Notes on Competitive Trade Theory

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These notes synthesize contributions from a wide spectrum of writers concerning competitive trade models. They bear the mark of two of my own teachers, Jagdish Bhagwati and Ron Findlay, as well as writings too numerous to note here. The reader may also like to consult the excellent texts by R. Caves, R. Jones, and J. Frankel, *World Trade and Payments*, or that by P. Krugman and M. Obstfeld, *International Economics: Theory and Policy*. More advanced treatments of many of the topics may be found in J. Bhagwati and T.N. Srinivasan, *Lectures on International Trade*, or E. Helpman and P. Krugman, *Market Structure and Foreign Trade*.

I. Introduction: Microeconomic Foundations of Competitive Trade Theory

Principal Questions in Trade Theory

Why doesn't Kenya export supercomputers to the United States? Why doesn't the United States export coffee to Colombia? Why doesn't Colombia export VCRs to Japan? Why does the United States export cars to Germany and Germany *also* export cars to the United States? Why do some countries not trade with each other at all? These are all questions about the *pattern of trade*.

If the United States exports a bushel of wheat, what determines how much coffee can be had in exchange for it? Will it be an ounce, a pound, a ton? This is a question of relative prices of exports and imports, or the *terms of trade*.

If countries trade, do they benefit from this trade or are they harmed by it? Do some countries gain at the expense of others? Will they be best off with free trade or by implementing taxes and other restrictions on trade? Will the market, left alone, lead to the best solution, or are there market failures? If there are market failures, are there measures the government can (or will) take to reach such an optimum? How do we decide which measure is appropriate? These are questions regarding the *welfare* results of trade and the potential for improving it via *commercial policy*.

Trade theory, as economic theory, has typically been distinguished according to positive or normative analysis. In this framework, positive theory seeks to understand the determinants of the pattern of trade and the terms at which trade takes place. The normative seeks to ascertain whether agents and/or countries gain or lose by trading. This seems a worthwhile distinction -- it is possible to determine the results of certain conditions apart from whether they are desirable or not. If anything, the reader is advised to be careful that those who assert the distinction apply it scrupulously.

Different Approaches

If our social reality were transparent, there would be no need for a specialized field of economics. It is precisely because this reality is exceedingly complex that we need economic theory. Typically, economists aim at articulating key aspects of a problem by analyzing it in simplified models. It is probably too much to hope that we will develop a single model that will illuminate every aspect of international trade. So we develop a variety of models with the hope that each will lend insight to a different aspect of the reality that we seek to understand. From here, it is a matter of the quality of our discretion whether we use the models appropriately.

In principle, a theory of international trade could be developed from two different bases. One could start with specified *aggregate* behavioral relationships -- for example, as is done in Keynesian open economy models -- and then deduce the results of a world with two (or more) such economies. Alternatively, one could start with a description of *individual* agents in the economy, their objectives, choices and constraints, and then aggregate to derive the interactions of two (or more) such economies. Each approach allows for many possible specifications within this broad division. It is this latter approach -- that of specifying the individual agents, their objectives, choices, and constraints -- that we will follow in this course. In particular, it is within the framework most closely identified with the Neoclassical school of economics that we will work. Even as we develop the analysis, we will be at pains to point out the key assumptions necessary to draw strong conclusions in some parts of the analysis. Likewise we will aim to alert the reader to instances where -- applied to the wrong problem -- the results can be positively misleading. The hope is that the reader will find that, for all of its faults, this approach to analyzing international trade yields insights of fundamental importance. As well, we hope that the reader will keep in mind that it is intended to be only part of the story.

Our Approach to the Study of Trade Theory

We want to understand trade between economies. The first step must be to understand how the economies look in the absence of international trade, a state referred to as autarky. So, at a very general level, we want to specify the characteristics of an economy. To do this, we must specify:

1. The *agents* in the economy
2. The *objective* of each agent
3. The *choices* that each agent must make
4. The *constraints* that each agent acts under

In the first instance, the agents in our economy will be *individuals*, as consumer and suppliers of factors, and *firms*. Later we will add *government* as an independent agent. So, let us begin:

INDIVIDUALS

Objective: Each individual is assumed to maximize utility. At this level of generality, we do not yet specify all of the determinants of utility, except to note that it will include consumption of final goods. For example, utility could be extended to depend on the amount of work one has to do, the utility of others, the level of government spending on public goods, or anything else that they may care about.

Choices: Typically in the trade models that we work with, the only choice facing individuals is the levels of final goods that they will consume. A natural second choice would be the level of supply of factors (such as labor). However it is typically assumed (especially on the "real" side of trade) that all factors are supplied inelastically (i.e. in fixed amounts).

Constraints: The individual has a utility function which represents her preferences. She maximizes utility subject to a budget constraint. The budget constraint is given by the earnings on inelastically supplied labor and holdings of capital (if any). Prices of both goods and factors are taken as given by the individual.

FIRMS

Objective: Firms are assumed to maximize profits. For technical reasons, under conditions of perfect competition with a constant returns to scale technology, this leads to an indeterminate level of output. In this case we will assume that they are cost minimizers subject to achieving a certain level of output.

Choices: Firms face two decisions. First, they must choose the level of output of each good produced (say goods X and Y), Second, they must choose the combination of factors (say capital and labor; or capital, land and labor) with which to produce that output.

Constraints: Firms have a production function which represents all of the technical possibilities for production. When we have perfect competition in goods and factor markets, prices both of goods and factors will be taken as given by the individual firm. If in addition the production function is constant returns to scale, we can represent the entire productive sector as a single firm following these behavioral assumptions.

GOVERNMENT

Objective: (1) The "naive" assumption. In much discussion of trade policy, the government is viewed as if it were solely concerned with maximizing national welfare, appropriately defined. This is valuable insofar as it helps us to focus precisely on these issues of maximizing national welfare. However we may delude ourselves and misunderstand much of the real conflict in trade policy if we hew too closely to this description of actual policy. (2) The "Political Economy" assumption. An alternative set of views constitutes what we may call the political economy view of government. Here we may be able to develop a number of plausible theories of government trade policy objectives. One might be simply that in representative democracies, governments act so as to maximize the votes received by the ruling party in subsequent elections. Many other plausible objectives could be articulated. [It is not clear if we will have time to explore the political economy papers in this course, although of course it will have to be present in our discussions of real-world policy issues.]

Choices: The government will typically have a wide variety of measures that it can take. We note some of the more important ones here. It can decide whether there will be any international trade at all (although even here its hand may not be entirely free, as some economists have developed formal models of smuggling!). It can impose taxes or subsidies on production or consumption of one good or the other, to encourage or discourage its production or consumption. It can impose taxes or subsidies on the use of one factor or another, in one industry or many. It can impose tariffs, voluntary export restraints, quotas, and many other measures.

Constraints: (1) Under the naive assumption, the constraints that the government acts within are simply the objectives and constraints of the individual and firms in its own economy, and the opportunities to trade yielded by the rest of the world. (2) In the political economy view of the government, it may have its own "technology" for producing votes, or face other kinds of constraints.

We have now specified the agents in our economy, and the objective, choices and constraints that they work under. Yet we have not said anything yet about how they actually make their choices. How do they make decisions regarding their optimal levels of the variables that they control? For this, we need to take an extended sojourn into microeconomic theory. We will develop the basic principles and apply them to the consumer's and firm's problems, leaving the government to the side for now.

Economics and Optimal Choice: The Marginal Principle

Economics, it is often said, is the art (science?) of allocating scarce resources to competing ends in an optimal way. We frequently encounter such statements in the introduction to economics texts and brush by them to get to the "real" economics. But is worth taking a closer look at this statement, because it yields deep insight to virtually every problem that we encounter in economics. In this section we will develop a very common-sense, almost naive, approach to economics. We will ignore many complications that intrude on real world problems so as to understand very clearly some fundamental properties of economic optima. Later we will qualify some of the statements here to take account of the exceptions and special conditions. Here, though, we start with the core.

Let us begin with a simple principle: *every economic decision has both costs and benefits*. This is almost a trivial statement, since if there are only costs or only benefits, there is really no problem in choosing. So long as there are benefits, enjoy them, so long as there are costs, avoid them. But from this simple principle comes a great deal in economics.

Suppose I get to choose the level of a variable X. This might be the consumer deciding how much ice cream to purchase or a business deciding how many workers to hire or any other economic decision. How should you choose the level of X? If you don't buy enough ice cream, you lose out on all of the pleasure of slurping a cone; if the business hires too few workers, it doesn't produce enough and loses out on profits it might have made. On the other hand, if you buy too much ice cream, maybe you lose out by having to give up other things that matter to you (concert tickets, etc.); if the firms hire too many workers, it may actually lower profits from what it would have had with fewer workers. So how do you actually decide what level of X (ice cream, workers, etc.) to choose?

If every economic decision, like the choice of X, has both costs and benefits, then consider a simple rule:

1. Start with any level of X
2. Ask yourself: If I increase X by one unit, what will be the (marginal) benefits and costs of this last unit?
3. If the benefits exceed the costs, then go ahead and increase X by one unit and ask yourself the same question again for the next unit of X.
4. If the costs of the next unit of X outweigh the benefits, try reducing one unit of X and check to see if the benefits of this outweigh the costs. Continue repeating this so long as benefits of reducing X outweigh the costs.
5. When the benefits of increasing X by one unit exactly equal the costs of doing so, stop. This is the best point. Symmetrically, this will imply that the benefits are equal to the costs for small decreases in X (ice cream, workers). As I said, this is the optimal point.

We have derived an exceedingly important result. In economic decisions, we know the optimal point because this is where the costs of small changes in our choices exactly equal the benefits of these changes. You should treat the foregoing as a poem, a prized treasure, a revelation!

Yet it should be so obvious to you that it is beneath wonder. Of course! If I can change my choice by a little and the benefits exceed the costs, then I should do it. If the costs exceed the benefits, maybe I'd better retreat a little. Stand still when there is no further gain to be had.

This little insight, though, should be of wondrous value in your approach to economic problems. By this stage in you economic studies, you will have encountered unconstrained maximization, equality constrained maximization, inequality constrained maximization, and who knows what else. But this basic principle, with a few doo-dads and twinkles, should always characterize the solution to economic optimization problems.

The value of this insight is that we will often be able to guess the conditions that characterize our optimum even before we solve the formal problem. We just look at the problem, ask what are the decisions to be taken, and ask what are the benefits and costs. We choose the level of X such that small changes in X have the benefits of the change exactly equal to the costs of the change. We will refer to this as the *Marginal Principle*.

Intuitive Solution to Problems Given Perfect Competition in All Markets

Let's begin with a few familiar problems and just guess the solution. Since we are guessing, we may have to make a couple of adjustments to be sure that our guess is right. Take the problem of choosing the right amount of ice cream. Suppose you consume ice cream and theater tickets (a strange diet), how much ice cream should you buy? Well, what are the benefits and costs of changing your demand for ice cream by one unit? The benefit is the pleasure that the extra unit of ice cream brings to you which you will recognize as the marginal utility of ice cream: $\partial U(X, Y)/\partial X$ [we made explicit the fact that the marginal utility depends on the level of both X and Y]. So if the benefit of adding a unit of ice cream is the marginal utility of ice cream, what is its cost? A first guess would be P_x, the price of ice cream, since that's what we have to pay to get it. However we immediately run into a problem. We have to compare the marginal benefits and marginal costs. But how do we compare the sighs of pleasure that we get from the extra ice cream with the cold dollars and cents of the price? We can't. To make a comparison, there must be something that converts the benefits and costs to a common unit so they can be compared. Think harder. Suppose I ask you how much would you gain at the margin, in units of utility, if I were to give you an extra dollar of income (i.e. were to relax your budget constraint by one dollar). Let's call this number λ (recalling this is units of utility). Thus, if I want to know the cost to me in foregone utility of spending P_x dollars on this unit of ice cream (since by doing so I couldn't spend it on other things), it's simply λ, the marginal utility of a dollar of income, times the number of dollars spent for the last unit of ice cream, P_x. That is, the cost in units of utility is λP_x. But then what was our optimality condition? At the optimal X, the benefits of changing X by a unit ($\partial U/\partial X$) have to match the costs (λP_x). That is $\partial U/\partial X = \lambda P_x$. But this looks exactly like the first order condition of the Lagrangean expression for maximizing utility subject to a budget constraint.

You may ask, if we get the same answer as with the math, why did we bother to do this exercise? The reply is that it forces you to look with a much sharper eye at the tradeoffs that agents are facing. It is not a replacement for the formal mathematical derivation. Rather it allows you to see what the answer of the math will be even before you've done it. The advantage of this is that many economic problems share similar structures. Once you understand a few of these basic structures, basically the tradeoffs facing agents, you can simply write down the optimality conditions almost without having to think. They seem completely natural, even obvious. Here the computation, each time, of essentially the same problem is like a heavy burden on your mind which does not allow you to think creatively about the problem at hand. Understanding a few of these basic structures will free your mind for more interesting problems.

Now that we have a little experience, let's see if we can move through another problem by guessing the answer. Consider a competitive firm (why competitive? -- because then it can take P and W as given) that is trying to maximize profits. How many workers should it hire? Our approach is to ask, what is the benefit of hiring one more worker? What is the cost? The benefit of hiring one more worker, we may guess, is that she will raise output by the marginal product of labor, $\partial F(K, L)/\partial L$ [again we emphasize that marginal products depend on the levels of both K and L]. The cost, we may guess, is that we have to pay her the going wage, W. Now we should like to find the L where the marginal benefits just equal the marginal costs. But we have a problem, the marginal product is in physical units of output while the wage is in dollars. How are we to compare them? Obviously we have to take the value of the marginal product, P $\partial F/\partial L$ and compare this to the wage, W. So our optimality condition is:

$$W = P\,\partial F/\partial L \quad \text{or equivalently}$$

$$W/P = \partial F/\partial L.$$

As you can easily verify by doing the formal problem, this is exactly the correct condition. It's just a lot less work to know the answer, or be able to correctly guess it, rather than to have to derive it each time you need it (especially given that we run into *almost* identical problems so frequently in economics).

Tradeoffs: Market Opportunities and Private Willingness

The foregoing has yielded important insights. Yet it has neglected an extremely important problem. Suppose that we have alternative ways to achieve a certain objective. For example, we can obtain utility from consuming either X or Y. The firm can produce a certain level of output with different combinations of capital and labor, and so has to decide on the right mix. That is, there are two choice variables which can be substituted in aiming at a larger objective. How do you choose the right mix (of ice cream and theater tickets to maximize utility; of capital and labor to minimize costs)?

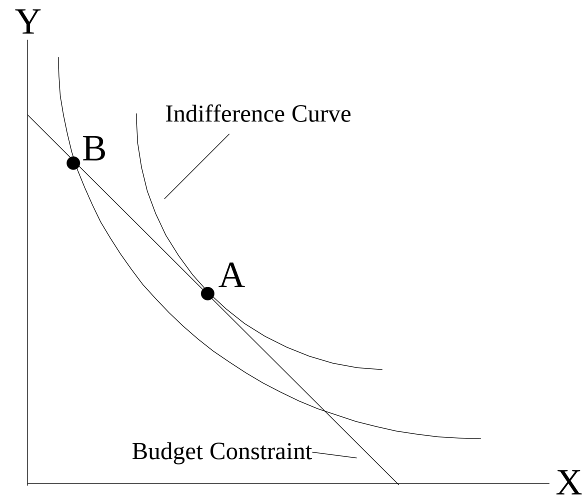
We will state the basic principle of optimality and then look at a few examples to be sure that we understand it. We must distinguish (1) market opportunities to trade from the (2) private willingness (or ability) to substitute. Whew! What is that? Basically, we cannot be at an optimum if the rate at which (secret?) agents are willing to trade is different form the rate at which the market allows them to trade. To understand this, we must turn to examples.

We will examine this question graphically and analytically in three contexts: (1) The consumer choice problem between two goods; (2) the producer's choice of factor inputs to minimize cost; and (3) the producer's choice of optimal output mix.

Let's start with the consumer's problem. We already know the answer: choose the combination of X and Y that puts us on the indifference curve just tangent to the budget line. Graphically, the answer is so intuitively pleasing as to bedazzle us. And this is our downfall. The answer is so obvious that we don't press hard enough to fully understand it.

In considering alternative potential points of optimum, we may first distinguish two sets of points: those strictly inside the budget line and those on the budget line. Those strictly inside the budget line do not really pose a problem of a tradeoff, since we could have more of both X and Y. Supposing (as we do here) that the consumer always would like more of both, we can immediately rule out these points as potential optimum points.

This leaves the set of points on the budget line as potential optimal points. Here we feel the pinch: if we want more of one we have to settle for less of the other. As we have already noted, we know the optimal point will be the point just tangent to the highest indifference curve. But we gain more insight into this by first looking at a non-optimal point on the budget constraint. Consider some point on the budget line (B) to the northwest of the optimal point. Draw the indifference curve through this point. You will note that the indifference curve through this point is steeper than the budget line. This is very important and we will want to look at it closely.



Consumer's Choice

An indifference curve, we know, gives combinations of X and Y such that we are just indifferent between any combination on the line. We are willing to give up such-and-such amount of Y if we can have one more unit of X. In fact, the slope of the indifference curve at a particular point measures exactly that. It tells us, at a given point, how much of Y we would need to be compensated for giving up one unit of X. The best way to think of it is that this slope tells us the *consumer's private valuation of good X* (in units of Y). This is such an important concept that it is given a special name, the marginal rate of substitution (MRS). That is, the MRS is just the slope of the indifference curve at a point, and it measures private valuation of X.

Private willingness is one thing and market opportunity is quite another. What do we mean by market opportunity here? We know that we have the opportunity to move anywhere along the budget line. But what does this signify? First, note that the slope of the budget line reflects the relative price $P = P_x/P_y$. The relative price answers exactly this question: in the market how much Y do I have to give up in order to obtain one more unit of X? We could think of it as the *market cost of X* (in units of Y).

Let's return to the consumer's problem now. As before, we look at point B, where the indifference curve is steeper than the budget line. Since the indifference curve is steeper there than the budget line, we know that the slope of the indifference curve (MRS) is greater than the slope of the budget line (the relative price), i.e. MRS > P. Thus, the consumer's private valuation of X is greater than the market cost of X. Not surprisingly, then, the consumer desires to take advantage of this to trade in the market to obtain more X. The consumer stops trading for more X just when the private valuation of X (the MRS) is equal to the market cost of X (the price ratio). Now we see what the tangency condition means. The private willingness to trade (slope of the indifference curve, i.e. the MRS) just equals the market opportunity to trade (slope of the budget line, i.e. the price ratio).

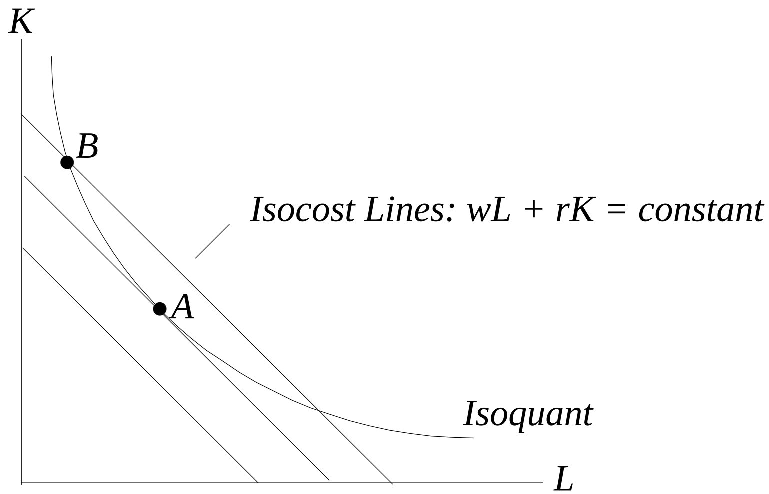
How can we be sure that we don't move past the optimum? Just look at a point on the budget line to the southeast of the optimum. We see that the indifference curve there cuts the budget line with a slope less steep than the budget line. That is MRS < P. The consumer's private valuation of X is less than the market cost of X. The consumer now trades for more Y.

