an understanding of what a variable represents

4.2 Demonstrate the use of a problem solving strategy to include multiple representations of the situation, organization of the information, and algebraic representation of linear equations or inequalities

4.3 Represent verbal statements as algebraic expressions, equations, and inequalities

4.4 Distinguish between problem events that use expressions, equations, or inequalities

4.5 Solve linear equations and inequalities in one variable using algebraic properties of equality

4.6 Demonstrate an understanding of the meaning of solutions to problems, i.e. identity, contradiction, conditional

4.7 Represent solutions of inequalities on a number line

Suggested Timeline

Week 1	Objectives 4.1 and 4.2
Week 2	Objectives 4.3 through 4.6 (equations)
Weeks 3 and 4	Objectives 4.3 through 4.7 (inequalities), review and test

Sample Introductory Application for DMA 040

1) In your own words, explain what a variable can represent.
   Consider the following problems that can be represented using algebraic expressions or equalities:
   a) Three more than six times a number is equal to seven more than four times a number. Find the number.
   b) In an isosceles triangle, two sides of the triangle are equal in length. One of the equal sides is three more than six times the third side which is represented by x. The other equal side is seven more than four times the third side. Find the measure of the two equal sides of the triangle.

2) What does it mean to have six x's?

3) Consider the following statements: These two algebraic expressions are equal to each other or you can say that they are in balance. To remain equal, they must remain in balance. Write a paragraph to describe the balance of the equation in the picture. Include the balance of the expressions and tell how that balance is confirmed with the value you found for x.

Parameters and Teaching Tips for DMA 040

- This module should be introduced with a contextualized application. Students may not have all of the skills yet to solve the application but it will give them the context from which the skills can be developed. Application problems should apply to curricula represented in the class as much as possible.
- An understanding of algebra involves the topics of variables, algebraic expressions, equality and how these topics relate to the problem solving process. Students frequently lack a numerical reference for the letter used as a variable, and they view algebraic expressions as incomplete statements that student often feel confusingly names a portion of the solution to the problem and also describes a process (i.e., x + 3). What the equal sign truly means in an equation is another area of confusion for many algebra students. These are important obstacles to clear up in the students thinking before they begin to create and solve equations to solve word problems.
- Keeping in mind the appropriate goals for algebra are conceptual understanding and problem-solving skills, this module focuses on helping students identify the structures of word problems that are common to many types of word problems, applying inference skills to the hidden questions in word problems and understand the appropriate use of problem solving strategies that can be applied during problem solving.
- Instruct on the vocabulary of expressions, equations and inequalities. Include distinguishing between like and unlike terms, distinguishing between a constant and a variable, distinguishing between an expression and an equation, distinguishing between a numerical coefficient and the variable. These skills will not be measured directly; students will demonstrate their understanding during the assessment of applied problems.
- Applications of linear equations may include but not limited to: percent of increase and/or decrease, sales tax, tips, commissions, discounts, odd/even/consecutive integer problems, age, distance, coin, mixture and simple interest. With respect to linear applications, students should be able to demonstrate an understanding of the question being asked, be able to assign variables to unknown quantities, translate the vocabulary to write an equation that models the application, solve and check solutions for reasonableness
- The idea of "balance" is useful in solving linear equations and inequalities. Illustrate the addition and multiplication of equalities, inverse operations, clearing fractions, clearing decimals and evaluating a solution using substitution as skills necessary to solve linear applications. These skills will be assessed in conceptual applications.
- Applications of linear inequalities may include but not limited to: comparison of two cell phone plans, comparison of two car rental plans and averages with intervals. With respect to linear inequality applications, students should be able to demonstrate an understanding of the question being asked, be able to assign variables to unknown quantities, translate the vocabulary to write an inequality that models the application, solve and check solutions for reasonableness.
- In this module it is understood that students will have a calculator available to use, whenever reasonable. Assessments must be carefully written to allow the use of the calculator and ensure the conceptual understanding of objectives, while at the same time, incorporating numeracy concepts when a calculator is not necessary.
- Typical algebra word problems are appropriate for this module. Helping the students

recognize the underlying structure of the word problem is the emphasis of this module. For example:

> John drives a car from Statesville, NC to Wilmington, NC. During the first part of the trip he drove at a rate of 60 mph, but later the traffic was very heavy and he could only drive at a rate of 40 mph. If it is 240 miles from Statesville to Wilmington and if the whole trip takes 5 hours, how far does John drive at 40 mph and how far does he drive at 60 mph?
>
> Students can use one or more of the following strategies to solve this problem:
> 1) Students act out the problem
> 2) Students draw a diagram to represent the events of the problem
> 3) Students use the diagram to determine the structure of the word problem (is this a problem where you are given parts to add and get a total or is this a problem in which you are given two equal quantities?)
> 4) Students create a chart to organize the information.
> 5) Students use the diagram, structure and chart data to create an equation.
> 6) Students solve the equation for the numerical value of the variable
> 7) Student answers the question, "Did you answer the question asked?"
> 8) If the student has not answered the question asked, the student looks at the chart data to find the information necessary to answer the problem question.
>
> Students will find that distance, investment, coin, consecutive integers, and mixture problems often have the same structure – just different problem events.

