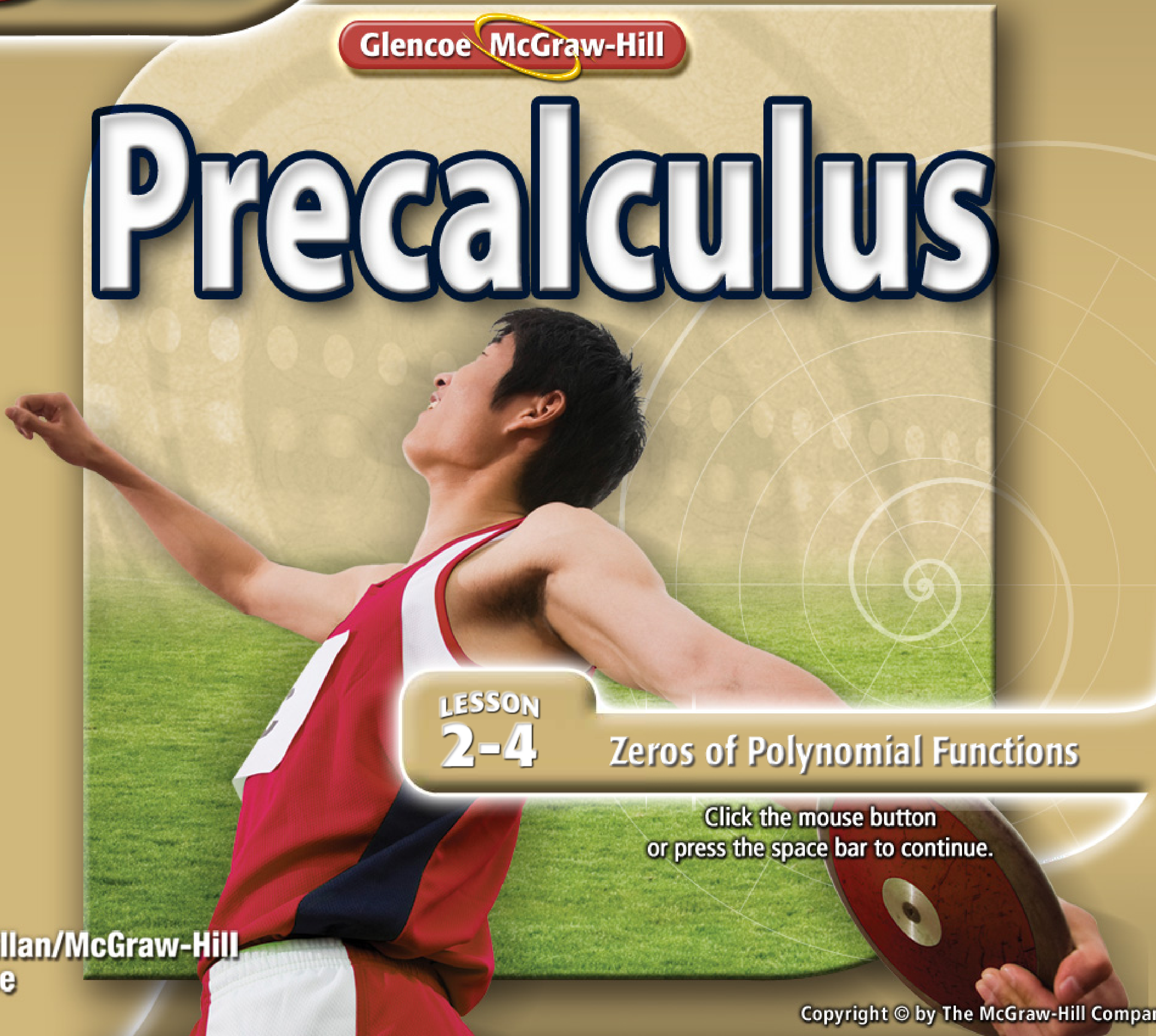
Interactive Classroom

Glencoe McGraw-Hill

Precalculus



LESSON 2-4

Zeros of Polynomial Functions

Click the mouse button or press the space bar to continue.

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New Vocabulary

- Rational Zero Theorem
- lower bound
- upper bound
- Descartes' Rule of Signs
- Fundamental Theorem of Algebra
- Linear Factorization Theorem
- Conjugate Root Theorem
- complex conjugates
- irreducible over the reals

Multilingual eGlossary

Key Concept **Rational Zero Theorem**

If f is a polynomial function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$, with degree $n \geq 1$, integer coefficients, and $a_0 \neq 0$, then every rational zero of f has the form $\frac{p}{q}$, where

- p and q have no common factors other than ± 1,
- p is an integer factor of the constant term a_0, and
- q is an integer factor of the leading coefficient a_n.

Corollary If the leading coefficient a_n is 1, then any rational zeros of f are integer factors of the constant term a_0.

Note: this technique only finds roots that can be written as fractions.

EXAMPLE 1 Leading Coefficient Equal to 1

A. List all possible rational zeros of $f(x) = x^3 - 3x^2 - 2x + 4$. Then determine which, if any, are zeros.

Step 1 Identify possible rational zeros.

