Notes for:

AP Physics 1: Algebra-Based

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Lynn English High School

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http://www.mrbigler.com/AP-Physics-1/Notes-AP-Physics-1.pdf

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This is a set of class notes for AP Physics 1: Algebra-Based. This hardcopy is provided so that you can fully participate in class discussions without having to worry about writing everything down.

These notes are meant to complement the textbook discussion of the same topics. In some cases, the notes and the textbook differ in method or presentation, but the physics is the same. There may be errors and/or omissions in the textbook. There are certainly errors and omissions in these notes, despite my best efforts to make them clear, correct, and complete.

As we discuss topics in class, you will almost certainly want to add your own notes to these. If you have purchased this copy, you are encouraged to write directly in it, just as you would write in your own notebook. However, if this copy was issued to you by the school and you intend to return it at the end of the year, you will need to write your supplemental notes on separate paper. If you do this, be sure to write down page numbers in your notes, to make cross-referencing easier.

You should bring these notes to class every day, because lectures and discussions will follow these notes, which will be projected onto the SMART board.

Table of Contents

Cornell Notes

Unit: Introduction

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- how to take advantage of the Cornell note-taking system

Language Objectives:

- Understand the term Cornell Notes and be able to describe how Cornell Notes are different from ordinary note-taking.

Notes:

The Cornell note-taking system was developed about fifty years ago at Cornell University. I think it's a great way to get more out of your notes. I think it's an especially useful system for adding your comments to someone else's notes (such as mine).

The main features of the Cornell Notes system are:

1. The main section of the page is for what actually gets covered in class.

2. The left section (officially 2½ inches, though I have shrunk it to 2 inches for these notes) is for "cues"—questions or comments of yours that will help you find, remember, or effectively use these notes.

3. The bottom section (2 inches) is officially for you to add a 1–2 sentence summary of the page in your own words. This is a good idea. However, because the rest of the page is my notes, not yours, you may also want to use that space for anything else you want to remember that wasn't in the pre-printed notes.

Use this space for summary and/or additional notes:

Reading & Taking Notes from a Textbook

Unit: Introduction

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- pre-reading and reading a section of a textbook and taking notes

Language Objectives:

- understand and be able to describe the reading and note-taking strategies presented in this section

Notes:

If you read a textbook the way you would read a novel, you probably won't remember much of what you read. Before you can understand anything, your brain needs enough context to know how to file the information. This is what Albert Einstein was talking about when he said, "It is the theory which decides what we are able to observe."

When you read a section of a textbook, you need to create some context in your brain, and then add a few observations to solidify the context before reading in detail.

René Descartes described this process in 1644 in the preface to his *Principles of Philosophy*:

> "I should also have added a word of advice regarding the manner of reading this work, which is, that I should wish the reader at first go over the whole of it, as he would a romance, without greatly straining his attention, or tarrying at the difficulties he may perhaps meet with, and that afterwards, if they seem to him to merit a more careful examination, and he feels a desire to know their causes, he may read it a second time, in order to observe the connection of my reasonings; but that he must not then give it up in despair, although he may not everywhere sufficiently discover the connection of the proof, or understand all the reasonings—it being only necessary to mark with a pen the places where the difficulties occur, and continue reading without interruption to the end; then, if he does not grudge to take up the book a third time, I am confident that he will find in a fresh perusal the solution of most of the difficulties he will have marked before; and that, if any remain, their solution will in the end be found in another reading."

Use this space for summary and/or additional notes:

Reading & Taking Notes from a Textbook

The following 4-step system takes about the same amount of time you're used to spending on reading and taking notes, but it will probably make a tremendous difference in how much you understand and remember.

1. Copy the titles/headings of each section. Leave about ¼ page of space after each one. (Don't do anything else yet.) This should take about 2–3 minutes.
2. Do not write anything yet! Look through the section for pictures, graphs, and tables. Take a minute to look at these—the author must have thought they were important. Also read over (but don't try to answer) the homework questions/problems at the end of the section. (For the visuals, the author must think these things illustrate something that is important enough to dedicate a significant amount of page real estate to it. For the homework problems, these illustrate what the author thinks you should be able to do once you know the content.) This process should take about 10–15 minutes.
3. Actually read the text, one section at a time. For each section, jot down keywords and sentence fragments that remind you of the key ideas. You are not allowed to write more than the ¼ page allotted. (You don't need to write out the details—those are in the book, which you already have!) This process is time consuming, but shorter than what you're probably used to doing for your other teachers.
4. Read the summary at the end of the chapter or section—this is what the author thinks you should know now that you've finished the reading. If there's anything you don't recognize, go back and look it up. This process should take about 5–10 minutes.

You shouldn't need to use more than about one sheet of paper (both sides) per 10 pages of reading!

Use this space for summary and/or additional notes:

Taking Notes on Math Problems

Unit: Introduction

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- taking notes on a mathematical problem

Language Objectives:

- understand and be able to describe the strategies presented in this section

Notes:

If you were to copy down a math problem and look at it a few days or weeks later, chances are you'll recognize the problem, but you won't remember how you solved it.

Solving a math problem is a process. For notes to be useful, they need to describe the process as it happens, not just the final result.

If you want to take good notes on how to solve a problem, you need your notes to show what you did at each step.

For example, consider the following physics problem:

> A 25 kg cart is accelerated from rest to a velocity of $3.5\,\frac{\text{m}}{\text{s}}$ over an interval of 1.5 s. Find the net force applied to the cart.

The process of solving this problem involves applying two equations:
$v = v_o + at$ and $F = ma$.

Use this space for summary and/or additional notes:

Taking Notes on Math Problems

A good way to document the process is to use a two-column format, in which you show the steps of the solution in the left column, and you write an explanation of what you did and why for each step in the right column.

For this problem, a two-column description would look like the following:

Step	Description/Explanation
$m = 25\ \text{kg}$ $v_o = 0$ $v = 3.5\ \frac{\text{m}}{\text{s}}$ $t = 1.5\ \text{s}$ F = quantity desired	Define variables.
$\underline{\underline{F}} = \underline{m}a$	Choose a formula that contains ***F***. See if we have the other variables. ➔ No. We need to find ***a*** before we can solve the problem.
$\underline{v} = \underline{v_o} + \underline{\underline{a}}\underline{t}$	Choose a formula that contains ***a***. See if we have the other variables. ➔ Yes.
$3.5 = 0 + a(1.5)$ $3.5 = 1.5a$ $\frac{3.5}{1.5} = \frac{\cancel{1.5}}{\cancel{1.5}}a$ $2.33 = a$	Substitute ***v***, $\boldsymbol{v_o}$ and ***t*** into the 1st equation and solve for ***a***.
$F = ma$ $\text{F} = (25)(2.33)$ $\text{F} = 58.33$	Substitute ***m*** and ***a*** into the 2nd equation and solve for ***F***.
$\text{F} = \boxed{58.33\ \text{N}}$	Include the units and box the final answer.

Use this space for summary and/or additional notes:

Introduction: Laboratory & Measurement

Unit: Laboratory & Measurement

Topics covered in this chapter:

The purpose of this chapter is to teach skills necessary for designing and carrying out laboratory experiments, recording data, and writing summaries of the experiment in different formats.

- *Designing & Performing Experiments* discusses strategies for coming up with your own experiments and carrying them out.
- *Accuracy & Precision*, *Uncertainty & Error Analysis*, and *Recording and Analyzing Data* discuss techniques for working with the measurements taken during laboratory experiments.
- *Keeping a Laboratory Notebook* and *Formal Laboratory Reports* discuss ways in which you might communicate (write up) your laboratory experiments.

Calculating uncertainty (instead of relying on significant figures) is a new and challenging skill that will be used in lab write-ups throughout the year.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Massachusetts Curriculum Frameworks (2006):

Use this space for summary and/or additional notes:

No MA curriculum frameworks are specifically addressed in this chapter.

Skills learned & applied in this chapter:

- Designing laboratory experiments
- Error analysis (calculation & propagation of uncertainty)
- Formats for writing up lab experiments

Use this space for summary and/or additional notes:

The Scientific Method

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- Understand the scientific method

Language Objectives:

- Understand and correctly use terms relating to the scientific method, such as "peer review"

Notes:

The scientific method is a fancy name for "figure out what happens by trying it."

In the middle ages, "scientists" were called "philosophers." These were church scholars who decided what was "correct" by a combination of observing the world around them and then arguing and debating with each other about the mechanisms and causes.

During the Renaissance, scientists like Galileo Galilei and Leonardo da Vinci started using experiments instead of argument to decide what really happens in the world.

Steps:

1. Observe something interesting.
2. Figure out and perform an experiment that will have different outcomes depending on the parameter(s) being tested. You can make a ***claim*** that describes what you expect will happen (sometimes called a hypothesis), or you can just perform the experiment to see what happens.
3. Repeat the experiment, varying your conditions as many ways as you can.
4. If you are testing a claim, ***assume that your claim is wrong.*** Try every experiment and make every observation you can think of that might refute your claim.

Use this space for summary and/or additional notes:

5. If your claim holds, try to come up with a model that explains and predicts the behavior you observed. This model is called a ***theory***. If your claim holds but you cannot come up with a model, try to completely and accurately describe the conditions under which your claim successfully predicts the outcomes. This description is called a ***law***.
6. Share your theory, your experimental procedures, and your data with other scientists. Some of these scientists may:
    a. Look at your experiments to see whether the experiments really can distinguish between the different outcomes.
    b. Look at your data to see whether the data really do support your theory.
    c. Try your experiments or other related experiments themselves and see if the results are consistent with your theory.
    d. Add to, modify, limit, refute (disprove), or suggest an alternative to your theory.
    e. Not care in the slightest about your theory or your experiments. (Scientists are in no way obligated to spend their time testing someone else's theories.)

    This process is called "peer review." If a significant number of scientists have reviewed your claims and agree with them, and no one has refuted your theory, your theory may gain acceptance within the scientific community.

Note that the word "theory" in science has a different meaning from the word "theory" in everyday language. In science, a theory is a model that:

- Has never failed to explain a collection of related observations
- Has never failed to successfully predict the outcomes of related experiments

For example, the theory of evolution has never failed to explain the process of changes in organisms because of factors that affect the survivability of the species.

If a repeatable experiment contradicts a theory, and the experiment passes the peer review process, the theory is deemed to be wrong. If the theory is wrong, it must be either modified to explain the new results or discarded completely.

Use this space for summary and/or additional notes:

Note that, despite what your ninth-grade science teacher may have taught you, it is possible (and often useful) to have a hypothesis or claim before performing an experiment, but an experiment is just as valid and just as useful whether or not a hypothesis was involved.

Theories vs. Natural Laws

The terms "theory" and "law" developed organically, so any definition of either term must acknowledge that common usage, both within and outside of the scientific community, will not always be consistent with the definitions.

A theory is a model that attempts to explain *why* or *how* something happens. A law simply describes what happens without attempting to provide an explanation. Theories and laws can both be used to predict the outcomes of related experiments.

For example, the Law of Gravity states that objects attract other objects based on their masses and distances from each other. It is a law and not a theory because the Law of Gravity does not explain *why* masses attract each other.

Atomic Theory states that matter is made of atoms, and that those atoms are themselves made up of smaller particles. The interactions between the particles that make up the atoms (particularly the electrons) are used to explain certain properties of the substances. This is a theory because it gives an explanation for *why* the substances have the properties that they do.

Note that a theory cannot become a law any more than a definition can become a measurement or a postulate can become a theorem.

Use this space for summary and/or additional notes:

Designing & Performing Experiments

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- determine what you are testing
- figure out how to get from what you can measure to what you want to determine

Language Objectives:

- Understand and correctly use the terms “dependent variable” and “independent variable.”
- Understand and be able to describe the strategies presented in this section.

Notes:

Most high school physics experiments are relatively simple to understand, set up and execute—much more so than in chemistry or biology. This makes physics well-suited for teaching you how to design experiments.

The education “buzzword” for this is ***inquiry-based experiments***, which means you (or your lab group) will need to figure out what to do to perform an experiment that answers a question about some aspect of physics. In this course, you will usually be given only an objective or goal and a general idea of how to go about achieving it. You and your lab group (with help) will decide the specifics of what to do, what to measure (and how to measure it), and how to make sure you are getting good results. This is a form of *guided* inquiry.

Use this space for summary and/or additional notes:

Framing Your Experiment

Experiments are motivated by something you want to find out, observe, or calculate.

Independent vs. Dependent Variables

In an experiment, there is usually something you are doing, and something you are measuring or observing.

independent variable: the conditions you are setting up. These are the numbers you pick. Because you pick the numbers, they are *independent* of what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are choosing the heights, so height is the *independent* variable.

dependent variable: the things that happen in the experiment. These are the numbers you measure, which are *dependent* on what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are measuring the time, which depends on the height. This means time is the *dependent* variable.

control variable: other things that could vary but are being kept constant. These are usually parameters that could be independent variables in other experiments, but are kept constant so they do not affect the relationship between the independent variable being tested and the dependent variable being measured.

If someone asks what your independent and dependent variables are, the question simply means, "What did you change (independent variable)?" and "What did you measure (dependent variable)?"

Use this space for summary and/or additional notes:

Qualitative Experiments

If the goal of your experiment is to find out ***whether or not*** something happens at all, you need to set up a situation in which the phenomenon you want to observe can either happen or not, and then observe whether or not it does. The only hard part is making sure the conditions of your experiment don't bias whether the phenomenon happens or not.

If you want to find out ***under what conditions*** something happens, what you're really testing is whether or not it happens under different sets of conditions that you can test. In this case, you need to test three situations:

1. A situation in which you are sure the thing will happen, to make sure you can observe it. This is your **positive control**.
2. A situation in which you sure the thing cannot happen, to make sure your experiment can produce a situation in which it doesn't happen and you can observe its absence. This is your **negative control**.
3. A condition or situation that you want to test to see whether or not the thing happens. The condition is your independent variable, and whether or not the thing happens is your dependent variable.

Quantitative Experiments

If the goal of your experiment is to quantify (find a numerical relationship for) the extent to which something happens (the dependent variable), you need to figure out a set of conditions under which you can measure the thing that happens. Once you know that, you need to figure out how much you can change the parameter you want to test (the independent variable) and still be able to measure the result. This gives you the highest and lowest values of your independent variable. Then perform the experiment using a range of values for the independent value that cover the range from the lowest to the highest (or *vice-versa*).

For quantitative experiments, a good rule of thumb is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Use this space for summary and/or additional notes:

Determining What to Measure

Determining what to measure usually means determining what you need to know and figuring out how to get from there to *quantities that you can measure*.

For a quantitative experiment, if you have a mathematical formula that includes the quantity you want to measure, you simply need to find the values of the other quantities in the equation. For example, if you want to calculate the force needed to move an object, the equation is:

$$F = ma$$

In order to calculate the force, you would need to know:

- **mass**: you can measure this using a balance
- **acceleration**: you could measure this with an accelerometer, but we do not have one in the lab. This means you will need to calculate acceleration from another formula.

Notice that we have figured out that we can measure mass directly, but we need some way to figure out the acceleration. This means that our experiment to determine the force consists of two steps:

1. Measure the mass.
2. Perform an experiment to determine the acceleration.
3. Calculate the force on the object from the mass and the acceleration.

So now we need to perform an experiment for the acceleration. A useful formula is:

$$d = v_o t + \tfrac{1}{2} a t^2$$

This means in order to calculate acceleration, we need to know:

- **displacement (*d*)**: the change in the objects position. You can measure this with a meter stick or tape measure.
- **initial velocity (v_o)**: you either need to measure this or set up your experiment so the initial velocity is zero (the object starts out at rest).
- **time (*t*)**: you can measure this with a stopwatch.

Use this space for summary and/or additional notes:

Designing & Performing Experiments

Now, every variable we need in the equation is something we can measure, so we're all set. Our experiment is:

1. Measure the **mass** of the object.
2. Determine the **acceleration** of the object:
   a. Set up an experiment in which the object starts out at rest (**initial velocity is zero**), and then it **accelerates**.
   b. Measure the **displacement** (change in position) of the object and the **time** it takes.
   c. **Calculate** the **acceleration**.
3. Now you have enough information to **calculate** the **force** on the object from the mass and the acceleration.

Use this space for summary and/or additional notes:

Graphical Solutions

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- Use a graph to calculate the relationship between two variables.

Language Objectives:

- Understand and use terms relating to graphs.

Notes:

Most experiments in a high-school physics class involve finding a mathematical relationship between two quantities. While it is possible to simply measure the two quantities once and calculate, an approach that measures the relationship across a range of values will provide a better result.

As mentioned above, a good rule of thumb for quantitative experiments is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Once you have your data points, arrange the equation into $y=mx+b$ form, such that the slope is the quantity of interest. Then accurately plot your data and draw a best-fit line. The slope of this line will be the quantity of interest (or its reciprocal).

Use this space for summary and/or additional notes:

Accuracy & Precision

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Understand what accuracy and precision mean and the difference between the two.

Language Objectives:

- Understand and be able to differentiate between accuracy and precision.

Notes:

Science relies on making and interpreting measurements, and the accuracy and precision of these measurements affect what you can conclude from them.

Random vs. Systematic Errors

Random errors are natural uncertainties in measurements because of the limits of precision of the equipment used. Random errors are assumed to be distributed around the actual value, without bias in either direction. Systematic errors occur from specific problems in your equipment or your procedure. Systematic errors are often biased in one direction more than another, and can be difficult to identify.

Accuracy vs. Precision

The words "accuracy" and "precision" have specific meanings in science.

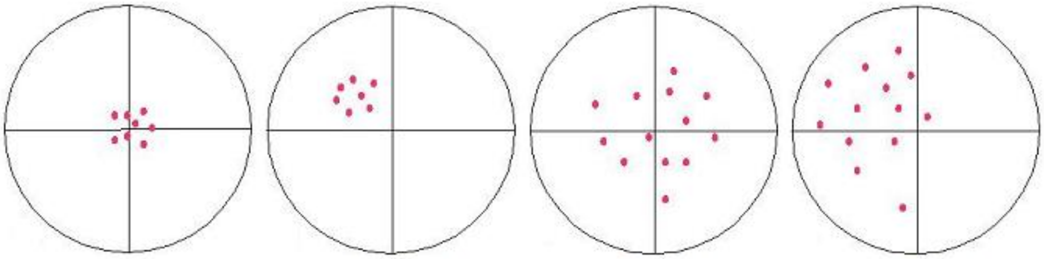
accuracy: for a single measurement, how close the measurement is to the "correct" or accepted value. For a group of measurements, how close the average is to the accepted value.

precision: for a single measurement, how finely the measurement was made. (How many decimal places it was measured to.) For a group of measurements, how close the measurements are to each other.

Use this space for summary and/or additional notes:

Examples:

Suppose the following drawings represent arrows shot at a target.



accurate precise

not accurate precise

accurate not precise

not accurate not precise

The first set is both accurate (the average is close to the center) and precise (the data points are all close to each other.)

The second set is precise (close to each other), but not accurate (the average is not close to the correct value). This is an example of *systemic* error—some problem with the experiment caused all of the measurements to be off in the same direction.

The third set is accurate (the average is close to the correct value), but not precise (the data points are not close to each other). This is an example of *random* error—the measurements are not biased in any particular direction, but there is a lot of scatter.

The fourth set is neither accurate nor precise, which means that there are significant random and systematic errors present.

4. For another example, suppose two classes estimate Mr. Bigler's age. The first class's estimates are 73, 72, 77, and 74 years old. These measurements are fairly precise (close together), but not accurate. (Mr. Bigler is actually about 50 years old.) The second class's estimates are 0, 1, 97 and 98. This *set* of data is accurate (because the average is 49, which is close to correct), but the *set* is not precise because the individual values are not close to each other.

Use this space for summary and/or additional notes:

Uncertainty & Error Analysis

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Understand what uncertainty of measurement means.

Skills:

- Read and interpret uncertainty values.
- Estimate measurement errors.
- Propagate estimate of error/uncertainty through calculations.

Language Objectives:

- Understand and correctly use the terms “uncertainty,” “standard uncertainty,” and “relative error.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

In science, unlike mathematics, there is no such thing as an exact answer. Ultimately, every quantity is limited by the precision and accuracy of the measurements that it came from. If you can only measure a quantity to within 10%, that means any number that is derived from that measurement can’t be any better than ±10%.

Error analysis is the practice of determining and communicating the causes and extents of possible errors or uncertainty in your results. Error analysis involves understanding and following the uncertainty in your data from the initial measurement to the final report.

Use this space for summary and/or additional notes:

Uncertainty

The uncertainty of a measurement describes how different the correct value is likely to be from the measured value. For example, if a length was measured to be 22.336 cm, and the uncertainty was 0.030 cm (meaning that the measurement is only known to within ±0.030 cm), we could represent this measurement in either of two ways:

(22.336 ± 0.030) cm   22.336(30) cm

The first of these states the variation (±) explicitly in cm (the actual unit). The second is shows the variation in the last digits shown.

Absolute Error

Absolute error (or absolute uncertainty) refers to the uncertainty in the actual measurement, such as (3.64 ± 0.22) cm.

Relative Error

Relative error shows the error or uncertainty as a fraction of the measurement. (Percent error, which you used in chemistry last year, is related to relative error.)

The formula for relative error is $\text{R.E.} = \dfrac{\text{uncertainty}}{\text{measured value}}$

For example, a measurement of (3.64 ± 0.22) cm would have a relative error of 0.22 out of 3.64. Mathematically, we express this as:

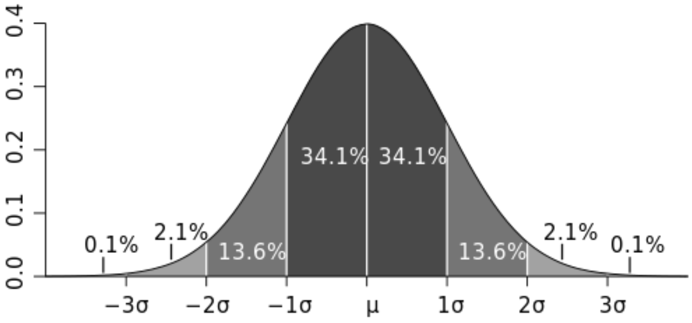
$$\text{R.E.} = \frac{0.22}{3.64} = 0.0604$$

To turn relative error into percent error, multiply by 100. A relative error of 0.0604 is the same as 6.04% error.

Use this space for summary and/or additional notes:

<u>standard uncertainty</u> (*u*): the standard deviation of the mean of a set of measurements.

Statistically, if the measurements follow a Gaussian (normal, "bell-shaped") distribution, plus or minus one standard deviation (±*u*) should include 68.2% of the measurements, plus or minus two standard deviations (±2*u*) should include 95.4% of the measurements, and plus or minus three standard deviations (±3*u*) should include 99.6% of the measurements.



Most scientists use the standard deviation to represent uncertainty, which means roughly 68.2% of the individual measurements fall within the uncertainty.

E.g., if we report a measurement of (22.336 ± 0.030) cm, that means:

- The mean value is $\bar{x} = 22.336\text{ cm}$
- It is 68% likely that any single measurement falls between between (22.336 − 0.030) cm and (22.336 + 0.030) cm, *i.e.,* between 22.306 cm and 22.366 cm.

However, because the reported measurement is an average of a set of measurements, if there are a lot of data points, the likelihood that the *average* of the measurements is close to the actual value is much higher than the likelihood that any individual measurement is close.

Use this space for summary and/or additional notes:

Calculating the Uncertainty of a Set of Measurements

When you have several independent measurements of the same data point, the uncertainty is calculated using statistics, so that some specific percentage of the measurements will fall within the average plus or minus the uncertainty.

Ten or More Data Points

If you have a large enough set of data points (at least 10), then the standard uncertainty is the standard deviation of the mean. That formula is:

$$u = \sigma_m = \frac{s}{\sqrt{n}}$$

where:

u = standard uncertainty

σ_m = standard deviation of the mean

s = sample standard deviation $= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

n = number of measurements

If the variable x represents the measured quantity and $\bar{x}$ represents the arithmetic mean (average) value, you would express your result as:

$$\text{reported value} = \bar{x} \pm u = \bar{x} \pm \frac{s}{\sqrt{n}}$$

While this would be the correct formula to use when possible, often we have too few data points (small values of *n*), which causes the formula to predict a much larger uncertainty than we probably actually have.

Use this space for summary and/or additional notes:

Approximation for Fewer than Ten Data Points

If you have only a few data points (fewer than 10), then you have too few data points to accurately calculate the population standard deviation. In this case, we can estimate of the standard uncertainty by using the formula:

$$u \approx \frac{r}{\sqrt{3}}$$

where:

u = uncertainty

r = range (the difference between the largest and smallest measurement)

If the variable x represents the measured quantity, you would express your result as:

$$\text{reported value} = \bar{x} \pm \frac{r}{\sqrt{3}}$$

Note that we are treating $\sqrt{3}$ as a constant. Whenever you have more than one but fewer than ten data points, find the range and divide it by $\sqrt{3}$ to get the estimated uncertainty.

Example:

Suppose you measured a mass on a balance and the reading drifted between 3.46 g and 3.58 g:

$$\bar{x} = \frac{3.46 + 3.58}{2} = 3.52$$

$$r = 3.58 - 3.46 = 0.12$$

$$u \approx \frac{r}{\sqrt{3}} \approx \frac{0.12}{1.732} \approx 0.07$$

You would record the balance reading as (3.52 ± 0.07) g.

Use this space for summary and/or additional notes:

Uncertainty of a Single Measurement

If you have the ability to measure a quantity that is not changing (such as the mass or length of an object), you will get the same value every time you measure it. This means you have only one data point.

When you have only one data point, you can often assume that the standard uncertainty is the limit of how precisely you can measure it (including any estimated digits). This will be your best educated guess, based on how closely you think you actually measured the quantity. This means you need to take measurements as carefully and precisely as possible, because *every careless measurement needlessly increases the uncertainty of the result*.

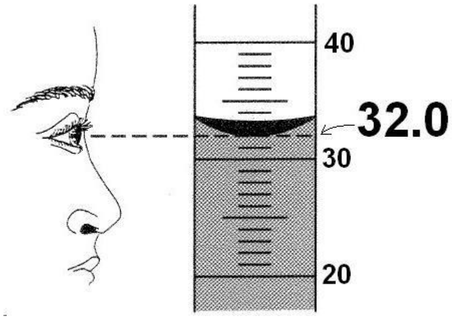
Digital Measurements

For digital equipment, if the reading is *stable* (not changing), look up the published precision of the instrument in its user's manual. (For example, many balances used in high schools have a readability of 0.01 g but are only precise to within ± 0.02 g.) If there is no published value (or the manual is not available), assume the uncertainty is ± 1 in the last digit.

Use this space for summary and/or additional notes:

Analog Measurements

When making analog measurements, always estimate to one extra digit beyond the finest markings on the equipment. For example, in the diagram below, the graduated cylinder is marked in 1 mL increments. When measuring volume in this graduated cylinder, you would estimate and write down the volume to the nearest 0.1 mL, as shown:



In the above experiment, you should record the volume as 32.0 ± 0.1 mL. It would be inadequate to write the volume as 32 mL. (Note that the zero at the end of the reading of 32.0 mL is not extra. It is necessary because *you measured the volume to the nearest 0.1 mL and not to the nearest 1 mL.*)

When estimating, you can generally assume that the estimated digit has an uncertainty of ±1. This means the uncertainty of the measurement is usually $\pm \frac{1}{10}$ of the finest markings on the equipment. Here are some examples:

Equipment	**Markings**	**Estimate To**	**Uncertainty**
triple-beam balance	0.01 g	0.001 g	± 0.001 g
ruler	1 mm	0.1 mm	± 0.1 mm
100 mL graduated cylinder	1 mL	0.1 mL	± 0.1 mL
thermometer	1°C	0.1°C	± 0.1°C

Use this space for summary and/or additional notes:

Propagating Uncertainty in Calculations

When you perform calculations using numbers that have uncertainty, you need to propagate the uncertainty through the calculation.

Crank Three Times

The simplest way of doing this is the "crank three times" method. The "crank three times" method involves:

1. Perform the calculation using the actual numbers. This gives the result (the part before the ± symbol).
2. Perform the calculation again, using the end of the range for each value that would result in the smallest result. (Note that with fractions, this means you need to subtract the uncertainty for values in the numerator and add the uncertainty for values in the denominator.) This gives the lower limit of the range.
3. Perform the calculation again using the end of the range for each value that would result in the largest result. This gives the upper limit of the range.
4. If you have fewer than ten data points, use the approximation that the uncertainty = $u \approx \frac{r}{\sqrt{3}}$, where *r* is the range.

The advantage to "crank three times" is that it's easy to understand and you are therefore less likely to make a mistake. The disadvantage is that it can become unwieldy when you have multi-step calculations.

Use this space for summary and/or additional notes:

Error Propagation

This method, which we will use throughout the year, requires applying a set of rules based on the formulas you use for the calculations. This is probably the method your college professors will expect you to use in lab experiments.

The rules are:

1. If the calculation involves ***addition or subtraction***, add the *absolute* errors. For example, (8.45 ± 0.15) cm + (5.43 ± 0.12) cm means:

$$\begin{array}{r} (8.45 \pm 0.15)\text{ cm} \\ +\ \underline{(5.43 \pm 0.12)\text{ cm}} \\ \boxed{(13.88 \pm 0.27)\text{ cm}} \end{array}$$

   (Uncertainty calculations involving addition or subtraction are intuitively easy.)

2. If the calculation involves ***multiplication or division***, add the *relative* errors, and multiply the total relative error by the result to get the uncertainty. For example, if we have the problem (2.51 ± 0.16) kg × $(0.37 \pm 0.04)\,\frac{\text{m}}{\text{s}^2}$), we would do the following:

   The result is 2.51 kg × $0.37\,\frac{\text{m}}{\text{s}^2} = 0.9287\,\frac{\text{kg·m}}{\text{s}^2}$

   For the error analysis:

   The relative error of (2.51 ± 0.16) kg is $\frac{0.16}{2.51} = 0.0637$

   The relative error of $(0.37 \pm 0.04)\,\frac{\text{m}}{\text{s}^2}$ is $\frac{0.04}{0.37} = 0.10810$

   The total relative error is 0.0637 + 0.1081 = 0.1718.

   The total relative error is what we multiply the result by to get its (absolute) uncertainty:

   $0.9287\,\frac{\text{kg·m}}{\text{s}^2} \times 0.1718 = \pm\ 0.1596\,\frac{\text{kg·m}}{\text{s}^2}$

   The answer with its uncertainty is $(0.9287 \pm 0.1596)\,\frac{\text{kg·m}}{\text{s}^2}$, which we can round to $\boxed{(0.93 \pm 0.16)\,\frac{\text{kg·m}}{\text{s}^2}}$.

Use this space for summary and/or additional notes:

In general, it's good practice to show the uncertainty to one or two significant figures. The result should be rounded to the same place value as the uncertainty.

3. For calculations that involve ***exponents***, use the same rule as for multiplication and division, but multiply the relative error by the exponent. For example, consider the problem:
   ½ × (12.2 ± 0.8) kg × $((5.8 \pm 0.2)\,\frac{\text{m}}{\text{s}})^2$

   The result of the calculation is: $\frac{1}{2}(12.2)(5.8)^2 = 205.2\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$

   The relative errors of the two measurements are:

   $$\frac{0.8}{12.2} = 0.0656 \quad \text{and} \quad \frac{0.2}{5.8} = 0.0345$$

   Because the $5.8\,\frac{\text{m}}{\text{s}}$ is squared in the calculation, we need to multiply its relative error by two (the exponent). This gives a total relative error of:

   0.0656 + (0.0345 × 2) = 0.1345

   Now multiply the total relative error by the result to get the uncertainty:

   $$205.2\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \times 0.1345 = 27.6\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

   We would round this to $\boxed{(205 \pm 28)\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}}$.

Use this space for summary and/or additional notes:

How Significant Figures Work (as an Approximation)

Note that the rules you learned in chemistry for calculations with significant figures come from these rules, with the assumption that the uncertainty is ± 1 in the last digit.

For example, consider the following problem:

problem:		“sig figs” equivalent:
23000	± 1000	23 ???.??? ?
0.0075	± 0.0001	0.007 5
+ 1650	± 10	+ 1 65?.??? ?
24650.0075	± 1010.0001	24 ???.??? ?

(Check this digit for rounding)

If we rounded our answer of 24 650 ± 1 010 to 25 000 ± 1 000, it would agree with the number we got by using the rules for significant figures.

Note, however, that our answer of 24 650 ± 1 010 suggests that the actual value is between 23 640 and 25 660, whereas the answer based on significant figures suggests that the actual value is between 24 000 and 26 000. This is why scientists frown upon use of “sig figs” instead of genuine error analysis. In the laboratory, if you insist on using sig figs, it is better to report answers to *one more sig fig* than the problem calls for, in order to minimize round-off answers in your results.

Use this space for summary and/or additional notes:

As another example, consider the problem:

$$34.52 \times 1.4$$

Assume this means $(34.52 \pm 0.01) \times (1.4 \pm 0.1)$.

The answer (without its uncertainty) is $34.52 \times 1.4 = 48.328$

Because the calculation involves multiplication, we need to add the relative errors, and then multiply the total relative error by the answer to get the uncertainty.

The relative uncertainties are:

$$\frac{0.01}{34.52} = 0.00029$$

$$\frac{0.1}{1.4} = 0.0714$$

Adding them gives: 0.00029 + 0.0714 = 0.0717

Multiplying by our answer gives: 0.0717 × 48.328 = 3.466

The uncertainty is: ±3.466.

Our answer including the uncertainty is therefore 48.328 ± 3.466, which we can round to 48.3 ± 3.5.

Doing the calculation with sig figs would give us an answer of 48, which implies 48 ± 1. Again, this is in the ballpark of the actual uncertainty, but you should notice (and be concerned) that the sig figs method underrepresents the actual uncertainty by a factor of 3.5!

Despite the inherent problems with significant figures, *on the AP Exam, you will need to report calculations using the same sig fig rules that you learned in chemistry*.

Use this space for summary and/or additional notes:

Rules for Using Significant Figures

The first significant digit is where the "measured" part of the number begins—the first digit that is not zero.

The last significant digit is the last "measured" digit—the last digit whose true value is known or accurately estimated (usually ±1).

- If the number doesn't have a decimal point, the last significant digit will be the last digit that is not zero.
- If the number does have a decimal point, the last significant digit will be the last digit shown.
- If the number is in scientific notation, the above rules tell us (correctly) that all of the digits before the "times" sign are significant.

For any measurement that does not have an explicitly stated uncertainty value, assume the uncertainty is ±1 *in the last significant digit.*

In the following numbers, the significant figures have been underlined:

- $\underline{13}{,}000$
- $0.0\underline{275}$
- $0.0\underline{150}$
- $\underline{6804.30500}$
- $\underline{6.0} \times 10^{23}$
- $\underline{3400.}$ (note the decimal point at the end)

Use this space for summary and/or additional notes:

Math with Significant Figures

Addition & Subtraction:

Line up the numbers in a column. Any column that has an uncertain digit—a zero from rounding—is an uncertain column. (Uncertain digits are shown as question marks in the right column below.) You need to round off your answer to get rid of all of the uncertain columns.

For example:

problem:	meaning:
23000	23???.????
0.0075	0.0075
+ 1650	+ 165?.????
24650.0075	24???.????

Because we can't know which digits go in the hundreds, tens, ones, and decimal places of ***all*** of the addends, the exact values of those digits must therefore be unknown in the sum.

This means we need to round off the answer to the nearest 1,000, which gives a final answer of 25,000 (which actually means 25,000 ± 1,000).

A silly (but correct) example of addition with significant digits is:

$$100 + 37 = 100$$

Use this space for summary and/or additional notes:

Multiplication, Division, Etc.

For multiplication, division, and just about everything else (except for addition and subtraction, which we have already discussed), round your answer off to the same number of significant digits as the number that has the fewest.

For example, consider the problem 34.52 × 1.4 = 48.328

The number 1.4 has the fewest significant digits (2). Remember that 1.4 really means 1.4 ± 0.1. This means the actual value, if we had more precision, could be anything between 1.3 and 1.5. This means the actual answer could be anything between 34.52 × 1.3 = 44.876 and 34.52 × 1.5 = 51.780.

This means the actual answer is 48.328 ± 3.452. This means that the ones digit is approximate, and everything beyond it is unknown. Therefore, it would make the most sense to report the number as 48 ± 3.

In this problem, notice that the least significant term in the problem (1.4) had 2 significant digits, and the answer (48) also has 2 significant digits. This is where the rule comes from.

A silly (but correct) example of multiplication with significant digits is:

$$147 \times 1 = 100$$

Use this space for summary and/or additional notes:

Summary of Uncertainty Calculations

Uncertainty of a Single Quantity

Measured Once

Make your best educated guess of the uncertainty based on how precisely you were able to measure the quantity and the uncertainty of the instrument(s) that you used.

Measured Multiple Times (Independently)

- If you have a lot of data points, the uncertainty is the standard deviation of the mean.
- If you have few data points, use the approximation $u \approx \frac{r}{\sqrt{3}}$.

Uncertainty of a Calculated Value

- For addition & subtraction, add the uncertainties of each of the measurements. The sum is the uncertainty of the result.
- For multiplication, division and exponents:
    1. Find the relative error of each measurement.
       $$\text{R.E.} = \frac{\text{uncertainty}\ (\pm)}{\text{measured value}}$$
    2. Multiply the relative error by the exponent (if any).
    3. Add each of the relative errors to find the total relative error.
    4. The uncertainty (±) is the total R.E. times the result.

Use this space for summary and/or additional notes:

Keeping a Laboratory Notebook

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Understand the purpose of an informal lab write-up, such as you would write in a notebook.
- rules for recording and working with lab data

Skills:

- Write up an experiment in laboratory notebook format.

Language Objectives:

- Understand and be able to describe the sections of a laboratory notebook write-up, and which information goes in each section.

Notes:

A laboratory notebook serves two important purposes:

1. It is a legal record of what you did and when you did it.
2. It is a diary of what you did in case you want to remember the details later.

In a research laboratory, you would normally do a write-up in your lab notebook whenever you do a significant experiment that you believe you might want to refer back to sometime in the future.

Your Notebook as an Official Record

Laboratory notebooks are kept by scientists in research laboratories and high tech companies. If a company or research institution needs to prove that you did a particular experiment on a particular date and got a particular set of results, your lab notebook is the primary evidence. This means you need to maintain your lab notebook in a way that gives it the best chance of being able to prove beyond a reasonable doubt exactly what you did and when you did it.

Use this space for summary and/or additional notes:

Here are some general rules for working with data. (Most of these are courtesy of Dr. John Denker, at http://www.av8n.com/physics/uncertainty.htm):

Recording Data

- Write something about what you did on the same page as the data, even if it is a very rough outline. Your procedure notes should not get in the way of actually performing the experiment, but there should be enough information to corroborate the detailed summary of the procedure that you will write afterwards. (Also, for evidence's sake, the sooner after the experiment that you write the detailed summary, the more weight it will carry in court.)
- Keep *all* of the raw data, whether you will use it or not.
- Don't discard a measurement, even if you think it is wrong. Record it anyway and put a "?" next to it. You can always choose not to use the data point in your calculations (as long as you give an explanation).
- Never erase or delete a measurement. The only time you should ever cross out recorded data is if you accidentally wrote down the wrong number.
- Record all digits. Never round off original data measurements. If the last digit is a zero, you must record it anyway!
- For analog readings (*e.g.,* ruler, graduated cylinder, thermometer), always estimate and record one extra digit.
- Always write down the units with each measurement!
- Record *every* quantity that will be used in a calculation, whether it is changing or not.
- Don't convert in your head before writing down a measurement. Record the original data in the units you actually measured it in, and convert in a separate step.

Use this space for summary and/or additional notes:

Calculations

- Use enough digits to avoid unintended loss of significance. (Don't introduce round-off errors in the middle of a calculation.) This usually means use at least two more digits than the number of "significant figures" you expect your answer to have.
- Use few enough digits to be reasonably convenient.
- Record uncertainty separately from the measurement. (Don't rely on "sig figs" to express uncertainty.)
- Leave digits in the calculator between steps. (Don't round until the end.)
- When in doubt, keep plenty of "guard digits" (digits after the place where you think you will end up rounding).

Integrity of Data

Your data are your data. In classroom settings, people often get the idea that the goal is to report an uncertainty that reflects the difference between the measured value and the "correct" value. That idea certainly doesn't work in real life—if you knew the "correct" value you wouldn't need to make measurements!

In all cases—in the classroom and in real life—you need to determine the uncertainty of your own measurement by scrutinizing your own measurement procedures and your own analysis. Then you judge how well they agree.

For example, we would say that the quantities 10 ± 2 and 11 ± 2 agree reasonably well, because there is considerable overlap between their probability distributions. However, 10 ± 0.2 does not agree with 11 ± 0.2, because there is no overlap.

If your results disagree with well-established results, you should look for and comment on possible problems with your procedure and/or measurements that could have caused the differences you observed. You must *never* fudge your data to improve the agreement.

Use this space for summary and/or additional notes:

Notebook Format

Every lab you work in, whether in high school, college, research, or industry, will have its own preferred laboratory notebook format. It is much more important to understand what *kinds* of information you need to record and what you will use it for than it is to get attached to any one format.

In physics class this year, you will record data on loose paper, but in the same format as you would use for a laboratory notebook. (Otherwise, the teacher would need to take home three bushels of lab notebooks weighing more than 100 pounds to grade for every lab experiment.)

The format we will use is meant to follow an outline of the actual experiment. Don't worry too much about following the format exactly. It is much more important that the information you have is complete and recorded correctly than it is to follow the format.

Title & Date

Each experiment should have the title and date the experiment was performed written at the top. The title should be a descriptive sentence fragment (usually without a verb) that gives some information about the purpose of the experiment.

Objective

This should be a one or two-sentence description of what you are trying to determine or calculate by performing the experiment.

Use this space for summary and/or additional notes:

Background

Your background or experimental plan needs to explain:

- the process of determining what you are measuring (starting from the quantity you are looking for and using equations and other relationships to relate that quantity to quantities that you can measure)
- the specific quantities that you are going to vary (your independent variable)
- the specific quantities that you are going to keep constant (your control variables)
- the specific quantities that you are going to measure or observe (your dependent variables), and how you are going to measure or observe them
- how you are going to calculate or interpret your results.

Notes

Your notes are your original data. In every experiment, you must write down a brief description of the procedure and all of your data (measurements and observations) *during the experiment*, and you must include *the same piece of paper that you wrote on during the experiment* as part of your notebook, no matter how messy it is! (If I catch you turning in a copied over set of notes for this section, you will lose credit.)

If you were keeping an actual notebook, your notebook would be the only place you are allowed to write anything during the experiment. However, because you are not keeping a physics notebook this year, the piece of paper with your original data serves the same purpose.

Procedure

This is a detailed description of exactly what you did in order to take your measurements. You need to include:

- A *labeled* sketch of your experimental set-up, even if the experiment is simple. The sketch will serve to answer many questions about how you set up the experiment and most of the key equipment you used.
- A list of any equipment that you used other than what you labeled in your sketch.

Use this space for summary and/or additional notes:

- A step-by-step description of everything you did. The description needs to include the actual values of quantities you used in the experiment (your control and independent variables). For a repeated procedure, write the steps once, then list the differences from one trial to the next. *E.g.,* "Repeat steps 1–4 using distances of 1.5 m, 2.0 m, 2.5 m, and 3.0 m."

Data & Observations

This is a section in which you present all of your data. This section is a clean copy of what you wrote down in your notes section during the experiment.

For a high school lab, it is usually sufficient to present a single data table that includes your measurements for each trial and the quantities you calculated from them. However, if you have other data or observations that you recorded during the lab, they must be listed here.

Analysis

The analysis section is where you interpret your data. (Any calculated values in the table in the Data & Observations section are actually analysis, even though it is permissible to include them above.)

Your analysis needs to include:

- Any calculated values that did not appear in the data table in your Data & Observations section
- A carefully-plotted graph showing the data points you took for your dependent vs. independent variables. Often, the quantity you are calculating will be the slope. You need the graph to show the region in which the slope is linear, which tells the range over which your experiment is valid. Note that any graphs you include in your write-up must be drawn accurately to scale, using graph paper, and using a ruler whenever a straight line is needed. (When a graph is required, you will lose points if you include a freehand sketch instead.)
- One (and only one) sample calculation for each separate formula that you used. For example, if you calculated acceleration for each of five data points, you would write down the formula, and then choose one set of data to plug in and show how you got the answer.

Use this space for summary and/or additional notes:

- Quantitative error analysis. You need to:
    1. Measure or estimate the uncertainty of each your measurements.
    2. Calculate the relative error for each measurement.
    3. Combine your relative errors to get the total relative error for your calculated value(s).
    4. Multiply the total relative error by your calculated values to get the uncertainty (±) for each one.
- Sources of uncertainty: this is a list of factors *inherent in your procedure* that limit how precise your answer can be. ***Never include hypothetical human error!*** A statement like “We might have written down the wrong number.” or “We might have done the calculations incorrectly.” is really saying, “We might be stupid and you shouldn’t believe anything we say in this report.” (You will lose points if you include statements like this.)

  Note, however, that if a problem *actually occurred*, and if you *used that data point in your calculations anyway*, you need to explain what happened and calculate an estimate of the effects on your results.

Conclusions

Your conclusion should be worded the same way as your objective, this time including your calculated results with their uncertainties. You do not need to restate sources of uncertainty in your conclusions unless you believe they were significant enough to create some doubt about your results.

Use this space for summary and/or additional notes:

Formal Laboratory Reports

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Understand the purpose of a formal lab write-up, such as you would submit to a scientific journal

Skills:

- Write up an experiment formally in journal article format.

Language Objectives:

- Understand and correctly use the term "abstract," as it pertains to a formal laboratory report.
- Understand and be able to describe the sections of a formal laboratory report, and which information goes in each section.

Notes:

A formal laboratory report serves one important purpose: to communicate the results of your experiment to other scientists outside of your laboratory or institution.

A formal report is a significant undertaking. In a research laboratory, you might submit as many as one or two articles to a scientific journal in a year. Some college professors require students to submit lab reports in journal article format.

The format of a formal journal article-style report is as follows:

Use this space for summary and/or additional notes:

Abstract

This is the most important part of your report. It is a (maximum) 200-word executive summary of everything about your experiment—the procedure, results, analysis, and conclusions. In most scientific journals, the abstracts are searchable via the internet, so it needs to contain enough information to enable someone to find your abstract, and after reading it, to know enough about your experiment to determine whether or not to purchase a copy of the full article (which can sometimes cost $100 or more). It also needs to be short enough that the person doing the search won't just say "TL; DR" ("Too Long; Didn't Read") and move on to the next abstract.

Because the abstract is a complete summary, it is always best to wait to write it until you have already written the rest of your report.

Introduction

Your introduction is actually an entire research paper on its own, with citations. (For a high school lab report, it should be 1–3 pages; for scientific journals, 5–10 pages is not unheard of.) Your introduction needs to describe any general background information that another scientist might not know, plus all of the background information that specifically led up to your experiment. The introduction is usually the most time-consuming part of the report to write.

Materials and Methods

This section is describes both the background and experimental procedure sections of a lab notebook write-up. Unlike a lab notebook write-up, the Materials and Methods section of a formal report is written in paragraph form, in the past tense. Again, a labeled drawing of your apparatus is a necessary part of the section, but you need to *also* describe the set-up in the text.

Also unlike the lab notebook write-up, your Materials and Methods section needs to give some explanation of your choices of the values of your independent and control variables.

Use this space for summary and/or additional notes:

Data and Observations

This section is similar to the same section in the lab notebook write-up, except that:

1. You should present only data you actually recorded/measured in this section. (Calculated values are presented in the Discussion section.)
2. You need to *introduce* the data table. (This means you need to describe the important things someone should notice in the table first, and then say something like "Data are shown in Table 1.")

Note that all figures and tables in the report need to be numbered consecutively.

Discussion

This section is similar to the Analysis section in the lab notebook write-up, but with some important differences.

Your discussion is essentially a long essay discussing your results and what they mean. You need to introduce and present a table with your calculated values and your uncertainty. After presenting the table, you should discuss the results, uncertainties, and sources of uncertainty in detail. If your results relate to other experiments, you need to discuss the relationship and include citations for those other experiments.

Your discussion needs to include all of the formulas that you used as part of your discussion, but you do not need to show your work for each calculation.

Conclusions

Your conclusions should start by presenting the same information that you included in the same section in your lab notebook write-up. However, in the formal report, you should mention significant sources of uncertainty, and suggest how future experiments might follow up on or expand on your experiment.

Works Cited

As with a research paper, you need to include a complete list of bibliography entries for the references you cited in your introduction and/or discussion sections.

Use this space for summary and/or additional notes:

Introduction: Mathematics

Unit: Mathematics

Topics covered in this chapter:

The purpose of this chapter is to familiarize you with mathematical concepts and skills that will be needed in physics.

- *Standard Assumptions in Physics* discusses what you can and cannot assume to be true in order to be able to solve the problems you will encounter in this class.
- *Assigning & Substituting Variables* discusses how to determine which quantity and which variable apply to a number given in a problem based on the units, and how to choose which formula applies to a problem.
- *The Metric System* and *Scientific Notation* briefly review skills that you are expected to remember from your middle school math and science classes.
- *Trigonometry*, *Vectors*, *Vectors* vs. *Scalars in Physics*, and *Vector Multiplication* discuss important mathematical concepts that are widely used in physics, but may be unfamiliar to you.

Use this space for summary and/or additional notes:

Depending on your math background, some of the topics, such as trigonometry and vectors, may be unfamiliar. These topics will be taught, but in a cursory manner.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Massachusetts Curriculum Frameworks (2006):

No MA curriculum frameworks are specifically addressed in this chapter. However, this chapter addresses the following mathematical understandings explicitly listed in the MA Curriculum Frameworks as prerequisites for this course:

- Construct and use tables and graphs to interpret data sets.
- Solve simple algebraic expressions.
- Perform basic statistical procedures to analyze the center and spread of data.
- Measure with accuracy and precision (*e.g.,* length, volume, mass, temperature, time)
- Convert within a unit (*e.g.,* centimeters to meters).
- Use common prefixes such as milli-, centi-, and kilo-.
- Use scientific notation, where appropriate.
- Use ratio and proportion to solve problems.

In addition, this chapter addresses the following mathematical understandings. The MA frameworks state that “the following skills are not detailed in the Mathematics Framework, but are necessary for a solid understanding in this course.”

- Determine the correct number of significant figures.
- Determine percent error from experimental and accepted values.
- Use appropriate metric/standard international (SI) units of measurement for mass (kg); length (m); time (s); force (N); speed (m/s); acceleration (m/s^2); frequency (Hz); work and energy (J); power (W); momentum (kg·m/s); electric current (A); electric potential difference/voltage (V); and electric resistance (Ω).

Use this space for summary and/or additional notes:

- Use the Celsius and Kelvin scales.

Skills learned & applied in this chapter:

- Estimating uncertainty in measurements
- Propagating uncertainty through calculations
- Identifying quantities in word problems and assigning them to variables
- Choosing a formula based on the quantities represented in a problem
- Using trigonometry to calculate the lengths of sides and angles of triangles
- Representing quantities as vectors
- Adding and subtracting vectors
- Multiplying vectors using the dot product and cross product

Use this space for summary and/or additional notes:

Standard Assumptions in Physics

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- what are the usual assumptions in physics problems

Language Objectives:

- understand and correctly use the term "assumption," as it pertains to setting up and solving physics problems.

Notes:

Many of us have been told not to make assumptions. There is a popular expression that states that "when you assume, you make an ass of you and me":

ass|u|me

In science, particularly in physics, this adage is crippling. Assumptions are part of everyday life. When you cross the street, you assume that the speed of cars far away is slow enough for you to walk across without getting hit. When you eat your lunch, you assume that the food won't cause an allergic reaction. When you run down the hall and slide across the floor, you assume that the friction between your shoes and the floor will be enough to stop you before you crash into your friend.

<u>assumption</u>: something that is unstated but considered to be fact for the purpose of making a decision or solving a problem. Because it is impossible to measure and/or calculate everything that is going on in a typical physics or engineering problem, it is almost always necessary to make assumptions.

Use this space for summary and/or additional notes:

Standard Assumptions in Physics

In a first-year physics course, in order to make problems and equations easier to understand and solve, we will often assume that certain quantities have a minimal effect on the problem, even in cases where this would not actually be true. The term used for these kinds of assumptions is "ideal". Some of the ideal physics assumptions we will use include the following. Over the course of the year, you can make each of these assumptions unless you are explicitly told otherwise.

- Constants (such as acceleration due to gravity) have the same value in all parts of the problem.
- Variables change in the manner described by the relevant equation(s).
- Ideal machines and other objects that are not directly considered in the problem have negligible mass, inertia, and friction. (Note that these idealizations may change from problem-to-problem. A pulley may have negligible mass in one problem, but another pulley in another problem may have significant mass that needs to be considered as part of the problem.)
- If the problem does not mention air resistance and air resistance is not a central part of the problem, friction due to air resistance is negligible.
- The mass of an object can often be assumed to exist at a single point in 3-dimensional space. (This assumption does not hold for problems where you need to calculate the center of mass, or torque problems where the way the mass is spread out is part of the problem.)
- Sliding (kinetic) friction between surfaces is negligible. (This will not be the case in problems involving friction, though even in friction problems, ice is usually assumed to be frictionless unless you are explicitly told otherwise.)
- Force can be applied in any direction using an ideal rope. (You can even push on it!)
- Collisions between objects are perfectly elastic or perfectly inelastic unless the problem states otherwise.
- No energy is lost when energy is converted from one form to another. (This is always true, but in an ideal collision, energy lost to heat is usually assumed to be negligible.)
- The amount that solids and liquids expand or contract due to temperature differences is negligible. (This will not be the case in problems involving thermal expansion.)

Use this space for summary and/or additional notes:

- The degree to which solids and liquids can be compressed or expanded due to changes in pressure is negligible.
- Gas molecules do not interact when they collide or are forced together from pressure. (Real gases can form liquids and solids or participate in chemical reactions.)
- Electrical wires have negligible resistance.
- All physics students do all of their homework. ☺ (Of course, real physics students do not always do their homework, which can lead to much more interesting (in a bad way) results on physics tests.)

In some topics, a particular assumption may apply to some problems but not others. In these cases, the problem needs to make it clear whether or not you can make the relevant assumption. (For example, in the "forces" topic, some problems involve friction and others do not. A problem that does not involve friction might state that "a block slides across a frictionless surface.")

If you are not sure whether you can make a particular assumption, you should ask the teacher. If this is not practical (such as an open response problem on a standardized test), you should decide for yourself whether or not to make the assumption, and ***explicitly state what you are assuming as part of your answer***.

Use this space for summary and/or additional notes:

Assigning & Substituting Variables

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- be able to declare (assign) variables from a word problem
- be able to substitute values for variables in an equation

Language Objectives:

- understand and correctly use the terms “variable” and “subscript.”
- accurately describe and apply the concepts described in this section, using appropriate academic language

Notes:

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word	meaning	word	meaning	word	meaning
and, more than (but not “is more than”)	+	percent (“per” + “cent”)	÷ 100	is at least	≥
less than (but not “is less than”)	−	change in *x*, difference in *x*	Δx	is more than	>
of	×	is	=	is at most	≤
per, out of	÷			is less than	<

Use this space for summary and/or additional notes:

Assigning & Substituting Variables

Identifying Variables

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	S.I. unit	variable
mass	kg	m
distance, length	m	d , ℓ
area	m^2	A
acceleration	m/s^2	a
volume	m^3	V
velocity (speed)	m/s	v
pressure	Pa	P^*
momentum	N·s	p^*
density	kg/m^3	ρ^*
moles	mol	n
time	s	t
temperature	K	T
heat	J	Q
electric charge	C	q

*Note the subtle differences between uppercase "P", lowercase "p", and the Greek letter ρ ("rho").

Any time you see a number in a word problem that has a unit you recognize (such as one listed in this table), notice which quantity the unit is measuring and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

Use this space for summary and/or additional notes:

Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$\rho = \frac{m}{V}$ and you're given: $m = 12.3\text{ g}$ and $V = 2.8\text{ cm}^3$

simply substitute 12.3 g for m, and 2.8 cm^3 for V, giving:

$$\rho = \frac{12.3\text{ g}}{2.8\text{ cm}^3} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often easier to leave the units out when you substitute numbers for variables and then add them back in at the end:

$$\rho = \frac{12.3}{2.8} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

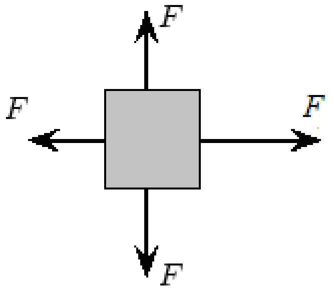
Use this space for summary and/or additional notes:

Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

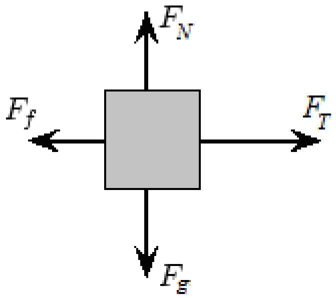
1. The force of gravity, pulling downward.
2. The "normal" force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

The variable for force is "F", so the diagram would look like this:



In order to distinguish between the forces and make the diagram easier to understand, we add subscripts to the variables:

1. F_g is the force of gravity.
2. F_N is the normal force.
3. F_T is the tension in the rope.
4. F_f is friction.



Use this space for summary and/or additional notes:

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—it needs to be smaller than the other letters and lowered slightly. For example, when we write F_g, the variable is F (force) and the subscript $_g$ attached to it tells which kind of force it is (gravity). This might occur in the following equation:

$$F_g = mg$$ ← right ☺

It is important that the subscript $_g$ on the left does not get confused with the variable g on the right. Otherwise, the following error might occur:

$$Fg = mg$$

$$F\cancel{g} = m\cancel{g}$$ ← wrong! ☹

$$F = m$$

Another common use of subscripts is the subscript "0" to mean "initial". For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity "v_o" where the subscript "0" means "at time zero", *i.e.,* at the beginning of the problem, when the "time" on the "problem clock" would be zero. The final velocity is simply "v" without the zero.

Use this space for summary and/or additional notes:

The Problem-Solving Process

1. Identify the quantities in the problem, based on the units and any other information in the problem.
2. Assign the appropriate variables to those quantities.
3. Find an equation that relates all of the variables.
4. Substitute the values of the variables into the equation.
    a. If you have only one variable left, it should be the one you're looking for.
    b. If you have more than one variable left, find another equation that uses one of the variables you have left, plus other quantities that you know.
5. Solve the equation(s), using basic algebra.
6. Apply the appropriate unit(s) to the result.

Use this space for summary and/or additional notes:

Sample Problem

A force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object?

We have units of N and kg, and we're looking for acceleration. We need to look these up in our reference tables.

From Table D ("Quantities, Variables and Units") on page 3 of our reference tables, we find:

Symbol	**Unit**	**Quantity**	**Variable**
N	newton	force	$\vec{\boldsymbol{F}}$
kg	kilogram	mass	m
		acceleration	$\vec{\boldsymbol{a}}$

Now we know that we need an equation that relates the variables $\vec{\boldsymbol{F}}$, m, and a. ($\vec{\boldsymbol{F}}$ and $\vec{\boldsymbol{a}}$ *are* in boldface with an arrow above them because they are vectors. We'll discuss vectors a little later in the course.)

Now that we have the variables, we find a formula that relates them. From the second formula box in Table E ("Mechanics Formulas and Equations") on page 4 of our reference tables, we find that:

$$\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

So we substitute:

$$30 = 1.5\,a$$
$$20 = a$$

Again from Table D, we find that acceleration has units of meters per second squared, so our final answer is $\boxed{20\,\frac{\text{m}}{\text{s}^2}}$.

Use this space for summary and/or additional notes:

The Metric System

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Understand how units behave and combine algebraically.
- Know the 4 common prefixes and their numeric meanings.

Skills:

- Be able to describe quantities using metric units with & without prefixes.

Language Objectives:

- Understand and correctly use the terms “unit” and “prefix.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

Notes:

A unit is a specifically defined measurement. Units describe both the type of measurement, and a base amount.

For example, 1 cm and 1 inch are both lengths. They are used to measure the same dimension, but the specific amounts are different. (In fact, 1 inch is exactly 2.54 cm.)

Every measurement is a number multiplied by its units. In algebra, the term “3*x*” means “3 times *x*”. Similarly, the distance “75 m” means “75 times the distance 1 meter”.

The number and the units are <u>*both*</u> *necessary to describe any measurement.* You *always* need to write the units. Saying that “12 is the same as 12 g” would be as ridiculous as saying “12 is the same as 12 × 3”.