Sample Conceptual Questions for DMA 040

SLO	Sample question
4.1	Cara is working on her algebra homework and is trying to write an equation to solve a problem. The problem concerns the amount of money earned over a number of years. She has determined that the number of years is the variable. Which of the following cannot be used to represent the variable in this problem and why? X, x, y, n, 7, A, 0
4.2	1) A 21 foot board is to be divided to make supports. The longer piece is one foot more than three times the shorter piece. What length is the longer piece? a) What is the application asking you to find? b) How would you represent the shorter piece; the longer piece? c) Write an equation to solve this application. d) Solve the equation and check for validity. 2) El has \$1020 in a savings account at the beginning of the summer. She wants to have at least \$250 in the account by the end of the summer to pay her part of a beach house rental. She withdraws \$70 each week for food, clothes and entertainment. How many weeks can El withdraw from her account and have at least \$250 left? a) What is the application asking you to find? b) How would you represent the money left in her savings? c) Write an inequality to solve this application. d) Solve the inequality and check for validity.
4.3	Represent the following statements as an expression, equation, or inequality. a) Three times four equals twelve. b) Four times a number subtracted from six is less than twice the number. Let "N" represent the number. c) If x represents Judy's age and Julie is seven years older. Represent Julie's age using x.
4.4	Explain in full sentences, as if you were teaching another student, the difference between an algebraic expression and an algebraic equation.
4.5	Debbie is selling her house through a real estate agent who charges 6.5% of the sale price. Debbie must have at least \$150,750 left after she pays her agent so she can pay off her mortgage and have enough to put a down payment on her new home. a) Should this situation be represented with an equation or inequality? b) Identify the variable and write an expression for the amount left after the agent is paid the commission. c) Set up the problem to find out how much Debbie needs to sell her house for so that she is left with enough money to meet her needs. d) Write the answer in a full sentence.
4.6	Solve the following equations and describe the solution of each using full sentences: $4(x+5)-8=4x-12$ $4(x+5)-8=4x-11$ $4(x+5)-8=5x-12$

4.7	The revenue R for selling x chicken dinners at the church fundraiser is found using the equation $R = 7.50x$. The cost C to make x dinners is $C = 4.25x + 273$. If the church sells 120 dinners, will they make a profit? If they sell 75 dinners will they make a profit? What is the number of dinners that must be sold to break even? Draw a number line graph to illustrate the possible number of dinners that can be sold so that the church makes a profit.

Student Success Tips for DMA 040

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students:

- At the beginning of the module, students should create a time line in order to gain an understanding of the amount of time that the student will spend working on the course assignments. Each week, students should evaluate the time that they are spending on course assignments.
- Apply Pólya's four-step process:
    a. The first and most important step in solving a problem is to understand the problem, identify exactly which quantity the problem is asking you to find. Always read math problems completely before beginning any calculations. If you glance too quickly at a problem, you may misunderstand what really needs to be done to complete the problem.
    b. Next you need to devise a plan by identifying which problem solving strategies and procedures you have learned that can be applied to solve the problem at hand.
    c. Carry out the plan – don't start in the middle of the plan. Work the problem step by step, drawing diagrams, organizing information, and determining the structure of the problem that will help you get the correct equation.
    d. Look back**.** Does the answer you found seem reasonable? Have you answered the question that was asked or does the solution to the variable represent the answer to a hidden question the problem?

DMA 050
Graphs and Equations of Lines

Brief Description

In this module, the emphasis will be on a conceptual understanding of the problem events that result in graphic and algebraic representations of lines. Once students understand these concepts they will solve contextual application problems. Students will also interpret basic graphs (line, bar, circle, etc.) to solve problems.

Course Competencies

At the completion of this module, the student will be able to:

- Read and interpret basic graphs to solve problems
- Apply the concept of slope as a rate of change in real-world situations
- Write and graph linear equations in two variables to model real-world situations
- Represent real world situations as linear equations in two variables in tabular form, graphically, and algebraically.

Conceptual Student Learning Objectives

5.1 Analyze and interpret basic graphs to solve problems

5.2 Represent real world situations in tabular, graphical, and algebraic equation form using two variables

5.3 Generate a table of values given an equation in two variables and plot in Cartesian plane to graph a line

5.4 Demonstrate an understanding of the concept of slope as a rate of change in real world situations using the slope formula

5.5 Find and interpret the x- and y-intercepts of linear models in real world situations

5.6 Graph linear equations using a variety of strategies

5.7 Given a contextual application, write a linear equation and use the equation to make predictions

5.8 Demonstrate a conceptual understanding of horizontal and vertical lines in terms of slope and graphically

Suggested Timeline

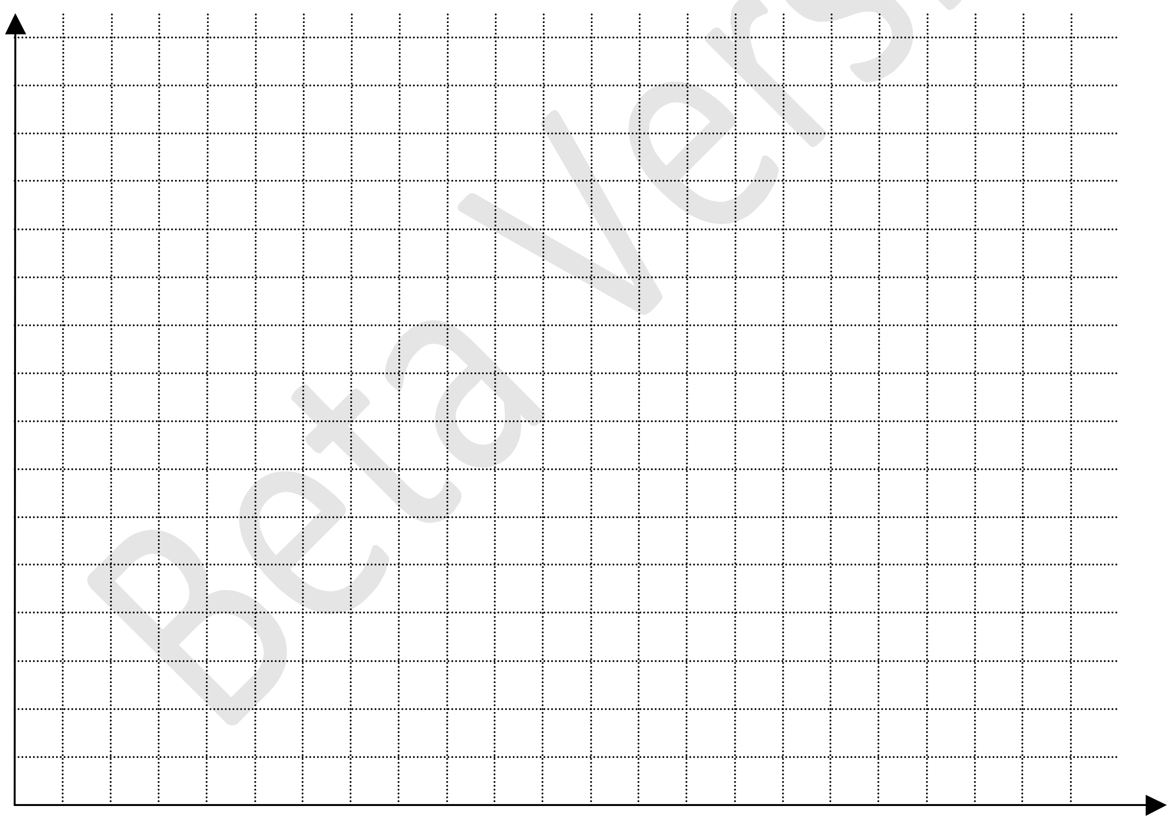
Week 1	Objective 5.1
Week 2	Objectives 5.2 through 5.4
Weeks 3 and 4	Objectives 5.5 through 5.8, review and test