You may wonder how this relates to our results in the first section. As you will recall, we derived some optimal conditions for the choice of X (ice cream) that set $\partial U/\partial X = \lambda P_x$. We could symmetrically derive conditions for good Y (theater tickets) that $\partial U/\partial Y = \lambda P_y$. If we want to look at the relative benefits of choosing X and Y, then, we need to take the ratio of these two expressions:

$$\frac{\partial U/\partial X}{\partial U/\partial Y} = \frac{\lambda P_X}{\lambda P_Y}$$

But notice that the left hand side is exactly the MRS and the right hand side is just the relative price P. That is the optimum is where MRS = P.

Let us look at the firms's choice of factor mix for given factor prices to achieve a certain level of output of one good. Again, the solution is very familiar. The factor price ratio gives us the slope of the isocost lines, and we choose the capital and labor mix where the isoquant is just tangent to the lowest isocost line.

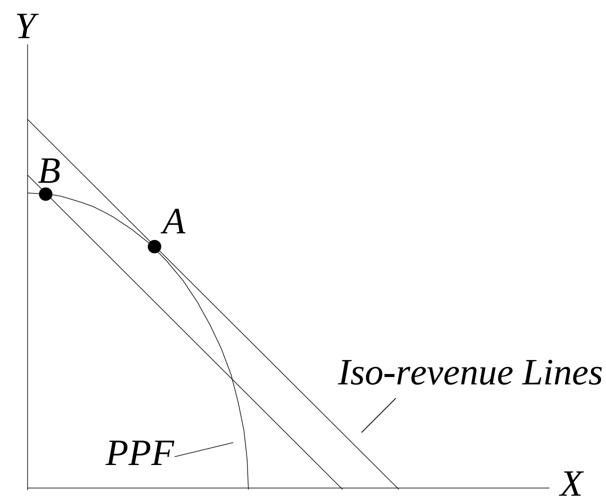


Producer's Input Problem

Again, though, the simplicity and intuitive appeal stops us short of a complete understanding. Consider some other point on the isoquant with a different factor mix, as at B. Why not produce with this mix? Recall that the slope of the isoquant is called the *marginal rate of technical substitution*. It tells us how much capital the firm is able to forego in exchange for an additional unit of labor, while keeping output constant. Another way to think about it is as the *firm's technical valuation of labor* (in units of capital). The *factor price ratio* (w/r), which is the slope of the isocost line, should then be called the *market cost of labor* (in units of capital). Now, why isn't this other point optimal? Because the technical valuation of labor differs from the market cost of labor. Again, consider a point on the isoquant to the northwest of the optimal point, as at B. Here the slope of the isoquant is greater (in absolute value) than that of the isocost lines. Thus the firm's technical valuation of labor exceeds the market cost of labor. Not surprisingly, it is optimal for the firm, then, to employ more labor (and so less capital) to produce the given level of output.

We also mentioned earlier that with perfect competition in all markets and constant returns to scale technology, we are justified in modeling the productive sector as if it were a single firm following the competitive behavioral assumptions. With factor supplies inelastic, the firm should use them to produce the bundle with the maximum revenue (in equilibrium profits are zero in any case). Again, the optimum is at A. However, insight is gained by noting that at B, P does not equal MRT.

P is the value in the market of X, while MRT is the technical cost of X. With P > MRT, the market valuation exceeds the technical cost, so we do better by raising output of X.



Producer's Output Choice

Thus far, we have been concerned with how agents choose. We would like to know also what the welfare effects are of the outcomes of the market system. Do countries gain from trade or lose by trading? We will want to know whether a certain policy raises national welfare or lowers it. We may want to know whether it raises world welfare. How are we to answer these questions?

Much of the analysis will be conducted in terms of the concept of pareto optimality. This is a very powerful concept, yet as we will show, also a very weak criterion. Popular discussions of economic issues are fraught with fallacies based on the misuse of the concept, and so we will want to be very careful how we use it.

First we want to define two terms. An allocation of goods among individuals is said to be *pareto optimal* (also pareto efficient) if it is impossible to redistribute them in such a way as to improve the situation of at least one without worsening the situation of someone else. In comparing two allocations, an allocation A is said to *pareto dominate* an allocation B if some people are better off at A than B and no one is worse off.

Let us turn to the power of the idea of pareto optimality. Consider an allocation that is not a pareto optimum. By definition, there exists an allocation in which some people can be made better off without others being made worse off. On first blush, it would appear crazy to prefer the first allocation to the pareto optimal one. This is the power of the concept of pareto optimality: if we are not at a pareto optimum there always exists an allocation that could help some without hurting others (maybe the first group could even share some of their gains so that everyone benefits).

Having articulated the power of the concept of pareto optimum, we now must explain why it is also a very weak criterion. Perhaps an example will help to understand its limitations. Assuming that her desire for more goods was never satiated, an allocation that gave everything in the economy to one person would be pareto optimal. It meets the criterion that we cannot improve the situation of any without hurting at least one person (viz. the person who initially has everything). If one believes that this is not an optimal situation for the society, then we must admit that pareto optimality cannot be a complete criterion.

This points to the general fact that pareto optima are typically not unique. In a standard exchange economy (which we will examine shortly), we have a whole contract curve, every point of which is a pareto optimum. How are we to rank among the pareto optima? To accomplish this -- as is daily faced in real-world policy decisions -- we would have to face the additional question of tradeoffs between individuals. For the most part, economics has professed no special insight about these tradeoffs. However the notion that economics does not have something special to contribute to this problem (perhaps philosophy, sociology or other disciplines would have more to say) should be clearly distinguished from the idea that it is unimportant or a matter of indifference.

An even deeper problem may be articulated. Is it true that any pareto optimal point is to be preferred to any non-pareto optimal point? The answer is no. Many points are strictly non-comparable. We have already shown this above by showing that we have many pareto optima, which by definition are non-comparable on pareto optimality grounds. But more deeply, consider a move from a non-pareto optimal point to some pareto optimal point. Will the second point pareto dominate the first? Again, no. We can easily show cases in which a move from one non-pareto optimal point to one that is pareto optimal makes some worse off. Thus we cannot in general say that a move from an inefficient point to an efficient (pareto optimal) point is even a pareto improvement.

This should strike you as a most disturbing result. Much of economics (and much of this course) is concerned precisely with these issues of efficiency (in the pareto sense). If we cannot be assured that a movement from an inefficient allocation to an efficient one is a pareto improvement, then how are we to proceed?

One approach (see A. Takayama, *International Trade*, Ch. 17) has been to argue that in moving between allocations we have an improvement if the winners are *in principle able to compensate* the losers and still be better off. This is appealing, and has much to be said for it. In particular, it draws on the notion that few policies are likely to benefit absolutely everyone, and if this were required, then we may actually be able to implement very little. Thus we say that if the winners gain more than the losers lose, that this is an improvement. However this argument is not absolutely compelling. Strictly, we do not have a pareto improvement unless *actual compensation is made*. To this it may be responded that if there are many other interventions, some of which benefit one sector at the expense of the other, and vice versa, that the principle above will assure that (almost) everybody will gain, and so it is justified. This is a reply that traces back at least to John Locke, and is interesting if hard to verify due to the counterfactual nature of the assertion.

The question of *actual compensation* is very important. If you wonder why the prescriptions of economists (such as those articulated in this course) are too infrequently heeded, you might take a first look at the question of *actual compensation*. In real-world particular policy choices, there will typically be gains by some at the expense of others; the fact that compensation for the losses is infrequently forthcoming may help to explain why the economist's "first-best" option is often neglected. The problem, though, is thorny. Why should the initial distribution be given priority over the resulting one, so that in switching compensation is required? Why should we compensate those who lose via trade in a manner different than those injured by economic shifts of a purely domestic manner? Does this invite appeals for compensation rather than encourage economic adjustment?

How is this general problem dealt with in trade theory? Unfortunately it becomes too cumbersome to incorporate these (very serious) issues explicitly in every model that we develop. As it turns out, we abstract away from most of these problems via a device that we will articulate later. The advantage of this is that it allows us to take a detailed look at problems that are already very complex without these additional complications. These will be left as distributional issues that each country must work out in its own fashion. Of course, in thinking about real-world controversies, we must always have these problems in mind.

The Social Planner, Optimality Conditions, and Competitive Equilibrium

We are about to launch into an extremely interesting and important problem. Every society has resources at its disposal: land, people, machines, etc. Given these resources, and the existing state of technology, it can use these resources to produce goods that the people in the society desire. Suppose there were an omniscient and beneficent Social Planner: how would she answer the following questions of resource allocation:

How much of each good should be produced?

What combinations of factors should be used to produce each good?

How shall these goods be allocated among the individuals?

A complete solution would have to face directly the problem of how the society weighs the utility of each individual. We do not do this here. Yet we may plausibly argue that a *necessary* condition for attaining the social optimum is that of pareto optimality. As before, if we are at a non-pareto optimal situation, by definition it is possible to make someone better off without making someone else worse off. It is then hard to imagine a social welfare function that would make such a situation the social optimum.

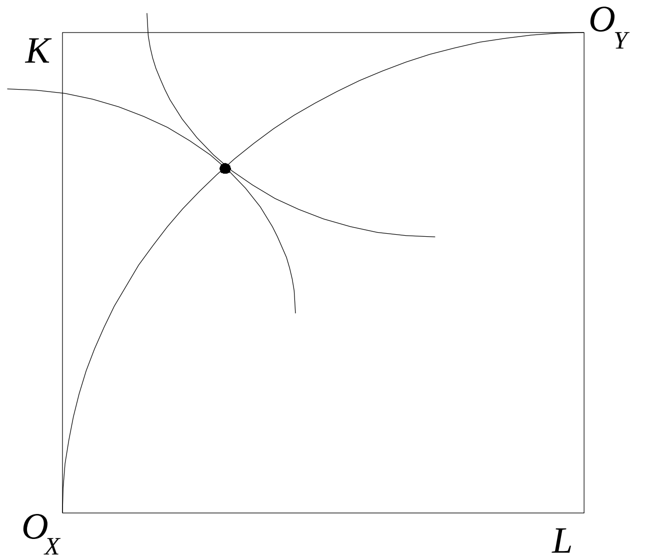
We want to start with a simple version of this problem. Let there be two individuals, A and B who together supply a fixed amount of labor, L. Let the society have a fixed amount of capital, K. There are two goods, X and Y with corresponding production functions:

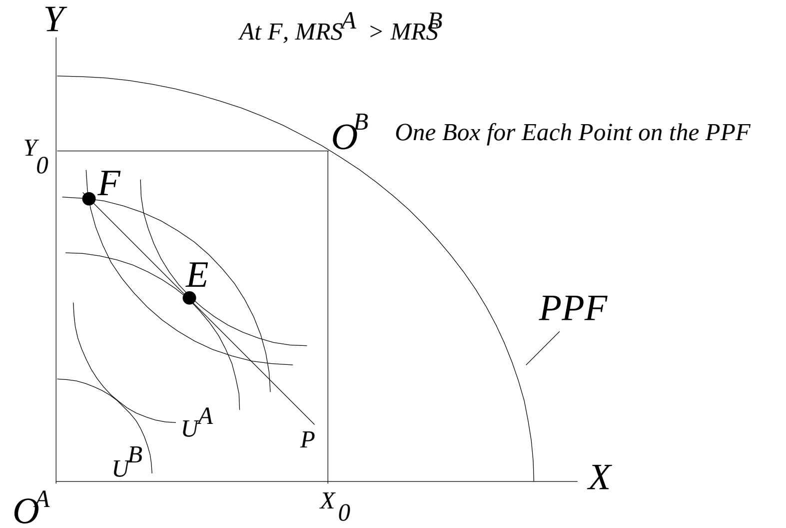
$$X = X(K_x, L_x), \quad Y = Y(K_y, L_y)$$

Consider the case of an omniscient social planner, who could make all of the economic decisions outlined above. How would she arrange economic affairs so as best to benefit her people? We begin by looking at the problem of factor allocation. In an Edgeworth-Bowley box we can draw respective isoquants for goods X and Y. Assuming that our people like both X and Y, our social planner can never be doing her best if it is possible to increase the output of both X and Y. It is this principle that leads to the familiar condition that the isoquants be tangent for technical efficiency. This yields a set of points known as the efficiency locus.

But we have to push harder to understand this. First, recall that the slope of an isoquant is the marginal rate of technical substitution (MRTS). The MRTS tells you how much capital you must give up at the margin for another unit of labor while maintaining output of that good constant. Thus the tangency condition is that we equate $MRTS^x = MRTS^y$. Why is a point of non-tangency not an efficient point? Consider a case where the isoquants intersect with the X isoquant steeper. Graphically it is obvious that there are allocations of the factors that increase output of both goods. But more importantly from our analytical perspective is to note that the slope of the X isoquant is greater than that of the Y isoquant -- i.e. $MRTS^x > MRTS^y$. In this circumstance, we can move to the efficiency locus by shifting labor towards X and capital towards Y,and have more of each good.

Each point of the efficiency locus maps to a point on the PPF





Production Choice and Consumption Efficiency

Note that for each point on the efficiency locus, it is impossible to increase the output of one good without decreasing that of the other. We can read off the isoquants and map this space into goods space (X, Y), which will yield us a production possibility frontier. We will represent it here in the form typically encountered in most trade theory, where it is bowed outward, reflecting increasing opportunity costs in moving from one good to the other in production [a sufficient condition would be constant returns to scale with some substitution possibilities between capital and labor, as we will discuss later].

We will also be very concerned with the slope of the PPF. It has the special name of the marginal rate of transformation (MRT). The MRT tells the amount of Y that must be given up at the margin to produce one more X, *assuming that factors continue to be used in optimal proportions*.

Now consider a single point on the PPF. If this was the amount that we produced of the only two goods, then we could again form an Edgeworth Bowley box diagram, only now with goods X and Y on the axes. Note that the slope of an indifference curve is equal to the marginal rate of substitution (MRS), which tells us the private valuation of X. By symmetry to our earlier exposition we see that the only situation where it is impossible to make one better off without injuring the other is if $MRS^A = MRS^B$. Note that at a point where A's indifference curve is steeper than B's, we do not have optimality since $MRS^A > MRS^B$. Of course, we could draw a similar box for each point on the PPF. The condition would continue to hold true.

Another consideration for the social planner is whether she is producing the right mix of goods: could welfare be improved by producing some different mix of goods? The simplest way to look at this is to recall that the MRT is the cost to the society to get another X, and the MRS is the private valuation of X . Thus, for example, if MRS > MRT, then the valuation of X exceeds the cost of providing it, and the society can be made better off by producing more X (and so less Y). If MRS < MRT, we move the opposite direction. Thus only when MRS = MRT will we no longer be able to improve things.

A final necessary condition, which should be obvious, is that all factors must be fully employed so long as marginal productivities are positive.

Thus the necessary conditions for the Social Planner's optimum, which we will refer to as the *Pareto Conditions* are:

1. $MRS^A = MRS^B$
2. $MRTS_X = MRTS_Y$
3. MRT = MRS
4. Full employment

Principle of the Invisible Hand

Arguably the single most important result in all of economics is Adam Smith's Principle of the Invisible Hand. What does it say? It claims that individuals as consumers and producers, each pursuing her own interest as she sees best, wholly ignorant of the aims and desires of others, reacting only to externally given prices, will attain a social outcome that cannot be improved upon. This is an astounding, almost literally incredible claim. If it were true, we could dispense not only with any government intervention, but the omniscient social planner as well -- there would be nothing for her to do that could improve on the outcome the market yields.

Is it true? Let us first talk about the kind of world in which it would be true. Consider a world in which there is perfect competition in goods and factor markets, with no externalities, no increasing returns to scale, and no uncertainty. This is exactly the kind of world in which we already derived how individuals and firms would make their choices. What were the results of those choices?

1. Each individual chose goods X and Y so that MRS = P
2. Each firm chose factors K and L so that MRTS = w/r
3. Each firm chose output levels so that MRT = P
4. Full employment was guaranteed by perfect competition in factor markets.

But these can easily be seen as precisely the Pareto Conditions! In this world the Principle of the Invisible Hand would hold. This is a result known as the *First Welfare Theorem*: under the conditions noted above, *the competitive outcome is pareto efficient*.

We have noted the weaknesses of pareto efficiency as a social welfare criterion. An additional argument for the beneficial aspects of the market, which we will not demonstrate, is what is known as the *Second Welfare Theorem*. It says that, given certain additional restrictions, any pareto optimal outcome can be achieved as a competitive equilibrium by suitable lump sum transfers. Thus if we don't like the particular pareto efficient outcome that results, we can enact lump sum transfers and achieve one that has a better social outcome (given some social welfare criterion). Of course, when thinking about the real world, we had better also be concerned about whether such transfers are economically or politically feasible.

The Principle of the Invisible Hand is an astoundingly powerful result. Accordingly, we must pause to ask, what is the true sense that lies behind this? *Why* does it work? We noted that each agent equates marginal benefits and costs of small changes in their decisions. But the farmer may not know the industrialist, both of which draw on the same labor pool. One consumer need pay no attention to the choices of other consumers, nor need she pay attention to the production conditions that lead producers to deliver milk and shoes to the stores. How do we know that the relevant margins are equalized across the various agents? Here the key is the role of prices in conveying information about social scarcity -- it is prices that links the decisions of all of the agents, equalizing all of the relevant margins.

Under what circumstances does the Principle of the Invisible Hand fail? It remains true that every decision maker equates marginal benefits and costs. The rub comes from the fact that the private costs and benefits that the decision maker perceives may differ from the full costs and benefits to society (e.g. a chemical manufacturer may not care about the pollution that reduces other people's enjoyment of a river). This suggests that a sufficient condition for the Principle of the Invisible Hand to be operative is that the private costs and benefits perceived by decision makers correspond precisely to the true social costs and benefits of the decision. As we will see, much of our discussion of trade policy will be of how best to align perceived private tradeoffs with true social tradeoffs, so that again decentralized decisionmakers will attain the social optimum.

Earlier we discussed the conditions for optimum as the equality between market opportunity and private willingness to trade. Now we must amend this. Implicitly we assumed that the decision maker perceived the same private costs and benefits as the true social costs and benefits. The amendment that we must offer is that the market opportunity to trade must reflect true social costs and benefits; in such a case we can be sure that decentralized decision makers will make precisely the same decision as the social planner would.

When Private Cost/Benefit Diverges from Social Cost/Benefit

We have introduced the problem of when social costs/benefits may diverge from the private costs/benefits, and suggested that this may prevent us from attaining a pareto optimal outcome. But we would like to look at some of the conditions commonly treated in the trade literature that may give rise to this divergence.

I. Increasing Returns to Scale

With increasing returns to scale, one large firm can undercut two small firms, and so there is a tendency toward monopoly. For a monopoly, a necessary condition for the optimum is that marginal revenue equals marginal cost. However, because demand slopes down, price P is greater than marginal revenue ($P + X P'$), hence greater than marginal cost. In our framework, the marginal social benefit is equal to the price, yet the marginal private benefit is equal to the marginal revenue. Since the latter equals the marginal cost, it follows that marginal social benefit is greater than the marginal cost, hence we cannot be at the social optimum. In fact, as is well known, the problem is that the monopoly will produce less output than is socially optimal.

II. External Effects

A. In Production

1. Level of output

Suppose that the level of output of good X affects how much Y we can produce for given levels of capital and labor. A simple example would be having an oil refinery upwind of a laundry: the more oil refined, the less cleaning of laundry is achieved for given inputs of capital and labor. Thus the level of oil output must enter the production function of the laundry. However, the oil refiner will not on his own take into account the (negative) effect of her output on the laundry, so there is the divergence between private and social costs/benefits. Note however, that there is no particular interest attached to the fact that it was a negative external effect -- the same divergence of private and social benefits would have occurred with a positive one as well (in this case we would underproduce the good with the positive external effect).