EXAMPLE 1 Leading Coefficient Equal to 1

Step 2 Use direct substitution to test each possible zero.

$f(1) = (1)^3 - 3(1)^2 - 2(1) + 4$ or 0

$f(-1) = (-1)^3 - 3(-1)^2 - 2(-1) + 4$ or 2

$f(2) = (2)^3 - 3(2)^2 - 2(2) + 4$ or -4

$f(-2) = (-2)^3 - 3(-2)^2 - 2(-2) + 4$ or -12

$f(4) = (4)^3 - 3(4)^2 - 2(4) + 4$ or 12

$f(-4) = (-4)^3 - 3(-4)^2 - 2(-4) + 4$ or -100

The only rational zero is 1.

Answer: $\pm 1, \pm 2, \pm 4$; 1

EXAMPLE 1 Leading Coefficient Equal to 1

B. List all possible rational zeros of $f(x) = x^3 - 2x - 1$. Then determine which, if any, are zeros.

Step 1

EXAMPLE 1 Leading Coefficient Equal to 1

Step 2 Begin by testing 1 and –1 using synthetic substitution.

1 |

–1 |

EXAMPLE 1 Leading Coefficient Equal to 1

Because $f(-1) = 0$, you can conclude that $x = -1$ is a zero of f. Thus $f(x) = (x + 1)(x^2 - x - 1)$.

Because the factor $x^2 - x - 1$ yields no rational zeros, we can conclude that f has only one rational zero, $x = -1$.

Answer: ± 1; 1

EXAMPLE 1 Guided Practice

List all possible rational zeros of $f(x) = x^4 - 12x^2 - 15x - 4$. Then determine which, if any, are zeros.

A. $\pm 1, \pm 4; \pm 1, \pm 4$

B. ± 2; none

C. $\pm 1, \pm 2, \pm 4; -1, 4$

D. $\pm 1, \pm 2; -1, 2$

EXAMPLE 2 Leading Coefficient not Equal to 1

List all possible rational zeros of $f(x) = 2x^3 - 5x^2 - 28x + 15$. Then determine which, if any, are zeros.

Step 1 The leading coefficient is 2 and the constant term is 15. Possible rational zeros:

$$\frac{\text{Factors of 15}}{\text{Factors of 2}} = \frac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1, \pm 2} \text{ or}$$

$$\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$$

Hint: start finding roots by testing integers first!

EXAMPLE 2 Leading Coefficient not Equal to 1

Step 2 By synthetic substitution, you can determine that $x = -3$ is a rational zero.

Now, break down the polynomial into linear factors like we did in the last section.

EXAMPLE 2 Leading Coefficient not Equal to 1

Answer: $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}; \frac{1}{2}, -3, 5$

EXAMPLE 2 Guided Practice

List all possible rational zeros of $f(x) = 4x^3 - 20x^2 + x - 5$. Then determine which, if any, are zeros.

A. $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}; 5$

B. $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}; \frac{5}{4}$

C. $\pm 1, \pm 5; 5$

D. $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{1}{4}; -1, 5, \frac{1}{4}$

Real-World EXAMPLE 3 **Solve a Polynomial Equation**

WATER LEVEL The water level in a bucket sitting on a patio can be modeled by $f(x) = x^3 + 4x^2 - 2x + 7$, where $f(x)$ is the height of the water in millimeters and x is the time in days. On what day(s) will the water reach a height of 10 millimeters?

Because $f(x)$ represents the day when the water will reach a given height, you need to solve $f(x) = 10$ to determine what day the water will reach a height of 10 millimeters.

Real-World EXAMPLE 3 Solve a Polynomial Equation

$f(x)$	$= 10$	Write the equation.
$x^3 + 4x^2 - 2x + 7$	$= 10$	Substitute $x^3 + 4x^2 - 2x + 7$ for $f(x)$.
$x^3 + 4x^2 - 2x - 3$	$= 0$	Subtract 10 from each side.

Apply the Rational Zeros Theorem to this new polynomial function, $f(x) = x^3 + 4x^2 - 2x - 3$.

Step 1 Possible rational zeros: factors of $-3 = \pm1, \pm3$.

Real-World EXAMPLE 3 Solve a Polynomial Equation

Step 2 Because the number of the day cannot be negative, check each of the positive rational zeros using synthetic substitution. Doing, so, you can determine that $x = 1$ is the only positive rational zero of f.

1	1	4	–2	–3
		1	5	3
	1	5	3	0

Because $x = 1$ is a zero of f, $x = 1$ is a solution of $f(x) = 0$. So, it was day 1 when the water reached a height of 10 millimeters.

Answer: day 1

Real-World EXAMPLE 3 Guided Practice

PHYSICS **The path of a ball is given by the function $f(x) = -4.9x^2 + 21.5x + 40$, where x is the time in seconds and $f(x)$ is the height above the ground in meters. After how many seconds will the ball reach a height of 25 meters?**

A. 4 seconds, 10 seconds

B. 4 seconds

C. 5 seconds, $\frac{30}{49}$ seconds

D. 5 seconds