Sample Introductory Application for DMA 050

The following table describes the median household income in North Carolina from 1984-2005

Income	20,569	21,451	21,861	22,760	24,415	26,406	26,329	26,853	27,771	28,820	30,114
x-value	0	1	2	3	4	5	6	7	8	9	10
Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
Income	31,979	35,601	35,840	35,838	37,254	38,317	38,162	36,515	37,279	40,238	42,056
x-value	11	12	13	14	15	16	17	18	19	20	21
Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005

1. Examine the data and describe the trends in words:
2. Construct a plot of the data. Since we are examining how Income changes "with respect to" time, we use time as our independent variable, or x-value, and plot it on the horizontal axis; Income as our dependent variable, or y-value, and plot it on the vertical axis. NOTE: To streamline your calculations for this lab, use x = 0 for 1984, use x = 1 for 1985, x = 2 for 1986, x = 3 for 1987, etc.



3. Predict the median household income for the year 2010. How accurate do you think your prediction is?

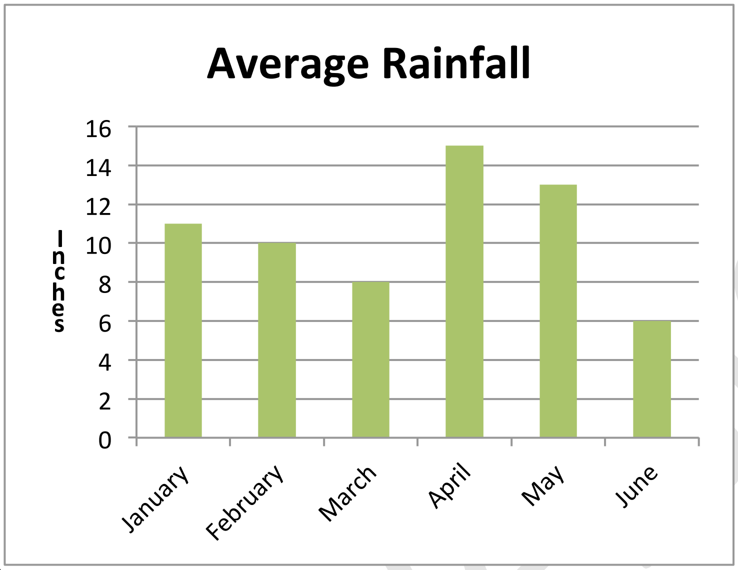
Parameters and Teaching Tips for DMA 050

- This module should be introduced using a contextual approach. Even though students may not have the skills necessary to solve the application, the application will provide a reason or basis for learning the new mathematical concepts.
- Instruction should entail methodologies that include but are not limited to: visualization, manipulatives, cooperative learning and problem solving techniques.
- An emphasis should be placed on the connection between tabular, graphical and algebraic representations of the same event.
- When instructing on basic graphs include analyzing bar graphs, line graphs, circle graphs and histograms. Student's ability to analyze said graphs will be assessed in context of an application problem.
- Applications of the concept of slope may include but are not limited to: pitch of a roof, grade of a road, rate of depreciation and constant velocity.
- Students will have access to technological resources, such as, computers, software or calculators in their study of the lessons.