The metric system is a set of units of measurement that is based on natural quantities (on Earth) and powers of 10.

Use this space for summary and/or additional notes:

The Metric System

The metric system has 7 fundamental "base" units:

- meter (m): length
- kilogram (kg): mass (even though "kilo" is actually a prefix, mass is defined based on the kilogram, not the gram)
- second (s): time
- Kelvin (K): temperature
- mole (mol): amount of substance
- ampere (A): electric current
- candela (cd): intensity of light

Each of these base units is defined in some way that could be duplicated in a laboratory anywhere on Earth (except for the kilogram, which is defined by a physical object that is locked in a vault in the village of Sevres, France). All other metric units are combinations of one or more of these seven.

For example:

Velocity (speed) is a change in distance over a period of time, which would have units of distance/time (m/s).

Force is a mass subjected to an acceleration. Acceleration has units of distance/time^2 (m/s^2), and force has units of mass × acceleration. In the metric system this combination of units (kg·m/s^2) is called a newton (symbol "N"), which means: $1\text{ N} \equiv 1\text{ kg}\cdot\text{m/s}^2$

Use this space for summary and/or additional notes:

The Metric System

The metric system uses prefixes to indicate multiplying a unit by a power of ten. There are prefixes for powers of ten from 10^{-18} to 10^{18}. The most commonly used prefixes are:

- mega (M) = $10^{6} = 1\,000\,000$
- kilo (k) = $10^{3} = 1\,000$
- centi (c) = $10^{-2} = \frac{1}{100} = 0.01$
- milli (m) = $10^{-3} = \frac{1}{1{,}000} = 0.001$
- micro (μ) = $10^{-6} = \frac{1}{1\,000\,000} = 0.000\,001$

These prefixes can be used in combination with any metric unit, and they work just like units. “35 cm” means “35 times c times m” or “$(35)(\frac{1}{100})(\text{m})$”. If you multiply this out, you get 0.35 m.

Any metric prefix is allowed with any metric unit.

For example, standard atmospheric pressure is 101 325 Pa. This same number could be written as 101.325 kPa or 0.101 325 MPa.

There is a popular geek joke based on the ancient Greek heroine Helen of Troy. She was said to have been the most beautiful woman in the world, and she was an inspiration to the entire Trojan fleet. She was described as having “the face that launched a thousand ships.” Therefore a milliHelen must be the amount of beauty required to launch one ship.

Use this space for summary and/or additional notes:

Conversions

If you need to convert from one prefix to another, the rule of thumb is that if the prefix gets larger, then the number needs to get smaller and vice-versa.

For example, suppose we need to convert 0.25 mg to µg.

The prefix "m" means 10^{-3} and "µ" means 10^{-6}. The prefix is getting smaller by 3 decimal places, so the number needs to get bigger by 3 decimal places. The answer is therefore 250 µg.

You can think of the rule in the following way:

$$0.25\text{ mg} = (0.25)\,(0.001)\text{ g} = 0.000\,25\text{ g}$$
$$250\ \mu\text{g} = (250)\,(0.000\,001)\text{ g} = 0.000\,25\text{ g}$$

As you can see, when the prefix got smaller, the number had to get bigger in order for the value to remain equal to 0.000 25 g.

Use this space for summary and/or additional notes:

The MKS vs. cgs Systems

Because physics heavily involves units that are derived from other units, it is important that you make sure all quantities are expressed in the appropriate units before applying formulas. (This is how we get around having to do factor-label unit-cancelling conversions—like you learned in chemistry—for every single physics problem.)

There are two measurement systems used in physics. In the MKS, or "meter-kilogram-second" system, units are derived from the S.I. units of meters, kilograms, seconds, moles, Kelvins, amperes, and candelas. In the cgs, or "centimeter-gram-second" system, units are derived from the units of centimeters, grams, seconds, moles, Kelvins, amperes, and candelas. The following table shows some examples:

Quantity	**MKS Unit**	**S.I. Equivalent**	**cgs Unit**	**S.I. Equivalent**
force	newton (N)	$\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	dyne (dyn)	$\frac{\text{g}\cdot\text{cm}}{\text{s}^2}$
energy	joule (J)	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$	erg	$\frac{\text{g}\cdot\text{cm}^2}{\text{s}^2}$
magnetic flux density	tesla (T)	$\frac{\text{N}}{\text{A}}, \frac{\text{kg}\cdot\text{m}}{\text{A}\cdot\text{s}^2}$	gauss (G)	$\frac{0.1\,\text{dyn}}{\text{A}}, \frac{0.1\,\text{g}\cdot\text{cm}}{\text{A}\cdot\text{s}^2}$

In this class, we will use exclusively MKS units. This means you only have to learn one set of derived units. However, you can see the importance, when you solve physics problems, of making sure all of the quantities are in MKS units before you plug them into a formula!

Use this space for summary and/or additional notes:

Scientific Notation

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- Be able to convert numbers to and from scientific notation.
- Be able to enter numbers in scientific notation correctly on your calculator.

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

Scientific notation is a way of writing a very large or very small number in compact form. The value is always written as a number between 1 and 10 multiplied by a power of ten.

For example, the number 1 000 would be written as 1×10^{3}. The number 0.000 075 would be written as 7.5×10^{-5}. The number 602 000 000 000 000 000 000 000 would be written as 6.02×10^{23}. The number 0.000 000 000 000 000 000 000 000 000 000 000 663 would be written as 6.6×10^{-34}.

(Note: in science, large numbers are typeset with a space after every three digits, both before and after the decimal point. To avoid confusion, commas are not used because in some countries, the comma is used as a decimal point.)

Scientific notation is really just math with exponents, as shown by the following examples:

$$5.6 \times 10^{3} = 5.6 \times 1000 = 5600$$

$$2.17 \times 10^{-2} = 2.17 \times \frac{1}{10^{2}} = 2.17 \times \frac{1}{100} = \frac{2.17}{100} = 0.0217$$

Use this space for summary and/or additional notes:

Notice that if 10 is raised to a positive exponent means you're multiplying by a power of 10. This makes the number larger, and the decimal point moves to the right. If 10 is raised to a negative exponent, you're actually dividing by a power of 10. This makes the number smaller, and the decimal point moves to the left.

Significant figures are easy to use with scientific notation: all of the digits before the "×" sign are significant. The power of ten after the "×" sign represents the (insignificant) zeroes, which would be the rounded-off portion of the number. In fact, the mathematical term for the part of the number before the "×" sign is the *significand*.

Math with Scientific Notation

Because scientific notation is just a way of rewriting a number as a mathematical expression, all of the rules about how exponents work apply to scientific notation.

Adding & Subtracting: adjust one or both numbers so that the power of ten is the same, then add or subtract the significands.

$$(3.50\times 10^{-6})+(2.7\times 10^{-7})=(3.50\times 10^{-6})+(0.27\times 10^{-6})$$
$$=(3.50+0.27)\times 10^{-6}=3.77\times 10^{-6}$$

Multiplying & dividing: multiply or divide the significands. If multiplying, add the exponents. If dividing, subtract the exponents.

$$\frac{6.2\times 10^{8}}{3.1\times 10^{10}}=\frac{6.2}{3.1}\times 10^{8-10}=2.0\times 10^{-2}$$

Exponents: raise the significand to the exponent. Multiply the exponent of the power of ten by the exponent to which the number is raised.

$$(3.00\times 10^{8})^{2}=(3.00)^{2}\times (10^{8})^{2}=9.00\times 10^{(8\times 2)}=9.00\times 10^{16}$$

Use this space for summary and/or additional notes:

Using Scientific Notation on Your Calculator

Scientific calculators are designed to work with numbers in scientific notation. It's possible to can enter the number as a math problem (always use parentheses if you do this!) but math operations can introduce mistakes that are hard to catch.

Scientific calculators all have some kind of scientific notation button. The purpose of this button is to enter numbers directly into scientific notation and make sure the calculator stores them as a single number instead of a math equation. (This prevents you from making PEMDAS errors when working with numbers in scientific notation on your calculator.) On most Texas Instruments calculators, such as the TI-30 or TI-83, you would do the following:

What you type	What the calculator shows	What you would write
6.6 $\boxed{\text{EE}}$ −34	6.6E−34	6.6×10^{-34}
1.52 $\boxed{\text{EE}}$ 12	1.52E12	1.52×10^{12}
−4.81 $\boxed{\text{EE}}$ −7	−4.81E−7	-4.81×10^{-7}

On some calculators, the scientific notation button is labeled $\boxed{\text{EXP}}$ or $\boxed{\times 10^{x}}$ instead of $\boxed{\text{EE}}$.

***Important note**: many high school students are afraid of the $\boxed{\text{EE}}$ button because it is unfamiliar. If you are afraid of your $\boxed{\text{EE}}$ button, you need to get over it and start using it anyway. However, if you insist on clinging to your phobia, you need to at least use parentheses around all numbers in scientific notation, in order to minimize the likelihood of PEMDAS errors in your calculations.*

Use this space for summary and/or additional notes:

Trigonometry

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- trigonometry functions that are used heavily in physics

Skills:

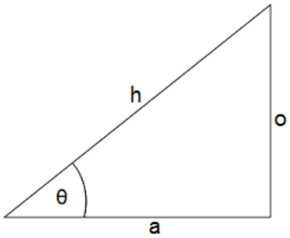
- find the x-component and y-component of a vector

Language Objectives:

- Understand and correctly use the terms "sine," "cosine," and "tangent."
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

Notes:

If we have the following triangle:



- side "h" (the longest side, opposite the right angle) is the *<u>hypotenuse</u>*.
- side "o" is the side of the triangle that is *<u>opposite</u>* (across from) angle θ.
- side "a" is the side of the triangle that is *<u>adjacent</u>* to (connected to) angle θ (and is not the hypotenuse).

Use this space for summary and/or additional notes:

Trigonometry

In a right triangle, the ratios of the lengths of the sides will be a function of the angles, and vice-versa.

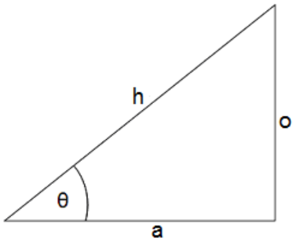
Trigonometry (from "trig" = "triangle" and "ometry" = "measurement") is the study of these relationships.

The three primary trigonometry functions are defined as follows:

$$\text{sine}\,\theta = \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{h}$$

$$\text{cosine}\,\theta = \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{h}$$

$$\text{tangent}\,\theta = \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{o}{a}$$

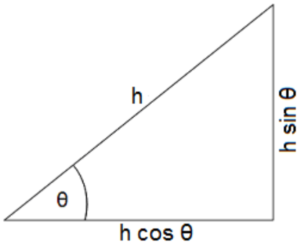


There are a lot of stupid mnemonics for remembering which sides are involved in which functions. My favorite of these is "**O**h **h**ell, **a**nother **h**our **o**f **a**lgebra!"

The most common use of trigonometry functions in physics is to decompose a vector into its components in the x- and y-directions. In this situation, the vector is the hypotenuse. If we know the angle of the vector, we can use trigonometry and algebra to find the components of the vector in the x- and y-directions:

$$\cos\theta = \frac{a}{h} \quad \text{which means } a = h\cos\theta$$

$$\sin\theta = \frac{o}{h} \quad \text{which means } o = h\sin\theta$$



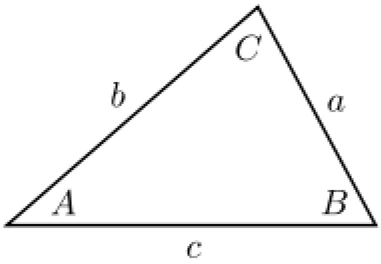
This is often necessary in physics problems involving gravity. Because gravity acts only in the y-direction, the formulas that apply in the y-direction are often different from the ones that apply in the x-direction.

Use this space for summary and/or additional notes:

The Laws of Sines and Cosines

The Law of Sines and the Law of Cosines are often needed to calculate distances or angles in physics problems.

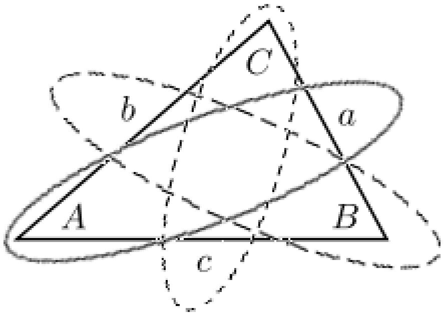
Consider the following triangle *ABC*, with sides *a*, *b*, and *c*, and angles *A*, *B*, and *C*. Angle *A* has its vertex at point *A*, and side *a* is opposite vertex *A* (and hence is also opposite angle *A*).



The Law of Sines

The law of sines states that, for any triangle:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$



You can use the law of sines if you know one angle and the length of the opposite side. If you know those two things plus any other side or any other angle, you can work your way around the triangle and calculate every side and every angle.

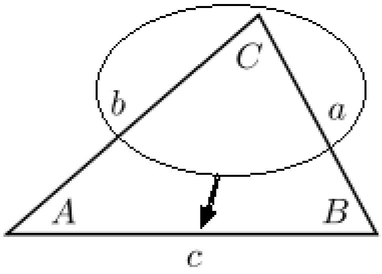
Use this space for summary and/or additional notes:

The Law of Cosines

The law of cosines states that, for any triangle:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

You can use the law of cosines to find any angle or the length of the third side of a triangle as long as you know any two sides and the included angle:



You can also use the law of cosines to find one of the angles if you know the lengths of all three sides.

Remember that which sides and angles you choose to be *a*, *b* and *c*, and *A*, *B* and *C* are arbitrary. This means you can switch the labels around to fit your situation, as long as angle *C* is opposite side *c* and so on.

Note that the Pythagorean Theorem is simply the law of cosines in the special case where *C* = 90° (because cos 90° = 0).

The law of cosines is algebraically less convenient than the law of sines, so a good strategy would be to use the law of sines whenever possible, reserving the law of cosines for situations when it is not possible to use the law of sines.

Use this space for summary and/or additional notes:

Vectors

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- what a vector is

Skills:

- adding & subtracting vectors

Language Objectives:

- Understand and correctly use the terms "vector," "scalar," and "magnitude."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

vector: a quantity that has both a magnitude (value) and a direction.

scalar: a quantity that has a value but does not have a direction. (A scalar is what you think of as a "regular" number, including its unit.)

magnitude: the scalar part of a vector (*i.e.,* the number and its units, but without the direction). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if $\vec{\boldsymbol{F}}$ is 25 N to the east, then $\|\vec{\boldsymbol{F}}\| = 25\,\text{N}$

resultant: a vector that results from a mathematical operation (such as the addition of two vectors).

Use this space for summary and/or additional notes:

unit vector: a vector that has a magnitude of 1.

Unit vectors are typeset as vectors, but with a "hat" instead of an arrow.

The purpose of a unit vector is to turn a scalar into a vector without changing its magnitude (value). For example, if d represents the scalar quantity 25 cm, and $\hat{\boldsymbol{n}}$[*] represents a unit vector pointing southward, then $d\hat{\boldsymbol{n}}$ would represent a vector of 25 cm to the south.

The letters $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$, and $\hat{\boldsymbol{k}}$ are often used to represent unit vectors along the x, y, and z axes, respectively.

Variables that represent vectors are traditionally typeset in ***bold Italics***. Vector variables may also optionally have an arrow above the letter:

$$\boldsymbol{J}, \vec{\boldsymbol{F}}, \boldsymbol{v}$$

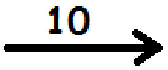
Variables that represent scalars are traditionally typeset in *plain Italics:*

$$V,\ t,\ \lambda$$

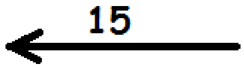
Note that a variable that represents only the magnitude of a vector quantity is generally typeset as a scalar:

For example, $\vec{\boldsymbol{F}}$ is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount ***and*** the direction.) The magnitude would be 25 N, and would be represented by the variable F.

Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:



magnitude 10
direction: 0°
(to the right)



magnitude 15
direction: +180°
(to the left)

magnitude 7
direction: +90° (up)

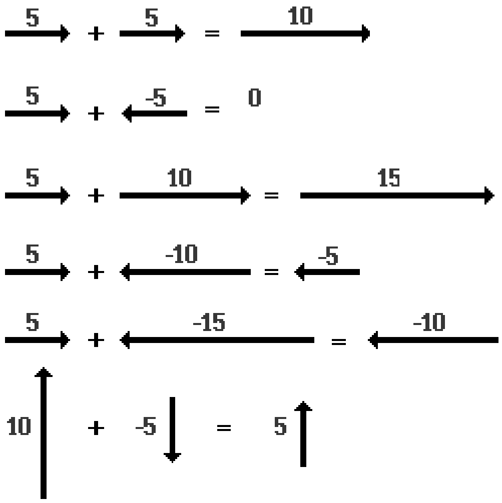
---

[*] $\hat{n}$ is pronounced "n hat"

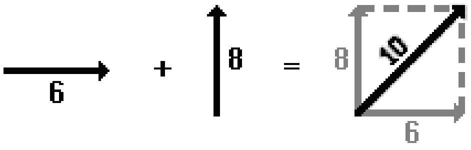
Use this space for summary and/or additional notes:

Adding & Subtracting Vectors

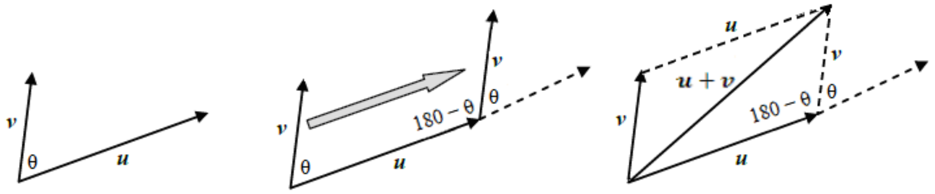
If the vectors have the same direction or opposite directions, the resultant is easy to envision:



If the vectors are not in the same direction, we move them so they start from the same place and complete the parallelogram. If they are perpendicular, we can add them using the Pythagorean theorem:



The same process applies to adding vectors that are not perpendicular:

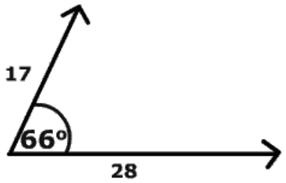


(1) (2) (3)

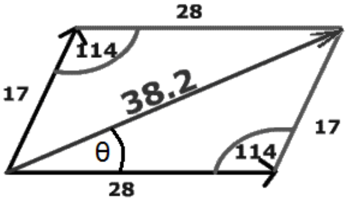
However, the trigonometry needed for the calculations is more involved.

Use this space for summary and/or additional notes:

If the vectors have different (and not opposite) directions, the resultant vector is given by the diagonal of the parallelogram created from the two vectors. For example, the following diagram shows addition of a vector with a magnitude of 28 and a direction of 0° added to a vector with a magnitude of 17 and a direction of 66°:



The resultant vector is given by the parallelogram created by the two vectors:



The magnitude can be calculated using the law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$c^2 = 28^2 + 17^2 - 2(28)(17)\cos(114°)$$

$$c^2 = 1460$$

$$c = \sqrt{1460} = 38.2$$

For the direction, use the law of sines:

$$\frac{38.2}{\sin 114°} = \frac{17}{\sin\theta}$$

$$\sin\theta = \frac{17\sin 114°}{38.2} = \frac{(17)(0.914)}{38.2} = 0.407$$

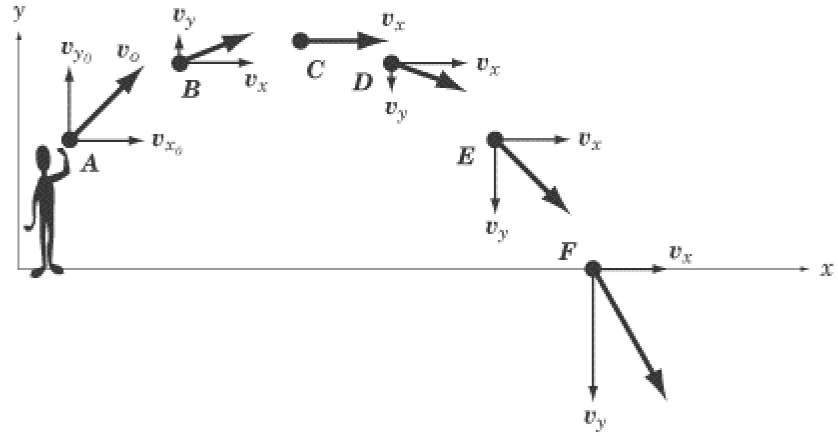
$$\theta = \sin^{-1} 0.407 = 24.0°$$

Thus the resultant vector has a magnitude of 38.2 and a direction of +24.0° (or 24.0° above the horizontal).

Use this space for summary and/or additional notes:

One type of physics problem that commonly uses vectors in this way is two-dimensional projectile motion. If the motion of the projectile is represented by a vector, $\vec{\boldsymbol{v}}$, at angle θ, the vector can be represented as the sum of a horizontal vector and a vertical vector. This is useful because the horizontal vector gives us the component (portion) of the vector in the x-direction, and the vertical vector gives us the component of the vector in the y-direction.

For example, in the following diagram, the velocity of the projectile is $\vec{\boldsymbol{v}}$. The component of the velocity in the x-direction is $\vec{\boldsymbol{v}}_x$, and the component in the y-direction is $\vec{\boldsymbol{v}}_y$.



Notice that $\vec{\boldsymbol{v}}_x$ remains constant, but $\vec{\boldsymbol{v}}_y$ changes (because of the effects of gravity).

Use this space for summary and/or additional notes:

Vectors vs. Scalars in Physics

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- how vectors are used in an algebra-based physics course

Language Objectives:

- Accurately describe and apply the concepts described in this section, including the difference between "vector" and "scalar," using appropriate academic language.

Notes:

In physics, most numbers represent quantities that can be measured or calculated from measurements. Most of the time, there is no concept of a "deficit" of a measured quantity. For example, quantities like mass, energy, and power can only be nonnegative, because in classical mechanics there is no such thing as "anti-mass," "anti-energy," or "anti-power."

However, vector quantities have a direction as well as a magnitude, and direction can be positive or negative.

A rule of thumb that works *most* of the time in this class is:

Scalar quantities. These are almost always positive. (Note, however, that we will encounter some exceptions during the year. An example is electric charge, which can be positive or negative.)

Vector quantities. Vectors can be positive or negative. In any given problem, you will choose which direction is positive. Vectors in the positive direction will be expressed as positive numbers, and vectors in the opposite (negative) direction will be expressed as negative numbers.

Use this space for summary and/or additional notes:

<u>Differences</u>. The difference or change in a variable is indicated by the Greek letter Δ in front of the variable. Any difference can be positive or negative. However, note that a difference can be a vector, indicating a change relative to the positive direction (*e.g.,* $\boldsymbol{\Delta x}$, which indicates a change in position), or scalar, indicating an increase or decrease (*e.g.,* ΔV, which indicates a change in volume).

In some cases, you will need to split a vector in two vectors, one vector in the x-direction, and a separate vector in the y-direction. In these cases, you will need to choose which direction is positive and which direction is negative for *<u>both</u>* the x- and y-axes. Once you have done this, every vector quantity must be assigned a positive or negative value, according to the directions you have chosen.

Use this space for summary and/or additional notes:

Example:

Suppose you have a problem that involves throwing a ball straight upwards with a velocity of $15\,\frac{\text{m}}{\text{s}}$. Gravity is slowing the ball down with a downward acceleration of $9.8\,\frac{\text{m}}{\text{s}^2}$. You want to know how far the ball has traveled in 0.5 s.

Displacement, velocity, and acceleration are all vectors. The motion is happening in the y-direction, so we need to choose whether "up" or "down" is the positive direction. Suppose we choose "up" to be the positive direction. This means:

- When the ball is first thrown, it is moving upwards. This means its velocity is in the *positive* direction, so we would represent the initial velocity as $\vec{\boldsymbol{v}}_o = +15\,\frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the *negative* direction. We would therefore represent the acceleration as $\vec{\boldsymbol{a}} = -9.8\,\frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so it can only be positive.

If we had to substitute the numbers into the formula:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2$$

we would do so as follows:

$$\vec{\boldsymbol{d}} = (+15)(0.5) + (\tfrac{1}{2})(-9.8)(0.5)^2$$

and we would find out that $\vec{\boldsymbol{d}} = +6.275\,\text{m}$.

The answer is *positive*. Earlier, we defined positive as "up", so the answer tells us that the displacement is upwards from the starting point.

Use this space for summary and/or additional notes:

Vectors vs. Scalars in Physics

What if, instead, we had chosen "down" to be the positive direction?

- When the ball is first thrown, it is moving upwards. This means its velocity is now in the *negative* direction, so we would represent the initial velocity as $\vec{\boldsymbol{v}}_o = -15\,\frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the *positive* direction. We would therefore represent the acceleration as $\vec{\boldsymbol{a}} = +9.8\,\frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so it can only be positive.

If we had to substitute the numbers into the formula:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2$$

we would do so as follows:

$$\vec{\boldsymbol{d}} = (-15)(0.5) + (\tfrac{1}{2})(9.8)(0.5)^2$$

and we would find out that $\vec{\boldsymbol{d}} = -6.275\,\text{m}$.

The answer is *negative*. Remember that "down" was positive, which means "up" is the negative direction. This means the displacement is upwards from the starting point, as before.

Remember: in any problem you solve, the choice of which direction is positive *vs.* negative is arbitrary. The only requirement is that every vector quantity in the problem needs to be consistent with your choice.

Use this space for summary and/or additional notes:

Vector Multiplication

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- dot product & cross product of two vectors

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

With scalar (ordinary) numbers, there is only one way to multiply them, which you learned in elementary school. Vectors, however, can be multiplied in three different ways.

dot product: multiplication of two vectors that results in a scalar.

cross product: multiplication of two vectors that results in a new vector.

tensor product: multiplication of two vectors that results in a tensor. (A tensor is an array of vectors that describes the effect of each vector on each other vector within the array. We will not use tensors in a high school physics course.)

Use this space for summary and/or additional notes:

Multiplying a Vector by a Scalar

Multiplying a vector by a scalar is like multiplying a variable by a number. The magnitude changes, but the direction does not. For example, in physics, displacement equals velocity times time:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}t$$

Velocity is a vector; time is a scalar. The magnitude is the velocity times the time, and the direction of the displacement is the same as the direction of the velocity.

The Dot (Scalar) Product of Two Vectors

The scalar product of two vectors is called the "dot product". Dot product multiplication of vectors is represented with a dot:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}}$$[*]

The dot product of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = AB\cos\theta$$

where A is the magnitude of $\vec{\boldsymbol{A}}$, B is the magnitude of $\vec{\boldsymbol{B}}$, and θ is the angle between the two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

For example, in physics, *work* (a scalar quantity) is the dot product of the vectors *force* and *displacement* (distance):

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = Fd\cos\theta$$

---

[*] pronounced "A dot B"

Use this space for summary and/or additional notes:

The Cross (Vector) Product of Two Vectors

The vector product of two vectors is called the cross product. Cross product multiplication of vectors is represented with a multiplication sign:

$$\vec{A}\times\vec{B}$$*

The cross product of vectors $\vec{A}$ and $\vec{B}$ that have an angle of θ between them is given by the formula:

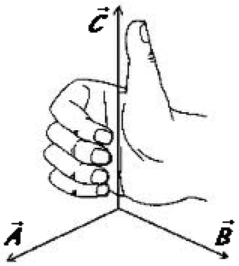
$$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$$

where the magnitude is $AB\sin\theta$, and the vector $\hat{n}$ is the direction. ($AB\sin\theta$ is a scalar. The unit vector $\hat{n}$ is what gives the vector its direction.)

The direction of the cross product is a little difficult to make sense out of. You can figure it out using the "right hand rule":

Position your right hand so that your fingers curl from the first vector to the second. Your thumb points in the direction of the resultant vector ($\hat{n}$).

Note that this means that the resultant vectors for $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ will point in *opposite* directions, *i.e.,* the cross product of two vectors is <u>*not*</u> commutative!



$$\vec{A}\times\vec{B} = \vec{C}$$
$$\vec{B}\times\vec{A} = -\vec{C}$$

A vector coming out of the page is denoted by a series of ⊙ ⊙ ⊙ ⊙ ⊙ symbols, and a vector going into the page is denoted by a series of ⊗ ⊗ ⊗ ⊗ ⊗ symbols. The symbols represent an arrow inside a tube. The dot represents the tip of the arrow coming toward you, and the "X" represents the fletches (feathers) on the tail of the arrow going away from you.)

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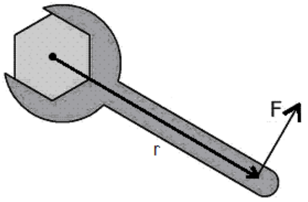
[*] pronounced "A cross B"

Use this space for summary and/or additional notes:

In physics, torque is a vector quantity that is derived by a cross product.



The torque produced by a force $\vec{F}$ acting at a radius $\vec{r}$ is given by the equation:

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta\,\hat{n}$$

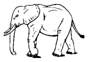
Because the direction of the force is usually perpendicular to the displacement, it is usually true that $\sin\theta = \sin 90° = 1$. This means the magnitude $rF\sin\theta = rF(1) = rF$. Using the right-hand rule, we determine that the *direction* of the resultant torque vector ($\hat{n}$) is coming out of the page.

Thus, if you are tightening or loosening a nut or bolt that has right-handed (standard) thread, the torque vector will be in the direction that the nut or bolt moves.

Vector Jokes

Now that you understand vectors, here are some bad vector jokes:

Q: What do you get when you cross an elephant with a bunch of grapes?

A:   $\sin\theta\,\hat{n}$

Q: What do you get when you cross an elephant with a mountain climber?

A: You can't do that! A mountain climber is a scalar ("scaler," meaning someone who scales a mountain).

Use this space for summary and/or additional notes:

Degrees, Radians and Revolutions

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- express angles and arc length in degrees, radians, and full revolutions

Skills:

- convert between degrees, radians and revolutions

Language Objectives:

- Understand and correctly use the terms "degree," "radian," and "revolution"
- Accurately describe and apply the concepts described in this section using appropriate academic language.

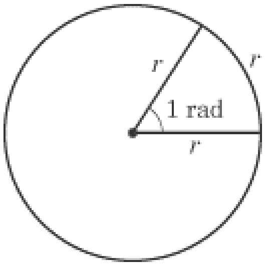
Notes:

degree: an angle equal to $\frac{1}{360}$ of a full circle. A full circle is therefore 360°.

revolution: a rotation of exactly one full circle (360°) by an object.

radian: the angle that results in an arc length that equal to the radius of a circle. *I.e.*, one "radius" of the way around the circle. Because the distance all the way around the circle is 2π times the radius, a full circle (or one rotation) is therefore 2π radians.

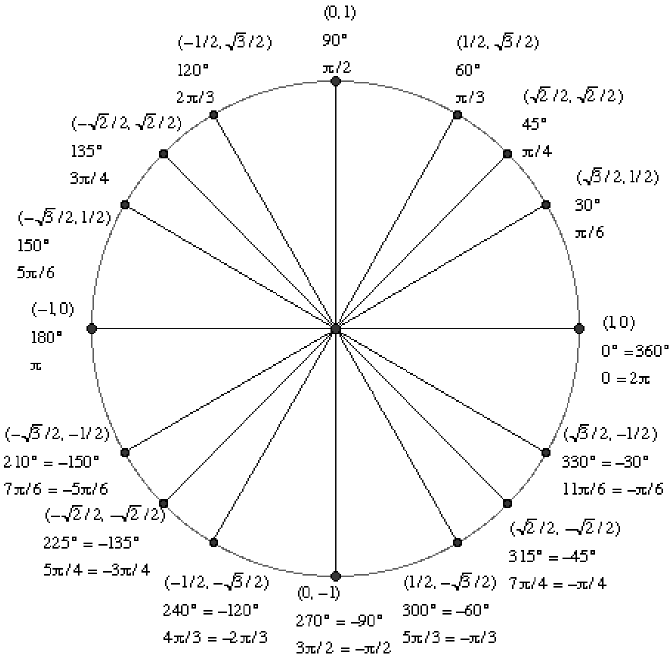
We are used to measuring angles in degrees. However, trigonometry functions are often more convenient if we express the angle in radians:



Use this space for summary and/or additional notes:

This is often convenient because if we express the angle in radians, the angle is equal to the arc length (distance traveled around the circle) times the radius, which makes much easier to switch back and forth between the two quantities.

On the following unit circle (a circle with a radius of 1), several of the key angles around the circle are marked in radians, degrees, and the (*x,y*) coördinates of the corresponding point around the circle.



In each case, the angle in radians is equal to the distance traveled around the circle, starting from the point (1,0).

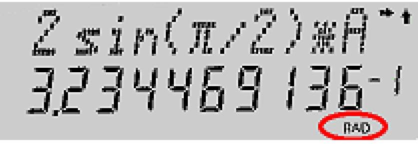
It is useful to memorize the following:

Degrees	0°	90°	180°	270°	360°
Rotations	0	¼	½	¾	1
Radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

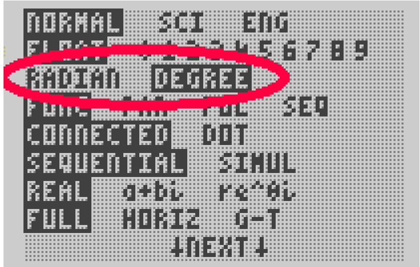
Use this space for summary and/or additional notes:

In algebra 2 and precalculus, students learn and make use of the conversion between degrees and radians, but often lose sight of the most important feature of radians—that their purpose is to allow you to easily determine the arc length (distance traveled around the circle) by simply multiplying the angle (in radians) times the radius.

In physics, you will need to be able to solve rotational problems using degrees, radians, and/or rotations. For this reason, when using trigonometry functions it will be important to make sure your calculator mode is set correctly for degrees or radians, as appropriate to each problem:



TI-30 scientific calculator



TI-84+ graphing calculator

If you convert your calculator to degrees, don't forget to convert it back to radians before you use it for precalc!

Use this space for summary and/or additional notes:

Polar, Cylindrical & Spherical Coördinates

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- expressing a location in Cartesian, polar, cylindrical, or spherical coördinates

Skills:

- convert between Cartesian coördinates and polar, cylindrical and/or spherical coördinates

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

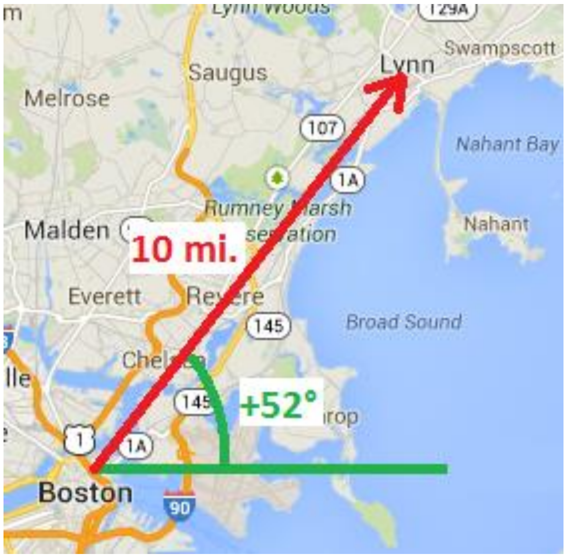
In your math classes so far, you have expressed the location of a point using Cartesian coördinates—either (x, y) in two dimensions or (x, y, z) in three dimensions.

Cartesian coördinates: (or rectangular coördinates): a two- or three-dimensional coordinate system that specifies locations by separate distances from each two or three axes (lines). These axes are labeled x, y, and z, and a point is specified using its distance from each axis, in the form (x, y) or (x, y, z).

Use this space for summary and/or additional notes:

<u>polar coördinates</u>: a two-dimensional coördinate system that specifies locations by their distance from the origin (radius) and angle from some reference direction. The radius is labeled *r*, and the angle is θ (the Greek letter "theta"). A point is specified using the distance and angle, in the form (r, θ).

For example, when we say that Lynn is 10 miles from Boston at an angle of +52°, we are using polar coördinates:



(Note: cardinal or "compass" direction is traditionally specified with North at 0° and 360°, and clockwise as the positive direction, meaning that East is 90°, South is 180°, West is 270°. This means that the compass heading from Boston to Lynn would be 38° to the East of true North. However, in this class we will specify angles as mathematicians do, with 0° indicating the direction of the positive *x*-axis.)

<u>cylindrical coördinates</u>: a three-dimensional coördinate system that specifies locations by distance from the origin (radius), angle from some reference direction, and height above the origin. The radius is labeled *r*, the angle is θ, and the height is *z*. A point is specified using the distance and angle, and height in the form (r, θ, z).

Use this space for summary and/or additional notes:

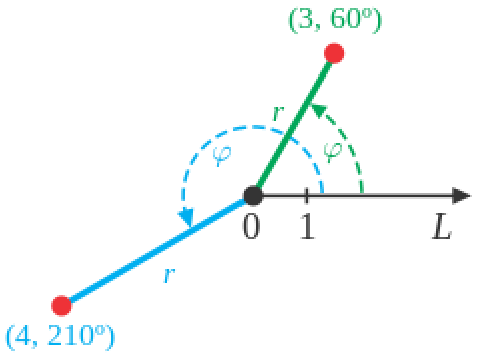
<u>spherical coördinates</u>: a three-dimensional coördinate system that specifies locations by distance from the origin (radius), and two separate angles, one from some horizontal reference direction and the other from some vertical reference direction. The radius is labeled *r*, the horizontal angle is θ, and the vertical angle is ϕ (the Greek letter "phi"). A point is specified using the distance and angle, and height in the form (r, θ, ϕ).

When we specify a point on the Earth using longitude and latitude, we are using spherical coördinates. The distance is assumed to be the radius of the Earth (because the interesting points are on the surface), the longitude is θ, and the latitude is ϕ. (Note, however, that latitude on the Earth is measured up from the equator. In AP Physics 1, we will use the convention that $\phi = 0°$ is straight upward, meaning ϕ will indicate the angle *downward* from the "North pole".)

In AP Physics 1, the problems we will see are one- or two-dimensional. For each problem, we will use the simplest coördinate system that applies to the problem: Cartesian (x, y) coördinates for linear problems and polar (r, θ) coördinates for problems that involve rotation.

Note that while mathematicians prefer to express angles in radians, physicists often use degrees, which are more familiar.

The following example shows the locations of the points (3, 60°) and (4, 210°):



Use this space for summary and/or additional notes:

Converting Between Cartesian and Polar Coordinates

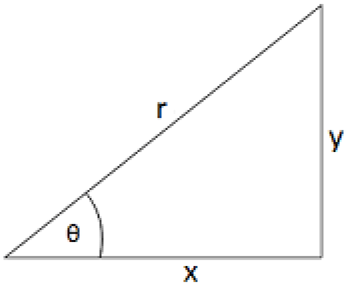
If vectors make sense to you, you can simply think of polar coördinates as the magnitude (*r*) and direction (θ) of a vector.

Converting from Cartesian to Polar Coördinates

If you know the *x*- and *y*-coördinates of a point, the radius (*r*) is simply the distance from the origin to the point. You can calculate *r* from *x* and *y* using the distance formula:

$$r = \sqrt{x^2 + y^2}$$

The angle comes from trigonometry:



$\tan\theta = \frac{y}{x}$, which means $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Sample Problem:

Q: Convert the point (5,12) to polar coördinates.

A: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{12}{5}\right) = \tan^{-1}(2.4) = 67.4^\circ = 1.18\text{ rad}$$

$\boxed{(13, 67.4°)}$ or $\boxed{(13, 1.18\text{ rad})}$

Use this space for summary and/or additional notes:

Converting from Polar to Cartesian Coördinates

As we saw in our review of trigonometry, if you know r and θ, then $x = r\cos\theta$ and $y = r\sin\theta$.

Sample Problem:

Q: Convert the point (8, 25°) to Cartesian coördinates.

A: $x = 8\cos(25^\circ) = (8)(0.906) = 7.25$
$y = 8\sin(25^\circ) = (8)(0.423) = 3.38$

$\boxed{(7.25, 3.38)}$

In practice, you will rarely need to convert between the two coördinate systems. The reason for using polar coördinates in a rotating system is because the quantities of interest are based on the rotational angle and the distance from the center of rotation. Using polar coördinates for these problems *avoids* the need to use trigonometry to convert between systems.

Use this space for summary and/or additional notes:

Introduction: Kinematics (Motion)

Unit: Kinematics (Motion)

Topics covered in this chapter:

In this chapter, you will study how things move and how the relevant quantities are related.

- *Motion, Speed & Velocity* and *Acceleration* deal with understanding and calculating the velocity (change in position) and acceleration (change in velocity) of an object, and with representing and interpreting graphs involving these quantities.
- *Projectile Motion* deals with an object that has two-dimensional motion—moving horizontally and also affected by gravity.

Some of the challenging tasks include identifying quantities from their units, choosing the equation that relates the quantities of interest, and keeping track of positive and negative directions when working with vector quantities.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Use this space for summary and/or additional notes:

Massachusetts Curriculum Frameworks (2006):

1.4 Interpret and apply Newton's three laws of motion.

1.5 Use a free-body force diagram to show forces acting on a system consisting of a pair of interacting objects. For a diagram with only co-linear forces, determine the net force acting on a system and between the objects.

1.6 Distinguish qualitatively between static and kinetic friction, and describe their effects on the motion of objects.

1.7 Describe Newton's law of universal gravitation in terms of the attraction between two objects, their masses, and the distance between them.

Topics from this chapter assessed on the SAT Physics Subject Test:

Kinematics, such as velocity, acceleration, motion in one dimension, and motion of projectiles

1. Displacement
2. Speed, velocity and acceleration
3. Kinematics with graphs
4. One-dimensional motion with uniform acceleration
5. Two-dimensional motion with uniform acceleration

Skills learned & applied in this chapter:

- Choosing from a set of equations based on the quantities present.
- Working with vector quantities.
- Relating the slope of a graph and the area under a graph to equations.
- Using graphs to represent and calculate quantities.
- Keeping track of things happening in two directions at once.

Use this space for summary and/or additional notes:

Linear Motion, Speed & Velocity

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- understand terms relating to position, speed & velocity
- understand the difference between speed and velocity

Language Objectives:

- Understand and correctly use the terms “position,” “distance,” “displacement,” “speed,” and “velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

coördinate system: a framework for describing an object’s ***position*** (location), based on its distance (in one or more directions) from a specifically-defined point (the *origin*). (You should remember these terms from math.)

direction: which way an object is oriented or moving within its coördinate system. Note that direction can be positive or negative.

position (x): the location of an object relative to the origin (zero point) of its coördinate system. We will consider position to be a zero-dimensional vector, which means it can be positive or negative with respect to the chosen coördinate system.

distance (d): [scalar] how far an object has moved.

displacement ($\vec{\boldsymbol{d}}$ or $\Delta \boldsymbol{x}$): [vector] how far an object’s current position is from its starting position (“initial position”). Displacement can be positive or negative (or zero), depending on the chosen coördinate system.

rate: the change in a quantity over a specific period of time.

motion: when an object’s position is changing over time.

Use this space for summary and/or additional notes:

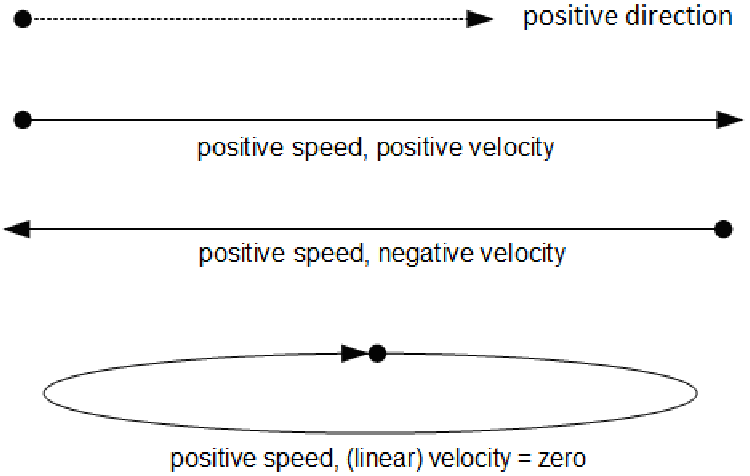
Linear Motion, Speed & Velocity

speed: [scalar] the rate at which an object is moving at an instant in time. Speed does not depend on direction, and is always nonnegative.

velocity: ($\vec{\boldsymbol{v}}$) [vector] an object's displacement over a given period of time. Because velocity is a *vector*, it has a *direction* as well as a *magnitude*. Velocity can be positive, negative, or zero.

uniform motion: motion at a constant velocity (*i.e.,* with constant speed and direction)

An object that is moving has a positive speed, but its velocity may be positive, negative, or zero, depending on its position.



Use this space for summary and/or additional notes:

Linear Motion, Speed & Velocity

Variables Used to Describe Linear Motion

Variable	**Quantity**	**MKS Unit**
x	position	m
d, Δx	distance	m
$\vec{\boldsymbol{d}}$, $\Delta\boldsymbol{x}$	displacement	m
h	height	m
$\vec{\boldsymbol{v}}$	velocity	$\frac{\text{m}}{\text{s}}$
$\bar{\vec{\boldsymbol{v}}}$	average velocity	$\frac{\text{m}}{\text{s}}$

The average velocity of an object is its displacement divided by the time, or its change in position divided by the (change in) time:

$$\bar{\vec{\boldsymbol{v}}} = \frac{\vec{\boldsymbol{d}}}{t} = \frac{x - x_o}{t} = \frac{\boldsymbol{\Delta x}}{t} = \frac{\boldsymbol{\Delta x}}{\Delta t}$$

(Note that elapsed time is always a difference (Δt), though we usually use t rather than Δt as the variable.)

We can use calculus to turn $\bar{v}$ into v by taking the limit as Δt approaches zero:

$$v = \underset{\Delta t \to 0}{Lim} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

i.e., velocity is the first derivative of displacement with respect to time.

We can rearrange this formula to show that displacement is average velocity times time:

$$\vec{\boldsymbol{d}} = \bar{\vec{\boldsymbol{v}}} t$$

Position is the object's starting position plus its displacement:

$$x = x_o + \vec{\boldsymbol{d}} = x_o + \bar{\vec{\boldsymbol{v}}} t$$

where x_0[*] means "position at time = 0". This formula is often expressed as:

$$x - x_o = \vec{\boldsymbol{d}} = \bar{\vec{\boldsymbol{v}}} t$$

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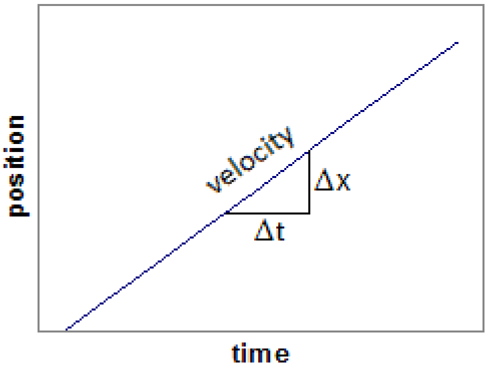
[*] x_0 is pronounced "x-zero" or "x-naught".

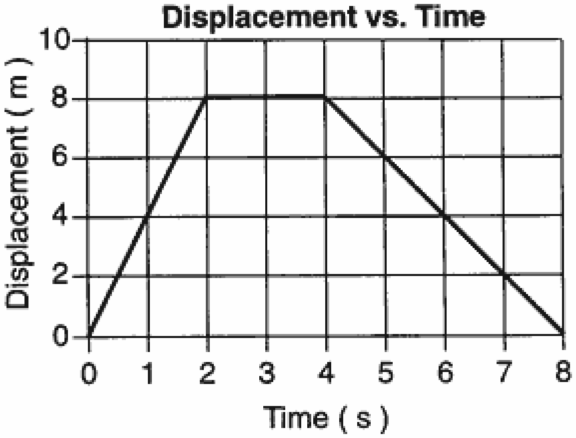
Use this space for summary and/or additional notes:

Note that $\frac{\Delta x}{\Delta t}$ is the slope of a graph of position (x) *vs.* time (t). Because $\frac{\Delta x}{\Delta t} = v$, this means that the *slope* of a graph of position *vs.* time is equal to the velocity.



In fact, *on any graph, the quantity you get when you divide the quantity on the* x-*axis by the quantity on the* y-*axis is, by definition, the slope. I.e.,* the slope is $\frac{\Delta y}{\Delta x}$, which means the quantity defined by $\frac{y\text{-axis}}{x\text{-axis}}$ will always be the slope.

Recall that velocity is a vector, which means it can be positive, negative, or zero. On the graph below, the velocity is $+4\,\frac{\text{m}}{\text{s}}$ from 0 s to 2 s, zero from 2 s to 4 s, and $-2\,\frac{\text{m}}{\text{s}}$ from 4 s to 8 s.



Use this space for summary and/or additional notes:

Linear Motion, Speed & Velocity

Sample problems:

Q: A car travels 1200 m in 60 seconds. What is its average velocity?

A: $\bar{v} = \frac{d}{t}$

$$\bar{v} = \frac{1200\text{ m}}{60\text{ s}} = 20\ \tfrac{\text{m}}{\text{s}}$$

Q: A person walks 320 m at an average velocity of $1.25\ \tfrac{\text{m}}{\text{s}}$. How long did it take?

A: "How long" means what length of ***time***.

$$\bar{v} = \frac{d}{t}$$

$$1.25 = \frac{320}{t}$$

$$t = 256\ s$$

It took 256 seconds for the person to walk 320 m.

Use this space for summary and/or additional notes:

Linear Acceleration

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- what linear acceleration means
- what positive *vs.* negative acceleration means

Skills:

- calculate position, velocity and acceleration for problems that involve movement in one direction

Language Objectives:

- Understand and correctly use the term "acceleration."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

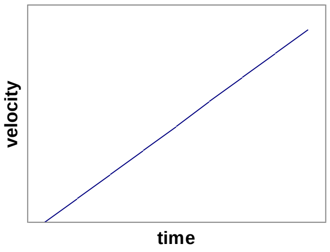
Notes:

acceleration: a change in velocity over a period of time.

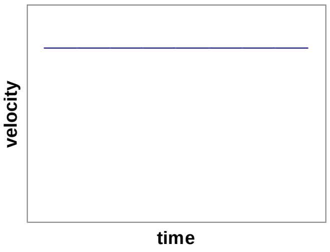
uniform acceleration: when an object's rate of acceleration (*i.e.,* the rate at which its velocity changes) is constant.

If an object's velocity is increasing, we say it has *positive* acceleration.

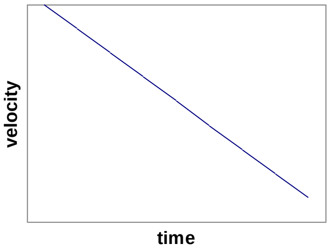
If an object's velocity is decreasing, we say it has *negative* acceleration.



positive acceleration



acceleration = zero



negative acceleration

Use this space for summary and/or additional notes:

Note that if the object's velocity is negative, then increasing velocity (positive acceleration) would mean that the velocity is getting *less negative*, *i.e.,* the object would be slowing down in the negative direction.

Variables Used to Describe Acceleration

Variable	Quantity	MKS Units
$\vec{a}$	acceleration	$\frac{\text{m}}{\text{s}^2}$
$\vec{g}$	acceleration due to gravity	$\frac{\text{m}}{\text{s}^2}$

By convention, physicists use the variable $\vec{g}$ to mean acceleration due to gravity, and $\vec{a}$ to mean acceleration caused by something other than gravity.

Because acceleration is a change in velocity over a period of time, the formula for acceleration is:

$$\bar{a} = \frac{v - v_o}{t} = \frac{\Delta v}{t} = \frac{\Delta v}{\Delta t} \qquad \text{and, from calculus:} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The units must match the formula, which means the units for acceleration must be velocity (distance/time) divided by time, which equals distance divided by time squared.

Because $v = \frac{dx}{dt}$, this means that acceleration is the second derivative of position with respect to time: $a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

However, in an algebra-based physics course, we will limit ourselves to problems in which acceleration is constant.

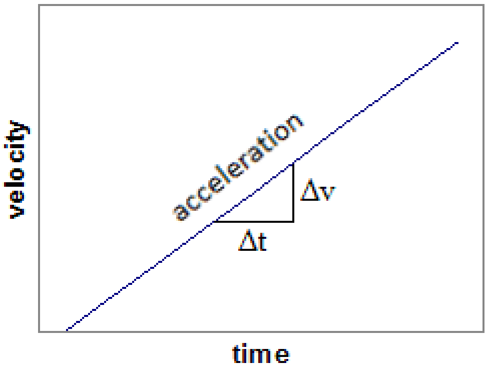
We can rearrange this formula to show that the change in velocity is acceleration times time:

$$\Delta v = v - v_o = at$$

Use this space for summary and/or additional notes:

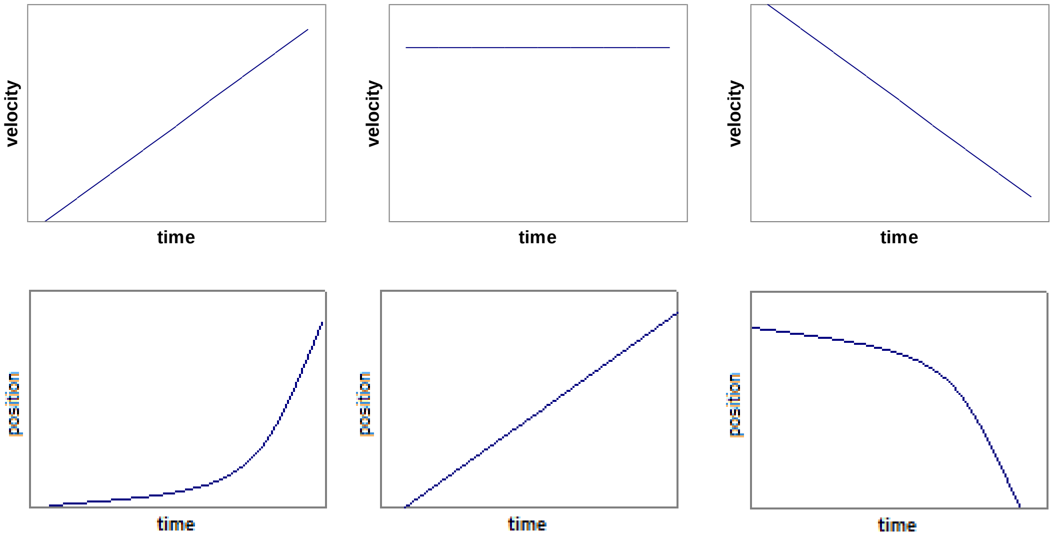
Note that when an object's velocity is changing, the final velocity, v, is not the same as the average velocity, $\bar{v}$. (This is a common mistake that first-year physics students make.)

$\frac{\Delta v}{\Delta t}$ is the slope of a graph of velocity (v) *vs.* time (t). Because $\frac{\Delta v}{\Delta t} = a$, this means that acceleration is the ***slope*** of a graph of velocity *vs.* time:



Note the relationship between velocity-time graphs and position-time graphs.

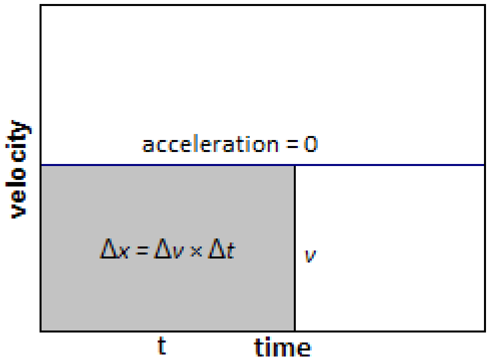
positive acceleration
acceleration = zero
negative acceleration

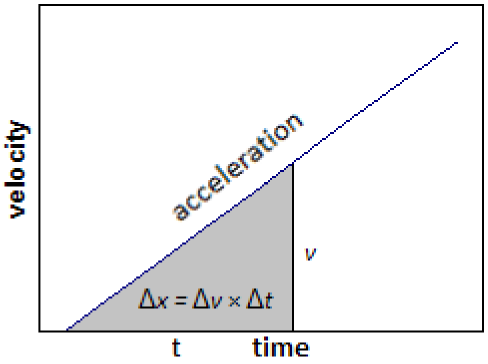


concave up
linear
concave down

Use this space for summary and/or additional notes:

Note also that $\bar{v}t$ is the area under a graph (*i.e.,* the area between the curve and the x-axis) of velocity (v) *vs.* time (t). Because $\bar{v}t = d$, this means the *area under* a graph of velocity *vs.* time is the displacement (Δx). Note that this works both for constant velocity (the graph on the left) and changing velocity (as shown in the graph on the right).





In fact, *on any graph, the quantity you get when you multiply the quantities on the* x- *and* y-*axes is, by definition, the area under the graph.*

In calculus, the area under a curve is the integral of the equation for the curve. This means:

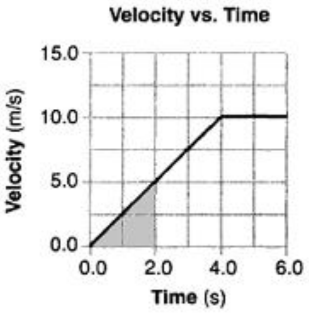
$$d = \int_0^t v\,dt$$

where v can be any function of t.

Use this space for summary and/or additional notes:

In the graph below, between 0 s and 4 s the object is accelerating at a rate of $+2.5\,\frac{\text{m}}{\text{s}^2}$.

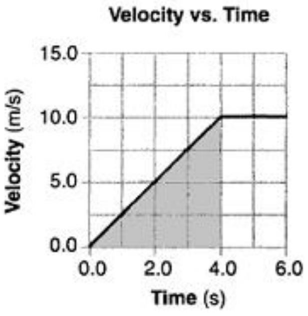
Between 4 s and 6 s the object is moving at a constant velocity (of $+10\,\frac{\text{m}}{\text{s}}$), so the acceleration is zero.



$a = 2.5\,\frac{\text{m}}{\text{s}^2}$

$d = \frac{1}{2}(2.5)(2^2) = 5\text{ m}$

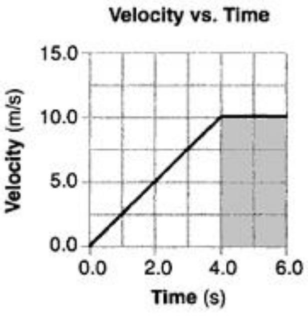
$A = \frac{1}{2}(2)(5) = 5\text{ m}$



$a = 2.5\,\frac{\text{m}}{\text{s}^2}$

$d = \frac{1}{2}(2.5)(4^2) = 20\text{ m}$

$A = \frac{1}{2}(4)(10) = 20\text{ m}$



$a = 0$

$d = \bar{v}t = (10)(2) = 20\text{ m}$

$A = (2)(10) = 20\text{ m}$

In each case, the area under the velocity-time graph equals the total distance traveled.

Use this space for summary and/or additional notes:

To show the relationship between v and $\bar{v}$, we can combine the formula for average velocity with the formula for acceleration in order to get a formula for the position of an object that is accelerating.

$$d = \bar{v}t$$

$$v = at$$

However, the problem is that v in the formula $v = at$ is the velocity at the *end*, which is not the same as the *average* velocity $\bar{v}$.

If the velocity of an object is changing (*i.e.,* the object is accelerating), the average velocity is given by the formula:

$$\bar{v} = \frac{v_o + v}{2}$$

If the object starts at rest (not moving, which means $v_o = 0$) and it accelerates at a constant rate, the average velocity is therefore

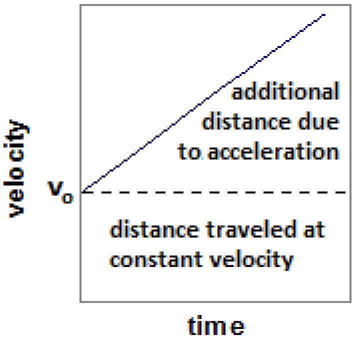
$$\bar{v} = \frac{v_o + v}{2} = \frac{0 + v}{2} = \frac{v}{2} = \tfrac{1}{2}v$$

Combining all of these gives, for an object starting from rest:

$$d = \bar{v}t = \tfrac{1}{2}vt = \tfrac{1}{2}(at)\,t = \tfrac{1}{2}at^2$$

If an object was moving before it started to accelerate, it had an initial velocity, or a velocity at time = 0. We will represent this initial velocity as v_o[*]. Now, the formula becomes:

$$x - x_o = d = \underbrace{v_o t}_{\text{distance the object would travel at its initial velocity}} + \underbrace{\tfrac{1}{2}at^2}_{\underline{\textit{additional}}\text{ distance the object will travel because it is accelerating}}$$



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[*] pronounced "v-zero" or "v-naught"

Use this space for summary and/or additional notes:

This equation can be combined with the equation for velocity to give the following equation, which relates initial and final velocity and distance:

$$v^2 - v_o^2 = 2ad$$

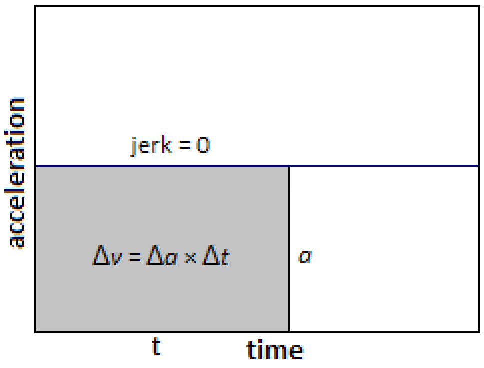
Finally, when an object is accelerating because of gravity, we say that the object is in "free fall".