2. Level of Factor Input

We can distinguish an external effect that depends not on the level of output in a sector, but rather on the level of a particular factor input in that sector. Suppose that both goods require capital and labor, but as the amount of capital in the X industry increases, so does the amount of acid rain, which has a negative impact on industry Y (fish farming). Note that here we assume that it is specifically the level of capital in industry X that affects industry Y (presumably the level of labor employed in X has no effect on Y).

B. In Consumption

In principle, the presence of possible externalities in consumption differs little from what we have said regarding production. A first form of externality might be when my level of utility depends not only on what I consume, but also on your level of utility. Maybe I hate to see you at low levels of utility (then again, maybe I love this). This is parallel to the externality associated with production levels. A second form of externality is when my level of utility depends on how much of one of the goods you consume. [Additional Reading: A.K. Dixit, *Optimization in Economic Theory*]

II. RECIPROCAL DEMAND AND THE WORLD TRADING EQUILIBRIUM

Introduction

International trade theory should not, in principle, be very different from general economic theory. The approach to it taken here could easily be retitled "supply and demand in the open economy." The differences from general economic theory come not in basic approach, but in particular assumptions that make it a special case worth considering in detail. These differences include, limits to international factor mobility, the ability of governments to impose discriminatory taxes and/or regulations on foreign producers relative to domestic producers for the home market, different available technologies, and questions of welfare divisions between countries.

We noted earlier that much of trade theory is geared toward determining the *pattern of trade*, the *terms of trade*, and the *welfare consequences of trade*. We will make considerable progress toward answering these questions in this chapter in a framework of demand and supply that is very general. In fact, virtually all of the trade theory that will follow can be interpreted as special cases of this more general framework. As you might guess, the cost of this generality is that we beg a lot of important questions that can only be answered when we turn to models with more specific assumptions, especially as to the structure of production.

The approach developed in this chapter determines *world trading equilibrium* such that the import demand of each is equal to the export supply of the other. As in general models, the pattern of trade (quantities) and the terms of trade (prices) are determined simultaneously.

Before we can launch into the main topic of this chapter, we have to make a detour to talk about the representation of a society's preferences.

Assumptions on Preferences

Most of the literature on international trade has focussed on the effects of different production structures on trade. The representation of individual consumers and their preferences is, in most instances, very rudimentary. To outline the principle results of this body of theory, we are obliged to follow the assumptions typically made within it. But the reader should be strongly cautioned that in real-world policy issues, we must be aware of the possible violation of these assumptions, and consider how this would modify the results obtained.

In much of the work that we will do in this course, we will be making some very special assumptions about the preferences of individuals within a country. Specifically, we will assume that all individuals within a country have the same preferences over goods, i.e. they have the same utility function. This is referred to as *homogeneous preferences*. A second assumption, is that the *proportions* in which individuals demand the available goods depends only on relative prices -- not on income levels. This is referred to as the *homotheticity of preferences*. Someone with homothetic preferences whose income rises while relative prices stay the same will continue to purchase the goods in the same proportion; i.e. the income expansion path is a ray from the origin. Caution: these are not innocent assumptions. They are very powerful assumptions that at once greatly simplify the analysis and allow us to draw very strong welfare implications of the theories we develop, and at the same time suppress very important issues related to the heterogeneity within societies that may be crucial to many real-world conflicts over trade policy.

Notice that the assumptions of homogeneity and homotheticity are logically independent. We can easily write down indifference curves which are taken to be the same for each individual yet do not have the special homotheticity property. As well, we can write down multiple sets of distinct indifference curves, each of which does have the homotheticity property.

The joint assumption of homogeneity and homotheticity of preferences allows us to make a dramatic simplification--that of the *representative consumer*. That is, we will be able to draw indifference curves that represent the choices over goods of the society as a whole for each price ratio and income level. Without the assumption of homogeneity, we would have to calculate separately the demands of each individual and then aggregate each time there was any change in the economic environment. Without the assumption of homotheticity, we would have to recalculate the demand of the society as a whole each time there was a redistribution of income within the country, which will occur with virtually every change in the economic environment. Thus, representing the society's preferences as if it were a single individual greatly simplifies the analysis. However, in considering the welfare implications, and even the positive implications, of our analysis, we may be obliged to return to this assumption.

Aside from the simplicity that it allows us in developing the theory, what other argument could be advanced for making these simplifying assumptions? What they allow us to do is to separate issues of international distribution from the internal issues of distribution within each society. Insofar as we do want to focus on these international issues of distribution, this is a more reasonable assumption. However, insofar as we are concerned about the real impact on individuals within each society, the simplification can be quite misleading, and care should be taken in interpreting the results.

World Trading Equilibrium

In our simplest closed economy models, we required for equilibrium that demand equal supply in each market (technically, we could have equilibrium with excess supply, but then the price must be zero). How do we characterize equilibrium in open (trading) economies? We will look at this in a world with only two goods. Unfortunately for the student first trying to learn these models, there are several equivalent ways of characterizing world equilibrium. Before we discuss them, we must say a word about the budget constraint that countries face.

In general, countries need not maintain balanced trade. A country that is running a trade surplus will be accumulating assets or claims on the rest of the world; correspondingly, countries running trade deficits will be financing this by selling assets to the rest of the world. These concerns of the composition of the balance of payments are of great practical import. However, they are the province of the branch of international trade dealing with monetary issues (sometimes called open economy macro). In most, if not all, of the material that we will cover in this course, we are abstracting from this set of problems to concentrate on what is typically termed the "real" side of international trade. (The contrast is similar to the difference between macroeconomics and general equilibrium microeconomics). Countries here barter goods for goods. The reason is simple: in these models there are no assets apart from the goods themselves, so they cannot run trade surpluses or deficits, which would imply an accumulation of some asset (money, gold, etc.). Here each country is bound by a budget constraint that requires balanced trade:

$$P\,D_X + D_Y = P\,X + Y$$

$$P\,D_X^* + D_Y^* = P\,X^* + Y^*$$

Adding these together, we find that:

$$P(D_X^W - X^W) + (D_Y^W - Y^W) \equiv 0$$

You may recognize this as a version of Walras' Law -- the value of world excess demand is identically zero. Returning to the question of characterizing the world trading equilibrium, we see that -- given balanced trade -- this can be done in a number of ways:

1. World demand equals world supply of X
2. World demand equals world supply of Y
3. Home import demand for Y equals foreign export supply of Y
4. Home imports (at equil. prices) are valued equally to foreign imports

It is important to realize that these are not independent conditions--rather they are different characterizations of the equilibrium. In trade theory, it is the latter two conditions which tend to get used frequently.

World Equilibrium in the Pure Exchange Model

We want to develop a world trading equilibrium. We will look at this first in a model with the simplest possible production structure--the pure exchange model. We will look at this equilibrium in three different settings: (1) the structure of import demand/export supply; (2) offer curves; and (3) the Bowley-Edgeworth box diagram. While the last setting is ultimately the most natural place for looking at the pure exchange economy, working through the first two has the advantage of allowing us to look more closely at changes in demand in a setting that is familiar, and also allowing us to make the initial development of these two frameworks without the additional complication of changes in supply.

Lets look first at the home economy with a fixed endowment $(\bar{X}, \bar{Y})$ This can be shown in (X, Y) space as in the figure. Let imports of

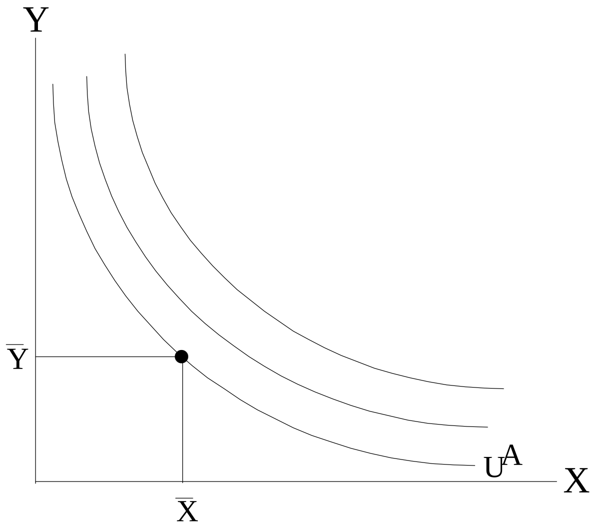
Y and exports of X:

$$M_Y = D_Y - \bar{Y} \qquad and \qquad E_X = \bar{X} - D_X$$

The budget constraint requires $M_Y = P E_X$.

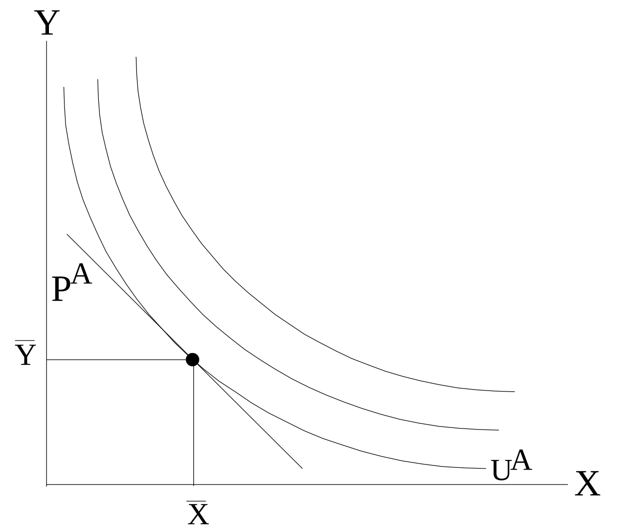
Our first task is to derive an import demand curve. We begin by asking what this person would do given the opportunity to trade at a variety of prices. Note that we can represent the trade geometrically by movements along a line with slope minus P which runs through the endowment point. Now there is some indifference curve which runs through the endowment point, call it U_A (here we are labelling the indifference curve by the level of utility attained). As we know, the slope of an indifference curve is the marginal rate of substitution. So, at the endowment point we have:

$MRS(X, Y) = [\partial U(X, Y)/\partial D_X]/[\partial U(X, Y)/\partial D_Y]$



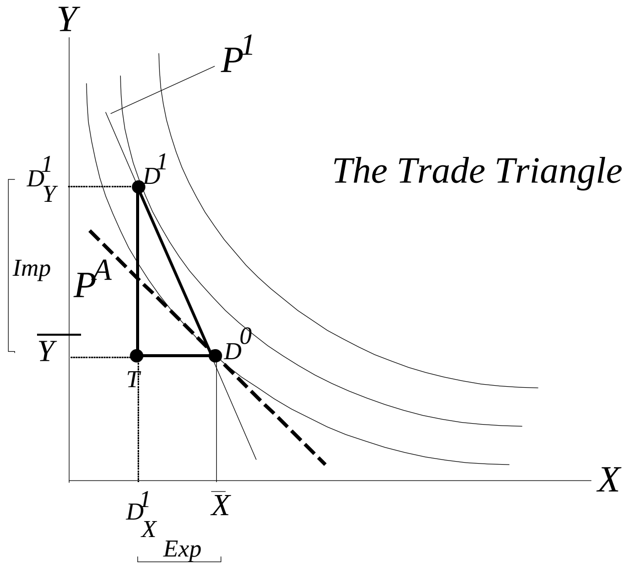
Autarky Endowments (Supplies)

Now consider a price P^A, where it is chosen so that it equals the MRS at the endowment point. [see figure] That is, if prices are P^A then the market opportunity to trade already equals the private willingness to trade (MRS) at the endowment point. It is clear, then, that the consumer finds no advantage in trading away from the initial endowment in either direction at these prices.



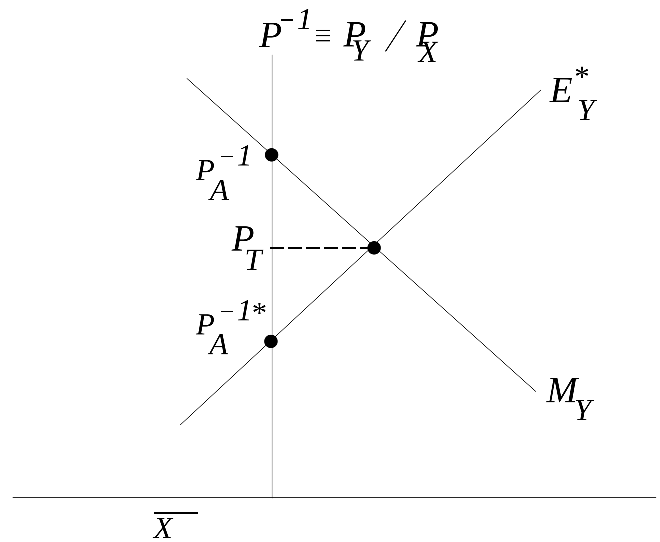
Autarky Equilibrium Price

What would happen if, instead, the consumer faced prices $P^1 > P^A$. This corresponds to a steeper price line through the initial endowment. Since at the initial position, $P^A = MRS$, we must have at that same point $P^1 >$ MRS. That is, the rate at which we would be willing to trade Y for another X is strictly less than the rate at which we are able to make this trade. Correspondingly, the amount of X we are obliged to surrender for another Y is strictly less than the amount we must surrender in the market for another Y. It then makes sense to trade (export) X in exchange for (imports of) Y when $P > P^A$. The optimal trade is clear graphically [see figure].



If trade is allowed at prices $P^1 > P^A$ we will move to D_1 where we export X and import Y. The triangle D_0 T D_1 is referred to as the *trade triangle*, indicating what will be exported and what imported (if these were the prices. Notice that in moving from P^A to P^1 we move from D_0 to D_1, and we can divide this into the standard substitution and income effects. For a rise in P, the substitution effect will always lead to a rise in the demand for Y relative to X. The income effect depends on the normality of the good. Typically (not always) we will assume normality, so the income effect will be positive for both goods when the terms of trade improve.

The remaining task of deriving the import demand curve is very simple. We simply vary the price and note the corresponding level of imports. Now we want to see how to represent this graphically. It will be convenient to use a graph where the horizontal axis is M_Y and the vertical axis is P^{-1}. Recalling that at P^A we would not trade, we can easily verify that this corresponds to $M_Y = 0$. In our example, as P rose (P^{-1} fell) home imports M_Y rose [see figure].



Trade Equilibrium Price

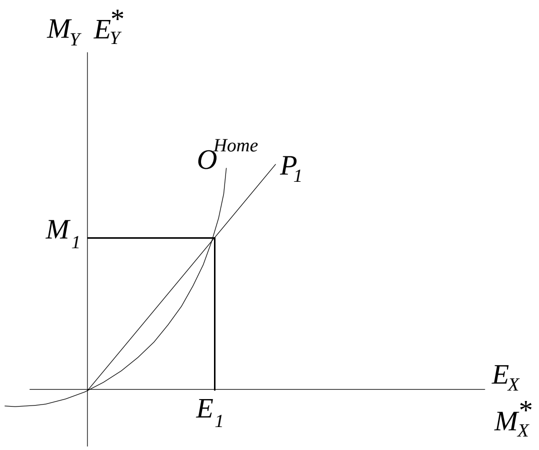
Now we look at the choices facing the other country. *A priori* there is no reason to suppose that pre-trade MRS = MRS*. Thus there is no reason to expect that $P^A = P^{A*}$. Thus for definiteness, suppose $P^{A*} > P^A$ (so $P^{A*\,-1} < P^{A\,-1}$), and that in fact $P^{A*} > P^1 > P^A$. Now consider the choices that the foreign consumer will make when faced with P^1 [see figure]. It is clear that the foreign consumer will export Y ($E_Y* > 0$) and import X ($M_X* > 0$). Tracing this out for varying price ratios, and graphing P^{-1} against E_Y* gives us the export supply curve.

We have now derived an import demand curve and an export supply curve. We have already shown that we will have world equilibrium when home import demand equals foreign export supply (both of the same good, here Y). Thus the intersection tells us the reigning terms of trade and the volume of home imports of Y, so implicitly also home exports of Y (via the balanced trade constraint). The equilibrium is as depicted in the figure.

Offer Curves in the Pure Exchange Economy

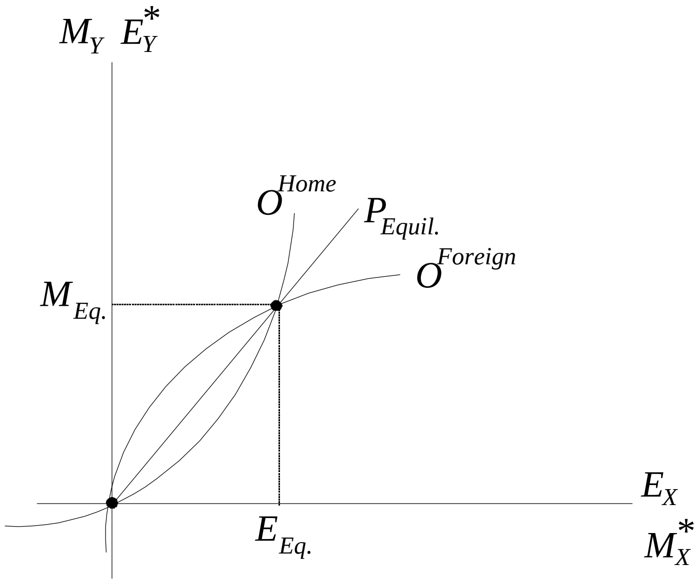
Here we will develop another way of representing world equilibrium, known as offer curves. In principle, they contain precisely the same information as our import demand/export supply framework. However, they reveal some of this information in a geometrically lucid manner, and so are of additional value for that. As well, the use of offer curves has a long tradition (dating to Marshall) and so is a necessary tool for reading the trade literature.

We have done virtually all of the hard work to derive the offer curves. All that remains is to assemble what we've learned in the proper manner. Recall that in deriving the import demand curve, we faced our home consumer with various prices. To each price there corresponded an optimal consumption point, hence optimal imports and exports from the given initial endowment. In fact, we could form a series of ordered triplets, that would give the price and corresponding imports and exports. It is preciesely this set of triplets that we will use to plot the offer curve. Now we need to show how to do it geometrically.



We can draw a graph with home exports of X, E_X, on the horizontal axis, and home imports of Y, M_Y, on the vertical axis. Now consider a ray emanating from the origin with slope P_1. We have already determined what the optimal exports and imports would be. Thus, in this space, the triplet (P_1, E^1_X, M^1_Y) gives us a single point representing the desired trades. This point lies on the ray from the origin with slope P_1. How can we be sure? Recall that balanced trade requires $M_Y^1 = P^1 E_X^1$, which evidently lies on the ray described. If we do this for every possible price, we trace out the offer curve. Quick! What is the slope of the ray such that the offer curve is at the origin here (i.e. no trade)? Easy -- it must be P^A, the price at which we had no interest in trading, corresponding to the slope of the indifference curve through our endowment point.

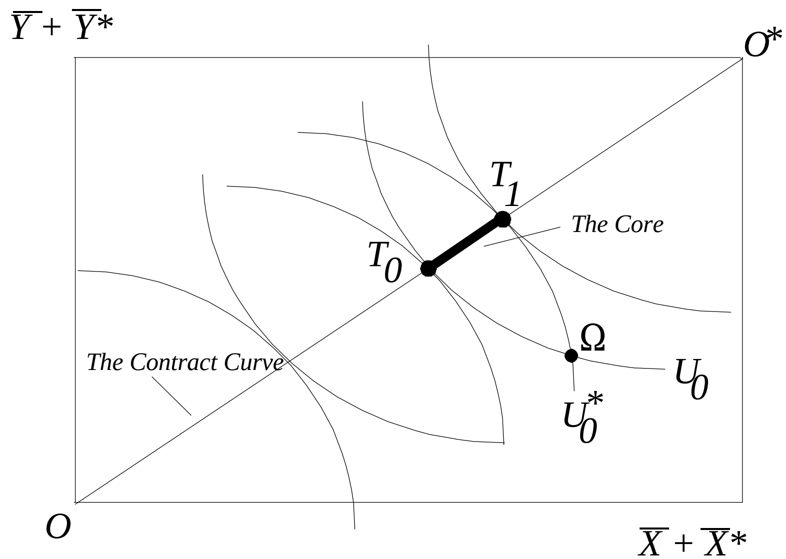
We can perform a similar experiment for the foreign country, tracing out the offer of exports and imports at each price. Obviously, our full equilibrium requires a price that matches our desired imports with their exports and vice versa. This is represented as the intersection of the two offer curves.



