20

REFLECTION AND REFRACTION OF LIGHT

Light makes us to see things and is responsible for our visual contact with our immediate environment. It enables us to admire and adore various beautiful manifestations of mother nature in flowers, plants, birds, animals, and other forms of life. Can you imagine how much shall we be deprived if we were visually impaired? Could we appreciate the brilliance of a diamond or the majesty of a rainbow? Have you ever thought how light makes us see? How does it travel from the sun and stars to the earth and what is it made of? Such questions have engaged human intelligence since the very beginning. You will learn about some phenomena which provide answers to such questions.

Look at light entering a room through a small opening in a wall. You will note the motion of dust particles, which essentially provide simple evidence that light travels in a straight line. An arrow headed straight line represents the direction of propagation of light and is called a ray; a collection of rays is called a **beam**. The ray treatment of light constitutes **geometrical optics**. In lesson 22, you will learn that light behaves as a wave. But a wave of short wavelength can be well approximated by the ray treatment. When a ray of light falls on a mirror, its direction changes. This process is called *reflection*. But when a ray of light falls at the boundary of two dissimilar surfaces, it bends. This process is known as *refraction*. You will learn about reflection from mirrors and refraction from lenses in this lesson. You will also learn about *total internal reflection.* These phenomena find a number of useful applications in daily life from automobiles and health care to communication.

OBJECTIVES

After studying this lesson, you should be able to:

- *explain reflection at curved surfaces and establish the relationship between the focal length and radius of curvature of spherical mirrors;*

- *state sign convention for spherical surfaces;*
- *derive the relation between the object distance, the image distance and the focal length of a mirror as well as a spherical refractive surface;*
- *state the laws of refraction;*
- *explain total internal reflection and its applications in everyday life;*
- *derive Newton's formula for measuring the focal length of a lens;*
- *describe displacement method to find the focal length of a lens; and*
- *derive an expression for the focal length of a combination of lenses in contact.*

20.1 REFLECTION OF LIGHT FROM SPHERICAL SURFACES

In your earlier classes, you have learnt the laws of reflection at a plane surface. Let us recall these laws :

Law 1 –*The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence always lie in the same plane.*

Law 2 –*The angle of incidence is equal to the angle of reflection* :

$$\angle i = \angle r$$

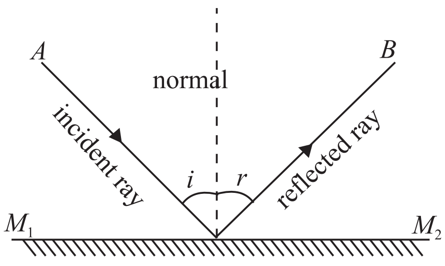


Fig. 20.1 : Reflection of light from a plane surface

These are illustrated in Fig. 20.1. Though initially stated for plane surfaces, these laws are also true for spherical mirrors. This is because a spherical mirror can be regarded as made up of a large number of extremely small plane mirrors. A well-polished spoon is a familiar example of a spherical mirror. Have you seen the image of your face in it? Fig. 20.2(a) and 20.2 (b) show two main types of spherical mirrors.

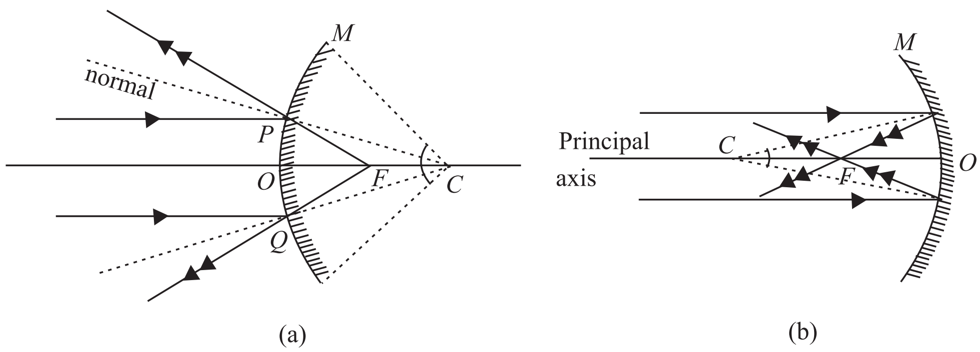


Fig. 20.2 : Spherical mirrors : a) a convex mirror, and b) a concave mirror

Note that the reflecting surface of a convex mirror curves outwards while that of a concave mirror curves inwards. We now define a few important terms used for spherical mirrors.

The centre of the sphere, of which the mirror is a part, is called the ***centre of curvature*** of the mirror and the radius of this sphere defines its ***radius of curvature***. The middle point O of the reflecting surface of the mirror is called its ***pole***. The straight line passing through C and O is said to be the ***principal axis*** of the mirror. The circular outline (or periphery) of the mirror is called its ***aperture*** and the angle ($\angle MCM'$) which the aperture subtends at C is called the ***angular aperture*** of the mirror. Aperture is a measure of the size of the mirror.

A beam of light incident on a spherical mirror parallel to the principal axis converges to or appears to diverge from a common point after reflection. This point is known as ***principal focus*** of the mirror. The distance between the pole and the principal focus gives the ***focal length*** of the mirror. A plane passing through the focus perpendicular to the principal axis is called the ***focal plane***.

We will consider only small aperture mirrors and rays close to the principal axis, called **paraxial rays.** (The rays away from the principal axis are called **marginal** or **parapheral rays**.)

INTEXT QUESTIONS 20.1

1. Answer the following questions :
   (a) Which mirror has the largest radius of curvature : plane, concave or convex?
   (b) Will the focal length of a spherical mirror change when immersed in water?
   (c) What is the nature of the image formed by a plane or a convex mirror?
   (d) Why does a spherical mirror have only one focal point?
2. Draw diagrams for concave mirrors of radii 5cm, 7cm and 10cm with common centre of curvature. Calculate the focal length for each mirror. Draw a ray parallel to the common principal axis and draw reflected rays for each mirror.
3. The radius of curvature of a spherical mirror is 30cm. What will be its focal length if (i) the inside surface is silvered? (ii) outside surface is silvered?
4. Why are dish antennas curved?

20.1.1 Ray Diagrams for Image Formation

Let us again refer to Fig. 20.2(a) and 20.2(b). You will note that

- *the ray of light through centre of curvature retraces its path.*

- *the ray of light parallel to the principal axis, on reflection, passes through the focus; and*
- *the ray of light through F is reflected parallel to the principal axis.*

To locate an image, any two of these three rays can be chosen. The images are of two types : real and virtual.

Real image of an object is formed when reflected rays actually intersect. These images are inverted and can be projected on a screen. They are formed on the same side as the object in front of the mirror (Fig. 20.3(a)).

Virtual image of an object is formed by reflected rays that appear to diverge from the mirror. Such images are always erect and virual; these cannot be projected on a screen. They are formed behind the mirror (Fig. 20.3(b)).

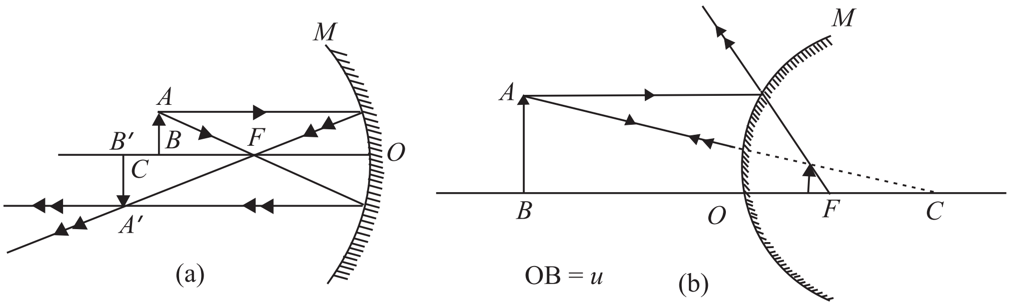


Fig. 20.3 : Image formed by a) concave mirror, and b) convex mirror

20.1.2 Sign Convention

We follow the sign convention based on the cartesian coordinate system. While using this convention, the following points should be kept in mind:

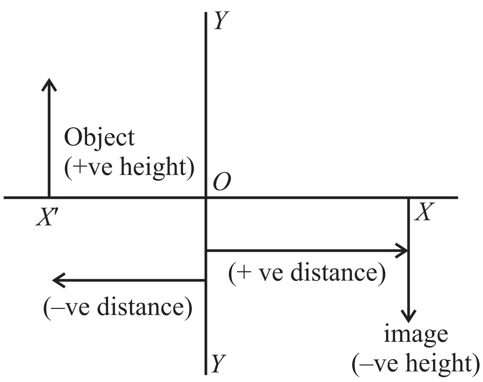


Fig. 20.4 : Sign convention

1. All distances are measured from the pole (*O*) of the mirror. The object is always placed on the left so that the incident ray is always taken as travelling from left to right.
2. All the distances on the left of *O* are taken as negative and those on the right of *O* as positive.
3. The distances measured above and normal to the principal axis are taken as positive and the downward distances as negative.

The radius of curvature and the focal length of a concave mirror are negative and those for a convex mirror are positive.

20.2 DERIVATION OF MIRROR FORMULA

We now look for a relation between the object distance (u), the image distance (v) and the local length f of a spherical mirror. We make use of simple geometry to arrive at a relation, which surprisingly is applicable in all situations. Refer to Fig. 20.5,which shows an object AB placed in front of a concave mirror. The mirror produces an image $A'B'$.

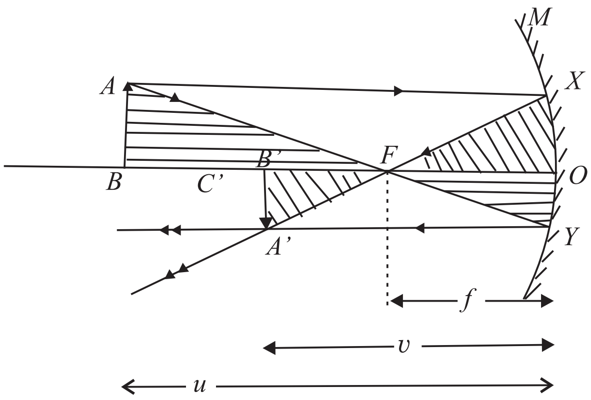


Fig. 20.5 : Image formation by a concave mirror: mirror formula

AX and AY are two rays from the point A on the object AB, M is the concave mirror while XA' and YA' are the reflected rays.

Using sign conventions, we can write

In optics it is customary to denote object distance by v. You should not confuse it with velocity.

object distance, $OB = -u$,

focal length, $OF = -f$,

image distance, $OB' = -v$,

and radius of curvature $OC = -2f$

Consider ΔABF and ΔFOY. These are similar triangles. We can, therefore, write

$$\frac{AB}{OY} = \frac{FB}{OF} \qquad (20.1)$$

Similarly, from similar triangles ΔXOF and $\Delta B'A'F$, we have

$$\frac{XO}{A'B'} = \frac{OF}{FB'} \qquad (20.2)$$

But $AB = XO$, as AX is parallel to the principal axis. Also $A'B'= OY$. Since left hand sides of Eqns. (20.1) and (20.2) are equal, we equate their right hand sides. Hence, we have

$$\frac{FB}{OF} = \frac{OF}{FB'} \qquad (20.3)$$

Putting the values in terms of u, v and f in Eqn. (20.3), we can write

$$\frac{-u-(-f)}{-f} = \frac{-f}{-v-(-f)}$$

$$\frac{-u+f}{-f} = \frac{-f}{-v+f}$$

On cross multiplication, we get

$$uv - uf - vf + f^2 = f^2$$

or

$$uv = uf + vf$$

Dividing throughout by uvf, we get the desired relation between the focal length and the object and image distances :

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \qquad (20.4)$$

We next introduce another important term ***magnification***. This indicates the ratio of the size of image to that of the object :

$$m = \frac{\text{size of the image}}{\text{size of the object}} = \frac{h_2}{h_1}$$

But

$$\frac{A'B'}{AB} = \frac{-v}{-u}$$

∴

$$m = -\frac{h_2}{h_1} = \frac{v}{u} \qquad (20.5)$$

Since a real image is inverted, we can write

$$m = \frac{A'B'}{AB} = -\frac{v}{u} \qquad (20.5b)$$

To *solve numerical problems, remember the following steps :.*

1. For any spherical mirror, use the mirror formula:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

2. Substitute the numerical values of the given quantities with proper signs.
3. Do not give any sign to the quantity to be determined; it will automatically be obtained with the relevant sign.
4. Remember that the linear magnification is negative for a real image and positive for a virtual image.
5. It is always better to draw a figure before starting the (numerical) work.