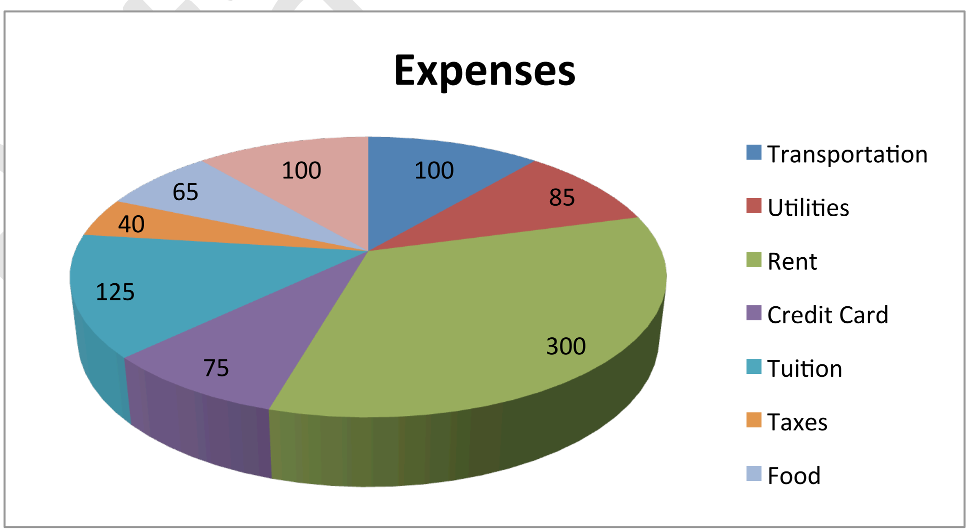
Sample Conceptual Questions for DMA 050

<table>
<tr><th>SLO</th><th>Sample question</th></tr>
<tr><td>5.1</td><td>

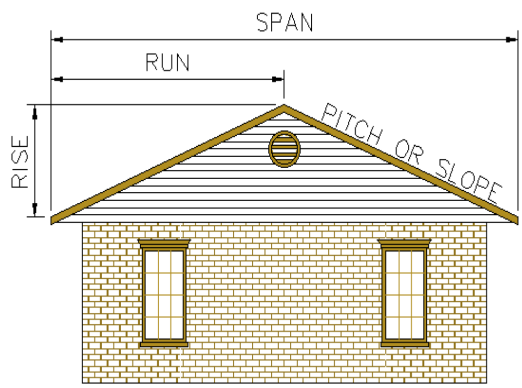
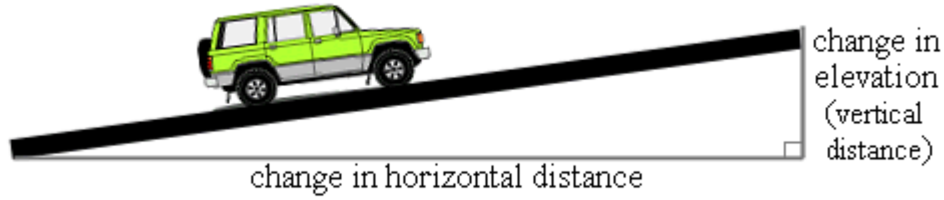
a) Calculate the rainfall difference between the April and June. Should this change be represented with a positive or negative value? Why?

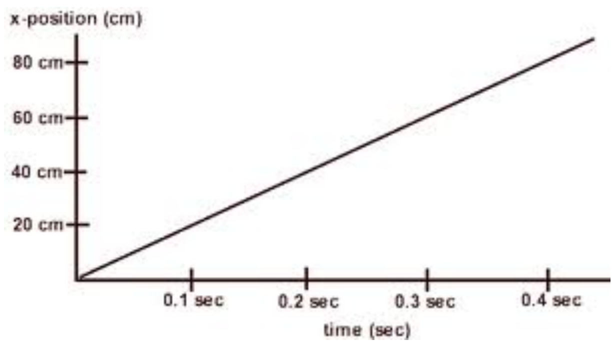


b) Calculate the sum of the smallest and the largest expense amounts. What percentage of total expenses is this sum?



</td></tr>
</table>

5.2	For billing purposes, the Pure Water Company measures water used by hundred cubic feet or HCF. One HCF equals 750 gallons. The Pure Water Company charges a flat base fee of $48 per month for water usage up to 15 HCF. They then charge $3.97 for every HCF used after the initial 15 included in the base fee. If last month your Pure Water Company bill was $79.76, how many total HCF's did you use? How many gallons does this represent?
5.3	According to *Human Physiology in Space* (Lujan &White, 1994) a person's height can be expressed as a function of the length of the femur (the thigh bone). For males, a person with a 17.9-inch femur would be estimated to be about 65.65 inches tall. Each additional inch in femur length corresponds to a 1.88 inches in additional height. a) Write an equation that gives height as a function of femur length. b) Make a table of values with femur lengths from 15 inches to 20 inches in one column and the corresponding heights in a second column. Plot the points from your table with the horizontal axis representing femur length and the vertical axis representing heights.
5.4	a) In the figure below, if the rise (Δy) is 10 and the run (Δx) is 4, calculate the pitch of the roof.  b) Calculate the grade of the road if the vertical distance is 10 ft. and the horizontal distance is 95 ft.  c) A new car is purchased for $25,000. Five years later, the value of the car has decreased to $9,500. Calculate the rate of depreciation for the five years. $\frac{\Delta value}{\Delta\, years} =$ rate of depreciation.

	d) Calculate the average velocity between 0.2 sec and 0.1 sec. Calculate the average velocity between 0.3 sec. and 0.4 sec. 
5.5	The value of an new SUV is given by the equation: $y = -5000x + 30000$, where y is the value of the SUV and x is the number of years after purchase. Find both the x-intercept and y-intercept. Interpret the meaning of each in the context of this problem.
5.6	Consider the linear equation $2y - 3x = 12$. Discuss how you would graph the equation using two different methods and why you would choose one method over the other. Use complete sentences.
5.7	The bullet (Shinkansen) train in Japan travels at 300 km per hour. a) Write a function that describes the distance traveled, *d*, as a function of length of the trip in hours, *h* on the bullet train. b) It is 9 AM and Jane is in Hakata, which is 1180 km from Tokyo. She has a job interview in Tokyo at 2 PM at an address that is about 1.5 km from the train station. The bullet is scheduled to leave at 9:15. Can she make it?
5.8	You borrowed a bicycle from your friend for a few weeks, but now it is time to return it. He lives 3 miles from you. You ride to his house, traveling at 15 miles per hour. When you arrive, it is dinnertime and his mom invites you to stay for dinner. Since she makes the best fried chicken you have ever tasted, you accept the invitation. You stay 45 minutes. Then you walk home at a rate of 3.5 miles per hour. Make a graph that represents this situation, with distance from your home on the vertical axis and time since you left home on the horizontal axis. a) What units did you use on the vertical axis? b) What units did you use on the horizontal axis? c) What is the slope of your graph that represents the time you spent eating at your friend's house? d) What units are used to measure this slope?