THE PURE EXCHANGE ECONOMY IN THE EDGEWORTH-BOWLEY BOX

The treatment of the pure exchange economy is frequently developed within a framework referred to as the Edgeworth-Bowley box. First look at how the home country's problem looks. In (X, Y) space, we can represent the initial endowment as a point, Ω. Also in the same space we can draw the indifference curves representing the preferences of the consumer at home. We can do the same for the foreign representative consumer(see figure), only now we add the twist that we will revolve the box around so it is upside down.

Now we will combine the two diagrams, creating a box diagram, whose width is given by (X + X*) and whose height is given by (Y + Y*) [see figure]. In this box, a single point now represents *both* countries' endowments. For example, the endowments could be given by point Ω. Note that in principle each point in the box represents a possible division of the world endowments among the two countries. Further note that point Ω is not an optimal point in the sense that there are other distributions of the world's goods such that *both* countries could be made better off.

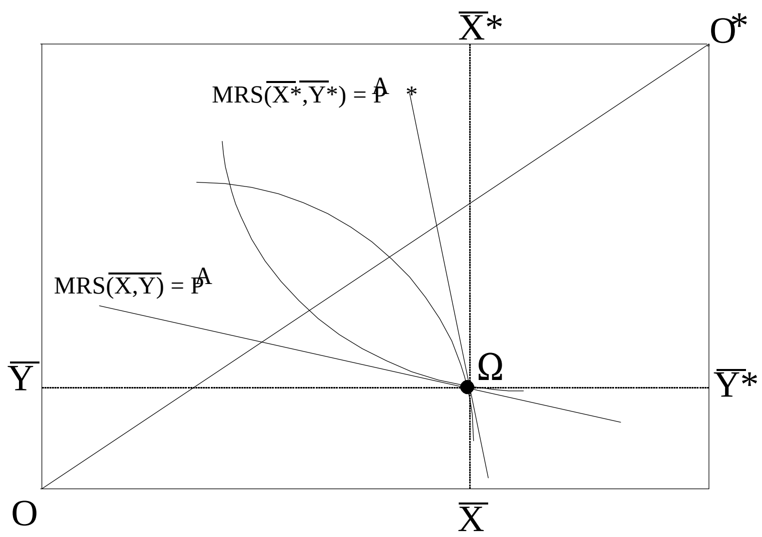


Recall that pareto optimality requires that MRS be the same for all individuals. Equivalently, at an interior optimum in the Edgeworth box, the indifference curves of consumers must be tangent. If we drew in all of the indifference curves, and traced out the set of points where the indifference curves were tangent, this would give us the so-called *contract curve*. In the diagram, this stretches from O to O^* as indicated, passing through points T^0 and T^1. The contract curve represents the set of points that are pareto optimal. Recall that a point is pareto optimal if there is no way to improve the situation of one without worsening the situation of the other. In this context, the points along the contract curve between T^0 and T^1 are referred to as the *core*. These are the set of pareto optimal points that pareto dominate the endowment (i.e. only among these pareto optimal points can we make one better off without hurting the other, relative to the endowment).

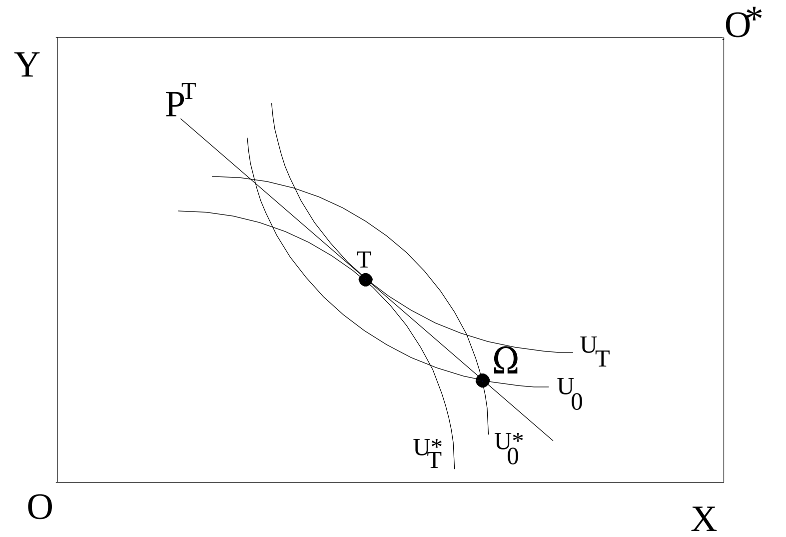
Recall also that price-taking consumers optimize by arranging purchases such that P = MRS. Since free trade establishes a common international price ratio, we may already surmise that this will lead to a pareto optimal outcome (demonstrating this and stating the conditions will come shortly).

We would like to work within this diagram to see what trade might take place. First, note that a straight line with negative slope may be interpreted as a price ratio. It shows the tradeoff of Y for each unit of X. If it passes through the original endowment, it may be looked at as a budget line *for each* of the countries. That is, if this were to be the reigning price ratio, it would demark an opportunity set for consumption after trading. Since our diagram has one consumer standing on her head, we may wonder if increases in the slope of this budget line correspond to increasing the relative price of X to both consumers. As it turns out, this is correct. It is easiest to see if you simply note that it changes the opportunity set for both, which must represent the same set of available tradeoffs.

Let us look more closely at the endowment point Ω. Note that the slope of home's indifference curve is less steep than foreign's indifference curve. That is, MRS < MRS*. This implies that home values X less highly relative to Y than foreign does. We will look more formally soon at the determination of equilibrium. For now, note that if at Ω, $P^A = MRS(X, Y)$, then home will have no desire to trade, yet the foreign country will find $P^A < MRS(\bar{X}*, \bar{Y}*)$, and so will want to supply Y in exchange for X (see figure).



On the world market, this must correspond to an excess supply of Y. If instead, at Ω the trade price were $P^{A*} = MRS*$, then foreign would have no incentive to trade, but the home country will find $P^{A*} > MRS$ and so will want to trade X to obtain more Y. This must correspond to a world excess demand for Y. Given certain conditions that will be specified later, we may suspect that there is an intermediate price for which the market for X (and so Y) has cleared. This would correspond to a point such as T (see figure).



Interpreting the Pure Exchange Economy

We have limited our exposition to two countries. How should we interpret this? The simplest interpretation might be to look on it as the home country relative to the rest of the world. Another way would be to think of them as groups of countries, say industrialized countries and developing countries. There are similarities within each group in endowments and tastes, but more important differences between groups. Are there other significant ways to interpret the two-country model?

How should we interpret the restriction to only two goods? We may first think of it as groups of goods, those we export versus those we import. Perhaps we could think of it as capital intensive goods versus labor intensive goods. How else?

Notice that in the pure exchange model, we abstract away from the effects on production in each economy to focus on the pure consumption gains or losses.

Production with Increasing Opportunity Costs

Typically we draw the production possibilities frontier (PPF) as bowed out from (concave to) the origin. This is not a necessary aspect of a PPF. It is possible that the PPF is bowed in towards (convex to) the origin. We will work here, though with the typically encountered shape. We want to explain the significance of this assumption and give sufficient conditions for it occurring without entering into a formal proof of the sufficiency of these conditions.

The basic idea of the concave PPF is that of increasing opportunity costs. That is, as we move from producing, say, a given amount of Y and X, to producing less Y and more X, we have to give up greater and greater amounts of Y to obtain each succeeding X. A good reason why this might be the case is if some factors tend to be more specialized toward the production of one good (Y) as opposed to the other (X). We may think of this relative specialization in the sense that -- for given relative factor prices -- one of the goods uses one of the factors relatively more intensively. Thus as we shift production more and more towards X and away from Y, the factors that are released tend to be those used more intensively in Y and the factors in greater demand are those used intensively in X. Ultimately, the economy is forced to adapt to this by shifting the technique used in X production to adjust to the use of factors relatively more specialized toward the production of good Y.

A sufficient condition for the concave PPF -- and conditions frequently utilized in trade theory -- is that production take place under constant returns to scale, that there be some possibilities of substitution between factors in production, and that for any given factor price ratio, the goods not use the factors in the same proportions.

Reciprocal Demand in a Model with Production

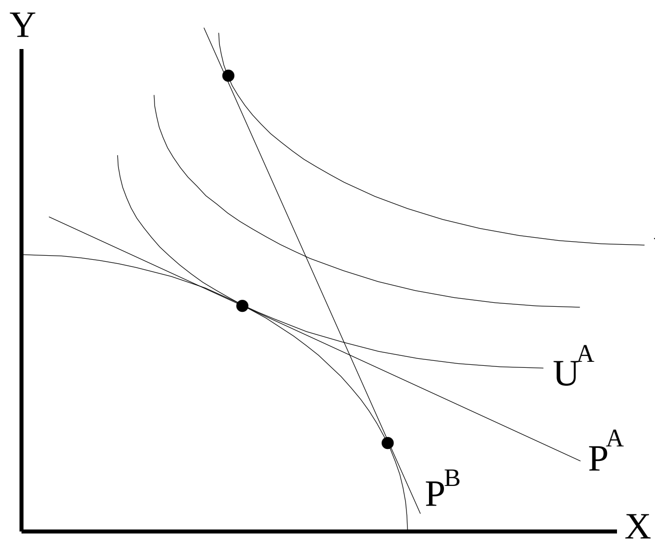
We are interested now in looking at how the world equilibrium looks when we move from the pure exchange economy to an economy with production. Fortunately, we have already done most of the hard work, and need add only some small modifications to our earlier representation of the world equilibrium. The model that we are going to look at now reflects increasing opportunity costs in the sense outlined above that yielded the concave PPF. That is, as we move along the PPF from specialization in Y to specialization in X, the marginal rate of transformation is increasing.

We will be looking at this in the context of a competitive economy. We know that if the factor market is competitive that we will have full employment. Does full employment itself guarantee that we will be on the PPF? No, since if factors are not apportioned efficiently, we may have full employment yet still lie inside the PPF. Recall, though that the firms choose factors in each sector such that the marginal rate of technical substitution equals the factor price ration, that is MRTS $=$ w/r. Since they do this in each sector, we know $MRTS_X = MRTS_Y$, which is the required efficiency condition. In addition, under perfect competition, producers acted such that the marginal rate of transformation equalled the goods price ratio -- MRT $=$ P. Since the MRT is the slope of the PPF, this corresponds to producing at the tangency between a price line and the PPF.

By looking at the exchange model, we have already developed a relatively sophisticated notion of demand. Now we need to add production, which it turns out is very simple here. As price changes, production moves along the PPF. Thus we can summarize supply of X as $X = X(P)$. Since $P \equiv P_X/P_Y$ we easily verify that the supply of X is increasing in P. Similarly $Y = Y(P^{-1})$. Note that we have written Y as a function (increasing) of P^{-1}. Since $P^{-1} \equiv P_Y/P_X$ this should not be surprising. A rise in P necessarily means a fall in P^{-1}, so a rise in production of X and a fall in production of Y.

We can now insert this directly into the framework of import demand/export supply that we developed earlier -- only now with production variable. Let us look at the home country first. What would the autarky (pre-trade) situation look like? We can see that in this competitive economy, production and consumption would occur at a common point, where MRS $=$ MRT $= P^A$, and where MRTS $=$ w/r (since we have full employment and are on the PPF). [see figure] This would put us at the autarky equilibrium, point A. In the import demand/export supply diagram, this implies $M_Y = 0$ just when $P = P^A$.

We continue to derive the import demand schedule just as before, only now we allow for production to vary. For example, we could ask what would happen if P were $P^B > P^A$ (i.e. $P^{B-1} < P^{A-1}$). It is clear that production shifts toward X. Consumption moves in the manner developed in the earlier section. Our economy would import Y and export X.

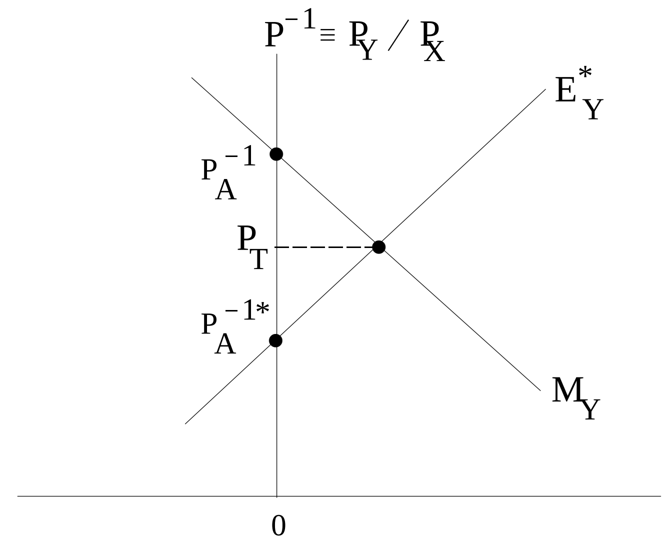


We can equivalently define the foreign export supply curve as:

$E_Y^* = Y^*(P^{-1}) - D_Y^*(P^{-1}, R^*)$.

The same comments regarding the effects of prices on supply and demand hold here for the foreign country.

Balanced trade obtains exactly where our demand for imports equals their supply of exports (both of Y). Note in the derivation that the home import demand cuts the vertical curve at the autarky price. Correspondingly, export supply is $E_X = X(P) - D_X(P, R) = E_X(P, R)$.



Trade Equilibrium Price

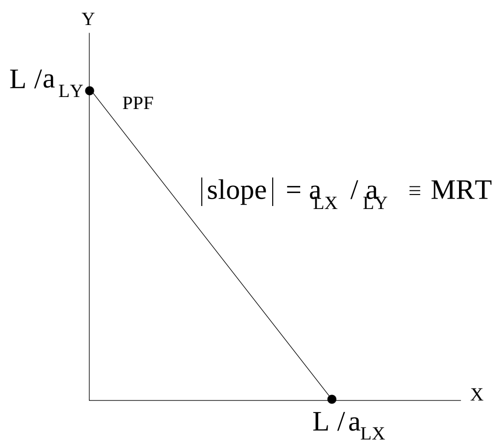
It is crucial to note that the import demand and export supply curves are not themselves typical demand and supply curves. Each of them already has within it a whole structure of production and demand.

III. THE RICARDIAN MODEL

In the last chapter we established an important principle: If autarky relative prices differ, it is possible for countries to trade at an intermediate price ratio such that both may gain. Our development of the world equilibrium was in very general terms based on demand and supply in each country. However it was left unexplained why autarky prices should differ. The present chapter develops the Ricardian model of trade, and uses the concept of *comparative advantage* as the basis of trade. We will see that in the Ricardian model, this comparative advantage derives from underlying differences in the technology available to each country.

Derivation of the Ricardian PPF

One factor (labor) in fixed supply
Two Goods: X and Y
The technology is fully described by the constant unit labor inputs: a_{LX} and $a_{LY.}$
Factor market clearing requires:
$L_X + L_Y = L$ $\quad X = L_X/a_{LX}$ $\quad Y = L_Y/a_{LY}$
thus $a_{LX} X + a_{LY} Y = L$.
We can solve this for Y in standard linear form as:
$Y = L/a_{LY} - a_{LX}/a_{LY} X$



$MRT = a_{LX}/a_{LY}$ defines the marginal rate of transformation, which tells how much Y must be reduced to produce one more unit of X (note that it is constant here).

Autarky Equilibrium in a Ricardian World

We will assume homogeneous homothetic preferences so that we can use the construct of a single representative consumer.

Assuming that both goods are produced, the optimality conditions in a competitive economy insure that:
$P = MRT = MRS$ where $P \equiv P_X/P_Y$ and

$MRT \equiv a_{LX}/a_{LY}$	$MRS \equiv [\partial U/\partial X]/[\partial U/\partial Y]$
Ability to Transform	Willingness to Substitute

Homogeneous preferences means everybody in the society has the same preferences. Homothetic preferences mean that for a given relative price P, the proportion in which the goods are consumed is independent of the level of income. [These allow us to put aside *for the moment* problems of income distribution.]

Ricardian Supply and Demand

The prices of the goods produced depend on labor costs. If any price was strictly above its labor costs, then assuming a given wage and fixed input coefficients, we could have infinite profits by expanding production of that good to an infinite level. Thus the assumption of competition and finite production levels insures that the price must be less than or equal to the labor costs; that is:
$P_X \leq w\, a_{LX}$ and $P_Y \leq w\, a_{LY}$ (equality if actually produced)
If both goods are produced, it follows that:
$P = P_X/P_Y = (w\, a_{LX})/(w\, a_{LY}) = a_{LX}/a_{LY} = MRT$
So we can draw the Ricardian Supply Curves for X and Y [see figure].

Note that, the autarky price has been determined even before considering demand. The division of output between the two goods, then will correspond to that demanded at the supply-determined price.

Comparative Advantage

Why do countries trade? What determines which countries produce which goods? A first idea might occur: since labor is the only input, home should produce those goods which it can produce with less labor than the foreign country. It should import the other goods. This is the so-called *Principle of Absolute Advantage*. By this principle, if $a_{LX} < a_{LX}*$ then we should export good X. But is this an adequate characterization of the determinants of trade? What should happen if the home country has an absolute advantage in producing every good? In this case, $a_{Li} < a_{Li}*$ for all goods i. Should they trade at all?

To answer this, we must turn to the *Principle of Comparative Advantage*. This is arguably the most important result in international trade theory, and so must be studied closely. Recall that we have already defined the marginal rate of transformation as $MRT \equiv a_{LX}/a_{LY}$.

We will say that home has a *comparative advantage* in the production of good X just when:

$$MRT < MRT* \quad \text{that is,} \quad [a_{LX}/a_{LY}] < [a_{LX}*/a_{LY}*].$$

Note that in the autarky equilibrium, we had the condition that $P = MRT$, so also in the foreign country we had $P* = MRT*$. Thus, recalling that $P = P_X/P_Y$, we have a comparative advantage in good X when $P < P*$, that is when we have a lower autarky relative price of good X.

This may seem a bit hard to follow. But the basic idea is simple. We are the *relatively* efficient producer of X. When our autarky price is lower, this means that we have to give up less Y to get one more unit of X than is true in the foreign country. Conversely, if the foreign country were to produce one unit less of X, they could use the extra labor to produce more Y than we gave up. Intuitively, this extra amount of Y could be divided up and both would be better off.

Comparative Advantage: An Example

Suppose $a_{LX} = 1, \quad a_{LY} = 2,$
$a_{LX}* = 2, \quad a_{LY}* = 3$

Here home has an absolute advantage in each good (check!). However its comparative advantage lies in good X, since:

$$[a_{LX}/a_{LY}] = 1/2 < 2/3 = [a_{LX}*/a_{LY}*].$$

Thus from an autarky position, if home produced one less Y, it could produce two more X. If foreign produced one less X it could produce 3/2 more Y. The net gain to the world of shifting the site of production is one X and one-half Y. Both could be made better off.

Note, then, *absolute advantage* is not an adequate determinant of the pattern of trade. Although home has an absolute advantage in the production of good Y (as well as good X), it does not export Y. The reason is that in spite of the *absolute* advantage in Y, it has a *comparative* disadvantage in Y.