INTEXT QUESTIONS 20.2

1. A person standing near a mirror finds his head look smaller and his hips larger. How is this mirror made?
2. Why are the shaving mirrors concave while the rear view mirrors convex? Draw ray diagrams to support your answer.

3. As the position of an object in front of a concave mirror of focal length 25cm is varied, the position of the image also changes. Plot the image distance as a function of the object distance letting the latter change from $-x$ to $+x$. When is the image real? Where is it virtual? Draw a rough sketch in each case.

4. Give two situations in which a concave mirror can form a magnified image of an object placed in front of it. Illustrate your answer by a ray diagram.

5. An object 2.6cm high is 24cm from a concave mirror whose radius of curvature is 16cm. Determine (i) the location of its image, and (ii) size and nature of the image.

6. A concave mirror forms a real image four times as tall as the object placed 15cm from it. Find the position of the image and the radius of curvature of the mirror.

7. A convex mirror of radius of curvature 20cm forms an image which is half the size of the object. Locate the position of the object and its image.

8. A monkey gazes in a polished spherical ball of 10cm radius. If his eye is 20cm from the surface, where will the image of his eye form?

20.3 REFRACTION OF LIGHT

When light passes obliquely from a rarer medium (air) to a denser medium (water, glass), there is a change in its direction of propagation. ***This bending of light at the boundary of two dissimilar media is called refraction***.

When a ray of light is refracted at an interface, it obeys the following two laws :

Law I : The incident ray, the refracted ray and the normal to the surface at the point of incidence always lie in the same plane.

Law II : The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for a given pair of media. It is independent of the angle of incidence when light propagates from a rarer to a denser medium. Moreover, for a light of given colour, the ratio depends only on the pair of media.

This law was pronounced by the Dutch scientist Willebrord van Roijen Snell and in his honour is often referred to as ***Snell's law***. According to Snell's law

$$\frac{\sin i}{\sin r} = \mu_{12}$$

where μ_{12} is a constant, called the *refractive index* of second medium with respect to the first medium, and determines how much bending would take place at the interface separating the two media. It may also be expressed as the ratio of the velocity of light in the first medium to the velocity of light in the second medium

$$\mu_{12} = \frac{c_1}{c_2}$$

Refractive indices of a few typical substances are given in Table 20.1.Note that these values are with respect to air or vacuum. The medium having larger refractive index is optically denser medium while the one having smaller refractive index is called rarer medium. So water is denser than air but rarer than glass. Similarly, crown glass is denser than ordinary glass but rarer than flint glass.

Table 20.1 : Refractive indices of some common materials

Medium	μ
Vacuum/air	1
Water	1.33
Ordinary glass	1.50
Crown glass	1.52
Dense flint glass	1.65
Diamond	2.42

If we consider refraction from air to a medium like glass, which is optically denser than air[Fig. 20.6 (a)], then ∠*r* is less than ∠*i*. On the other hand, if the ray passes from water to air, ∠*r* is greater than ∠*i* [Fig. 20.6(b)]. That is, the refracted ray bends towards the normal on the air–glass interface and bends away from the normal on water–air interface.

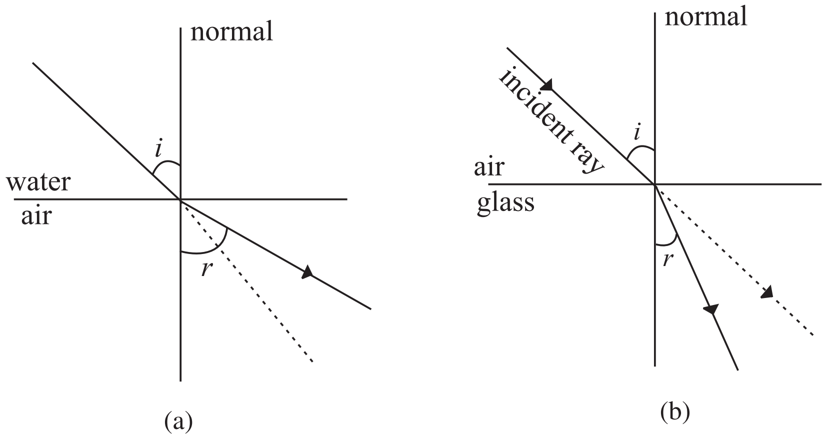


Fig. 20.6 : a) Refraction on air–glass interface, and **b)** refraction on water–air interface

Willebrord Van Roijen Snell (1580 – 1626)

Willebrord Snell was born in 1580 in Lieden. He began to study mathematics at a very young age. He entered the University of Leiden and initially studied law. But, soon he turned his attention towards mathematics and started teaching at the university by the time he was 20. In 1613, Snell succeded his father as Professor of Mathematics.

He did some important work in mathematics, including the method of calculating the approximate value of π by polygon. His method of using 96 sided polygon gives the correct value of π up to seven places while the classical method only gave this value upto two correct places. Snell also published some books including his work on comets. However, his biggest contribution to science was his discovery of the laws of refraction. However, he did not publish his work on refraction. It became known only in 1703, seventy seven years after his death, when Huygens published his results in "Dioptrics".

20.3.1 Reversibility of light

Refer to Fig. 20.6(b) again. It illustrates the principle of reversibility. It appears as if the ray of light is retracing its path. It is not always necessary that the light travels from air to a denser medium. In fact, there can be any combination of transparent media. Suppose, light is incident at a water-glass interface. Then, by applying Snell's law, we have

$$\frac{\sin i_w}{\sin i_g} = \mu_{wg} \tag{20.6}$$

Now, let us consider separate air-glass and air-water interfaces. By Snell's law, we can write

$$\frac{\sin i_a}{\sin i_g} = \mu_{ag}$$

and

$$\frac{\sin i_a}{\sin i_w} = \mu_{aw}$$

On combining these results, we get

$$\mu_{ag} \sin i_g = \mu_{aw} \sin i_w \tag{20.7}$$

This can be rewritten as

$$\frac{\sin i_w}{\sin i_g} = \frac{\mu_{ag}}{\mu_{aw}} \tag{20.8}$$

On comparing Eqns. (20.6) and (20.8), we get

$$\mu_{wg} = \frac{\mu_{ag}}{\mu_{aw}} \tag{20.9}$$

This result shows than when light travels from water to glass, the refractive index of glass with respect to water can be expressed in terms of the refractive indics of glass and water with respect to air.

Example 20.1 : A ray of light is incident at an angle of 30° at a water-glass interface. Calculate the angle of refraction. Given $\mu_{ag} = 1.5$, $\mu_{aw} = 1.3$.

Solution : From Eqn. (20.8), we have

$$\frac{\sin i_w}{\sin i_g} = \frac{\mu_{ag}}{\mu_{aw}}$$

$$\frac{\sin 30^0}{\sin i_g} = \frac{1.5}{1.3}$$

or $$\sin i_g = \left(\frac{1.3}{1.5}\right) \times \frac{1}{2}$$

$$= 0.4446$$

or $$i_g = 25°41'$$

Example 20.2 : Calculate the speed of light in water if its refractive index with respect to air is 4/3. Speed of light in vacuum = 3×10^8 ms^{-1}.

Solution : We know that

$$\mu = \frac{c}{\upsilon}$$

or $$\upsilon = \frac{c}{\mu}$$

$$= \frac{(3 \times 10^8 \text{ ms}^{-1})}{4/3}$$

$$= \frac{3 \times 10^8 \times 3}{4}$$

$$= 2.25 \times 10^8 \text{ ms}^{-1}$$

Example 20.3 : The refractive indices of glass and water are 1.52 and 1.33 respectively. Calculate the refractive index of glass with respect to water.

Solution : Using Eqn. (20.9), we can write

$$\mu_{wg} = \frac{\mu_{ag}}{\mu_{aw}} = \frac{1.52}{1.33} = 1.14$$

INTEXT QUESTIONS 20.3

1. What would be the lateral displacement when a light beam is incident normally on a glass slab?
2. Trace the path of light if it is incident on a semicircular glass slab towards its centre when $\angle i < \angle i_c$ and $\angle i > \angle i_c$.
3. How and why does the Earth's atmosphere alter the apparant shape of the Sun and Moon when they are near the horizon?
4. Why do stars twinkle?
5. Why does a vessel filled with water appear to be shallower (less deep) than when without water? Draw a neat ray diagram for it.
6. Calculate the angle of refraction of light incident on water surface at an angle of 52º. Take $\mu = 4/3$.

20.4 TOTAL INTERNAL REFLECTION

ACTIVITY 20.1

Take a stick, cover it with cycle grease and then dip it in water or take a narrow glass bottle, like that used for keeping Homeopathic medicines, and dip it in water. You will observe that the stick or the bottle shine almost like silver. Do you know the reason? This strange effect is due to a special case of refraction. We know that when a ray of light travels from an optically denser to an optically rarer medium, say from glass to air or from water to air, the refracted ray bends away from the normal. This means that the angle of refraction is greater than the angle of incidence. What happens to the refracted ray when the angle of incidence is increased? The bending of refracted ray also increases. However, the maximum value of the angle of refraction can be 90º. ***The angle of incidence in the denser medium for which the angle of refraction in rarer medium, air in this case, equals 90º is called the critical angle, i_C.*** The refracted ray then moves along the boundary separating the two media. If the angle of incidence is greater than the critical angle, the incident ray will be reflected back in the same medium, as shown in Fig. 20.7(c). Such a reflection is called ***Total Internal Reflection*** and the incident ray is said to be totally internally reflected. For total internal reflection to take place, the following two conditions must be satisfied.

- Light must travel from an optically denser to an optically rarer medium.
- The angle of incidence in the denser medium must be greater than the critical angle for the two media.

The glass tube in water in Activity 20.1 appeared silvery as total internal reflection took place from its surface.

An expression for the critical angle in terms of the refractive index may be obtained readily, using Snell's law. For refraction at the glass-air interface, we can write

$$\frac{\sin i}{\sin r} = \mu_{ga}$$

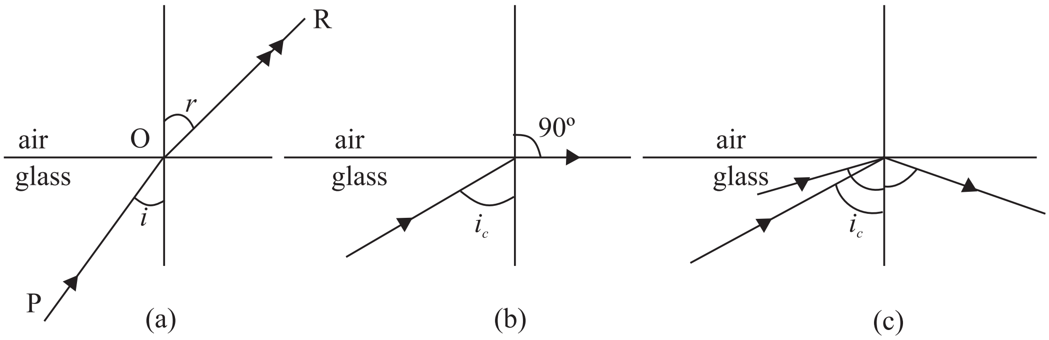


Fig. 20.7 : Refraction of light as it travels from glass to air for **a)** $i < i_c$, **b)** $i = i_c$ and **c)** $i > i_c$

Putting $r = 90°$ for $i = i_c$, we have

$$\frac{\sin i_c}{\sin 90°} = \mu_{ga}$$

or $$\sin i_c = \mu_{ga}$$

Hence $$\mu_{ag} = \frac{1}{\mu_{ga}} = \frac{1}{\sin i_c}$$

The critical angles for a few substances are given in Table 20.2

Table 20.2 : Critical angles for a few substances

Substance	*μ*	*Critical angle*
Water	1.33	48.75°
Crown glass	1.52	41.14°
Diamond	2.42	24.41°
Dense flint glass	1.65	37.31°

Example 20.4 : Refractive index of glass is 1.52. Calculate the critical angle for glass air interface.