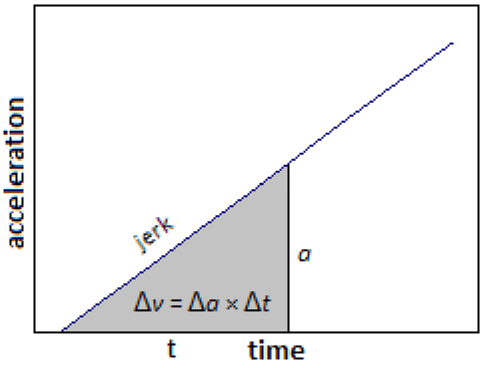
On earth, the average acceleration due to gravity is approximately $9.807\,\frac{\text{m}}{\text{s}^2}$ at sea level (which we will usually round to $9.8\,\frac{\text{m}}{\text{s}^2}$, or sometimes just $10\,\frac{\text{m}}{\text{s}^2}$). Any time gravity is involved (and the problem takes place on Earth), assume that $a = g = 9.8\,\frac{\text{m}}{\text{s}^2}$.

Extensions

Just as a change in velocity is called acceleration, a change in acceleration with respect to time is called "jerk": $\vec{\boldsymbol{j}} = \frac{\Delta\vec{\boldsymbol{a}}}{\Delta t}$.

While questions about jerk have not been seen on the AP exam, some AP problems do require you to understand that the area under a graph of acceleration *vs.* time would be the change in velocity (Δv), just as the area under a graph of velocity *vs.* time is the change in position.





Use this space for summary and/or additional notes:

Angular Motion, Speed and Velocity

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- understand terms relating to angular position, speed & velocity

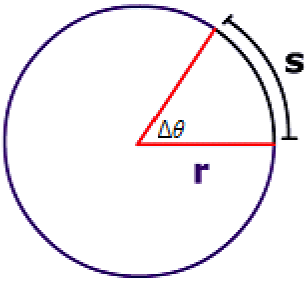
Language Objectives:

- Understand and correctly use the terms "angle" and "angular velocity."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

If an object is rotating (traveling in a circle), then its position at any given time can be described using polar coördinates by its distance from the center of the circle (*r*) and its angle (θ) relative to some reference angle (which we will call $\theta = 0$).

arc length (*s*): the length of an arc; the distance traveled around part of a circle.



$$s = r\Delta\theta$$

Use this space for summary and/or additional notes:

Angular Motion, Speed and Velocity

<u>angular velocity</u> (ω): the rotational velocity of an object as it travels around a circle, *i.e.,* its change in angle per unit of time. (For purposes of comparison, the definition of angular velocity is presented along with its linear counterpart.)

$$\vec{\boldsymbol{v}} = \frac{d}{t} = \frac{\Delta x}{\Delta t} = \frac{x - x_o}{t} \qquad \vec{\boldsymbol{\omega}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta_o}{t}$$

linear **angular**

In general, physicists use Greek letters for angular variables. The variable for angular velocity is the lower case Greek letter omega (ω). Be careful to distinguish in your writing between the Greek letter "ω" and the Roman letter "*w*".

<u>tangential velocity</u>: the linear velocity of a point on a rigid, rotating body. The term tangential velocity is used because the instantaneous direction of the velocity is tangential to the direction of rotation.

To find the tangential velocity of a point on a rotating (rigid) body, the point travels an arc length of *s* in time *t*. If angle θ is in radians, then $s = r\Delta\theta$. This means:

$$\bar{\vec{\boldsymbol{v}}}_T = \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} = r\bar{\vec{\boldsymbol{\omega}}} \quad \text{and therefore} \quad \vec{\boldsymbol{v}}_T = r\vec{\boldsymbol{\omega}}$$

Sample Problems:

Q: What is the angular velocity ($\frac{\text{rad}}{\text{s}}$) in of a car engine that is spinning at 2400 rpm?

A: 2400 rpm means 2400 revolutions per minute.

$$\left(\frac{2400\,\text{rev}}{1\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\frac{2\pi\,\text{rad}}{1\,\text{rev}} = \frac{4800\pi}{60} = 80\pi\,\frac{\text{rev}}{\text{s}} = 251\,\frac{\text{rev}}{\text{s}}$$

Use this space for summary and/or additional notes:

Q: Find the total distance traveled in 10 s by a penny sitting on a spinning disc with a radius of 0.25 m that is rotating at a rate of 1 revolution per 2 s.

A: We are looking for the distance around the circle, which is the arc length. (This means we need to work in radians.)

$$s = r\Delta\theta$$

We know that *r* = 0.25 m, but we need to find $\Delta\theta$.

$$\Delta\theta = \omega t$$

We know that *t* = 10 s, but we need to find the angular velocity *ω*.

1 revolution per 2 s is an angular velocity of $\omega = \frac{2\pi}{2} = \pi\,\frac{\text{rad}}{\text{s}}$.

Now we can solve:

$$\Delta\theta = \omega t = (\pi)(10) = 10\pi$$
$$s = r\Delta\theta = (0.25)(10\pi) = 2.5\pi\text{ m} = (2.5)(3.14) = 7.85\text{ m}$$

Extension

Just as jerk is the rate of change of linear acceleration, angular jerk is the rate of change of angular acceleration. $\vec{\boldsymbol{\zeta}} = \frac{\Delta\vec{\boldsymbol{\alpha}}}{\Delta t}$. (ζ is the Greek letter “zeta”. Many college professors cannot draw it correctly and just call it “squiggle”.) Angular jerk has not been seen on AP Physics exams.

Use this space for summary and/or additional notes:

Angular Acceleration

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- what angular acceleration means

Skills:

- calculate angle, angular velocity and angular acceleration for problems that involve rotational motion.

Language Objectives:

- Understand and correctly use the term “angular acceleration.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

If a rotating object starts rotating faster or slower, this means its rotational velocity is changing.

<u>angular acceleration</u> (α): the change in angular velocity with respect to time. (Again, the definition is presented with the linear equation for comparison.)

$$\vec{\boldsymbol{a}} = \frac{\Delta \vec{\boldsymbol{v}}}{\Delta t} = \frac{\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o}{t} \qquad \vec{\boldsymbol{\alpha}} = \frac{\Delta \vec{\boldsymbol{\omega}}}{\Delta t} = \frac{\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\omega}}_o}{t}$$

linear **angular**

As before, be careful to distinguish between the lower case Greek letter “α” and the lower case Roman letter “a”.

As with linear acceleration, if the object has angular velocity and then accelerates, the position equation looks like this:

$$\vec{\boldsymbol{x}} = \vec{\boldsymbol{x}}_o + \vec{\boldsymbol{v}}_o t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2 \qquad \vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\theta}}_o + \vec{\boldsymbol{\omega}}_o t + \tfrac{1}{2}\vec{\boldsymbol{\alpha}}t^2$$

linear **angular**

Use this space for summary and/or additional notes:

Angular Acceleration

<u>tangential acceleration</u>: the linear acceleration of a point on a rigid, rotating body. The term tangential acceleration is used because the instantaneous direction of the acceleration is tangential to the direction of rotation.

The tangential acceleration of a point on a rigid, rotating body is:

$$\vec{\boldsymbol{a}}_T = r\vec{\boldsymbol{\alpha}}$$

Sample Problem:

Q: A bicyclist is riding at an initial (linear) velocity of $7.5\,\frac{\text{m}}{\text{s}}$, and accelerates to a velocity of $10.0\,\frac{\text{m}}{\text{s}}$ over a duration of 5.0 s. If the wheels on the bicycle have a radius of 0.343 m, what is the angular acceleration of the bicycle wheels?

A: First we need to find the initial and final angular velocities of the bike wheel. We can do this from the tangential velocity, which equals the velocity of the bicycle.

$$\vec{\boldsymbol{v}}_{o,T} = r\vec{\boldsymbol{\omega}}_o$$
$$7.5 = (0.343)\vec{\boldsymbol{\omega}}_o$$
$$\vec{\boldsymbol{\omega}}_o = \frac{7.5}{0.343} = 21.87\,\frac{\text{rad}}{\text{s}}$$

$$\vec{\boldsymbol{v}}_T = r\vec{\boldsymbol{\omega}}$$
$$10.0 = (0.343)\vec{\boldsymbol{\omega}}$$
$$\vec{\boldsymbol{\omega}} = \frac{10.0}{0.343} = 29.15\,\frac{\text{rad}}{\text{s}}$$

Then we can use the equation:

$$\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\omega}}_o = \vec{\boldsymbol{\alpha}}t$$
$$29.15 - 21.87 = \vec{\boldsymbol{\alpha}}(5.0)$$
$$7.28 = 5.0\vec{\boldsymbol{\alpha}}$$
$$\vec{\boldsymbol{\alpha}} = \frac{7.28}{5.0} = 1.46\,\frac{\text{rad}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

An alternative method is to solve the equation by finding the linear acceleration first:

$$\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o = \vec{\boldsymbol{a}}t$$

$$10.0 - 7.5 = \vec{\boldsymbol{a}}(5)$$

$$2.5 = 5\vec{\boldsymbol{a}}$$

$$\vec{\boldsymbol{a}} = \frac{2.5}{5} = 0.5\,\tfrac{\text{m}}{\text{s}^2}$$

Then we can use the relationship between tangential acceleration and angular acceleration:

$$\vec{\boldsymbol{a}}_T = r\vec{\boldsymbol{\alpha}}$$

$$0.5 = (0.343)\vec{\boldsymbol{\alpha}}$$

$$\vec{\boldsymbol{\alpha}} = \frac{a}{0.343} = 1.46\,\tfrac{\text{rad}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

Centripetal Acceleration

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.8

Knowledge/Understanding Goals:

- why an object moving in a circle is constantly accelerating

Skills:

- calculate the centripetal force of an object moving in a circle

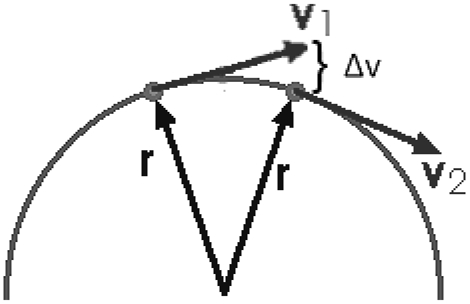
Language Objectives:

- Understand and correctly use the terms "rotation," "centripetal force," and "centrifugal force."
- Explain the difference between centripetal force and centrifugal force.

Notes:

If an object is moving at a constant speed around a circle, its speed is constant, its direction keeps changing as it goes around. Because *velocity* is a vector (speed and direction), this means its velocity is constantly changing. (To be precise, the magnitude is staying the same, but the direction is changing.)

Because a change in velocity over time is acceleration, this means the object is constantly accelerating. This continuous change in velocity is toward the center of the circle, which means *there is continuous acceleration toward the center of the circle.*



Use this space for summary and/or additional notes:

Centripetal Acceleration

centripetal acceleration (a_c): the constant acceleration of an object toward the center of rotation that keeps it rotating around the center at a fixed distance.

The formula for centripetal acceleration (a_c) is:

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

(The derivation of this formula requires calculus, so it will not be presented here.)

Sample Problem:

Q: A weight is swung from the end of a string that is 0.65 m long at a rate of rotation of 10 revolutions in 6.5 s. What is the centripetal acceleration of the weight? How many "g's" is that? (*I.e.,* how many times the acceleration due to gravity is the centripetal acceleration?)

A: The angular velocity is:

$$\left(\frac{10\,\text{rev}}{6.5\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right) = \frac{20\pi}{6.5} = 9.67\,\tfrac{\text{rad}}{\text{s}}$$

The centripetal acceleration is therefore:

$$a_c = r\omega^2$$
$$a_c = (0.65)(9.67)^2 = (0.65)(93.44) = 60.7\,\tfrac{\text{m}}{\text{s}^2}$$

This is $\frac{60.7}{9.8} = 6.2$ times the acceleration due to gravity.

Use this space for summary and/or additional notes:

Solving Linear & Rotational Motion Problems

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.2

Skills:

- solve problems involving motion in one or two dimensions

Language Objectives:

- Set up and solve word problems relating to linear or angular motion.

Notes:

The following is a summary of the variables used for motion problems:

Linear			Angular		
Var.	**Unit**	**Description**	**Var.**	**Unit**	**Description**
x	m	position	θ	— (rad)	angle; angular position
$\vec{d}, \Delta x$	m	displacement	$\Delta\theta$	— (rad)	angular displacement
$\vec{v}$	$\frac{m}{s}$	velocity	$\vec{\omega}$	$\frac{1}{s}$ $\left(\frac{rad}{s}\right)$	angular velocity
$\vec{a}$	$\frac{m}{s^2}$	acceleration	$\vec{\alpha}$	$\frac{1}{s^2}$ $\left(\frac{rad}{s^2}\right)$	angular acceleration
t	s	time	t	s	time

Notice that each of the linear variables has an angular counterpart.

Note that "radian" is not a unit. A radian is a ratio that describes an angle as the ratio of the arc length to the radius. This ratio is dimensionless (has no unit), because the units cancel. This means that an angle described in radians has no unit, and therefore never needs to be converted from one unit to another. However, we often write "rad" after an angle measured in radians to remind ourselves that the quantity describes an angle.

Use this space for summary and/or additional notes:

Solving Linear & Rotational Motion Problems

We have learned the following equations for solving motion problems:

Linear Equation	**Angular Equation**	**Relation**	**Comments**
$\vec{\boldsymbol{d}} = \boldsymbol{\Delta}\vec{\boldsymbol{x}} = \vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_o$	$\boldsymbol{\Delta}\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o$	$\vec{\boldsymbol{s}} = r\boldsymbol{\Delta}\vec{\boldsymbol{\theta}}$	Definition of displacement.
$\bar{\vec{\boldsymbol{v}}} = \frac{\vec{\boldsymbol{d}}}{t} = \frac{\Delta\vec{\boldsymbol{x}}}{t} = \frac{\vec{\boldsymbol{v}}_o + \vec{\boldsymbol{v}}}{2}$	$\bar{\vec{\boldsymbol{\omega}}} = \frac{\Delta\vec{\boldsymbol{\theta}}}{t} = \frac{\vec{\boldsymbol{\omega}}_o + \vec{\boldsymbol{\omega}}}{2}$	$\vec{\boldsymbol{v}}_T = r\vec{\boldsymbol{\omega}}$	Definition of *average* velocity. Note that you can't use $\overline{v}$ or $\overline{\omega}$ if there is acceleration.
$\vec{\boldsymbol{a}} = \frac{\Delta\vec{\boldsymbol{v}}}{t} = \frac{\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o}{t}$	$\vec{\boldsymbol{\alpha}} = \frac{\Delta\vec{\boldsymbol{\omega}}}{t} = \frac{\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\omega}}_o}{t}$	$\vec{\boldsymbol{a}}_T = r\vec{\boldsymbol{\alpha}}$	Definition of acceleration.
$\vec{\boldsymbol{x}} = \vec{\boldsymbol{x}}_o + \vec{\boldsymbol{v}}_o t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2$	$\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\theta}}_o + \vec{\boldsymbol{\omega}}_o t + \tfrac{1}{2}\vec{\boldsymbol{\alpha}}t^2$		Position formula.
$\vec{\boldsymbol{v}}^2 = \vec{\boldsymbol{v}}_o^2 + 2\vec{\boldsymbol{a}}\Delta\vec{\boldsymbol{x}}$	$\vec{\boldsymbol{\omega}}^2 = \vec{\boldsymbol{\omega}}_o^2 + 2\vec{\boldsymbol{\alpha}}\Delta\vec{\boldsymbol{\theta}}$		Relates velocities, acceleration and distance. Useful if time is not known.
$\vec{\boldsymbol{a}}_c = \frac{\vec{\boldsymbol{v}}^2}{r}$		$\vec{\boldsymbol{a}}_c = r\vec{\boldsymbol{\omega}}^2$	Centripetal acceleration (acceleration toward the center of a circle.)

Note that vector quantities (shown in bold) can be positive or negative, depending on direction.

Use this space for summary and/or additional notes:

Selecting the Right Equation

When you are faced with a problem, choose an equation based on the following criteria:

- The equation must contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.
  - If an object starts at rest (not moving), then $\vec{\boldsymbol{v}}_o = 0$ or $\vec{\boldsymbol{\omega}}_o = 0$.
  - If an object comes to a stop, then $\vec{\boldsymbol{v}} = 0$ or $\vec{\boldsymbol{\omega}} = 0$.
  - If gravity is involved (*e.g.,* the object is falling), $\vec{\boldsymbol{a}} = \vec{\boldsymbol{g}} = 9.8\,\frac{\text{m}}{\text{s}^2}$.
    (Applies to linear acceleration problems only.)

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

Use this space for summary and/or additional notes:

Strategies for Linear Motion Problems Involving Gravity

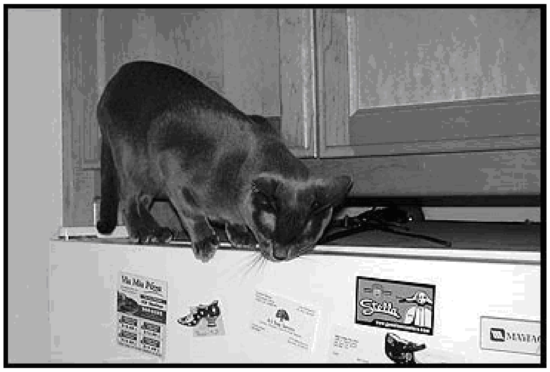
Linear motion problems in physics often involve gravity. These problems usually fall into one of two categories:

1. If you have an object in free fall, the problem will probably give you either the distance it fell ($x - x_o$), or the time it fell (t). Use the position equation $\vec{\boldsymbol{x}} = \vec{\boldsymbol{x}}_o + \vec{\boldsymbol{v}}_o t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$ to calculate whichever one you don't know. (If the object starts from rest, that means $\vec{\boldsymbol{v}}_o = 0$.)
2. If an object is thrown upwards, it will decelerate at a rate of $-9.8\,\frac{\text{m}}{\text{s}^2}$ (assuming "up" is the positive direction) until it stops moving ($\vec{\boldsymbol{v}} = 0$). Then it will fall. This means you need to split the problem into two parts:
   a. When the object is moving upward, the initial velocity, $\vec{\boldsymbol{v}}_o$, is usually given and $\vec{\boldsymbol{v}}$ (at the top) = zero. From these, you can use $\vec{\boldsymbol{v}}^2 = \vec{\boldsymbol{v}}_o^2 - 2\vec{\boldsymbol{a}}\Delta\vec{\boldsymbol{x}}$ to figure out the maximum height.
   b. Once you know the maximum height, you know the distance to the ground, $\vec{\boldsymbol{v}}_o = 0$, and you can use the position equation (this time with $\vec{\boldsymbol{a}} = +9.8\,\frac{\text{m}}{\text{s}^2}$) to find the time it spends falling. The total time (up + down) will be twice as much.

Use this space for summary and/or additional notes:

Sample Problems:

Q: If a cat jumps off a 1.8 m tall refrigerator, how long does it take to hit the ground?



A: The problem gives us d = 1.8 m. The cat is starting from rest ($v_a = 0$), and gravity is accelerating the cat at a rate of $a = g = 9.8\,\frac{\text{m}}{\text{s}^2}$. We need to find *t*.

Looking at the equations, the one that has what we need (t) and only quantities we know is:

$$d = v_o t + \tfrac{1}{2} a t^2$$

v_o = 0, so this reduces to:

$$d = \tfrac{1}{2} a t^2$$

$$1.8 = (\tfrac{1}{2})(9.8)\, t^2$$

$$\frac{1.8}{4.9} = 0.367 = t^2$$

$$t = \sqrt{0.367} = 0.61\, s$$

Use this space for summary and/or additional notes:

Q: An apple falls from a tree branch at a height of 5 m and lands on Isaac Newton's head. (Assume Isaac Newton was 1.8 m tall.)

How fast was the apple traveling at the time of impact?

A: We know $d = s - s_o = 5 - 1.8 = 3.2\,\text{m}$. We also know that the apple is starting from rest (v_a = 0), and gravity is accelerating the apple at a rate of $a = g = 9.8\,\frac{\text{m}}{\text{s}^2}$. We want to find *v*.

The equation that relates all of our variables is:

$$v^2 - v_o^2 = 2ad$$

Substituting, we get:

$$v^2 - 0 = (2)(9.8)(3.2)$$

$$v^2 = 62.7$$

$$v = \sqrt{62.7} = 7.9\,\frac{\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes:

Projectile Motion

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.2

Skills:

- solve problems involving linear motion in two dimensions

Language Objectives:

- Understand and correctly use the term "projectile."
- Set up and solve word problems involving projectiles.

Notes:

projectile: an object that is propelled (thrown, shot, *etc.*) horizontally and also falls due to gravity.

Gravity affects projectiles the same way regardless of whether the projectile is moving horizontally. Gravity does not affect the horizontal motion of the projectile. This means the vertical and horizontal motion of the projectile can be considered separately, using a separate set of equations for each.

Assuming we can neglect friction and air resistance (which is usually the case in first-year physics problems), we make two important assumptions:

- All projectiles have a constant horizontal velocity, v_h, in the positive horizontal direction. The equation for the horizontal motion is:
$$d_h = v_h t$$
- All projectiles have a constant downward acceleration of $g = 9.8\,\frac{\text{m}}{\text{s}^2}$ (in the vertical direction), due to gravity. (You can choose whether the positive vertical direction is up or down, depending on the situation.) The equation for the vertical motion is:
$$d_v = v_{o,v}t + \tfrac{1}{2}gt^2$$
- The time that the projectile spends falling must be the same as the time that the projectile spends moving horizontally. This means time (t) is the same in both equations, which means time is the variable that links the vertical problem to the horizontal problem.

Use this space for summary and/or additional notes.

Projectile Motion

The consequences of these assumptions are:

- The *time* that the object takes to fall is determined by its movement *only* in the vertical direction.
- The *horizontal distance* that the object travels is determined by the time (calculated above) and its velocity in the horizontal direction.

Therefore, the general strategy for most projectile problems is:

1. Solve the vertical problem first, to get the time.
2. Use the time from the vertical problem to solve the horizontal problem.

Sample problem:

Q: A ball is thrown horizontally at a velocity of $5\,\frac{\text{m}}{\text{s}}$ from a height of 1.5 m. How far does the ball travel (horizontally)?

A: We're looking for the horizontal distance, d_h. We know the vertical distance, $d_v = 1.5\,\text{m}$, and we know that $v_{o,v} = 0$ (there is no initial vertical velocity because the ball is thrown horizontally), and we know that $a = g = 9.8\,\frac{\text{m}}{\text{s}^2}$.

We need to separate the problem into the horizontal and vertical components.

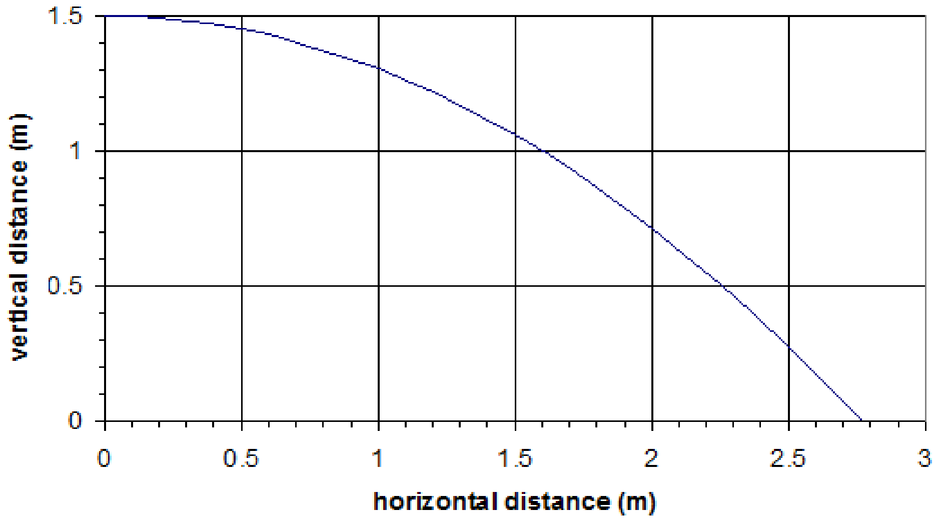
Horizontal:	Vertical:
$d_h = v_h t$ $d_h = 5\,t$	$d_v = v_{o,v} t + \frac{1}{2} g t^2$ $d_v = \frac{1}{2} g t^2$
At this point we can't get any farther, so we need to turn to the vertical problem.	$1.5 = (\frac{1}{2})(9.8)\,t^2$ $\frac{1.5}{4.9} = 0.306 = t^2$ $t = \sqrt{0.306} = 0.55\,s$

Now that we know the time, we can substitute it back into the horizontal equation, giving:

$$d_h = (5)(0.55) = 2.77\,\text{m}$$

Use this space for summary and/or additional notes:

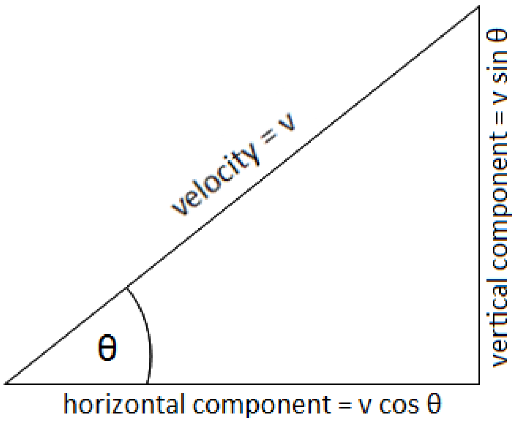
A graph of the vertical *vs.* horizontal motion of the ball looks like this:



Use this space for summary and/or additional notes:

Projectiles Launched at an Angle

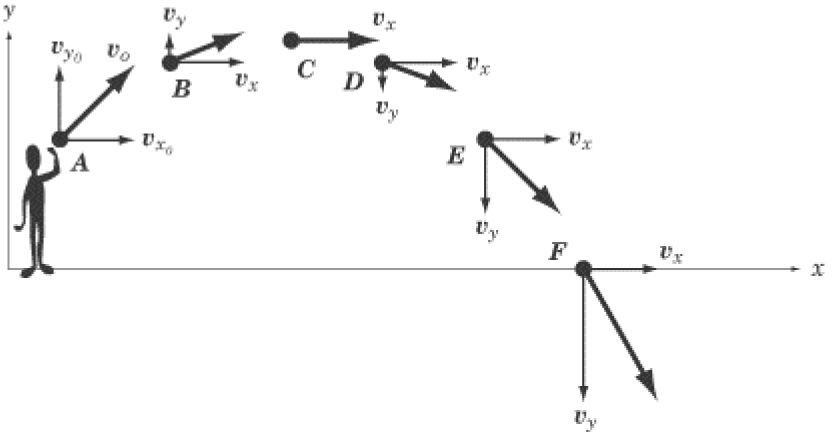
If the object is thrown/launched at an angle, you will need to use trigonometry to separate the velocity vector into its horizontal (x) and vertical (y) components:



Thus:

- horizontal velocity = $v_h = v \cos \theta$
- *initial* vertical velocity = $v_{o,v} = v \sin \theta$

Note that the vertical component of the velocity, v_y, is constantly changing because of acceleration due to gravity.



Use this space for summary and/or additional notes:

Sample Problems:

Q: An Angry Bird is launched upward from a slingshot at an angle of 40° with a velocity of $20\,\frac{\text{m}}{\text{s}}$. The bird strikes the pigs' fortress at the same height that it was launched from. How far away is the fortress?

A: We are looking for the horizontal distance, d_h.

We know the magnitude and direction of the launch, so we can find the horizontal and vertical components of the velocity using trigonometry:

$$v_h = v\cos\theta = 20\cos 40^\circ = (20)(0.766) = 15.3\,\tfrac{\text{m}}{\text{s}}$$
$$v_{o,v} = v\sin\theta = 20\sin 40^\circ = (20)(0.643) = 12.9\,\tfrac{\text{m}}{\text{s}}$$

We call the vertical component $v_{o,v}$ because it is both the initial velocity (v_o) and the vertical velocity (v_v), so we need both subscripts.

Let's make upward the positive vertical direction.

We want the horizontal distance (d_h), which is in the horizontal equation, so we start with:

$$d_h = v_h t$$
$$d_h = 15.3\,t$$

At this point we can't get any farther because we don't know the time, so we need to get it by solving the vertical equation.

The Angry Bird lands at the same height as it was launched, which means the vertical displacement (d_v) is zero. We already calculated that the initial vertical velocity is $12.9\,\frac{\text{m}}{\text{s}}$. If upward is the positive direction, acceleration due to gravity needs to be negative (because it's downward), so $a = g = -9.8\,\frac{\text{m}}{\text{s}^2}$.

Use this space for summary and/or additional notes:

Projectile Motion

The vertical equation is:

$$\begin{aligned}
d_v &= v_o t + \tfrac{1}{2}at^2 \\
0 &= 12.9\,t + (\tfrac{1}{2})(-9.8)\,t^2 \\
0 &= 12.9\,t - 4.9\,t^2 \\
0 &= t\,(12.9 - 4.9\,t) \\
t &= 0, \quad 12.9 - 4.9\,t = 0 \\
12.9 &= 4.9\,t \\
t &= \frac{12.9}{4.9} = 2.62\,\text{s}
\end{aligned}$$

Finally, we return to the horizontal equation to find d_h.

$$\begin{aligned}
d_h &= 15.3\,t \\
d_h &= (15.3)(2.62) = 40.2\,\text{m}
\end{aligned}$$

Q: A ball is thrown upward at an angle of 30° from a height of 1 m with a velocity of $18\,\frac{\text{m}}{\text{s}}$. How far does the ball travel?

A: As before, we are looking for the horizontal distance, d_h.

Again, we'll make upward the positive vertical direction.

Again we find the horizontal and vertical components of the velocity using trigonometry:

$$\begin{aligned}
v_h &= v\cos\theta = 18\cos 30° = (18)(0.866) = 15.6\,\tfrac{\text{m}}{\text{s}} \\
v_{o,v} &= v\sin\theta = 18\sin 30° = (18)(0.500) = 9.0\,\tfrac{\text{m}}{\text{s}}
\end{aligned}$$

Starting with the horizontal equation:

$$\begin{aligned}
d_h &= v_h t \\
d_h &= 15.6\,t
\end{aligned}$$

Again, we can't get any farther, so we need to get the time from the vertical problem.

Use this space for summary and/or additional notes:

The ball moves 1 m downwards. Its initial position is $s_o = +1\,\text{m}$, and its final position is s = 0, so we can use the equation:

$$s - s_o = v_o t + \tfrac{1}{2}at^2$$
$$0 - 1 = 9.0\,t + (\tfrac{1}{2})(-9.8)\,t^2$$
$$0 = 1 + 9.0\,t - 4.9\,t^2$$

This time we can't factor the equation, so we need to solve it using the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-9 \pm \sqrt{9^2 - (4)(-4.9)(1)}}{(2)(-4.9)}$$
$$t = \frac{9 \pm \sqrt{81 + 19.6}}{9.8} = \frac{9 \pm \sqrt{100.6}}{9.8}$$
$$t = \frac{9 \pm 10.03}{9.8} = \frac{19.03}{9.8} = 1.94\,\text{s}$$

Now we can go back to the horizontal equation and use the horizontal velocity $(15.6\,\tfrac{\text{m}}{\text{s}})$ and the time $(1.94\,\text{s})$ to find the distance:

$$d_h = v_h t = (15.6)(1.94) = 30.3\,\text{m}$$

Another way to solve the vertical problem is to realize that the ball goes up to its maximum height, then comes back down. The ball starts from a different height than it falls down to, so unfortunately we can't just find the time at the halfway point and double it.

At the maximum height, the vertical velocity is zero. For the ball going up, this gives:

$$v - v_o = at$$
$$0 - 9 = (-9.8)\,t_{up}$$
$$t_{up} = \frac{-9}{-9.8} = 0.918\,\text{s}$$

Use this space for summary and/or additional notes:

At this point, the ball has reached a height of:

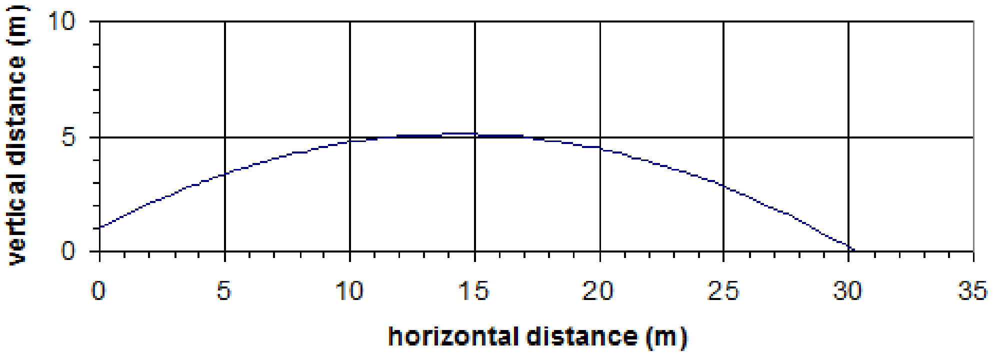
$$s = s_o + v_o t + \tfrac{1}{2}at^2$$
$$s = 1 + (9)(0.918) + (\tfrac{1}{2})(-9.8)(0.918)^2$$
$$s = 1 + 8.26 - 4.13 = 5.13\text{ m}$$

Now the ball falls from its maximum height of 5.13 m to the ground. The time this takes is:

$$d = \tfrac{1}{2}at_{down}^2$$
$$5.13 = (\tfrac{1}{2})(9.8)\,t_{down}^2$$
$$t_{down}^2 = \frac{5.13}{4.9} = 1.05$$
$$t_{down} = \sqrt{1.05} = 1.02\text{ s}$$

Thus the total elapsed time is $t_{up} + t_{down}$ = 0.918 + 1.02 = 1.94 s. (Q.E.D.)

The motion of this ball looks like this:



Use this space for summary and/or additional notes:

Introduction: Forces

Unit: Forces

Topics covered in this chapter:

In this chapter you will learn about different kinds of forces and how they relate.

- *Newton's Laws* and *Forces* describe basic scientific principles of how objects affect each other.
- *Free-Body Diagrams* describes a way of drawing a picture that represents forces acting on an object.
- *Forces Applied at an Angle*, *Ramp Problems*, and *Pulleys & Tension* describe some common situations involving forces and how to calculate the forces involved.
- *Friction* and *Aerodynamic Drag* describe situations in which a force is created by the action of another force.
- *Newton's Law of Universal Gravitation* describes how to calculate the force of gravity caused by massive objects such as planets and stars.

Use this space for summary and/or additional notes.

One of the first challenges will be working with variables that have subscripts. Each type of force uses the variable ***F***. Subscripts will be used to keep track of the different kinds of forces. This chapter also makes extensive use of vectors.

Another challenge in this chapter will be to "chain" equations together to solve problems. This involves finding the equation that has the quantity you need, and then using a second equation to find the quantity that you are missing from the first equation.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-1. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

HS-PS2-3. Apply scientific and engineering ideas to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.

HS-PS2-4. Use mathematical representations of Newton's Law of Gravitation ~~and Coulomb's Law~~ to describe and predict the gravitational ~~and electrostatic~~ forces between objects.

Massachusetts Curriculum Frameworks (2006):

1.4 Interpret and apply Newton's three laws of motion.

1.5 Use a free-body force diagram to show forces acting on a system consisting of a pair of interacting objects. For a diagram with only co-linear forces, determine the net force acting on a system and between the objects.

1.6 Distinguish qualitatively between static and kinetic friction, and describe their effects on the motion of objects.

1.7 Describe Newton's law of universal gravitation in terms of the attraction between two objects, their masses, and the distance between them.

Use this space for summary and/or additional notes:

Introduction: Forces

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Dynamics**, such as force, Newton's laws, statics, and friction.
- **Gravity**, such as the law of gravitation, orbits, and Kepler's laws.

1. What are Forces?
2. Types of Forces
3. Newton's Laws
4. Problem Solving With Newton's Laws
5. Pulleys
6. Inclined Planes
7. Newton's Law of Universal Gravitation
8. Weightlessness

Skills learned & applied in this chapter:

- Solving chains of equations.
- Using trigonometry to extract a vector in a desired direction.
- Rotational forces (torque).
- Working with material-specific constants from a table.
- Estimating the effect of changing one variable on another variable in the same equation.

Use this space for summary and/or additional notes:

Newton's Laws of Motion

Unit: Forces

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.4

Knowledge/Understanding Goals:

- Newton's three laws of motion

Language Objectives:

- Understand and correctly use the terms "at rest," "in motion," and "force."
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to forces.

Notes:

force: a push or pull on an object.

Newton's First Law: (the law of inertia)

Everything keeps doing what it was doing unless a force acts to change it. "An object at rest remains at rest, unless acted upon by a net force. An object in motion remains in motion, unless acted upon by a net force."

For example, a brick sitting on the floor will stay at rest on the floor forever unless an outside force moves it.

Wile E. Coyote, on the other hand, remains in motion…

Inertia is a property of mass. Everything with mass has inertia. The more mass an object has, the more inertia it has.

Use this space for summary and/or additional notes.

Newton's Second Law: Forces cause acceleration (a change in velocity). "A net force, $\vec{F}$, acting on an object causes the object to accelerate in the direction of the net force." In equation form:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

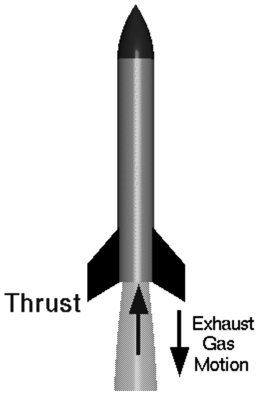
Newton's second law is probably the most important concept regarding forces.

- If there is a net force, the object accelerates (its velocity changes). If the object's velocity is changing (*i.e.,* if it accelerates), then there must be a net force acting on it.
- If there is no net force, the object's velocity will not change (Newton's First Law). If the object's velocity is not changing (*i.e.,* if it is not accelerating), there must be no net force. (If there are no net forces, either there are no forces acting on the object at all, or all of the forces on the object are equal and opposite, and their effects cancel.)

Newton's Third Law: Every force involves two objects. The first object exerts a force on the second, and the second object exerts the same force back on the first. "For every action, there is an equal and opposite reaction."

Burning fuel in a rocket causes exhaust gases to escape from the back of the rocket. The force from the gases exiting (the action) applies thrust to the rocket (the reaction), which propels the rocket forward.

If you punch a hole in a wall and break your hand in the process, your hand applied the force that broke the wall (the action). This caused a force from the wall, which broke your hand (the reaction). This may seem obvious, though you will find that someone who has just broken his hand by punching a hole in a wall is unlikely to be receptive to a physics lesson!



Use this space for summary and/or additional notes:

Systems

<u>system</u>: the collection of objects being considered in a problem.

For example, gravity is the force of attraction between two objects because of their mass. If you jump off the roof of the school, the Earth attracts you, and you attract the Earth. (Because the Earth has a lot more mass than you do, you move much farther toward the Earth than the Earth moves toward you.)

If the system is you, then the Earth exerts a net force on you, causing you to move. However, if the system is you plus the Earth, the force exerted by the Earth on you is equal to the force exerted by you on the Earth. Because the forces are equal in strength but in opposite directions ("equal and opposite"), their effects cancel, which means there is no net force on the system. (Yes, there are forces within the system, but that's not the same thing.)

Use this space for summary and/or additional notes:

Linear Forces

Unit: Forces

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Knowledge/Understanding Goals:

- what force is
- net force
- types of forces

Skills:

- identify the forces acting on an object

Language Objectives:

- Understand and correctly use the terms "force," "normal force," "contact force," "opposing force," and "weight."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

force: (vector) a push or pull on an object.

weight: the force of gravity pulling an object downward. In physics, we represent weight as the vector $\vec{F}_g$. Note that from Newton's second law, $\vec{F}_g = m\vec{g}$, which means on Earth, $\vec{F}_g = m(9.8)$.

opposing force: a force in the opposite direction of another force that reduces the effect of the original force.

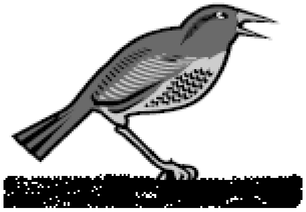
net force: the overall force on an object after opposing forces cancel out.

contact force: a opposing force that exists only while another force is acting on an object. Examples include friction and the normal force.

normal force: a force exerted by a surface (such as the ground or a wall) that resists the force of gravity on an object.

Use this space for summary and/or additional notes.

Because force is a vector, two forces in opposite directions counteract each other; if the magnitudes are equal, the forces completely cancel. For example, in the following picture:



the forces acting on the bird are:

- gravity, which is pulling the bird down, and
- the force from the ground, which is pushing the bird up.

The force from the ground is called the ***normal*** force. (The term "normal" is borrowed from math and means "perpendicular".)

The normal force is the force exerted by a surface (such as the ground, a table, or a wall) that counteracts another force on the object. The normal force is called a ***contact force***, because it is caused by the action of another force, and it exists *only* while the objects are in contact. The normal force is also an ***opposing force*** because it acts in the opposite direction from the applied force, and acts to lessen or diminish the applied force.

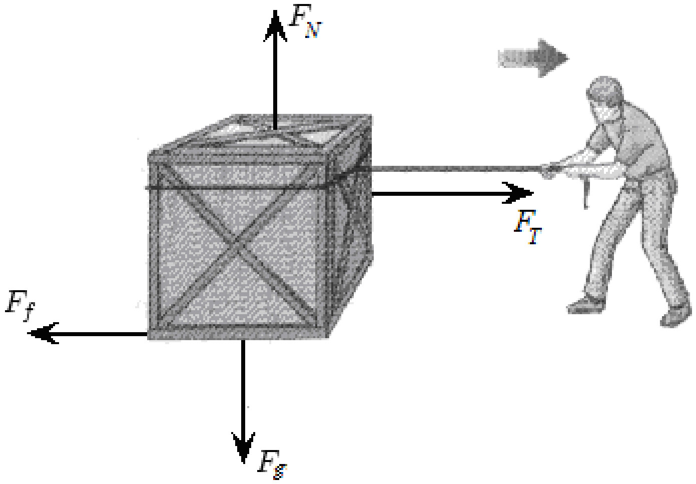
For example, if you push on a wall with a force of 10 N and the wall doesn't move, that means the force you apply to the wall causes a normal force of 10 N pushing back from the wall. This normal force continues for as long as you continue pushing.

friction: Like the normal force, friction is also both a contact force (caused by the action of another force) that opposes the force that causes it. The direction is parallel to the surfaces that are in contact, and opposite to the direction of motion.

Use this space for summary and/or additional notes:

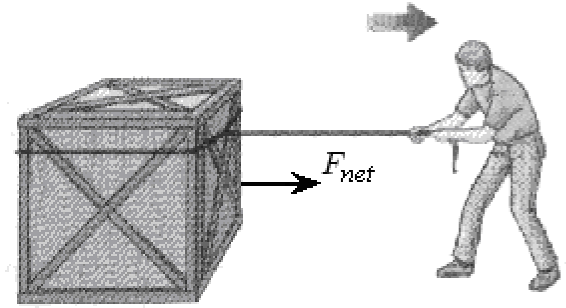
Linear Forces

An object can have several forces acting on it at once:



On the box in the above diagram, the forces are gravity ($\vec{F}_g$), the normal force ($\vec{F}_N$), the tension in the rope ($\vec{F}_T$), and friction ($\vec{F}_f$). Notice that in this problem, the arrow for tension is longer than the arrow for friction, because the force of tension is stronger than the force of friction.

net force: the remaining force on an object after canceling opposing forces. The net force on the box (after canceling out gravity and the normal force, and subtracting friction from the tension) would be represented as:

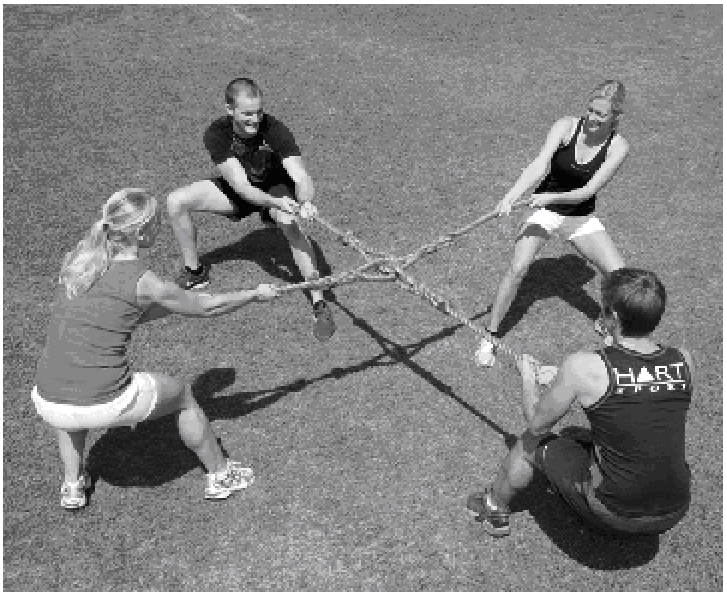


Because there is a net force to the right, the box will accelerate to the right as a result of the force.

Use this space for summary and/or additional notes:

Linear Forces

You can think of forces as participants in a multi-direction tug-of-war:



In the above situation, the net force is in the direction that the ropes will move.

<u>Forces cause acceleration.</u> If a net force acts on an object, it will speed up, slow down or change direction. Remember that *if the object's <u>velocity</u> is not changing, there is no <u>net</u> force, which means all of the forces on the object cancel.*

In the MKS system, the unit of force is the newton (N). One newton is defined as the amount of force that it would take to cause a 1 kg object to accelerate at a rate of $1\,\frac{\text{m}}{\text{s}^2}$.

$$1\,\text{N} \equiv 1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

Because the acceleration due to gravity on Earth is $9.8\,\frac{\text{m}}{\text{s}^2}$, $F = mg$ indicates that a 1 kg mass has a weight on Earth of 9.8 N.

In more familiar terms, one newton is approximately 3.6 ounces, which is the weight of an average-sized apple. One pound is approximately 4.45 N.

Use this space for summary and/or additional notes:

Common Types of Forces

Force	Symbol	Definition	Direction
gravity (weight)	$\vec{F}_g$	pull of gravity between two objects with mass, one of which is usually the Earth	between the centers of mass of the objects (usually toward the center of the Earth)
tension	$\vec{F}_T$	pull exerted by a rope/string/cable	away from the object in the direction of the string/rope/cable
normal	$\vec{F}_N$	contact force by a surface on an object	perpendicular to and away from surface
friction	$\vec{F}_f$	contact force that opposes sliding between surfaces	parallel to surface; opposite to direction of applied force
thrust	$\vec{F}_t$	push that accelerates objects such as rockets, planes & cars	in the same direction as acceleration
spring	$\vec{F}_s$	the push or pull exerted by a spring	opposite the displacement of the object
buoyancy	$\vec{F}_B$	the upward force by a fluid on objects less dense than the fluid	opposite to gravity
drag	$\vec{F}_D$	friction caused by the molecules of a fluid as an object moves through it	opposite to the direction of motion
lift	$\vec{F}_\ell$	the upward push (reaction force) by a fluid on an object (such as an airplane wing) moving through it at an angle	opposite to gravity.

Use this space for summary and/or additional notes:

Force	Symbol	Definition	Direction
electrostatic	$\vec{\boldsymbol{F}}_e$	the attraction or repulsion between two objects with an electrical charge	like charges repel; opposite charges attract
magnetic	$\vec{\boldsymbol{F}}_B$	the magnetic attraction or repulsion between two objects	like poles repel; opposite poles attract

You may have noticed that both buoyancy and magnetic force use the same subscript (both are represented as $\vec{\boldsymbol{F}}_B$). This should not cause too much confusion, because there are very few situations in which both of these forces would be applied to the same object. If that were to happen, we would need to change one of the subscripts. (For example, we might use $\vec{\boldsymbol{F}}_b$ for the buoyant force and $\vec{\boldsymbol{F}}_B$ for the magnetic force.)

Extension

The rate of change of force with respect is called "yank": $\vec{\boldsymbol{Y}} = \frac{\Delta\vec{\boldsymbol{F}}}{\Delta t}$. Just as $\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$, yank is the product of mass times jerk: $\vec{\boldsymbol{Y}} = m\vec{\boldsymbol{j}}$. Problems involving yank have not been see on the AP exam.

Use this space for summary and/or additional notes:

Free-Body Diagrams

Unit: Forces

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Skills:

- draw a free-body diagram representing the forces on an object

Language Objectives:

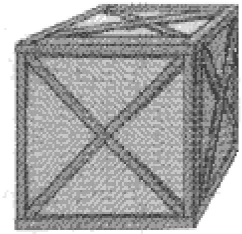
- Understand and correctly use the term “free-body diagram.”
- Identify forces by name and draw a free-body diagram.

Notes:

free-body diagram (force diagram): a diagram representing the forces acting on an object.

In a free-body diagram, we represent the object as a dot, and each force as an arrow. The direction of the arrow represents the direction of the force, and the relative lengths of the arrows represent the relative magnitudes of the forces.

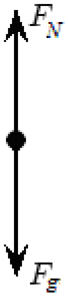
Consider a box that is at rest on the floor:



The forces on the box are:

- gravity, which is pulling the box toward the center of the Earth
- the normal force from the floor opposing gravity and holding up the box.

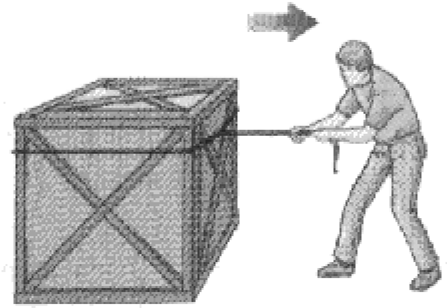
The free-body diagram for the box looks like this:



Use this space for summary and/or additional notes.

Free-Body Diagrams

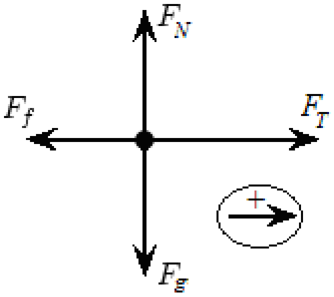
Now consider the following situation of a box that *accelerates* to the right as it is pulled across the floor by a rope:



From the picture and description, we can assume that:

- The box has weight, which means gravity is pulling down on it.
- The floor is holding up the box.
- The rope is pulling on the box.
- Friction between the box and the floor is resisting the force from the rope.
- Because the box is accelerating to the right, the force applied by the rope must be stronger than the force from friction.

In the free-body diagram, we again represent the object (the box) as a point, and the forces (vectors) as arrows. Because there is a net force, we need to include a legend that shows which direction is positive.



The forces are:

$\vec{F}_g$ = the force of gravity pulling down on the box

$\vec{F}_N$ = the normal force (the floor holding the box up)

$\vec{F}_T$ = the force of tension from the rope. (This might also be designated F_a because it is the force *applied* to the object.)

$\vec{F}_f$ = the force of friction resisting the motion of the box.

Use this space for summary and/or additional notes:

In the bottom right corner of the diagram, the arrow with the "+" sign shows that we have chosen to make the positive direction to the right.

Notice that the arrows for the normal force and gravity are equal in length, because in this problem, these two forces are equal in magnitude.

Notice that the arrow for friction is shorter than the arrow for tension, because in this problem, the tension is stronger than the force from friction. The difference between the lengths of these two vectors represents the net force, which is what causes the box to accelerate to the right.

Use this space for summary and/or additional notes.

Steps for Drawing Free-Body Diagrams

In general, the following are the steps for drawing most free-body diagrams.

1. Is gravity involved?
    - Represent gravity as $\vec{F}_g$ pointing straight down.
2. Is something holding the object up?
    - If it is a flat surface, it is the normal force ($\vec{F}_N$), perpendicular to the surface.
    - If it is a rope, chain, *etc.,* it is the force of tension ($\vec{F}_T$) acting along the rope, chain, *etc.*
3. Is there an external force pulling or pushing on the object?
4. Is there an opposing force?
    - If there are two surfaces in contact, there is almost always friction ($\vec{F}_f$), unless the problem specifically states that the surfaces are frictionless.
    - At low velocities, air resistance is very small and can usually be ignored, even if the problem does not explicitly say so.
    - Usually, all sources of friction are shown as one combined force. *E.g.,* if there is sliding friction along the ground and also air resistance, the $\vec{F}_f$ vector includes both.
5. Are forces acting in opposing directions?
    - If the problem requires calculations involving opposing forces, you need to indicate which direction is positive.

Use this space for summary and/or additional notes:

Newton’s Second Law

Unit: Forces

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.4

Skills:

- Solve problems relating to Newton’s Second Law ($F = ma$)
- Solve problems that combine kinematics (motion) and forces

Language Objectives:

- Identify and correctly use the quantities involved in a Newton’s Second Law problem.
- Identify and correctly use the quantities involved in a problem that combines kinematics and forces.

Notes:

Newton’s Second Law: Forces cause acceleration (a change in velocity). “A net force, $\vec{\boldsymbol{F}}$, acting on an object causes the object to accelerate in the direction of the net force.”

If there is a net force, the object accelerates (its velocity changes). If there is no net force, the object’s velocity remains the same.

If an object accelerates (its velocity changes), there was a net force on it. If an object’s velocity remains the same, there was no net force on it.

Remember that forces are vectors. No net force can mean no forces at all, or it can mean that there were equal forces in opposite directions and their effects cancel.

Use this space for summary and/or additional notes.

In equation form:

$$\vec{\boldsymbol{a}} = \frac{\vec{\boldsymbol{F}}_{\text{net}}}{m}$$

This form is preferred, because acceleration is what results from a force applied to a mass. (*I.e.,* force and mass are the independent variables and acceleration is the dependent variable. Forces cause acceleration, not the other way around.) However, the equation is more commonly written:

$$\vec{\boldsymbol{F}}_{\text{net}} = m\vec{\boldsymbol{a}}$$

Sample Problems

Most of the physics problems involving forces require the application of Newton's Second Law, $\vec{\boldsymbol{F}}_{\text{net}} = m\vec{\boldsymbol{a}}$.

Q: A net force of 50 N in the positive direction is applied to a cart that has a mass of 35 kg. How fast does the cart accelerate?

A: Applying Newton's Second Law:

$$\vec{\boldsymbol{a}} = \frac{\vec{\boldsymbol{F}}_{net}}{m}$$

$$\vec{\boldsymbol{a}} = \frac{50}{35}$$

$$\vec{\boldsymbol{a}} = 1.43\,\frac{\text{m}}{\text{s}^2}$$

Q: What is the weight of (*i.e.,* the force of gravity acting on) a 10 kg block?

A: $\vec{\boldsymbol{F}} = m\vec{\boldsymbol{g}}$

$\vec{\boldsymbol{F}} = (10.)(9.8) = 98\,\text{N}$

(Remember that we use the variable $\vec{\boldsymbol{g}}$ instead of $\vec{\boldsymbol{a}}$ when the acceleration is caused by gravity.)

Use this space for summary and/or additional notes:

Free Body Diagrams and Newton's Second Law

Free-body diagrams are often used in combination with Newton's second law ($\vec{a} = \frac{\vec{F}_{net}}{m}$); the free-body diagram enables you to calculate the net force, from which you can calculate mass or acceleration.

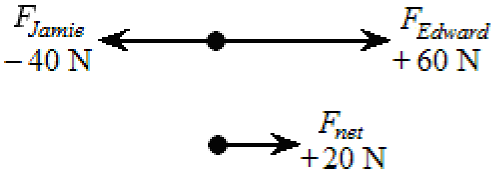
Sample Problem:

Q: Two children are fighting over a toy.

Jamie pulls to the left with a force of 40 N, and Edward pulls to the right with a force of 60 N. If the toy has a mass of 0.6 kg, what is the resulting acceleration of the toy?

A: Let us decide that the positive direction is to the right. (This is convenient because the force to the right is larger, which means the net force will come out to a positive number.)

The force diagram looks like this:



$$\vec{F}_{net} = m\vec{a}$$

$$+20 = (0.6)\,\vec{a}$$

$$\vec{a} = \frac{20}{0.6} = +33.3\,\tfrac{\text{m}}{\text{s}^2}\ \text{(to the right)}$$

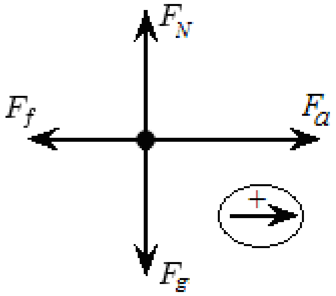
Use this space for summary and/or additional notes:

Newton's Second Law

Q: A 5.0 kg block is resting on a horizontal, flat surface. How much force is needed to overcome a force of 2.0 N of friction and accelerate the block from rest to a velocity of $6.0\,\frac{\text{m}}{\text{s}}$ over a 1.5-second interval?

A: This is a combination of a Newton's second law problem, and a motion problem. We will need a free-body diagram to be able to visualize what's going on.

The free-body diagram for the block looks like this:



1. The net force is given by:

$$F_{net} = F_a - F_f = F_a - 2$$
$$F_a = F_{net} + 2$$

   This means we need to find the net force, and then add 2 N to get the applied force.

2. To find the net force, we need the equation:

$$F_{net} = ma$$

3. We know that $m =$ 5.0 kg, but we don't know a. We need to find a in order to calculate F_{net}. For this, we will turn to the motion problem.

4. The problem tells us that $v_o = 0$, $v = 6.0\,\frac{\text{m}}{\text{s}}$, and $t = 1.5\,\text{s}$. Looking at the motion equations, we see that we have all of the variables except for a in the equation:

$$v - v_o = at$$

Use this space for summary and/or additional notes:

Newton's Second Law

5. Our strategy is to solve this equation for a, then substitute into $F_{net} = ma$ to find F_{net}, then use the relationship we found from the free-body diagram to find F_a.

   The acceleration is:

$$
\begin{aligned}
v - v_o &= at \\
6.0 - 0 &= a(1.5) \\
6.0 &= 1.5\,a \\
a &= 4.0\,\tfrac{\text{m}}{\text{s}^2}
\end{aligned}
$$

   Substituting back into $F_{net} = ma$ gives:

$$
\begin{aligned}
F_{net} &= ma \\
F_{net} &= (5.0)(4.0) \\
F_{net} &= 20.\,\text{N}
\end{aligned}
$$

   Finally:

$$
\begin{aligned}
F_a &= F_{net} + 2 \\
F_a &= 20 + 2 \\
F_a &= 22\,\text{N}
\end{aligned}
$$

Problems like this are straightforward to solve, but they are challenging because you need to keep chasing the quantities that you don't know until you have enough information to calculate them. However, you need to keep track of each step, because once you have found the last equation you need, you have to follow the steps in reverse order to get back to the answer.

Use this space for summary and/or additional notes:

Force Applied at an Angle

Unit: Forces

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.5

Skills:

- calculate forces applied at different angles using trigonometry

Language Objectives:

- Set up and solve word problems involving forces applied at an angle.

Notes:

An important property of vectors is that a vector has no effect on a second vector that is perpendicular to it. As we saw with projectiles, this means that the velocity of an object in the horizontal direction has no effect on the velocity of the same object in the vertical direction. This allowed us to solve for the horizontal and vertical velocities as separate problems.

The same is true for forces. If forces are perpendicular to each other, they act independently, and the two can be separated into separate, independent mathematical problems:

$$\vec{\boldsymbol{F}}_{\text{net,h}} = m\vec{\boldsymbol{a}}_{\text{h}}$$

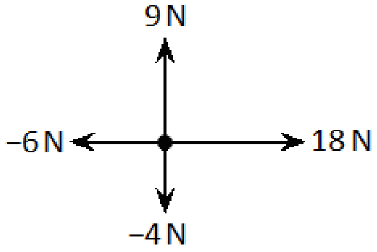
$$\vec{\boldsymbol{F}}_{\text{net,v}} = m\vec{\boldsymbol{a}}_{\text{v}}$$

Note that the above is for linear situations. Two-dimensional rotational problems require calculus, and are therefore outside the scope of this course.