Student Success Tips for DMA 050

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students:

- At the beginning of the module, students should create a time line in order to gain an understanding of the amount of time that the student will spend working on the course assignments. Each week, students should evaluate the time that they are spending on course assignments.
- When students are asked to solely work problems in an equation-to-graph direction, the instructor makes the implicit assumption that when students are taught how to produce a graph, they will automatically understand how it relates to the table and to the symbolic representation. Students must be given opportunities to work problems from tables-to-graphs-to-equations and graphs-to-tables-equations in order to insure that students understand the relationships among the multiple representations.
- Don't be afraid to ask questions, any question. The better your question, however, the better chance you have of getting the answer that you need.

OK questions	*Better questions*	*Rationale*
I don't understand this section	I don't understand why some equations have one variable and some have two variables.	This is a very specific remark that will get a very specific response and hopefully clear up your difficulty.
I don’t understand slope.	How can I tell the difference between the slope of a horizontal line and the slope of a vertical line?	This helps the instructor determine what you do and don’t understand about slope.
How do you do #17?	This is how I tried to do #17. What went wrong?	This gives the instructor insight into your thinking process.

DMA 060
Polynomials and Quadratic Applications

Brief Description

In this module, the emphasis will be on a conceptual understanding of the problem events that result in graphic and algebraic representations of quadratics. Once the students have a solid understanding of these concepts they will be ready to learn the procedures for finding algebraic solutions to contextual quadratic applications. Within the problem solving process, students will perform polynomial operations, factor polynomials, and apply factoring to solve polynomial equations. The concept of functions will be introduced. Reasonableness of solutions will be emphasized. The graphing calculator will be used in this module to examine the graphic representation of quadratic functions in addition to the algebraic skills developed.

Course Competencies

- Represent real world applications as quadratic equations in tabular, graphic, and algebraic forms
- Apply exponent rules
- Solve application problems involving polynomial operations
- Apply the principles of factoring when solving problems
- Represent contextual applications using function notation
- Analyze graphs of quadratic functions to solve problems

Conceptual Student Learning Outcomes

6.1 Demonstrate the use of a problem solving strategy to include multiple representations of the situation, organization of the information, and algebraic representation of quadratic equations

6.2 Add and subtract polynomials

6.3 Apply exponent rules

6.4 Multiply polynomials

6.5 Divide a polynomial by a monomial

6.6 Factor trinomials using multiple methods

6.7 Factor the difference of two squares

6.8 Solve quadratic applications using the zero product property and critique the reasonableness of solutions found

6.9 Graph quadratic functions using the graphing calculator to identify and interpret the maximum, minimum, and y-intercept values and the domain and range in terms of the problem

Suggested Timeline

Week 1	Objectives 6.1 through 6.3
Week 2	Objectives 6.4 through 6.7
Weeks 3 and 4	Objectives 6.8 and 6.9, review and test

Parameters and Teaching Tips for DMA 060

- Introduce the module with a rich application. The application will provide opportunities to discuss with students the skills that they will eventually use to solve the application. Discussing the skills after the introduction of an application gives students a context from which the skills can be developed.
- The use of a graphing calculator is important in this module. Clear understanding of the connection between the quadratic function, the graph of the function, and the application of the function is facilitated by the use of the graphing calculator.
- Along with the increased use of calculators, students will need more opportunities to examine their solutions for reasonableness. This is an important concept when introducing students to quadratic equations.
- Instructors, through the use of questions and guided discovery, should help students understand the word problem events that lead to a quadratic equation.
- The use of visualization skills will be important for students to use while learning to use multiple representations and quadratic word problems.
- Instruct on the vocabulary of polynomials, identifying a term and parts of a term as well as classifying polynomials. These concepts will not be assessed directly; the usefulness is in solving applied problems.
- When instructing on operations with polynomials assist the student with identifying like terms. Although a necessary skill when simplifying, this concept will not be assessed alone but within the context of operations with polynomials.
- When instructing on division include division of a polynomial by a monomial; and, demonstrate how to split a fraction with a polynomial numerator and monomial denominator into multiple terms.
- When instructing on factoring, begin by assisting the student in identifying the greatest common factor. Then demonstrate writing a polynomial as a product of factors.
- When instructing on factoring trinomials, factor trinomials with a leading coefficient of one using multiple methods. Also, factor trinomials with a leading coefficient other than one using multiple methods.
- Factoring problems should be carefully chosen so that substitution method is not required. For instance, the following problem would not be included in this module: $(x+h)^2 + 2(x+h) + 1 = 0$.
- When solving quadratic equations, students should not be required to complete the square.
- The quadratic formula will be introduced in the radicals module (DMA 080) and should not be introduced in this module.
- Students should not factor the sum or difference of cubes.

Sample Introductory Application for DMA 060

A baseball player hits a foul ball into the air from a height of 4 feet off the ground. The initial velocity as the ball comes off the bat is 130 feet per second. The motion of the ball can be modeled by the polynomial equation $f(x) = -16x^2 + 130x + 4$, where x is the time in seconds.

1. What is the height of the ball after 1 second? What is the height of the ball after 2 seconds? What is the height of the ball after 10 seconds?