Solution : We know that

$$\mu = 1/\sin i_c$$

$$\sin i_c = 1/\mu = \frac{1}{1.52}$$

$$\therefore \quad i_c = 42°$$

Much of the shine in transparent substances is due to total internal reflection. Can you now explain why diamonds sparkle so much? This is because the critical angle is quite small and most of the light entering the crystal undergoes multiple internal reflections before it finally emerges out of it.

In ordinary reflection, the reflected beam is always weaker than the incident beam, even if the reflecting surface is highly polished. This is due to the fact that some light is always transmitted or absorbed. But in the case of total internal reflection, cent percent (100%) light is reflected at a transparent boundary.

20.4.1 Applications of Refraction and Total Internal Reflection

There are many manifestations of these phenomena in real life situations. We will consider only a few of them.

(a) Mirage : Mirage is an optical illusion which is observed in deserts or on tarred roads in hot summer days. This, you might have observed, creates an illusion of water, which actually is not there.

Due to excessive heat, the road gets very hot and the air in contact with it also gets heated up. The densities and the refractive indices of the layers immediately above the road are lower than those of the cooler higher layers. Since there is no abrupt change in medium (see Fig. 20.9), a ray of light from a distant object, such as a tree, bends more and more as it passes through these layers. And when it falls on a layer at an angle greater than the critical angle for the two consecutive layers, total internal reflection occurs. This produces an inverted image of the tree giving an illusion of reflection from a pool of water.

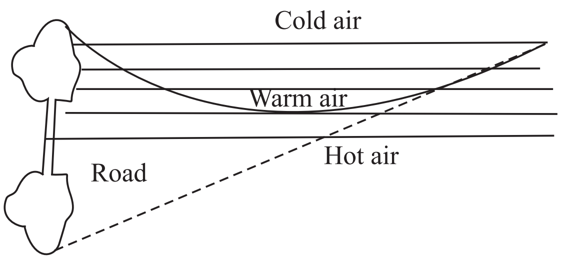


Fig. 20.8 : Formation of mirage

Totally Reflecting Prisms : A prism with right angled isosceles trianglur base or a totally reflecting prism with angles of 90°, 45° and 45° is a very useful device for reflecting light.

Refer to Fig. 20.9(a). The symmetry of the prism allows light to be incident on O at an angle of 45°, which is greater than the critical angle for glass i.e. 42°. As a result, light suffers total internal reflection and is deviated by 90°.

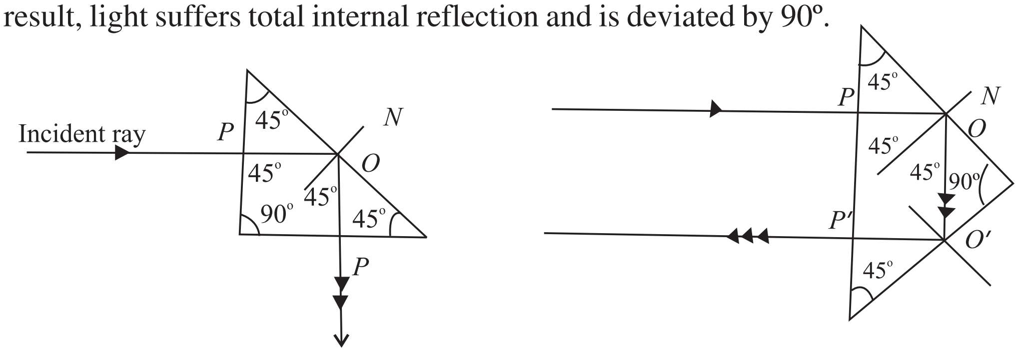


Fig. 20.9 : Totally reflecting prisms

Choosing another face for the incident rays, it will be seen (Fig. 20.9(b)) that the ray gets deviated through 180^0 by two successive total internal reflections taking place at O and O'.

Optical Fibres

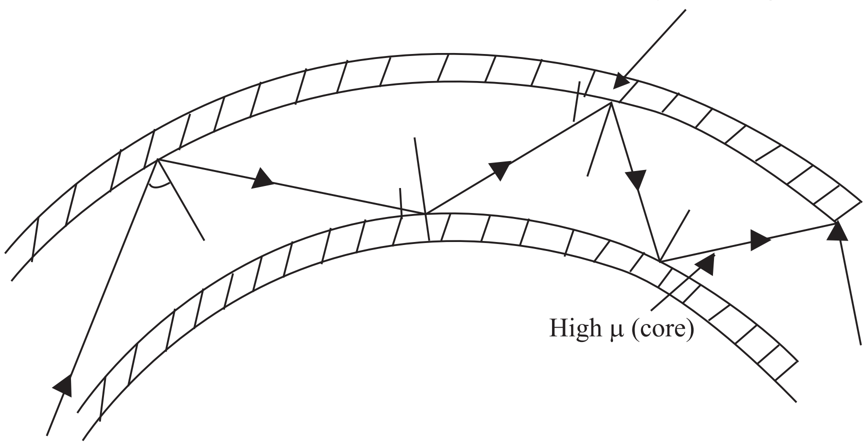


Fig. 20.10 : Multiple reflection in an optical fibre

An optical fibre is a hair-thin structure of glass or quartz. It has an inner core which is covered by a thin layer (called **cladding**) of a material of slightly lower refractive index. For example, the refractive index of the core is about 1.7 and that of the cladding is 1.5. This arrangement ensures total internal reflection. You can easily understand it, if you recall the conditions for total internal reflection.

When light is incident on one end of the fibre at a small angle, it undergoes multiple total internal reflections along the fibre (Fig. 20.10). The light finally emerges with undiminished intensity at the other end. Even if the fibre is bent, this process is not affected. Today optical fibres are used in a big way. A flexible light pipe using optical fibres may be used in the examination of inaccessible parts of the body e.g. laproscopic examination of stomach, urinary bladder etc. Other medical applications of optical fibres are in neurosurgery and study of bronchi. Besides medical applications, optical fibres have brought revolutionary changes in the way we communicate now. Each fibre can carry as many as 10,000 telephone messages without much loss of intensity, to far off places. That is why millions of people across continents can interact simultaneously on a fibre optic network.

INTEXT QUESTIONS 20.4

1. Why can't total internal reflection take place if the ray is travelling from a rarer to a denser medium?
2. Critical angle for glass is 42°. Would this value change if a piece of glass is immersed in water? Give reason for your answer.

Notes

3. Show, with the help a ray diagram how, a ray of light may be deviated through 90° using a i) plane mirror, and ii) totally reflecting prism. Why is the intensity of light greater in the second case?

4. A liquid in a container is 25cm deep. What is its apparant depth when viewed from above, if the refractive index of the liquid is 1.25? What is the critical angle for the liquid?

20.5 REFRACTION AT A SPHERICAL SURFACE

We can study formation of images of objects placed around spherical surfaces such as glass marbles (Kanchas), water drops, glass bottle, etc. For measuring distances from spherical refracing surfaces, we use the same sign convention as applicable for spherical mirrors. Refer to Fig. 20.11.

SPS′ is a convex refracting surface separating two media, air and glass. *C* is its centre of curvature. *P* is a point on *SPS′* almost symmetrically placed. You may call it the *pole*. *CP* is then the *principal axis*. *O* is a point object. *OA* is an incident ray and *AB* is the refracted ray. Another ray *OP* falls on the surface normally and goes undeviated after refraction. *PC* and *AB* appear to come from *I*. Hence *I* is the virtual image of *O*.

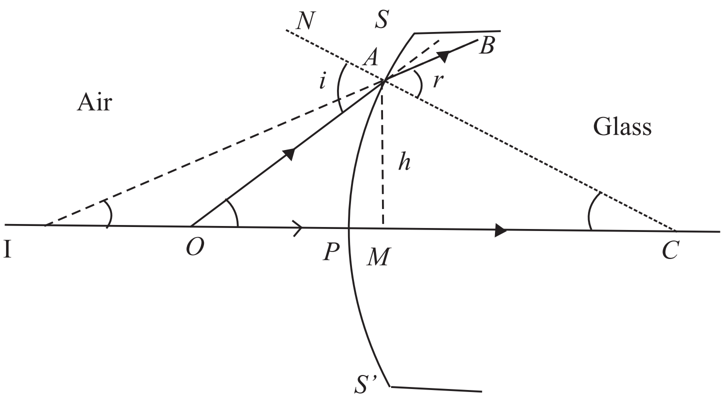


Fig. 20.11 : Refraction at a spherical surface

Let $\angle OAN = i$, the angle of incidence and $\angle CAB = r$, the angle of refraction. Using the proper sign convention, we can write

$$PO = -u \; ; PI = -v \; ; PC = +R$$

Let α, β, and γ be the angles subtended by *OA*, *IA* and *CA*, respectively with the principal axis and *h* the height of the normal dropped from *A* on the principal axis. In ΔOCA and ΔICA, we have

$$i = \alpha + \gamma \quad (i \text{ is exterior angle}) \qquad (20.10)$$

and

$$r = \beta + \gamma \quad (r \text{ is exterior angle}) \qquad (20.11)$$

From Snell's law, we recall that

$$\frac{\sin i}{\sin r} = \mu$$

where μ is the refractive index of the glass surface with respect to air. For a surface of small aperture, P will be close to A and so i and r will be very small (sin $i \simeq i$, sin $r \simeq r$). The above equation, therefore, gives

$$i = \mu r \qquad (20.12)$$

Substituting the values of i and r in Eqn. (20.12) from Eqns. (20.10) and (20.11) respectively, we get

$$\alpha + \gamma = \mu (\beta + \gamma)$$

or

$$\alpha - \mu\beta = \gamma (\mu - 1) \qquad (20.13)$$

As α, β and γ are very small, we can take tan $\alpha \simeq \alpha$, and tan $\beta \simeq \beta$, and tan $\gamma \simeq \gamma$. Now referring to ΔOAM in Fig. 20.11, we can write

$$\alpha \approx \tan \alpha = \frac{AM}{MO} = \frac{AP}{PO} = \frac{h}{-u} \quad \text{(if } M \text{ is very near to } P\text{)}$$

$$\beta \approx \tan \beta = \frac{AM}{MI} = \frac{AM}{PI} = \frac{h}{-v}$$

and

$$\gamma \simeq \tan \gamma = \frac{AM}{MC} = \frac{AM}{PC} = \frac{h}{R}$$

Substituting for α, β and γ in Eqn. (20.13), we get

$$\frac{h}{-u} - \frac{\mu h}{v} = (\mu - 1)\frac{h}{R}$$

or

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R} \qquad (20.14)$$

This important relationship correlates the object and image distances to the refractive index μ and the radius of curvature of the refracting surface.