Derivation of the Ricardian World Production Possibilities Frontier

The world PPF shows the maximum feasible combinations of X and Y, given the available technology and labor supplies. A combination of X and Y is said to be the maximum feasible if it is not possible to produce more of one without producing less of the other. These are also called points of production efficiency.

If each country is on its PPF, does it follow that the world is on the world PPF? No! It may be that the distribution of production across countries is accomplished inefficiently. In such a case, by shifting the site of production of each good to the country with comparative advantage of that good, we can increase the availability of some goods without reducing the availability of others. That is, we can move towards a point of world production efficiency.

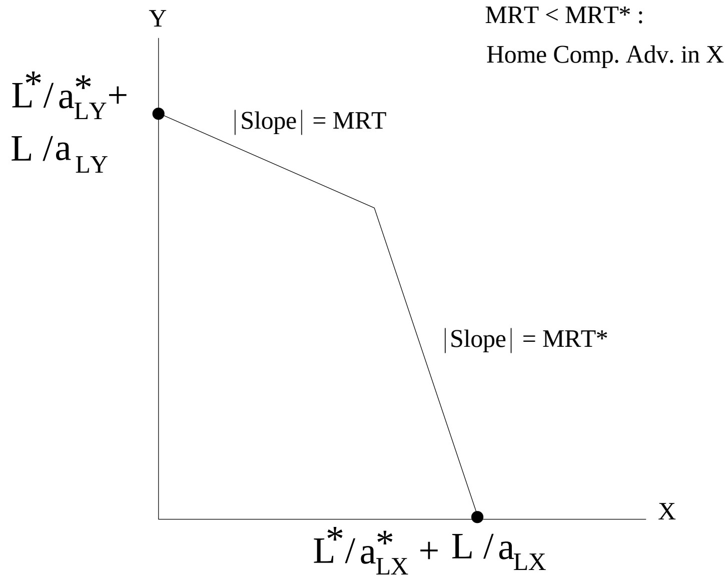
We would like to derive the world PPF. Two points are easily derivable, namely those when both countries produce only X, or only Y. In this case:

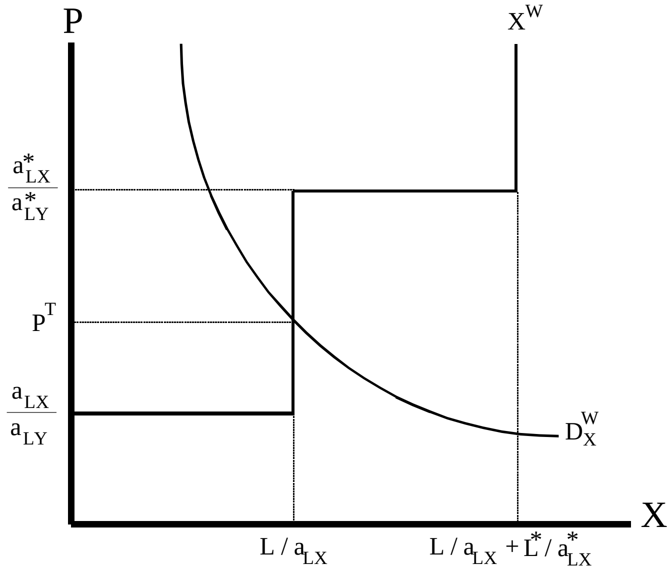
$$X_{max} = L/a_{LX} + L*/a_{LX}*$$

and $$Y_{max} = L/a_{LY} + L*/a_{LY}*$$

To proceed, we must know the pattern of comparative advantage. Suppose, as before, that home has a comparative advantage in good X. Then MRT < MRT*. Thus if any X at all should be produced, it should be produced in the home country. Thus, starting from a position where only Y is produced, have the home country produce one unit of X; the amount of Y we have to give up is exactly MRT (recall that is how we defined it!). It is obvious that we would have the home start producing X first, since the fact that it has a lower MRT *means* that we have to give up less Y for that first unit of X. Note, though, that since the labor input requirements are constant for each country, the same must be true of their ratio, the MRT. That is, for each extra X we produce, we have to give up MRT units of Y. This continues to be true up until the point that home is completely specialized in X. If we want the world to produce yet more X, then the foreign country will have to start producing it. But, now that the foreign country starts to produce it, we have to give up more Y to get a unit of X, since MRT* > MRT.

Note that if actual production is to the left of the kink in the world PPF, then the price under trade is equal to the home autarky price: $P^T = P^A = MRT$. If we actually produce to the right of the kink, then $P^T = P^{A*} = MRT^*$. If we actually produce at the kink, then all that we can say is that the trade price is in the range: $P^T \in [P^A, P^{A*}]$.

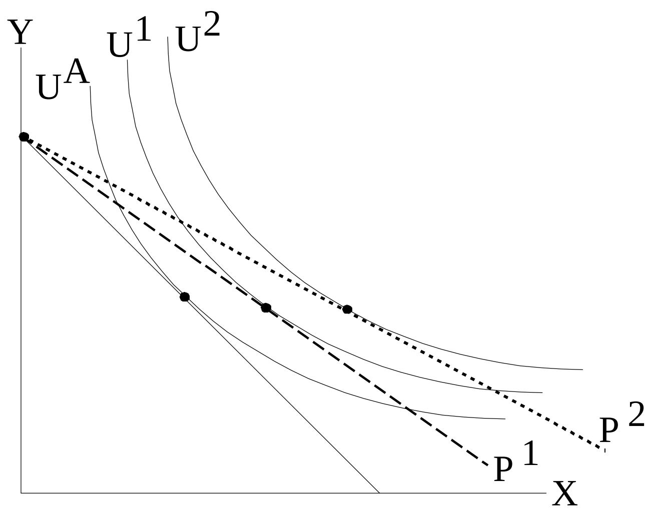




To determine the actual point on the world PPF where we will produce, we have to look at world demand and supply. As before, assume MRT < MRT*, and the prices are $P \equiv P_X/P_Y$. Note that the world supply curve is in the nature of a step function. This corresponds to the condition that a country will produce both goods only if P = MRT in that country. Thus the horizontal sections occur at the levels at which P = MRT in the respective countries. We can then define three regions in which both goods are supplied. At a price equal to the home autarky price, P^T = MRT only home will produce X (although it may produce some Y as well); foreign will specialize in Y. At a price such that MRT < P^T < MRT*, both countries specialize--home in good X and foreign in good Y. At a trade price such that P^T = MRT* the home country will specialize in good X and the foreign will be willing to produce any feasible combination of good X and Y. Of course, given the supply conditions, the prices and quantities depend on the demand conditions. However, note that small shifts in demand affect *production levels* only when one of the countries is producing both goods. Further, in this case, the *price is wholly unaffected*. On the other hand, if both countries are completely specialized, then small shifts in demand lead to *no change in production levels*, only a *change in the price level*!

Ricardian Gains From Trade

So long as the trade prices differ from autarky prices, there are gains to trade. In general, the gains will be larger the larger the divergence.



Relative Wages in a Ricardian World

The Ricardian model can be used to take a first look at what might give rise to the observed distribution of wages in the world.

If both countries specialize, which occurs when $MRT < P^T < MRT^*$, it must be true that $P_X = w\, a_{LX}$ and $P_Y = w^*\, a_{LY}^*$.

This implies that $w/w^* = [P_X (1/a_{LX})]/[P_Y (1/a_{LY}^*)]$. Note that $1/a_{LX}$ is the marginal product of labor, so $P_X (1/a_{LX})$ gives the value of the marginal product for good X. Thus the ratio of wages between the two countries equals the ratio of the value of the respective marginal products. Note that a shift of demand that raised $P = P_X/P_Y$ yet still left production completely specialized would raise home relative wages.

We can say a bit more about relative wages in this first case. So long as the trade prices are (weakly) between the autarky prices, we can give a bound for relative wages (also known as the factoral terms of trade):

w/w^* is at a minimum when $P^T = P^A$, so $P_Y = w\, a_{LY} = w^*\, a_{LY}^*$

w/w^* is at a maximum when $P^T = P^{A*}$, so $P_X = w\, a_{LX} = w^*\, a_{LX}^*$

Thus, $[(1/a_{LY})/(1/a_{LY}^*)] \le w/w^* \le [(1/a_{LX})/(1/a_{LX}^*)]$.

If home specialized, but foreign did not, then it must be the case that:

$P_X = w\, a_{LX} = w^*\, a_{LX}^*$ and so $w/w^* = a_{LX}^*/a_{LX}$.

That is, **here relative wages are fixed wholly by the technical differences in the good produced in common.** This is true so long as one does not specialize.

Small Ricardian Economy Capable of Producing N Goods

By definition, a *small* Ricardian economy faces fixed prices for its output. Even if it is capable of producing N goods, it will specialize in one (or several in the knife-edge case of a tie) for which there is the largest value of the marginal product: $P_i (1/a_{Li}) = VMP_i$ (recall $1/a_{Li}$ is the marginal product in the i'th good. There will be a tie if and only if $P_i/P_j = a_{Li}/a_{Lj}$.

Notice that this suggests that if we actually lived in a Ricardian world there would likely be a <u>very</u> high degree of specialization. As always in a Ricardian model, we have the technical coefficients (here the labor input requirements) independent of the level of output. Labor is assumed to be homogeneous and perfectly mobile domestically.

Nontraded Goods in the Ricardian Small Open Economy

Nontraded goods are those for which transport costs make trade in such goods prohibitively costly (e.g. land, haircuts, etc.). As before, the small open economy will produce only that traded good for which P_i/a_{Li} is a maximum. It will import all other tradable goods that it consumes. But we know that this gives the wage rate $w = P_i/a_{Li}$. Thus given the technology of the nontraded good, and specifically its unit labor requirement, we must have $P_{NT} = w\, a_{LNT}$ as its price. Note that our assumptions on how specialization in traded goods occurs implies that relatively small changes in relative traded prices can lead to a complete change in traded specialization. This is not surprising given the rigid Ricardian technical conditions that we have specified. What is noteworthy is that the small shift in prices will change the wage, and so the price of the nontraded good, but it will not lead to the same violent change in production. The nontraded sector is able to adjust prices and production levels with relatively minor fluctuations. The nontraded sector can pass on the increased labor costs, while the original traded sector cannot. Note that we could have done an essentially identical comparative static if there was technical change in the traded goods. What does this suggest about the possible advantages and disadvantages of an open economy?

Intraindustry Trade in the Ricardian Economy

In recent years, a number of international economists have made much of the fact that much of world trade is "intraindustry" trade -- that is, trade is in goods of similar factor intensity. As we will see later, they have argued that we can account for such trade only if we turn to increasing returns to scale and imperfect competition. However, they ignored the very simplest model -- that of Ricardo. Since there is only one factor, goods necessarily have identical factor "intensity." That is, the simple Ricardian model is the paradigmatic model of competitive intraindustry trade. As we will see later, this can be generalized to a world with more goods and factors. The important point is that when there are goods that are excellent substitutes in production, then technical differences matter. If these lead to strong specialization, then similar countries may engage in intraindustry trade.

Ricardian World with International Labor Mobility

It is left ambiguous whether the Ricardian technological differences are due to differences in the workers themselves or other technical conditions. Assuming the latter, perfect international labor mobility will insure that under conditions of competition, the site of production will be determined by <u>absolute advantage</u>. If one country has an absolute advantage in both goods, the other country ceases to exist. Assume $a_{LX} < a_{LX}^*$ but $a_{LY} > a_{LY}^*$. Then all X is produced at home and all Y is produced abroad. The new world PPF lies outside the old world PPF, touching it only at the point of initial complete specialization. The slope is $-a_{LX}/a_{LY}^*$.

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IV. The Heckscher-Ohlin-Samuelson Model: The Factor Proportions Theory

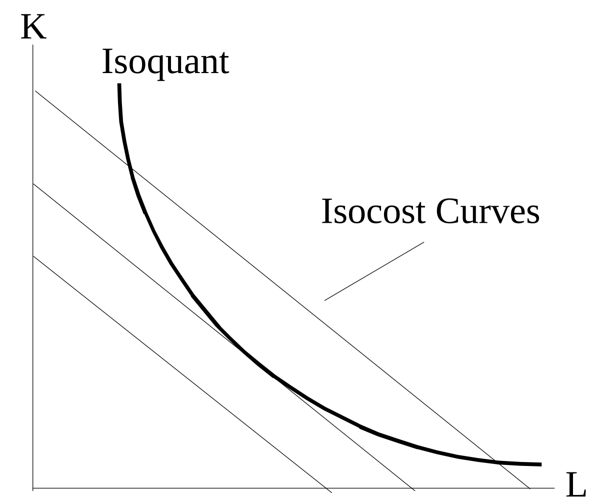
In this chapter we will develop the factor proportions theory of international trade. Our job of analyzing trade here will be easier if we first review a few microeconomic principles and develop a few useful diagrams that illustrate these.

Isoquants, Isocost Lines and the Lerner Diagram

In developing our intuition regarding the determinants of patterns of international trade, it is useful to have some basic diagrams that help us to think about the implications of changes in the economic environment. One basic building block is the Lerner diagram. To derive it, though, we must begin by reviewing the meaning of an isoquant.

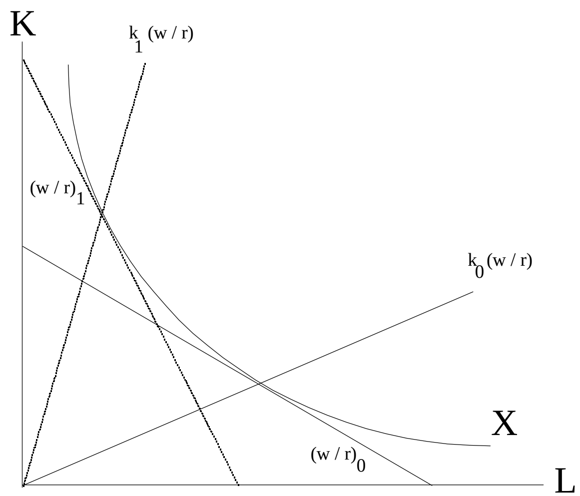
It may be worthwhile recalling that the term "iso-" means essentially "constant." So an isoquant is a line along which the quantity produced is constant. The isoquant for a good X is drawn with the axes given by the factor inputs used to produce that good. In our work, we will have two factors, capital and labor, and so these will correspond to our axes. If we draw a unit isoquant of X, we will show the combinations of K and L that are just capable of producing one unit of X. Note that in general as we use more labor, we can use less capital, but each time that we do this the amount of labor that we have to add increases as we have less and less capital, yet keep producing one unit of X. This gives the isoquant the familiar "bowed-in" shape, convex to the origin. Since the MRTS (marginal rate of technical substitution) is the slope of the isoquant, we can see that this corresponds to diminishing MRTS as labor is substituted for capital. [see figure]

Given that a unit of output could be produced with any of the different combinations of K and L along the isoquant curve, the producer is faced with the problem of choosing which to use. The obvious answer is to choose the one that produces the given output at least cost. From the perspective of the individual producer, the factor price ratio, w/r, is beyond her control. Thus, for any given factor price ratio we can draw a line in capital/labor space with slope -w/r that is an isocost line. That is, every combination of K and L on that line will be such that (w L + r K) equals the *same* constant number.[see figure] Obviously lines with the same slope but further from the origin represent higher levels of costs. [By now you should see the symmetry between these isocost lines and the consumer's typical budget line, or "iso-expenditure" line.] Thus the way to produce it at least cost is to pick the point of tangency between the isoquant and the isocost line for the given w/r. This point of tangency gives the optimal capital to labor ratio for the given w/r.[see figure]



Moreover, very significantly, we can note that as w/r rises (the isocost curves become steeper) the optimal capital to labor mix will rise.[see figure] This should not be surprising--as the cost of labor (the wage) rises relative to capital (the rental cost) we should substitute toward the factor that has become relatively cheaper (capital here). This establishes the fact that, say for good X, the optimal k_X will be monotonically increasing in w/r.

Geometrically this corresponds to a steeper line from the origin to the point of tangency.[see figure] Moreover, the fact that we are working with a constant returns to scale production function means that the optimal capital to labor mix depends *only* on the factor price ratio -- *not* on the scale of production.[see appendix on CRS production functions]

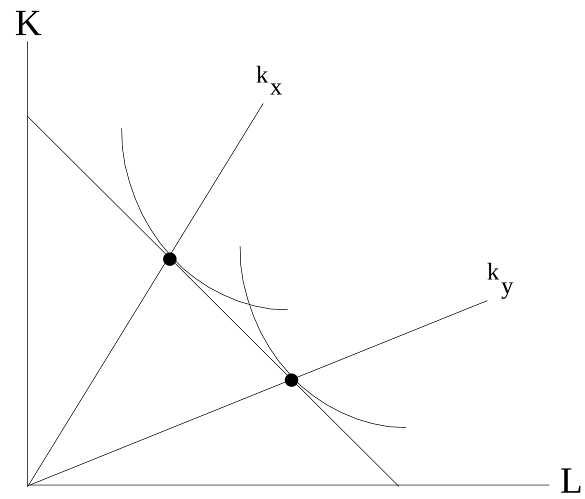


At any given w/r there is no reason to expect that the optimal capital to labor ratio in the X sector will be the same as that in the Y sector. We will frequently be interested in a special case. In this case, for *all* w/r the optimal capital to labor ratio is higher in the X sector than in the Y sector.[see figure] This is referred to as the *strong factor intensity assumption*. That is, no matter what w/r is, $k_X(w/r) > k_Y(w/r)$ [where these indicate functional dependence].

The Product Mix Choice

There is nothing to prevent us from writing the unit isoquants of goods X and Y in the same diagram, since they both use capital and labor. However, unless their production functions are identical, there is no reason to expect them to coincide. All that we have required of them so far is the regularity condition that they be convex to the origin.

We could as easily have defined a two-unit or three-unit, etc., isoquant and drawn it in the same space. We will define a special-unit isoquant for good Y which will be useful in our analysis. Let the units in which good X is measured be given, and draw the corresponding unit isoquant. Now, let the domestic price be given by the international price ratio P (we are a small country). Recall that P tells us how much of good Y we are willing to give up at the margin for one unit good X. Now draw the P-unit isoquant for good Y. Notice now that we have isoquants drawn where the quantities are chosen so that they are of equivalent value in the market (P being given exogenously). This is known as the *Lerner diagram*. [see figure]



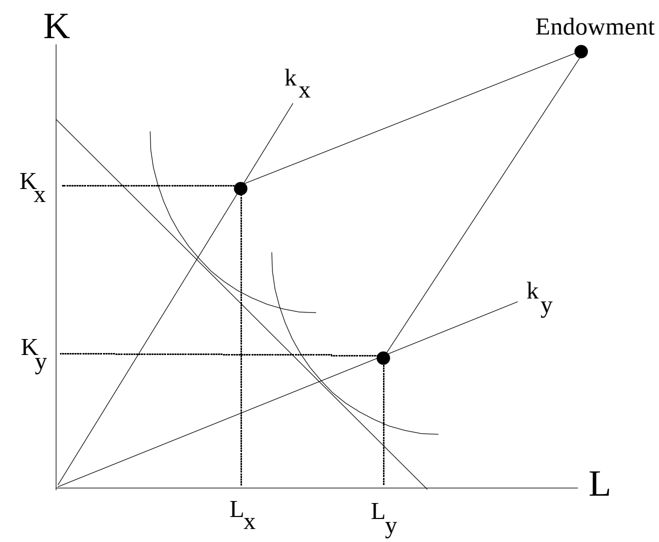
The Lerner Diagram

We turn now to the producer's problem. Given that P units of Y trade for one X in the market, which good should she produce? The answer is that if both bundles have the same value in the market, choose the one that costs less to produce. But how does she find this? To answer this, we have to turn again to our isocost curve. The problem is solved by finding the isoquant (with quantities as described before) which is tangent to the lowest isocost curve.[see figure] Why? Because this allows her to earn the given return (to either one X or P units of Y) at the least cost. We can further see the conditions under which producers in this small country would be willing to produce both goods--namely that the w/r ratio is such that both isoquants are tangent to the same minimum isocost line. [see figure] Otherwise producers would shift away from producing the good tangent to a higher isocost line. Usually we will be working in circumstances where both goods are produced.