2. Do all of the answers in question 1 make sense? Use complete sentences to explain your answer.

3. Complete the table of values that shows the height of the ball in terms of time.

Quantity Name	**Time**	**Height**
Unit	Seconds	Feet
Expression	x	
	0	
	2	
	4	
	6	
	8	
	10	

4. Create a graph that models the path of the ball on the grid below. Carefully label all parts of the graph.

5. What is the greatest height of the ball? How long does it take to get to this height? Answer in complete sentences.

6. Write an equation that you can use to find out how long it will take for the ball to be 250 feet above the ground.

7. Write an equation that you can use to find out how long it will take for the ball to hit the ground.

Sample Conceptual Questions for DMA 060

<table>
<tr><th>SLO</th><th>Sample Question</th></tr>
<tr><td>6.1</td><td>A) A water balloon is thrown upward from the top of an 80 foot tower with an initial velocity of 64 feet per second. The height of the object after t seconds is given by the equation $f(t) = -16t^2 + 64t + 80$.
a) Find $f(0)$. What does it mean in terms of the situation?
b) Complete a table showing the height of the balloon after 1, 2, 3, 4, and 5 seconds. What does the height after 5 seconds represent?
c) Draw a graph to model the path of the balloon.

B) Find the lengths of the sides of a right triangle if the lengths can be expressed as three consecutive even integers.
a) Create a table and use trial and error and the Pythagorean Theorem to try to find the lengths of the sides.
b) Let $x + 4$ represent the length of the hypotenuse. Draw a right triangle showing the lengths of the other two sides in terms of x.
c) Write and solve an equation to find the lengths.
d) What does the solution $x = -2$ mean in the context of this problem?</td></tr>
<tr><td>6.2</td><td>In developing his business plan, Don wrote an equation to calculate the cost C of producing cupcakes as $C = 250 + .78x$ where x is the number of cupcakes. If he sells the cupcakes for $1.50 each, his revenue R equation will be $R = 1.50x$. Profit is calculated by subtracting cost from revenue. Using what you know, write a new formula to calculate Don's profit for x cupcakes sold.</td></tr>
<tr><td>6.3</td><td>Evaluate the following and explain your answers:
a) $2x^0$
b) $(2x)^0$</td></tr>
<tr><td>6.4</td><td>Explain how you would multiply the binomials $(a+b)(a-b)(2a+b)(2a-b)$.</td></tr>
<tr><td>6.5</td><td>This is a skill based SLO and will be tested as such.</td></tr>
<tr><td>6.6</td><td>Tell what method you would use to factor $x^2 - 5x + 6$ and the reason why you would choose it.</td></tr>
<tr><td>6.7</td><td>This is a skill based SLO and will be tested as such.</td></tr>
<tr><td>6.8</td><td>Laura has decided to invest in an emerging markets fund. Her initial investment of $16,000 grew to $25,000 in two years. Answer the following questions if the formula
$16000(1 + x)^2 = 25000$ describes the situation when x is the rate of return.
a. Is this a linear or quadratic equation? Explain your answer.
b. If necessary, rewrite the equation to solve the problem.
c. Enter the equation in your graphing calculator and sketch the graph here.
d. Find the rate of return algebraically. Show all work.
e. State the answer to the problem. If your solution resulted in more than one possible answer, explain why other solutions were not chosen. Use your sketch for clarification if appropriate.</td></tr>
<tr><td>6.9</td><td>a) A company makes water heaters. The factory can produce x water heaters per year. The profit $f(x)$ the company makes is a function of the number of water heaters it produces.</td></tr>
</table>

	$$f(x) = -2x^2 + 1000x$$ a. Graph the function using your graphing calculator and sketch the graph here. b. Identify the maximum point on your graph and find the coordinates. c. Explain the meaning of the maximum in terms of the problem. d. Identify the zeros on your graph and find the coordinates. e. Explain the meaning of the zeros in terms of the problem. f. What is the domain and range of the graph? Write them in interval notation here. g. Explain the meaning of the domain and range in terms of the problem. b) If the graphs of two quadratic functions have the same x intercepts, will they also have the same vertex? Why or why not?

Student Success Tips for DMA 060

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students:

- At the beginning of the module, students should create a time line in order to gain an understanding of the amount of time that the student will spend working on the course assignments. Each week, students should evaluate the time that they are spending on course assignments.
- Instructors should help students understand the importance of goal setting. A student whose goal is to learn as much as possible will probably have very different behaviors than students whose goals are to get a passing grade with as little work as possible, or whose goals are to avoid embarrassment, or who is a perfectionist and whose goal is to avoid making mistakes. Helping students understand their course goals won't necessarily cause them to change the goal, but it will help the student understand the relationship between the goal and the results of their behaviors.

DMA 070
Rational Expressions and Equations

Brief Description

In this module, the emphasis will be on a conceptual understanding of the problem events that result in graphic and algebraic representations of rational equations. Once the students have a solid understanding of these concepts they will be ready to learn the procedures for finding algebraic solutions to contextual rational applications. Students will identify and simplify rational expressions. Reasonableness of solutions in an application will be emphasized. The use of the graphing calculator is important for students to understand the function concepts in the module.

Course Competencies

- Solve contextual application problems involving operations on rational expressions and/or equations
- Represent real world situations as rational equations and graphically using a graphing calculator
- Analyze the meaning of asymptotes using a graphing calculator
- Explain the reasonableness of solutions found

Conceptual Student Learning Outcomes

7.1 Demonstrate the use of a problem solving strategy to include multiple representations of the situation, organization of the information, and algebraic representation of rational equations

7.2 Graph rational functions using the graphing calculator to identify and interpret the y-intercept values and domain in terms of the problem

7.3 Multiply and divide rational expressions

7.4 Add and subtract rational expressions

7.5 Solve rational equations

Suggested Timeline

Week 1	Objectives 7.1 and 7.2
Week 2	Objectives 7.3 and 7.4
Weeks 3 and 4	Objective 7.5, review and test

Parameters and Teaching Tips for DMA 070

- Introduce the module with a rich application. The application will provide opportunities to discuss with students the skills that they will eventually use to solve the application. Discussing the skills after the introduction of an application gives students a context from which the skills can be developed.
- When finding like denominators, the problems should be chosen so that there is no more than three factors in the least common denominator.
- When instructing on multiplying and dividing rational expressions assist the student by identifying a rational expression and applying factoring methods to rational expression. These concepts will be assessed when students apply operations using rational expressions.
- Applications in this module can include, but are not limited to, work, distance, and direct variation.
- Complex rational expressions are not included in the module.
- Emphasis should be placed on reasonableness of solutions in application problems.
- The use of the graphing calculator is important for students to understand the function concepts in the module. Identifying zeros and the domain on a graph help students to make the connection between the equation and the graph.