20.5.1 Reflection through lenses

A lens is a thin piece of transparent material (usually glass) having two surfaces, one or both of which are curved (mostly spherical). You have read in your earlier classes that lenses are mainly of two types, namely, convex lens and concave lens. Each of them is sub-divided into three types as shown in Fig. 20.12. Thus, you can have plano-convex and plano-concave lenses too.

Basic Nomenclature

Thin lens : If the thickness of a lens is negligible in comparison to the radii of curvature of its curved surfaces, the lens is referred to as a thin lens. We will deal with thin lenses only.

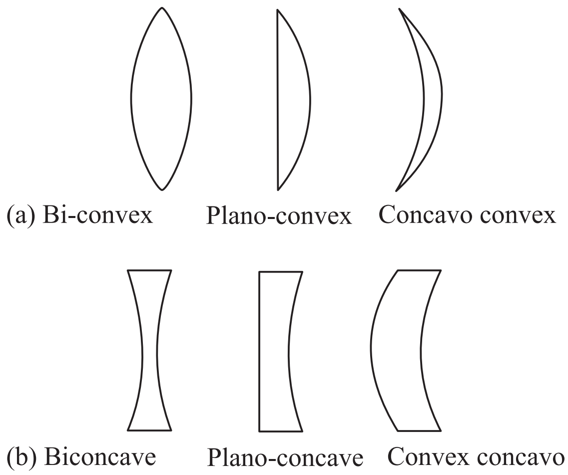


Fig. 20.12 : Types of lenses

Principal axis is the line joining the centres of curvature of two surfaces of the lens.

Optical centre is the point at the centere of the lens situated on the principal axis. The rays passing through the optical centre do not deviate.

Principal focus is the point at which rays parallel and close to the principal axis converge to or appear to diverge. It is denoted by *F* (Fig. 20.13) Rays of light can pass through a lens in either direction. So every lens has two principal focii, one on its either side.

Focal length is the distance between the optical centre and the principal focus. In Fig. 20.13, *OF* is focal length (*f*). As per the sign convention, *OF* is positive for a convex lens and negative for a concave lens.

Focal plane is the plane passing through the focus of a lens perpendicular to its principal axis.

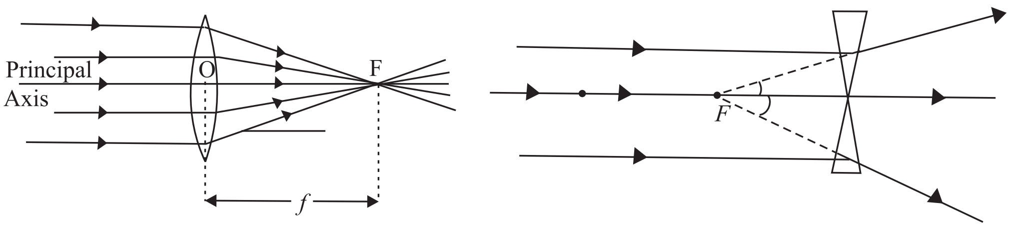


Fig. 20.13 : Foci of **a)** convex, and **b)** concave lenses

20.5.2 Lens Maker's Formula and Magnification

You can now guess that the focal length must be related to the radius of curvature and the refractive index of the material of the lens. Suppose that a thin convex lens *L* is held on an optical bench (Fig. 20.14). Let the refractive index of the material of the lens with respect to air be μ and the radii of curvatures of its two surfaces be R_1 and R_2, respectively. Let a point object be situated on the principal

Notes

axis at P. C_1 and C_2 are the centres of curvature of the curved surfaces 1 and 2, respectively.

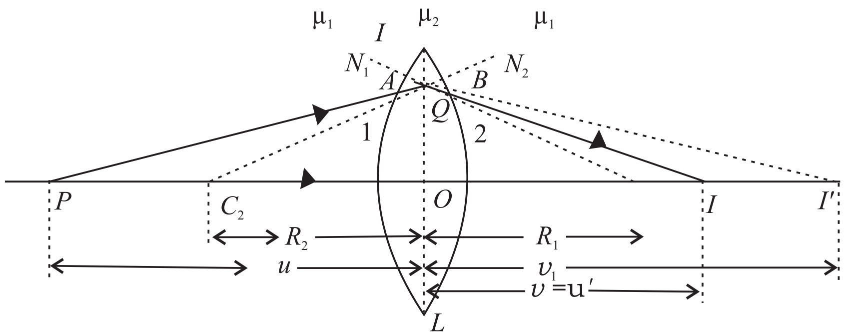


Fig. 20.14 : Point image of a point object for by a thin double convex lens

A ray from P strikes surface 1 at A. $C_1 N_1$ is normal to surface 1 at the point A. The ray PA travels from the rarer medium (air) to the denser medium (glass), and bends towards the normal to proceed in the direction AB. The ray AB would meet the principal axis C_2C_1 at the point I' *in the absence of the surface 2*. Similarly, another ray from P passing through the optical centre O passes through the Point I'. I' is thus the virtual image of the object P.

Then object distance $OP = u$ and image distance $OI' = v_1$ (say). Using Eqn. (20.14) we can write

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{R_1} \qquad (20.15)$$

Due to the presence of surface 2, the ray AB strikes it at B. C_2N_2 is the normal to it at point B. As the ray AB is travelling from a denser medium (glass) to a rarer medium (air), it bends away from the normal C_2N_2 and proceeds in the direction BI and meets another ray from P at I. Thus I is image of the object P formed by the lens. It means that image distance $OI = v$.

Considering point object O, its virtual image is I' (due to surface 1) and the final image is I. I' is the virtual object for surface 2 and I is the final image. Then for the virtual object I' and the final image I, we have, object distance $OI' = u' = v_1$ and image distance $OI = v$.

On applying Eqn. (20.12) and cosidering that the ray AB is passing from *glass to air,* we have

$$\frac{(1/\mu)}{v} + \frac{1}{v_1} = \frac{(1/\mu) - 1}{R_2}$$

or,

$$\frac{1}{\mu v} - \frac{1}{v_1} = \frac{1 - \mu}{\mu R_2}$$

Since the lens used is actually thin, points A and B may be considered very close to point a and hence C_1A is taken equal to C_1Q and C_2B as C_2Q.

Multiping both sides by μ, we get

$$\frac{1}{v} - \frac{\mu}{v_1} = \frac{\mu - 1}{R_2} \qquad (20.16)$$

Adding Eqns. (20.15) and (20.16), we have

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (20.17)$$

It $u = \infty$, that is the object is at infinity, the incoming rays are parallel and after refraction will converge at the focus ($v = f$). Then Eqn. (20.17) reduces to

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (20.18)$$

This is called lens maker's formula.

From Eqns. (20.17) and (20.18), we can conclude that

- *The focal length of a lens depends on the radii of curvature of spherical surfaces. Focal length of a lens of larger radii of curvature will be more.*
- *Focal length of a lens is smaller if the refractive index of its material is high.*

In case a lens is dipped in water or any other transparent medium, the value of μ changes and you can actually work out that focal length will increase. However, if the density of the medium is more than that of the material of the lens, say carbon disulphide, the rays may even diverge.

20.5.3 Newton's Formula

Fig. 20.5.3 shows the image of object AB formed at $A'B'$ by a convex lens F_1 and F_2 are the first and second principal focii respectively.

Let us measure the distances of the object and image from the first focus and second focus respectively. Let x_1 be the distance of object from the first focus and x_2 be the distance of image from the second focus and f_1 and f_2 the first and second focal lengths, respectively as shown in Fig. 20.5.3.

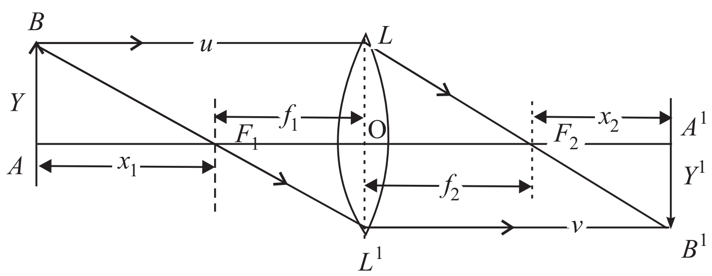


Fig. 20.5.3

Now, in similar “s, ABF_1 and $OL'F_1$

$$\frac{-y'}{y} = \frac{-f_1}{-x_1}$$

Also from similar Δs, OLF_2 and $A'B'F_2$

$$\frac{-y'}{y} = \frac{x_2}{f_2}$$

Comparing these two equations we get

$$x_1 x_2 = f_1 f_2$$

for $\quad f_1 \equiv f_2 \approx f$ (say), then $x_1 x_2 = +f^2$

or $\quad f = \sqrt{x_1 x_2}$

This relation is called Newton's formula and can be conveniently used to measure the focal length.

20.5.4 Displacement Method to find the Position of Images (Conjugates points)

In the figure 20.5.4, $A'B'$ is the image of the object AB as formed by a lens L. $OA = u$ and $OA' = v$.

The principle of reversibility of light rays tells us that if we move the lens towards the right such that $AO = v$, then again the image will be formed at the same place.

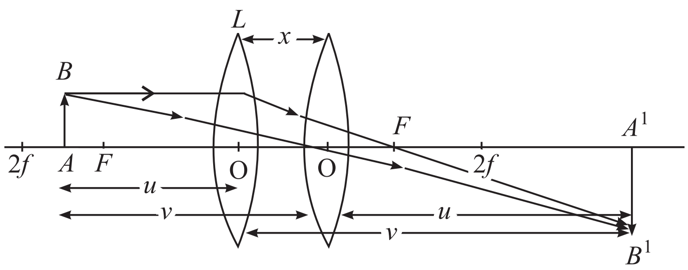


Fig. 20.5.4

Thus $\quad AA' = D = u + v \quad$ ...(i)

and the separation between the two positions of the lens:

$$OO' = x = (v - u) \quad \text{...(ii)}$$

Adding (i) and (ii) we get

$$v = \frac{x+D}{2}$$

and, subtracting (ii) from (i) we get

$$u = \frac{D-x}{2}$$

Substituting these values in the lens formula, we get.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{(-u)}$$

$$\frac{1}{f} = \frac{2}{x+D} + \frac{2}{D-x} = \frac{2}{D+x} + \frac{2}{D-x}$$

$$\frac{1}{f} = \frac{2(D-x+D+x)}{D^2-x^2}$$

$$\frac{1}{f} = \frac{4D}{D^2-x^2}$$

or $$f = \frac{D^2-x^2}{4D}$$

Thus, keeping the positions of the object and screen fixed we can obtain equally clear, bright and sharp images of the object on the screen corresponding to the two positions of the lens. This again is a very convenient way of finding f of a lens.

20.6 FORMATION OF IMAGES BY LENSES

The following properties of the rays are used in the formation of images by lenses:

- A ray of light through the optical centre of the lens passes undeviated.
- A parallel ray, after refraction, passes through the principal focus.
- A ray of light through F or F' is rendered parallel to the principal axis after refraction.

  Any two of these rays can be chosen for drawing ray diagrams.

The lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ suggests the dependence of the image distance (v) on the object distance (u) and the focal length (f) of the lens.

The magnification of a lens is defined as the ratio of the height of the image formed by the lens to the height of the object and is denoted by m :

$$m = \frac{I}{O} = \frac{v}{u}$$

where I is height of the image and O the height of the object.

Example 20.5 : The radii of curvature of a double convex lens are 15cm and 30cm, respectively. Calculate its focal length. Also, calculate the focal length when it is immersed in a liquid of refractive index 1.65. Take μ of glass = 1.5.