We will begin, then, by assuming that both goods are produced (later we will show the conditions under which this will hold). If price is given exogenously, then this fixes the isoquants in our Lerner diagram. This then also tells us what the w/r ratio is and tells us the K/L ratio in each sector. We have assumed that X is the capital intensive good, so for any w/r, $k_X > k_Y$. Moreover, for a given endowment factor ratio $k \in (k_Y, k_X)$ there is a unique $L_X/L \equiv \lambda \in (0, 1)$ that solves $k = \lambda k_X + (1 - \lambda) k_Y$. This implies that we also know the output proportions, and given the actual levels of K and L, the absolute levels in which each good is produced.

We have not yet specified the condition under which both goods will actually be produced. To do this we need to define the *cone of diversification*. As before, an exogenously given price fixes the isoquants in the Lerner diagram. There is then a unique factor price ratio under which both goods will be produced, call it $(w/r)_0$. Corresponding to this, for given technology, is an optimal factor input ratio for each good (viz. k_{X0} and k_{Y0}). We will say that our economy is in the diversification cone when the aggregate capital to labor ratio is $k \in (k_{Y0}, k_{X0})$ where the latter are defined on the basis of $(w/r)_0$. [see figure]

Suppose that k is in the diversification cone, but $(w/r) > (w/r)_0$.[see figure] As is evident graphically and analytically, with this factor price ratio, only X will be produced (since only it is tangent to the lowest isocost line), and it will be produced at a capital to labor ratio $k_{X1} > k_{X0}$. But note that $k_{X1} > k_{X0} > k$. If the X producer seeks to operate at this k_X, the economy will run out of capital before all labor is employed. The assumption of perfect competition in the factor markets then assures that w/r must fall. Why? Firms seeking to expand output find that with capital fully employed, they can hire capital away from other firms only by offering a higher rental; however there is an excess supply of labor, so wages are bid down. Thus w/r must be falling. As this occurs, the optimal k_X must also be falling. For precisely the same reasons, this process cannot stop before w/r falls to $(w/r)_0$. Note that if w/r fell any more, suddenly all production would shift to the Y good (which would now alone be tangent to the lowest isocost line). However, an exactly symmetric argument insures that w/r would again be driven to the level $(w/r)_0$. But then at this factor price ratio producers are indifferent to producing either X or Y (since both yield same revenue at equal costs). The only solution to the output mix, then, is when the proportions are such that we have full employment of both factors. Summing up, for an exogenously given price, we can define a cone of diversification. If the aggregate capital to labor ratio falls in this cone of diversification, then there is a unique factor price ratio, capital to labor ratio in each sector, and output mix that will appear in this country.[see figure]



Allocation in The Lerner Diagram

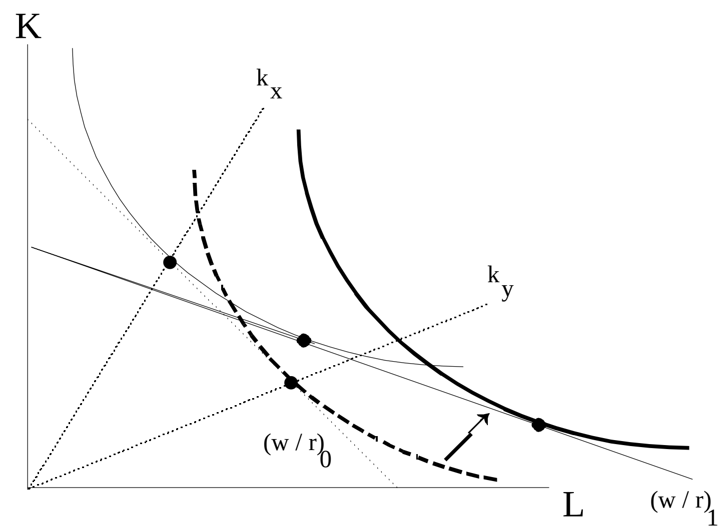
It is worth noting that the factor price ratio and optimal factor input ratios, for a given price, only required that k be in the cone of diversification. The surprising implication is that changes in the relative availability of capital and labor in the economy--due to population growth or capital accumulation--have *no* affect on the wages or rentals paid to these factors. For a given price, the only effect is to change the composition of output.

Outside the Cone of Diversification

Thus far, we have worked under the assumption that k was in the cone of diversification. What if it was not? Since the cases are symmetric, we will consider the case where $k > k_{X0}$. Consider first, what would happen if $w/r = w/r_0$. At this factor price ratio, producers would be indifferent between producing good X and good Y. However since both k_{Y0}, $k_{X0} < k$ we know that as they hire factors and expand output, they will run into a labor shortage before all of the capital is employed. This would be true even if only good X, assumed the more capital intensive, were to be produced. The implication, symmetrically to our previous arguments, is that here the w/r would have to rise. When this happened, we would immediately stop producing any of good Y (since only the X isoquant in the Lerner diagram would be tangent to the lowest isocost curve). The factor price ratio will continue to rise up to the point that the optimal k_X ratio exactly equals the k for the economy as a whole (this is a necessary condition for full employment of factors). Only good X is produced. Thus, if the economy's factor endowment ratio lies outside of the diversification cone, we will produce only that good that uses intensively the factor which is relatively abundant.

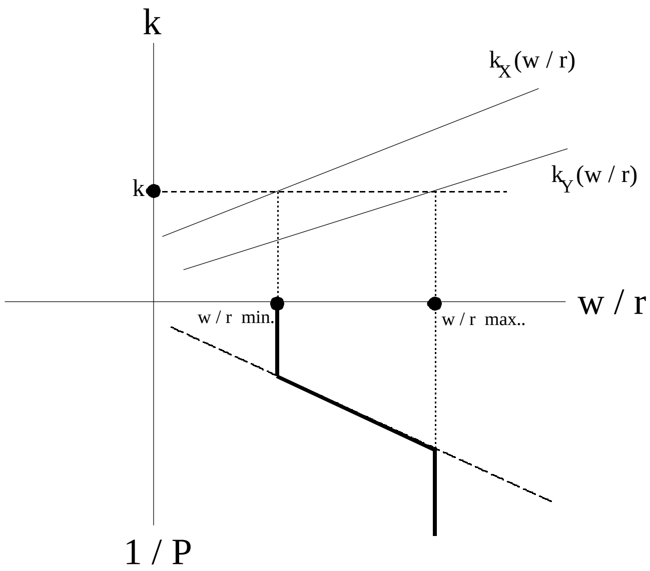
A Change in the World Price

By now we have gained sufficient facility in working with the Lerner diagram that we should be easily able to see the implications of a change in the world price level on factor prices and output levels. Recall that the quantities were selected so that they represented equivalent values for a given world price. This required that one X exchange for P units of Y. If P were to rise, then, one unit of X would now trade for more units of Y. In the Lerner diagram, this must correspond to a new isoquant for Y that is further from the origin. Assuming that we remain within the diversification cone, this must establish a new lower w/r.[see figure] For now we will be content with the observation that P has an inverse relation to w/r (this depending, of course, on our strong factor intensity assumption). Note that as P rises and w/r falls, k_X (as well as k_Y) is falling. If P were to continue rising, there is some point at which k_X would fall to equal k. At this point only X would be produced. What would happen to w/r if P continued to rise? Nothing. The w/r ratio would continue to be fixed at this level that we can call $(w/r)_{min}$.[see figure] The reason is that it is no longer possible to adjust the factor proportions (these are given by aggregate k), and so the marginal productivities cannot change, so neither can the competitive factor price ratio. Symmetrically, if the price were falling, we would establish some $(w/r)_{max}$ when we began producing only Y, above which we would not move. Note that these minimum and maximum levels of w/r are *not fixed by the technology alone, but by the aggregate factor proportions actually existing in the economy, given the technology*.



Goods Prices Affect Factor Prices

This wealth of information that relates output prices, endowments, technology, and factor prices can be summarized in a remarkable little graph. Given output prices, technology and endowments, we can derive the corresponding factor price ratios, factor input ratios, and product mix levels. [see figure]

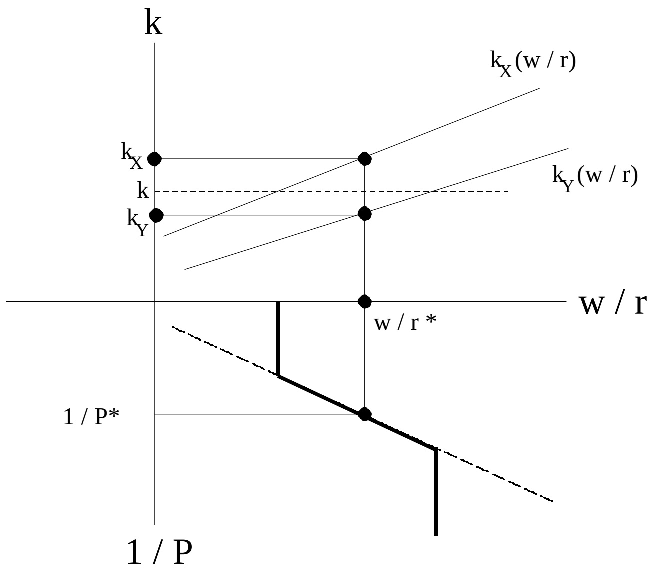


Factor Proportions and Output Proportions

The home economy is endowed with factors in levels K and L. Simple calculations (from the full employment conditions) will show that taking $k \equiv K/L$ as the aggregate capital/labor ratio, $k_X \equiv K_X/L_X$ and $k_Y \equiv K_Y/L_Y$ as the ratios in the X and Y sectors respectively,
and defining $\lambda \equiv L_X/L$ and correspondingly $(1-\lambda) = L_Y/L$, we can arrive at the following result:

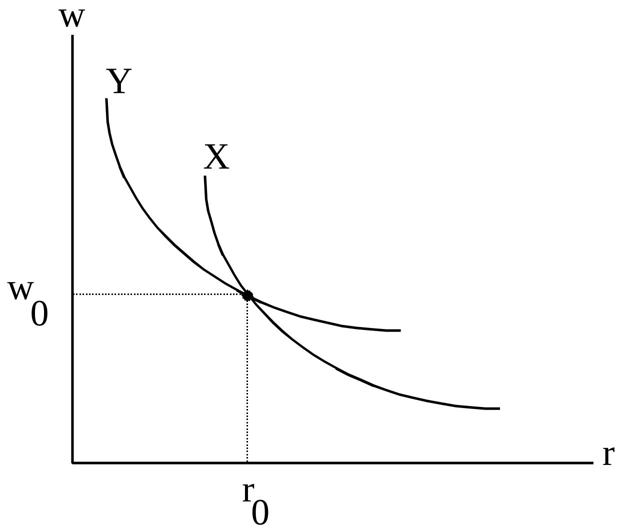
$$k = \lambda k_X + (1-\lambda)\, k_Y$$

This says that (given full employment--which is guaranteed by the competitive labor market assumption) the aggregate capital to labor ratio must be a weighted sum of the ratios in the two sectors, with the weights given by the share of the labor force employed in each sector.



The Mussa Factor Price Frontier Diagram

One additional diagram will he helpful in deriving the principal theorems in the Heckscher-Ohlin framework. This is the Mussa diagram. The competitive cost conditions insure that in equilibrium no good can have a price above its cost of production.. If it did, the firm could set output to infinity and have unbounded profit. Of course, the price could be less than the cost, but then the good would not be produced. If both goods are produced under constant returns, then price equals unit cost. We can represent this in a diagram whose axes are the prices of factors. The price equals unit cost curves are shaped similarly to indifference curves. For fixed prices, they give the highest combinations of rentals and wages consistent with production of the good.



Mussa Diagram

With w on the vertical axis, and r on the horizontal axis, the slope at a point is minus the capital to labor ratio used in that sector at those factor prices. (Note the symmetry with the case of the slope of an isoquant.) Thus, as depicted in the diagram, X is the capital intensive sector. A change in a good's prices would lead to a uniform shift of the respective curve, reflecting the possibility of paying higher returns consistent with the price equals unit cost condition. Note that when our concerns directly concern factor prices, this may be an interesting diagram in which to consider a problem.

The Heckscher-Ohlin (Factor Endowments) Model: Major Theorems

The Ricardian model allowed us to articulate an important potential determinant of trade: comparative advantage. That is, a country will export goods which it can produce relatively efficiently. But what accounts for the pattern of comparative advantage? In the Ricardian model it is differences in the technology available to each country. But what if the technology available was the same for all countries. Would there be any reason to trade? Is there any basis for comparative advantage?

The Heckscher-Ohlin model answers "yes" to both questions. It assumes a world in which countries are identical in every respect except one: they have different endowments of factors, i.e. of labor and capital. They argue that it is the factor endowments that give rise to the observed pattern of comparative advantage, so explain the actual pattern of trade.

There are four principal theorems associated with the Heckscher Ohlin model. They are:

1. *Heckscher Ohlin Theorem*: A country has a production bias towards, hence will tend to export, that good which uses intensively the factor which is relatively abundant in that country.

2. *Factor Price Equalization Theorem*: Under certain conditions, trade in goods alone is sufficient to replicate the outcome in a world in which both goods and factors move freely. One consequence is that under these conditions, the real returns to factors will be equalized.

3. *Rybczynski Theorem*: At constant relative goods prices, a rise in the endowment of one factor will lead to a more than proportional expansion of the output in the sector which uses that factor intensively, and an absolute decline of the output of the other good.

4. *Stolper-Samuelson Theorem*: A rise in the relative price of a good will lead to a more than proportional rise in the return of the factor used intensively in the favored sector, and a decline in the return to the other factor.

1. Prove Rybczynski

Given P, factor prices and factor input ratios are fixed (if both are to be produced). From $k = \lambda k_X + (1 - \lambda)k_Y$, we know $dk = (k_X - k_Y)\, d\lambda$. Since by assumption X is the capital intensive sector, and $dk > 0$, it follows that $d\lambda > 0$. That is, the share of the labor force in the X-sector increased. Since the total labor force is unchanged, that implies the share of the labor force in the Y sector has fallen. But recall that the capital to labor ratios are unchanged in each sector (since prices are assumed fixed). Thus if the labor in the Y-sector has fallen, so must it have lost capital to the X sector. Since it lost both capital and labor (in equal proportions), its output must have declined (again proportionately, since we have constant returns to scale). Thus, the growth in the capital use in the X sector must be greater than the aggregate addition to capital (dk). Moreover, since factor proportions don't change, it must have drawn enough labor from the Y sector to match the proportional growth of capital. Hence output in the X sector (by constant returns to scale) must have grown more than proportionally to the rise in aggregate capital. QED

2. Prove the Heckscher Ohlin Theorem

Imagine two countries, A and B, which are identical in every respect, except their aggregate factor endowment ratio $k^A > k^B$. In country B in autarky, there is some P^B that will prevail. The fact that this is autarky equilibrium means that producers and consumers must be maximizing profits and utility respectively, and supply equals demand in each market. What would happen in country A if it were faced with prices P^B? Since the consumers are assumed to have homogeneous homothetic preferences both within and between countries, it follows that consumers in A will demand goods X and Y in the same proportion as in country B. However, a simple application of the Rybczynski Theorem will show that this cannot be an equilibrium. Since $k^A > k^B$, we may interpret this as if we were looking at the conditions of the Rybczynski Theorem, with $dk > 0$ as we contemplate country A as opposed to B. As we recall, Rybczynski asserted this would lead to an absolute rise in the production of the capital intensive good (X here) and a fall in the production of the other. Thus country A produces proportionately more X and less Y than country B at prices P^B. But recall that A's consumers demanded X and Y in the same proportion as in country B. Thus if this cleared both markets in B, it must correspond to an excess supply of X and excess demand for Y in A. The only way to clear both markets in A in autarky is for there to be a fall in the relative price of X to $P' < P^B$. This establishes that the capital intensive country has a lower autarky relative price of the capital intensive good (X). Hence when we open to trade, it will tend to export that good. QED.

3. Prove the Factor Price Equalization Theorem

Free trade establishes a common international price. Both countries have identical constant returns to scale technologies. We have shown that given a goods price ratio, there is a unique factor ratio that will be used to produce each good (technically, we require that the Lerner isoquants not intersect twice -- a condition referred to as "no factor intensity reversals"). Thus, if both countries produce both goods, it will be done not only with access to the same technology, but with the same technique, characterized by the factor intensity, in each good across countries. But with a constant returns to scale technology, factor productivity is independent of scale, and to given common technique across countries must be the same. But with perfect competition, factors are paid their marginal product. But these are, as we have already shown, the same across countries. It follows that factor prices must be equal. QED.

4. Prove the Stolper-Samuelson Theorem

Consider a rise in the relative price P_X / P_Y. This can be depicted as an outward shift of the X curve in the Mussa diagram. As we see, wages fall and rentals rise. The result, though, is stronger yet. Rentals rise more than proportionally than the rise in the relative goods price (you can verify this diagramatically in the Mussa diagram or algebraically from the price equals unit cost conditions).

Heckscher-Ohlin in the Integrated Equilibrium

There is another way to think about the Heckscher-Ohlin model, in a framework due to Samuelson, and Dixit and Norman. This is the framework known as the "integrated equilibrium."

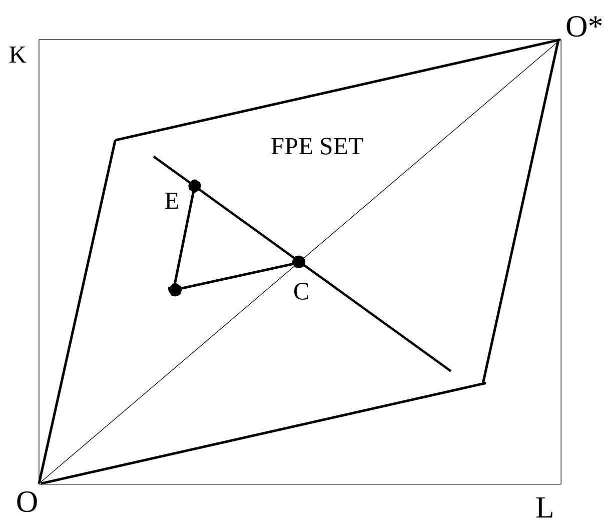
When Samuelson first established the factor price equalization theorem, many people found it hard to believe that trade not only reduced, but completely eliminated, the differences in factor prices. So he sought a way to explain it that would have strong intuitive appeal. Imagine, he suggested, that initially we live in a world with barriers neither to the movement of goods, nor of factors. Since factors can move, their prices must be equalized. Now imagine that an angel came down and divided the factors of production so as to create separate nations. When would this new world, in which factors cannot move across national boundaries, simply replicate the equilibrium that existed in the fully integrated world economy?

A few requirements can immediately be noted. First, to end up with the same equilibrium, we must have the same aggregate output. So we must be able to divide this aggregate output among the countries. Second, if they are to produce this output, then they must use factor inputs for each good, and in each country, exactly as they were employed in the integrated world (remember production was efficient in that world). Third, while using these techniques, each country must be able to fully employ its factors. If the above conditions hold, then we know that factor prices, so income to the world is unchanged. With identical, homothetic preferences, as before, the distribution of world income does not affect the pattern of demand for the world as a whole. Thus demand will equal supply, and the equilibrium is complete.

One nice feature of this framework is that it allows a simple graphical depiction of trade that emphasizes that trade is the implicit exchange of the services of factors -- goods being a way of embodying these factor services.

Let the FPE set be defined to be the set of distributions of the world endowments across the countries consistent with replicating the integrated equilibrium. In the 2x2x2 case, this is a parallelogram in factor space. The slopes of the sides reflect the factor intensities used in the X and Y sectors, and their length reflects the overall factor usage in the respective sectors.