Sample Introductory Application for DMA 070

Psychologists study memory and learning. In an experiment on memory, students in a language class are asked to memorize 40 vocabulary works in Latin, a language with which the students are not familiar. After studying the words for one day, students are tested each day after to see how many words they remember. The class average is then found. The function

$$f(x) = \frac{5x + 30}{x}$$

models the average number of Latin words remembered by the students, after x days.

1. Interpret the point (25, 6.2) in the context of this scenario.

2. According to the graph, between which two days do students forget the most?

3. Does this function have a minimum? If so, what is the minimum? If not, explain how you know that no minimum exists.

4. What does this graph tell psychologists about memory? Answer using full sentences using the terms of the problem.

5. Find x when $f(x) = 11$. Explain the meaning of the solution in the context of this problem. Check your answer using another method.

6. Find $f(365)$ and interpret its meaning in context. Explain how the graph supports your answer.

7. Use the formula to find the number of days it takes for the students to remember half of the original vocabulary list. Check the reasonableness of your answer by referring to the graph.

Sample Conceptual Questions for DMA 070

SLO	Sample question
7.1	
7.2	Scott and Ian design a cool t-shirt for snow boarders. Their friends are very impressed and everybody wants one, so Scott and Ian set up a t-shirt printing business in their garage. Total start-up costs are \$450 due to the availability of a used graphics machine. They estimate that is will cost \$5.50 to print each t-shirt. a) Write a linear function $f(x)$ to model the cost of producing □ t-shirts. b) Write a rational function f(x) to model the average cost of producing x t-shirts. c) Graph $f(x)$ in your graphing calculator. What is the domain of $f(x)$ in the context of the problem? Write the domain using interval notation. Explain your answer comparing the algebraic domain versus your answer. d) Does $f(x)$ have a vertical asymptote(s)? If so, what is its equation(s)? e) Identify the zeros of $f(x)$ and interpret them in terms of the problem. f) How many t-shirts will Scott and Ian have to make to earn back their start-up costs?
7.3	This is a skill based SLO and will be tested as such.
7.4	This is a skill based SLO and will be tested as such.
7.5	A boat takes 1.5 times longer to go 360 miles up a river than to return. If the boat cruises at 15 miles per hour in still water, what is the rate of the current?

Student Success Tips for DMA 070

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students:

- At the beginning of the module, students should create a time line in order to gain an understanding of the amount of time that the student will spend working on the course assignments. Each week, students should evaluate the time that they are spending on course assignments.
- Instructors can help students understand the importance of self-efficacy for solving mathematics problems and especially for solving word problems. Students make judgments about their mathematical capabilities based on comparisons of performance with peers, successful and unsuccessful outcomes on assessment measures, and feedback from others such as teachers, parents, and peers. This means that the words and tone used by an instructor, the types of classroom activities are crucial to students' self-efficacy. Believing that you can succeed is as important as gaining mathematical knowledge. Even if the student learns the course content, the brain can, and often does, talk itself out of knowing something.
    - Students whose perceptions of their capabilities are high often go on to challenge themselves, persevere in the face of difficulties, and expend greater effort resulting in more successful experiences. Self-doubters on the other hand often give up in the face of difficulty, or avoid mathematical experiences altogether to preserve self-worth. The art of teaching is to create instructional activities that are challenging but that are scaffolded to make concepts more understandable. When these learning experiences are connected to the students lives, introduced in a ways that are compatible to the students' diverse learning styles and structured so that students are active learners, it increases the likelihood that learners will perceive their capabilities as sufficient to the task.
    - Self-efficacy influences whether discrepancies between performance and goals are motivating or discouraging. Being over- or under- confident can undermine performance.For example, a student's goal grade for her upcoming math test is an A, and she earns a B. If she has high self-efficacy she will attribute her shortcoming to insufficient effort, "If I had studied more, I could have earned an A. Next time I will work harder." However, if she has low math self-efficacy she will attribute her shortcoming to lack of ability, "I just don't get this material; I am not capable of getting an A in this course" (Zimmerman, 2000).

DMA 080
Radical Expressions and Equations

Brief Description

In this module, students will understand how to manipulate radicals to solve real world applications involving radical equations. Students will simplify and perform operations with radical expressions and rational exponents. Reasonableness of solutions in an application will be emphasized. The graphing calculator will be used to examine the graphic representation of radical functions.

Course Competencies

- Solve contextual application problems involving operations on radical expressions and/or equations
- Represent real world situations as radical equations and graphically using a graphing calculator
- Explain the reasonableness of solutions found
- Correctly perform operations with radical expressions
- Use graphing calculator to analyze radical functions

Specific Student Learning Outcomes

8.1 Demonstrate the use of a problem solving strategy to include multiple representations of the situation, organization of the information, and algebraic representation of radical equations

8.2 Correctly use rational exponents to rewrite radical expressions

8.3 Simplify radical expressions

8.4 Add and subtract radical expressions

8.5 Multiply radical expressions

8.6 Divide radical expressions

8.7 Solve radical equations

8.8 Solve quadratic equations using the quadratic function.

8.9 Graph radical functions using the graphing calculator to identify and interpret the graph in terms of the problem

Suggested Timeline

Week 1	Objectives 8.1 and 8.2
Week 2	Objectives 8.3 through 8.5
Weeks 3 and 4	Objectives 8.6 through 8.9, review and test

Parameters and Teaching Tips for DMA 080

- Introduce the module with a rich application. Students may not have all of the skills yet to solve the application but it will give them the context from which the skills can be developed.
- Most problems should be square roots or cube roots. Higher indexes are not required in this module.
- All radicands should be positive. Imaginary numbers are not part of this module.
- Emphasis should be placed on reasonableness of solutions in application problems.
- The use of the graphing calculator is important for students to understand the function concepts in the module. Identifying zeros and the domain and range on a graph help students to make the connection between the equation and the graph.
- Students will be taught the skills of rationalizing denominators when dividing radicals.