Solution : From Eqn. (20.18) we recall that

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here R_1= + 15cm, and R_2 = –30cm. On substituting the given data, we get

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{15} - \frac{1}{-30}\right)$$

$$\Rightarrow \qquad f = 20 \text{ cm}$$

When the lens is immersed in a liquid, μ will be replaced by $\mu_{\ell g}$:

$$\mu_{\ell g} = \frac{\mu_{ag}}{\mu_{al}}$$

$$= \frac{1.5}{1.65} = \frac{10}{11}$$

Therefore

$$\frac{1}{f_l} = (\mu_{\ell g} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{10}{11} - 1\right)\left(\frac{1}{15} - \frac{1}{-30}\right)$$

$$= -\frac{1}{110}$$

$$\therefore \qquad f = -110\text{cm}$$

As f is negative, the lens indeed behaves like a concave lens.

20.7 POWER OF A LENS

A practical application of lenses is in the correction of the defects of vision. You may be using spectacles or seen other learners, parents and persons using spectacles. However, when asked about the power of their lens, they simply quote a positive or negative number. What does this number signify? This number is the power of a lens in dioptre. The power of a lens is defined as the reciprocal of its focal length in metre:

$$P = \frac{1}{f}$$

The S1 unit of power of a lens is m^{-1}. Dioptre is only a commercial unit generally used by opticians. The power of a convex lens is positive and that of a concave lens is negative. Note that greater power implies smaller focal length. Using lens maker's formula, we can relate power of a lens to its radii of curvature:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Example 20.6 : Calculate the radius of curvature of a biconvex lens with both surfaces of equal radii, to be made from glass ($\mu = 1.54$), in order to get a power of +2.75 dioptre.

Solution :

$$P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P = +2.75 \text{ dioptre}$$

$$\mu = 1.54$$

$$R_1 = R$$

and

$$R_2 = -R$$

Substuting the given values in lens makers formula, we get

$$2.75 = (0.54)\left(\frac{2}{R}\right)$$

$$R = \frac{0.54 \times 2}{2.75}$$

$$= 0.39 \text{ m}$$

$$= 39 \text{ cm}$$

20.8 COMBINATION OF LENSES

Refer to Fig. 20.15. Two thin convex lenses *A* and *B* having focal lengths f_1 and f_2, respectively have been kept in contact with each other. *O* is a point object placed on the common principal axis of the lenses.

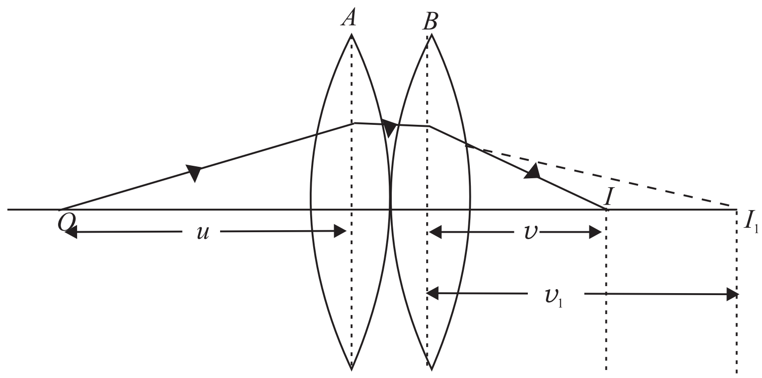


Fig. 20.15 : Two thin convex lenses in contact

Note that lens A forms the image of object O at I_1. This image serves as the ***virtual*** object for lens B and the final image is thus formed at I. If υ be the object distance and υ_1 the image distance for the lens A, then using the lens formula, we can write

$$\frac{1}{\upsilon_1} - \frac{1}{u} = \frac{1}{f_1} \qquad (20.19)$$

If υ is the final image distance for the lens B, we have

$$\frac{1}{\upsilon} - \frac{1}{\upsilon_1} = \frac{1}{f_2} \qquad (20.20)$$

Note that in writing the above expression, we have taken υ_1 as the object distance for the thin lens B.

Adding Eqns. (20.19) and (20.20), we get

$$\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (20.21)$$

If the combination of lenses is replaced by a single lens of focal length F such that it forms the image of O at the same position I, then this lens is said to be equivalent to both the lenses. It is also called the ***equivalent lens*** for the combination. For the equivalent lens, we can write

$$\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{F} \qquad (20.22)$$

where $$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}. \qquad (20.23)$$

If P is power of the equivalent lens and P_1 and P_2 are respectively the powers of individual lenses, then

$$P = P_1 + P_2 \qquad (20.24)$$

Note that Eqns.(20.23) and (20.24) derived by assuming two thin convex lenses in contact also hold good for any combination of two thin lenses in contact (the two lenses may both be convex, or concave or one may be concave and the other convex).

Example 20.7 : Two thin convex lenses of focal lengths 20cm and 40cm are in contact with each other. Calculate the focal length and the power of the equivalent lens.

Solution : The formula for the focal length of the combination $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ gives

$$\frac{1}{F} = \frac{1}{20} + \frac{1}{40}$$

$$= \frac{3}{40}$$

or

$$F = \frac{40}{3} = 13.3\text{cm} = 0.133\text{cm}$$

Power of the equivalent lens is

$$P = \frac{1}{F} = \frac{1}{0.133} = +7.5 \text{ dioptre.}$$

INTEXT QUESTIONS 20.5

1. On what factors does the focal length of a lens depend?
2. A lens, whose radii of curvature are different, is used to form the image of an object placed on its axis. If the face of the lens facing the object is reversed, will the position of the image change?
3. The refractive index of the material of an equi-double convex lens is 1.5. Prove that the focal length is equal to the radius of curvature.
4. What type of a lens is formed by an air bubble inside water?
5. A lens when immersed in a transparent liquid becomes invisible. Under what condition does this happen?
6. Calculate the focal length and the power of a lens if the radii of curvature of its two surfaces are +20cm and –25cm ($\mu = 1.5$).

7. Is it possible for two lenses in contact to produce zero power?
8. A convex lens of focal length 40cm is kept in contact with a concave lens of focal length 20cm. Calculate the focal length and the power of the combination.

Defects in image formation

Lenses and mirrors are widely used in our daily life. It has been observed that they do not produce a point image of a point object. This can be seen by holding a lens against the Sun and observing its image on a paper. You will note that it is not exactly circular. Mirrors too do not produce a perfect image. The defects in the image formation are known as ***aberrations***. The aberrations depend on (i) the quality of lens or mirror and (ii) the type of light used.

Two major aberrations observed in lenses and mirrors, are (a) ***spherical aberration*** and (b) ***chromatic aberration***. These aberration produce serious defects in the images formed by the cameras, telescopes and miscroscopes etc.

Spherical Aberration

This is a monochromatic defect in image formation which arises due to the sphericity and aperture of the refracting or reflecting surfaces. The paraxial rays and the marginal rays form images at different points I_p and I_m respectively (Fig. 20.16)

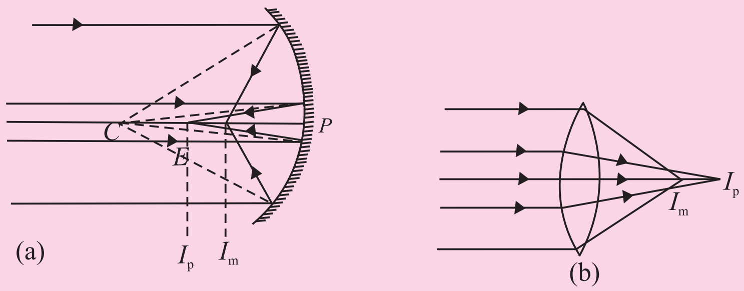


Fig. 20.16 : Spherical aberration in **a)** spherical mirror, and **b)** lens. I_p is image formed by the paraxial rays and I_m that formed by the marginal rays.

The ***spherical aberration*** in both mirrors and lenses can be reduced by allowing only the paraxial rays to be incident on the surface. It is done by using stops. Alternatively, the paraxial rays may be cut-off by covering the central portion, thus allowing only the marginal or parapheral rays to form the image. However, the use of stops reduces the brightness of the image.

A much appreciated method is the use of elliptical or parabolic mirrors.

The other methods to minimize spherical aberration in lenses are : use of plano convex lenses or using a suitable combination of a convex and a concave lens.

Chromatic Aberration in Lenses

A convex lens may be taken as equivalent to two small-angled prisms placed base to base and the concave lens as equivalent to such prisms placed vertex to vertex. Thus, a polychromatic beam incident on a lens will get dispersed. The parallel beam will be focused at different coloured focii. This defect of the image formed by spherical lenses is called ***chromatic aberration***. It occurs due to the dispersion of a polychromatic incident beam (Fig. 20.17. Obviously the red colour is focused farther from the lens while the blue colour is focused nearer the lens (in a concave lens the focusing of the red and blue colours takes place in the same manner but on the opposite side of it).

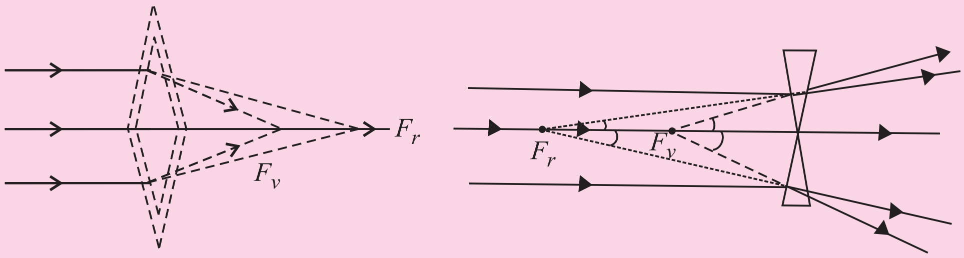


Fig. 20.17: Chromatic aberration

To remove this defect we combine a convergent lens of suitable material and focal length when combined with a divergent lens of suitable focal length and material. Such a lens combination is called an ***achromatic doublet*** . The focal length of the concave lens can be found from the necessary condition for achromatism given by

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

WHAT YOU HAVE LEARNT

- Real image is formed when reflected rays actually intersect after reflection. It can be projected on a screen.
- The focal length is half of the radius of curvature.

$$f = \frac{R}{2}$$

The object and image distances are related to magnification as

$$m = \frac{v}{u}$$

- Refraction of light results in change in the speed of light when it travels from one medium to another. This causes the rays of light to bend towards or away from the normal.
- The refractive index μ determines the extent of bending of light at the interface of two media.
- Snell's law is mathematically expressed as

$$\frac{\sin i}{\sin r} = \mu_{12}$$

where i is the angle of incidence in media 1 and r is the angle of refraction in media 2.

- Total internal reflection is a special case of refraction wherein light travelling from a denser to a rarer media is incident at an angle greater than the critical angle:

$$\mu = \frac{1}{\sin i_C}$$

- Any transparent media bounded by two spherical surfaces or one spherical and one plane surface forms a lens.
- Images by lenses depend on the local length and the distance of the object from it.
- Convex lenses are converging while concave lenses are diverging.
- $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$m = \frac{v}{u}$$

and $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

are simple relationships between the focal length (f), the refractive index, the radii of curvatures (R_1, R_2), the object distance (u) and the image distance (v).

- Newton's formula can be used to measure the focal length of a lens.
- Displacement method is a very convenient way of finding focal length of a lens.