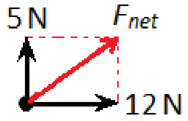
Use this space for summary and/or additional notes:

Force Applied at an Angle

For example, if we have the following forces acting on an object:



The net horizontal force (F_h) would be 18 N + (−6 N) = +12 N, and the net vertical force (F_V) would be 9 N + (−4 N) = +5 N. The total net force would be the resultant of the net horizontal and net vertical forces:



Using the Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = F_{net}^2$$

$$169 = F_{net}^2$$

$$\sqrt{169} = F_{net} = 13\,\text{N}$$

We can get the angle from trigonometry:

$$\tan\theta = \frac{opposite}{adjacent} = \frac{5}{12} = 0.417$$

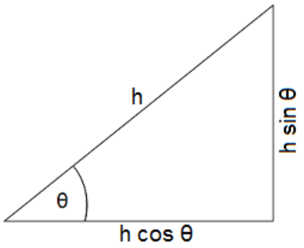
$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.417) = 22.6°$$

Use this space for summary and/or additional notes:

Force Applied at an Angle

If we have one or more forces that is neither vertical nor horizontal, we can use trigonometry to split the force into a vertical component and a horizontal component. Once all of the forces are either vertical or horizontal, we can add them as above.

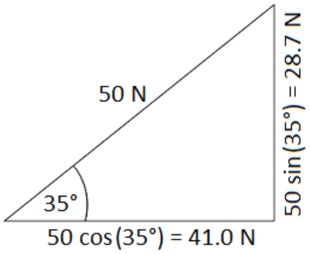
Recall the following relationships from trigonometry:



Suppose we have a force of 50 N at a direction of 35° above the horizontal. In the above diagram, this would mean that h = 50 N and θ = 35°:

The horizontal force is $h\cos(\theta) = 50\cos(35°) = 41.0\,\text{N}$

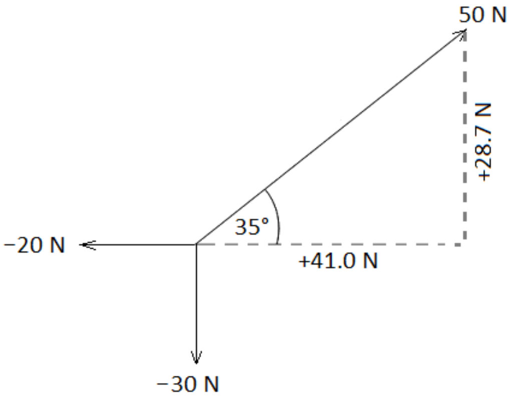
The vertical force is $h\cos(\theta) = 50\sin(35°) = 28.7\,\text{N}$



Now, suppose an object was subjected to the same 50 N force at an angle of 35° above the horizontal, but also a 20 N force to the left and a 30 N force downward.

The net horizontal force would therefore be 41 + (−20) = 21 N to the right.

The net vertical force would therefore be 28.7 + (−30) = −1.3 N upwards, which is the same as 1.3 N downwards.

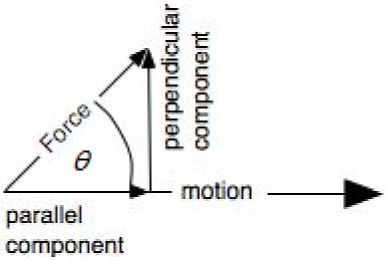


Once you have calculated the net vertical and horizontal forces, you can resolve them into a single net force, as in the above example.

Use this space for summary and/or additional notes:

Force Applied at an Angle

In some physics problems, a force is applied at an angle but the object can move in only one direction. A common problem is a force applied at an angle to an object resting on a flat surface, which causes the object to move horizontally:



In this situation, only the horizontal (parallel) component of the applied force actually causes the object to move. If magnitude of the total force is *F*, then the horizontal component of the force is given by:

$$F_h = F\cos\theta$$

If the object accelerates horizontally, that means <u>*only*</u> the horizontal component is causing the acceleration, which means the net force must be $F\cos\theta$ and we can ignore the vertical component.

For example, suppose the worker in the diagram at the right pushes on the hand truck with a force of 200 N, at an angle of 60°.

The force in the direction of motion (horizontally) would be:

$$F\cos\theta = 200\cos(60^\circ) = (200)(0.5) = 100\text{ N}$$

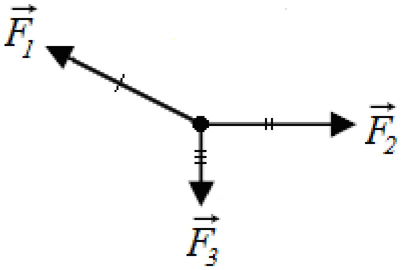
In other words, if the worker applies 200 N of force at an angle of 60°, the resulting horizontal force will be 100 N.

Use this space for summary and/or additional notes:

Force Applied at an Angle

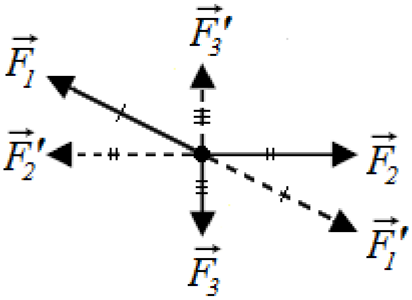
Static Problems Involving Forces at an Angle

Many problems involving forces at an angle are based on an object with no net force (either a stationary object or an object moving at constant velocity) that has three or more forces acting at different angles. In the following diagram, the forces are $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$.



$\vec{F}_1$ needs to cancel the combination of $\vec{F}_2$ and $\vec{F}_3$. This means:

1. If we split $\vec{F}_1$ into its horizontal and vertical components, those components would be exactly opposite $\vec{F}_2$ and $\vec{F}_3$, so we will call them $\vec{F}_2'$ and $\vec{F}_3'$.
2. If we calculate the resultant of vectors $\vec{F}_2$ and $\vec{F}_3$, this would be exactly opposite $\vec{F}_1$, so we will call this vector $\vec{F}_1'$.



As you can see, each vector is canceled by the resultant of the other two vectors, which shows why there is no net force.

Use this space for summary and/or additional notes:

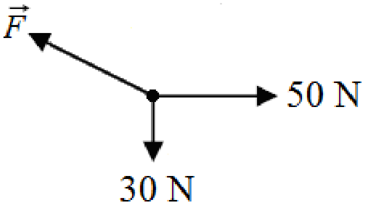
Force Applied at an Angle

Strategy

1. Resolve all known forces into their horizontal and vertical components.
2. Add the horizontal and vertical components separately.
3. Use the Pythagorean Theorem to find the magnitude of forces that are neither horizontal nor vertical.
4. Because you know the vertical and horizontal components of the resultant force, use arcsine ($\sin^{-1}$), arccosine ($\cos^{-1}$) or arctangent ($\tan^{-1}$) to find the angle.

Sample Problems:

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitude and direction of $\vec{F}$?

A: $\vec{F}$ is equal and opposite to the resultant of the other two vectors. The magnitude of the resultant is:

$$\|\vec{F}'\| = \sqrt{30^2 + 50^2} = \sqrt{3400} = 58.3\,\text{N}$$

The direction is:

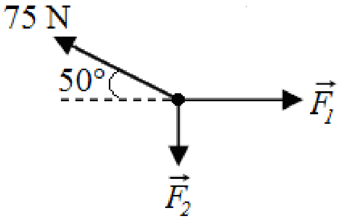
$$\tan\theta = \frac{30}{50} = 0.6$$

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.6) = 31.0^\circ \text{ up from the left (horizontal)}$$

Use this space for summary and/or additional notes:

Force Applied at an Angle

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitudes of $\vec{\boldsymbol{F}}_1$ and $\vec{\boldsymbol{F}}_2$?

A: $\vec{\boldsymbol{F}}_1$ and $\vec{\boldsymbol{F}}_2$ are equal and opposite to the vertical and horizontal components of the 75 N force, which we can find using trigonometry:

$$\|\vec{\boldsymbol{F}}_1\| = \text{horizontal} = 75\cos(50°) = (75)(0.643) = 48.2\text{ N}$$

$$\|\vec{\boldsymbol{F}}_2\| = \text{vertical} = 75\sin(50°) = (75)(0.766) = 57.5\text{ N}$$

Use this space for summary and/or additional notes:

Ramp Problems

Unit: Forces

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- calculate the forces on a ramp

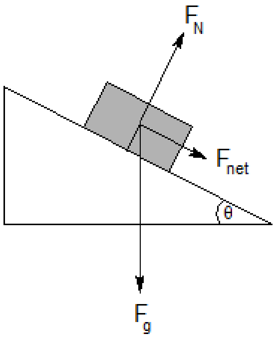
Language Objectives:

- Set up and solve word problems involving forces on a ramp.

Notes:

The direction of the normal force does not always directly oppose gravity. For example, if a block is resting on a (frictionless) ramp, the weight of the block is $\vec{F}_g$, in the direction of gravity. However, the normal force is perpendicular to the ramp, not to gravity.

If we were to add the vectors representing the two forces, we would see that the resultant—the net force—acts down the ramp:



Intuitively, we know that if the ramp is horizontal $(\theta = 0)$, the net force is zero and $\vec{F}_N = \vec{F}_g$, because they are equal and opposite.

We also know intuitively that if the ramp is vertical $(\theta = 90°)$, the net force is $\vec{F}_g$ and $\vec{F}_N = 0$.

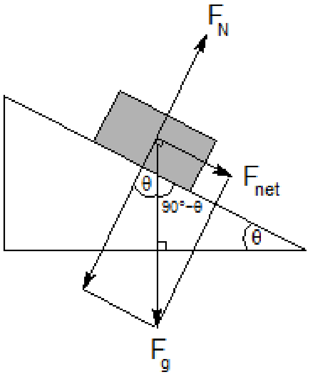
Use this space for summary and/or additional notes:

If the angle is between 0 and 90°, the net force must be between 0 and $\vec{F}_g$, and the proportion must be related to the angle (trigonometry!). Note that $\sin(0°)=0$ and $\sin(90°)=1$. Intuitively, it makes sense that multiplying $\vec{F}_g$ by the sine of the angle should give the net force down the ramp for any angle between 0 and 90°.

Similarly, If the angle is between 0 and 90°, the normal force must be between $\vec{F}_g$ (at 0) and 0 (at 90°). Again, the proportion must be related to the angle (trigonometry!). Note that $\cos(0°)=1$ and $\cos(90°)=0$. Intuitively, it makes sense that multiplying $\vec{F}_g$ by the cosine of the angle should give the normal force for any angle 0 and 90°.

Let's look at a geometric explanation:

From geometry, we can determine that the angle of the ramp, θ, is the same as the angle between gravity and the normal force.



From trigonometry, we can calculate that the component of gravity parallel to the ramp (which equals the net force down the ramp) is the side *opposite* angle θ. This means:

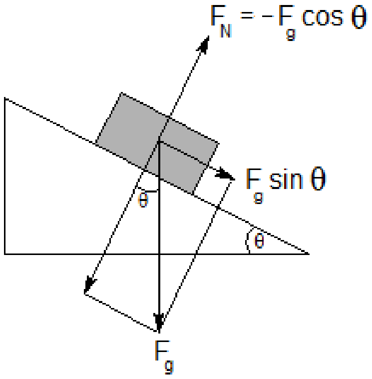
$$F_{net} = F_g \sin\theta$$

Use this space for summary and/or additional notes:

Ramp Problems

The component of gravity perpendicular to the ramp is $F_g \cos\theta$, which means the normal force is:

$$F_N = -F_g \cos\theta$$



(The negative sign is because we have chosen down to be the positive direction.)

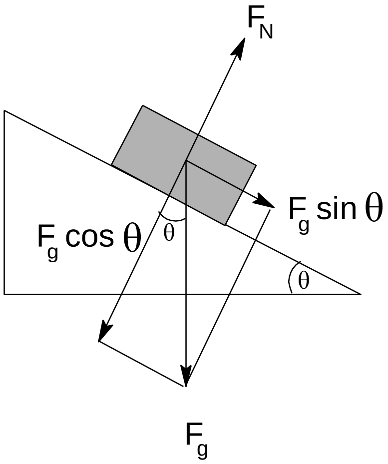
Use this space for summary and/or additional notes:

Ramp Problems

Sample Problem:

Q: A block with a mass of 2.5 kg sits on a frictionless ramp with an angle of inclination of 35°. How fast does the block accelerate down the ramp?

A: The weight of the block is $F_g = ma = (2.5)(9.8) = 24.5\,\text{N}$. However, the component of the force of gravity in the direction that the block slides down the ramp is $F_g \sin\theta$:



$$F_g \sin\theta = 24.5 \sin 35° = (24.5)(0.574) = 14.1\,\text{N}$$

Now that we know the net force (in the direction of motion), we can apply Newton's Second Law:

$$F = ma$$
$$14.1 = 2.5\,a$$
$$a = 5.64\,\tfrac{\text{m}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

Pulleys & Tension

Unit: Forces

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- set up & solve problems involving ropes under tension

Language Objectives:

- Understand and correctly use the terms "pulley" and "tension."
- Set up and solve word problems involving pulleys and/or tension.

Notes:

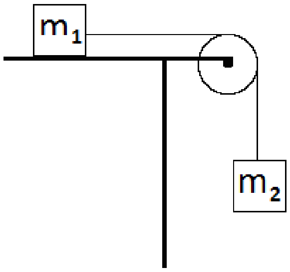
tension: the pulling force on a rope, cable, etc.

pulley: a wheel used to change the direction of tension on a rope

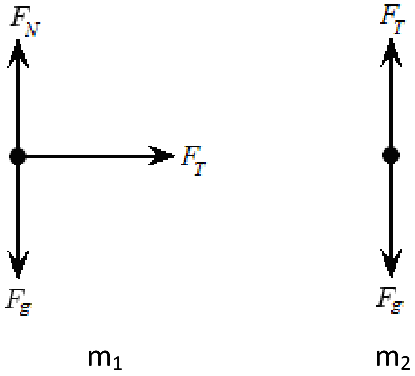
Use this space for summary and/or additional notes:

Pulleys & Tension

A typical problem involving pulleys and tension might be to find the acceleration of blocks m_1 and m_2 in the following situation. (Assume that the pulley has negligible mass and the surface and the pulley are frictionless.)



Free-body diagrams for the two masses would look like the following:



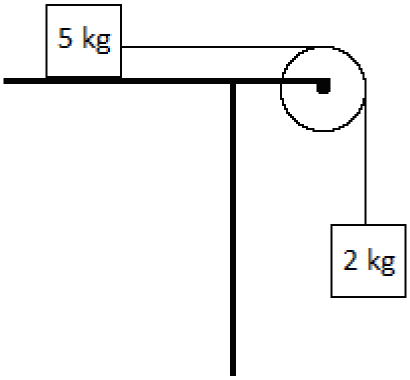
We know the following:

- F_T is the net force on m_1. Therefore, for m_1, $F_{net} = m_1 a$.
- For m_2, gravity and tension are pulling in opposite directions. The net force is therefore $F_{net} = F_g - F_T = m_2 a$
- Because the blocks are connected, both F_T and a must be the same for both blocks.

Use this space for summary and/or additional notes:

Sample Problem:

Q: Suppose we had the following situation:



Calculate the acceleration of the pair of blocks.

A: For the block on the table:

$$F_T = ma = (5)(a)$$

For the block hanging from the pulley:

$$F_{net} = F_g - F_T = ma = (2)(a)$$
$$(2)(9.8) - F_T = 2a$$
$$19.6 - F_T = 2a$$

Now we substitute $F_T = 5a$ into the second equation:

$$19.6 - 5a = 2a$$
$$19.6 = 7a$$
$$a = \frac{19.6}{7} = 2.8\,\tfrac{\text{m}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

Friction

Unit: Forces

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.6

Knowledge/Understanding Goals:

- difference between static & kinetic friction
- direction of the vector representing friction

Skills:

- calculate the frictional force on an object
- calculate net force in problems involving friction

Language Objectives:

- Understand and correctly use the terms “friction,” “static friction,” “kinetic friction,” and “coëfficient of friction.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving friction.

Notes:

friction: a contact force caused by the roughness of the materials in contact, deformations in the materials, and molecular attraction between materials. Frictional forces are always parallel to the plane of contact between two surfaces, and opposite to the direction of motion or applied force.

static friction: friction acting on an object at rest that resists its ability to start moving.

kinetic friction: friction resisting the motion of an object.

In almost all situations, static friction is a stronger force than kinetic friction.

Use this space for summary and/or additional notes:

Friction

<u>coëfficient of friction</u>: a constant that relates the frictional force on an object to the normal force. The coëfficient of friction is represented by the Greek letter μ (mu). It is a dimensionless number, which means that it has no units. (This is because μ is a ratio of two forces, so the units cancel.)

<u>coëfficient of static friction</u>: μ_s represents the coëfficient of friction for an object that is stationary (not moving)

<u>coëfficient of kinetic friction</u>: μ_k represents the coëfficient of friction for an object that is moving.

The coëfficient of friction takes into account the surface areas and surface characteristics of the objects in contact.

The force of friction on an object is given by the equations:

$$F_f \le \mu_s F_N \quad \text{for an object that is stationary, and}$$

$$F_f = \mu_k F_N \quad \text{for an object that is moving,}$$

Where F_f is the magnitude of the force of friction, μ_s and μ_k are the coëfficients of static and kinetic friction, respectively, and F_N is the magnitude of the normal force.

Note that the force of static friction is an inequality. For a stationary object, the force that resists sliding is, of course, equal to the force applied. However, once the applied force exceeds $\mu_s F_N$, the object starts moving and the equation for kinetic friction applies.

Use this space for summary and/or additional notes:

Friction as a Vector Quantity

Like other forces, the force of friction is, of course, actually a vector. Its direction is opposite to the direction of motion. However, the direction of the friction vector is opposite to the direction of the force attempting to cause sliding (in the case of static friction) or opposite to the direction of motion (in the case of kinetic friction). This is perpendicular to the normal force, but the direction of the normal force cannot tell us the direction of the force of friction.

We could write:

$\vec{\boldsymbol{F}}_f \leq -\mu_s F_N \hat{\boldsymbol{F}}_{applied}$	for an object that is stationary (because the force of friction is in the direction opposite to the applied force)
$\vec{\boldsymbol{F}}_f = -\mu_k F_N \hat{\boldsymbol{v}}$	for an object that is moving (because the force of friction is in the direction opposite to the velocity)

However, in practice, it is more cumbersome to try to explain the unit vectors than it is to simply declare that the force of static friction is opposite to the force that is attempting to cause sliding, and the force of kinetic friction is opposite to the direction of motion.

This means that whether the force of friction should be positive or negative needs to be determined directly from the coördinate system chosen for the problem.

Use this space for summary and/or additional notes:

Solving Simple Friction Problems

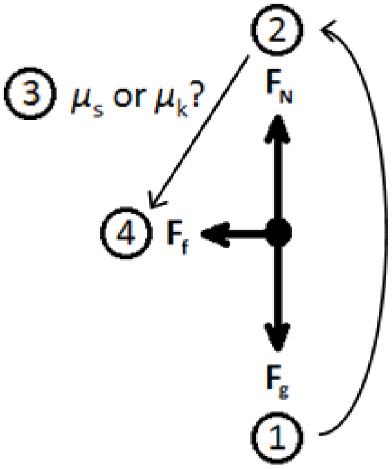
Because friction is a contact force, all friction problems involve friction in addition to some other (usually externally applied) force.

To calculate the force from friction, you need to:

1. Calculate the force of gravity. On Earth, $F_g = m(9.8)$
2. Calculate the normal force. If the object is resting on a horizontal surface (which is usually the case), the normal force is usually equal in magnitude to the force of gravity. This means that for an object sliding across a horizontal surface:

   $$F_N = F_g$$

3. Figure out whether the friction is static (there is an applied force, but the object is not moving), or kinetic (the object is moving). Look up the appropriate coëfficient of friction (μ_s for static friction, or for kinetic friction).



4. Calculate the force of friction from the equation:

   $$F_f \le \mu_s F_N \quad \text{or} \quad F_f = \mu_k F_N$$

   Make the force of friction positive or negative, as appropriate.

5. If the problem is asking for net force, remember to go back and calculate it now that you have calculated the force of friction.

   If friction is causing the object to slow down and eventually stop and there is no separate applied force, then:

   $$F_{net} = F_f$$

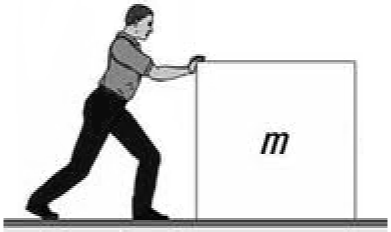
   However, if there is an applied force and friction is opposing it, then the net force would be:

   $$F_{net} = F_{applied} - F_f$$

Use this space for summary and/or additional notes:

Sample Problem:

Q: A person pushes a box at a constant velocity across a floor:



The box has a mass of 40 kg, and the coefficient of kinetic friction between the box and the floor is 0.35. What is the magnitude of the force that the person exerts on the box?

A: The box is moving at a constant velocity, which means there is no acceleration, and therefore no net force on the box. This means the force exerted by the person is exactly equal to the force of friction.

The force of friction between the box and the floor is given by the equation:

$$F_f = \mu_k F_N$$

The normal force is equal in magnitude to the weight of the box (F_g), which is given by the equation:

$$F_N = F_g = ma = (40)(9.8) = 392\,\text{N}$$

Therefore, the force of friction is:

$$F_f = \mu_k F_N$$
$$F_f = (0.35)(392) = 137\,\text{N}$$

Use this space for summary and/or additional notes:

Aerodynamic Drag

Unit: Forces

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- an intuitive sense of how aerodynamic drag works
- how aerodynamic drag is calculated

Language Objectives:

- Understand and correctly use the term “drag” when it refers to an object that is slowed down by a fluid.
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to aerodynamic drag.

Notes:

Most of the physics I problems that involve aerodynamic drag fall into two categories:

1. The drag force is small enough that we ignore it.
2. The drag force is equal to some other force that we can measure or calculate.

For simple situations involving aerodynamic drag, the drag force is given by the following equation:

$$\vec{\boldsymbol{F}}_D = -\tfrac{1}{2}\rho\vec{\boldsymbol{v}}^2 C_D A$$

where:

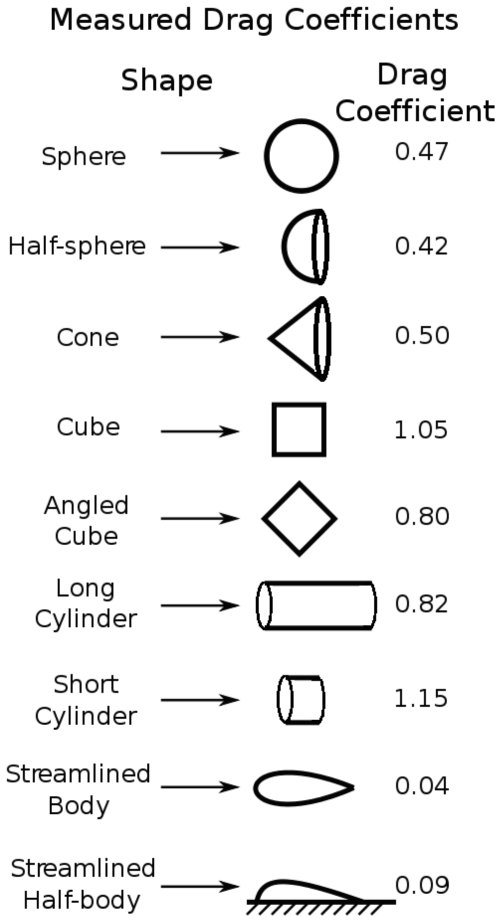
$\vec{\boldsymbol{F}}_D$ = drag force
ρ = density of the fluid
$\vec{\boldsymbol{v}}$ = velocity of the object (relative to the fluid)
C_D = drag coëfficient of the object (based on its shape)
A = cross-sectional area of the object

Use this space for summary and/or additional notes:

This equation applies when the object has a blunt form factor, and the object's velocity relative to the properties of the fluid (such as viscosity) causes turbulence in the object's wake (*i.e.,* behind the object).

The drag coëfficient, C_D, is a dimensionless number (meaning that it has no units) that encompasses all of the types of friction associated with aerodynamic drag. It serves the same purpose in drag problems that the coëfficient of friction, μ, serves in problems involving friction between solid surfaces.

Approximate drag coëfficients for simple shapes are given below, assuming that the fluid motion relative to the object is in the direction of the arrow.



Use this space for summary and/or additional notes:

Universal Gravitation

Unit: Forces

NGSS Standards: HS-PS2-4

MA Curriculum Frameworks (2006): 1.7

Skills:

- solve problems involving Newton's Law of Universal Gravitation

Language Objectives:

- Understand and correctly use the terms "gravity" and "gravitation."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Newton's Law of Universal Gravitation.

Notes:

Gravity is a force of attraction between two objects because of their mass. The cause of this attraction is not currently known. More mass causes a stronger force, but the force gets weaker as the object gets farther away.

This means that the gravitational pull of a single object is directly proportional to its mass, and inversely proportional to its distance, i.e.,

$$F_g \propto \frac{m}{d}$$

(The symbol $\propto$ means "is proportional to".)

If we have two objects, "1" and "2":

$$F_{g,1} \propto \frac{m_1}{d_1} \quad \text{and} \quad F_{g,2} \propto \frac{m_2}{d_2}$$

Because each object is pulling on the other one, the total force is therefore:

$$F_g \propto \frac{m_1}{d_1} \cdot \frac{m_2}{d_2} = \frac{m_1\, m_2}{d_1\, d_2} = \frac{m_1\, m_2}{d^2}$$

Use this space for summary and/or additional notes.

Finally, if we are using MKS units, the masses are in kilograms, the distance is in meters. If we want the force in newtons to be correct, we have to multiply by the appropriate conversion factor, which turns out to be $6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$. (The units are chosen because they cancel the m^2 and kg^2 from the formula and give newtons, which is the desired unit. Thus the formula becomes:

$$F_g = (6.67\times10^{-11}\ \tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2})\frac{m_1 m_2}{d^2}$$

The number $6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ is called the universal gravitation constant, and is represented by the symbol “G”. Thus we have Isaac Newton’s Law of Universal Gravitation in equation form:

$$F_g = \frac{G m_1 m_2}{d^2}$$

However, you may recall that on Earth, where the acceleration due to gravity is $g = 9.8\ \frac{\text{m}}{\text{s}^2}$, we also have the formula:

$$F_g = mg = m(9.8)$$

In the expression $g = \frac{G m_1}{d^2}$, G is $6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$, m_1 is the mass of the Earth $(5.98\times10^{24}\ \text{kg})$ and d is the radius of the Earth $(6.37\times10^{6}\ \text{m})$. If you plug these numbers into the formula, you get $g = 9.8\ \frac{\text{m}}{\text{s}^2}$ as expected, *i.e.,* near the surface of the Earth:

$$F_g = \boxed{\frac{G m_1}{d^2}}\, m_2 = g\cdot m_2 = 9.8\cdot m_2$$

Use this space for summary and/or additional notes:

Universal Gravitation

Sample Problems:

Q: Find the force of gravitational attraction between the Earth and a person with a mass of 75 kg. The mass of the Earth is 5.98 × 10^{24} kg, and its radius is 6.37 × 10^{6} m.

$$F_g = \frac{Gm_1m_2}{d^2}$$

A: $$F_g = \frac{(6.67\times10^{-11})(5.98\times10^{24})(75)}{(6.37\times10^{6})^2}$$

$$F_g = 737\ N$$

Q: Find the acceleration due to gravity on the moon.

A: $$g_{moon} = \frac{Gm_{moon}}{d^2_{moon}}$$

$$g_{moon} = \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2} = 1.62\ \tfrac{\text{m}}{\text{s}^2}$$

Q: If the distance between an object and the center of mass of a planet is tripled, what happens to the force of gravity between the planet and the object?

A: Starting with $F_g = \frac{Gm_1m_2}{d^2}$, if we replace d with 3d, we would get:

$$F'_g = \frac{Gm_1m_2}{(3d)^2} = \frac{Gm_1m_2}{9d^2} = \frac{1}{9}\cdot\frac{Gm_1m_2}{d^2}$$

Thus F'_g is $\frac{1}{9}$ of the original F_g.

Use this space for summary and/or additional notes:

Kepler's Laws of Planetary Motion

Unit: Forces

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- understand terms relating to angular position, speed & velocity

Skills Goals:

- solve problems using Kepler's Laws

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

The German mathematician and astronomer Johannes Kepler derived the laws and equations that govern planetary motion, which were published in three volumes between 1617 and 1621.

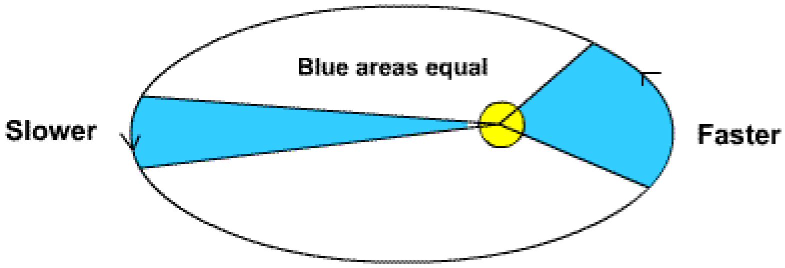
Kepler's First Law

The orbit of a planet is an ellipse, with the sun at one focus.

Use this space for summary and/or additional notes.

Kepler's Second Law

A line that joins a planet with the sun will sweep out equal areas in equal amounts of time.



I.e., the planet moves faster as it moves closer to the sun and slows down as it gets farther away. If the planet takes exactly 30 days to sweep out one of the blue areas above, then it will take exactly 30 days to sweep out the other blue area, and any other such area in its orbit.

While we now know that the planet's change in speed is caused by the force of gravity, Kepler's Laws were published fifty years before Isaac Newton published his theory of gravity.

Kepler's Third Law

If T is the period of time that a planet takes to revolve around a sun and $\bar{r}$ is the average radius of the planet from the sun (the length of the semi-major axis of its elliptical orbit) then:

$$\frac{T^2}{\bar{r}^3} = \text{constant for every planet in that solar system}$$

As it turns out, $\frac{T^2}{\bar{r}^3} = \frac{4\pi^2}{GM}$, where G is the universal gravitational constant and M is the mass of the star in question, which means this ratio is different for every planetary system. For our solar system, the value of $\frac{T^2}{\bar{r}^3}$ is approximately $9.5\times10^{-27}\,\frac{\text{s}^2}{\text{m}^3}$ or $3\times10^{-34}\,\frac{\text{y}^2}{\text{m}^3}$.

Use this space for summary and/or additional notes:

Introduction: Rotational Dynamics

Unit: Rotational Dynamics

Topics covered in this chapter:

This chapter deals with the ability of a moving object (or potential for an object to move) to affect other objects.

- *Linear Momentum* describes a way to represent the movement of an object and what happens when objects collide, and the equations that relate to it. *Impulse* describes changes in momentum.
- *Work* and *Energy* describe the ability to cause something to move and the related equations. *Power* describes the rate at which energy is applied.
- *Escape Velocity* and *Newton's Cradle* describe interesting applications of energy and momentum.

New challenges in this chapter involve keeping track of the same quantity applied to the same object, but at different times.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-2. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

Use this space for summary and/or additional notes:

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-3. Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

Massachusetts Curriculum Frameworks (2006):

2.1 Interpret and provide examples that illustrate the law of conservation of energy.

2.2 Interpret and provide examples of how energy can be converted from gravitational potential energy to kinetic energy and vice versa.

2.3 Describe both qualitatively and quantitatively how work can be expressed as a change in mechanical energy.

2.4 Describe both qualitatively and quantitatively the concept of power as work done per unit time.

2.5 Provide and interpret examples showing that linear momentum is the product of mass and velocity, and is always conserved (law of conservation of momentum). Calculate the momentum of an object.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Energy and Momentum**, such as potential and kinetic energy, work, power, impulse, and conservation laws.

1. What is Linear Momentum?
2. Impulse
3. Conservation of Momentum
4. Collisions
5. Center of Mass
6. Work
7. Energy
8. Forms of Energy
9. Power

Skills learned & applied in this chapter:

- Working with more than one instance of the same quantity in a problem.

Use this space for summary and/or additional notes:

Introduction: Rotational Dynamics

- Conservation laws (before/after problems).

Use this space for summary and/or additional notes:

Centripetal Force

Unit: Rotational Dynamics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.8

Knowledge/Understanding Goals:

- the difference between centripetal and centrifugal force

Skills:

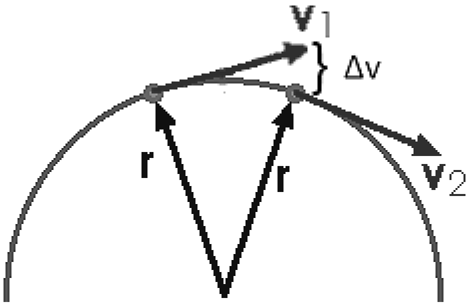
- calculate the centripetal force of an object moving in a circle

Language Objectives:

- Understand and correctly use the terms “rotation,” “centripetal force,” and “centrifugal force.”
- Explain the difference between centripetal force and centrifugal force.

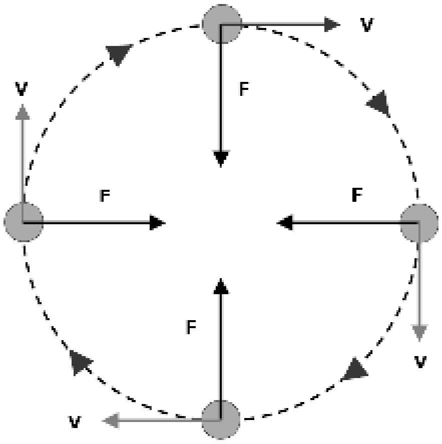
Notes:

As we saw previously, when an object is moving at a constant speed around a circle, its direction keeps changing toward the center of the circle as it goes around, which means *there is continuous acceleration toward the center of the circle.*



Because acceleration is caused by a net force (Newton’s second law of motion), if there is continuous acceleration toward the center of the circle, then there must be a continuous force toward the center of the circle.

This force is called “centripetal force”.



Use this space for summary and/or additional notes:

Centripetal Force

centripetal force: the inward force that keeps an object moving in a circle. If the centripetal force were removed, the object would fly away from the circle in a straight line that starts from a point tangent to the circle.

Recall that the formula for centripetal acceleration (a_c) is:

$$a_c = \frac{v^2}{r} = r\omega^2$$

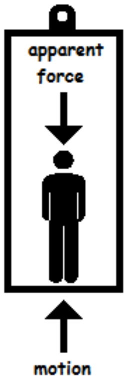
Given that $F = ma$, the equation for centripetal force is therefore:

$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$

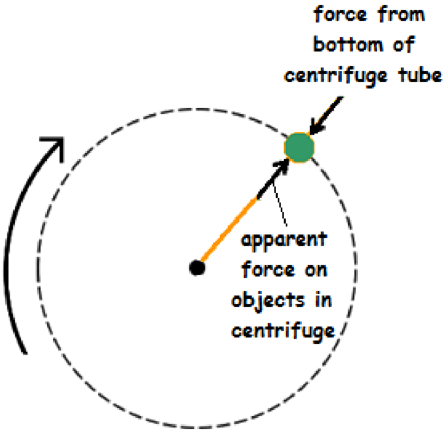
centrifugal "force": the apparent outward force felt by an object that is moving in a circle.

Centrifugal "force" is technically not a force as we would define it in physics. Centrifugal "force" is actually the inertia of objects resisting motion as they are continuously pulled toward the center of a circle by centripetal acceleration.

As an analogy, imagine that you are standing in an elevator. While the elevator is accelerating upward, the force between you and the floor of the elevator increases. An increase in the normal force from the floor because of the upward acceleration of the elevator feels the same as an increase in the downward force of gravity.



Similarly, a sample being spun in a centrifuge is subjected to the force *from the bottom of the centrifuge tube* as the tube is accelerated toward the center. The faster the rotation, the stronger the force. Again, an increase in the normal force from the bottom of the centrifuge tube would "feel" the same as a downward force toward the bottom of the centrifuge tube.



Use this space for summary and/or additional notes:

Sample Problems:

Q: A 300 kg roller coaster car reaches the bottom of a hill traveling at a speed of $20\,\frac{\text{m}}{\text{s}}$. If the track curves upwards with a radius of 50 m, what is the total force exerted by the track on the car?

A: The total force on the car is the normal force needed to resist the force of gravity on the car (equal to the weight of the car) plus the centripetal force exerted on the car as it moves in a circular path.

$$F_g = mg = (300)(9.8) = 2\,940\text{ N}$$

$$F_c = \frac{mv^2}{r} = \frac{(300)(20)^2}{50} = 2\,400\text{ N}$$

$$F_N = F_g + F_c = 2\,940 + 2\,400 = 5\,340\text{ N}$$

Q: A 20 g ball attached to a 60 cm long string is swung in a horizontal circle 80 times per minute. Neglecting gravity, what is the tension in the string?

A: Converting to MKS units, the mass of the ball is 0.02 kg and the string is 0.6 m long.

$$\omega = \frac{80\text{ revolutions}}{1\text{ min}} \times \frac{2\pi\text{ rad}}{\text{revolution}} \times \frac{1\text{ min}}{60\text{ s}} = \frac{8\pi}{3} = 8.38\,\frac{\text{rad}}{\text{s}}$$

$$F_T = F_c = mr\omega^2$$

$$F_T = F_c = (0.02)(0.6)(8.38)^2 = 0.842\text{ N}$$

Use this space for summary and/or additional notes:

Center of Mass

Unit: Rotational Dynamics

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Knowledge/Understanding Goals:

- center of mass

Skills:

- find the center of mass of an object

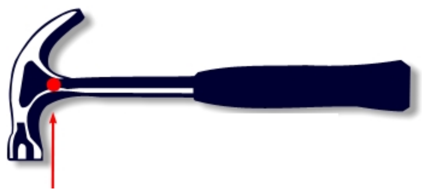
Language Objectives:

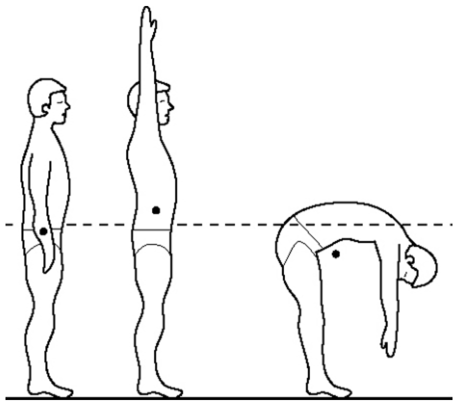
- Understand and correctly use the term "center of mass."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

center of mass: the point where all of an object's mass could be placed without changing the overall forces on the object or its rotational inertia.

Objects have nonzero volumes. For any object, some of the mass of the object will always be closer to the center of rotation, and some of the mass will always be farther away. In most of the problems that you will see in this course, we can simplify the problem by pretending that all of the mass of the object is at a single point.





Use this space for summary and/or additional notes:

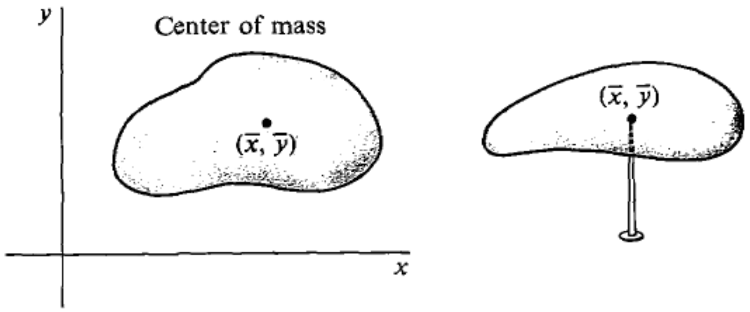
You can find the location of the center of mass of an object from the following formula:

$$r_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

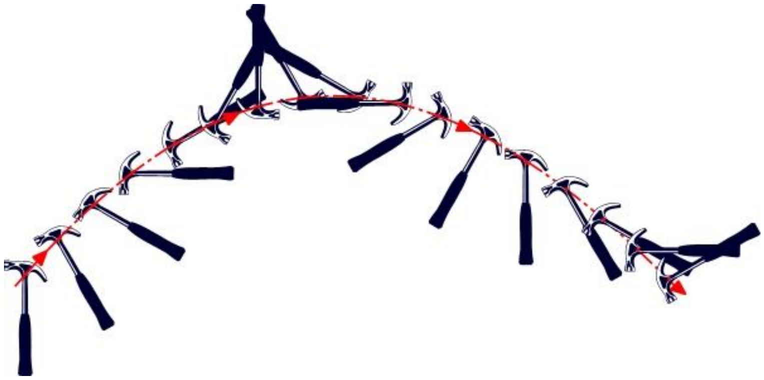
In this equation, the symbol Σ means "summation." When this symbol appears in a math equation, calculate the equation to the right of the symbol for each set of values, then add them up.

In this case, for each object (designated by a subscript), first multiply *mr* for that object, and then add up each of these products to get the numerator. Add up the masses to get the denominator. Then divide.

Because an object at rest remains at rest, this means that an object's center of mass is also the point at which the object will balance on a sharp point. (Actually, because gravity is involved, the object balances because the torques cancel. We will discuss that in detail later.)



Finally, note that an object that is rotating freely in space will always rotate about its center of mass:



Use this space for summary and/or additional notes:

Sample Problem:

Q: Two people sit at the ends of a massless 3.5 m long seesaw. One person has a mass of 59 kg, and the other has a mass of 71 kg. Where is their center of mass?

A: (Yes, there's no such thing as a massless seesaw. This is an idealization to make the problem easy to solve.)

In order to make this problem simple, let us place the 59-kg person at a distance of zero.

$$r_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

$$r_{cm} = \frac{(59)(0) + (71)(3.5)}{(59 + 71)}$$

$$r_{cm} = \frac{248.5}{130} = 1.91\,\text{m}$$

Their center of mass is 1.91 m away from the 59-kg person.

Use this space for summary and/or additional notes:

Rotational Inertia

Unit: Rotational Dynamics

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Knowledge/Understanding Goals:

- rotational inertia

Skills:

- calculate the rotational inertia of a system that includes one or more masses at different radii from the center of rotation

Language Objectives:

- Understand and correctly use the terms “rotational inertia” and “torque.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

inertia: the tendency for an object to continue to do what it is doing (remain at rest or remain in motion).

rotational inertia (or angular inertia): the tendency for a rotating object to continue rotating.

moment of inertia (I): a quantitative measure of the rotational inertia of an object. Moment of inertia is measured in units of kg·m^2.

Inertia in linear systems is a fairly easy concept to understand. The more mass an object has, the more it tends to remain at rest or in motion, and the more force is required to change its motion. *I.e.,* in a linear system, inertia depends only on mass.

Use this space for summary and/or additional notes:

Rotational Inertia

Rotational inertia is somewhat more complicated than inertia in a non-rotating system. Suppose we have a mass that is being rotated at the end of a string. (Let's imagine that we're doing this in space, so we can neglect the effects of gravity.) The mass's inertia keeps it moving around in a circle at the same speed. If you suddenly shorten the string, the mass continues moving at the *same speed through the air*, but because the radius is shorter, the mass makes more revolutions around the circle in a given amount of time.

In other words, the object has the same linear speed (*not* the same *velocity* because its *direction* is constantly changing), but its angular velocity (degrees per second) has increased.

This must mean that an object's moment of inertia (its tendency to continue moving at a constant angular velocity) must depend on its distance from the center of rotation as well as its mass.

The formula for moment of inertia is:

$$I = \sum_i m_i r_i^2$$

I.e., for each object or component (designated by a subscript), first multiply mr^2 for the object and then add up the rotational inertias for each of the objects to get the total.

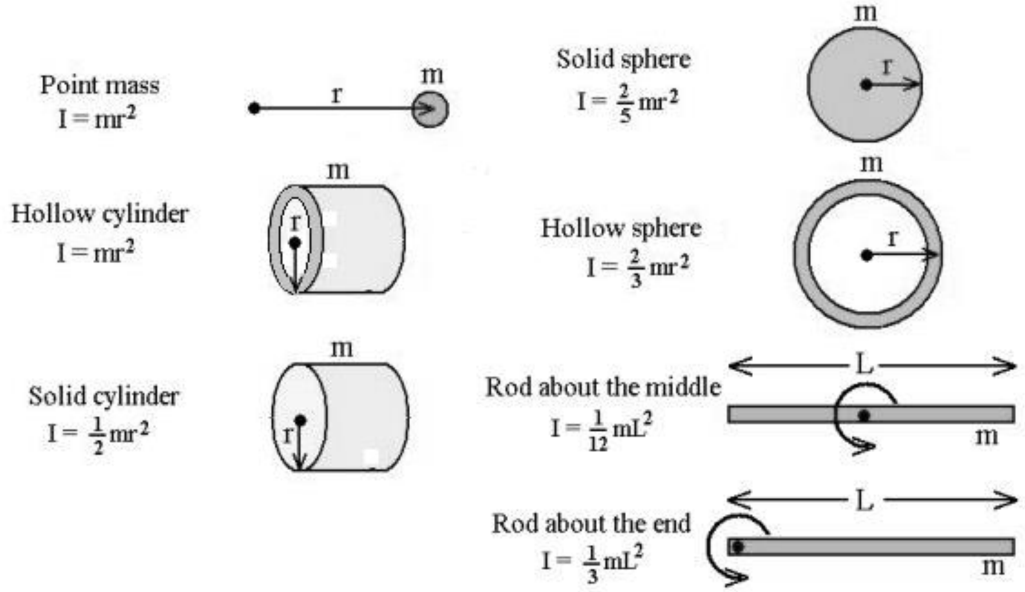
For a point mass (a simplification that assumes that the entire mass exists at a single point):

$$I = mr^2$$

This means the rotational inertia of the point-mass is the same as the rotational inertia of the object.

Use this space for summary and/or additional notes:

Calculating the moment of inertia for an arbitrary shape requires calculus. However, for solid, regular objects with well-defined shapes, their moments of inertia can be reduced to simple formulas:



In the above table, note that a rod can have a cross-section of any shape.

Sample Problem:

Q: A solid brass cylinder has a density of $8\,500\,\frac{\text{kg}}{\text{m}^3}$, a radius of 0.10 m and a height of 0.20 m and is rotated about its center. What is its moment of inertia?

A: In order to find the mass of the cylinder, we need to use the volume and the density.

$$V = \pi r^2 h = (3.14)(0.1)^2(0.2)$$

$$V = 0.00628\,\text{m}^3$$

$$\rho = \frac{m}{V}$$

$$8\,500 = \frac{m}{0.006\,28}$$

$$m = 53.4\,\text{kg}$$

Moment of inertia of a cylinder:

$$I = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}(53.4)(0.1)^2 = 0.534\,\text{kg}\cdot\text{m}^2$$

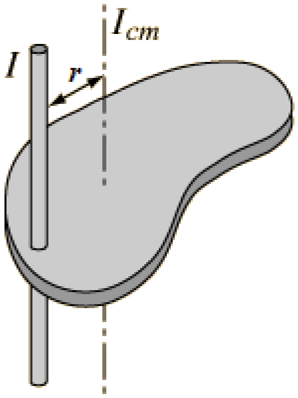
Use this space for summary and/or additional notes:

Parallel Axis Theorem

The moment of inertia of any object about an axis through its center of mass is always the minimum moment of inertia for any axis in that direction in space.

The moment of inertia about any axis that is parallel to the axis through the center of mass is given by:

$$I_{\text{parallel axis}} = I_{\text{cm}} + mr^2$$



Note that the formula for the moment of inertia of a point mass at a distance *r* from the center of rotation comes from the parallel axis theorem. The radius of the point mass itself is zero, which means:

$$I_{cm} = 0$$
$$I = I_{cm} + mr^2$$
$$I = 0 + mr^2$$

Use this space for summary and/or additional notes:

Torque

Unit: Rotational Dynamics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- center of mass and torque

Skills:

- calculate torque

Language Objectives:

- Understand and correctly use the terms “torque,” “axis of rotation,” “fulcrum,” “lever arm,” and “center of mass.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems involving torque.

Notes:

torque ($\vec{\boldsymbol{\tau}}$): a vector quantity that measures the effectiveness of a force in causing rotation. Take care to distinguish the Greek letter “τ” from the Roman letter “t”. Torque is measured in units of newton-meters:

$$1\,\text{N}\cdot\text{m} = 1\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

(Note that work and energy (which we will study later) are also measured in Newton-meters. However, work and energy are scalar quantities. The derived S.I. unit Joule (J) is a scalar unit that measures scalar Newton-meters. Because torque is a *vector* quantity, one Newton-meter of torque can *not* be described as one Joule. This is true even if we are only concerning ourselves with the magnitude.)

axis of rotation: the point around which an object rotates.

fulcrum: the point around which a lever pivots. Also called the pivot.

lever arm: the distance from the axis of rotation that a force is applied, causing a torque.

Use this space for summary and/or additional notes:

Torque

Just as force is the quantity that causes linear acceleration, torque is the quantity that causes angular acceleration (a change in angular velocity).

Because inertia is a property of mass, Newton's second law is the relationship between force and inertia. Newton's second law in rotational systems looks similar to Newton's second law in linear systems:

$\vec{\boldsymbol{a}} = \dfrac{\sum \vec{\boldsymbol{F}}}{m} = \dfrac{\vec{\boldsymbol{F}}_{net}}{m}$	$\vec{\boldsymbol{\alpha}} = \dfrac{\sum \vec{\boldsymbol{\tau}}}{I} = \dfrac{\vec{\boldsymbol{\tau}}_{net}}{I}$
$\vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}}$	$\vec{\boldsymbol{\tau}}_{net} = I\vec{\boldsymbol{\alpha}}$
linear	**rotational**

Torque is also the cross product of distance from the center of rotation ("lever arm") × force:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$$

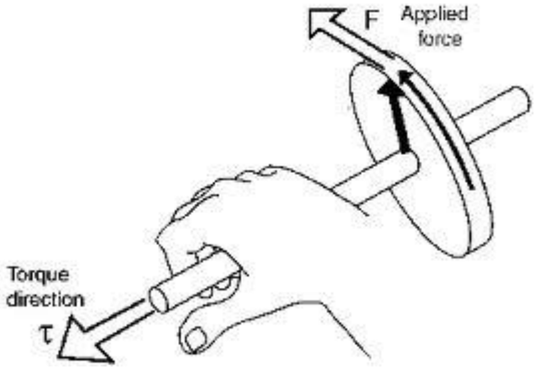
We use the variable *r* for the lever arm (which is a distance) because torque causes rotation, and *r* is the distance from the center of the circle (radius) at which the force is applied.

The magnitude of $\vec{\boldsymbol{\tau}}$ is $rF\sin\theta$, where θ is the angle between the lever arm and the applied force. $F\sin\theta$ is sometimes written as $F_{\perp}$ (the component of the force that is perpendicular to the radius) and sometimes $F_{||}$ (the component of the force that is parallel to the direction of motion). These notes will use $F_{\perp}$, because in many cases the force is applied to a lever, and the component of the force that causes the torque is perpendicular to the lever itself, so it is easy to think of it as "the amount of force that is perpendicular to the lever". This gives the equation:

$$\boldsymbol{\tau} = rF_{\perp} = I\alpha$$

Use this space for summary and/or additional notes:

Of course, because torque is the cross product of two vectors, it is a vector whose direction is perpendicular to both the lever arm and the force:



This is an application of the "right hand rule." If your fingers of your right hand curl from the first vector ($\vec{r}$) to the second ($\vec{F}$), then your thumb points in the direction of the resultant vector ($\vec{\tau}$). Note that the direction of the torque vector is parallel to the axis of rotation.

Note, however, that most people think of the "direction" of a torque as the direction of the rotation that the torque would produce (clockwise or counterclockwise) in the absence of any other torques.

Mathematically, the direction of the torque vector is needed only to cause torques in the same direction to add and torques in opposite directions to subtract. Most people find it easier to define the positive direction for torque in terms of the rotation (clockwise or counterclockwise) and ignore the direction of the vector.

Torque is measured in Newton-meters:

$$1\,\text{N}\cdot\text{m} = 1\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

Note that work and energy (which we will study later) are also measured in Newton-meters. However, work and energy are scalar quantities, whereas torque is a vector quantity. *I.e.,* one Newton-meter of torque is <u>*not*</u> the same as one newton-meter of work or energy. This is true even if we are only concerning ourselves with the magnitude.

Use this space for summary and/or additional notes:

Sample Problem

Q: If a perpendicular force of 20 N is applied to a wrench with a 25 cm handle, what is the torque applied to the bolt?

A: $\tau = rF_{\perp}$

$\tau = (0.25\text{m})(20\text{N})$

$\tau = 4\ \text{N}\cdot\text{m}$

Seesaw Problems

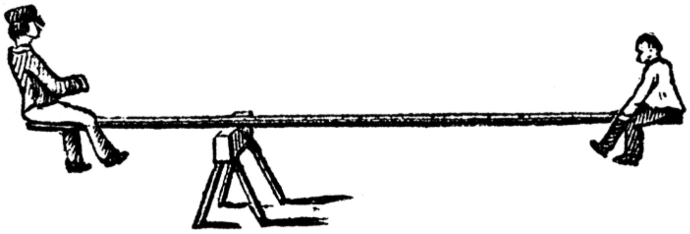
A seesaw problem is one in which objects on opposite sides of a lever (such as a seesaw) balance one another.

To solve seesaw problems, if the seesaw is not moving, then the torques must balance and the net torque must be zero.

The total torque on each side is the sum of the separate torques caused by the separate masses. Each of these masses can be considered as a point mass (infinitely small object) placed at the object's *center of mass*.

Sample Seesaw Problem

Q: In the following diagram, the mass of the person on the left is 90. kg and the mass of the person on the right is 50. kg. The board is 6.0 m long and has a mass of 20. kg.



Where should the board be positioned in order to balance the seesaw?

Use this space for summary and/or additional notes:

A: When the seesaw is balanced, the torques on the left have to equal the torques on the right.

Left Side (CCW)

Person

The person has a mass of 90 kg and is sitting at a distance *x* from the fulcrum:

$$\tau_{LP} = rF$$

$$\tau_{LP} = x(mg) = x(90\,\text{kg})(9.8\tfrac{\text{m}}{\text{s}^2})$$

$$\tau_{LP} = 882x$$

Board

The center of mass of the left part of the board is at a distance of $\frac{x}{2}$.

The weight (F_g) of the board to the left of the fulcum is $\left(\frac{x}{6}\right)(20)(9.8)$

$$\tau_{LB} = rF$$

$$\tau_{LB} = r(mg) = \left(\frac{x}{2}\right)\left(\frac{x}{6.0}\right)(20)(9.8)$$

$$\tau_{LB} = 16.\overline{3}\,x^2$$

Total

$$\tau_{ccw} = \tau_{LB} + \tau_{LP}$$

$$\tau_{ccw} = 16.3x^2 + 882x$$

Right Side (CW)

Person

The person on the right has a mass of 50 kg and is sitting at a distance of 6 − x from the fulcrum:

$$\tau_{RP} = rF$$

$$\tau_{RP} = r(mg) = (6-x)(50\,\text{kg})(9.8\tfrac{\text{m}}{\text{s}^2})$$

$$\tau_{RP} = 490\,(6-x)$$

$$\tau_{RP} = 2940 - 490x$$

Board

The center of mass of the right part of the board is at a distance of $\frac{6-x}{2}$.

The weight (F_g) of the board to the right of the fulcum is $\left(\frac{6-x}{6}\right)(20)(9.8)$

$$\tau_{RB} = rF$$

$$\tau_{RB} = r(mg) = \left(\frac{6-x}{2}\right)\left(\frac{6-x}{6}\right)(20)(9.8)$$

$$\tau_{RB} = 16.\overline{3}\,(36 - 12x + x^2)$$

$$\tau_{RB} = 588 - 196x + 16.\overline{3}x^2$$

Total

$$\tau_{cw} = \tau_{RB} + \tau_{RP}$$

$$\tau_{cw} = 16.3x^2 - 196x + 588 + 2940 - 490x$$

$$\tau_{cw} = 16.3x^2 - 686x + 3528$$

Use this space for summary and/or additional notes:

Because the seesaw is not rotating, the net torque must be zero. So we need to define the positive and negative directions. A common convention is to define counter-clockwise as the positive direction. (Most math classes already do this—a positive angle means counter-clockwise starting from zero at the *x*-axis.)

This gives:

$$\tau_{ccw} = 16.3x^2 + 882x$$
$$\tau_{cw} = -(16.3x^2 - 686x + 3528) = -16.3x^2 + 686x - 3528$$

And therefore:

$$0 = \tau_{net} = \sum \tau = \tau_{ccw} + \tau_{cw} = \cancel{16.3x^2} + 882x - \cancel{16.3x^2} + 686x - 3528$$
$$0 = 882x + 686x - 3528$$
$$0 = 1568x - 3528$$
$$1568x = 3528$$
$$x = \frac{3528}{1568} = 2.25\,\text{m}$$

The board should be placed with the fulcrum 2.25 m away from the person on the left.

Extension

Just as yank is the rate of change of force with respect to time, the rate of change of torque with respect to time is called rotatum: $\vec{\boldsymbol{P}} = \dfrac{\Delta \vec{\boldsymbol{\tau}}}{\Delta t} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{Y}}$.

Rotatum is also sometimes called the “moment of a yank,” because it is the rotational analogue to yank. Problems involving rotatum have not been seen on the AP exam.

Use this space for summary and/or additional notes:

Solving Linear & Rotational Dynamics Problems

Unit: Rotational Dynamics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.2

Skills:

- solve problems involving combinations of linear and rotational dynamics

Language Objectives:

- Set up and solve word problems relating to linear and/or rotational dynamics.

Notes:

The following is a summary of the variables used for dynamics problems:

Linear			**Angular**		
Var.	**Unit**	**Description**	**Var.**	**Unit**	**Description**
$\vec{x}$	m	position	$\vec{\theta}$	— (rad)	angle; angular position
$\vec{d}, \Delta\vec{x}$	m	displacement	$\Delta\vec{\theta}$	— (rad)	angular displacement
$\vec{v}$	$\frac{\text{m}}{\text{s}}$	velocity	$\vec{\omega}$	$\frac{1}{\text{s}}$ $(\frac{\text{rad}}{\text{s}})$	angular velocity
$\vec{a}$	$\frac{\text{m}}{\text{s}^2}$	acceleration	$\vec{\alpha}$	$\frac{1}{s^2}$ $\left(\frac{\text{rad}}{\text{s}^2}\right)$	angular acceleration
t	s	time	t	s	time
m	kg	mass	I	$\text{kg}\cdot\text{m}^2$	moment of inertia
$\vec{F}$	N	force	$\vec{\tau}$	N·m	torque

Notice that each of the linear variables has an angular counterpart.

Note that "radian" is not a unit. A radian is a ratio that describes an angle as the ratio of the arc length to the radius. This ratio is dimensionless (has no unit), because the units cancel. This means that an angle described in radians has no unit, and therefore never needs to be converted from one unit to another. However, we often write "rad" after an angle measured in radians to remind ourselves that the quantity describes an angle.

Use this space for summary and/or additional notes:

Solving Linear & Rotational Dynamics Problems

We have learned the following equations for solving motion problems:

Linear Equation	**Angular Equation**	**Relation**	**Comments**
$\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$	$\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}}$	$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = rF_{\perp}$	Quantity that produces acceleration
$\vec{\boldsymbol{F}}_c = m\vec{\boldsymbol{a}}_c = \frac{m\vec{\boldsymbol{v}}^2}{r}$		$\vec{\boldsymbol{F}}_c = m\vec{\boldsymbol{a}}_c = mr\vec{\boldsymbol{\omega}}^2$	Centripetal force (which causes centripetal acceleration)

Note that vector quantities (shown in bold) can be positive or negative, depending on direction.

Problems Involving Linear and Rotational Dynamics

The main points of the linear dynamics (forces) chapter were:

1. A net force produces acceleration. $\vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}}$
2. If there is no acceleration, then there is no net force, which means all forces must cancel in all directions. No acceleration may mean a static situation (nothing is moving) or constant velocity.
3. Forces are vectors. Perpendicular vectors do not affect each other, which means perpendicular forces do not affect each other.

The analogous points hold true for torques:

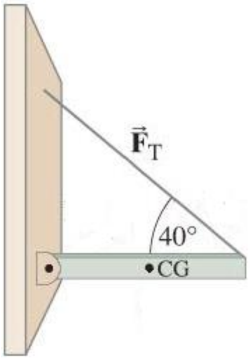
1. A net torque produces angular acceleration. $\vec{\boldsymbol{\tau}}_{net} = I\vec{\boldsymbol{\alpha}}$
2. If there is no angular acceleration, then there is no net torque, which means all torques must cancel. No angular acceleration may mean a static situation (nothing is rotating) or it may mean that there is rotation with constant angular velocity.
3. Torques are vectors. Perpendicular torques do not affect each other.
4. Torques and linear forces act independently.