Trade in this setting is the export of factors in the proportion used in the exportable sector, and the import of factors in the proportion used in the importable sectors. This must exactly bridge the difference between the factor content of production and the factor content of consumption. The former, with full employment, is simply the endowment. The latter is proportional to the world endowment (so on the diagonal of the factor box). If trade is balanced, it is also constrained to be on the budget (or "isoincome") line with a negative slope equal to − w/r that passes through the endowment.



V. The Specific Factors (Ricardo-Viner) Model

We developed the Heckscher-Ohlin model with two factors of production –– capital and labor. We assumed that the factors were freely mobile between sectors and would move in response to any differential rewards. However it is not necessarily the case that factors can always move from one sector to another. Capital equipment that today is used to produce jello cannot tomorrow be shifted to produce airplane engines. Perhaps over time we can allow one to wear down and instead of replacing it shift new investment toward the other sector. Nonetheless, in the short run this may be completely infeasible. There may also be factors, such as land, which even in the long run cannot be shifted to a different productive sector, say computer manufacture. Even labor, which we generally consider to be relatively mobile, may not move very quickly even in the face of substantial wage differences.

All of this suggests the potential usefulness of looking at a model that allows for the possibility of some factors being sector-specific, what we will call the *specific factors model*. The assumption here is that there are three factors of production –– labor, which is mobile between sectors, and two forms of sector-specific capital which are wholly immobile. As discussed below, this might be interpreted as a short-run version of the Heckscher-Ohlin model, or if factors truly are specialized, even as a long term model.

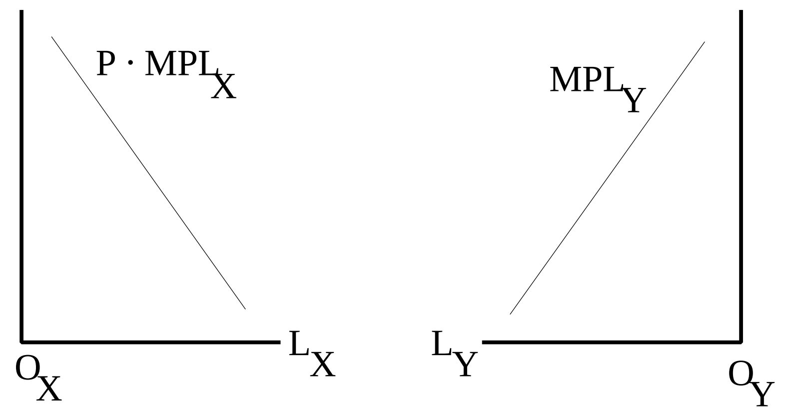
Output is produced again with a constant returns to scale production function, which also shows diminishing returns to labor taken alone. The fact that capital is immobile between sectors in no way affects the competitive conditions for capital *within* a sector. This condition guarantees that, with the CRS technology, capital (in each sector) receives its marginal product. Of course, the return to capital need not be the same across sectors, since it cannot move to take advantage of any differential. Labor, of course, is mobile between sectors, so will receive the same wage in each, corresponding to its marginal product.

Consider the problem of the producer. The only factor choice to be made is the level of labor to be employed. Formally:

$$\text{Choose L to} \quad \text{Max } \pi_y = P_Y Y(K_Y, L) - w L$$

$$\text{FOC} \quad P_Y \partial Y/\partial L = w$$

This gives the standard labor demand (marginal product of labor) curve (see figure). Thus for a given wage, $w = w_0$, the Y producer would choose to continue hiring labor up to the point where $P_Y \partial Y/\partial L = w_0$.



Value Marginal Product Curves

Of course, we get a symmetric optimality condition in the X sector requiring that $P_X \partial X/\partial L = w$. It will be convenient to write this out in a peculiar graph where L is increasing to the left. (see figure).

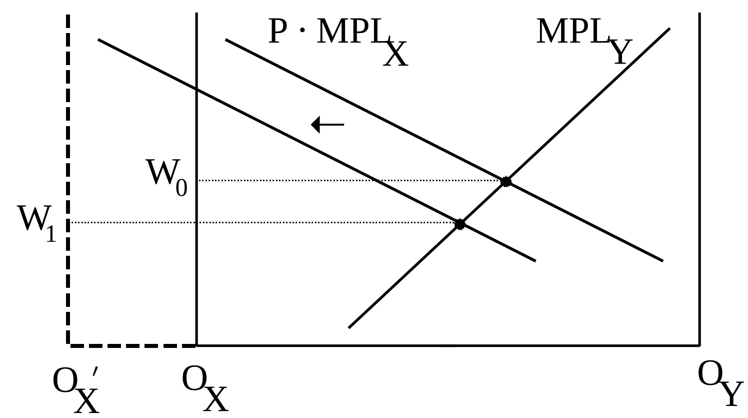
Now we want to combine the two graphs to determine the wage and how much labor is employed in each sector. Let the horizontal base of our graph (see figure) be equal to the economy's endowment of labor. Then letting labor employed in Y start from the left and labor employed in X start from the right, we can draw the first two figures together. Since competition implies that the wage will be the same between sectors, this implies that in equilibrium with both goods being produced, we must have $P_Y \partial Y/\partial L = P_X \partial X/\partial L$. This can be shown graphically as the intersection of the two curves.

It is often emphasized that an important reason for interest in the specific factors model is the insight that it provides into possible coalitions over trade and other policies given disturbances to the economy. (See, for example, Caves, Jones and Frankel). We will look at several comparative static experiments.

First let us look at what happens when there is a change in the factor endowment of the economy. Recall that in the Heckscher-Ohlin model, for a small open economy, small changes in factor endowments led to relatively large changes in the output mix, but remarkably had no effect at all on factor prices (so long as we stayed within the diversification cone). Here we will find that factor prices will change.

Expansion of Mobile Factor

Consider an increase in the availability of labor. Graphically this corresponds to an extension of the horizontal axis measuring the fixed amount of labor. Note that the curve $P_Y\ \partial Y/\partial L$ shifts right (see figure), but only because the origin has shifted for it. It really occupies the same position in its own (w,L) space. It is clear that the competitive conditions lead to an excess supply of labor at the initial wage, leading wages to fall and employment to rise in both sectors, until the marginal products (and so wages) in each sector had been driven to equality at a new lower point where the new labor was fully employed (see figure). The fact that more labor is employed with the original capital in each sector implies that (at original prices) the real return to both immobile factors increased. One can easily use this model to make predictions about the sentiments of laborers versus capital owners to immigration.

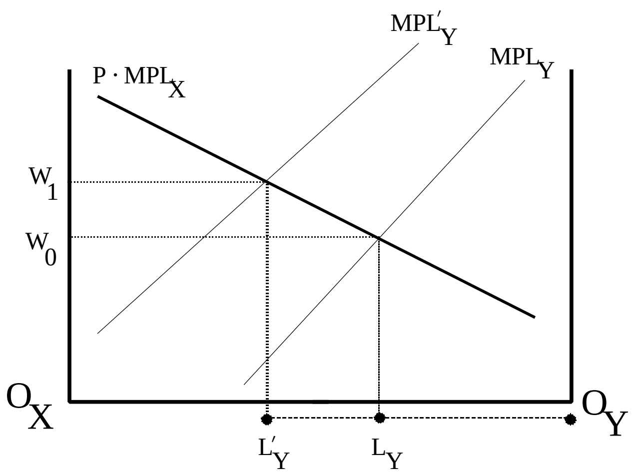


Increase in the Mobile Factor

Expansion of Specific Factor

Now consider an increase in the specific capital, say in the Y sector. With more capital available, the marginal product of labor is higher at each level of employment in that sector. Graphically, this corresponds to an upward shift in the marginal product (labor demand) curve (see figure).

Clearly, the result of this is to initially cause an excess demand for labor, and so cause wages to rise. Employment will rise in the sector with increased capital and fall in the other.



Expansion of a Specific Factor

What about the factor payments to each of the specific forms of capital? Recall that with constant returns to scale and perfect competition factor rewards must exhaust output. Thus a rise in the wage (with prices constant) must correspond to a fall in the reward to *both* specific factors. Perhaps it is not surprising that the rate of return to the sector whose capital is becoming more plentiful should fall, but why should the other fall? As C\J\F note, the other sector reaps no benefit from the expansion of capital of the first. Since wages rise and prices are fixed, the owners of capital in the other sector are squeezed, and so returns must fall. Suppose here that the X sector is agriculture and the Y sector is industry. How would we expect landlords to look on an expanding industrial sector? Suppose that all earnings on industrial capital are used to expand the capital stock in that area; what will happen to earnings over time, thus to future expansion opportunities?

Change in Relative Prices

Recall that our condition for equilibrium was $P_X\,\partial X/\partial L = P_Y \partial Y/\partial L = w$. A rise in P_X thus leads to an upward shift of the labor demand curve associated with the X sector, proportional to the change in price (see figure). What happens to factor returns? Graphically it is clear that the wage will rise, although less than proportionally to the change in price. Thus signing the welfare change is tricky. Note w rose less than the proportional rise in P_X, so fell in units of X. Whether those who depend only on wage income are better or worse off depends on whether they consume X (worse off) or Y (better off). Note that moving from an autarky position to free trade will change relative prices. How should labor (assumed the mobile factor here) respond to the proposition of free trade differently if the anticipated export is a staple item versus a luxury?

We might suppose that with more people now employed in the X sector with a fixed amount of specific capital that the return to capital must have risen. This is correct. Another way of looking at it is to recall the magnification effect, which holds that changes in factor prices will bound the change in goods prices. Since wages rose by less than the change in prices, it follows that the return to capital in the X sector must have risen by even more than the rate of change in prices. Similarly, the fall in employment in the Y sector, with unchanged specific capital suggests that the return to capital in that sector must have fallen. Again, the magnification effect implies that the return to specific capital in the Y sector fell by more than the fall in the relative price of the Y good.

Note the policy implications of the foregoing. Changes in trading prices have very dramatic effects on the returns to specific factors, and much less dramatic effects on the returns to mobile factors. Since many trade policies in effect shift the relative prices of goods (e.g. tariffs) we may expect that they could lead to more heated controversy when we are considering sectors with specific factors. If labor is relatively more mobile, the gain or loss will depend importantly on how trade affects the prices of the goods which figure more importantly in the laborer's consumption basket.

One of the most striking implication of the Heckscher Ohlin model was that commodity trade alone, under the specified conditions, could lead to complete international equalization of factor prices. Should we expect to find this here? As we have seen, even with given prices, small changes in the amount of the mobile factor change factor returns. Thus even in the event that two countries had identical endowments of the specific factors, any small differences in the mobile factor would prevent factor price equalization. So FPE does not hold here. What underlies the failure of FPE here? Recall that in the (2 good) Heckscher Ohlin model, every non-specialized country produced using the same factor intensity (for a given price). The differing factor endowments led to shifts of factors from one sector to another to adjust output such that there would be full employment at those factor intensities. Here two of the factors are not able to move, so it is not generally possible for different countries to work at the same factor intensity in both sectors, hence no FPE.

Now consider a more complex version of the same model. Let there be N sectors, each with its own specific capital. Let there also be one mobile factor, say labor. As before, labor will move such that wages, and so the marginal product of labor is constant across sectors. As before FPE will fail to hold, and for essentially identical reasons. Now consider a case where one of the factors (say labor) is mobile internationally. How will this affect our results regarding factor price equalization? If labor is mobile internationally, it will move to where the wage is highest. Competition then trivially insures that wages will be constant in all sectors of all countries. However, we have not yet shown FPE, since we would also have to show that the return to all specific factors in all countries was the same -- a proposition which on first blush may appear more dubious. However, recall that given identical technologies and constant returns to scale, each wage corresponds to a unique capital to labor ratio in a specific sector. The implication is that each good is produced under identical conditions in each country where it is produced (in regard to factor intensity). But, equally, if the capital to labor ratio, sector by sector, is the same across countries, then the returns to each specific form of capital must also be the same. That is, we get a form of FPE, taking each distinct form of specific capital as a different factor. As it turns out, this result does <u>not</u> depend on which factor is internationally mobile. So long as any factor is internationally mobile, we get the same result.

Now consider the effect of technical progress (say in the X sector) that increases labor productivity in a small specific factor economy. For fixed output prices, the technical progress shifts the labor demand curve upward (see figure). This will increase employment in the progressing sector, unambiguously raising the real return to labor and lowering the real return to the other specific factor.

One of the most interesting aspects of the specific factors model is the clear delineation of political alliances one would expect. When there are *changes in endowments*, the interests of specific capital move together, and are opposed to those of the sectorally mobile factor. When there are *changes in relative prices*, the interests of owners of specific capital are very sharply opposed, and the interest of the sectorally mobile factor is relatively unaffected.

VI. Policy in the Perfectly Competitive World

We turn now to trade policy in the perfectly competitive framework. We begin with an approach that emphasizes that policy properly works on the pareto marginal conditions for efficiency in the competitive framework. This allows a very simple statement of principles that has remarkably varied yet intuitive application. We turn next to develop a graphic approach to trade policy for the small country, showing both free and tariff-distorted trade. We then turn to policy for a large country -- i.e. one that affects world prices. We detour to develop some tools of comparative statics and welfare analysis. We then develop an example of import substitution that indicates why free trade is not going to be nationally optimal for a large country. We then develop a graphic approach, based on the offer curve, that illustrates optimal tariff policy. The optimal tariff in a two-good world is then derived formally, and related to foreign elasticities of import demand and export supply. Finally, we extend this to the world with many goods.

A. <u>The Pareto Conditions</u>

Let DRT be the domestic (marginal) rate of transformation.
Let DRS be the domestic (marginal) rate of substituion.
Let FRT be the foreign -- i.e. via trade -- (marginal) rate of transformation.

Then the pareto conditions for an open competitive economy are:

1. DRS = DRT = FRT
2. Production on the PPF

These conditions for pareto optimality in the competitive trading economy also may be used to indicate the potential problems to which policy may be properly addressed.

1. DRS = DRT ≠ FRT

This arises in two principal cases. The first is when the country is large, and so has some monopoly power in world markets (even though all firms are competitive). Now the country affects international prices, so P* does not represent the true opportunity cost of the marginal unit traded. So free trade, which allows DRS = P = DRT = P* ≠ FRT. Restoration of optimality (from the national perspective) requires an optimal tariff, which we will derive shortly.

The other case in which this occurs is when the small economy imposes a tariff. In this case, P* truly is the opportunity cost of the marginal unit traded, since the country does not affect world prices. Thus the imposition of a tariff leads to DRS = P = DRT ≠ P* = FRT. The first best policy here is simply to remove the tariff.

2. DRS = FRT ≠ DRT

The first equality would arise, for example, in a small country or a large country with an optimal tariff. The inequality might arise if there is an externality associated with the level of output of one of the goods. The first best policy will be to implement a production tax cum subsidy on the output of one of the goods so that producers perceive relative prices according to the true marginal impact of their choice.

3. FRT = DRT ≠ DRS

The equality would arise in a small economy with free trade or a large economy with an optimal tariff. The inequality might arise due to an externality in consumption. The first best policy will be to implement a tax cum subsidy on consumption so that consumers perceive the true marginal impact of their decisions when making choices.

4. Not on the PPF

This arises for two general reasons. First, there could be unemployment. This could arise as in efficiency wage models, as a result of an economywide minimum wage that exceeds the market clearing wage, or for a variety of other reasons. Second, there could be a failure of equivalence of MRTS across sectors. This could arise if there is an externality associated with the use of a factor in one sector. In the case of unemployment, the specific policy may depend on the reason for the unemployment. If it is due to an economywide minimum wage, the first best solution is a wage subsidy equal to the difference between the minimum wage and the market clearing wage. If we are off the PPF because of a factor externality, the first best solution is to implement a factor tax cum subsidy that leads producers to face the true costs of marginal changes in factor employment.

B. Implications for Policy, and Policy Equivalences

Several important principles emerge from the analysis. The first -- the importance of which is virtually impossible to overemphasize -- is that optimal policy addresses the source of the divergence or distortion. If the source is a trade distortion -- the failure to exploit market power by a large country -- the optimal policy will be one addressed to that market (here an optimal tariff).

A corollary is that if the distortion in the economy is based on domestic considerations, trade policy will not be the optimal instrument to correct it. Thus, a positive production externality in the importable sector would call for a production subsidy in that sector, or a production tax in the other sector. Imposition of a tariff to favor that sector would add a gratuitous consumption cost to the desired incentive for production (remember that the tariff is equivalent to a combined production subsidy and consumption tax). The crucial point is that domestic distortions call for domestic policies to correct them -- trade policies cannot be first best.

Trade policies may, in some cases of domestic distortion, be welfare improving when the first best policy is unavailable. However, the question of second best policies must be studied issue by issue. The reason is based on the theory of the second best: the relaxation of one distortion in an economy with more than one distortion need not raise welfare.

A complementary result to the idea of targetting derives from the work of Tinbergen and others: We will need as many instruments as there are targets. Hence if there are two distortions, we need two instruments to implement the optimal policy. For example, a large country with a distortion in the factor market requires both a tariff and a factor tax cum subsidy for optimality.

Finally, this framework also, and less obviously, suggests the least costly way to implement an objective that springs from essentially non-economic concerns. For example, suppose (for whatever reason) our country wanted to be sure to have a target level of labor in sector Y. One way to increase employment there would be to put a tariff on Y; an alternative would be to give a production subsidy to output in Y; finally, we could give a factor subsidy for employment of labor in Y. The last policy will be the least costly way to achieve the employment target in that sector. The dictum developed for eliminating distortions: go to the source of the distortion, thus extends to how to introduce a certain distortion at least cost.

C. The Small Country: A Graphic Approach

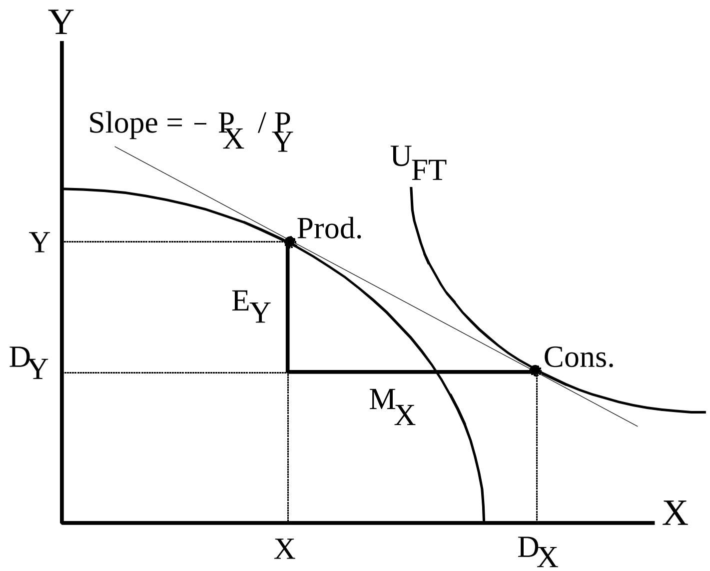
In international trade theory, we refer to a country as small if it cannot affect the international prices that it faces. We begin by pursuing the positive effects of policy for a small country.

First, we need to depict the closed economy and free trade equilibria. By assumption, we are working with a convex technology. Consumers have strictly convex preferences which can be aggregated into a social indifference curve. The equilibrium has certain optimality properties which we note. First, perfect competition in the factor markets insures full employment. Second, cost minimization and the absence of externalities or policy distortions insures that marginal rates of technical substitution are equalized across sectors. In combination, the two insure that we are on the domestic PPF. Optimal production decisions by competitive producers and optimizing consumers insure that the respective rates of substitution are equated to the goods price ratio, which then yields DRT = DRS. These conditions jointly insure the pareto optimality of the closed economy equilibrium.