- Power of a lens indicates how diverging or converging it is:

$$P = \frac{1}{f}$$

Power is expressed in dioptre. (or m^{-1} in SI units)

- The focal length F of an equivalent lens when two their lenses of focal lengths f_1 and f_2 one kept in contact is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

TERMINAL EXERCISES

1. List the uses of concave and convex mirrors.
2. What is the nature and position of image formed when the object is at (i) infinity (ii) $2f$ (iii) f in case of concave mirror and convex mirror.
3. List the factors on which lateral displacement of an incident ray depends as it suffers refraction through a parallel-sided glass slab? Why is the lateral displacement larger if angle of incidence is greater. Show this with the help of a ray diagram.
4. State conditions for total internal reflection of light to take place.
5. How is +1.5 dioptre different from –1.5 dioptre? Define dioptre.
6. Why does the intensity of light become less due to refraction?
7. A lamp is 4m from a wall. Calculate the focal length of a concave mirror which forms a five times magnified image of the lamp on the wall. How far from the wall must the mirror be placed?
8. A dentist's concave mirror has a radius of curvature of 30cm. How far must it be placed from a cavity in order to give a virtual image magnified five times?
9. A needle placed 45cm from a lens forms an image on a screen placed 90cm on the other side of the lens. Identify the type of the lens and determine its focal length. What is the size of the image, if the size of the needle is 5.0cm?
10. An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21 cm. Describe the nature of the image by the lens. What happens if the object is moved farther from the lens?
11. An object is placed at a distance of 100cm from a double convex lens which forms a real image at a distance of 20cm. The radii of curvature of the surfaces

of a lens are 25cm and 12.5 cm respectively. Calculate the refractive index of the material of the lens.

12. A ray of light is travelling from diamond to glass. Calculate the value of the critical angle for the ray, if the refractive index of glass is 1.51 and that of diamond 2.47.

13. A small object is placed at a distance of 15cm from two coaxial thin convex lenses in contact. If the focal length of each lens is 20cm. Calculate the focal length and the power of the combination and the distance between the object and its image.

14. While finding the focal length of a convex lens, an object was kept at a distance of 65.0 cm from the screen. Tow positions of the lens for which clear image of the object was formed on the screen were obtained. The distance between these two positions was found to be 15 cm. Calculate the focal length of the given lens.

ANSEWERS TO INTEXT QUESTIONS

20.1

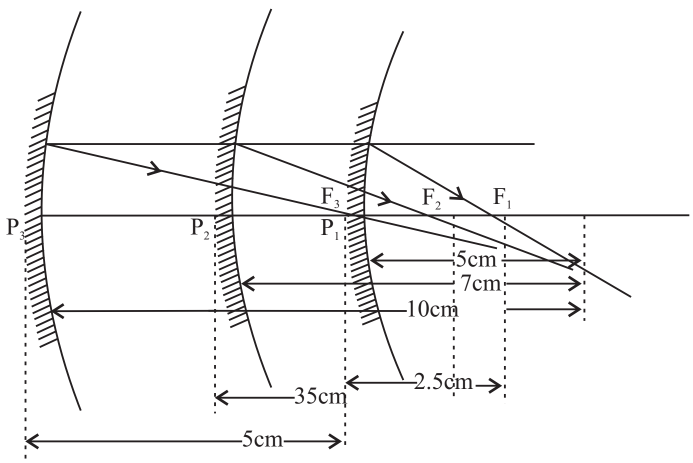
1. (a) plane mirror (its radius of curvature is infinitely large).

   (b) No. The focal length of a spherical mirror is half of its radius of curvature ($f \simeq R/2$) and has nothing to do with the medium in which it is immersed.

   (c) Virtual

   (d) This is because the rays parallel to the principal axis converge at the focal point F; and the rays starting from F, after reflection from the mirror, become parallel to the principal axis. Thus, F serves both as the first and the second focal point.

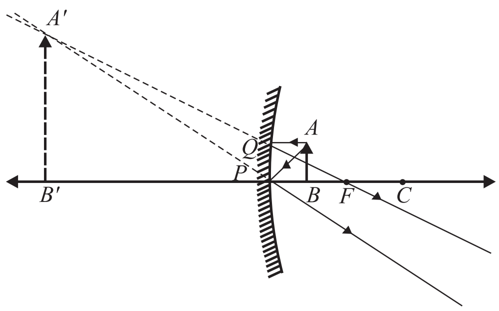
2. Focal lengths : 2.5cm, 3.5cm, 5cm.



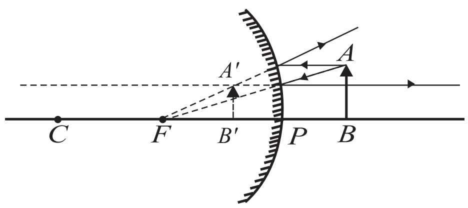
3. $f = -15\text{cm}; f = +15\text{cm}$.
4. The dish antennas are curved so that the incident parallel rays can be focussed on the receiver.

20.2

1. The upper part of the mirror must be convex and its lower part concave.
2. Objects placed close to a concave mirror give an enlarged image. Convex mirrors give a diminished erect image and have a larger field of view.

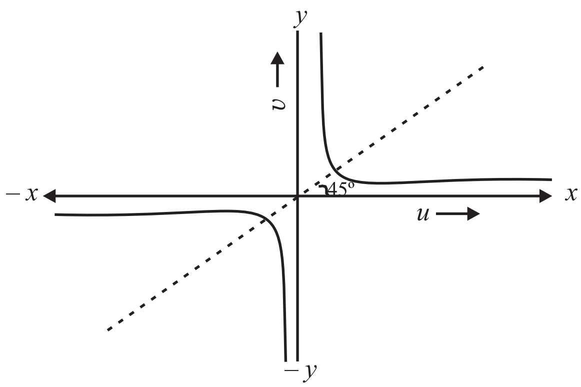


(a) Image formed by concave mirror

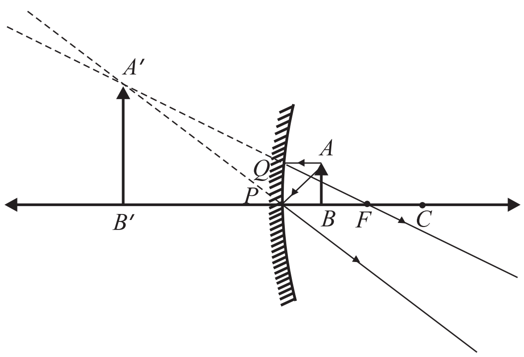


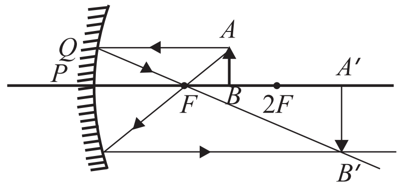
(b) Image formed by convex mirror

3. for $|u| > f$, we get real image; $u = -2f$ is a special case when an object kept as the centre of curvature of the mirror forms a real image at this point itself ($v = -2f$). For $u < f$, we get virtual image.



4. When (i) $u < f$, and (ii) $f < u < 2f$.

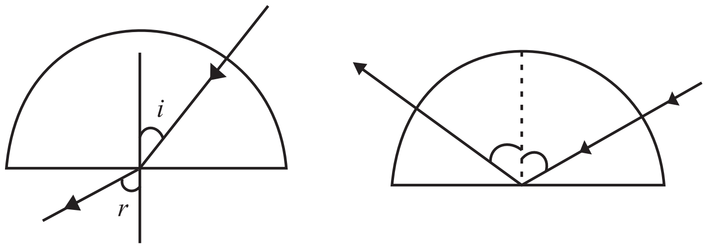




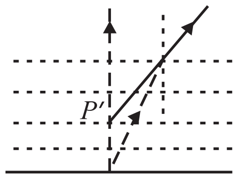
5. (i) 12cm in front of mirror, real and inverted, (ii) 0.8cm
6. $v = -60$cm, $R = -24$cm 7. $u = -10$cm, $v = +5$cm
8. $v = 4$cm

20.3

1. No lateral displacement.



2. $\angle r > \angle i$ when $\angle i < \angle i$ Total internal reflection where $\angle i > \angle i_c$
3. The density of air and hence its refractive index decrease as we go higher in altitude. As a result, the light rays from the Sun, when it is below the horizon, pass from the rarer to the denser medium and bend towards the normal, till they are received by the eye of the observer. This causes the shape to appear elongated.
4. Due to the change in density of the different layers of air in the atmosphere, μ changes continuously. Therefore, the refractive index of air varies at different levels of atmosphere. This along with air currents causes twinkling of stars.
5. Due to refraction point P appears at P'.

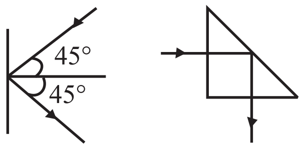


6. 36.2^0

20.4

1. Total internal reflection cannot take place if the ray travels from a rarer to a denser medium as the angle of refraction will always be less than the angle of incidence.
2. Yes the critical angle will change as

$$\mu_{ag} = \frac{1}{\sin i_c} \qquad \mu_{\omega g} = \frac{\mu_{ag}}{\mu_{aw}}$$

3. 

The intensity in the second case is more due to total internal reflection.

4. 20cm, $i_c = \sin^{-1} 0.8$

20.5

2. No. Changing the position of R_1 and R_2 in the lens maker's formula does not affect the value of f. So the image will be formed in the same position.

3. Substitute $R_1 = R$; $R_2 = –R$ and $\mu = 1.5$ in the lens maker's formula. You will get $f = R$.

4. Concave lens. But it is shaped like a convex lens.

5. This happens when the refractive index of the material of the lens is the same as that of the liquid.

6. $f = 22.2$ cm and $P = 4.5$ dioptre

7. Yes, by placing a convex and a concave lens of equal focal length in contact.

8. – 40cm, – 2.5 dioptre

Answers to Problems in Terminal Exercise

7. $f = –0.83$, 5m.   8. 12cm

9. $f = 30$cm, size of image = 10cm, converging lens

10. The image is erect, virtual and diminished in size, and located at 8.4cm from the lens on the same side as the object. As the object is moved away from the lens, the virtual image moves towards the focus of the lens but never beyond and progressively diminshes in size.

11. $\mu = 1.5$ 12. 37.7°

13. 10cm, 10 dioptre, 45 cm.

14. $f = 15.38$ cm

21

DISPERSION AND SCATTERING OF LIGHT

In the previous lesson you have learnt about reflection, refraction and total internal reflection of light. You have also learnt about image formation by mirrors and lenses and their uses in daily life. When a narrow beam of ordinary light is refracted by a prism, we see colour bands. This phenomenon has to be other than reflection or refraction. *The splitting of white light into its constituent colours or wavelengths by a medium is called* ***dispersion***. In this lesson, you will study about this phenomenon. A beautiful manifestation of this phenomenon in nature is in the form of rainbow. You will also learn in this lesson about the phenomenon of scattering of light, which gives sky its blue colour and the sun red colour at sunrise and sunset. Elementary idea of Raman effect will also be discussed in this lesson.

OBJECTIVES

After studying this lesson, you should be able to :

- *explain dispersion of light;*
- *derive relation between the angle of deviation* (δ), *angle of prism (A) and refractive index of the material of the prism* (μ);
- *relate the refractive index with wavelength and explain dispersion through a prism;*
- *explain formation of primary and secondary rainbows;*
- *explain scattering of light and list its applications.and; and*
- *explain Raman effect.*

21.1 DISPERSION OF LIGHT

Natural phenomena like rings around planets (halos) and formation of rainbow etc. cannot be explained by the rectilinear propagation of light. To understand such events, light is considered as having wave nature. (You will learn about it in the next lesson.) As you know, light waves are transverse electromagnetic waves which propagate with speed 3×10^8 ms^{-1} in vacuum. Of the wide range of electromagnetic spectrum, the visible light forms only a small part. Sunlight consists of seven different wavelengths corresponding to seven colours. Thus, colours may be identified with their wavelengths. You have already learnt *that the speed and wavelength of waves change when they travel from one medium to another.* The speed of light waves and their corresponding wavelengths also change with the change in the medium. The speed of a wave having a certain wavelength becomes less than its speed in free space when it enters an optically denser medium.