Use this space for summary and/or additional notes:

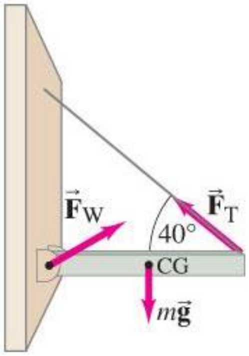
Solving Linear & Rotational Dynamics Problems

One of the most common types of problem involves a stationary object that has both linear forces and torques, both of which are in balance.

In the diagram at the right, a beam with a center of gravity (center of mass) in the middle (labeled "CG") is attached to a wall with a hinge. The end of the beam is held up with a rope at an angle of 40° above the horizontal.



The rope applies a torque to the beam at the end at an angle of rotation with a radius equal to the length of the beam. Gravity applies a force straight down on the beam.



1. Because the beam is not rotating, we know that $\vec{\tau}_{net}$ must be zero, which means the wall must apply a torque that counteracts the torque applied by the rope. (Note that the axis of rotation for the torque from the wall is the opposite end of the beam.)
2. Because the beam is not moving (translationally), we know that $\vec{F}_{net}$ must be zero in both the vertical and horizontal directions. This means that the vertical components of $\vec{F}_T$ and $\vec{F}_W$ must add up to $m\vec{g}$, and the horizontal components of $\vec{F}_T$ and $\vec{F}_W$ must cancel.

Use this space for summary and/or additional notes:

Introduction: Work, Energy & Momentum

Unit: Work, Energy & Momentum

Topics covered in this chapter:

This chapter deals with the ability of a moving object (or potential for an object to move) to affect other objects.

- *Linear Momentum* describes a way to represent the movement of an object and what happens when objects collide, and the equations that relate to it. *Impulse* describes changes in momentum.
- *Work* and *Energy* describe the ability to cause something to move and the related equations. *Power* describes the rate at which energy is applied.
- *Escape Velocity* and *Newton's Cradle* describe interesting applications of energy and momentum.

New challenges in this chapter involve keeping track of the same quantity applied to the same object, but at different times.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-2. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

Use this space for summary and/or additional notes:

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-3. Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

Massachusetts Curriculum Frameworks (2006):

2.1 Interpret and provide examples that illustrate the law of conservation of energy.

2.2 Interpret and provide examples of how energy can be converted from gravitational potential energy to kinetic energy and vice versa.

2.3 Describe both qualitatively and quantitatively how work can be expressed as a change in mechanical energy.

2.4 Describe both qualitatively and quantitatively the concept of power as work done per unit time.

2.5 Provide and interpret examples showing that linear momentum is the product of mass and velocity, and is always conserved (law of conservation of momentum). Calculate the momentum of an object.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Energy and Momentum**, such as potential and kinetic energy, work, power, impulse, and conservation laws.

10. What is Linear Momentum?
11. Impulse
12. Conservation of Momentum
13. Collisions
14. Center of Mass
15. Work
16. Energy
17. Forms of Energy

Use this space for summary and/or additional notes:

18. Power

Skills learned & applied in this chapter:

- Working with more than one instance of the same quantity in a problem.
- Conservation laws (before/after problems).

Use this space for summary and/or additional notes:

Work

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.3

Knowledge/Understanding Goals:

- understand the definition of work

Skills:

- calculate work done by a force applied to an object

Language Objectives:

- Understand and correctly use the term "work."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving work.

Notes:

work: the effort applied over a distance against a force.

For example, if you lift a heavy object off the ground, you are doing work against the force of gravity.

Mathematically, work is the dot product of the force vector and the displacement vector:

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}}$$

The dot product is one of three ways of multiplying vectors. The dot product is a scalar (a number without a direction), and is equal to the product of the magnitudes of the force and distance, and the cosine of the angle between them. This means:

$$W = Fd\cos\theta = F_{||}d$$

Where F is the magnitude of the force vector $\vec{\boldsymbol{F}}$, d is the magnitude of the displacement vector $\vec{\boldsymbol{d}}$, and θ is the angle between the two vectors. Sometimes $F\cos\theta$ is written as $F_{||}$, which means "the component of the force that is parallel to the direction of motion."

Use this space for summary and/or additional notes:

Work

Note that if the force and displacement are in the same direction, the angle $\theta = 0°$ which means $\cos\theta = \cos(0°) = 1$. In this case, $F_{||} = F\cos\theta = (F)(1) = F$ and the equation reduces to $W = Fd$.

Work is measured in newton-meters (N·m).

$$1\,\text{N}\cdot\text{m} \equiv 1\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

As we will see later, work is equivalent to energy, which means a newton-meter is equivalent to (but not the same as) a Joule.

Note that:

- If the displacement is zero, no work is done by the force. *E.g.,* if you hold a heavy box without moving it, you are exerting a force (counteracting the force of gravity) but you are not doing work.
- If the net force is zero, no work is done by the displacement (change in location) of the object. *E.g.,* if a cart is sliding across a frictionless air track at a constant velocity, the net force on the cart is zero, which means no work is being done.
- If the displacement is perpendicular to the direction of the applied force (θ = 90°, which means cos θ = 0), no work is done by the force. *E.g.,* you can slide a very heavy object along a roller conveyor (pictured below), because the force of gravity is acting vertically and the object's displacement is horizontal, which means gravity is doing no work, and therefore you do not have to do any work against gravity.

Use this space for summary and/or additional notes:

Work

In industry, gravity roller conveyors are usually set up at a slight downward angle:

In this situation, gravity is doing work to pull the boxes downward. (Vertical force; vertical displacement). There is friction in the bearings of the rollers, which keeps the boxes from accelerating. However, friction cannot *cause* displacement, which means friction cannot do work. The angle is designed so that the work done by gravity is exactly the work done against friction, which means the workers do not have to do any work (in the physics sense ☺) to keep the boxes moving.

Use this space for summary and/or additional notes:

Rotational Work

In a rotating system, distance is the arc length:

$$s = r\Delta\theta$$

Therefore, if the force is perpendicular to the radius:

$$W = Fs = rF_{\perp}\Delta\theta$$

For a force in any direction:

$$F_{\perp} = F\sin\theta$$

Recall that $\tau = rF\sin\theta$. This means:

$$W = r(F\sin\theta)\Delta\theta = (rF\sin\theta)\Delta\theta = \tau\Delta\theta$$

Think of torque as the rotational counterpart to force and the angle as the rotational counterpart to the distance. This means:

$W = F_{\parallel}d = F_{\parallel}\Delta x$	$W = \tau\Delta\theta$
translational	**rotational**

Use this space for summary and/or additional notes:

Sample Problems

Q: If you lifted a 60. kg box 1.5 m off the ground at a constant velocity over a period of 3.0 s, how much work did you do?

A: The box is being lifted, which means the work is done against the force of gravity.

$$W = F_{||} \cdot d = F_g d$$

$$W = F_g d = [mg]d$$

$$W = [(60)(9.8)](1.5)$$

$$W = [588](1.5) = 882\,\mathrm{N \cdot m}$$

Note that the amount of time it took to lift the box has nothing to do with the amount of work done.

It may be tempting to try to use the time to calculate velocity and acceleration in order to calculate the force. However, because the box is lifted at a constant velocity, the only force needed to lift the box is enough to overcome the weight of the box (F_g).

In general, if work is done to move an object vertically, the work is done against gravity, and you need to use $a = g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$ for the acceleration when you calculate $F = ma$.

Similarly, if work is done to move an object horizontally, the work is *not* against gravity and you need to find out the acceleration of the object before you can calculate $F = ma$.

Use this space for summary and/or additional notes:

Work

Q: In the picture below, the adult is pulling on the handle of the wagon with a force of 150. N at an angle of 60.0°.

If the adult pulls the wagon for a distance of 500. m, how much work does he do?

A: $W = F_{||}d$

$W = [F\cos\theta]d$

$W = [(150.)\cos 60.0^\circ](500.)$

$W = [(150.)(0.500)](500.\text{ m}) = 37\,500\text{ N}\cdot\text{m}$

Q: The lug nut on a wheel needs to be tightened with an applied torque of 150 N·m. If this torque is applied through a rotation of 30°, how much work is done.

A: The equation for work is:

$$W = \tau\Delta\theta$$

$$\tau = 150\text{ N}\cdot\text{m}$$

$$\Delta\theta = 30^\circ \times \frac{2\pi\text{ rad}}{360^\circ} = \frac{\pi}{6} = 0.524\text{ rad}$$

$$W = \tau\Delta\theta = (150)(0.524) = 78.5\text{ N}\cdot\text{m of work}$$

Notice that newton-meters of torque and newton-meters of work are completely different quantities, even though they have the same units. In this problem, 150 N·m of torque results in 78.5 N·m of work!

Use this space for summary and/or additional notes:

Energy

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

Knowledge/Understanding Goals:

- define energy
- conservation of energy
- work-energy theorem

Skills:

- calculate gravitational potential energy
- calculate kinetic energy

Language Objectives:

- Understand and correctly use the terms "energy," "kinetic energy," and "potential energy."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving energy and the conservation of energy.

Notes:

energy: the ability to do work

Energy is a scalar quantity that exists in several forms, including:

kinetic energy (K): the energy an object has because of its motion.

heat (Q): the energy an object has because of the kinetic energy of its molecules.

electrical work (W): the work done by applying an electric current over a period of time. Measured in joules (J) or kilowatt-hours (kWh).

potential energy (U) the unrealized energy that an object has because of its position, temperature, chemical reactions that could occur, *etc.*

Use this space for summary and/or additional notes:

In mechanics, kinetic energy usually refers to the energy of an object because of its mass and velocity. Potential energy usually refers to an object's position, and the ability of the force of gravity to cause it to move.

Energy is measured in Joules (J):

$$1\ \mathrm{J} \equiv 1\ \mathrm{N \cdot m} \equiv 1\ \tfrac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}}$$

Gravitational Potential Energy

The gravitational potential energy of an object is determined by the force of gravity and its distance above the ground (which is the distance over which the force of gravity is able to expend energy to do work on the object).

$$U = F_g h = mgh$$

(Remember that g is the variable that we use for acceleration when it equals the acceleration due to gravity, which is $9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$ on Earth.)

Translational Kinetic Energy

The translational (linear) kinetic energy of an object is related to its mass and velocity:

$$K = \tfrac{1}{2} m v^2$$

Use this space for summary and/or additional notes:

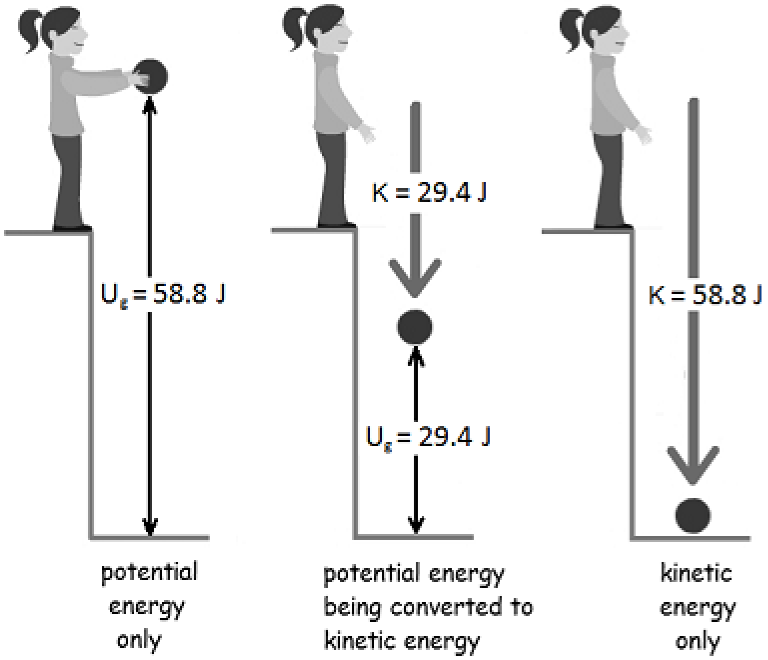
Conservation of Energy

In a *closed system*, the total energy is constant. Energy can be converted from one form to another. When this happens, the increase in any one form of energy is the result of a corresponding decrease in another form of energy.

In a system that has potential energy, kinetic energy and heat, the total energy is given by:

$$E_{total} = U + K + Q$$

In the following diagram, suppose that the girl drops a ball with a mass of 2 kg from a height of 3 m.



Before the girl lets go of the ball, it has 58.8 J of potential energy. As the ball falls to the ground, potential energy is gradually converted to kinetic energy. The potential energy continuously decreases and the kinetic energy continuously increases, but the total energy is always 58.8 J. After the ball hits the ground, 58.8 N·m of work was done by gravity, and the 58.8 J of kinetic energy is converted to other forms, such as thermal energy (the temperatures of the ball and the ground increase infinitesimally), sound, *etc.*

Use this space for summary and/or additional notes:

Work-Energy Theorem

Work and energy are equivalent quantities. The amount of work done by an object that has kinetic energy is given by the change in its kinetic energy:

$$W = \Delta K$$

Because kinetic energy lost equals potential energy gained (and *vice versa*), we can say that:

$$W = \Delta K = -\Delta U$$

Note that the units for work and energy—newton-meters and joules—are equivalent.

Work-energy theorem problems will give you information related to the kinetic energy of an object (such as its mass and a change in velocity) and ask you how much work was done.

A simple rule of thumb (meaning that it is *not always strictly* true) is:

- Potential energy is energy in the *future* (energy that is available for use).
- Kinetic energy is energy in the *present* (the energy of an object that is currently in motion).
- Work is the result of energy in the *past* (the result of kinetic energy having acted on an object).

Use this space for summary and/or additional notes:

Solving Conservation of Energy Problems

Conservation of energy problems involve recognizing that energy is changing from one form to another. Once you have figured out what is being converted, calculate the amount energy that is converted, and use the equation for the new form to calculate the desired quantity.

In mechanics, conservation of energy problems usually involve work, gravitational potential energy, and kinetic energy:

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = Fd\cos\theta \quad (= Fd \text{ if force \& displacement are in the same direction})$$

$$U_g = mgh$$

$$K = \tfrac{1}{2}mv^2$$

There are two common types of conservation of energy problems:

One type gives you the energy at the start, and you have to calculate the desired quantity (such as velocity or height) from the energy after the conversion.

$$\textit{Energy of one type} = \textit{Energy of another type}$$

$$\textit{Equation for first type} = \textit{Equation for second type}$$

The second type requires you to calculate the same type of energy at two different times or locations. You have to use the difference to calculate the desired quantity (such as velocity or height).

$$\textit{Energy after} - \textit{Energy before} = \textit{Energy Change}$$

$$\textit{Energy change of one type} = \textit{Energy change of another type} \text{ or } \textit{work done}$$

Use this space for summary and/or additional notes:

Sample Problems

Q: An 875 kg car accelerates from $22\,\frac{\text{m}}{\text{s}}$ to $44\,\frac{\text{m}}{\text{s}}$. What were the initial and final kinetic energies of the car? How much work did the engine do to accelerate it?

A: $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(875\,\text{kg})(22\,\frac{\text{m}}{\text{s}})^2 = 211\,750\text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(875\,\text{kg})(44\,\frac{\text{m}}{\text{s}})^2 = 847\,000\text{ J}$

$W = \Delta K = 847\,000\text{ J} - 211\,750\text{ J} = 635\,250\text{ J} = 635\,250\,\text{N}\cdot\text{m}$

The engine did $\boxed{635\,250\text{ N}\cdot\text{m}}$ of work.

Q: An 80. kg physics student falls off the roof of a 15 m high school building. How much kinetic energy does he have when he hits the ground?

A: There are two approaches to answer this question.

1. Recognize that the student's potential energy at the top of the building is entirely converted to kinetic energy when he hits the ground. Calculate his potential energy at the top of the building, and that will be his kinetic energy when he hits the ground.

$$U = mgh = (80\,\text{kg})(9.8\,\tfrac{\text{m}}{\text{s}^2})(15\,\text{m}) = 11\,760\text{ J}$$

The student has $\boxed{11\,760\text{ J}}$ of kinetic energy when he hits the ground.

Use this space for summary and/or additional notes:

2. Use motion equations to find the student's velocity when he hits the ground, based on the height of the building and acceleration due to gravity. Then use the formula $K = \frac{1}{2}mv^2$.

$$d = \tfrac{1}{2}at^2$$

$$15\,\text{m} = \tfrac{1}{2}(9.8\,\tfrac{\text{m}}{\text{s}^2})t^2$$

$$t^2 = 3.061$$

$$t = \sqrt{3.061} = 1.750\,\text{s}$$

$$v = at$$

$$v = (9.8\,\tfrac{\text{m}}{\text{s}^2})(1.750\,\text{s}) = 17.15\,\tfrac{\text{m}}{\text{s}}$$

$$K = \tfrac{1}{2}mv^2$$

$$K = \tfrac{1}{2}(80\,\text{kg})(17.15\tfrac{\text{m}}{\text{s}})^2$$

$$K = 11\,760\,\text{J}$$

As before, the student has approximately $\boxed{11\,760\text{ J}}$ of kinetic energy when he hits the ground.

Use this space for summary and/or additional notes:

Rotational Work

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

Knowledge/Understanding Goals:

- what rotational work is
- difference between rotational and translational work

Skills:

- calculate the rotational work done of an object

Language Objectives:

- Understand and correctly use the term “rotational work .”
- Set up and solve word problems involving work done on rotating objects.

Notes:

Just as work is done when a force causes an object to translate (move in a straight line), work is also done when a torque causes an object to rotate.

In a rotating system, the formula for work looks similar to the equation for work in linear systems, with force replaced by torque, and (translational) distance replaced by rotational distance (angle) angular velocity:

$W = F_{\parallel} d$	$W = \tau \Delta \theta$
translational	**rotational**

Use this space for summary and/or additional notes:

Sample Problem

Q: How much work is done on a bolt when it is turned 30° by applying a perpendicular force of 100 N to the end of a 36 cm long wrench?

A: The equation for work is:

$$W = \tau \Delta\theta$$

The torque is:

$$\tau = rF_{\perp}$$
$$\tau = (0.36)(100) = 36\,\mathrm{N\cdot m}$$

The angle, in radians, is:

$$\theta = 30° \times \frac{2\pi\,\mathrm{rad}}{360°} = \frac{\pi}{6}\,\mathrm{rad}$$

The work done on the bolt is therefore:

$$W = \tau \Delta\theta$$
$$W = (36)\left(\frac{\pi}{6}\right)$$
$$W = 6\pi = (6)(3.14) = 18.8\,\mathrm{N\cdot m}$$

Note that torque and work are different, unrelated quantities that both happen to have the same unit (N·m). However, *torque and work are not interchangeable*! 36 N·m of torque produced 18.8 N·m of work because of the angle through which the torque was applied. If the angle had been different, the amount of work would have been different.

This is an example of why you cannot rely exclusively on dimensional analysis to set up and solve problems!

Use this space for summary and/or additional notes:

Rotational Kinetic Energy

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

Knowledge/Understanding Goals:

- what rotational kinetic energy is
- difference between rotational and translational kinetic energy

Skills:

- calculate the rotational kinetic energy of an object
- solve conservation of energy problems involving objects that have linear motion (translational kinetic energy) and are also rotating (rotational kinetic energy)

Language Objectives:

- Understand and correctly use the term “rotational kinetic energy .”
- Set up and solve word problems involving rotational kinetic energy and conservation of energy with rotating objects.

Notes:

Just as an object that is moving in a straight line has kinetic energy, a rotating object also has kinetic energy.

When the log in the above picture is rotating, it transfers some of its kinetic energy to the people trying to stand on it, throwing one of them into the water.

Use this space for summary and/or additional notes:

Rotational Kinetic Energy

In a rotating system, the formula for kinetic energy looks similar to the equation for kinetic energy in linear systems, with mass (translational inertia) replaced by moment of inertia (rotational inertia), and linear (translational) velocity replaced by angular velocity:

$K_t = \frac{1}{2}mv^2$	$K_r = \frac{1}{2}I\omega^2$
translational	**rotational**

Sample Problem

Q: What is the rotational kinetic energy of a tenpin bowling ball that has a mass of 7.25 kg and a radius of 10.9 cm as it rolls down a bowling lane at $8.0\,\frac{\text{m}}{\text{s}}$?

A: The rotational kinetic energy is:

$$K_r = \tfrac{1}{2}I\omega^2$$

We can find the angular velocity from the translational velocity:

$$v = r\omega$$
$$8.0 = (0.109)\,\omega$$
$$\omega = \frac{8.0}{0.109} = 73.3\,\tfrac{\text{rad}}{\text{s}}$$

The bowling ball is a solid sphere. The moment of inertia of a solid sphere is:

$$I = \tfrac{2}{5}mr^2$$
$$I = \tfrac{2}{5}(7.25)(0.109)^2$$
$$I = 0.0345\,\text{kg}\cdot\text{m}^2$$

To find the rotational kinetic energy, we plug these numbers into the equation:

$$K_r = \tfrac{1}{2}I\omega^2$$
$$K_r = (\tfrac{1}{2})(0.0345)(73.3)^2$$
$$K_r = 185.6\text{ J}$$

Use this space for summary and/or additional notes:

Total Kinetic Energy

If an object (such as a ball) is rolling, then it is rotating and also moving (translationally). Its total kinetic energy must therefore be the sum of its translational kinetic energy and its rotational kinetic energy:

$$K_{total} = K_t + K_r$$
$$K_{total} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Sample problem:

Q: A standard Type 2 (medium) tennis ball is hollow and has a mass of 58 g and a diameter of 6.75 cm. If the tennis ball rolls 5.0 m across a floor in 1.25 s, how much total energy does the ball have?

A: The translational velocity of the tennis ball is:

$$v = \frac{d}{t} = \frac{5.0}{1.25} = 4.0\,\tfrac{\text{m}}{\text{s}}$$

The translational kinetic energy of the ball is therefore:

$$K_t = \tfrac{1}{2}mv^2 = (\tfrac{1}{2})(0.058)(4)^2 = 0.464\text{ J}$$

The angular velocity of the tennis ball can be calculated from:

$$v = r\omega$$
$$4 = (0.03375)\,\omega$$
$$\omega = \frac{4}{0.03375} = 118.5\,\tfrac{\text{rad}}{\text{s}}$$

The moment of inertia of a hollow sphere is:

$$I = \tfrac{2}{3}mr^2 = (\tfrac{2}{3})(0.058)(0.03375)^2 = 4.40\times 10^{-5}\text{ kg}\cdot\text{m}^2$$

The rotational kinetic energy is therefore:

$$K_r = \tfrac{1}{2}I\omega^2 = (\tfrac{1}{2})(4.40\times 10^{-5})(118.5)^2 = 0.309\text{ J}$$

Finally, the total kinetic energy is the sum of the translational and rotational kinetic energies:

$$K = K_t + K_r = 0.464 + 0.309 = 0.773\text{ J}$$

Use this space for summary and/or additional notes:

Escape Velocity

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

Knowledge/Understanding Goals:

- how fast a rocket or space ship needs to travel to escape Earth's (or any other planet's) gravity
- how escape velocity relates to gravitational potential energy, kinetic energy, and Newton's Law of Universal Gravitation

Language Objectives:

- Understand and correctly use the term "escape velocity."
- Set up and solve word problems involving escape velocity.

Notes:

Escape Velocity

If you want to send a rocket or space ship to explore the rest of the solar system or beyond, the rocket needs enough kinetic energy to escape from the force of Earth's gravity.

To explain the calculation, it is necessary to understand that our formula for gravitational potential energy is actually a simplification. Because the force of gravity attracts objects to the center of the Earth, the equation should be:

$$U_g = mg\Delta h = mg(h - h_o)$$

where h_o is the distance to the center of the Earth (radius of the Earth), and h is the total distance between the object and the center of the Earth. Normally, we calculate gravitational potential energy based on a difference in height between some height above the ground (h) and the ground ($h = 0$).

However, when we consider the problem of escaping from the gravity of the entire planet, we need to consider the potential energy relative to the center of the Earth, where the total force of gravity would be zero.

Use this space for summary and/or additional notes:

Escape Velocity

To do this, the rocket's kinetic energy ($\frac{1}{2}mv^2$) must be greater than or equal to the Earth's gravitational potential energy (*mgh*). This means $\frac{1}{2}mv^2 = F_g h$. We can apply Newton's Law of Universal Gravitation:

$$\tfrac{1}{2}mv^2 = F_g h = \left(\frac{Gm_1 m_2}{d^2}\right)h$$

However, because we need to escape the gravitational pull of the entire planet, we need to measure h from the *<u>center</u>* of the Earth, not the surface. This means that at the surface of the earth, h in the above equation is the same as d in Newton's Law of Universal Gravitation. Similarly, the mass of the spaceship is one of the masses (let's choose m_2) in Newton's Law of Universal Gravitation. This gives:

$$F_g h = F_g d = \frac{Gm_1 m_2}{d^2} d = \frac{Gm_1 m_2}{d}$$

$$\tfrac{1}{2}m_2 {v_e}^2 = \frac{Gm_1 m_2}{d}$$

$$\tfrac{1}{2}{v_e}^2 = \frac{Gm_1}{d}$$

$$v_e = \sqrt{\frac{2Gm_{planet}}{d}}$$

At the surface of the Earth, where $m_{planet} = 5.98 \times 10^{24}\text{ kg}$ and $d = 6.37 \times 10^6\text{ m}$, $v_e = 1.12 \times 10^4 \,\frac{\text{m}}{\text{s}} = 11200\,\frac{\text{m}}{\text{s}}$. This equals approximately 25 100 miles per hour.

Use this space for summary and/or additional notes:

Sample Problem:

Q: When Apollo 11 went to the moon, the space ship needed to achieve the Earth's escape velocity of $11200\,\frac{\text{m}}{\text{s}}$ to escape Earth's gravity. What velocity did the space ship need to achieve in order to escape the moon's gravity and return to Earth? (*I.e.,* what is the escape velocity on the surface of the moon?)

A: $$v_e = \sqrt{\frac{2Gm_{moon}}{d_{moon}}}$$

$$v_e = \sqrt{\frac{(2)(6.67\times10^{-11})(7.35\times10^{22})}{1.74\times10^{6}}}$$

$$v_e = 2\,370\,\tfrac{\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes:

Power

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.4

Skills:

- calculate power

Language Objectives:

- Understand and correctly use the term "power."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving power.

Notes:

power: a measure of the rate at which energy is applied or work is done. Power is calculated by dividing work (or energy) by time.

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{\Delta U}{t}$$

Power is a scalar quantity and is measured in Watts (W).

$$1\text{ W} = 1\,\tfrac{\text{J}}{\text{s}} = 1\,\tfrac{\text{N}\cdot\text{m}}{\text{s}} = 1\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$$

Note that utility companies measure energy in kilowatt-hours. This is because $P = \frac{W}{t}$, which means energy = $W = Pt$.

Because 1 kW = 1000 W and 1 h = 3600 s, this means
1 kWh = (1000 W)(3600 s) = 3 600 000 J

Because $W = Fd$, this means $P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$

Use this space for summary and/or additional notes:

Power in Rotational Systems

In a rotational system, the formula for power looks similar to the equation for power in linear systems, with force replaced by torque and linear velocity replaced by angular velocity:

$P = Fv$	$P = \tau\omega$
linear	**rotational**

Solving Power Problems

Many power problems require you to calculate the amount of work done or the change in energy, which you should recall is:

$W = F_{\|\|}d$	if the force is caused by linear displacement
$W = \tau\Delta\theta$	if the work is produced by a torque
$\Delta K_t = \frac{1}{2}m(v^2 - v_o^2)$	if the change in energy was caused by a change in velocity
$\Delta K_r = \frac{1}{2}I(\omega^2 - \omega_o^2)$	if the change in energy was caused by a change in angular velocity
$\Delta U_g = mg\,\Delta h$	if the change in energy was caused by a change in height

Once you have the work or energy, you can plug it in for either W, ΔE_k or ΔU, use the appropriate parts of the formula:

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{\Delta U}{t} = Fv = \tau\omega$$

and solve for the missing variable.

Use this space for summary and/or additional notes:

Power

Sample Problems

Q: What is the power output of an engine that pulls with a force of 500. N over a distance of 100. m in 25 s?

A: $W = Fd = (500)(100) = 50\,000\text{ J}$

$$P = \frac{W}{t} = \frac{50\,000}{25} = 2\,000\text{ W}$$

Q: A 60. W light bulb is powered by a generator that is powered by a falling 1.0 kg mass on a rope. Assuming the generator is 100% efficient (*i.e.,* no energy is lost when the generator converts its motion to electricity), how far must the mass fall in order to power the bulb at full brightness for 1.0 minute?

A:
$$P = \frac{\Delta U_g}{t} = \frac{mg\,\Delta h}{t}$$
$$60 = \frac{(1)(9.8)\,\Delta h}{60}$$
$$3600 = 9.8\,\Delta h$$
$$\Delta h = \frac{3600}{9.8} = 367\text{ m}$$

Note that 367 m is approximately the height of the Empire State Building. Think about that the next time you decide that it doesn't matter if you leave the lights on when you're not in a room!

Use this space for summary and/or additional notes:

Linear Momentum

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS2-2

MA Curriculum Frameworks (2006): 2.5

Knowledge/Understanding Goals:

- definition of momentum

Skills:

- calculate the momentum of an object
- solve problems involving the conservation of momentum

Language Objectives:

- Understand and correctly use the terms "collision," "elastic collision," "inelastic collision," "inertia" and "momentum."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving collisions and momentum.

Notes:

collision: when two or more objects come together and hit each other.

elastic collision: a collision in which the objects bounce off each other after they collide, without any loss of kinetic energy.

inelastic collision: a collision in which the objects remain together after colliding. In an inelastic collision, total energy is still conserved, but some of the energy is changed into other forms, so the amount of kinetic energy is different before vs. after the collision.

Any macroscopic collision (meaning a collision between objects that are larger than individual atoms or molecules) will convert some of the kinetic energy into internal energy and other forms of energy. This means that no large scale impacts are ever perfectly elastic.

inertia: an object's ability to resist the action of a force.

Use this space for summary and/or additional notes:

Linear Momentum

<u>momentum</u> ($\vec{\boldsymbol{p}}$): the quantity of movement of an amount of matter. You can also think of momentum as the amount of force that a moving object could transfer each second in a collision. Momentum is a vector quantity given by the formula:

$$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$$

and is measured in units of $\text{N}\cdot\text{s}$, or $\frac{\text{kg}\cdot\text{m}}{\text{s}}$.

Note that an object at rest $(\vec{\boldsymbol{v}} = 0)$ has a momentum of zero.

The net force on an object is its change in momentum with respect to time:

$$\vec{\boldsymbol{F}}_{net} = \frac{\Delta\vec{\boldsymbol{p}}}{\Delta t} = \frac{d\vec{\boldsymbol{p}}}{dt}$$

Inertia and momentum are related, but are not the same thing; an object has inertia even at rest. An object's momentum changes if either its mass or its velocity changes, but an the inertia of an object can change only if its mass changes.

In a closed system, momentum is conserved. This means that unless there is an outside force, the combined momentum of objects after they collide is equal to the combined momentum of the objects before the collision.

Use this space for summary and/or additional notes:

Solving Momentum Problems

Almost all momentum problems involve the conservation of momentum law:

$$\sum \vec{\boldsymbol{p}}_i = \sum \vec{\boldsymbol{p}}_f$$

The symbol ∑ is the Greek capital letter "sigma". In mathematics, the symbol ∑ means "summation". $\sum \vec{\boldsymbol{p}}$ means the sum of the momentums. The subscript "*I*" means initial (before the collision), and the subscript "*f*" means final (after the collision). In plain English, find each individual value of $\vec{\boldsymbol{p}}$ (positive or negative, depending on the direction) and then add them all up to find the total. The conservation of momentum law means that the total before a collision must be equal to the total after.

For example, if you had a momentum problem with two objects, the law of conservation of momentum becomes:

$$\vec{\boldsymbol{p}}_{1,i} + \vec{\boldsymbol{p}}_{2,i} = \vec{\boldsymbol{p}}_{1,f} + \vec{\boldsymbol{p}}_{2,f}$$

Notice that we have two subscripts after each "***p***", because we have two separate things to keep track of. The "*i*" and "*f*" mean "initial" and "final," and the "1" and "2" mean object #1 and object #2.

Because $\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$, we can replace each $\vec{\boldsymbol{p}}$ with $m\vec{\boldsymbol{v}}$.

For our momentum problem with two objects, this becomes:

$$m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} = m_1\vec{\boldsymbol{v}}_{1,f} + m_2\vec{\boldsymbol{v}}_{2,f}$$

Note that there are six separate quantities in this problem: m_1, m_2, $\vec{\boldsymbol{v}}_{1,i}$, $\vec{\boldsymbol{v}}_{2,i}$, $\vec{\boldsymbol{v}}_{1,f}$, and $\vec{\boldsymbol{v}}_{2,f}$. A typical momentum problem will give you (or enable you to calculate) five of these, and will ask you for the sixth.

Note also that most momentum problems do not mention the word "momentum." The problems usually give information about masses and velocities before and after some sort of collision, and it is up to you to realize that the problem involves conservation of momentum.

Use this space for summary and/or additional notes:

Most momentum problems involve collisions. Usually, there are two objects initially, and the objects either bounce off each other (elastic collision) or stick together (inelastic collision).

For an elastic collision between two objects, the problem is exactly as described above: there are six quantities to consider: the two masses, the two initial velocities, and the two final velocities. The equation relating them is:

$$m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} = m_1\vec{\boldsymbol{v}}_{1,f} + m_2\vec{\boldsymbol{v}}_{2,f}$$

To solve the problem, you need to obtain the quantities given in the word problem and solve for the missing one.

For an inelastic collision between two objects, the objects stick together after the collision, which means there is only one "object" afterwards. The total mass of the object is $m_T = m_1 + m_2$, and there is only one "object" with a final velocity.

there are five quantities: the two masses, the two initial velocities, and the final velocity of the combined object. The equation relating them is:

$$m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} = m_T\vec{\boldsymbol{v}}_f$$

To solve this problem, you need to obtain the quantities given in the word problem, add the two masses to find m_T, and solve for the missing quantity.

Use this space for summary and/or additional notes:

Sample Problems

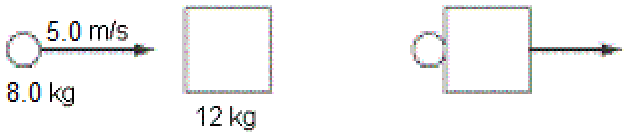
Q: What is the momentum of a 15 kg object moving at a velocity of $+3.0\,\frac{\text{m}}{\text{s}}$?

A: $\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$

$$\vec{\boldsymbol{p}} = (15\,\text{kg})(+3.0\,\tfrac{\text{m}}{\text{s}}) = 45\,\tfrac{\text{kg}\cdot\text{m}}{\text{s}} = +45\,\text{N}\cdot\text{s}$$

The answer is given as +45 N·s because momentum is a vector, and we indicate the direction as positive or negative.

Q: An object with a mass of 8.0 kg moving with a velocity of $+5.0\,\frac{\text{m}}{\text{s}}$ collides with a stationary object with a mass of 12 kg. If the two objects stick together after the collision, what is their velocity?



A: The momentum of the moving object before the collision is:

$$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}} = (8.0)(+5.0) = +40\,\text{N}\cdot\text{s}$$

The stationary object has a momentum of zero, so the total momentum of the two objects combined is +40 N·s.

After the collision, the total mass is 8.0 kg + 12 kg = 20 kg. The momentum after the collision must still be +40 N·s, which means the velocity is:

$$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}} \qquad 40 = 20\vec{\boldsymbol{v}} \qquad \vec{\boldsymbol{v}} = +2\,\tfrac{\text{m}}{\text{s}}$$

Using the equation, we would solve this as follows:

$$m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} = m_T\vec{\boldsymbol{v}}_f$$

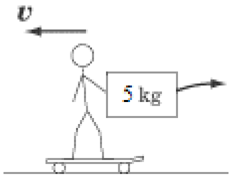
$$(8)(5) + (12)(0) = (8+12)\,\vec{\boldsymbol{v}}_f$$

$$40 = 20\,\vec{\boldsymbol{v}}_f$$

$$\vec{\boldsymbol{v}}_f = \frac{40}{20} = +2\,\tfrac{\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes:

Q: Mr. Stretchy has a mass of 60. kg and is holding a 5.0 kg box as he rides on a skateboard toward the west at a speed of $3.0\,\frac{\text{m}}{\text{s}}$. (Assume the 60. kg is the mass of Mr. Stretchy and the skateboard combined.) He throws the box behind him, giving it a velocity of $2.0\,\frac{\text{m}}{\text{s}}$ to the east.



What is Mr. Stretchy's velocity after throwing the box?

A: This problem is like an inelastic collision in reverse; Mr. Stretchy and the box are together before the "collision" and apart afterwards. The equation would therefore look like this:

$$m_T\vec{\boldsymbol{v}}_i = m_s\vec{\boldsymbol{v}}_{s,f} + m_b\vec{\boldsymbol{v}}_{b,f}$$

Where the subscript "*s*" is for Mr. Stretchy, and the subscript "*b*" is for the box. Note that after Mr. Stretchy throws the box, he is moving one direction and the box is moving the other, which means we need to be careful about our signs. Let's choose the direction Mr. Stretchy is moving (west) to be positive. Because the box is thrown to the east, this means the final velocity of the box will be $\vec{\boldsymbol{v}}_{b,f} = -2.0\,\frac{\text{m}}{\text{s}}$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned} m_T\vec{\boldsymbol{v}}_i &= m_s\vec{\boldsymbol{v}}_{s,f} + m_b\vec{\boldsymbol{v}}_{b,f} \\ (60+5)(+3) &= 60\,\vec{\boldsymbol{v}}_{s,f} + (5)(-2) \\ +195 &= 60\,\vec{\boldsymbol{v}}_{s,f} + (-10) \\ +205 &= 60\,\vec{\boldsymbol{v}}_{s,f} \\ \vec{\boldsymbol{v}}_{s,f} &= \frac{+205}{60} = +3.4\,\frac{\text{m}}{\text{s}} \end{aligned}$$

Use this space for summary and/or additional notes:

Q: A soccer ball that has a mass of 0.43 kg is rolling east with a velocity of $5.0\,\frac{\text{m}}{\text{s}}$. It collides with a volleyball that has a mass of 0.27 that is rolling west with a velocity of $6.5\,\frac{\text{m}}{\text{s}}$. After the collision, the soccer ball is rolling to the west with a velocity of $3.87\,\frac{\text{m}}{\text{s}}$. Assuming the collision is perfectly elastic and friction between both balls and the ground is negligible, what is the velocity (magnitude and direction) of the volleyball after the collision?

A: This is an elastic collision, so the soccer ball and the volleyball are separate both before and after the collision. The equation is:

$$m_s\vec{\boldsymbol{v}}_{s,i} + m_v\vec{\boldsymbol{v}}_{v,i} = m_s\vec{\boldsymbol{v}}_{s,f} + m_v\vec{\boldsymbol{v}}_{v,f}$$

Where the subscript "*s*" is for the soccer ball and the subscript "*v*" is for the volleyball. In all elastic collisions, assume we need to keep track of the directions, which means we need to be careful about our signs. We don't know which direction the volleyball will be moving after the collision (though a good guess would be that it will probably bounce off the soccer ball and move to the east). So let us arbitrarily choose east to be positive and west to be negative. This means:

quantity	**direction**	**value**
initial velocity of soccer ball	east	$+5.0\,\frac{\text{m}}{\text{s}}$
initial velocity of volleyball	west	$-6.5\,\frac{\text{m}}{\text{s}}$
final velocity of soccer ball	west	$-3.87\,\frac{\text{m}}{\text{s}}$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned}
m_s\vec{\boldsymbol{v}}_{s,i} + m_v\vec{\boldsymbol{v}}_{v,i} &= m_s\vec{\boldsymbol{v}}_{s,f} + m_v\vec{\boldsymbol{v}}_{v,f}\\
(0.43)(5.0) + (0.27)(-6.5) &= (0.43)(-3.87) + (0.27)\,\vec{\boldsymbol{v}}_{v,f}\\
2.15 + (-1.755) &= -1.664 + 0.27\,\vec{\boldsymbol{v}}_{v,f}\\
0.395 &= -1.664 + 0.27\,\vec{\boldsymbol{v}}_{v,f}\\
2.059 &= 0.27\,\vec{\boldsymbol{v}}_{v,f}\\
\vec{\boldsymbol{v}}_{s,f} &= \frac{+2.059}{0.27} = +7.63\,\frac{\text{m}}{\text{s}}
\end{aligned}$$

Positive means east, so the volleyball is moving with a velocity of $7.63\,\frac{\text{m}}{\text{s}}$ to the east.

Use this space for summary and/or additional notes:

Momentum and Kinetic Energy

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.5

Knowledge/Understanding Goals:

- how the conservation of momentum and conservation of energy are related

Language Objectives:

- Accurately describe how Newton's cradle illustrates the principles of conservation of energy and momentum, using appropriate academic language.

Notes:

For those of you taking calculus, you may recognize that momentum is the derivative of kinetic energy with respect to velocity.

$$\frac{d}{dv}\left(\tfrac{1}{2}mv^2\right) = mv$$

$$\frac{d}{dv}(K) = p$$

In an algebra-based physics class, this becomes:

$$p = \frac{\Delta K}{\Delta v}$$

which means momentum is the slope of a graph of kinetic energy as a function of velocity.

We can also use $p = mv$ to eliminate *v* from the kinetic energy equation, giving the equation:

$$K = \frac{p^2}{2m}$$

Use this space for summary and/or additional notes:

Momentum and Kinetic Energy

The relationship between momentum and kinetic energy explains why the velocities of objects after a collision are determined by the collision.

Because kinetic energy and momentum must *both* be conserved in an elastic collision, the two final velocities are actually determined by the masses and the initial velocities. The masses and initial velocities are determined before the collision. The only variables are the two velocities after the collision. This means there are two equations (conservation of momentum and conservation of kinetic energy) and two unknowns ($v_{1,f}$ and $v_{2,f}$).

For a perfectly elastic collision, conservation of momentum states:

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

and conservation of kinetic energy states:

$$\tfrac{1}{2} m_1 v_{1,i}^2 + \tfrac{1}{2} m_2 v_{2,i}^2 = \tfrac{1}{2} m_1 v_{1,f}^2 + \tfrac{1}{2} m_2 v_{2,f}^2$$

If we use these two equations to solve for $v_{1,f}$ and $v_{2,f}$ in terms of the other variables, the result is the following:

$$v_{1,f} = \frac{v_{1,i}(m_1 - m_2) + 2m_2 v_{2,i}}{m_1 + m_2}$$

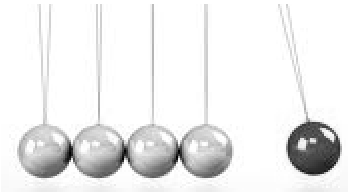
$$v_{2,f} = \frac{v_{2,i}(m_2 - m_1) + 2m_1 v_{1,i}}{m_1 + m_2}$$

For an inelastic collision, there is no solution that satisfies both the conservation of momentum and the conservation of kinetic energy; the total kinetic energy after the collision is al less than the total kinetic energy before. This matches what we observe, which is that momentum is conserved, but some of the kinetic energy is converted to heat energy during the collision.

Use this space for summary and/or additional notes:

Newton's Cradle

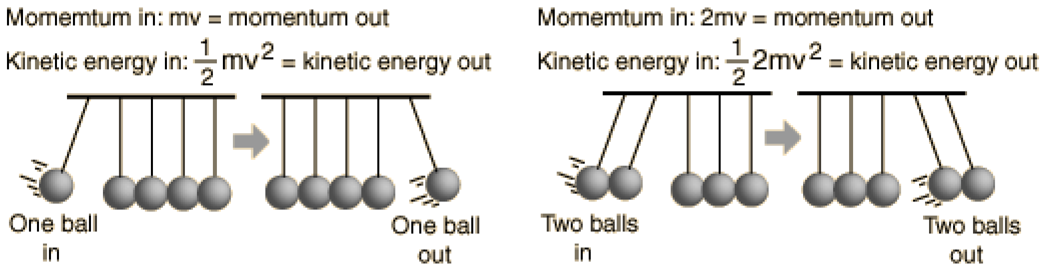
Newton's Cradle is the name given to a set of identical balls that are able to swing suspended from wires:



When one ball is swung and allowed to collide with the rest of the balls, the momentum transfers through the balls and one ball is knocked out from the opposite end. When two balls are swung, two balls are knocked out from the opposite end. And so on.

This apparatus demonstrates the relationship between the conservation of momentum and conservation of kinetic energy. When the balls collide, the collision is mostly elastic collision, meaning that all of the momentum and most of the kinetic energy are conserved.

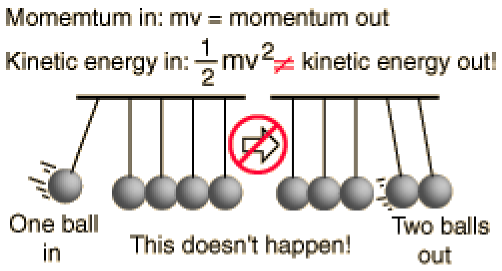
Before the collision, the moving ball(s) have momentum (*mv*) and kinetic energy ($\frac{1}{2}mv^2$). There are no external forces, which means *momentum* must be conserved. The collision is mostly elastic, which means *kinetic energy* is mostly also conserved. The only way for the same momentum and kinetic energy to be present after the collision is for the same number of balls to swing away from the opposite end with the same velocity.



Use this space for summary and/or additional notes:

Momentum and Kinetic Energy

If only momentum had to be conserved, it would be possible to pull back one ball but for two balls to come out the other side at ½ of the original velocity. However, this can't actually happen.



Conserving momentum in this case requires that the two balls come out with half the speed.

$$\text{Momentum out} = 2m\frac{v}{2}$$

But this gives

$$\text{Kinetic energy out} = \frac{1}{2}2m\frac{v^2}{4}$$

Which amounts to a loss of half of the kinetic energy!

Note also that if there were no friction, the balls would continue to swing forever. However, because of friction (between the balls and air molecules, within the strings as they stretch, *etc.*) and conversion of some of the kinetic energy to other forms (such as heat), the balls in a real Newton's Cradle will, of course, slow down and eventually stop.

Use this space for summary and/or additional notes:

Impulse

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.5

Knowledge/Understanding Goals:

- what impulse is

Skills:

- calculate the impulse given to an object
- calculate the change in momentum as the result of an impulse

Language Objectives:

- Understand and correctly use the term "impulse."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving impulse.

Notes:

impulse ($\vec{\boldsymbol{J}}$): a force applied to an object over an interval of time. Impulse is a vector quantity.

An impulse causes a change in momentum, and the amount of the impulse is equal to the change in momentum. The impulse also equals force times time:

$$\vec{\boldsymbol{J}} = \vec{\boldsymbol{F}}\Delta t = \Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_f - \vec{\boldsymbol{p}}_i$$

Impulse is measured in the same unit as momentum (newton-seconds):

$$1\,\text{N}\cdot\text{s} = 1\,\tfrac{\text{kg}\cdot\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes:

Sample Problem

Q: A baseball has a mass of 0.145 kg and is pitched with a velocity of $-38\,\frac{\text{m}}{\text{s}}$ toward home plate. After the ball is hit, its velocity is $+52\,\frac{\text{m}}{\text{s}}$ toward the outfield fence. If the impact between the ball and bat takes place over an interval of 3.0 ms, find the impulse given to the ball by the bat, and the force applied to the ball by the bat.

A: The impluse is:

$$\vec{\boldsymbol{J}} = \Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{p}}_f - \vec{\boldsymbol{p}}_i$$

$$\vec{\boldsymbol{J}} = m\vec{\boldsymbol{v}}_f - m\vec{\boldsymbol{v}}_i = m(\vec{\boldsymbol{v}}_f - \vec{\boldsymbol{v}}_i)$$

$$\vec{\boldsymbol{J}} = (0.145)(52-(-38)) = +13.05\,\text{N}\cdot\text{s}$$

The collision takes place over a time interval of 3.0 ms = 0.0030 s.

$$\vec{\boldsymbol{J}} = \vec{\boldsymbol{F}}\Delta t$$

$$+13.05 = \vec{\boldsymbol{F}}(0.0030)$$

$$\vec{\boldsymbol{F}} = \frac{13.05}{0.0030} = +4350\,\text{N}$$

Use this space for summary and/or additional notes:

Angular Momentum

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.3

Knowledge/Understanding Goals:

- understand angular momentum

Skills:

- calculate the angular momentum of an object
- apply the law of conservation of angular momentum

Language Objectives:

- Understand and correctly use the term "angular momentum"
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving angular momentum.

Notes:

angular momentum ($\vec{L}$) : the momentum of a rotating object in the direction of rotation. Angular momentum is the property of an object that resists changes in the speed or direction of rotation. Angular momentum is measured in units of $\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$.

Just as linear momentum is the product of mass (linear inertia) and (linear) velocity, angular momentum is also the product of rotational inertia and rotational velocity:

$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
linear	**rotational**

Angular momentum is also the cross-product of radius and linear momentum:

$$\vec{L} = \vec{r} \times \vec{p} = rp\sin\theta$$

Use this space for summary and/or additional notes:

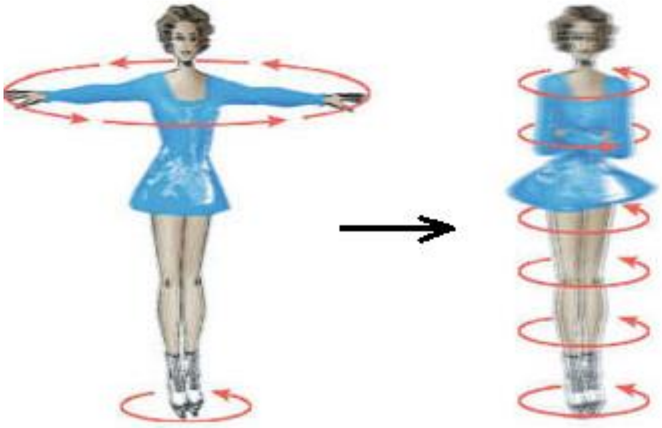
Just as a force produces a change in linear momentum, a torque produces a change in angular momentum. The net external torque on an object is its change in angular momentum with respect to time:

$$\vec{\boldsymbol{\tau}}_{net} = \frac{\Delta \vec{\boldsymbol{L}}}{\Delta t} = \frac{d\vec{\boldsymbol{L}}}{dt}$$

Conservation of Angular Momentum

Just as linear momentum is conserved unless an external force is applied, angular momentum is conserved unless an external torque is applied. This means that the total angular momentum before some change (that occurs entirely within the system) must equal the total angular momentum after the change.

An example of this occurs when a person spinning (*e.g.,* an ice skater) begins the spin with arms extended, then pulls the arms closer to the body. This causes the person to spin faster. (In physics terms, it increases the angular velocity, which means it causes angular acceleration.)



When the skater's arms are extended, the moment of inertia of the skater is greater (because there is more mass farther out) than when the arms are close to the body. Conservation of angular momentum tells us that:

$$L_i = L_f$$

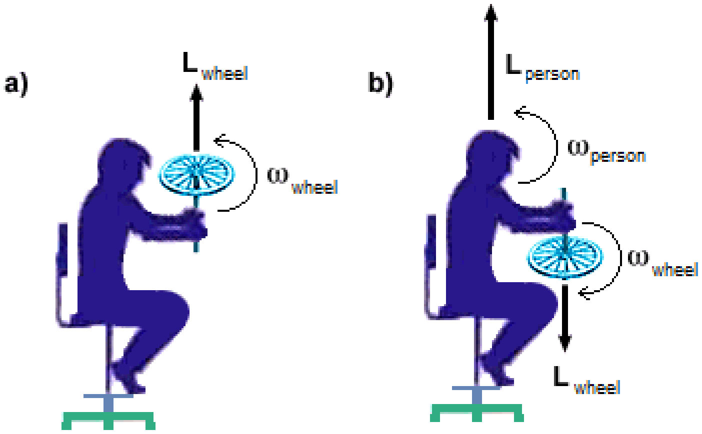
$$I_i \omega_i = I_f \omega_f$$

I.e., if I decreases, then ω must increase.

Use this space for summary and/or additional notes:

Angular Momentum

Another popular example, which shows the vector nature of angular momentum, is the demonstration of a person holding a spinning bicycle wheel on a rotating chair. The person then turns over the bicycle wheel, causing it to rotate in the opposite direction:



Initially, the direction of the angular momentum vector of the wheel is upwards. When the person turns over the wheel, the angular momentum of the wheel reverses direction. Because the person-wheel-chair system is an isolated system, the total angular momentum must be conserved. This means the person must rotate in the opposite direction as the wheel, so that the total angular momentum (magnitude and direction) of the person-wheel-chair system remains the same as before.

Use this space for summary and/or additional notes:

Sample Problem:

Q: A "Long-Playing" (LP) phonograph record has a radius of 15 cm and a mass of 150 g. A typical phonograph could accelerate an LP from rest to its final speed in 0.35 s.

(a) Calculate the angular momentum of a phonograph record (LP) rotating at $33\frac{1}{3}$ RPM.

(b) What average torque would be exerted on the LP?

A: The angular momentum of a rotating body is $L = I\omega$. This means we need to find I (the moment of inertia) and ω (the angular velocity).

An LP is a solid disk, which means the formula for its moment of inertia is:

$$I = \tfrac{1}{2}mr^2$$

$$I = (\tfrac{1}{2})(0.15\,\text{kg})(0.15\,\text{m})^2 = 1.69\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\omega = \frac{33\frac{1}{3}\,\text{rev}}{1\,\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} = 3.49\,\tfrac{\text{rad}}{\text{s}}$$

$$L = I\omega$$

$$L = (1.69\times10^{-3}\,\text{kg}\cdot\text{m}^2)(3.49\,\tfrac{\text{rad}}{\text{s}})$$

$$L = 5.89\times10^{-3}\ \tfrac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L - L_o}{\Delta t} = \frac{5.89\times10^{-3} - 0}{0.35} = 1.68\times10^{-2}\,\text{N}\cdot\text{m}$$

Use this space for summary and/or additional notes:

Introduction: Electricity

Unit: Electricity

Topics covered in this chapter:

This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Electric Change*, *Coulomb's Law*, and *Electric Fields* describe the behavior of individual charged particles and how to calculate the effects of these particles on each other.
- *Electric Current & Ohm's Law* describes equations and calculations involving the flow of charged particles (electric current).
- *Electrical Components*, *EMF & Internal Resistance of a Battery*, *Circuits*, *Series Circuits*, *Parallel Circuits*, *Mixed Series & Parallel Circuits*, and *Measuring Voltage, Current & Resistance* describe the behavior of electrical components in a circuit and how to calculate quantities relating to the individual components and the entire circuit, based on the way the components are arranged.
- *Magnetism* describes properties of magnets and what causes objects to be magnetic. *Electricity & Magnetism* describes how electricity and magnetism affect each other.

Use this space for summary and/or additional notes:

One of the new challenges encountered in this chapter is interpreting and simplifying circuit diagrams, in which different equations may apply to different parts of the circuit.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-4. Use mathematical representations of ~~Newton's Law of Gravitation and~~ Coulomb's Law to describe and predict the ~~gravitational and~~ electrostatic forces between objects.

HS-PS2-5. Plan and conduct an investigation to provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-3. Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

HS-PS3-5. Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction.

Massachusetts Curriculum Frameworks (2006):

5.1 Recognize that an electric charge tends to be static on insulators and can move on and in conductors. Explain that energy can produce a separation of charges.

5.2 Develop qualitative and quantitative understandings of current, voltage, resistance, and the connections among them (Ohm's law).

5.3 Analyze simple arrangements of electrical components in both series and parallel circuits. Recognize symbols and understand the functions of common circuit elements (battery, connecting wire, switch, fuse, resistance) in a schematic diagram.

Use this space for summary and/or additional notes:

5.4 Describe conceptually the attractive or repulsive forces between objects relative to their charges and the distance between them (Coulomb's law).

5.5 Explain how electric current is a flow of charge caused by a potential difference (voltage), and how power is equal to current multiplied by voltage.

5.6 Recognize that moving electric charges produce magnetic forces and moving magnets produce electric forces. Recognize that the interplay of electric and magnetic forces is the basis for electric motors, generators, and other technologies.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Electric Fields, Forces, and Potentials**, such as Coulomb's law, induced charge, field and potential of groups of point charges, and charged particles in electric fields
- **Capacitance**, such as parallel-plate capacitors and time-varying behavior in charging/ discharging
- **Circuit Elements and DC Circuits**, such as resistors, light bulbs, series and parallel networks, Ohm's law, and Joule's law
- **Magnetism**, such as permanent magnets, fields caused by currents, particles in magnetic fields, Faraday's law, and Lenz's law.

1. Electric Charge
2. Electric Force
3. Electric Field
4. Electric Potential
5. Conductors and Insulators
6. Voltage
7. Current
8. Resistance
9. Energy, Power, and Heat
10. Circuits
11. Capacitors
12. Permanent Magnets
13. Magnetic Force on Charges

Use this space for summary and/or additional notes:

14. Magnetic Force on Current-Carrying Wires
15. The Magnetic Field Due to a current
16. Motional EMF
17. Faraday's Law

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Identifying electric circuit components.
- Simplifying circuit diagrams.

Use this space for summary and/or additional notes:

Electric Charge

Unit: Electricity

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.1, 5.4

Knowledge/Understanding Goals:

- electric charge
- properties of electric charges
- conductors *vs.* insulators

Language Objectives:

- Understand and correctly use the terms “electricity,” “charge,” “current,” “conductor,” “insulator,” and “induction.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

electric charge: a physical property of matter which causes it to experience a force when near other electrically charged matter.

positive charge: the charge of a proton. Originally defined as the charge left on a piece of glass when rubbed with silk. The glass becomes positively charged because the silk pulls electrons off the glass.

negative charge: the charge of an electron. Originally defined as the charge left on a piece of amber (or rubber) when rubbed with fur (or wool). The amber becomes negatively charged because the amber pulls the electrons off the fur.

static electricity: stationary electric charge, such as the charge left on silk or amber in the above definitions.

Use this space for summary and/or additional notes:

electric current (sometimes called electricity): the movement of electrons through a medium (substance) from one location to another. Note, however, that electric current is defined as the direction a positively charged particle would move. Thus electric current "flows" in the opposite direction from the actual electrons.

WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.

Devices that Produce, Use or Store Charge

capacitor: a device that stores electric charge.

battery: a device that uses chemical reactions to produce an electric current.

generator: a device that converts mechanical energy (motion) into an electric current.

motor: a device that converts an electric current into mechanical energy.

Use this space for summary and/or additional notes:

Electric Charge

Conductors *vs.* Insulators

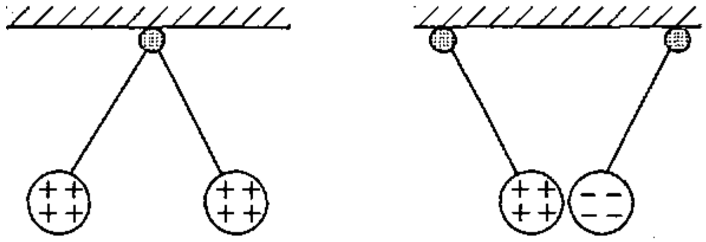
<u>conductor</u>: a material that allows charges to move freely through it. Examples of conductors include metals and liquids with positive and negative ions dissolved in them (such as salt water). When charges are transferred to a conductor, the charges distribute themselves evenly throughout the substance.

<u>insulator</u>: a material that does not allow charges to move freely through it. Examples of insulators include nonmetals and most pure chemical compounds (such as glass or plastic). When charges are transferred to an insulator, they cannot move, and remain where they are placed.

Behavior of Charged Particles

Like charges repel. A pair of the same type of charge (two positive charges or two negative charges) exert a force that pushes the charges away from each other.

Opposite charges attract. A pair of opposite types of charge (a positive charge and a negative charge) exert a force that pulls the charges toward each other.



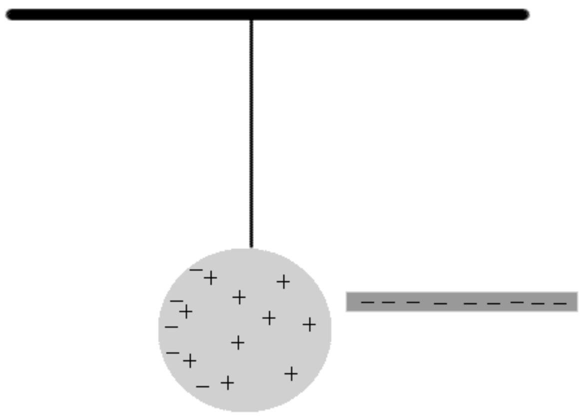
Charge is conserved. Electric charges cannot be created or destroyed, but can be transferred from one location or medium to another. (This is analogous to the laws of conservation of mass and energy.)