National income at factor cost is represented by the intersection of the price line tangent to the PPF with either axis. Domestic consumers face a budget constraint:

$$PD_X + D_Y = PX + Y = wL + rK \qquad \text{where } P = P_X/P_Y.$$

Of course, market clearing for the closed economy requires as well that $D_X = X$ and $D_Y = Y$.



Free Trade in the Small Open Economy

We now consider this small economy when it has the opportunity to engage in free trade. With free trade, international prices will prevail in the small economy. Producers choose a point on the PPF tangent to the international price line. Consumers independently choose a consumption bundle. The budget constraint that consumers face now depends on international prices.

$$P^*D_X + D_Y = P^*X + Y = wL + rK$$

Of course, there is no longer a constraint that domestic supply equal demand, but only that there be balanced trade. Rearranging the budget constraint:

$$P^* M_X = P^* (D_X - X) = E_Y = (Y - D_Y)$$

This gives rise to the "trade triangle." While we will focus on the welfare effects of policy later, it is evident that the free trade opportunities dominate autarky, and that this does not depend on any ability to reapportion production.

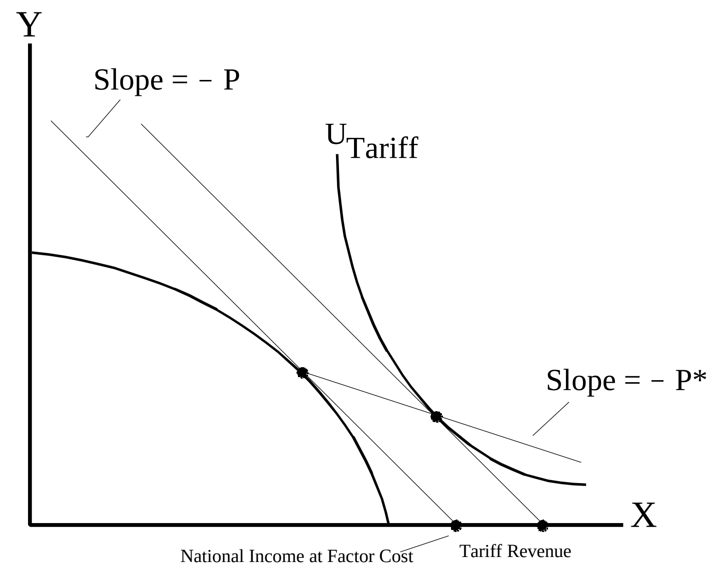
We now consider the positive effect of a tariff on the imported X good by the small country. The tariff drives a wedge between the international and domestic prices. Since world prices are unaffected by actions of the small country, this must serve to raise the price of X to $P_X = (1 + t) P_X^*$. Thus the domestic price ratio P must be higher. The higher price ratio induces additional production of X (up to the point at which the tariff is prohibitive). The budget constraint that consumers face now includes factor income equal to output at domestic market prices, and rebated tariff revenue.

$$PD_X + D_Y = PX + Y + tP^*M$$

Of course, balanced trade constrains the country to consume at world prices only the value of output at these prices:

$$P^*D_X + D_Y = P^*X + Y$$

Optimization by producers and consumers facing the tariff-ridden prices leads to an equalization of the respective rates of transformation and substitution to the price ratio that they face. If both goods are normal, the volume of imports must fall.



Tariff-Ridden Trade in the Small Open Economy

The graph also serves to illustrate a couple of important equivalences. The first is the equivalence of the tariff to a combined production subsidy and consumption tax at the same rate as the tariff. The point is that producers respond to the relative price, irrespective of whether this is due to a tariff or a production subsidy, so a subsidy of the same proportion leads them to make the same output choice. This output choice then generates the same factor incomes as before. Consumers face a consumption tax in the same proportion as the tariff, so face the same relative price. So long as their budget constraint is unchanged, they will act precisely as before.

Their factor income is $wL + rK = PX + Y$. The consumption tax raises revenues equal to t P^*D_X, while the production subsidy is tP^*X. The difference, equal to tP^*M, is rebated lump sum to the consumers. Thus, consumers again face the budget constraint:

$$PD_X + D_Y = PX + Y + tP^*M$$

Thus a tariff may be decomposed into a production subsidy and a production tax at equal rates.

A second equivalence should be clear from the theory of general equilibrium and the geometry of the problem. The choices of producers and consumers depend on the structure of relative prices -- not their absolute level. Thus a tariff on imports of X will be equivalent to a tax on exports at the same rate on Y. The revenue raised will be t E_Y. But by balanced trade, we know that $P^*M_X = E_Y$, so the revenue is tP^*M as before. So producers and consumers face the same relative prices and consumers face the same budget constraint, insuring the same equilibrium. This is known as the Lerner Symmetry theorem.

It is straightforward, then, to illustrate the effects of pursuing just a production subsidy, or just a consumption tax.

While I will not demonstrate this, it is evident that other equivalences could be established: a production subsidy may be looked on as a uniform factor subsidy in one sector; a uniform factor subsidy in one sector is equivalent to a uniform factor tax in the other; a subsidy to the use of labor in all sectors is equivalent to a tax in the same proportion on the use of capital in all sectors. The key, as always, is that the optimal choices depend only on the structure of relative prices.

Quotas in a competitive world

We would like to see how quotas compare with tariffs in their positive effects in the competitive framework. The first thing to note is that quotas that bind create scarcity rents. As we will see, these are the counterparts of tariff revenue. The relative effect of the quota, then, will depend on who receives these rents. Assume the government auctions the quota licenses in a competitive setting and so is able to appropriate the scarcity rents. What will the equilibrium look like? Assume that the quota is set so as to restrict the imports of Y to be no larger than under the tariff. Imagine that the quota is just fulfilled and that domestic supply to the home and foreign markets is unchanged. Then the supplies available are in accord with the tariff-ridden relative price, provided that consumers' income is sufficient for this output. With the same relative goods prices, factor income will be unchanged. The quota rents equal the excess of the domestic price over the foreign price times the quantity imported. But this yields exactly the revenue we had from the tariff, which is then rebated to the consumer. This means that the consumers' income is indeed just sufficient to purchase the initial bundle. In effect, the quota in a competitive setting is exactly equivalent to a tariff that restricts imports equally, provided we dispose of the quota rents in the same manner as the tariff revenue.

D. Policy for the Large Competitive Country

I have talked about the fact that (nationally) optimal policy requires that DRS = DRT = FRT. But what is FRT? An answer to this leads us in the direction of defining the optimal tariff. But before we do this, it is worth looking at the free trade outcome and considering why this might be suboptimal for the large country -- which will then indicate the concerns that policy is trying to balance.

In order to work out this story, we need three results:

1. The Marshall-Lerner Stability condition
2. The effect of parameter changes on the terms of trade
3. An indicator of welfare changes

Marshall-Lerner stability condition

Assume that home imports X and exports Y. By balanced trade:

$$PM_X(P) = M_Y^*(P^{-1})$$

If the world is Walrasian stable, we require that a rise in the relative price of X call forth an excess supply of X, . This requires that (where a ^ represents a proportional change):

$$\frac{\hat{M}_X}{\hat{P}}\ \hat{P} + \hat{P} < \frac{\hat{M}_Y^*}{\hat{P}^{-1}}\ \hat{P}^{-1}$$

$$\frac{\hat{M}_X}{\hat{P}}\ \hat{P} + \hat{P} < -\frac{\hat{M}_Y^*}{\hat{P}^{-1}}\ \hat{P}$$

$$-\epsilon + 1 < \epsilon *$$

$$\Delta \equiv \epsilon + \epsilon^* - 1 > 0$$

This is referred to as the Marshall-Lerner condition, and is a fundamental element of comparative static exercises. It says that Walrasian stability holds if the sum of the import demand elasticities exceeds one.

Parameter Changes and the Terms of Trade

The aim here is to consider the impact on the terms of trade of a shift in a parameter α that may affect the two countries' import demand functions. We begin with the balanced trade condition:

$$P\ M(P,\alpha) = M^*(P^{-1},\alpha)$$

$$\hat{P} + \hat{M} = \hat{M}^*$$

$$\hat{M} = -\epsilon\ \hat{P} + \hat{M}|_P$$

$$\hat{M}^* = -\epsilon^*\ \hat{P}^{-1} + \hat{M}^*|_P = \epsilon^*\ \hat{P} + \hat{M}^*|_P$$

$$\hat{P} - \epsilon\ \hat{P} + \hat{M}|_P = \epsilon^*\ \hat{P} + \hat{M}^*|_P$$

$$\{\epsilon + \epsilon^* - 1\}\ \hat{P} = \hat{M}|_P - \hat{M}^*|_P$$

$$\hat{P} = \frac{\hat{M}|_P - \hat{M}^*|_P}{\Delta} \qquad \textit{where } \Delta > 0 \textit{ by MLC}$$

Note that the less sensitive import demands are to changes in relative prices (Δ small), the more must prices move to clear markets. The numerator reflects the effect of the parameter change on world excess demand for home imports for fixed prices. If this net effect is positive at the initial price, then prices will have to rise to clear the market.

Two Forms for the Budget Constraint

The budget constraint facing the economy is of two forms -- one expressed in terms of international and one in terms of domestic prices. By balanced trade:

$$P^*\ D_X + D_Y = P^*\ X + Y$$

A tariff on X implies:

$$P = (1 + t)\ P^* \quad \text{or} \qquad P^* - P = -\ t\ P^*$$

Adding and subtracting:

$$P^*\ D_X - P\ D_X + P\ D_X + D_Y = P^*\ X - P\ X + P\ X + Y$$
$$P\ D_X + D_Y = -\ t\ P^*\ (X - D_X) + P\ X + Y$$
$$P\ D_X + D_Y = P\ X + Y + t\ P^*\ M_X$$

So the two forms to remember are:

$$P^*\ D_X + D_Y = P^*\ X + Y$$

i.e. demand is constrained to the value of output when evaluated at international prices; and

$$P\,D_X + D_Y = P\,X + Y + t\,P^*\,M_X$$

i.e. demand is constrained to be equal to the value of output plus the tariff revenue when measuring demand and supply in domestic prices.

An Indicator of Welfare Changes

We will also need a characterization of welfare changes. Utility is given as:

$$U = U(D_X,\ D_Y)$$

$$dU = U_{D_X}\ dD_X + U_{D_Y}\ dD_Y$$

$$dR \equiv \frac{dU}{U_{D_Y}} = \frac{U_{D_X}}{U_{D_Y}}\ dD_X + dD_Y = P\ dD_X + dD_Y$$

That is, we can measure welfare changes as a function of its effect on the quantities consumed, where the weight on D_X is given by the relative price (ratio of marginal utilities at the optimum).

Recall that the budget constraint can be written in terms of domestic or international prices. Taking differentials:

$$P\ D_X + D_Y = P\ X + Y + tP{*}M_X$$

$$P\ dD_X + dD_Y = P\ dX + dY + (X - D_X)\ dP + d\{tP{*}M_X\}$$

$$dR = -\ M_X\ dP + d\{tP{*}M_X\}$$

Note that the first term on the right hand side fell out because of competitive producers' optimum choice in production. We will come back to this later when we introduce imperfect competition. The expression says that changes in welfare can be looked at as depending on two factors: an alteration in the domestic price of importables and tariff revenue changes.

Alternatively we can develop this in terms of international prices.

$$P{*}\ D_X + D_Y = P{*}\ X + Y$$

$$+\ dD_Y = P\ dX + dY + (X - D_X)\ dP{*} + (P - P{*})\ (dD_X$$

$$dR = -\ M_X\ dP{*} + (P - P{*})\ dM_X$$

$$dR = -\ M_X\ dP{*} + tP{*}\ dM_X$$

Again, the first term falls out because competitive producers are optimizing. This expression says that we can look at the change in welfare as being composed of two effects: The first is a terms of trade effect, welfare rising with a fall in the international price of our importable. The second is the so-called tariff wedge effect -- tP* is the excess of P over P*, so the excess of consumer or producer valuation of the importable over its true cost to the economy (for fixed prices). A rise in imports, due either to a contraction of production or a rise in consumption of the importable, raises welfare.

E. The Sub-Optimality of Free Trade for the Large Country

We now want to develop intuition regarding the sub-optimality of free trade for the large country. First, note that the solution to the producer's and consumer's problems are such that small shifts in the production or consumption bundle leave their positions unchanged, to the first order, at unchanged prices. So, without loss of generality, we can consider a small shift of production in the direction of import substitution: $dX > 0$ and $dY < 0$ along the transformation curve. Our welfare indicator from above can be written (noting $t = 0$ for free trade):

$$dR = -\ M_X\ dP + \{\ P\ dX + dY\ \}$$

The term in brackets is zero, as described above. We need to use our formula for the effect of parameter changes on the terms of trade to determine dP:

$$dR = -M\ dP = -M\ P\ \hat{P} = -M\ P \left[\frac{\hat{M}|_P - \hat{M}^*|_P}{\Delta} \right]$$

At unchanged relative prices, nothing has happened to foreign import demand. Similarly, we have postulated no change in consumer behavior at unchanged prices (the production bundle also leaves incomes unchanged). The only change is that we have increased production of our importable, causing importables to fall at the proportional rate -dX/M. So the change in welfare is:

$$dR = -M\ P\ \hat{P} = M\ P\ \frac{dX}{M\ \Delta} = P\ \frac{dX}{\Delta} > 0$$

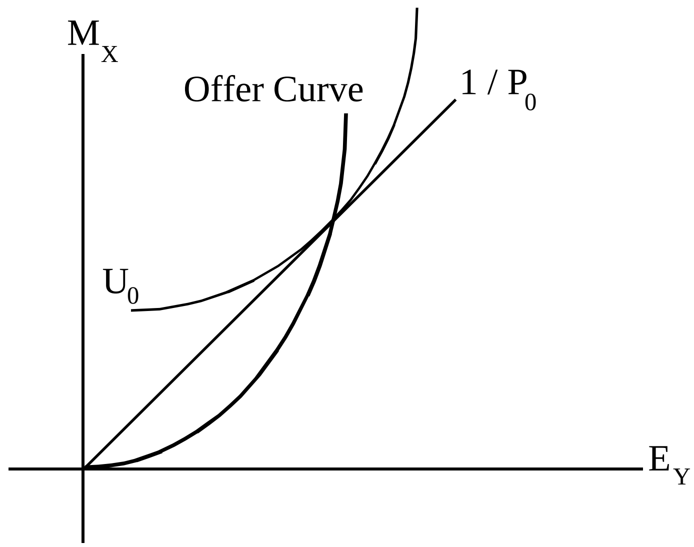
This suggests the possibility of national gains from restricting imports and improving the terms of trade. Here it did not matter whether we cut imports by reducing consumption or increasing production, since the opportunity costs were the same for each. In the general setting, we want to act on both margins. This can be achieved via the device of tariffs on the importable, which tends to shift production toward the importable and consumption toward the exportable, and to maintain the equality of domestic consumer and producer prices, which assures efficiency along this dimension.

F. <u>The Optimal Tariff in Offer Curves</u>

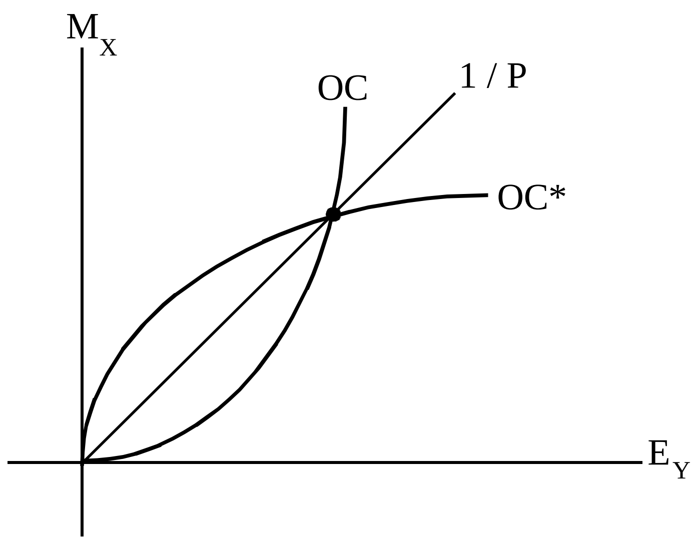
My aim here is to develop a little intuition about the optimal tariff. We start by developing one of the oldest geometric tools employed in trade theory -- the offer curve.

We can place indifference curves in a space where we are perhaps not used to them, that of net trades. Let the horizontal axis be E_Y and the vertical axis be M_X. The preference directions are North and West, as we prefer fewer exports for more imports.

A relative price for trade appears here as a ray from the origin. If we vary this price radially from the origin it traces out a locus of tangencies with our trade indifference curves. This locus is called the offer curve. These are the set of net trades that price-taking agents in this economy will be willing to make.



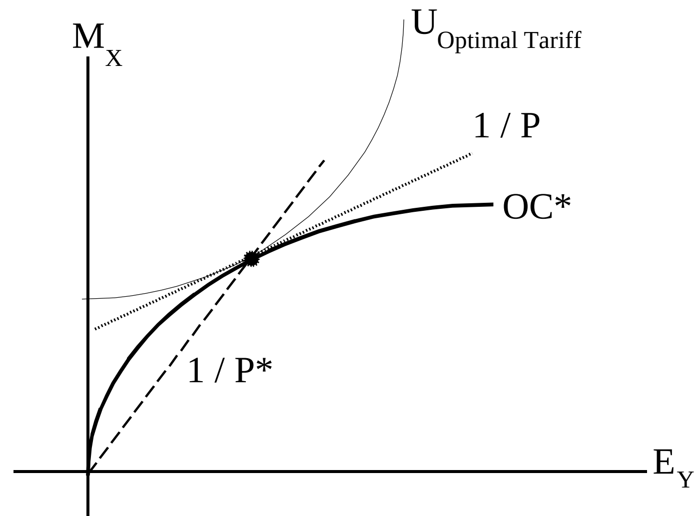
We can also develop the foreign offer curve, noting that the preference directions for the foreign offer curve are the opposite -- South and East -- as our exports are their imports and vice versa. The intersection of the offer curves (not necessarily unique) represents the free trade equilibrium.



Optimal Tariff and Net Trade (Offer) Curve

The slope of the trade indifference curve equivalently is the DRS and DRT, since with employment and consumer choice optimal these will be equal. The free trade price line is tangent to the trade indifference curve through the equilibrium point by the fact that this is on each offer curve. Thus, with free trade, P = DRT = DRS = DRS* = DRT* = P*. This implies that free trade is pareto optimal for the world as a whole (both have full employment).

Note, though, that DRT ≠ FRT here. The point, of course, is that since the country is large, it need not take the world price as given. If the foreign country is passive, then its offer curve represents the true opportunity set available to the home country. If the home country is small, then the foreign offer curve is simply the price line. Otherwise the slope of the foreign offer curve will differ from the slope of the price line, which means that the economy's marginal opportunities differ from those of the private agents in the economy. The government can use tariff policy to attain the desired equilibrium. The best feasible outcome for the home country is easily discerned: the point of tangency of the highest home trade indifference curve with the foreign offer curve.



The Optimal Tariff

If this is to be an equilibrium, then the foreign country must face the relative price indicated by the ray from the origin to this point, while the home economy must face the relative price that corresponds to the slope of the offer curve at this point. The difference implicitly defines the optimal tariff.

G. <u>The Optimal Tariff in the Two-Good Case</u>

We have already derived an expression for changes in welfare for an open economy, given as dR. This can be used to derive the optimal tariff. At the optimum dR = 0, reflecting a trade off between further improvement in terms of trade and loss of tariff revenue due to the constriction of imports. Thus

$$dR = -M_X\, dP* + tP*\, dM_X = 0$$

$$t_{opt.} = \frac{M_X\, dP*}{P*\, dM_X} = \frac{\hat{P}*}{\hat{M}_X}$$

$$Note: \quad P*\, M_X = M_Y^*, \quad so \quad \hat{P}* + \hat{M}_X = \hat{M}_Y^*$$

$$t_{opt.} = \frac{1}{\epsilon * - 1}$$