The refractive index μ has been defined as the ratio of the speed of light in vacuum to the speed of light in the medium. It means that the refractive index of a given medium will be different for waves having wavelengths 3.8×10^{-7} m and 5.8×10^{-7} m because these waves travel with different speeds in the same medium. This ***variation of the refractive index of a material with wavelength is known as dispersion.*** This phenomenon is different from refraction. In free space and even in air, the speeds of all waves of the visible light are the same. So, they are not separated. (Such a medium is called a non-dispersive medium.) But in an optically denser medium, the component wavelengths (colours) travel with different speeds and therefore get separated. Such a medium is called *dispersive medium*. Does this suggest that light will exhibit dispersion whenever it passes through an optically denser medium. Let us learn about it now.

21.1.1 Dispersion through a Prism

The separation of colours by a medium is not a sufficient condition to observe dispersion of light. These colours must be widely separated and should not mix up again after emerging from the dispersing medium. A glass slab (Fig. 21.1) is not suitable for observing dispersion as the rays of the emergent beam are very close and parallel to the incident beam

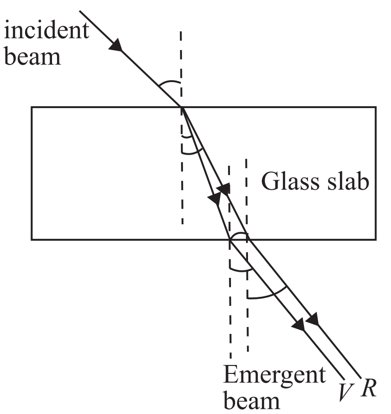


Fig. 21.1 : Passage of light through a glass slab

Newton used a prism to demonstrate dispersion of light. Refer to Fig. 21.2. White light from a slit falls on the face *AB* of the prism and light emerging from face *AC* is seen to split into different colours. Coloured patches can be seen on a screen. The face *AC* increases the separation between the rays refracted at the face *AB*. The incident white light *PQ* thus splits up into its component seven colours : Violet, indigo, blue, green, yellow, orange and red (VIBGYOR). The wavelengths travelling with different speeds are refracted through different angles and are thus separated. This *splitting of white light into component colours is known as dispersion*. *MR* and *MV* correspond to the red and violet light respectively. These colours on the screen produce the *spectrum.*

The bending of the original beam *PQN* along *MR* and *MV* etc. is known as ***deviation***. The angle between the emergent ray and the incident ray is known as the **angle of deviation.** Thus δ_v and δ_r represent the angles of deviation for violet light and red light, respectively.

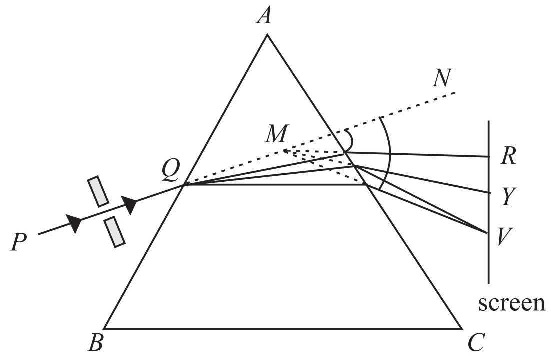


Fig. 21.2 : Dispersion of light by a prism

Read the following example carefully to fix the ideas on variation of the refractive index with the wavelength of light.

Example 21.1: A beam of light of average wavelength 600nm, on entering a glass prism, splits into three coloured beams of wavelengths 384 nm, 589 nm and 760 nm respectively. Determine the refractive indices of the material of the prism for these wavelengths.

Solution : The refractive index of the material of the prism is given by

$$\mu = \frac{c}{\upsilon}$$

where *c is* speed of light in vacuum, and υ is speed of light in the medium (prism).

Since velocity of a wave is product of frequency and wavelength, we can write

$$c = \nu\lambda_a \qquad \text{and} \qquad \upsilon = \nu\lambda_m$$

where λ_a and λ_m are the wavelengths in air and medium respectively and ν is the frequency of light waves. Thus

$$\mu = \frac{\nu\lambda_a}{\nu\lambda_m} = \frac{\lambda_a}{\lambda_m}$$

For 384 nm wavelength, the refractive index is

$$\mu_1 = \frac{600\times10^{-9}\,\text{m}}{384\times10^{-9}\,\text{m}} = 1.56$$

For wave length of 589 nm :

$$\mu_2 = \frac{600\times10^{-9}\,\text{m}}{58.9\times10^{-9}\,\text{m}} = 1.02$$

and for 760nm wavelength :

$$\mu_3 = \frac{600\times10^{-9}\,\text{m}}{760\times10^{-9}\,\text{m}} = 0.8$$

We have seen that the refractive index of a material depends on

- the nature of the material, and
- the wavelength of light.

An interesting outcome of the above example is that the variation in wavelength ($\Delta\lambda = \lambda_2 - \lambda_1$) produces variation in the refractive index ($\Delta\mu = \mu_2 - \mu_1$). The ratio $\frac{\Delta\mu}{\Delta\lambda}$ is known as the spectral *dispersive power of the material of prism* .

21.1.2 The Angle of Deviation

We would now establish the relation between the angle of incidence i, the angle of deviation δ and the angle of prism A. Let us consider that a monochromatic beam of light PQ is incident on the face AB of the principal section of the prism ABC [Fig.21.3]. On refraction, it goes along QR inside the prism and emerges along RS from face AC. Let $\angle A \equiv \angle BAC$ be the refracting angle of the prism. We draw normals NQ and MR on the faces AB and AC, respectively and produce them backward to meet at O. Then you can easily convince yourself that $\angle NQP = \angle i$, $\angle MRS = \angle e$, $\angle RQO = \angle r_1$, and $\angle QRO = \angle r_2$ are the angle of incidence, the angle of emergence and the angle of refraction at the faces AB and AC, respectively. The angle between the emergent ray RS and the incident ray PQ at D is known as the angle of deviation (δ).

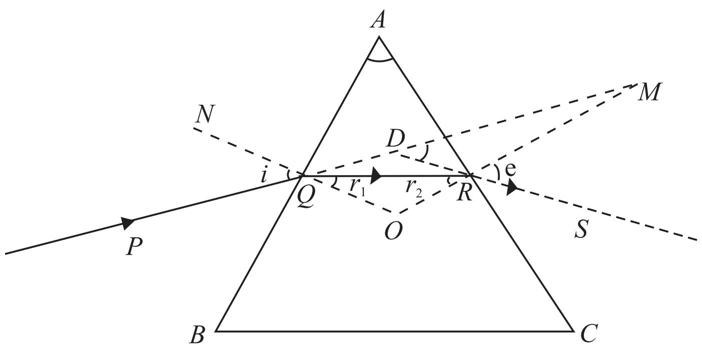


Fig. 21.3 : Refraction through a prism

Since $\angle MDR = \angle\delta$, As it is the external angle of the triangle QDR, we can write

$$\angle\delta = \angle DQR + \angle DRQ$$

$$= (\angle i - \angle r_1) + (\angle e - \angle r_2)$$

or $$\angle\delta = (\angle i + \angle e) - (\angle r_1 + \angle r_2) \qquad (21.1)$$

You may recall that the sum of the internal angles of a quadrilateral is equal to 360°. In the quadrilateral $AQOR$, $\angle AQO = \angle ARO = 90^0$, since NQ and MR are normals on faces AB and AC, respectively. Therefore

$$\angle QAR + \angle QOR = 180^0$$

or $$\angle A + \angle QOR = 180^0 \qquad (21.2)$$

But in ΔQOR

$$\angle OQR + \angle QRO + \angle QOR = 180^0$$

or $$\angle r_1 + \angle r_2 + \angle QOR = 180^0 \qquad (21.3)$$

On comparing Eqns. (21.2) and (21.3), we have

$$\angle r_1 + \angle r_2 = \angle A \qquad (21.4)$$

Combining this result with Eqn. (21.1), we have

$$\angle\delta = (\angle i + \angle e) - \angle A$$

or $$\angle i + \angle e = \angle A + \angle\delta \qquad (21.5)$$

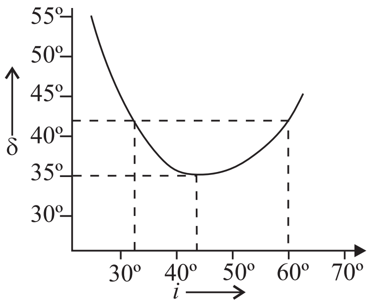


Fig. 21.4 : Plot between angle of incidence i and angle of deviation δ

Angle of Minimum Deviation

If we vary the angle of incidence i, the angle of deviation δ also changes; it becomes minimum for a certain value of i and again starts increasing as i increases further (Fig. 21.4). The minimum value of the angle of deviation is called *angle of minimum deviation* (δ_m). It depends on the material of the prism and the wavelength of light used. In fact, one angle of deviation may be obtained corresponding to two values of the angles of incidence. Using the principle of reversibility of light, we find that the second value of angle of incidence corresponds to the angle of emergence (e). In the minimum deviation position, there is only one value of the angle of incidence. So we have

$$\angle e = \angle i$$

Using this fact in Eqn.(21.5) and replacing δ by δ_m, we have

$$\angle i = \frac{\angle A + \angle\delta_m}{2} \qquad (21.6)$$

Applying the principle of reversibility of light rays and under the condition $\angle e = \angle i$, we can write $\angle r_1 = \angle r_2 = \angle r$, say

On substituting this result in Eqn. (21.4), we get

$$\angle r = \frac{\angle A}{2} \tag{21.7}$$

The light beam inside the prism, under the condition of minimum deviation, passes symmetrically through the prism and is parallel to its base. The refractive index of the material of the prism is therefore given by

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} \tag{21.8}$$

The refractive index μ can be calculated using Eqn.(21.8) for a monochromatic or a polychromatic beam of light. The value of δ_m is different for different colours. It gives a unique value of the angle of incidence and the emergent beam is brightest for this incidence.

For a prism of small angle *A*, keeping *i* and *r* small, we can write

$$\sin i = i, \sin r = r, \text{ and } \sin e = e$$

Hence

$$\mu = \frac{\sin i}{\sin r_1} = \frac{i}{r_1} \quad \text{or} \quad i = \mu r_1$$

Also

$$\mu = \frac{\sin e}{\sin r_2} = \frac{e}{r_2} \qquad \text{or} \qquad e = \mu r_2$$

Therefore,

$$\angle i + \angle e = \mu\,(\angle r_1 + \angle r_2)$$

Using this result in Eqns. (26.4) and (26.5), we get

$$\mu \angle A = \angle A + \angle \delta$$

or

$$\angle \delta = (\mu - 1)\angle A \tag{21.9}$$

We know that μ depends on the wavelength of light. So deviation will also depend on the wavelength of light. That is why δ_V is different from δ_R. Since the velocity of the red light is more than that of the violet light in glass, the deviation of the red light would be less as compared to that of the violet light.

$$\delta_V > \delta_R.$$

This implies that $\mu_V > \mu_R$. This change in the refractive index of the material with the wavelength of light is responsible for dispersion phenomenon.

21.1.3 Angular Dispersion and Dispersive Power

The difference between the angles of deviation for any two wavelengths (colours) is known as the *angular dispersion* for those wavelengths. The angular dispersion between the red and violet wavelengths is $\delta_V - \delta_R$. In the visible part of the spectrum, the wavelength of the yellow colour is nearly the average wavelength of the spectrum. The deviation for this colour δ_Y may, therefore, be taken as the average of all deviations.

The ***ratio of the angular dispersion to the mean deviation is taken as the dispersive power*** (ω) of the material of the prism :

$$\omega = \frac{\delta_V - \delta_R}{\delta_Y}$$

We can express this result in terms of the refractive indices using Eqn. (21.9) :

$$\omega = \frac{(\mu_V - 1)\angle A - (\mu_R - 1)\angle A}{(\mu_V - 1)\angle A}$$

$$= \frac{\mu_V - \mu_R}{\mu_Y - 1} = \frac{\Delta\mu}{\mu - 1} \qquad (21.10)$$

Example 21.2 : The refracting angle of a prism is 30′ and its refractive index is 1.6. Calculate the deviation caused by the prism.