Use this space for summary and/or additional notes:

Charging by Induction

<u>induction</u>: when an electrical charge on one object causes a charge in a second object.

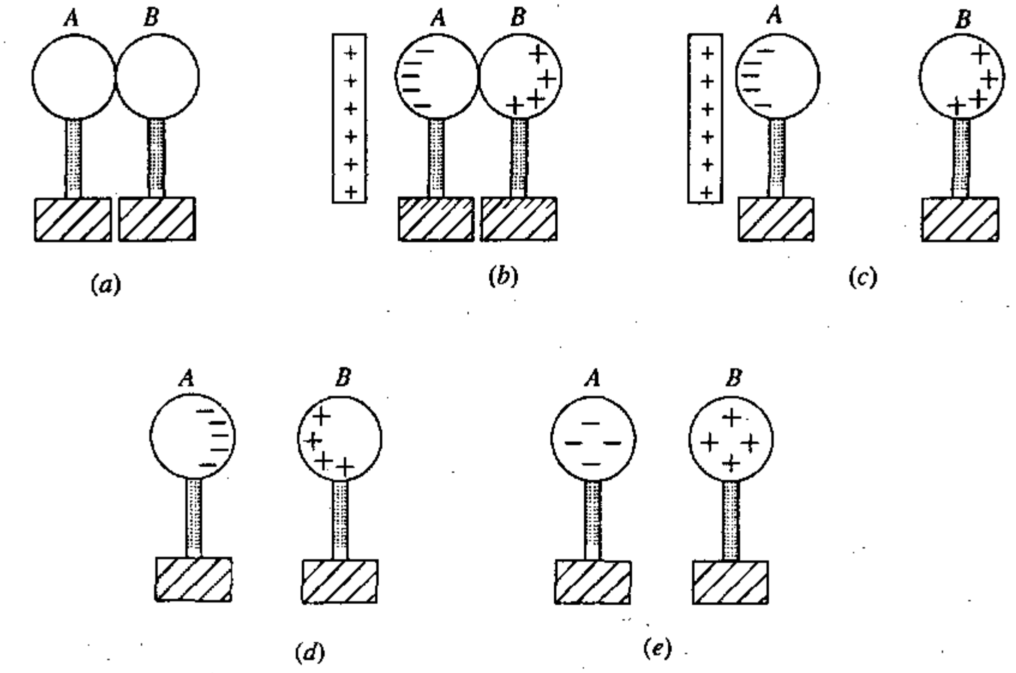
When a charged rod is brought near a neutral object, the charge on the rod attracts opposite charges and repels like charges that are near it. The diagram below shows a negatively-charged rod repelling negative charges.



If the negatively-charged rod above were touched to the sphere, some of the charges from the rod would be transferred to the sphere at the point of contact, and the sphere would acquire an overall negative charge.

Use this space for summary and/or additional notes:

A process for inducing charges in a pair of metal spheres is shown below:



(a) Metal spheres *A* and *B* are brought into contact.

(b) A positively charged object is placed near (but not in contact with) sphere *A*. This induces a negative charge in sphere *A*, which in turn induces a positive charge in sphere *B*.

(c) Sphere *B* (which is now positively charged) is moved away.

(d) The positively charged object is removed.

(e) The charges distribute themselves throughout the metal spheres.

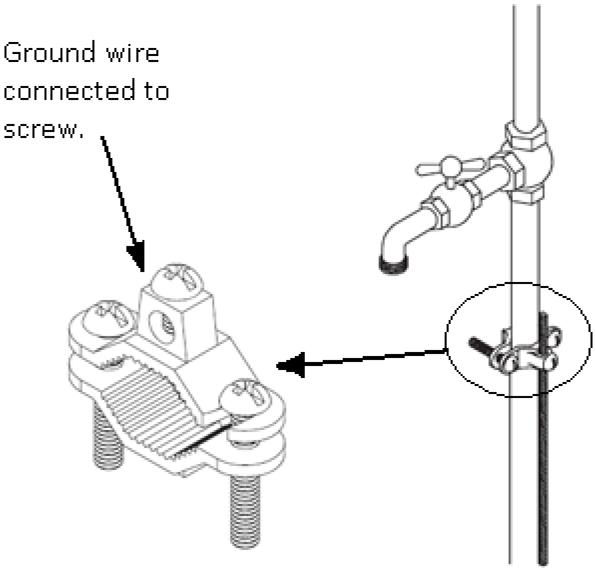
Use this space for summary and/or additional notes:

Grounding

For the purposes of our use of electric charges, the ground (Earth) is effectively an endless supply of both positive and negative charges. Under normal circumstances, if a charged object is touched to the ground, electrons will move to neutralize the charge, either by flowing from the object to the ground or from the ground to the object.

Grounding a charged object or circuit means neutralizing the electrical charge on an object or portion of the circuit.

In buildings, the metal pipes that bring water into the building are often used to ground the electrical circuits. The metal pipe is a good conductor of electricity, and carries the unwanted charge out of the building and into the ground outside.



Use this space for summary and/or additional notes:

Coulomb's Law

Unit: Electricity

NGSS Standards: HS-PS2-4, HS-PS3-5

MA Curriculum Frameworks (2006): 5.4

Skills:

- understand & solve problems using Coulomb's Law

Language Objectives:

- Accurately describe Coulomb's Law using appropriate academic language.
- Set up and solve word problems relating to Coulomb's Law.

Notes:

Electric charge is measured in Coulombs (abbreviation "C"). One Coulomb is the amount of electric charge transferred by a current of 1 ampere for a duration of 1 second.

1 C is the charge of 6.2415×10^{18} protons.

−1 C is the charge of 6.2415×10^{18} electrons.

One proton or electron (elementary charge) therefore has a charge of 1.6022×10^{-19} C.

Use this space for summary and/or additional notes:

Coulomb's Law

Because charged particles exert a force on each other, that force can be measured and quantified. The force is directly proportional to the strengths of the charges and inversely proportional to the square of the distance. The formula is therefore:

$$\vec{\boldsymbol{F}}_e = \frac{kq_1q_2}{d^2}\hat{\boldsymbol{d}}_{12}$$

(vector form)

$$F_e = \frac{kq_1q_2}{d^2}$$

(scalar form)

where:

$\vec{\boldsymbol{F}}_e$ = electrostatic force of repulsion between electric charges. A positive value of $\vec{\boldsymbol{F}}_e$ denotes that the charges are repelling (pushing away from) each other; a negative value of $\vec{\boldsymbol{F}}_e$ denotes that the charges are attracting (pulling towards) each other.

k = electrostatic constant = $8.9876 \times 10^9 \, \frac{\text{N·m}^2}{\text{C}^2}$

q_1 and q_2 = charges 1 and 2 respectively

d = distance between the centers of the two charges

$\hat{\boldsymbol{d}}_{12}$ = unit vector from q_1 towards q_2.

This formula is Coulomb's Law, named for its discoverer, the French physicist Charles-Augustin de Coulomb.

Use this space for summary and/or additional notes:

Coulomb's Law

Sample problem:

Q: Find the force of electrostatic attraction between the proton and electron in a hydrogen atom if the radius of the atom is 37.1 pm

A: The charge of a single proton is 1.60×10^{-19} C, and the charge of a single electron is -1.60×10^{-19} C.

$$37.1\,\text{pm} = 3.71 \times 10^{-11}\,\text{m}$$

$$F_e = \frac{kq_1q_2}{d^2}$$

$$F_e = \frac{(8.99 \times 10^{9})(1.60 \times 10^{-19})(-1.60 \times 10^{-19})}{(3.71 \times 10^{-11})^2}$$

$$F_e = -1.67 \times 10^{-7}\,\text{N}$$

The value of the force is negative, which signifies that the force is attractive.

Use this space for summary and/or additional notes:

Electric Current & Ohm’s Law

Unit: Electricity

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.2, 5.5

Knowledge/Understanding Goals:

- electric potential (voltage)
- electric current
- resistance

Skills:

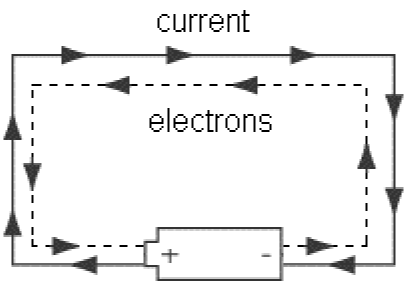
- perform calculations involving voltage, current, resistance and power

Language Objectives:

- Understand and correctly use the terms “current,” “direct current,” “alternating current,” “potential difference,” “voltage,” “resistance,” “power,” and “work.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Ohm’s Law.

Notes:

<u>electric current</u>: the flow of charged particles from one place to another, caused by a difference in electric potential. The direction of the electric current is defined as the direction that a positively-charged particle would move. Note, however, that the particles that are actually moving are electrons, which are negatively charged. This means that electric current “travels” in the *<u>opposite</u>* direction from the electrons.



Use this space for summary and/or additional notes:

Electric current ($\vec{I}$) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere is one coulomb per second.

$$I = \frac{\Delta q}{t}$$

voltage (potential difference): the difference in electric potential energy between two locations, per unit of charge.

Potential difference is the work (W) done on a charge per unit of charge (q). Potential difference (V) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

$$V = \frac{W}{q}$$

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

resistance: the amount of electromotive force (electric potential) needed to force a given amount of current through an object.

Resistance (R) is a scalar quantity and is measured in ohms (Ω). One ohm is one volt per ampere.

$$R = \frac{V}{I}$$

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{V}{R} \qquad \text{or} \qquad V = IR$$

Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.

Resistance is an intrinsic property of a substance. In the situations we will study in this course, the resistance of an object does not change.

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use resistors in circuits to reduce the amount of current that flows through the components.

Use this space for summary and/or additional notes:

Electric Current & Ohm's Law

<u>resistivity</u>: the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance (ρ), and of the length (L) and cross-sectional area (A) of the object. In MKS units, resistivity is measured in ohm-meters (Ω·m).

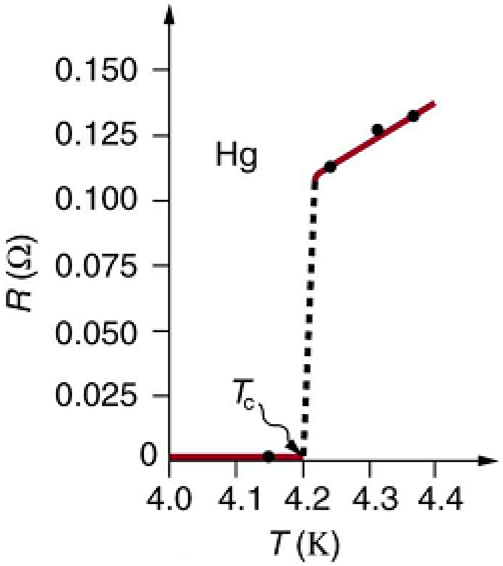
$$R = \frac{\rho L}{A}$$

Resistivity changes with temperature. For small temperature differences (less than 100°C), resistivity is given by:

$$\rho = \rho_o(1 + \alpha \Delta T)$$

where ρ_o is the resistivity at some reference temperature and α is the temperature coëfficient of resistivity for that substance. For conductors, α is positive (which means their resistivity increases with temperature). For metals, resistivity typically varies from +0.003 to +0.006 K^{-1}.

Some materials become superconductors (essentially zero resistance) at very low temperatures. The temperature where a superconductor's resistivity drops sharply is called the critical temperature (T_c). For example, mercury becomes a superconductor at a temperature of 4.2 K:



Use this space for summary and/or additional notes:

conductivity: the innate ability of a substance to conduct electricity. Conductivity (σ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol ℧). (Note that "mho" is "ohm" spelled backwards. Note also that conductivity is not tested on the AP Physics 1 exam.)

$$\sigma = \frac{1}{\rho}$$

capacitance: the ability of an object to hold an electric charge. Capacitance (C) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt. (Note that capacitance is not tested on the AP Physics 1 exam.)

$$C = \frac{q}{V}$$

power: as discussed in the mechanics section of this course, power (P) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$$

work: recall from mechanics that work (W) equals power times time, and is measured in either newton-meters (N·m) or joules (J):

$$W = Pt = VIt = I^2Rt = \frac{V^2t}{R} = Vq$$

Electrical work or energy is often measured in kilowatt-hours (kW·h).

$$1\,\text{kW}\cdot\text{h} \equiv 3.6\times10^6\ \text{J} \equiv 3.6\,\text{MJ}$$

Summary of Terms and Variables

Term	Variable	Unit	Term	Variable	Unit
charge	q or Q	coulomb (C)	resistance	R	ohm (Ω)
current	I	ampere (A)	power	P	watt (W)
voltage	V	volt (V)	work	W	joule (J)

Use this space for summary and/or additional notes:

Alternating Current vs. Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

alternating current: electric current flows back and forth in one direction and then the other, like a sine wave. The current alternates at a particular frequency. In the U.S., household current is 110 volts AC with a frequency of 60 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Note that alternating current is not tested on the AP Physics 1 exam.

Use this space for summary and/or additional notes:

Electrical Components

Unit: Electricity

NGSS Standards: HS-PS2-6

MA Curriculum Frameworks (2006): 5.3

Knowledge/Understanding Goals:

- recognize common components of electrical circuits

Language Objectives:

- Recognize and be able to name and draw symbols for each of the electrical components described in this section.

Notes:

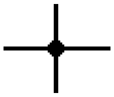
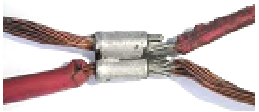
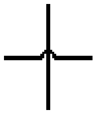
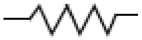
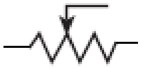
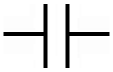
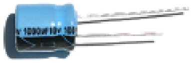
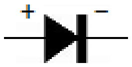
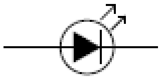
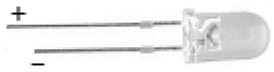
electrical component: an object that performs a specific task in an electric circuit. A circuit is a collection of components connected together so that the tasks performed by the individual components combine in some useful way.

circuit diagram: a picture that represents a circuit, with different symbols representing the different components.

Use this space for summary and/or additional notes:

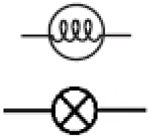
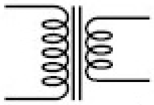
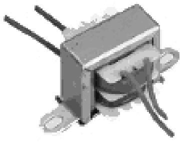
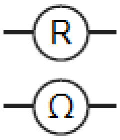
Electrical Components

The following table describes some of the common components of electrical circuits, what they do, and the symbols that are used to represent them in circuit diagrams.

Component	Symbol	Picture	Description
wire			Carries current in a circuit.
junction			Connection between two or more wires.
unconnected wires			Wires cross but are not connected.
battery			Supplies current at a fixed voltage.
resistor			Resists flow of current.
potentiometer (rheostat, dimmer)			Provides variable (adjustable) resistance.
capacitor			Stores charge.
diode			Allows current to flow in only one direction (from + to −).
light-emitting diode (LED)			Diode that gives off light.

Use this space for summary and/or additional notes:

Electrical Components

Component	Symbol	Picture	Description
switch			Opens / closes circuit.
incandescent lamp (light)			Provides light (and resistance).
transformer		 	Increases or decreases voltage.
voltmeter			Measures voltage (volts).
ammeter			Measures current (amperes).
ohmmeter			Measures resistance (ohms).
fuse			Opens circuit if too much current flows through it.
ground		 (clamps to water pipe)	Neutralizes charge.

Use this space for summary and/or additional notes:

Circuits

Unit: Electricity

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Knowledge/Understanding Goals:

- how resistance limits current in a circuit
- the difference between series and parallel circuits

Language Objectives:

- Explain why resistors are necessary in electric circuits.
- Understand and correctly use the terms "series" and "parallel" as applied to electric circuits.

Notes:

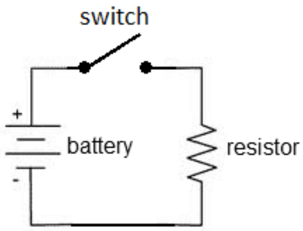
circuit: an arrangement of electrical components that allows electric current to pass through them so that the tasks performed by the individual components combine in some useful way.

closed circuit: a circuit that has a complete path for current to flow from the positive terminal of the battery or power supply through the components and back to the negative terminal.

open circuit: a circuit that has a gap such that current cannot flow from the positive terminal to the negative terminal.

short circuit: a circuit in which the positive terminal is connected directly to the negative terminal with no load (resistance) in between.

A diagram of a simple electric circuit might look like the diagram to the right.



When the switch is closed, the electric current flows from the positive terminal of the battery through the switch, through the resistor, and into the negative terminal of the battery.

Use this space for summary and/or additional notes:

Circuits

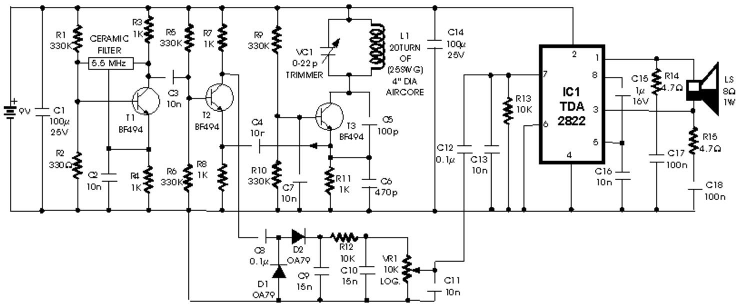
An electric circuit needs a power supply (often a battery) that provides current at a specific difference in electric potential (voltage), and one or more components that use the energy provided by the battery.

The battery continues to supply current, provided that:

1. There is a path for the current to flow from the positive terminal to the negative terminal, and
2. The total resistance of the circuit is small enough to allow the current to flow.

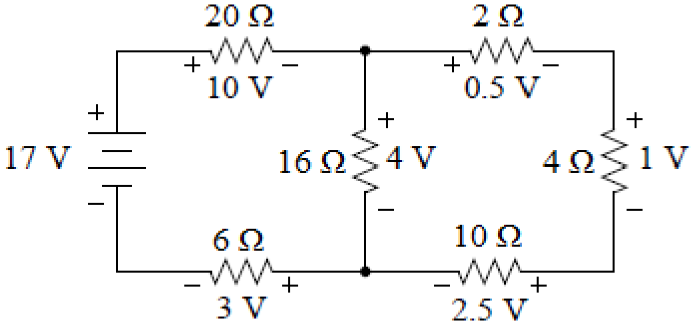
If the circuit is broken, current cannot flow and the chemical reactions inside the battery stop.

Of course, as circuits become more complex, the diagrams reflect this increasing complexity. The following is a circuit diagram for a metal detector:



Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance contributed by each component of a circuit.

The following is an example of a circuit with one 17 V battery and six resistors. Notice that the voltages and resistances are all labeled.

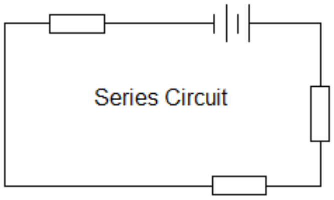


Use this space for summary and/or additional notes:

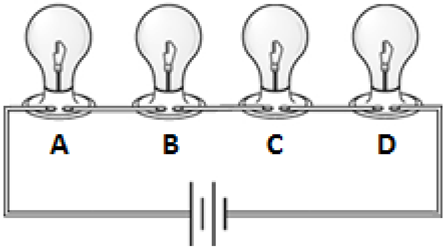
Series vs. Parallel Circuits

If a circuit has multiple components, they can be arranged in series or parallel.

series: Components in **series** lie along the same path, one after the other.



In a series circuit, all of the current flows through every component, one after another. If the current is interrupted anywhere in the circuit, no current will flow. For example, in the following series circuit, if any of light bulbs A, B, C, or D is removed, no current can flow and none of the light bulbs will be illuminated.

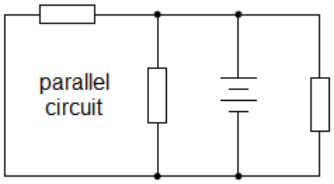


Because some voltage is "used up" by each bulb in the circuit, each additional bulb means the voltage is divided among more bulbs and is therefore less for each bulb. This is why light bulbs get dimmer as you add more bulbs in series.

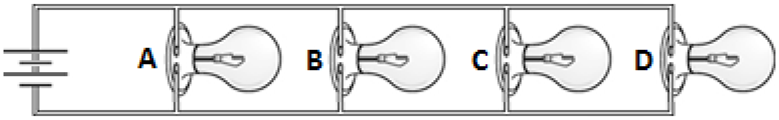
Christmas tree lights used to be wired in series. This caused a lot of frustration, because if one bulb burned out, the entire string went out, and it could take several tries to find which bulb was burned out.

Use this space for summary and/or additional notes:

parallel: Components in **parallel** lie in separate paths.

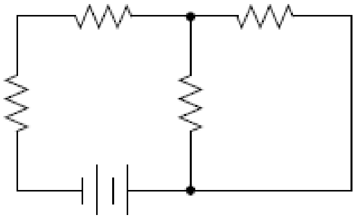


In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths. For example, in the following parallel circuit, if any of light bulbs A, B, C, or D is removed, current still flows through the remaining bulbs.



Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, the total current will increase with each additional branch.)

Note that complex circuits may have some components that are in series with each other and other components that are in parallel.



Use this space for summary and/or additional notes:

Series Circuits

Unit: Electricity

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Knowledge/Understanding Goals:

- the difference between series and parallel circuits

Skills:

- calculate voltage, current, resistance, and power in series circuits.

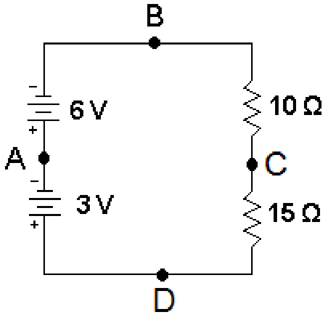
Language Objectives:

- Understand and correctly use the term “series circuit.”
- Set up and solve word problems relating to electrical circuits with components in series.

Notes:

Analyzing Series Circuits

The following circuit shows two batteries and two resistors in series:



Current

Because there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit:

$$I_{total} = I_1 = I_2 = I_3 = \ldots$$

Use this space for summary and/or additional notes:

Series Circuits

Voltage

In a series circuit, if there are multiple voltage sources (*e.g.,* batteries), the voltages add:

$$V_{total} = V_1 + V_2 + V_3 + \ldots$$

In the above circuit, there are two batteries, one that supplies 6 V and one that supplies 3 V. The voltage from A to B is +6 V, the voltage from A to D is −3 V (note that A to D means measuring from negative to positive), and the voltage from D to B is (+3 V) + (+6 V) = +9 V.

Resistance

If there are multiple resistors, each one contributes to the total resistance and the resistances add:

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

In the above circuit, the resistance between points B and D is 10Ω + 15Ω = 25Ω.

Power

In all circuits (series and parallel), any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Calculations

You can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

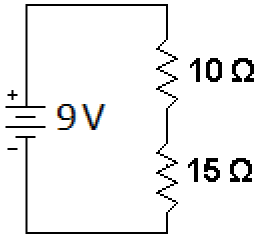
$$V = IR \qquad\qquad P = VI = I^2R = \frac{V^2}{R}$$

“Solving” the circuit for these quantities is much like solving a Sudoku puzzle. You systematically decide which variables (for each component and/or the entire circuit) you have enough information to solve for. Each result enables you to determine more and more of the, until you have found all of the quantities you need.

Use this space for summary and/or additional notes:

Sample Problem:

Suppose we are given the following circuit:



and we are asked to fill in the table:

	Unit	R_1	R_2	Total
Voltage (V)	V			9 V
Current (I)	A			
Resistance (R)	Ω	10 Ω	15 Ω	
Power (P)	W			

First, we recognize that resistances in series add, so we have:

	Unit	R_1	R_2	Total
Voltage (V)	V			9 V
Current (I)	A			
Resistance (R)	Ω	10 Ω	15 Ω	*25 Ω*
Power (P)	W			

Now, we know two variables in the "Total" column, so we use $V = IR$ to find the current. Because this is a series circuit, the total current is also the current through R_1 and R_2.

$$V = IR$$
$$9 = (I)(25)$$
$$I = \frac{9}{25} = 0.36\text{ A}$$

	R_1	R_2	Total
Voltage (V)			9 V
Current (I)	*0.36 A*	*0.36 A*	*0.36 A*
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

Use this space for summary and/or additional notes:

Series Circuits

As soon as we know the current, we can find the voltage across R_1 and R_2, again using $V = IR$.

	R_1	R_2	Total
Voltage (V)	*3.6 V*	*5.4 V*	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

Finally, we can fill in the power, using $P = VI$, $P = I^2R$, or $P = \frac{V^2}{R}$:

	R_1	R_2	Total
Voltage (V)	3.6 V	5.4 V	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)	*1.30 W*	*1.94 W*	*3.24 W*

Use this space for summary and/or additional notes:

Parallel Circuits

Unit: Electricity

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Skills:

- calculate voltage, current, resistance, and power in parallel circuits.

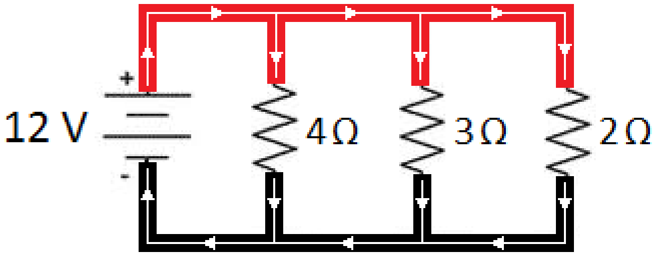
Language Objectives:

- Understand and correctly use the term "parallel circuit."
- Set up and solve word problems relating to electrical circuits with components in parallel.

Notes:

Parallel Circuits

The following circuit shows a battery and three resistors in parallel:



Current

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

Use this space for summary and/or additional notes:

Parallel Circuits

Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$V_{total} = V_1 = V_2 = V_3 = \ldots$$

Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Some students find it confusing that the combined resistance of a group of resistors in series is always less than any single resistor by itself.

Use this space for summary and/or additional notes:

Parallel Circuits

Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (i.e., how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.

The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

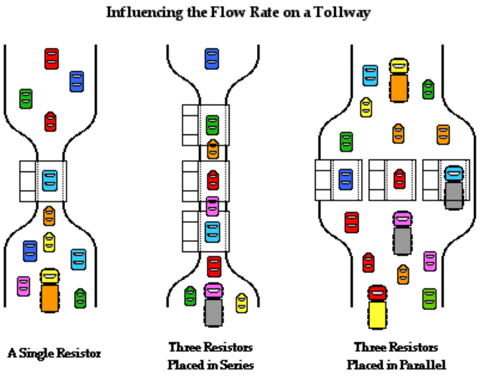
The water will flow through the *set* of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

Another common analogy is to compare resistors with toll booths on a highway.

One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in series would slow traffic down more.

Multiple toll booths in parallel make traffic flow faster because there are more paths for the cars to follow. Each additional toll booth further reduces the resistance to the flow of traffic.



Use this space for summary and/or additional notes:

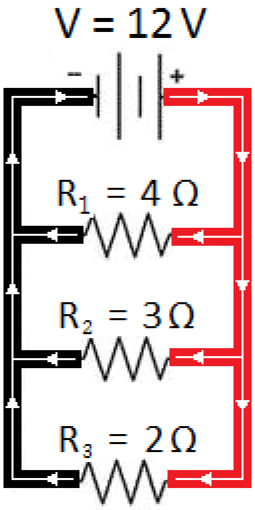
Calculations

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

$$V = IR \qquad P = VI = I^2R = \frac{V^2}{R}$$

Sample Problem

Suppose we are given the following circuit:



and we are asked to fill in the table:

	R_1	R_2	R_3	Total
Voltage (V)				12 V
Current (I)				
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches.

The first thing we can do is use $V = IR$ to find the current through each resistor:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	*3 A*	*4 A*	*6 A*	*13 A*
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

In a parallel circuit, the current adds, so the total current is 3 + 4 + 6 = 13 A.

Use this space for summary and/or additional notes:

Parallel Circuits

Now, we have two ways of finding the total resistance.

We can use $V = IR$ for the voltage and total current:

$$V = IR$$
$$12 = 13\,R$$
$$R = \frac{12}{13} = 0.923\,\Omega$$

Or we can use the formula for resistances in parallel:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$
$$\frac{1}{R_{total}} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$
$$R_{total} = \frac{12}{13} = 0.923\,\Omega$$

Now we have:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	*0.923 Ω*
Power (P)				

Use this space for summary and/or additional notes:

As we did with series circuits, we can calculate the power, using $P = VI$, $P = I^2R$, or $P = \frac{V^2}{R}$:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)	*36 W*	*48 W*	*72 W*	*156 W*

Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts and the chemicals leak out.

Use this space for summary and/or additional notes:

Mixed Series & Parallel Circuits

Unit: Electricity

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Skills:

- analyze circuits by replacing networks of resistors with a single resistor of equivalent resistance.

Language Objectives:

- Set up and solve word problems involving electrical circuits with some components in series and others in parallel with each other.

Notes:

Mixed Series and Parallel Circuits

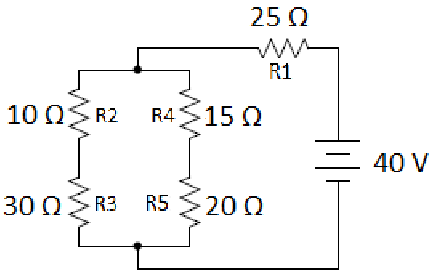
If a circuit has mixed series and parallel sections, you can simplify the circuit in order to determine the voltage, current, and/or resistance. The circuit can be simplified by replacing resistors in series or parallel with a single resistor of equivalent resistance, as in the following example:

Suppose we need to solve a mixed series & parallel circuit for voltage, current, resistance and power for each resistor.

Because the circuit has series and parallel sections, we cannot simply use the series and parallel rules across the entire table.

Use this space for summary and/or additional notes:

Mixed Series & Parallel Circuits



	Unit	R_1	R_2	R_3	R_4	R_5	Total
Voltage (V)	V						40 V
Current (I)	A						
Resistance (R)	Ω	25 Ω	10 Ω	30 Ω	15 Ω	20 Ω	
Power (P)	W						

We can use Ohm's Law $(V = IR)$ and the power equation $(P = VI)$ on each individual resistor and the totals for the circuit (columns), but we need two pieces of information for each resistor in order to do this.

Our strategy will be:

1. Simplify the resistor network until all resistances are combined into one equivalent resistor to find the total resistance.
2. Use $V = IR$ to find the total current.
3. Work backwards through your simplification, using the equations for series and parallel circuits in the appropriate sections of the circuit until you have all of the information.

Step 1: If we follow the current through the circuit, we see that it goes through resistor R1 first. Then it splits into two parallel pathways. One path goes through R2 and R3, and the other goes through R4 and R5.

Use this space for summary and/or additional notes:

There is no universal shorthand for representing series and parallel components, so let's define the symbols "—" to show resistors in series, and "||" to show resistors in parallel. The above network of resistors could be represented as:

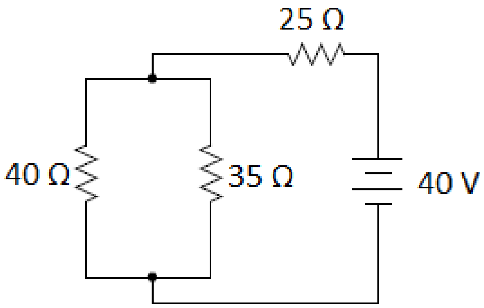
R1 — [(R2 — R3) || (R4 — R5)]

Now, we simplify the network just like a math problem—start with the innermost parentheses and work your way out.

Step 2: Replace R2 — R3 with a single resistor. These are in series, so we just add 10 Ω + 30 Ω = 40 Ω.

Step 3: Replace R4 — R5 with a single resistor. These are also in series, so we just add 15 Ω + 20 Ω = 35 Ω.

Now we have the following slightly simpler circuit:



Step 4: Combine the parallel 40Ω and 35Ω resistors into a single equivalent resistance:

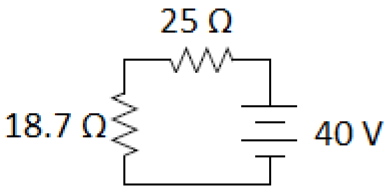
$$\frac{1}{R_{total}} = \frac{1}{40} + \frac{1}{35}$$

$$\frac{1}{R_{total}} = 0.0250 + 0.0286 = 0.0536$$

$$R_{total} = \frac{1}{0.0536} = 18.\overline{6}\,\Omega$$

Use this space for summary and/or additional notes:

Now, our circuit is equivalent to:



Step 5: Add the two resistances in series to get the total combined resistance of the circuit:

$$18.\overline{6} + 25 = 43.\overline{6}\,\Omega$$

Step 6: Now that we know the total voltage and resistance, we can use Ohm's Law to find the current:

$$V = I R$$

$$40 = I(43.\overline{6})$$

$$I = \frac{40}{43.\overline{6}} = 0.916\text{ A}$$

While we're at it, let's use $P = VI = (40)(0.916) = 36.6\,\text{W}$ to find the total power:

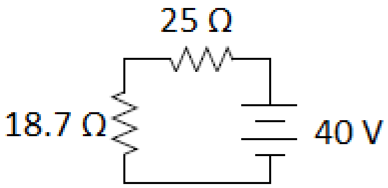
Now we have:

	Unit	R_1	R_2	R_3	R_4	R_5	Total
Voltage (V)	V						40 V
Current (I)	A						*0.916 A*
Resistance (R)	Ω	25 Ω	10 Ω	30 Ω	15 Ω	20 Ω	*$43.\overline{6}$ Ω*
Power (P)	W						*36.6 W*

Now we work backwards.

Use this space for summary and/or additional notes:

The next-to-last simplification step was:



The 25 Ω resistor is R1. All of the current goes through it, so the current through R1 must be 0.916 A. Using Ohm's Law, this means the voltage drop across R1 must be:

$$V = IR$$
$$V = (0.916)(25) = 22.9\text{ V}$$

and the power must be:

$$P = VI$$
$$P = (0.916)(25) = 22.9\text{ V}$$

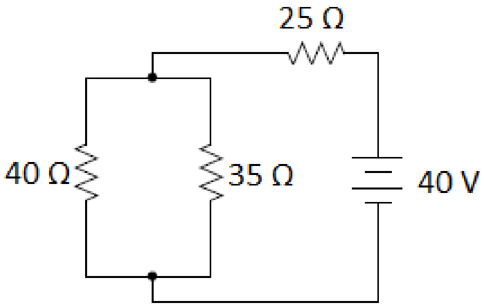
This means that the voltage across the parallel portion of the circuit $\big[$(R2 — R3) || (R4 — R5)$\big]$ must be 40 – 23 = 17 V.

	Unit	R_1	R_2	R_3	R_4	R_5	Total
Voltage (V)	V	*22.9 V*					40 V
Current (I)	A	*0.916 A*					0.916 A
Resistance (R)	Ω	25 Ω	10 Ω	30 Ω	15 Ω	20 Ω	$43.\overline{6}$ Ω
Power (P)	W	*22.9 W*					36.6 W

Use this space for summary and/or additional notes:

Mixed Series & Parallel Circuits

Backing up another step, this means that the voltage is 17 V across both parallel branches (because voltage is the same across parallel branches).



We can use this plus Ohm's Law to find the current through each branch.

Use this space for summary and/or additional notes:

Measuring Voltage, Current & Resistance

Unit: Electricity

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Knowledge & Understanding:

- how voltage, current and resistance are measured.
- how positive and negative numbers for voltage and current correspond with the direction of current flow.

Skills:

- measure voltage, current and resistance.

Language Objectives:

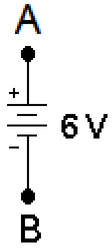
- Accurately describe how to measure voltage, current and resistance in an electric circuit, using appropriate academic language.

Notes:

Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance in each component of a circuit. In order to analyze actual circuits, it is necessary to be able to measure these quantities.

Measuring Voltage

Suppose we want to measure the electric potential (voltage) across the terminals of a 6 V battery. The diagram would look like this:



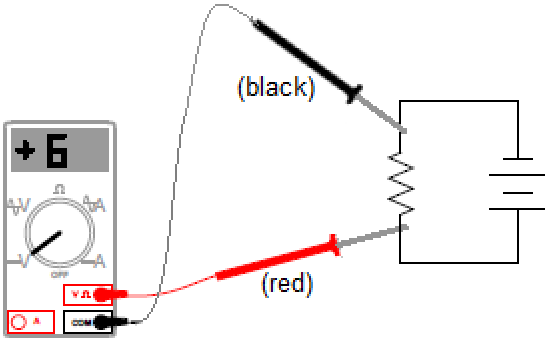
The voltage between points A and B is either +6 V or −6 V, depending on the direction. The voltage from A to B (positive to negative) is +6 V, and the voltage from B to A (negative to positive) is −6 V.

Use this space for summary and/or additional notes:

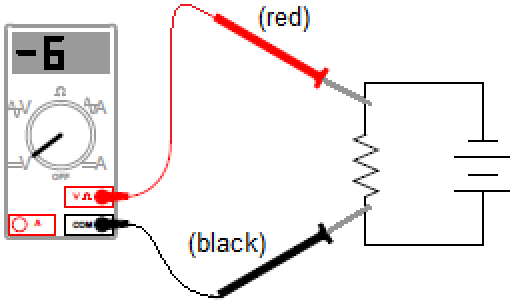
Measuring Voltage, Current & Resistance

When measuring voltage, the circuit needs to be powered up with current flowing through it. Make sure that the voltmeter is set for volts (DC or AC, as appropriate) and that the red lead is plugged into the V Ω socket (for measuring volts or ohms). Then touch the two leads *in parallel* with the two points you want to measure the voltage across. (Remember that voltage is the same across all branches of a parallel circuit. You want the voltmeter to be one of the branches, and the circuit to be the other branch.)

On a voltmeter (a meter that measures volts or voltage), the voltage is measured assuming the current is going from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the more positive end of a resistor and the black (−) lead on the more negative end, the voltage reading would be positive. In this circuit, the voltmeter reads +6 V:



If you switch the leads, so the black (−) lead is on the more positive end and the red (+) lead is on the more positive end, the voltage reading would be negative. In this circuit, the voltmeter reads −6 V:



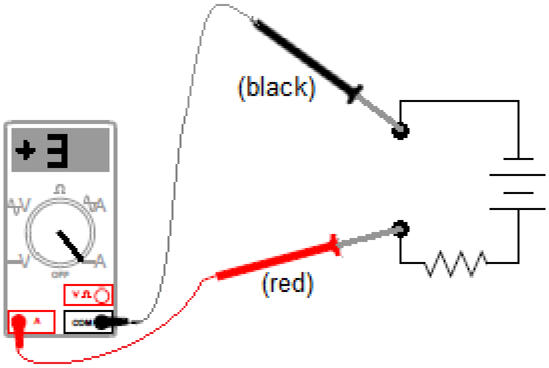
The reading of −6 V indicates that the current is actually flowing in the opposite direction from the way the voltmeter is measuring—from the black (−) lead to the red (+) lead.

Use this space for summary and/or additional notes:

Measuring Current

When measuring current, the circuit needs to be open between two points. Make sure the ammeter is set for amperes (A), milliamperes (mA) or microamperes (μA) AC or DC, depending on what you expect the current in the circuit to be. Make sure the red lead is plugged into appropriate socket (A if the current is expected to be 0.5 A or greater, or mA/μA if the current is expected to be less than 0.5 A). Then touch one lead to each of the two contact points, so that the ammeter is *in series* with the rest of the circuit. (Remember that current is the same through all components in a series circuit. You want all of the current to flow through the ammeter.)

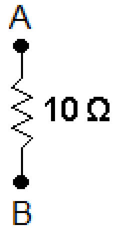
On an ammeter (a meter that measures current), the current is measured assuming that it is flowing from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the side that is connected to the positive terminal and the black (−) lead on the end that is connected to the negative terminal, the current reading would be positive. In this circuit, the current is +3 A:



As with the voltage example above, if you switched the leads, the reading would be −3 A instead of +3 A.

Use this space for summary and/or additional notes:

Measuring Resistance



Resistance does not have a direction. If you placed an ohmmeter (a meter that measures resistance) across points A and B, it would read 10 Ω regardless of which lead is on which point.

However, because an ohmmeter needs to supply a small amount of current across the component and measure the resistance, the reading is more susceptible to measurement problems, such as the resistance of the wire itself, how well the probes are making contact with the circuit, etc. It is often more reliable to measure the voltage and current and calculate resistance using Ohm's Law ($V = IR$).

Use this space for summary and/or additional notes:

Introduction: Simple Harmonic Motion

Unit: Simple Harmonic Motion

Topics covered in this chapter:

This chapter discusses properties of waves that travel through a medium (mechanical waves).

- *Simple Harmonic Motion* (SHM) describes the concept of repetitive back-and-forth motion and situations that apply to it.
- *Springs* and *Pendulums* describe specific examples of SHM and the specific equations relating to each.
- *Waves* gives general information about waves, including vocabulary and equations. *Reflection and Superposition* describes what happens when two waves share space within a medium.
- *Sound & Music* describes the properties and equations of waves that relate to music and musical instruments.
- *The Doppler Effect* describes the effects of motion of the source or receiver (listener) on the perception of sound.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

Massachusetts Curriculum Frameworks (2006):

4.1 Describe the measurable properties of waves (velocity, frequency, wavelength, amplitude, period) and explain the relationships among them. Recognize examples of simple harmonic motion.

4.2 Distinguish between mechanical and electromagnetic waves.

Use this space for summary and/or additional notes:

4.3 Distinguish between the two types of mechanical waves, transverse and longitudinal.

4.4 Describe qualitatively the basic principles of reflection and refraction of waves.

4.5 Recognize that mechanical waves generally move faster through a solid than through a liquid and faster through a liquid than through a gas.

4.6 Describe the apparent change in frequency of waves due to the motion of a source or a receiver (the Doppler effect).

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Simple Harmonic Motion**, such as mass on a spring and the pendulum
- **General Wave Properties**, such as wave speed, frequency, wavelength, superposition, standing wave diffraction, and the Doppler effect.

1. Periodic Motion (Simple Harmonic Motion)
2. Frequency and Period
3. Springs
4. Pendulums
5. Wave Motion
6. Transverse Waves and Longitudinal Waves
7. Superposition
8. Standing Waves and Resonance
9. The Doppler Effect

Skills learned & applied in this chapter:

- Visualizing wave motion.

Use this space for summary and/or additional notes:

Simple Harmonic Motion

Unit: Simple Harmonic Motion

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.1

Knowledge/Understanding Goals:

- what simple harmonic motion is
- examples of simple harmonic motion

Language Objectives:

- Understand and correctly use the term "simple harmonic motion" and be able to give examples.

Notes:

simple harmonic motion; motion consisting of regular, periodic back-and-forth oscillation.

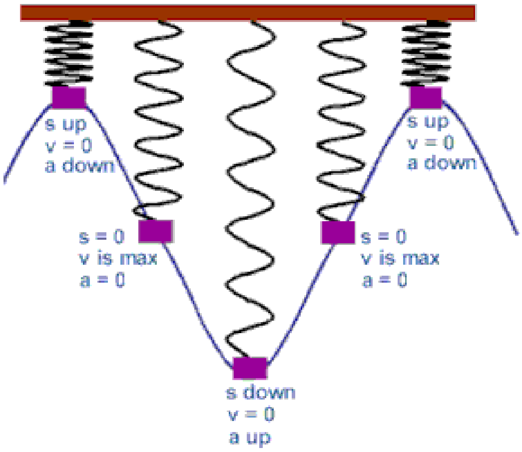
Requirements:

- The acceleration is always in the opposite direction from the displacement. This means the acceleration always slows down the motion and reverses the direction.
- In an ideal system (no friction), once simple harmonic motion is started, it would continue forever.
- A graph of displacement *vs.* time will result in the trigonometric function sine or cosine.

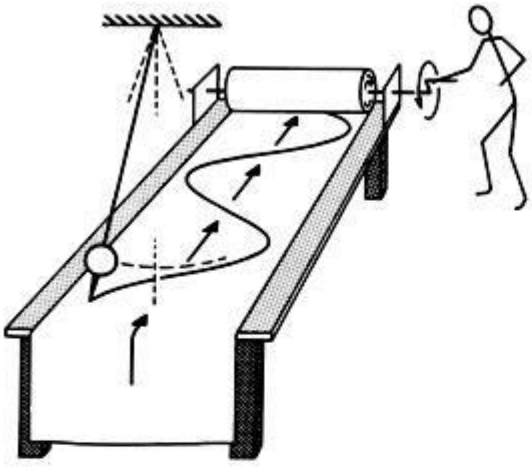
Use this space for summary and/or additional notes:

Examples of Simple Harmonic Motion

- **Springs**; as the spring compresses or stretches, the spring force accelerates it back toward its equilibrium position.



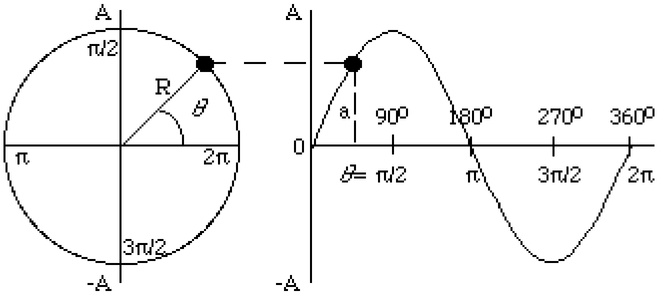
- **Pendulums**: as the pendulum swings, gravity accelerates it back toward its equilibrium position.



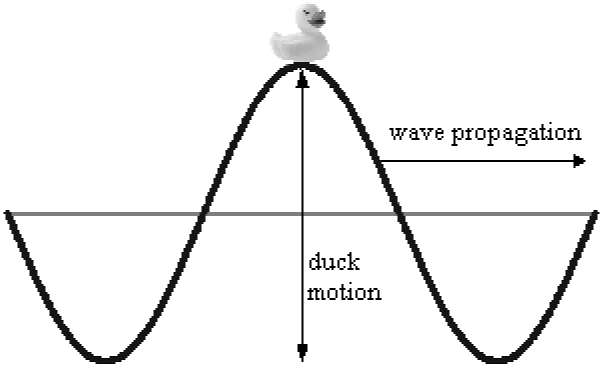
Use this space for summary and/or additional notes:

Simple Harmonic Motion

- **Uniform circular motion**: as an object moves around a circle, its vertical position (y-position) is continuously oscillating between $+r$ and $-r$.



- **Waves**: waves passing through some medium (such as water or air) cause the medium to oscillate up and down, like a duck sitting on the water as waves pass by.



Use this space for summary and/or additional notes:

Springs

Unit: Simple Harmonic Motion

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- springs and spring constants
- spring force as a vector quantity

Skills:

- calculate the force and potential energy of a spring

Language Objectives:

- Understand and correctly use the term "spring."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

spring: a coiled object that resists motion parallel with the direction of propagation of the coil.

Spring Force

The equation for the force (vector) from a spring is given by Hooke's Law, named for the British physicist Robert Hooke:

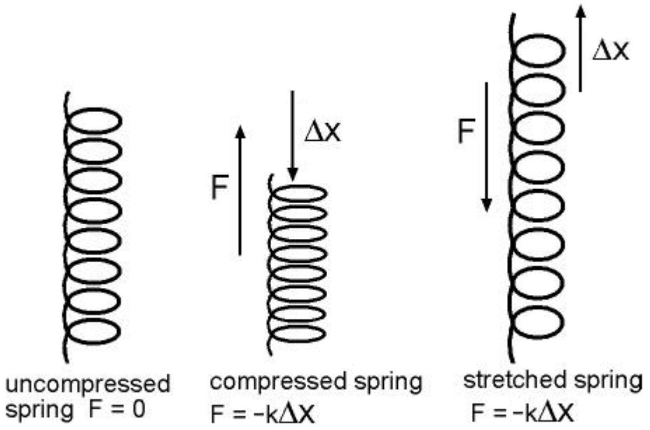
$$\vec{F}_s = -k\vec{x}$$

Where $\vec{F}_s$ is the spring force (vector quantity representing the force exerted by the spring), $\vec{x}$ is the displacement of the end of the spring (also a vector quantity), and k is the spring constant, an intrinsic property of the spring based on its mass, thickness, and the elasticity of the material that it is made of.

The negative sign in the equation is because the force is always in the opposite (negative) direction from the displacement.

A Slinky has a spring constant of $0.5\,\frac{\text{N}}{\text{m}}$, while a heavy garage door spring might have a spring constant of $500\,\frac{\text{N}}{\text{m}}$.

Use this space for summary and/or additional notes:



Potential Energy

The potential energy stored in a spring is given by the equation:

$$U = \tfrac{1}{2}kx^2$$

Where U is the potential energy (measured in joules), k is the spring constant, and x is the displacement. Note that the potential energy is always positive (or zero); this is because energy is a scalar quantity. A stretched spring and a compressed spring both have potential energy.

The Period of a Spring

period or period of oscillation: the time it takes a spring to move from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is T, and the unit is usually seconds.

The period of a spring depends on the mass of the spring and its spring constant, and is given by the equation:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

Use this space for summary and/or additional notes:

Frequency

frequency: the number of times something occurs in a given amount of time. Frequency is usually given by the variable f, and is measured in units of hertz (Hz). One hertz is the inverse of one second:

$$1\,\text{Hz} \equiv \frac{1}{1\,\text{s}} \equiv 1\,\text{s}^{-1}$$

Sample Problem:

Q: A spring with a mass of 0.1 kg and a spring constant of $2.7\,\frac{\text{N}}{\text{m}}$ is compressed 0.3 m. Find the force needed to compress the spring, the potential energy stored in the spring when it is compressed, and the period of oscillation.

A: The force is given by Hooke's Law.

Substituting these values gives:

$$\vec{\boldsymbol{F}} = -k\vec{\boldsymbol{x}}$$
$$\vec{\boldsymbol{F}} = -(2.7\,\tfrac{\text{N}}{\text{m}})(+0.3\,\text{m}) = -0.81\,\text{N}$$

The potential energy is:

$$U = \tfrac{1}{2}kx^2$$
$$U = (0.5)(2.7\,\tfrac{\text{N}}{\text{m}})(0.3\,\text{m})^2 = 0.12\,\text{J}$$

The period is:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$
$$T_s = (2)(3.14)\sqrt{\frac{0.1}{2.7}}$$
$$T_s = 6.28\sqrt{0.037} = (6.28)(0.19) = 1.2\,\text{s}$$

Use this space for summary and/or additional notes:

Pendulums

Unit: Simple Harmonic Motion

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- factors that affect the period and motion of a pendulum

Skills:

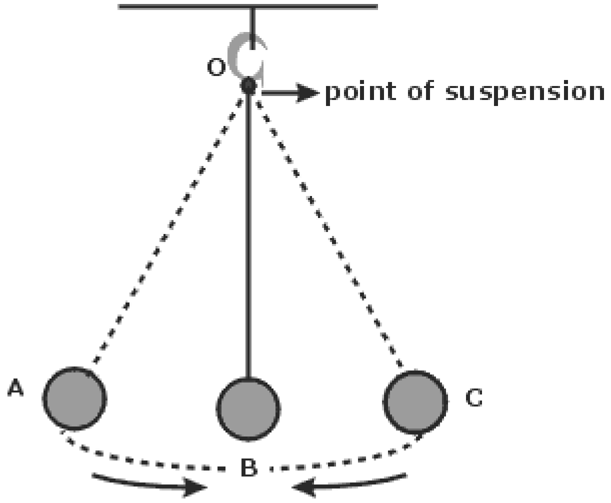
- calculate the period of a pendulum

Language Objectives:

- Understand and correctly use the terms "pendulum" and "period."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

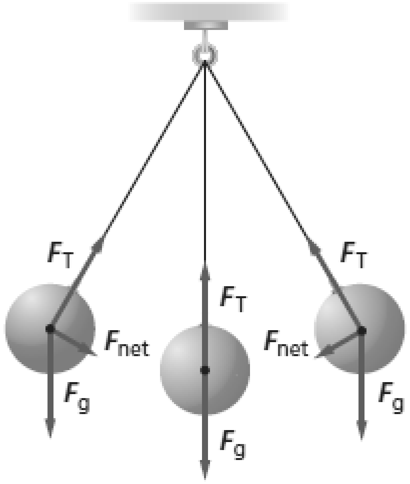
pendulum: a lever that is suspended from a point such that it can swing back and forth.



Use this space for summary and/or additional notes:

The Forces on a Pendulum

As the pendulum swings, its mass remains constant, which means the force of gravity pulling it down remains constant. The tension on the pendulum (which we can think of as a rope or string, though the pendulum can also be solid) also remains constant as it swings.



However, as the pendulum swings, the angle of the tension force changes. When the pendulum is not in the center (bottom), the vertical component of the tension is $F_T \cos \theta$, and the horizontal component is $F_T \sin \theta$. Because the angle is between 0° and 90°, $\cos \theta < 1$, which means F_g is greater than the upward component of F_T. This causes the pendulum to eventually stop. Also because the angle is between 0° and 90°, $\sin \theta > 0$, This causes the pendulum to start swinging in the opposite direction.

Use this space for summary and/or additional notes:

The Period of a Pendulum

period or period of oscillation: the time it takes a pendulum to travel from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is T, and the unit is usually seconds.

Note that the time between pendulum "beats" (such as the tick-tock of a pendulum clock) are ½ of the period of the pendulum. Thus a "grandfather" clock with a pendulum that beats seconds has a period $T = 2$ s.

The period of a pendulum depends on the force of gravity, the length of the pendulum, and the maximum angle of displacement. For small angles ($\theta < 15°$), the period is given by the equation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where T is the period of oscillation, $\boldsymbol{\ell}$ is the length of the pendulum in meters, and g is the acceleration due to gravity ($9.8\,\frac{\text{m}}{\text{s}^2}$ on Earth).

Note that the potential energy of a pendulum is simply the gravitational potential energy of the pendulum's center of mass at its maximum displacement.

The velocity of the pendulum at its lowest point (where the potential energy is zero and all of the energy is kinetic) can be calculated using the conservation of energy.

Use this space for summary and/or additional notes:

Pendulums

Sample Problem:

Q: An antique clock has a pendulum that is 0.20 m long. What is its period?

A: The period is given by the equation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T = 2(3.14)\sqrt{\frac{0.20}{9.8}}$$

$$T = 6.28\sqrt{0.0204}$$

$$T = (6.28)(0.142)$$

$$T = 0.898\text{ s}$$

Use this space for summary and/or additional notes:

Introduction: Mechanical Waves

Unit: Mechanical Waves

Topics covered in this chapter:

This chapter discusses properties of waves that travel through a medium (mechanical waves).

- *Simple Harmonic Motion* (SHM) describes the concept of repetitive back-and-forth motion and situations that apply to it.
- *Springs* and *Pendulums* describe specific examples of SHM and the specific equations relating to each.
- *Waves* gives general information about waves, including vocabulary and equations. *Reflection and Superposition* describes what happens when two waves share space within a medium.
- *Sound & Music* describes the properties and equations of waves that relate to music and musical instruments.
- *The Doppler Effect* describes the effects of motion of the source or receiver (listener) on the perception of sound.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

Massachusetts Curriculum Frameworks (2006):

4.1 Describe the measurable properties of waves (velocity, frequency, wavelength, amplitude, period) and explain the relationships among them. Recognize examples of simple harmonic motion.

4.2 Distinguish between mechanical and electromagnetic waves.

Use this space for summary and/or additional notes:

4.3 Distinguish between the two types of mechanical waves, transverse and longitudinal.

4.4 Describe qualitatively the basic principles of reflection and refraction of waves.

4.5 Recognize that mechanical waves generally move faster through a solid than through a liquid and faster through a liquid than through a gas.

4.6 Describe the apparent change in frequency of waves due to the motion of a source or a receiver (the Doppler effect).

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Simple Harmonic Motion**, such as mass on a spring and the pendulum
- **General Wave Properties**, such as wave speed, frequency, wavelength, superposition, standing wave diffraction, and the Doppler effect.

10. Periodic Motion (Simple Harmonic Motion)
11. Frequency and Period
12. Springs
13. Pendulums
14. Wave Motion
15. Transverse Waves and Longitudinal Waves
16. Superposition
17. Standing Waves and Resonance
18. The Doppler Effect

Skills learned & applied in this chapter:

- Visualizing wave motion.

Use this space for summary and/or additional notes:

Waves

Unit: Mechanical Waves

NGSS Standards: HS-PS4-1

MA Curriculum Frameworks (2006): 4.1, 4.3

Knowledge/Understanding:

- what waves are & how they move/propagate
- transverse *vs.* longitudinal waves
- mechanical *vs.* electromagnetic waves

Skills:

- calculate wavelength, frequency, period, and velocity of a wave

Language Objectives:

- Understand and correctly use the terms “wave,” “medium,” “propagation,” “mechanical wave,” “electromagnetic wave,” “transverse,” “longitudinal,” “torsional,” “crest,” “trough,” “amplitude,” “frequency,” “wavelength,” and “velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the wavelength, frequency and velocity of a wave.

Notes:

wave: a disturbance that travels from one place to another.

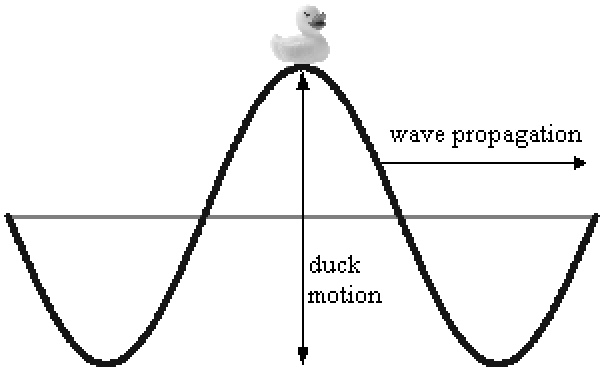
medium: a substance that a wave travels through.

propagation: the process of a wave traveling through space.

Use this space for summary and/or additional notes:

mechanical wave: a wave that propagates through a medium via contact between particles of the medium. Some examples of mechanical waves include ocean waves and sound waves.

- The energy of the wave is transmitted via the particles of the medium as the wave passes through it.
- The wave travels through the medium. The particles of the medium are moved by the wave passing through, and then return to their original position. (The duck sitting on top of the wave below is an example.)



- The denser the medium, the more frequently the particles come in contact, and therefore the faster the wave propagates. For example,

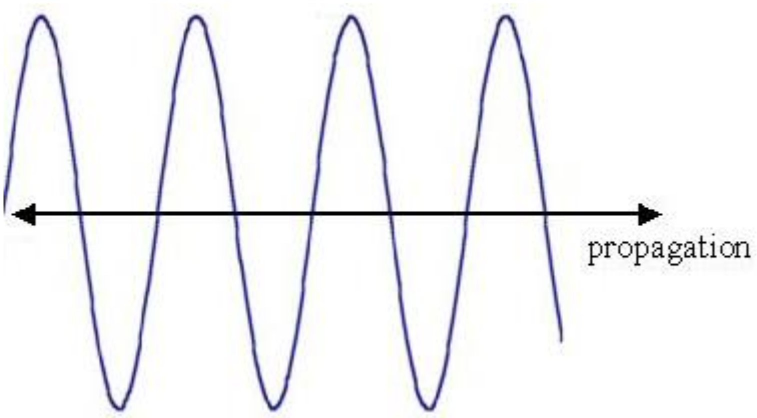
medium	**density ($\frac{kg}{m^3}$)**	**velocity of sound waves**
air (20°C and 1 atm)	1.2	$343\,\frac{m}{s}$ ($768\,\frac{mi}{hr}$)
water (20°C)	998	$1\,481\,\frac{m}{s}$ ($3\,317\,\frac{mi}{hr}$)
steel (longitudinal wave)	7 800	$6\,000\,\frac{m}{s}$ ($13\,000\,\frac{mi}{hr}$)

electromagnetic wave: a wave of electricity and magnetism interacting with each other. Electromagnetic waves can propagate through empty space.