Sample Introductory Application for DMA 080

Police sometimes use the formula $s = \sqrt{30df}$ to estimate the speed, s, in miles per hour, of a car that skidded d feet upon braking. The variable f is the coefficient of friction determined by the kind of road and the wetness or dryness of the road. The following table gives some values of f:

	Concrete	**Tar**
Wet	0.4	0.5
Dry	0.8	1.0

a) How fast was a car going on a concrete road if the skid marks on a rainy day were 80 feet long?

b) How fast was a car going on a dry tar road if the skid marks were 100 feet long?

c) How long would be the skid marks for a car driving on a wet tar road traveling at 50 mph?

d) Make a table of reasonable values for speed and feet skidded and create a graph for wet tar. Discuss the domain and the range within the context of this problem. Use the graph to verify your answer to part c.

Sample Conceptual Questions for DMA 080

SLO	Sample question
8.1.	The distance a person can see to the horizon depends on their height from the ground. The distance can be found using the formula $d = 1.5\sqrt{h}$, where d is the distance in miles and h is the height in feet to the person's eye. a) Find the distance someone can see to the horizon if their eye is 6 feet off the ground. b) Find the distance someone can see to the horizon if their eye is 24 feet off the ground. c) A person in an airplane can see 175 miles to the horizon, what is the altitude of the plane? d) How high off the ground must a person's eye be to see 15 miles to the horizon?
8.2	This is a skill based SLO and will be tested as such.
8.3	This is a skill based SLO and will be tested as such.
8.4	Alex is paid at a rate of $\sqrt{18h}$ and Kim is paid at a rate of $\sqrt{2h}$ where h represents the number of hours worked. Find an expression that represents the sum of their rates. Find an expression that represents the difference in their rates.
8.5	This is a skill based SLO and will be tested as such.
8.6	The speed of sound V near the Earth's surface is found using the equation V$= 20\sqrt{t} + 273$ where t is the surface temperature in degrees Celsius. The speed of sound at the Earth's surface is often given at 340 meters per second, but that is only accurate at a certain temperature. On what temperature is this figure based?
8.7	a) Graph each equation. State the domain of the graph using interval notation then describe how it differs from the parent graph $y = \sqrt{x}$. 1) $y = \sqrt{3x}$ 2) $y = \sqrt{x} + 3$ 3) $y = \sqrt{3x} - 3$ b) On a graphing calculator, when Jonathan graphs y$= \sqrt{x} \cdot \sqrt{x}$, he does not get the complete line $y = x$. Explain why.
8.8	A payroll can be completed in four hours by two computers working together. How many hours are needed for each computer to do the job alone if the older model requires three hours longer than the newer one? Round to the nearest hundredth, if necessary.
8.9	The time t (in seconds) that it takes for a cliff diver to reach the water is function of the height h (in feet) from which he dives: $t = \sqrt{\frac{h}{16}}$. a) Graph the equation on your calculator. You may have to adjust the window settings to make the graph easy to read. b) Where does the graph cross the horizontal axis? Interpret this point in terms of the problem. c) Find the time that it takes for a diver to hit the water when diving from a cliff that is 40 feet high. How did you use the calculator to find this value.

Student Success Tips for DMA 080

During this four-week module, it is suggested that the instructor take a few minutes each day to work on the following skills with the students:

- At the beginning of the module, students should create a time line in order to gain an understanding of the amount of time that the student will spend working on the course assignments. Each week, students should evaluate the time that they are spending on course assignments.
- One of the goals of developmental education is to produce self-regulated problem solvers who will succeed in curriculum mathematics courses. Instructors can help students understand the importance of self-regulation for academic success.
  Self-regulated problem solvers typically:
  - Has a course goal directed toward learning and understanding the course concepts because the students sees the importance of learning mathematics
  - Has high efficacy for solving word problems
  - Completes homework on time and asks questions to fully understand the homework
  - Maintains behaviors consistent with the completion of course goal; such as attending class, preparing for test, taking good notes

  Is a skilled problem solver who:
  - Takes time to understand the problem before beginning to solve the problem
  - Creates a solution plan before beginning to solve the problem
  - Monitors the success of the problem solving process
  - Monitors resources, especially time (Schoenfeld, 1987)
  - Evaluates strategies in order to adapt them in future problem solving methods (Zimmerman, 1995)
  - Attributes causation to results
  - Controls emotions generated by any previous negative experiences with mathematics

APPENDIX

Recommended Pre-Requisites for NCCCS Gateway Curriculum Math Courses

Gateway Math Course	Required Modules (place out of or take)
MAT 101	DMA 010-030
MAT 110	DMA 010-030
MAT 115	DMA 010-050
MAT 120	DMA 010-040
MAT 121	DMA 010-050
MAT 140	DMA 010-040
MAT 141	DMA 010-040
MAT 145	DMA 010-050
MAT 151*	DMA 010-050
MAT 155	DMA 010-050
MAT 161	DMA 010-080
MAT 171	DMA 010-080
MAT 175	DMA 010-080

Note: Core geometry competencies are threaded through DMA 010 through DMA 040.

*DEI Math Task Force is recommending that MAT 140 no longer be a pre-req for this course