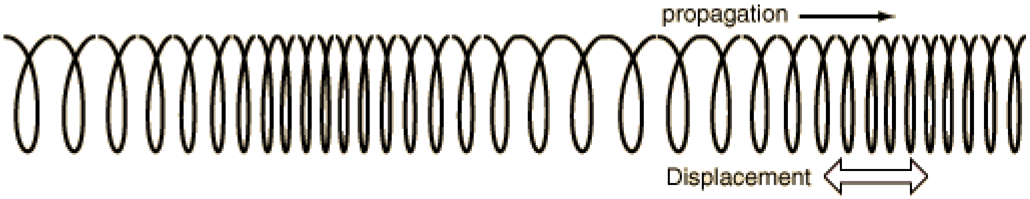
Use this space for summary and/or additional notes:

Types of Waves

transverse wave: moves its medium up & down (or back & forth) as it travels through. Examples: light, ocean waves



longitudinal wave (or compressional wave): compresses and decompresses the medium as it travels through. Example: sound.



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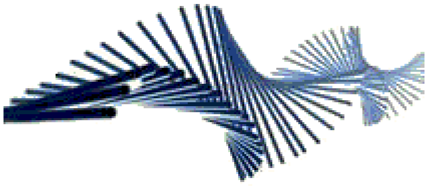
torsional wave: a type of transverse wave that propagates by twisting about its direction of propagation.



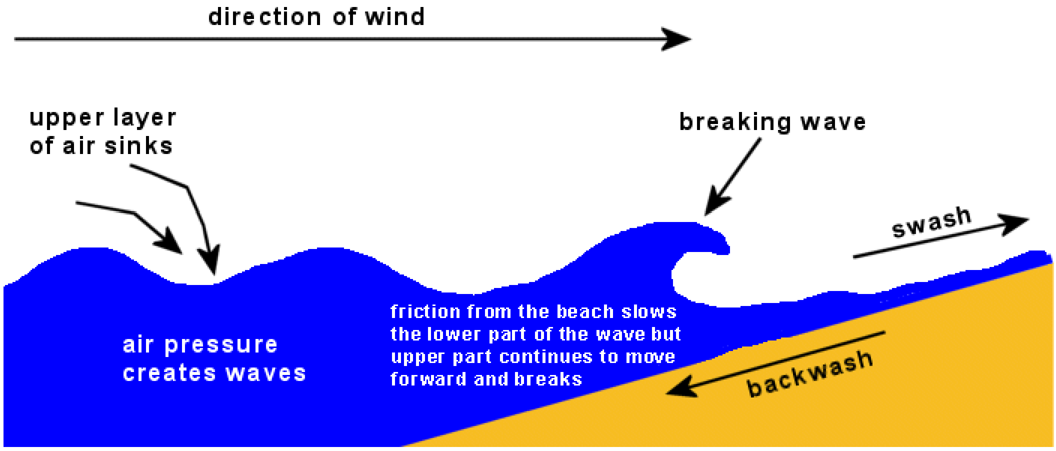
The most famous example of the destructive power of a torsional wave was the Tacoma Narrows Bridge, which collapsed on November 7, 1940. On that day, strong winds caused the bridge to vibrate torsionally. At first, the edges of the bridge swayed about eighteen inches. (This behavior had been observed previously, resulting in the bridge acquiring the nickname "Galloping Gertie".) However, after a support cable snapped, the vibration increased significantly, with the edges of the bridge being displaced up to 28 feet! Eventually, the bridge started twisting in two halves, one half twisting clockwise and the other half twisting counterclockwise, and then back again. This opposing torsional motion eventually caused the bridge to twist apart and collapse.

The bridge's collapse was captured on film. Video clips of the bridge twisting and collapsing are available on YouTube. There is a detailed analysis of the bridge's collapse at http://www.vibrationdata.com/Tacoma.htm

Use this space for summary and/or additional notes:

surface wave: a transverse wave that travels at the interface between two mediums.

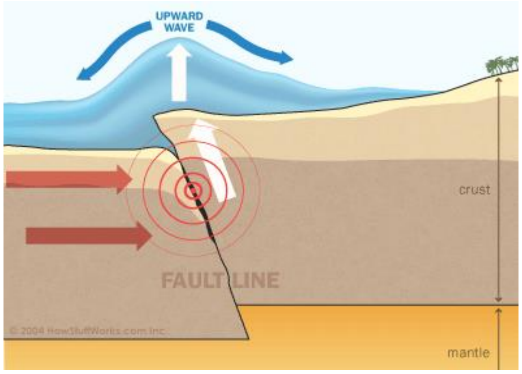
Ocean waves are an example of surface waves, because they travel at the interface between the air and the water. Surface waves on the ocean are caused by wind disturbing the surface of the water. Until the wave gets to the shore, surface waves have no effect on water molecules far below the surface.

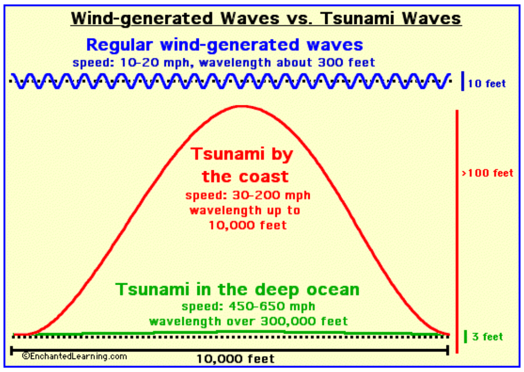


Use this space for summary and/or additional notes:

Tsunamis

The reason tsunamis are much more dangerous than regular ocean waves is because tsunamis are created by earthquakes on the ocean floor. The tsunami wave propagates through the entire depth of the water, which means tsunamis carry many times more energy than surface waves.

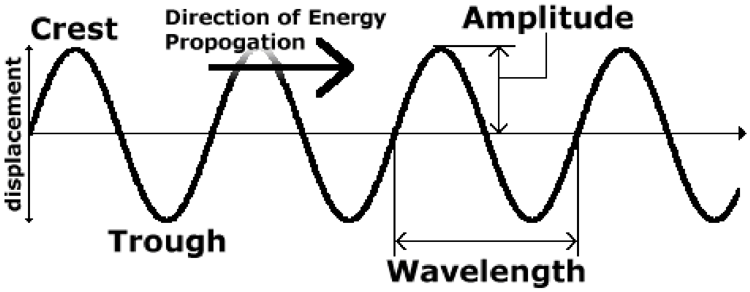




This is why a 6–12 foot high surface wave breaks harmlessly on the beach; however, a tsunami that extends 6–12 feet above the surface of the water includes a significant amount of energy below the surface and can destroy an entire city.

Use this space for summary and/or additional notes:

Properties of Waves



crest: the point of maximum positive displacement of a transverse wave. (The highest point.)

trough: the point of maximum negative displacement of a transverse wave. (The lowest point.)

amplitude: the distance of maximum displacement of a point in the medium as the wave passes through it.

wavelength: the length of the wave, measured from a specific point in the wave to the same point in the next wave. Symbol = λ (lambda); unit = distance (m, cm, nm, *etc.*)

frequency: the number of waves that travel past a point in a given time. Symbol = f; unit = $^1/_{\text{time}}$ (Hz = $^1/_s$)

Note that while high school physics courses generally use the variable f for frequency, college courses usually use ν (the Greek letter "nu", which is different from the Roman letter "v").

period or time period: the amount of time between two adjacent waves. Symbol = T; unit = time (usually seconds)

$$T = {}^1/_f$$

Use this space for summary and/or additional notes:

velocity: the velocity of a wave depends on its frequency (f) and its wavelength (λ):

$$v = \lambda f$$

The velocity of electromagnetic waves (such as light, radio waves, microwaves, X-rays, *etc.*) is called the speed of light, which is $3.00\times 10^{8}\ \frac{\text{m}}{\text{s}}$ in a vacuum. The speed of light is slower in a medium that has an index of refraction greater than 1. (We will discuss index of refraction in more detail in the light and optics topic.)

The velocity of a wave traveling through a string under tension (such as a piece of string, a rubber band, a violin/guitar string, *etc.*) depends on the tension and the ratio of the mass of the string to its length:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where F_T is the tension on the string, L is the length, and m is the mass.

Sample Problem:

Q: The radio station WZLX broadcasts waves with a frequency of 100.7 MHz. If the waves travel at the speed of light, what is the wavelength?

A: $f = 100.7\,\text{MHz} = 100\,700\,000\,\text{Hz} = 1.007\times 10^{8}\ \text{Hz}$

$v = c = 3.00\times 10^{8}\ \frac{\text{m}}{\text{s}}$

$v = \lambda f$

$3.00\times 10^{8} = \lambda\,(1.007\times 10^{8})$

$\lambda = \dfrac{3.00\times 10^{8}}{1.007\times 10^{8}} = 2.98\,\text{m}$

Use this space for summary and/or additional notes:

Reflection and Superposition

Unit: Mechanical Waves

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.1, 4.3

Knowledge/Understanding Goals:

- what happens when a wave reflects ("bounces") off an object or surface
- what happens when two or more waves occupy the same space

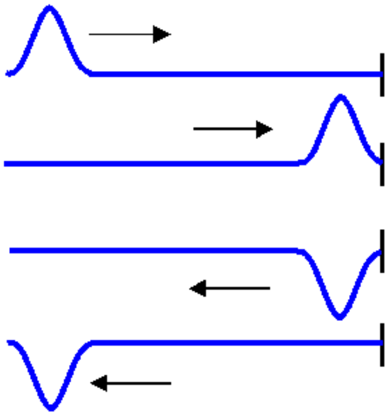
Language Objectives:

- Understand and correctly use the terms "reflection," "superposition," "constructive interference," and "destructive interference."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

Reflection of Waves

reflection: when a wave hits a fixed (stationary) point and "bounces" back.



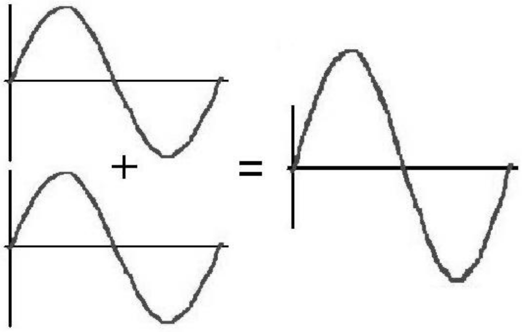
Notice that when the end of the rope is fixed, the reflected wave is inverted. (If the end of the rope was free, the wave would not invert.)

Use this space for summary and/or additional notes:

Superposition of Waves

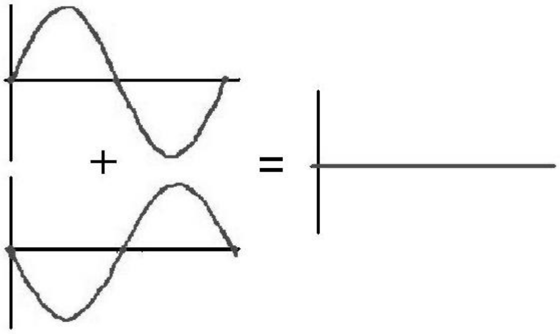
When waves are superimposed (occupy the same space), their amplitudes add:

constructive interference: when waves add in a way that the amplitude of the resulting wave is larger than the amplitudes of the component waves.



Because the wavelengths are the same and the maximum, minimum, and zero points all coïncide (line up), the two component waves are said to be "in phase" with each other.

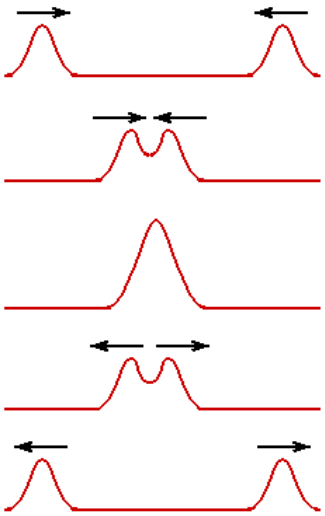
destructive interference: when waves add in a way that the amplitude of the resulting wave is smaller than the amplitudes of the component waves. (Sometimes we say that the waves "cancel" each other.)

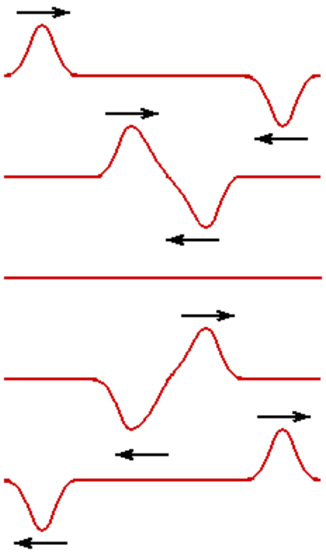


Because the wavelengths are the same but the maximum, minimum, and zero points do not coïncide, the waves are said to be "out of phase" with each other.

Use this space for summary and/or additional notes:

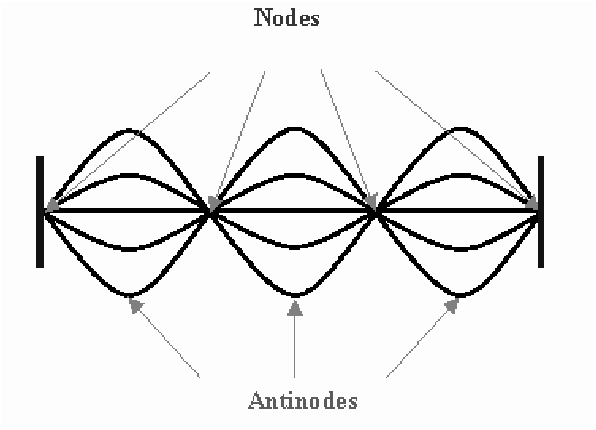
Note that waves can travel in two opposing directions at the same time. When this happens, the waves pass through each other, exhibiting constructive and/or destructive interference as they pass:





Standing Waves

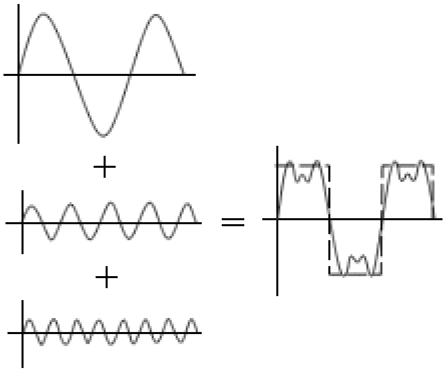
<u>standing wave</u>: when the wavelength is an exact fraction of the length of a medium that is vibrating, the wave reflects back and the reflected wave interferes constructively with itself. This causes the wave to appear stationary. Points along the wave that are not moving are called “nodes”. Points of maximum displacement are called “antinodes”.



Use this space for summary and/or additional notes:

Reflection and Superposition

When we add waves with different wavelengths and amplitudes, the result can be complex:

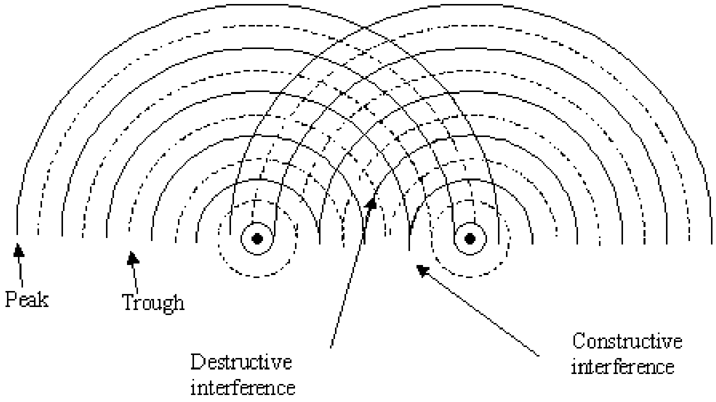


This is how radio waves encode a signal on top of a "carrier" wave. Your radio's antenna receives ("picks up") radio waves within a certain range of frequencies. Imagine that the bottom wave (the one with the shortest wavelength and highest frequency) is the "carrier" wave. If you tune your radio to its frequency, the radio will filter out other waves that don't include the carrier frequency. Then your radio subtracts the carrier wave, and everything that is left is sent to the speakers.

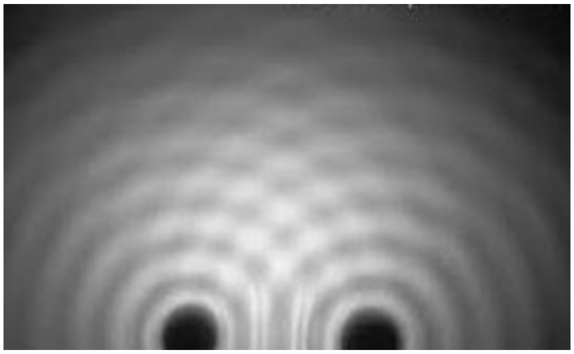
Use this space for summary and/or additional notes:

Interference Patterns

When two progressive waves propagate into each other's space, the waves produce interference patterns. This diagram shows how interference patterns form:



The resulting interference pattern looks like the following picture:



In this picture, the bright regions are wave peaks, and the dark regions are troughs. The brightest intersections are regions where the peaks interfere constructively, and the darkest intersections are regions where the troughs interfere constructively.

Use this space for summary and/or additional notes:

Sound & Music

Unit: Mechanical Waves

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- how musical notes are produced and perceived

Skills:

- calculate the frequency of the pitch produced by a string or pipe

Language Objectives:

- Understand and correctly use the terms "resonance," "frequency," and "harmonic series."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the frequencies and pitches (notes) produced by musical instruments.

Notes:

Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).

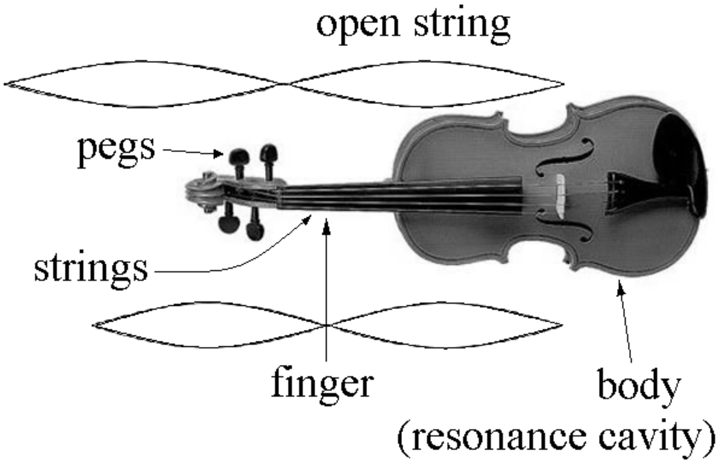
pitch: how "high" or "low" a musical note is. The pitch is determined by the frequency of the sound wave.

resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.

Use this space for summary and/or additional notes:

String Instruments

A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.



The vibration of the string creates a half-wave, *i.e.,* λ = 2L. The musician changes the half-wavelength by using a finger to shorten the length of the string that is able to vibrate. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string (such as a violin string) is given by the equation:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where:

f = frequency (Hz)

F_T = tension (N)

m = mass of string (kg)

L = length of string (m) $= \frac{\lambda}{2}$

The frequency (pitch) is therefore:

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$

Use this space for summary and/or additional notes:

Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. Something in the design of the mouthpiece causes the air inside the instrument to vibrate. When air is blown through the instrument, the air molecules compress and spread out at regular intervals that correspond with the length of the instrument, which determines the wavelength.

Most wind instruments use one of three ways of causing the air to vibrate:

Brass Instruments

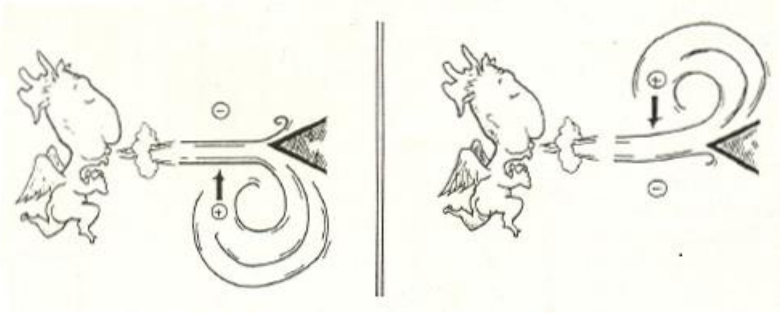
With brass instruments like trumpets, trombones, French horns, *etc.*, the player presses his/her lips tightly against the mouthpiece, and the player's lips vibrate at the appropriate frequency.

Reed Instruments

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons ("double-reed instruments") use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

Fipples

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple causes a pressure difference on one side vs. the other. The pressure builds more on one side, which forces air past the sharp edge. Then the pressure builds on the other side and the air switches back:



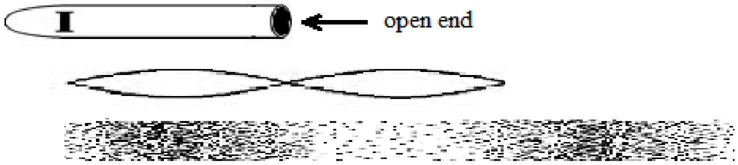
The frequency of this back-and-forth motion is what determines the pitch.

Use this space for summary and/or additional notes:

Open vs. Closed-Pipe Instruments

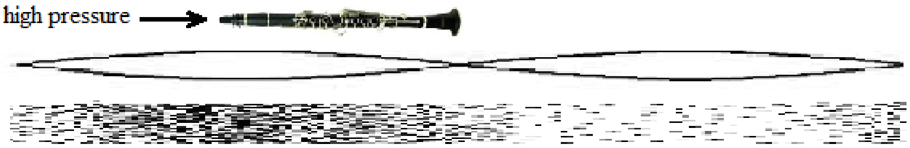
An open pipe has an opening at each end. A closed pipe has an opening at one end, and is closed at the other.

Examples of open pipes include uncapped organ pipes, whistles, recorders and flutes;



Notice that the two openings determine where the nodes are—these are the regions where the air pressure must be equal to atmospheric pressure (*i.e.,* the air is neither compressed nor expanded). Notice also that as with strings, the wavelength of the sound produced is twice the length of the pipe, *i.e.,* $\lambda = 2L$.

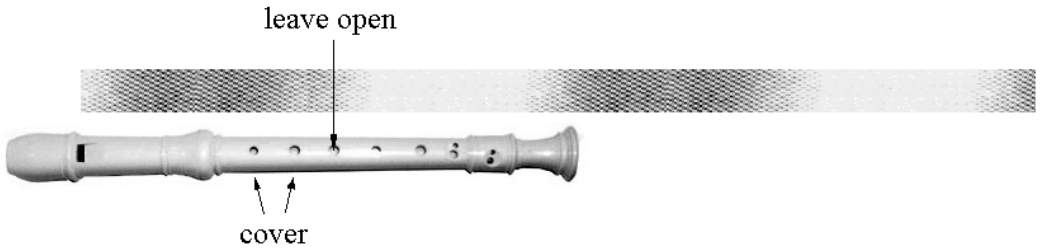
If the pipe is open to the atmosphere at only one end, such as a clarinet or brass instrument, there is only one node, at the mouthpiece. The opening, where the person is blowing into the instrument, is an antinode—a region of high pressure. This means that the body of the instrument is ¼ of a wave instead of ½, *i.e.,* $\lambda = 4L$.



This is why a closed-pipe instrument (*e.g.,* a clarinet) sounds an octave lower than an open-pipe instrument of similar length (*e.g.,* a flute).

Use this space for summary and/or additional notes:

For an instrument with holes, like a flute or recorder, the first open hole creates a node at that point, which determines the half-wavelength (or quarter-wavelength):



The speed of sound in air is v_s ($343\,\frac{\text{m}}{\text{s}}$ at 20°C and 1 atm), which means the frequency of the note (from the formula $v_s = \lambda f$) will be:

$f = \frac{v_s}{2L}$ for an open-pipe instrument (flute, recorder, whistle), and

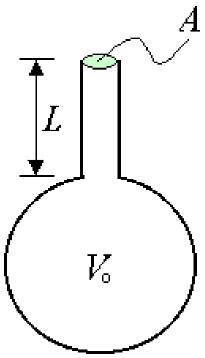
$f = \frac{v_s}{4L}$ for an closed-pipe instrument (clarinet, brass instrument).

Note that the speed of sound in air increases as the temperature increases. This means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. When this happens, it's not the instrument that is going out of tune, but the speed of sound!

Use this space for summary and/or additional notes:

Helmholtz Resonators (Bottles)

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate, because it depends on the resonance frequencies of the air in the large cavity, the air in the neck of the bottle, and the cross-sectional area of the opening.



The formula works out to be:

$$f = \frac{v_s}{2\pi}\sqrt{\frac{A}{V_o L}}$$

where:

- f = resonant frequency
- v_s = speed of sound in air ($343\,\frac{\text{m}}{\text{s}}$ at 20°C and 1 atm)
- A = cross-sectional area of the neck of the bottle (m^2)
- V_o = volume of the main cavity of the bottle (m^3)
- L = length of the neck of the bottle (m)

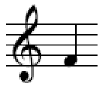
(Note that it may be more convenient to use measurements in cm, cm^2, and cm^3, and use $v_s = 34{,}300\,\frac{\text{cm}}{\text{s}}$.)

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You change the pitch by changing the size of the opening.

Use this space for summary and/or additional notes:

Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

pitch		**frequency (Hz)**	**pitch**		**frequency (Hz)**
	C	261.6		G	392.0
	D	293.7		A	440.0
	E	329.6		B	493.9
	F	349.2		C	523.2

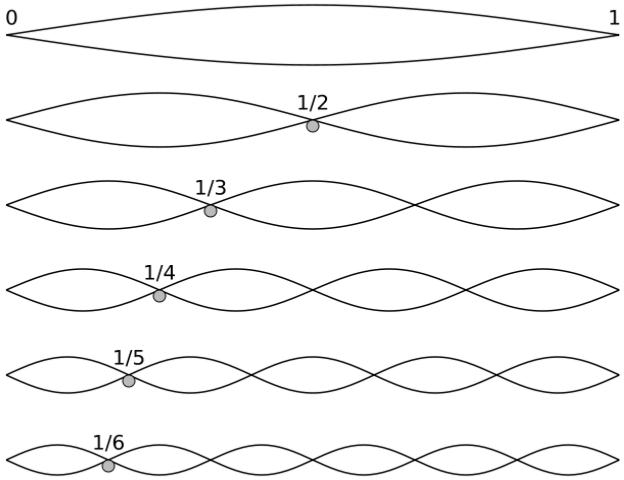
Note that a note that is an octave above another note has exactly twice the frequency of the lower note.

Harmonic Series

<u>harmonic series</u>: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves. The natural frequency is called the fundamental frequency, and the harmonics above it are numbered—1st harmonic, 2nd harmonic, *etc.*) Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.

Use this space for summary and/or additional notes:

The following diagram shows the fundamental frequency and the first five harmonics produced by a pipe or a vibrating string:

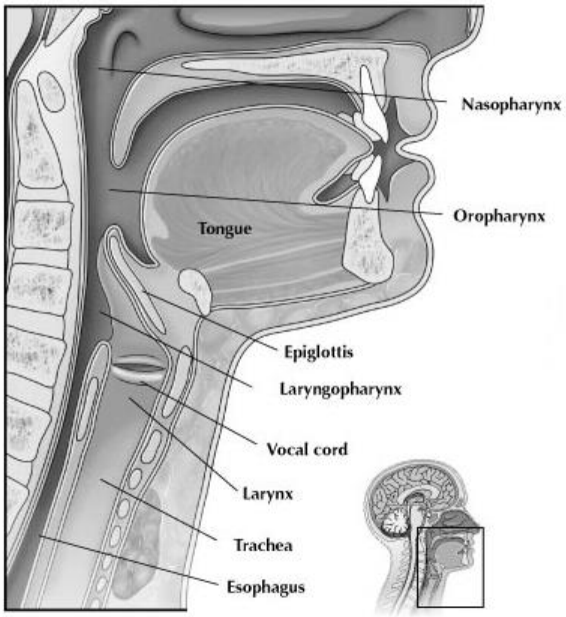


Fraction of String	**Wave-length**	**Harmonic**	**Frequency**	**Pitch (relative to fundamental)**
1	$2L$	0	f_o	Fundamental.
$^1/_2$	L	1st	$2f_o$	One octave above.
$^1/_3$	$\frac{2L}{3}$	2nd	$3f_o$	One octave + a fifth above.
$^1/_4$	$\frac{2L}{4}$	3rd	$4f_o$	Two octaves above.
$^1/_5$	$\frac{2L}{5}$	4th	$5f_o$	Two octaves + approximately a major third above.
$^1/_6$	$\frac{2L}{6}$	5th	$6f_o$	Two octaves + a fifth above.
$^1/_n$	$\frac{2L}{n}$	(n-1)th	$n\,f_o$	*etc.*

Use this space for summary and/or additional notes:

The Biophysics of Sound

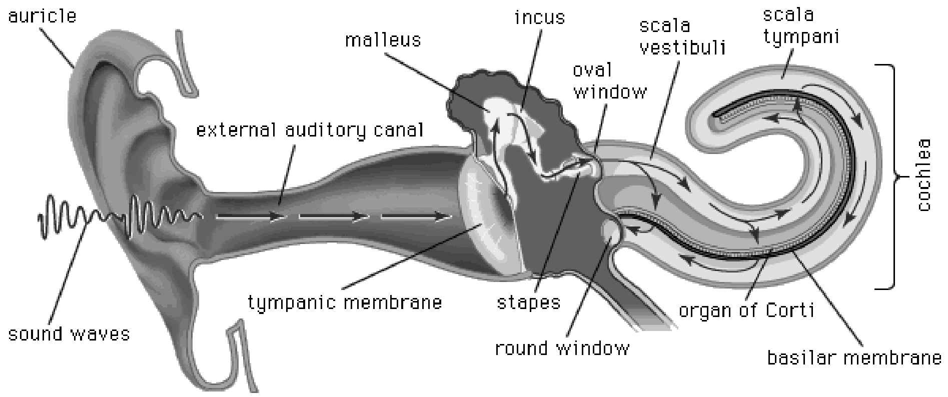
When a person speaks, stomach muscles force air from the lungs through the larynx.



The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency with which it vibrates. Just like with a string instrument, the change in tension changes the pitch. Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

Use this space for summary and/or additional notes:

When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.



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The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. The membrane is thickest close to the oval window and it gets thinner as the distance from the oval window increases. These differences in thickness cause sounds of different frequencies (pitches) to resonate in different regions of the cochlea. The brain uses the precise locations of these resonant frequencies to determine the pitch of a sound.

Use this space for summary and/or additional notes:

The Doppler Effect

Unit: Mechanical Waves

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.6

Knowledge/Understanding Goals:

- the Doppler effect

Skills:

- calculate the apparent shift in wavelength/frequency due to a difference in velocity between the source and receiver

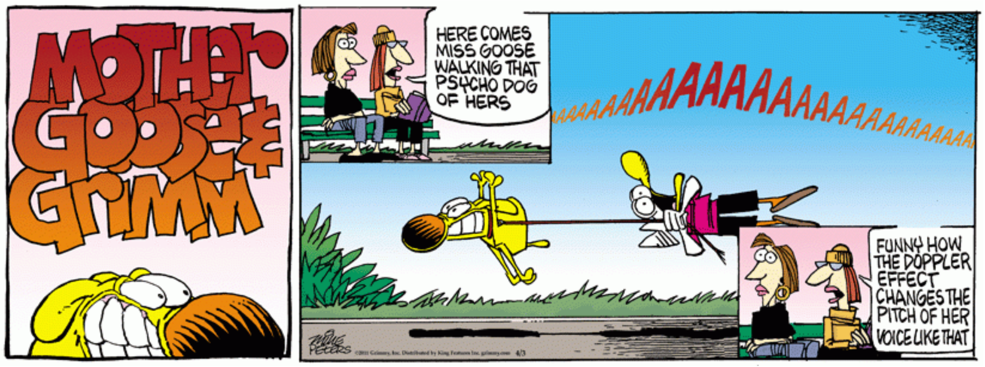
Language Objectives:

- Understand and correctly use the terms “Doppler effect” and “mach number.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the Doppler effect.

Notes:

<u>Doppler effect</u> or <u>Doppler shift</u>: the apparent change in frequency/wavelength of a wave due to a difference in velocity between the source of the wave and the observer. The effect is named for the Austrian physicist Christian Doppler.

You have probably noticed the Doppler effect when an emergency vehicle with a siren drives by.



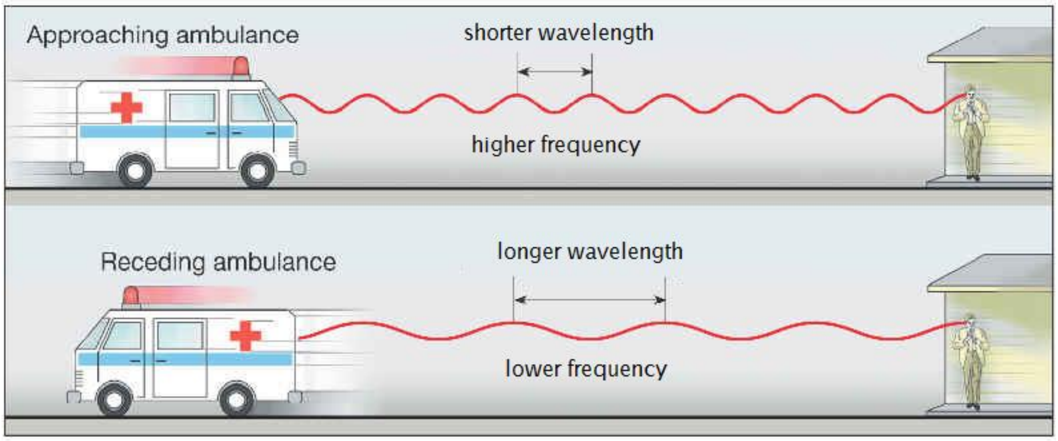
Use this space for summary and/or additional notes:

Why the Doppler Shift Happens

The Doppler shift occurs because a wave is created by a series of pulses at regular intervals, and the wave moves at a particular speed.

If the source is approaching, each pulse arrives sooner than it would have if the source had been stationary. Because frequency is the number of pulses that arrive in one second, the moving source results in an increase in the frequency observed by the receiver.

Similarly, if the source is moving away from the observer, each pulse arrives later, and the observed frequency is lower.



Use this space for summary and/or additional notes:

Calculating the Doppler Shift

The change in frequency is given by the equation:

$$f = f_o\left(\frac{v_w \pm v_r}{v_w \pm v_s}\right)$$

where:

f = observed frequency
f_o = frequency of the original wave
v_w = velocity of the wave
v_r = velocity of the receiver (you)
v_s = velocity of the source

The rule for adding or subtracting velocities is:

- The receiver's (your) velocity is in the numerator. If you are moving toward the sound, this makes the pulses arrive sooner, which makes the frequency higher. So if you are moving ***toward*** the sound, ***add*** your velocity. If you are moving ***away*** from the sound, ***subtract*** your velocity.
- The source's velocity is in the denominator. If the source is moving toward you, this makes the frequency higher, which means the denominator needs to be smaller. This means that if the source is moving ***toward*** you, ***subtract*** its velocity. If the source is moving ***away*** from you, ***add*** its velocity.

Don't try to memorize a rule for this—you will just confuse yourself. It's safer to reason through the equation. If something that's moving would make the frequency higher, that means you need to make the numerator larger or the denominator smaller. If it would make the frequency lower, that means you need to make the numerator smaller or the denominator larger.

Use this space for summary and/or additional notes:

Sample Problem:

Q: The horn on a fire truck sounds at a pitch of 350 Hz. What is the perceived frequency when the fire truck is moving toward you at $20\,\frac{\text{m}}{\text{s}}$? What is the perceived frequency when the fire truck is moving away from you at $20\,\frac{\text{m}}{\text{s}}$? Assume the speed of sound in air is $343\,\frac{\text{m}}{\text{s}}$.

A: The observer is not moving, so $v_r = 0$.

The fire truck is the source, so its velocity appears in the denominator.

When the fire truck is moving toward you, that makes the frequency higher. This means we need to make the denominator smaller, which means we need to ***subtract*** v_s:

$$f = f_o\left(\frac{v_w}{v_w - v_s}\right)$$

$$f = 350\left(\frac{343}{343-20}\right) = 350\,(1.062) = 372\,\text{Hz}$$

When the fire truck is moving away, the frequency will be lower, which mean we need to make the denominator larger. This means we need to ***add*** v_s:

$$f = f_o\left(\frac{v_w}{v_w + v_s}\right)$$

$$f = 350\left(\frac{343}{343+20}\right) = 350\,(0.9449) = 331\,\text{Hz}$$

Note that the pitch shift in each direction corresponds with about one half-step on the musical scale.

Use this space for summary and/or additional notes:

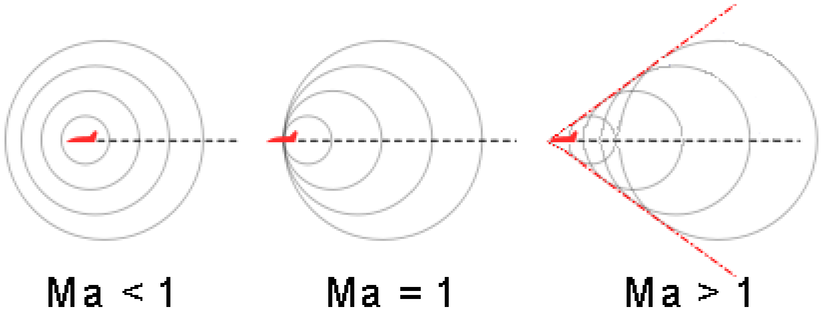
Exceeding the Speed of Sound

The speed of an object relative to the speed of sound in the same medium is called the Mach number (abbreviation Ma), named after the Austrian physicist Ernst Mach.

$$Ma = \frac{v_{object}}{v_{sound}}$$

Thus "Mach 1" or a speed of Ma = 1 is the speed of sound. An object such as an airplane that is moving at 1.5 times the speed of sound would be traveling at "Mach 1.5" or Ma = 1.5.

When an object such as an airplane is traveling slower than the speed of sound, the jet engine noise is Doppler shifted just like any other sound wave.

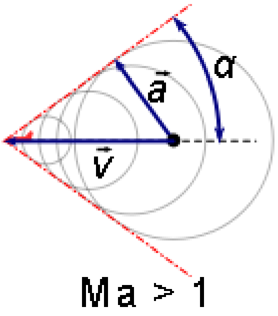


When the airplane's velocity reaches the speed of sound (Ma = 1), the leading edge of all of the sound waves produced by the plane coincides. These waves amplify each other, producing a loud shock wave called a "sonic boom".

(The "crack" of a bullwhip is a sonic boom—when a bullwhip is snapped sharply, the end of the bullwhip travels faster than sound.)

Use this space for summary and/or additional notes:

When an airplane is traveling faster than sound, the sound waves coincide at points behind the airplane at a specific angle, α:



The angle α is given by the equation:

$$\sin(\alpha) = \frac{1}{Ma}$$

I.e., the faster the airplane is traveling, the smaller the angle α, and the narrower the cone.

Use this space for summary and/or additional notes:

Introduction: Pressure & Fluid Mechanics

Unit: Pressure & Fluid Mechanics

Topics covered in this chapter:

Fluid mechanics is the study of behaviors that are specific to fluids (liquids and gases).

- *Pressure* is the property that is central to the topic of fluid mechanics.
- *Hydrostatics* and *Buoyancy* describe and give equations for the effects of gravity on pressure.
- *Gas Laws* describes behaviors and equations involving temperature, pressure and volume, as related to gases.
- *Fluid Motion & Bernoulli's Law* describes the effects of fluid motion on pressure.

This chapter focuses on real-world applications of fluids and pressure, including more demonstrations than most other topics. One of the challenges in this chapter is relating the equations to the behaviors seen in the demonstrations.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Massachusetts Curriculum Frameworks (2006):

No MA curriculum standards are addressed in this chapter.

Use this space for summary and/or additional notes:

Introduction: Pressure & Fluid Mechanics

Skills learned & applied in this chapter:

- Before & after problems.

Use this space for summary and/or additional notes:

Pressure

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- pressure

Skills:

- calculate pressure as an applied force

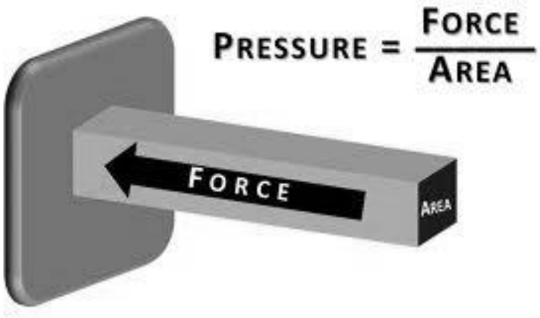
Language Objectives:

- Understand and correctly use the term "pressure" as it applies to situations in physics.
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to pressure and hydraulics.

Notes:

pressure: the exertion of force upon a surface by an object, fluid, *etc.* that is in contact with it.

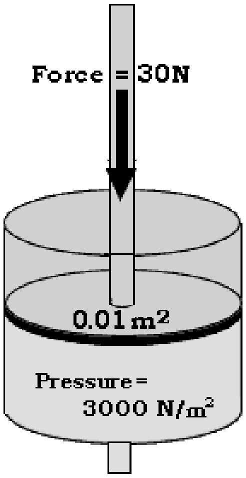
Mathematically, pressure is defined as force divided by area:



$$P = \frac{F}{A}$$

Use this space for summary and/or additional notes:

Pressure



The S.I. unit for pressure is the pascal (Pa).

$$1\,\text{Pa} \equiv 1\tfrac{\text{N}}{\text{m}^2} \equiv 1\tfrac{\text{kg}}{\text{ms}^2}$$

(Note that Pa is a two-letter symbol.)

Some other common pressure units are:

- bar: $1\,\text{bar} \equiv 100\,000\,\text{Pa} \equiv 10^5$ Pa
- pound per square inch (psi)
- atmosphere (atm): the average atmospheric pressure on Earth at sea level.

  $1\,\text{atm} \equiv 101325\,\text{Pa} \equiv 101.325\,\text{kPa} \equiv 1.01325\,\text{bar} = 14.696\,\text{psi}$

Air pressure can be described relative to a total vacuum (absolute pressure), but is more commonly described relative to atmospheric pressure (gauge pressure):

- absolute pressure: the total pressure on a surface. An absolute pressure of zero means there is zero force on the surface.
- gauge pressure: the difference between the pressure on a surface and atmospheric pressure. A gauge pressure of zero means the same as atmospheric pressure. The pressure in car tires is measured as gauge pressure. For example, a tire pressure of 30 psi (30 pounds per square inch, or $30\tfrac{\text{lb.}}{\text{in.}^2}$) would mean that the air inside the tires is pushing against the air outside the tires with a pressure of 30 psi. A flat tire would have a gauge pressure of zero.

Sample Problem

Q: What is the pressure caused by a force of 25 N acting on a piston with an area of 0.05 m^2?

A: $P = \dfrac{F}{A} = \dfrac{25\,\text{N}}{0.05\,\text{m}^2} = 500\,\text{Pa}$

Use this space for summary and/or additional notes:

Hydraulic Pressure

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- hydraulic pressure

Skills:

- calculate the force applied by a piston given the force on another piston and areas of both in a hydraulic system

Language Objectives:

- Understand and correctly use the term "hydraulic pressure."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to hydraulic pressure.

Notes:

Pascal's Principle, which was discovered by the French mathematician Blaise Pascal, states that any pressure applied to a fluid is transmitted uniformly throughout the fluid.

Because $P=\frac{F}{A}$, if the pressure is the same everywhere in the fluid, then $\frac{F}{A}$ must be the same everywhere in the fluid.

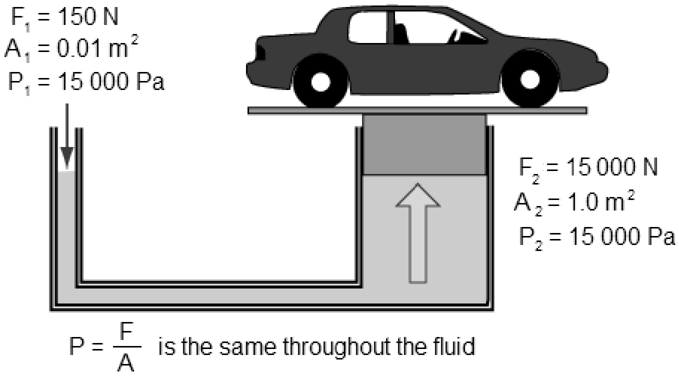
Use this space for summary and/or additional notes:

Hydraulic Pressure

If you have two pistons whose cylinders are connected, the pressure is the same throughout the fluid, which means the force on each piston is proportional to its own area. Thus:

$$P_1 = P_2 \text{ which means } \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

This principle is called "hydraulics." If you have a lift that has two pistons, one that is 100 times larger than the other, the larger one can supply 100 times as much force.



Conservation of energy tells us that the work done by F_1 must equal the work done by F_2, which means F_1 must act over a considerably larger distance than F_2. This also makes sense when you consider the volume of fluid transferred as a fraction of both the smaller and larger cylinders.

This is how hydraulic brakes work in cars. When you step on the brake pedal, the hydraulic pressure is transmitted to the master cylinder and then to the slave cylinders. The master cylinder is much smaller in diameter than the slave cylinders, which means the force applied to the brake pads is considerably greater than the force from your foot.

Sample Problem

Q: In a hydraulic system, a force of 25 N will be applied to a piston with an area of 0.50 m^2. If the force needs to lift a weight of 500. N, what must be the area of the piston supporting the 500. N weight?

A: $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ $\qquad \frac{25}{0.50} = \frac{500.}{A_2}$

$$25A_2 = (500)(0.50)$$
$$25A_2 = 250$$
$$A_2 = 10.\text{ m}^2$$

Use this space for summary and/or additional notes:

Hydrostatic Pressure

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- hydrostatic pressure

Skills:

- calculate pressure exerted by a column of fluid

Language Objectives:

- Understand and correctly use the term "hydrostatic pressure."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to hydrostatic pressure.

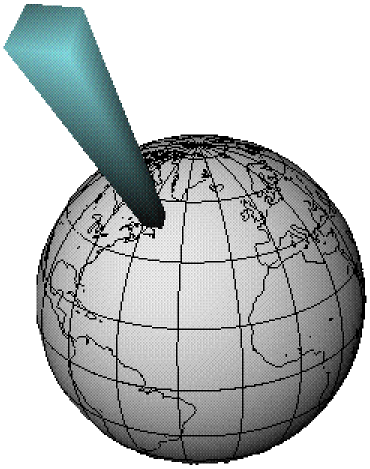
Notes:

Hydrostatic Pressure

hydrostatic pressure: the pressure caused by the weight of a column of fluid

The force of gravity pulling down on the molecules in a fluid creates pressure. The more fluid there is above a point, the higher the pressure at that point.

The atmospheric pressure that we measure at the surface of the Earth is caused by the air above us, all the way to the highest point in the atmosphere, as shown in the picture at right:



Use this space for summary and/or additional notes:

Hydrostatic Pressure

Assuming the density of the fluid is constant, the pressure in a column of fluid is caused by the weight (force of gravity) acting on an area. Because the force of gravity is *mg* (where $g = 9.8\,\frac{\text{m}}{\text{s}^2}$), this means:

$$P = \frac{F_g}{A} = \frac{mg}{A}$$

where:

P = pressure
g = acceleration due to gravity ($9.8\,\frac{\text{m}}{\text{s}^2}$ on Earth)
A = area of the surface the fluid is pushing on

We can cleverly multiply and divide our equation by volume:

$$P = \frac{mg}{A} = \frac{mg \cdot V}{A \cdot V} = \frac{m}{V} \cdot \frac{gV}{A}$$

Then, recognizing that density, ρ[*], is mass divided by volume, we can substitute:

$$P = \rho \cdot \frac{gV}{A}$$

Finally, if the volume of an object is the area of the base times the height (*h*), we can rewrite the equation as:

$$P = \frac{\rho g \cancel{A} h}{\cancel{A}} = \rho g h$$

---

[*] Note that physicists use the Greek letter ρ for density. You need to pay careful attention to the difference between the Greek letter ρ ("rho") and the Roman letter "p".

Use this space for summary and/or additional notes:

Finally, if there is an external pressure, P_o, above the fluid, we have to add it to the pressure from the fluid itself, which gives us the equation:

$$P = P_o + \rho g h$$

where:

P_o = pressure above the fluid (if relevant)

ρ = density of the fluid (this is the Greek letter "rho")

g = acceleration due to gravity ($9.8\,\frac{\text{m}}{\text{s}^2}$ on Earth)

h = height of the fluid above the point of interest

Sample Problem

Q: What is the water pressure in the ocean at a depth of 25 m? The density of sea water is $1025\,\frac{\text{kg}}{\text{m}^3}$.

A: $P = \rho g h = (1025\frac{\text{kg}}{\text{m}^3})(9.8\frac{\text{m}}{\text{s}^2})(25\,\text{m})$

$P = 251\,125\text{ Pa} = \boxed{250\,000\text{ Pa}}$ or $\boxed{2.50\text{ bar}}$.

Use this space for summary and/or additional notes:

Buoyancy

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Buoyancy & Archimedes' Principle

Skills:

- Calculate the buoyant force on an object

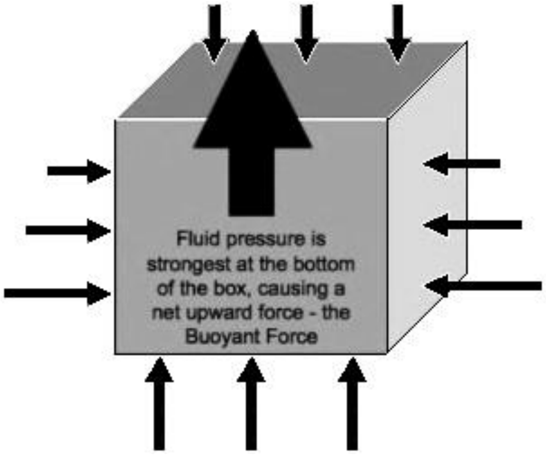
Language Objectives:

- Understand and correctly use the terms "displace" and "buoyant" or "buoyancy."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to buoyancy.

Notes:

displace: to push out of the way

buoyancy: a net upward force caused by the differences in hydrostatic pressure at different levels within a fluid:



The hydrostatic pressure is stronger at the bottom of the object than at the top, which causes a net upward force on the object.

Use this space for summary and/or additional notes:

Buoyancy

When an object displaces a fluid:

1. The volume of the fluid displaced equals the volume of the submerged part of the object: $V_d = V_{submerged}$
2. The weight of the fluid displaced equals the buoyant force (F_B).
3. The net force on the object, if any, is the difference between its weight and the buoyant force: $F_{net} = F_g - F_B$.

The buoyant force is caused by the difference in hydrostatic pressure from the bottom of the object to the top, *i.e.,* $F_B = PA_d$. However, the hydrostatic pressure is a function of the depth and the area is a function of the shape of the object. This means the buoyant force is given by the following double surface integral:

$$F_B = \oint\!\!\oint \rho g\, dA\, dh = \rho g \oint\!\!\oint dA\, dh$$

Conveniently, $\oint\!\!\oint dA\, dh$ is simply the volume of the part of the object that is submerged, which is equal to the volume of fluid displaced (item #1 above). Therefore:

$$F_B = \rho g \oint\!\!\oint dA\, dh = \rho g V_d$$

This gives the familiar equation for the buoyant force:

$$F_B = \rho V_d g$$

If the object floats, there is no net force, which means the weight of the object is equal to the buoyant force. This means:

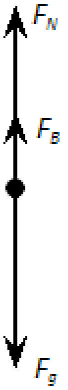
$$F_g = F_B$$
$$mg = \rho V_d g$$

Cancelling *g* from both sides gives $m = \rho V_d$, which can be rearranged to give the equation for density:

$$\rho = \frac{m}{V_d}$$

Use this space for summary and/or additional notes:

If the object sinks, the weight of the object is greater than the buoyant force. This means $F_B = \rho V_d g$, $F_g = mg$, and the weight of the submerged object is $F_{net} = F_g - F_B$.

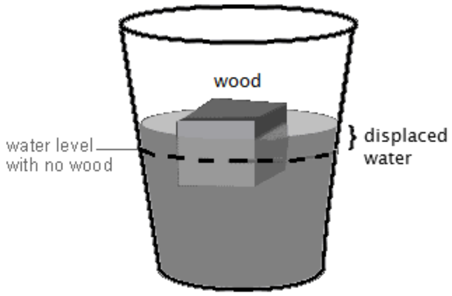


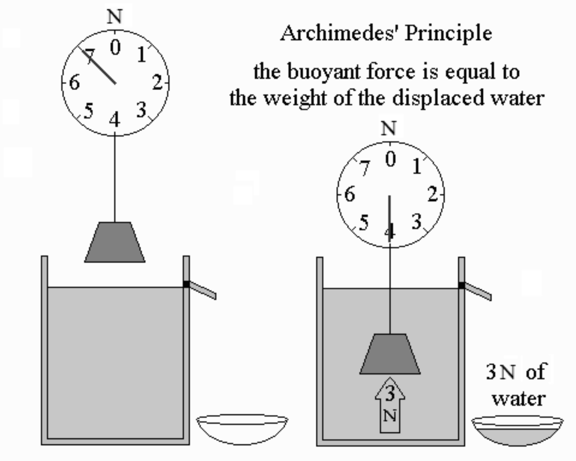
Note that if the object is resting on the bottom of the body of water, the net force must be zero, which means the normal force and the buoyant force combine to supply the total upward force. *I.e.,* for an object resting on the bottom:

$$F_{net} = 0 = F_g - (F_B + F_N)$$

which means:

$$F_g = F_B + F_N$$





This concept is known as Archimedes' Principle, named for the ancient Greek scientist who discovered it.

Use this space for summary and/or additional notes:

Buoyancy

The buoyant force can be calculated from the following equation:

$$F_B = m_d g = \rho V_d g$$

where:

F_B = buoyant force
m_d = mass of fluid displaced by the object
g = acceleration due to gravity ($9.8\,\frac{\text{m}}{\text{s}^2}$ on Earth)
ρ = density of the fluid applying the buoyant force (*e.g.,* water, air)
V_d = volume of fluid displaced by the object

Sample Problems:

Q: A cruise ship displaces 35 000 tonnes of water when it is floating. (1 tonne = 1 000 kg) If sea water has a density of $1\,025\,\frac{\text{kg}}{\text{m}^3}$, what volume of water does the ship displace? What is the buoyant force on the ship?

A: $\rho = \dfrac{m}{V}$

$$1\,025\,\tfrac{\text{kg}}{\text{m}^3} = \frac{35000000\,\text{kg}}{V}$$

$$V = 34\,146\,\text{m}^3$$

$$F_B = \rho V_d g$$

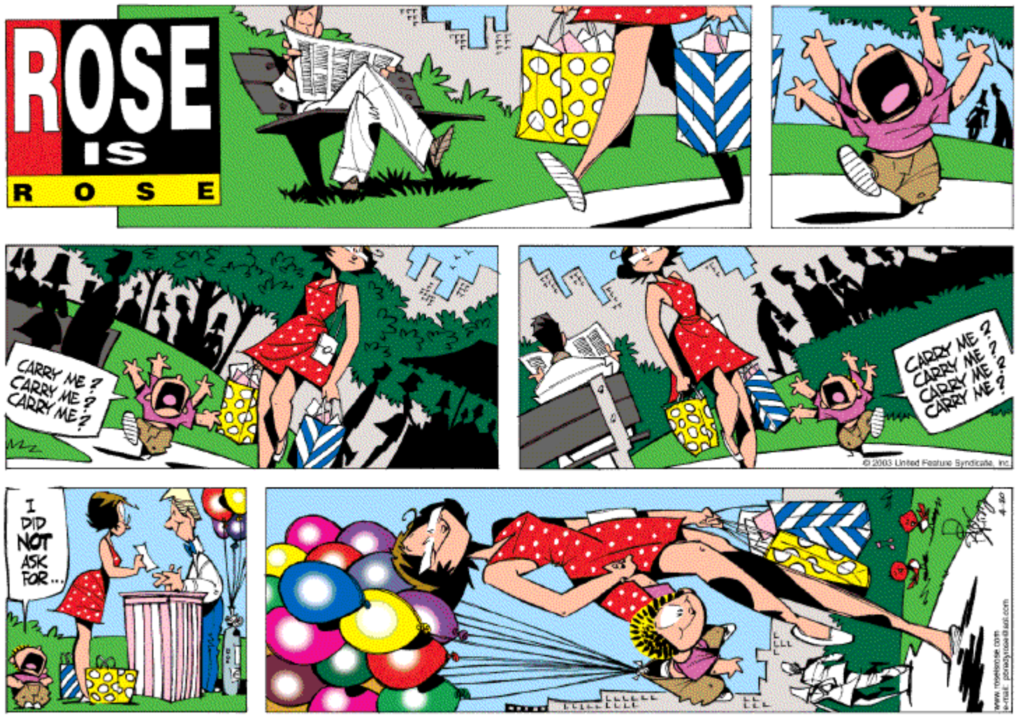
$$F_B = (1\,025\,\tfrac{\text{kg}}{\text{m}^3})(34\,146\,\text{m}^3)(9.8\,\tfrac{\text{m}}{\text{s}^2})$$

$$F_B = \boxed{3.43 \times 10^8\ \text{N}}.$$

Use this space for summary and/or additional notes:

Buoyancy

Q: Consider the following cartoon:



Given the following assumptions:

- The balloons are standard 11″ ballons, meaning that they have a diameter of 11 inches (28 cm).
- The temperature is 20°C. At this temperature, air has a density of $1.200\,\frac{\text{kg}}{\text{m}^3}$, and helium has a density of $0.166\,\frac{\text{kg}}{\text{m}^3}$.
- Pasquale is probably about four years old. The average mass a four-year-old boy is about 16 kg.
- The mass of an empty balloon plus string is 2.37 g = 0.00237 kg

How many balloons would it actually take to lift Pasquale?

Use this space for summary and/or additional notes:

A: In order to lift Pasquale, $F_B = F_g$.

$$F_g = mg = (16)(9.8) = 156.8\text{ N}$$

$$F_B = \rho_{air} V_d g = (1.200) V_d (9.8)$$

Because $F_B = F_g$, this means:

$$156.8 = (1.200) V_d (9.8)$$

$$V_d = 13.33\text{ m}^3$$

Assuming spherical balloons, the volume of one balloon is:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}(3.14)(0.14)^3 = 0.0115\text{ m}^3$$

Therefore, we need $\frac{13.33}{0.0115} = 1\,160\text{ balloons}$ to lift Pasquale.

However, the problem with this answer is that it doesn't account for the mass of the helium, the balloons or the strings.

Each balloon contains $0.0115\text{ m}^3 \times 0.166\,\frac{\text{kg}}{\text{m}^3} = 0.00191\text{ kg} = 1.91\text{ g}$ of helium. Each balloon (including the string) has a mass of 2.37 g = 0.00237 kg, so the total mass of each balloon full of helium is 0.00428 kg.

This means if we have *n* balloons, the total mass of Pasquale plus the balloons is $16 + 0.00428n$ kilograms. The total weight of Pasquale plus the balloons is therefore this number times 9.8, which equals 156.8 + 0.0419*n.*

The buoyant force of one balloon is:

$$F_B = \rho_{air} V_d g = (1.200)(0.0115)(9.8) = 0.135\text{ N}$$

Therefore, the buoyant force of *n* balloons is 0.135*n* newtons.

For Pasquale to be able to float, $F_B = F_g$, which means

$$0.135n = 0.0419n + 156.8$$

$$0.093n = 156.8$$

$n = \boxed{\text{1 685 balloons}}$

Use this space for summary and/or additional notes:

Gas Laws

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Boyle's Law (pressure *vs.* volume)
- Amontons' Law (pressure *vs.* temperature)
- Charles' Law (temperature *vs.* volume)
- Avogadro's Principle (number of particles *vs.* volume)
- ideal gas law
- combined gas law

Skills:

- Solve problems using the gas laws

Language Objectives:

- Understand and correctly use the terms "pressure," "volume," and "temperature," and "ideal gas."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using the combined gas law and the ideal gas law.

Notes:

ideal gas: a gas that behaves according to the Kinetic-Molecular Theory of Gases. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost. Most gases behave ideally when the temperature is high enough and the pressure is low enough that the molecules are not likely to form a liquid or solid, and when the molecules are made of substances that do not react chemically.

Use this space for summary and/or additional notes:

Boyle's Law

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional. If temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1 V_1 = P_2 V_2$$

(Note that by convention, gas laws use subscripts "1" and "2" instead of "i" and "f".)

Amontons' Law

In 1702, French physicist Guillaume Amontons discovered that the pressure and temperature of a gas were directly proportional. If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

(This law is often erroneously attributed to the French chemist Joseph Louis Gay-Lussac.)

Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional. If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Avogadro's Principle

In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of molecules:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Use this space for summary and/or additional notes:

The Combined Gas Law

If each of the above principles is combined, the following relationship is found for an ideal gas:

$$\frac{P_1V_1}{N_1T_1} = \frac{P_2V_2}{N_2T_2} = \text{constant}$$

If pressure is in Pa, volume is in m^3, and temperature is in Kelvin, the value of the constant turns out to be $1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}$. This number is called Boltzmann's constant, named for the German physicist Ludwig Boltzmann. Physicists usually use the variable k_B to represent Boltzmann's constant.

Note, however, that in most problems, the number of particles of gas doesn't change. This means $N_1 = N_2$, and we can cancel it from the equation, which gives:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

This equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

When using the combined gas law, any quantity that is not changing may therefore be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.) For example, suppose a problem doesn't mention anything about temperature. That means T is constant and you can cancel it. When you cancel T from both sides of the combined gas law, you get:

$$\frac{P_1V_1}{\cancel{T_1}} = \frac{P_2V_2}{\cancel{T_2}} \text{ which simplifies to } P_1V_1 = P_2V_2 \text{ (Boyle's Law)}$$

Use this space for summary and/or additional notes:

Solving Problems Using the Combined Gas Law

You can use this method to solve any "before/after" gas law problem:

1. Determine which variables you have
2. Determine which values are *inital* (#1) *vs. final* (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don't forget that temperatures *must* be in Kelvin!)

Use this space for summary and/or additional notes:

The Ideal Gas Law

The definition of Boltzmann's constant is called the ideal gas law:

$$\frac{PV}{NT} = k_B$$

Multiplying both sides of this equation by NT gives the ideal gas law in its most common form:

$$PV = Nk_BT$$

where:

P = absolute pressure (Pa)

V = volume (m^3)

N = number of particles (or molecules)

k_B = Boltzmann's constant ($1.38 \times 10^{-23}\ \frac{J}{K}$).

T = absolute temperature (K)

Note that the units on both sides of this equation are equivalent to Newton-meters, which are equivalent to joules (energy). This means that the ideal gas law is really just a mathematical statement of the work-energy theorem.

The left side of the equation, PV, represents the work that the gas can do on its surroundings. This is because:

$$P = \frac{F}{A}\text{, which means } PV = \frac{FV}{A} = Fd = W$$

The right side of the equation, Nk_BT, represents the number of particles (N) times the average kinetic energy of each particle (T), which equals the total kinetic energy of the particles. (Boltzmann's constant, k_B, is just the number that makes the units work out correctly.)

In other words, the work that a gas can do equals the kinetic energy of its particles.

Use this space for summary and/or additional notes:

Chemists more commonly work with moles rather than particles. In chemistry, the ideal gas law is more commonly written:

$$PV = nRT$$

where:

n = moles of gas

R = gas constant ($8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}$)

P, V, and T are as above

The gas constant, R, is equal to Boltzmann's constant times the conversion from moles to particles (Avogadro's constant):

$$R = k_B N_A = (1.38\times10^{-23}\,\tfrac{\text{J}}{\text{K}})(6.02\times10^{23}\,\text{mol}^{-1}) = 8.31\,\tfrac{\text{J}}{\text{mol}\cdot\text{K}}$$

Solving Problems Using the Ideal Gas Law

If a gas behaves according to the ideal gas law, simply substitute the values for pressure, volume, number of particles (or number of moles), and temperature into the equation. Be sure your units are correct (especially that temperature is in Kelvin), and that you use the correct constant, depending on whether you know the number of particles or the number of moles of the gas.

Use this space for summary and/or additional notes:

Sample problems:

Q: A sample of an ideal gas contains 2.10×10^{24} particles, has a pressure of 1.20 bar, and a temperature of 35°C. What is its volume?

A: First, we need to declare our variables.

Note that for the units to work out correctly (because of the units of Boltzmann's constant), pressure must be in Pa, volume must be in m^3, and temperature must be in Kelvin.

$P = 1.20\,\text{bar}\times\frac{100\,000\,\text{Pa}}{1\,\text{bar}} = 120\,000\,\text{Pa}$

$V = V$ (because we don't' know it yet)

$N = 2.1\times10^{24}$

$k_B = 1.38\times10^{-23}\,\frac{\text{m}^2\cdot\text{kg}}{\text{s}^2\cdot\text{K}}$

$T = 35°\text{C} + 273 = 308\,\text{K}$

Then we substitute these numbers into the ideal gas law and solve:

$$PV = Nk_BT$$

$$(120\,000\,\text{Pa})V = (2.10\times10^{24})(1.38\times10^{-23}\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2\cdot\text{K}})(308\,\text{K})$$

$$120\,000\,V = 8926$$

$$V = 0.0744\,\text{m}^3$$

Use this space for summary and/or additional notes:

Gas Laws

Q: A gas has a temperature of 25°C and a pressure of 1.5 bar. If the gas is heated to 35°C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25°C and 35°C), and two pressures (1.5 bar and the new pressure that we're looking for).

2. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

Time 1 ("before"):	**Time 2 ("after")**:
$P_1 = 1.5$ bar	$P_2 = P_2$
$T_1 = 25°\text{C} + 273 = 298$ K	$T_2 = 35°\text{C} + 273 = 308$ K

3. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Amontons' Law)}$$

4. Plug in our values and solve:

$$\frac{1.5\,\text{bar}}{298\,\text{K}} = \frac{P_2}{308\,\text{K}} \quad \rightarrow \quad \boxed{P_2 = 1.55\text{ bar}}$$

Use this space for summary and/or additional notes:

Fluid Motion & Bernoulli's Law

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Bernoulli's Law/Equation

Language Objectives:

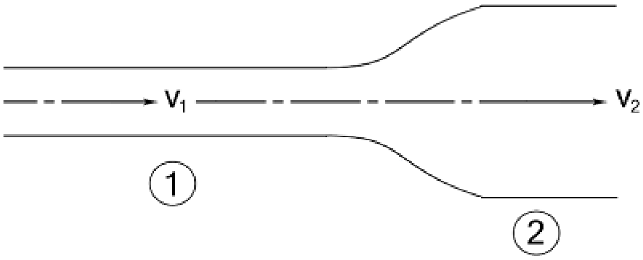
- Understand and correctly use the term "flow."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using Bernoulli's equation.

Notes:

<u>flow</u>: the net movement of a fluid

<u>velocity of a fluid</u>: the average velocity of a particle of fluid as the fluid flows past a reference point.

When a fluid flows through a pipe, the total volume of fluid per unit of time $\left(\frac{m^3}{s}\right)$ is equal to the velocity $\left(\frac{m}{s}\right)$ times the cross-sectional area (m^2). If the total flow remains the same, but the, but the diameter of the pipe changes:



then the cross-sectional area (A) times the fluid velocity (v) at point 1 equals the cross-sectional area times the fluid velocity at point 2:

$$A_1 v_1 = A_2 v_2$$

Use this space for summary and/or additional notes:

According to the Dutch-Swiss mathematician Daniel Bernoulli, the pressures in a moving fluid are caused by:

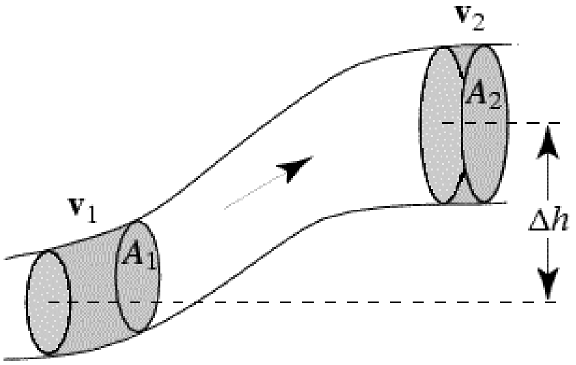
- The pressure exerted by the fluid, P (This is the pressure we would measure with a pressure gauge.)
- The hydrostatic pressure, ρgh.
- The dynamic pressure, $\frac{1}{2}\rho v^2$, which results from the force that the moving fluid particles exert on the other fluid particles around them. (Note that $\frac{1}{2}\rho v^2$ has units of pressure: $\left(\frac{\text{kg}}{\text{m}^3}\right)\cdot\left(\frac{\text{m}}{\text{s}}\right)^2 = \frac{\text{kg}}{\text{m}\cdot\text{s}^2} = \text{Pa}$.)

A change in any of these pressures affects the others, which means:

$$P + \rho gh + \tfrac{1}{2}\rho v^2 = \text{constant}$$

This equation is Bernoulli's equation.

If the velocity and height of the fluid are changing, as in the following diagram, then the pressure must also change as a result:

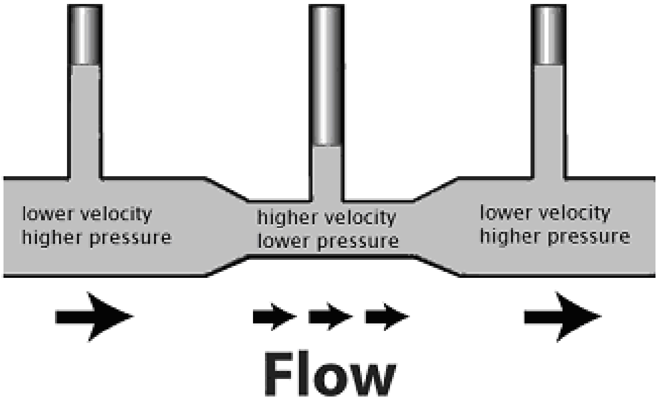


In this situation, Bernoulli's equation becomes:

$$P_1 + \rho gh_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \tfrac{1}{2}\rho v_2^2$$

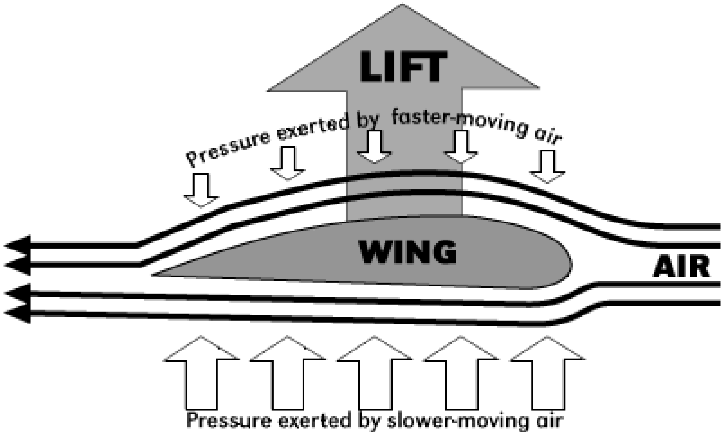
Use this space for summary and/or additional notes.

In Bernoulli's equation, increasing the fluid velocity (v) increases the $\frac{1}{2}\rho v^2$ term. Consider the following example:



This pipe is horizontal, which means h is constant; therefore ρgh is constant. This means that if $\frac{1}{2}\rho v^2$ increases, then pressure (P) must decrease so that $P+\rho gh+\frac{1}{2}\rho v^2$ remains constant.

This decrease in pressure caused by an increase in fluid velocity explains one of the ways in which an airplane wing provides lift:



Use this space for summary and/or additional notes.

A common demonstration of Bernoulli's Law is to blow across a piece of paper:

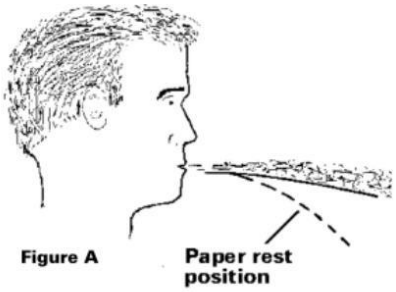


Figure A

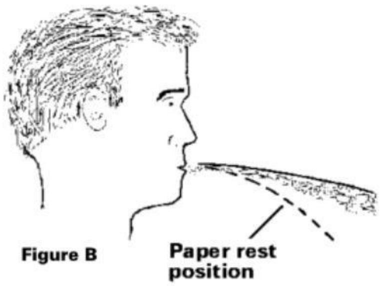


Figure B

The air moving across the top of the paper causes a decrease in pressure, which causes the paper to lift.

Sample Problems:

Q: A fluid in a pipe with a diameter of 0.40 m is moving with a velocity of $0.30\,\frac{\text{m}}{\text{s}}$. If the fluid moves into a second pipe with half the diameter, what will the new fluid velocity be?

A: The cross-sectional area of the first pipe is:

$A_1 = \pi r^2 = (3.14)(0.20)^2 = 0.126\text{ m}^2$

The cross-sectional area of the second pipe is:

$A_2 = \pi r^2 = (3.14)(0.10)^2 = 0.0314\text{ m}^2$

$A_1 v_1 = A_2 v_2$

$(0.126)(0.30) = (0.0314)v_2$

$v_2 = \boxed{1.2\,\frac{\text{m}}{\text{s}}}$.

Use this space for summary and/or additional notes.

Fluid Motion & Bernoulli's Law

Q: A fluid with a density of $1250\,\frac{\text{kg}}{\text{m}^3}$ has a pressure of 45 000 Pa as it flows at $1.5\,\frac{\text{m}}{\text{s}}$ through a pipe. The pipe rises to a height of 2.5 m, where it connects to a second, smaller pipe. What is the pressure in the smaller pipe if the fluid flows at a rate of $3.4\,\frac{\text{m}}{\text{s}}$ through it?

A:

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

$$45\,000 + (1250)(9.8)(0) + (½)(1250)(1.5)^2 =$$

$$P_2 + (1250)(9.8)(2.5) + (½)(1250)(3.4)^2$$

$$45\,000 + 1406 = P_2 + 30\,625 + 7225$$

$$46\,406 = P_2 + 37\,850$$

$$P_2 = 8\,556 \text{ Pa} = \boxed{8\,600 \text{ Pa}}$$

Use this space for summary and/or additional notes.

Appendix: AP Physics 1 Equation Tables

ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2015

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67\times10^{-27}$ kg	Electron charge magnitude,	$e = 1.60\times10^{-19}$ C
Neutron mass,	$m_n = 1.67\times10^{-27}$ kg	Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0\times10^{9}\ \mathrm{N\cdot m^2/C^2}$
Electron mass,	$m_e = 9.11\times10^{-31}$ kg	Universal gravitational constant,	$G = 6.67\times10^{-11}\ \mathrm{m^3/kg\cdot s^2}$
Speed of light,	$c = 3.00\times10^{8}\ \mathrm{m/s}$	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \mathrm{m/s^2}$

UNIT SYMBOLS	meter, m	kelvin, K	watt, W	degree Celsius, °C
	kilogram, k	hertz, Hz	coulomb, C	
	second, s	newton, N	volt, V	
	ampere, A	joule, J	ohm, Ω	

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan\theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. Assume air resistance is negligible unless otherwise stated.
III. In all situations, positive work is defined as work done on a system.
IV. The direction of current is conventional current: the direction in which positive charge would drift.
V. Assume all batteries and meters are ideal unless otherwise stated.

ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2015

MECHANICS

$v_x = v_{xo} + a_x t$

$x = x_o + v_{xo}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{xo}^2 + 2a_x(x - x_o)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\left|\vec{F}_f\right| \le \mu\left|\vec{F}_n\right|$

$a_c = \frac{v^2}{r}$

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = \vec{F}\Delta t$

$K = \frac{1}{2}mv^2$

$\Delta E = W = F_{\parallel}d = Fd\cos\theta$

$P = \frac{\Delta E}{\Delta t}$

$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$\omega = \omega_o + \alpha t$

$x = A\cos(2\pi ft)$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\tau = r_{\perp}F = rF\sin\theta$

$L = I\omega$

$\Delta L = \tau\Delta t$

$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$

$U_s = \frac{1}{2}kx^2$

$\rho = \frac{m}{V}$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
$\theta/$ = angle
ρ = density
τ = torque
ω = angular speed

$\Delta U_g = mg\Delta y$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$

$\vec{g} = \frac{\vec{F}_g}{m}$

$U_g = G\frac{m_1 m_2}{r}$

ELECTRICITY

$\left|\vec{F}_E\right| = k\left|\frac{q_1 q_2}{r^2}\right|$

$I = \frac{\Delta q}{\Delta t}$

$R = \frac{\rho\ell}{A}$

$I = \frac{\Delta V}{R}$

$P = I\Delta V$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

A = area
F = force
I = current
ℓ = length
P = power
q = charge
R = resistance
r = separation
t = time
V = electric potential
ρ = resistivity

WAVES

$\lambda = \frac{v}{f}$

f = frequency
v = speed
λ = wavelength

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = bh$

Circle
$A = \frac{1}{2}bh$

Rectangle
$A = bh$

Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2\ell$
$S = 2\pi r\ell + 2\pi r^2$

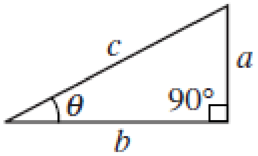
Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

Right triangle
$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



Appendix: Reference Tables

Table A. Metric Prefixes

Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000	10^{24}	yotta	Y
1 000 000 000 000 000 000 000	10^{21}	zeta	Z
1 000 000 000 000 000 000	10^{18}	exa	E
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deca	da
1	10^{0}	—	—
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f
0.000 000 000 000 000 001	10^{-18}	atto	a
0.000 000 000 000 000 000 001	10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001	10^{-24}	yocto	y

Table B. Physical Constants

Description	Symbol	Precise Value	Common Approximation
universal gravitational constant	G	$6.67384(80)\times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$6.67\times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$
acceleration due to gravity on Earth's surface	g	$9.7639\ \frac{\text{m}}{\text{s}^2}$ to $9.8337\frac{\text{m}}{\text{s}^2}$ average value at sea level is $9.80665\ \frac{\text{m}}{\text{s}^2}$	$9.8\ \frac{\text{m}}{\text{s}^2}$
speed of light in a vacuum	c	$299792458\frac{\text{m}}{\text{s}}$ *	$3.00\times 10^{8}\ \frac{\text{m}}{\text{s}}$
elementary charge (proton or electron)	e	$\pm 1.602176565(35)\times 10^{-19}$ C	$\pm 1.6\times 10^{-19}$ C
1 coulomb (C)		$6.24150965(16)\times 10^{18}$ elementary charges	6.24×10^{18} elementary charges
(electric) permittivity of a vacuum	ε_o	$8.85418782\times 10^{-12}\ \frac{\text{A}^2\cdot\text{s}^4}{\text{kg}\cdot\text{m}^3}$	$8.85\times 10^{-12}\ \frac{\text{A}^2\cdot\text{s}^4}{\text{kg}\cdot\text{m}^3}$
(magnetic) permeability of a vacuum	μ_o	$4\pi\times 10^{-7} = 1.25663706\times 10^{-6}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$	$1.26\times 10^{-6}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$
electrostatic constant	k	$\frac{1}{4\pi\varepsilon_o} = 8.9875517873681764\times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ *	$8.99\times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
1 electron volt (eV)		$1.602176565(35)\times 10^{-19}$ J	1.6×10^{-19} J
Planck's constant	h	$6.62606957(29)\times 10^{-34}\ \text{J}\cdot\text{s}$	$6.6\times 10^{-34}\ \text{J}\cdot\text{s}$
1 universal mass unit (u)		$931494061(21)\ \text{MeV}/c^2$	$931\text{MeV}/c^2$
Avogadro's constant	N_A	$6.02214129(27)\times 10^{23}\ \text{mol}^{-1}$	$6.02\times 10^{23}\ \text{mol}^{-1}$
Boltzmann constant	k_B	$1.3806488(13)\times 10^{-23}\ \frac{\text{J}}{\text{K}}$	$1.38\times 10^{-23}\ \frac{\text{J}}{\text{K}}$
universal gas constant	R	$8.3144621(75)\ \frac{\text{J}}{\text{mol}\cdot\text{K}}$	$8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}$
Rydberg constant	R_H	$\frac{m_e e^4}{8\varepsilon_o^2 h^3 c} = 10973731.6\frac{1}{\text{m}}$	$1.1\times 10^{7}\ \frac{1}{\text{m}}$
standard atmospheric pressure at sea level		101 325 Pa ≡ 1.01325 bar*	100 000 Pa ≡ 1.0 bar
rest mass of an electron	m_e	$9.10938215(45)\times 10^{-31}$ kg	9.11×10^{-31} kg
mass of a proton	m_p	$1.672621777(74)\times 10^{-27}$ kg	1.67×10^{-27} kg
mass of a neutron	m_n	$1.674927351(74)\times 10^{-27}$ kg	1.67×10^{-27} kg

*denotes an exact value (by definition)

Table C. Approximate Coëfficients of Friction

Substance	Static (μ_s)	Kinetic (μ_k)	Substance	Static (μ_s)	Kinetic (μ_k)
rubber on concrete (dry)	0.90	0.68	wood on wood (dry)	0.42	0.30
rubber on concrete (wet)		0.58	wood on wood (wet)	0.2	
rubber on asphalt (dry)	0.85	0.67	wood on metal	0.3	
rubber on asphalt (wet)		0.53	wood on brick	0.6	
rubber on ice		0.15	wood on concrete	0.62	
steel on ice	0.03	0.01	Teflon on Teflon	0.04	0.04
waxed ski on snow	0.14	0.05	Teflon on steel	0.04	0.04
aluminum on aluminum	1.2	1.4	graphite on steel	0.1	
cast iron on cast iron	1.1	0.15	leather on wood	0.3–0.4	
steel on steel	0.74	0.57	leather on metal (dry)	0.6	
copper on steel	0.53	0.36	leather on metal (wet)	0.4	
diamond on diamond	0.1		glass on glass	0.9–1.0	0.4
diamond on metal	0.1–0.15		metal on glass	0.5–0.7	

Table D. Quantities, Variables and Units

Quantity	**Variable**	**MKS Unit Name**	**MKS Unit Symbol**	**S.I. Base Unit**
position	$\boldsymbol{x}$	meter*	m	m
distance/displacement, (length, height)	$d, \boldsymbol{d}, (\ell, h)$	meter*	m	m
angle	ϑ	radian, degree	—, °	—
area	A	square meter	m^2	m^2
volume	V	cubic meter, liter	m^3, ℓ, L	m^3
time	t	second*	s	s
velocity	$\boldsymbol{v}$	meter/second	$\frac{m}{s}$	$\frac{m}{s}$
speed of light	c			
angular velocity	$\boldsymbol{\omega}$	radians/second	$\frac{1}{s}$	$\frac{1}{s}$
acceleration	$\boldsymbol{a}$	meter/second ^{2}	$\frac{m}{s^2}$	$\frac{m}{s^2}$
acceleration due to gravity	$\boldsymbol{g}$			
mass	m	kilogram*	kg	kg
force	$\boldsymbol{F}$	newton	N	$\frac{kg \cdot m}{s^2}$
pressure	P	pascal	Pa	$\frac{kg}{m \cdot s^2}$
energy	E	joule	J	$\frac{kg \cdot m^2}{s^2}$
potential energy	U			
heat	Q			
work	W	newton-meter	N·m	$\frac{kg \cdot m^2}{s^2}$
torque	$\boldsymbol{\tau}$	newton-meter	N·m	$\frac{kg \cdot m^2}{s^2}$
power	P	watt	W	$\frac{kg \cdot m^2}{s^3}$
momentum	$\boldsymbol{p}$	newton-second	N·s	$\frac{kg \cdot m}{s}$
impulse	$\boldsymbol{J}$			
moment of inertia	I	kilogram-meter ^{2}	$kg \cdot m^2$	$kg \cdot m^2$
angular momentum	$\boldsymbol{L}$	newton-meter-second	N·m·s	$\frac{kg \cdot m^2}{s}$
frequency	f	hertz	Hz	s^{-1}
wavelength	λ	meter	m	m
period	T	second	s	s
index of refraction	n	—	—	—
electric current	$\boldsymbol{I}$	ampere*	A	A
electric charge	q	coulomb	C	A·s
potential difference (voltage) electromotive force (emf)	V ε	volt	V	$\frac{kg \cdot m^2}{A \cdot s^3}$
electrical resistance	R	ohm	Ω	$\frac{kg \cdot m^2}{A^2 \cdot s^3}$
capacitance	C	farad	F	$\frac{A^2 \cdot s^4}{m^2 \cdot kg}$
electric field	$\boldsymbol{E}$	netwon/coulomb volt/meter	$\frac{N}{C}, \frac{V}{m}$	$\frac{kg \cdot m}{A \cdot s^3}$
magnetic field	$\boldsymbol{B}$	tesla	T	$\frac{kg}{A \cdot s^2}$
temperature	T	kelvin*	K	K
amount of substance	n	mole*	mol	mol
luminous intensity	I_v	candela*	cd	cd

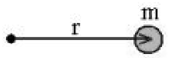
Variables representing vector quantities are typeset in ***bold italics***. * = S.I. base unit

Table E. Mechanics Formulas and Equations

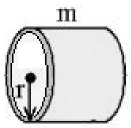
Kinematics (Distance, Velocity & Acceler-ation)	$\mathbf{d} = \Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_o$ $\bar{\mathbf{v}} = \frac{\mathbf{d}}{t} = \frac{\Delta\mathbf{x}}{\Delta t} = \frac{\mathbf{v}_o + \mathbf{v}}{2}$ $\Delta\mathbf{v} = \mathbf{v} - \mathbf{v}_o = \mathbf{a}t$ $\mathbf{x} - \mathbf{x}_o = \mathbf{d} = \mathbf{v}_o t + \frac{1}{2}\mathbf{a}t^2$ $\mathbf{v}^2 - \mathbf{v}_o^2 = 2\mathbf{a}\mathbf{d}$
Circular Motion	$s = r\Delta\theta \quad \mathbf{v}_T = r\boldsymbol{\omega} \quad \mathbf{a}_T = r\boldsymbol{\alpha}$ $a_c = \frac{v^2}{r} = \omega^2 r$ $\theta - \theta_0 = \vec{\boldsymbol{\omega}}_0 t + \frac{1}{2}\vec{\boldsymbol{\alpha}}t^2$
Forces & Dynamics	$\mathbf{a} = \frac{\mathbf{F}_{net}}{m} = \frac{\sum \mathbf{F}}{m} \qquad \mathbf{g} = \frac{\mathbf{F}_g}{m}$ $F_f \le \mu_s F_N \qquad F_f = \mu_k F_N$ $\mathbf{F}_g = \frac{Gm_1 m_2}{r^2}$
Rotational Dynamics	$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$ $I = \int_0^m r^2 dm = mr^2$ $F_c = ma_c = \frac{mv^2}{r}$ $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \qquad \tau = rF\sin\theta = r_\perp F$ $\boldsymbol{\alpha} = \frac{\boldsymbol{\tau}_{net}}{I} = \frac{\sum \boldsymbol{\tau}}{I}$
Simple Harmonic Motion	$T = \frac{2\pi}{\omega} = \frac{1}{f}$ $T_s = 2\pi\sqrt{\frac{m}{k}} \qquad T_p = 2\pi\sqrt{\frac{L}{g}}$ $\mathbf{F}_s = -k\mathbf{x}$ $U_s = \frac{1}{2}kx^2$
Momentum	$\mathbf{p} = m\mathbf{v}$ $\Sigma m_i \mathbf{v}_i = \Sigma m_f \mathbf{v}_f$ $\mathbf{J} = \Delta\mathbf{p} = \mathbf{F}_{net}\Delta t$ $\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega} \qquad L = rp\sin\theta = I\omega$ $\Delta\mathbf{L} = \boldsymbol{\tau}\Delta t$
Energy, Work & Power	$W = \mathbf{F} \bullet \mathbf{d} = Fd\cos\theta = F_{\parallel} d$ $W = \tau\Delta\theta$ $U_g = mgh = \frac{Gm_1 m_2}{r}$ $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $E_k = \frac{1}{2}I\omega^2$ $E_{total} = U + E_k + Q$ $W = \Delta K = -\Delta U$ $P = \frac{W}{t} = \mathbf{F} \bullet \mathbf{v} = Fv\cos\theta = \tau\omega$

Δ = change, difference
Σ = sum
d = distance (m)
$\mathbf{d}$ = displacement (m)
$\mathbf{x}$ = position (m)
s = arc length (m)
t = time (s)
$\mathbf{v}$ = velocity $\left(\frac{\text{m}}{\text{s}}\right)$
$\bar{\mathbf{v}}$ = average velocity $\left(\frac{\text{m}}{\text{s}}\right)$
$\mathbf{a}$ = acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
f = frequency $\left(\text{Hz} = \frac{1}{\text{s}}\right)$
a_c = centripetal acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
$\mathbf{F}$ = force (N)
F_f = force due to friction (N)
$\mathbf{F}_g$ = force due to gravity (N)
F_N = normal force (N)
F_c = centripetal force (N)
m = mass (kg)
$\mathbf{g}$ = acceleration due to gravity $\left(\frac{\text{m}}{\text{s}^2}\right)$
G = gravitational constant $\left(\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)$
r = radius (m)
$\mathbf{r}$ = radius (vector)
μ = coefficient of friction (*dimensionless*)
θ = angle (°, rad)
ω = angular velocity $\left(\frac{\text{rad}}{\text{s}}\right)$
k = spring constant $\left(\frac{\text{N}}{\text{m}}\right)$
$\mathbf{x}$ = displacement of spring (m)
L = length of pendulum (m)
$\boldsymbol{\tau}$ = torque (N·m)
K = kinetic energy (J)
U = potential energy (J)
h = height (m)
Q = heat (J)
P = power (W)
W = work (N·m)
T = (time) period (Hz)
$\mathbf{p}$ = momentum (N·s)
$\mathbf{J}$ = impulse (N·s)
$\mathbf{L}$ = angular momentum (N·m·s)

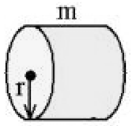
Table F. Moments of Inertia



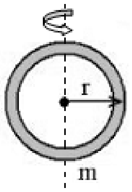
Point Mass:
$I = mr^2$



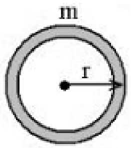
Hollow Cylinder:
$I = mr^2$



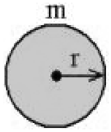
Solid Cylinder:
$I = \frac{1}{2}mr^2$



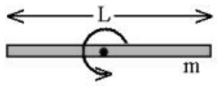
Hoop About Diameter:
$I = \frac{1}{2}mr^2$



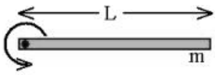
Hollow Sphere:
$I = \frac{2}{3}mr^2$



Solid Sphere:
$I = \frac{2}{5}mr^2$



Rod About the Middle:
$I = \frac{1}{12}mr^2$



Rod About the End:
$I = \frac{1}{3}mr^2$

Table G. Heat and Thermal Physics Formulas and Equations

Temperature	$^\circ F = 1.8(^\circ C) + 32$ $K = {^\circ C} + 273.15$	Δ = change $^\circ$F = Fahrenheit temperature (°F) $^\circ$C = Celsius temperature (°C) K = Kelvin temperature (K) Q = heat (J, kJ) m = mass (kg) C = specific heat capacity $\left(\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}\right)$ (C_p = const. pressure; C_v = const. volume) T = temperature (K) t = time (s) L = length (m) k = coëfficient of thermal conductivity $\left(\frac{\text{J}}{\text{m}\cdot\text{s}\cdot{}^\circ\text{C}}, \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\right)$ V = volume (m^3) α = linear coëfficient of thermal expansion ($^\circ\text{C}^{-1}$) β = volumetric coëfficient of thermal expansion ($^\circ\text{C}^{-1}$) R_i = "R" value of insulation R = gas constant $\left(\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)$ U = internal energy (J) W = work (N·m)
Heat	$Q = mC\,\Delta T$ $Q_{melt} = m\Delta H_{fus}$ $Q_{boil} = m\Delta H_{vap}$ $C_p - C_v = R$ $\Delta L = \alpha L_i \Delta T$ $\Delta V = \beta V_i \Delta T$ $\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \frac{Q}{\Delta t} = kA\frac{\Delta T}{L}$ $\frac{Q}{t} = -\frac{1}{R_i} A \Delta T$	
Thermodynamics	$\Delta U = \Delta Q + \Delta W$ $K = \frac{3}{2} k_B T$ $W = -\Delta(PV)$	

Table H. Thermal Properties of Selected Materials

Substance	**Melting Point (°C)**	**Boiling Point (°C)**	**Heat of Fusion ΔH_{fus} $\left(\frac{\text{kJ}}{\text{kg}}\right)$**	**Heat of Vaporization ΔH_{vap} $\left(\frac{\text{kJ}}{\text{kg}}\right)$**	**Specific Heat Capacity C_p $\left(\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}\right)$ at 25°C**	**Thermal Conductivity k $\left(\frac{\text{J}}{\text{m}\cdot\text{s}\cdot{}^\circ\text{C}}\right)$ at 25°C**	**Coefficients of Expansion at 20°C**	
							Linear α ($^\circ\text{C}^{-1}$)	**Volumetric β ($^\circ\text{C}^{-1}$)**
air (gas)	—	—	—	—	1.012	0.024	—	—
aluminum (solid)	659	2467	395	10460	0.897	250	2.3×10^{-5}	6.9×10^{-5}
ammonia (gas)	−75	−33.3	339	1369	4.7	0.024	—	—
argon (gas)	−189	−186	29.5	161	0.520	0.016	—	—
carbon dioxide (gas)	−78		574		0.839	0.0146	—	—
copper (solid)	1086	1187	134	5063	0.385	401	1.7×10^{-5}	5.1×10^{-5}
brass (solid)	—	—	—	—	0.380	120	1.9×10^{-5}	5.6×10^{-5}
diamond (solid)	3550	4827	10 000	30 000	0.509	2200	1×10^{-6}	3×10^{-6}
ethanol (liquid)	−117	78	104	858	2.44	0.171	2.5×10^{-4}	7.5×10^{-4}
glass (solid)	—	—	—	—	0.84	0.96–1.05	8.5×10^{-6}	2.55×10^{-5}
gold (solid)	1063	2660	64.4	1577	0.129	310	1.4×10^{-5}	4.2×10^{-5}
granite (solid)	1240	—	—	—	0.790	1.7–4.0		
helium (gas)	—	−269	—	21	5.193	0.142	—	—
hydrogen (gas)	−259	−253	58.6	452	14.30	0.168	—	—
iron (solid)	1535	2750	289	6360	0.450	80	1.18×10^{-5}	3.33×10^{-5}
lead (solid)	327	1750	24.7	870	0.160	35	2.9×10^{-5}	8.7×10^{-5}
mercury (liquid)	−39	357	11.3	293	0.140	8	6.1×10^{-5}	1.82×10^{-4}
paraffin wax (solid)	46–68	~300	~210	—	2.5	0.25	—	—
silver (solid)	962	2212	111	2360	0.233	429	1.8×10^{-5}	5.4×10^{-5}
steam (gas) @	0	100	—	2260	2.080	0.016	—	—
water (liq.) @ 25°C			334		4.181	0.58	6.9×10^{-5}	2.07×10^{-4}
ice (solid) @ −10°C				—	2.11	2.18	—	—

Table I. Electricity Formulas & Equations

Electrostatic Charges & Electric Fields	$\mathbf{F}_e = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$ $\mathbf{E} = \frac{\mathbf{F}_e}{q} = \frac{Q}{\varepsilon_o A}$ $\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} = \frac{\Delta V}{\Delta r}$ $W = q\,\mathbf{E}\bullet\mathbf{d} = qEd\cos\theta$ $V = \frac{W}{q} = \mathbf{E}\bullet\mathbf{d} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$ $\Delta U_E = q\Delta V$ $U_E = \frac{kq_1q_2}{r}$	$\Delta =$ change $\mathbf{F}_e =$ force due to electric field (N) $k =$ electrostatic constant $\left(\frac{\mathrm{N{\cdot}m^2}}{\mathrm{C^2}}\right)$ $q =$ point charge (C) $Q =$ charge (C) $\mathbf{E} =$ electric field $\left(\frac{\mathrm{N}}{\mathrm{C}}, \frac{\mathrm{V}}{\mathrm{m}}\right)$ $V =$ voltage $=$ electric potential difference (V) $W =$ work (N·m) $d =$ distance (m) $r =$ radius (m) $\boldsymbol{I} =$ current (A) $t =$ time (s) $R =$ resistance (Ω) $P =$ power (W) $Q_H =$ heat (J) $\rho =$ resistivity ($\Omega\cdot$m) $\ell =$ length (m) $A =$ cross-sectional area (m^2) $U =$ potential energy (J) $C =$ capacitance (F) $\mathbf{v} =$ velocity (of moving charge or wire) $\left(\frac{\mathrm{m}}{\mathrm{s}}\right)$ $\mathbf{B} =$ magnetic field (T) $\mu_o =$ magnetic permeability of free space r = radius (distance) from wire
Circuits	$I = \frac{\Delta Q}{\Delta t} = \frac{V}{R}$ $P = VI = I^2R = \frac{V^2}{R}$ $W = Q_H = Pt = VIt = I^2Rt = \frac{V^2t}{R}$ $R = \frac{\rho\ell}{A}$ $V = \frac{Q}{C}$ $C = k\varepsilon_o\frac{A}{d}$ $U_{capacitor} = \frac{1}{2}QV = \frac{1}{2}CV^2$	
Series Circuits	$I = I_1 = I_2 = I_3 = \ldots$ $V = V_1 + V_2 + V_3 + \ldots = \sum V_i$ $R_{eq} = R_1 + R_2 + R_3 + \ldots = \sum R_i$ $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots = \sum \frac{1}{C_i}$ $P_{total} = P_1 + P_2 + P_3 + \ldots = \sum P_i$	
Parallel Circuits	$I = I_1 + I_2 + I_3 + \ldots = \sum I_i$ $V = V_1 = V_2 = V_3 = \ldots$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \sum \frac{1}{R_i}$ $C_{total} = C_1 + C_2 + C_3 + \ldots = \sum C_i$ $P_{total} = P_1 + P_2 + P_3 + \ldots = \sum P_i$	

Table J. Electricity & Magnetism Formulas & Equations

Magnetism	$F_M = q(\boldsymbol{v}\times\boldsymbol{B})$ $\quad F_M = q\,vB\sin\theta$ $F_M = \ell(\boldsymbol{I}\times\boldsymbol{B})$ $\quad F_M = \ell\,\boldsymbol{I}\,\boldsymbol{B}\sin\theta$ $V = \ell(\boldsymbol{v}\times\boldsymbol{B})$ $\quad V = \ell vB\sin\theta$ $B = \frac{\mu_o}{2\pi}\frac{I}{r}$ $\Phi_B = \boldsymbol{B}\bullet\boldsymbol{A} = BA\cos\theta$ $\varepsilon = \frac{\Delta\Phi_B}{\Delta t} = B\ell v$
Electromagnetic Induction	$\frac{\#turns_{in}}{\#turns_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$ $P_{in} = P_{out}$

Δ = change
F_e = force due to electric field (N)
k = electrostatic constant $\left(\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)$
q = point charge (C)
V = voltage = electric potential difference (V)
ε = emf = electromotive force (V)
r = radius (m)
$\boldsymbol{I}$ = current (A)
ℓ = length (m)
t = time (s)
A = cross-sectional area (m^2)
$\boldsymbol{v}$ = velocity (of moving charge or wire) $\left(\frac{\text{m}}{\text{s}}\right)$
$\boldsymbol{B}$ = magnetic field (T)
μ_o = magnetic permeability of free space
Φ_B = magnetic flux

Table K. Resistor Color Code

Color	Digit	Multiplier
black	0	$\times 10^0$
brown	1	$\times 10^1$
red	2	$\times 10^{]2}$
orange	3	$\times 10^3$
yellow	4	$\times 10^4$
green	5	$\times 10^5$
blue	6	$\times 10^6$
violet	7	$\times 10^7$
gray	8	$\times 10^8$
white	9	$\times 10^9$
gold		± 5%
silver		± 10%

Table L. Symbols Used in Electrical Circuit Diagrams

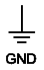
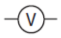
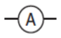
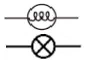
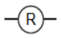
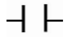
Component	Symbol	Component	Symbol
wire		battery	
switch		ground	
fuse		resistor	
voltmeter		variable resistor (rheostat, potentiometer, dimmer)	
ammeter		lamp (light bulb)	
ohmmeter		capacitor	
		diode	

Table M. Resistivities at 20°C

Conductors		Semiconductors		Insulators	
Substance	**Resistivity** $(\Omega\cdot\text{m})$	**Substance**	**Resistivity** $(\Omega\cdot\text{m})$	**Substance**	**Resistivity** $(\Omega\cdot\text{m})$
silver	1.59×10^{-8}	germanium	0.001 to 0.5	deionized water	1.8×10^5
copper	1.72×10^{-8}	silicon	0.1 to 60	glass	1×10^9 to 1×10^{13}
gold	2.44×10^{-8}	sea water	0.2	rubber, hard	1×10^{13} to 1×10^{13}
aluminum	2.82×10^{-8}	drinking water	20 to 2 000	paraffin (wax)	1×10^{13} to 1×10^{17}
tungsten	5.60×10^{-8}			air	1.3×10^{16} to 3.3×10^{16}
iron	9.71×10^{-8}			quartz, fused	7.5×10^{17}
nichrome	1.50×10^{-6}				
graphite	3×10^{-5} to 6×10^{-4}				

Table N. Waves & Optics

Waves	$\lambda = \frac{v}{f}$ $f = \frac{1}{T}$ $v_{\text{wave on a string}} = \sqrt{\frac{F_T}{\mu}}$ $f_{\text{doppler shifted}} = f\left(\frac{v_{\text{wave}} + v_{\text{detector}}}{v_{\text{wave}} + v_{\text{source}}}\right)$	v = velocity of wave $\left(\frac{\text{m}}{\text{s}}\right)$ f = frequency (Hz) λ = wavelength (m) T = period (of time) (s) F_T = tension (force) on string (N) μ = elastic modulus of string $\left(\frac{\text{kg}}{\text{m}}\right)$ θ_i = angle of incidence (°, rad) θ_r = angle of reflection (°, rad) θ_c = critical angle (°, rad) n = index of refraction (*dimensionless*) c = speed of light in a vacuum $\left(\frac{\text{m}}{\text{s}}\right)$ s_f = distance to the focus of a mirror or lens (m) r_c = radius of curvature of a spherical mirror (m) s_i = distance from the mirror or lens to the image (m) s_o = distance from the mirror or lens to the object (m) h_i = height of the image (m) h_o = height of the object (m) M = magnification (*dimensionless*) d = separation (m) L = distance from the opening (m) m = an integer
Reflection, Refraction & Diffraction	$\theta_i = \theta_r$ $n = \frac{c}{v}$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ $\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ $\Delta L = m\lambda = d\sin\theta$	
Mirrors & Lenses	$s_f = \frac{r_c}{2}$ $\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{s_f}$ $M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$	

Figure O. The Electcromagnetic Spectrum

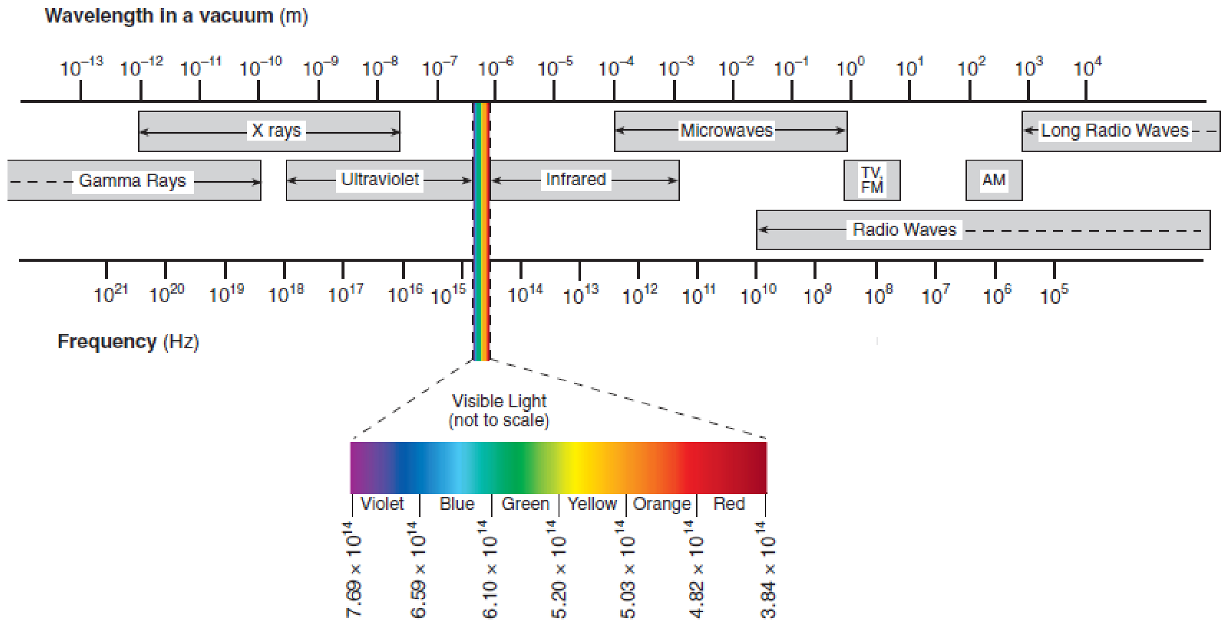


Table P. Properties of Water and Air

Temp. (°C)	Water			Air	
	Density $\left(\frac{kg}{m^3}\right)$	Speed of Sound $\left(\frac{m}{s}\right)$	Vapor Pressure (Pa)	Density $\left(\frac{kg}{m^3}\right)$	Speed of Sound $\left(\frac{m}{s}\right)$
0	999.78	1 403	611.73	1.288	331.30
5	999.94	1 427	872.60	1.265	334.32
10	999.69	1 447	1 228.1	1.243	337.31
20	998.19	1 481	2 338.8	1.200	343.22
25	997.02	1 496	3 169.1	1.180	346.13
30	995.61	1 507	4 245.5	1.161	349.02
40	992.17	1 526	7 381.4	1.124	354.73
50	990.17	1 541	9 589.8	1.089	360.35
60	983.16	1 552	19 932	1.056	365.88
70	980.53	1 555	25 022	1.025	371.33
80	971.79	1 555	47 373	0.996	376.71
90	965.33	1 550	70 117	0.969	382.00
100	954.75	1 543	101 325	0.943	387.23

Table Q. Absolute Indices of Refraction

Measured at $f = 5.09 \times 10^{14}$ Hz (yellow light)

Substance	Index of Refraction	Substance	Index of Refraction
air	1.000293	silica (quartz), fused	1.459
ice	1.309	plexiglass	1.488
water	1.3330	Lucite	1.495
ethyl alcohol	1.36	glass, borosilicate (Pyrex)	1.474
human eye, cornea	1.38	glass, crown	1.50–1.54
human eye, lens	1.41	glass, flint	1.569–1.805
safflower oil	1.466	sodium chloride, solid	1.516
corn oil	1.47	PET (#1 plastic)	1.575
glycerol	1.473	zircon	1.777–1.987
honey	1.484–1.504	cubic zirconia	2.173–2.21
silicone oil	1.52	diamond	2.417
carbon disulfide	1.628	silicon	3.96

Table R. Fluid Mechanics Formulas and Equations

Density & Pressure	$\rho = \frac{m}{V}$ $P = \frac{F}{A}$ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ $P = P_o + \rho g h$ $A_1 v_1 = A_2 v_2$ $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 =$ $P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$	$\Delta =$ change $\rho =$ density $\left(\frac{\text{kg}}{\text{m}^3}\right)$ $m =$ mass (kg) $V =$ volume (m^3) $P =$ presure (Pa) $g =$ acceleration due to gravity $\left(\frac{\text{m}}{\text{s}^2}\right)$ $h =$ height or depth (m) $A =$ area (m^2) $v =$ velocity (of fluid) $\left(\frac{\text{m}}{\text{s}}\right)$ $F =$ force (N) $n =$ number of moles (mol) $R =$ gas constant $\left(\frac{\text{J}}{\text{mol K}}\right)$ $N =$ number of molecules $k_B =$ Boltzmann's constant $\left(\frac{\text{J}}{\text{K}}\right)$ $T =$ temperature (K) $M =$ molar mass $\left(\frac{\text{g}}{\text{mol}}\right)$ $\mu =$ molecular mass (kg) $E_k =$ kinetic energy (J) $W =$ work (N·m)
Forces, Work & Energy	$F_B = \rho V_d g$ $PV = N k_B T = nRT$ $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ $E_{k\,(molecular)} = \frac{3}{2} k_B T$ $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 k_B T}{\mu}}$ $W = -P \Delta V$	

Table S. Planetary Data

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance from Sun (m)	5.79×10^{10}	1.08×10^{11}	1.50×10^{11}	2.28×10^{11}	7.78×10^{11}	1.43×10^{12}	2.87×10^{12}	4.50×10^{12}
Radius (m)	2.44×10^{6}	6.05×10^{6}	6.37×10^{6}	3.39×10^{6}	6.99×10^{7}	5.82×10^{7}	2.54×10^{7}	2.46×10^{7}
Mass (kg)	3.30×10^{23}	4.87×10^{24}	5.97×10^{24}	6.42×10^{23}	1.90×10^{27}	5.68×10^{26}	8.68×10^{25}	1.02×10^{26}
Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	5430	5250	5520	3950	1330	690	1290	1640
Orbit (years)	0.24	0.62	1.00	1.88	11.86	84.01	164.79	248.54
Rotation Period (hours)	1408	5832	23.9	24.6	9.9	10.7	17.2	16.1
Tilt of axis	2°	177.3°	23.5°	25.2°	3.1°	26.7°	97.9°	29.6°
# of observed satellites	0	0	1	2	67	62	27	13

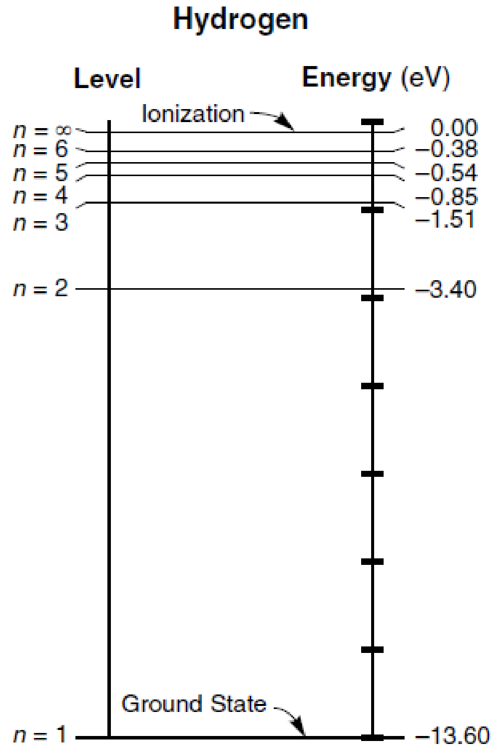
Table T. Sun & Moon Data

Radius of the sun (m)	6.96×10^{8}
Mass of the sun (kg)	1.99×10^{30}
Radius of the moon (m)	1.74×10^{6}
Mass of the moon (kg)	7.35×10^{22}
Distance of moon from Earth (m)	3.84×10^{8}

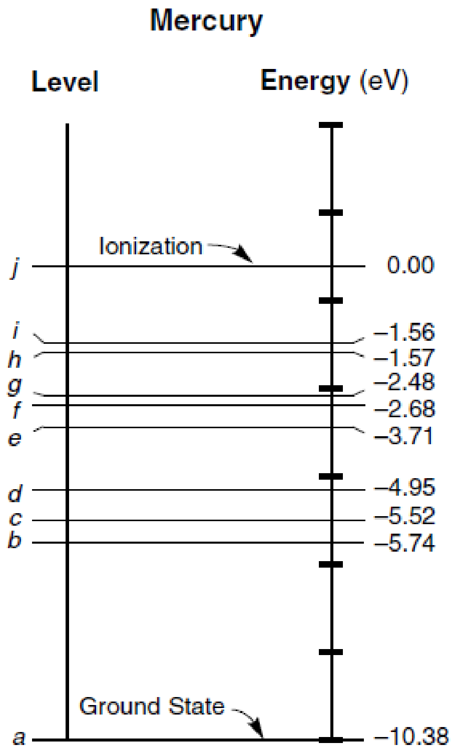
Table U. Atomic & Particle Physics (Modern Physics)

Energy	$E_{photon} = hf = \frac{hc}{\lambda} = pc = \hbar\omega$ $E_{k,\max} = hf - \phi$ $\lambda = \frac{h}{p}$ $E_{photon} = E_i - E_f$ $E^2 = (pc)^2 + (mc^2)^2$ $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$	$E =$ energy (J) $h =$ Planck's constant (J·s) $\hbar =$ reduced Planck's constant $= \frac{h}{2\pi}$ (J·s) $f =$ frequency (Hz) $c =$ speed of light $\left(\frac{m}{s}\right)$ $\lambda =$ wavelength (m) $p =$ momentum (N·s) $m =$ mass (kg) $E_k =$ kinetic energy (J) $\phi =$ work function $R_H =$ Rydberg constant $\left(\frac{1}{m}\right)$ $\gamma =$ Lorentz factor (*dimensionless*) $L =$ length in moving reference frame (m) $L_o =$ length in stationary reference frame (m) $\Delta t' =$ time in stationary reference frame (s) $\Delta t =$ time in moving reference frame (s) $m_o =$ mass in stationary reference frame (kg) $m_{rel} =$ apparent mass in moving reference frame (kg) $v =$ velocity $\left(\frac{m}{s}\right)$
Special Relativity	$\gamma = \frac{1}{\sqrt{1-{v^2}/{c^2}}}$ $\gamma = \frac{L_o}{L}$ $\gamma = \frac{\Delta t'}{\Delta t}$ $\gamma = \frac{m_{rel}}{m_o}$	

Figure V. Quantum Energy Levels



Energy Levels for the Hydrogen Atom



A Few Energy Levels for the Mercury Atom

Figure W. Particle Sizes

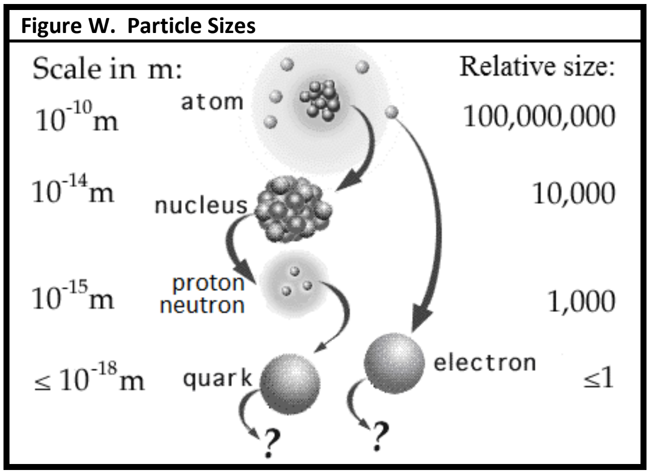


Table X. The Standard Model

	Generation				
	I	II	III		
mass → charge → spin →	2.4 MeV/c^2 +⅔ ½ **u** up quark	1.27 GeV/c^2 +⅔ ½ **c** charm quark	171.2 GeV/c^2 +⅔ ½ **t** top quark	0 MeV/c^2 0 1 γ photon	125.3 GeV/c^2 0 0 H^0 Higgs boson
quarks	4.8 MeV/c^2 −⅓ ½ **d** down quark	104 MeV/c^2 −⅓ ½ **s** strange quark	4.2 GeV/c^2 −⅓ ½ **b** bottom quark	0 MeV/c^2 0 1 **g** gluon	
	< 2.2 eV/c^2 0 ½ ν_e electron neutrino	< 0.17 MeV/c^2 0 ½ ν_μ muon neutrino	< 15.5 MeV/c^2 0 ½ ν_τ tau neutrino	91.2 GeV/c^2 0 1 Z^0 Z boson	gauge bosons
leptons	0.511 MeV/c^2 −1 ½ e electron	105.7 MeV/c^2 −1 ½ μ muon	1.777 GeV/c^2 −1 ½ τ tau	80.4 GeV/c^2 ±1 1 $W^\pm$ W boson	

<table>
<tr><th colspan="3">Table Y. Geometry & Trigonometry Formulas</th></tr>
<tr><td>Triangles</td><td>

$A=\frac{1}{2}bh$

$c^2=a^2+b^2-2ab\cos C$

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

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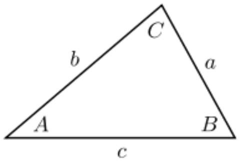

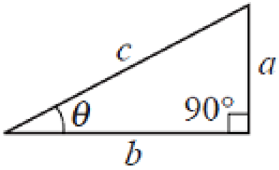

$a,b,c=$ length of a side of a triangle
$\theta=$ angle
$A=$ area
$C=$ circumference
$S=$ surface area
$V=$ volume
$b=$ base
$h=$ height
$\ell=$ length
$w=$ width
$r=$ radius

</td></tr>
<tr><td>Right Triangles</td><td>

$c^2=a^2+b^2$

$\sin\theta=\frac{a}{c}=\frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta=\frac{b}{c}=\frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{a}{b}=\frac{\text{opposite}}{\text{adjacent}}$

$b=c\cos\theta$

$a=c\sin\theta$

</td></tr>
<tr><td>Rectangles, Parallelograms and Trapezoids</td><td>

$A=\bar{b}h$

</td></tr>
<tr><td>Rectangular Solids</td><td>

$V=\ell wh$

</td></tr>
<tr><td>Circles</td><td>

$C=2\pi r$

$A=\pi r^2$

</td></tr>
<tr><td>Cylinders</td><td>

$S=2\pi r\ell+2\pi r^2=2\pi r(\ell+r)$

$V=\pi r^2\ell$

</td></tr>
<tr><td>Spheres</td><td>

$S=4\pi r^2$

$V=\frac{4}{3}\pi r^3$

</td></tr>
</table>

Table Z. Values of Trigonometric Functions

degree	radian	sine	cosine	tangent	degree	radian	sine	cosine	tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.747
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.331
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.430
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.301
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.081
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.636
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.290
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	∞

Table AA. Some Exact and Approximate Conversions

Length	1 cm	≈	width of a small paper clip		
	1 inch (in.)	≡	2.54 cm		
	length of a US dollar bill	=	6.14 in.	=	15.6 cm
	12 in.	≡	1 foot (ft.)	≈	30 cm
	3 ft.	≡	1 yard (yd.)	≈	1 m
	1 m	=	0.3048 ft.	=	39.37 in.
	1 km	≈	0.6 mi.		
	5,280 ft.	≡	1 mile (mi.)	≈	1.6 km
Mass/ Weight	1 small paper clip	≈	0.5 gram (g)		
	US 1¢ coin (1983–present)	=	2.5 g		
	US 5¢ coin	=	5 g		
	1 oz.	≈	30 g		
	one medium-sized apple	≈	1 N	≈	3.6 oz.
	1 pound (lb.)	≡	16 oz.	≈	454 g
	1 pound (lb.)	≈	4.45 N		
	1 ton	≡	2000 lb.	≈	0.9 tonne
	1 tonne	≡	1000 kg	≈	1.1 ton
Volume	1 pinch	=	≤ $^{1}/_{8}$ teaspoon (tsp.)		
	1 mL	≈	10 drops		
	1 tsp.	≈	5 mL	≈	60 drops
	3 tsp.	≡	1 tablespoon (Tbsp.)	≈	15 mL
	2 Tbsp.	≡	1 fluid ounce (fl. oz.)	≈	30 mL
	8 fl. oz.	≡	1 cup (C)	≈	250 mL
	16 fl. oz.	≡	1 U.S. pint (pt.)	≈	500 mL
	20 fl. oz.	≡	1 Imperial pint (UK)	≈	600 mL
	2 pt.	≡	1 U.S. quart (qt.)	≈	1 L
	4 qt. (U.S.)	≡	1 U.S. gallon (gal.)	≈	3.8 L
	4 qt. (UK) ≡ 5 qt. (U.S.)	≡	1 Imperial gal. (UK)	≈	4.7 L
Speed	1 $^{\text{m}}/_{\text{s}}$	≈	2.24 $^{\text{mi.}}/_{\text{h}}$		
	60 $^{\text{mi.}}/_{\text{h}}$	≈	100 $^{\text{km}}/_{\text{h}}$	≈	27 $^{\text{m}}/_{\text{s}}$
Energy	1 cal	≈	4.18 J		
	1 Calorie (food)	≡	1 kcal	≈	4.18 kJ
	1 BTU	≈	1.05 kJ		
Power	1 hp	≈	746 W		
	1 kW	≈	1.34 hp		
Temperature	0 K	≡	−273.15°C	=	absolute zero
	0°F	≈	−18°C		
	32°F	=	0°C = 273.15 K	=	water freezes
	70°F	≈	21°C	≈	room temp.
	212°F	=	100°C	=	water boils
Speed of light	300 000 000 $^{\text{m}}/_{\text{s}}$	≈	186 000 $^{\text{mi.}}/_{\text{s}}$	≈	1 $^{\text{ft.}}/_{\text{ns}}$

Table BB. Greek Alphabet

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	ο	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

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