Solution : We know that $\delta = (\mu - 1)\angle A$

On substituting the given data, we get

$$\delta = (1.6 - 1) \times \frac{1^\circ}{2} = \frac{0.6}{2} = 0.3^\circ = 18'$$

Example 21.3 : For a prism of angle A, the angle of minimum deviation is $A/2$. Calculate its refractive index, when a monochromatic light is used. Given $A = 60^\circ$

Solution : The refractive index is given by

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin(A/2)}$$

Now $\delta_m = A/2$ so that

$$\mu = \frac{\sin\left(\frac{A + A/2}{2}\right)}{\sin(A/2)} = \frac{\sin\left(\frac{3}{4}A\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{3}{4}A\right)}{\sin\left(\frac{A}{2}\right)} = \sqrt{2} = 1.4$$

INTEXT QUESTIONS 21.1

1. Most ordinary gases do not show dispersion with visible light. Why?
2. With your knowledge about the relative values of μ for the component colours of white light, state which colour is deviated more from its original direction?
3. Does dispersion depend on the size and angle of the prism?
4. Calculate the refractive index of an equilateral prism if the angle of minimum deviation is equal to the angle of the prism.

Rainbow formation

Dispersion of sunlight through suspanded water drops in air produces a spectracular effect in nature in the form of rainbow on a rainy day. With Sun at our back, we can see a brighter and another fainter rainbow. The brighter one is called the ***primary*** rainbow and the other one is said to be ***secondary rainbow***. Sometimes we see only one ***rainbow***. The bows are in the form of coloured arcs whose common centre lies at the line joining the Sun and our eye. Rainbow can also be seen in a fountain of water in the evening or morning when the sun rays are incident on the water drops at a definite angle.

Primary Rainbow

The primary rainbow is formed by two refractions and a single internal reflection of sunlight in a water drop. (See Fig. 21.5(a)). Descartes explained that rainbow is seen through the rays which have suffered minimum deviation. Parallel rays from the Sun suffering deviation of 137º.29′ or making an angle of 42º.31′ at the eye with the incident ray, after emerging from the water drop, produce bright shining colours in the bow. Dispersion by water causes different colours (red to violet) to make their own arcs which lie within a cone of 43º for red and 41º for violet rays on the outer and inner sides of the bow (Fig. 21.5 (b)).

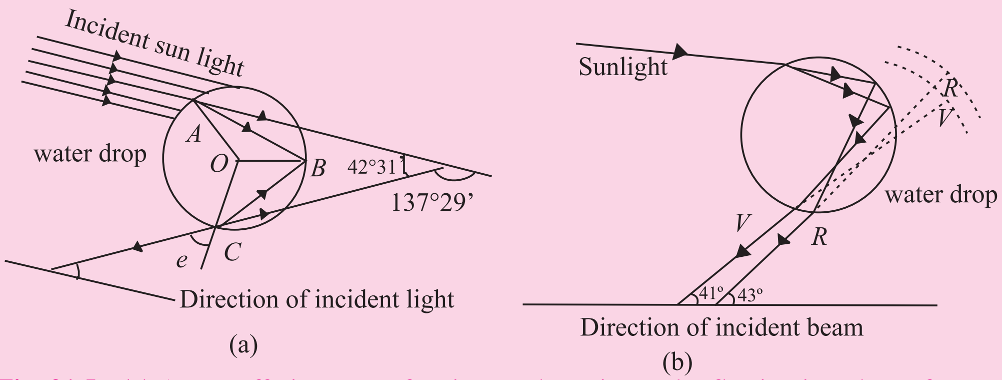


Fig. 21.5 : (a) A ray suffering two refractions and one internal reflection in a drop of water. Mean angle of minimum deviation is 137º29′, and **(b)** dispersion by a water drop.

Secondary Rainbow

The secondary rainbow is formed by two refractions and two internal reflections of light on the water drop. The angles of minimum deviations for red and violet colours are 231°. and 234°. respectively, so they subtend a cone of 51°. for the red and 54°. for the violet colour. From Fig.21.6 it is clear that the red colour will be on the inner and the violet colour on the outer side of the bow.

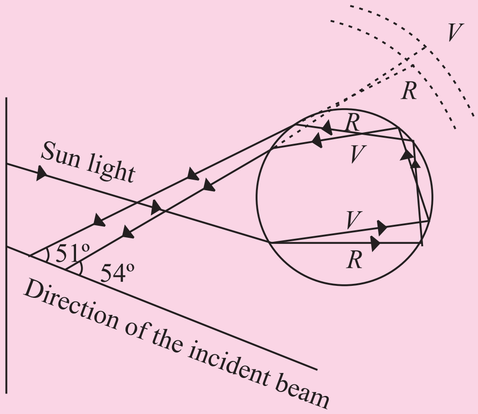


Fig. 21.6 : Formation of the secondary rainbow

The simultaneous appearance of the primary and secondary rainbows is shown in Fig.21.7. The space between the two bows is relatively dark. Note that the secondary rainbow lies above the primary bow.

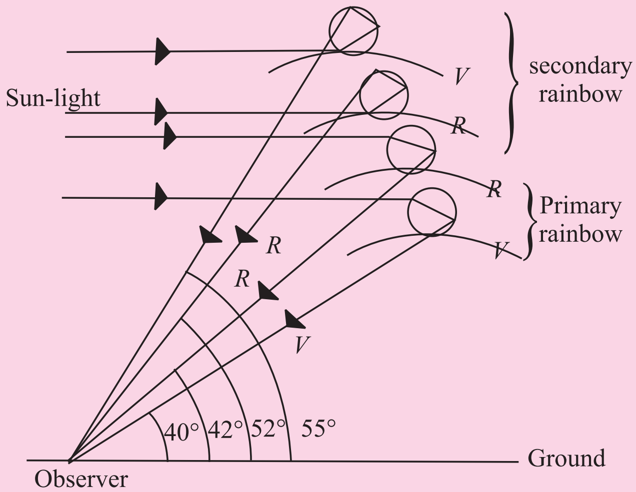


Fig. 21.7 : Simultaneous formation of the primary and secondary rainbow.

21.2 SCATTERING OF LIGHT IN ATMOSPHERE

On a clear day when we look at the sky, it appears blue. But the clouds appear white. Similarly, production of brilliant colours when sunlight passes through jewels and crystals also attracts our attention. You may like to know : How and why does it happen? These phenomena can be explained in terms of *scattering of light*. A solution of dust or particle-free benzene exposed to sunlight gives brilliant blue colour when looked sideways.

21.2.1 Scattering of Light

This phenomenon involves interaction of radiation with matter. Tiny dust particles are present in Earth's atmosphere. When sunlight falls on them, it gets diffused in all directions. That is why light reaches even those nooks and corners where it normally is not able to reach straight from the source.

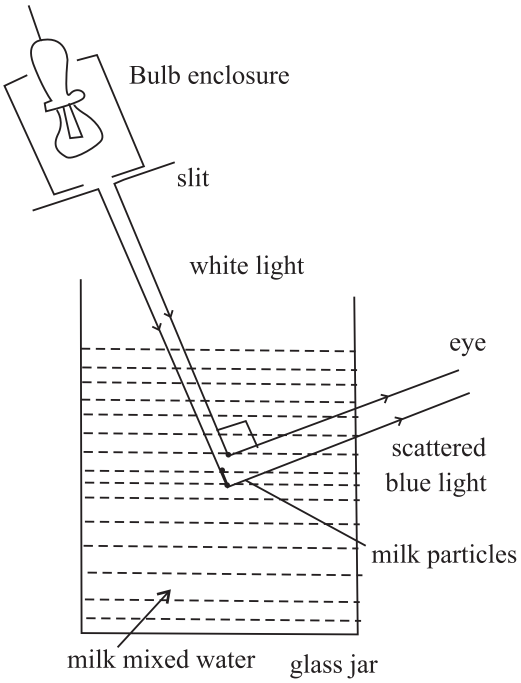


Fig. 21.8 : The scattering of light from milk particles

Let us perform a simple activity.

ACTIVITY 21.1

Take a glass jar or a trough, fill it with water and add a little milk to it. Now allow a narrow beam of light from a white bulb to fall on it. Observe the light at 90^o. You will see a bluish beam through water. This experiment shows that after scattering, the wavelenghts of light become a peculiarly different in a given direction (Fig. 21.14).

The phenomenon of scattering is a two step process : absorption of light by the scattering particle and then instant re-emission by it in all possible directions. Thus, this phenomenon is different from reflection. The scattered light does not obey the laws of reflection. It is important to note that the size of the particle must be less than the wavelength of light incident on it. A bigger sized particle will scatter all the wavelengths equally. The intensity of scattered light is given by *Rayleigh's law* of scattering. According to this law, ***the intensity of scattered light is inversely proportional to the fourth power of its wavelength***:

$$I \ \alpha \ \frac{1}{\lambda^4}$$

Here I is intensity and λ is wavelength of the scattered light. Thus, when white light is incident on the scattering particle, the blue light is scattered the most and the red light is scattered the least.

Example 21.4 : Waves of wavelength 3934Å, 5890Å and 6867Å are found in the scattered beam when sunlight is incident on a thin layer of chimeny smoke. Which of these is scattered more intensely?

Solution : The intensity of scattered light is given by

$$I \alpha \frac{1}{\lambda^4}$$

Since 3934Å is the smallest wavelength, it will be scattered most intensely.

On the basis of scattering of light, we can explain why sky appears blue, clouds appear white and the sun appears red at sunrise as well as at sunset.

C.V. Raman
(1888 – 1970)

Chandra Shekhar Venkat Raman is the only Indian national to receive Nobel prize (1930) in physics till date. His love for physics was so intense that he resigned his job of an officer in Indian finance department and accepted the post of Palit Professor of Physics at the Department of Physics, Calcutta University. His main contributions are : Raman effect on scattering of light, molecular diffraction of light, mechanical theory of bowed strings, diffraction of X-rays, theory of musical instruments and physics of crystals.

As Director of Indian Institute of Science, Bangalore and later as the founder Director of Raman Research Institute, he did yeoman's to Indian science and put it on firm footings in pre-independence period.

(A) Blue Colour of the Sky

We know that scattering of light by air molecules, water droplets or dust particles present in the atmosphere can be explained in accordance with Rayleigh's law. The shorter wavelengths are scattered more than the longer wavelengths. Thus, the blue light is scattered almost six times more intensely than the red light as the wavelength of the blue light is roughly 0.7 times that of the red. The scattered light becomes rich in the shorter wavelengths of violet, blue and green colours. On further scattering, the violet light does not reach observe's eye as the eye is comparatively less sensitive to violet than blue and other wavelengths in its neighbourhood. So, when we look at the sky far away from the sun, it appears blue.

Example 21.5 : What will be the colour of the sky for an astronant in a spaceship flying at a high attitude.

Solution : At a high attitude, in the absence of dust particle and air molecules, the sunlight is not scattered. So, the sky will appear black.

(B) White colour of the clouds

The clouds are formed by the assembly of small water drops whose size becomes more than the average wavelength of the visible light (5000Å). These droplets scatter all the wavelengths with almost equal intensity. The resultant scattered light is therefore white. So, a thin layer of clouds appears white. What about dense clouds?

(C) Red colour of the Sun at Sunrise and Sunset

We are now able to understand the red colour of the Sun at sunrise and sunset. In the morning and evening when the Sun is near the horizon, light has to travel a greater distance through the atmosphere. The violet and blue wavelengths are scattered by dust particles and air molecules at an angle of about 90°. The sunlight thus becomes devoid of shorter wavelengths and the longer wavelength of red colour reaches the observer (Fig. 21.9). So the Sun appears to us as red.

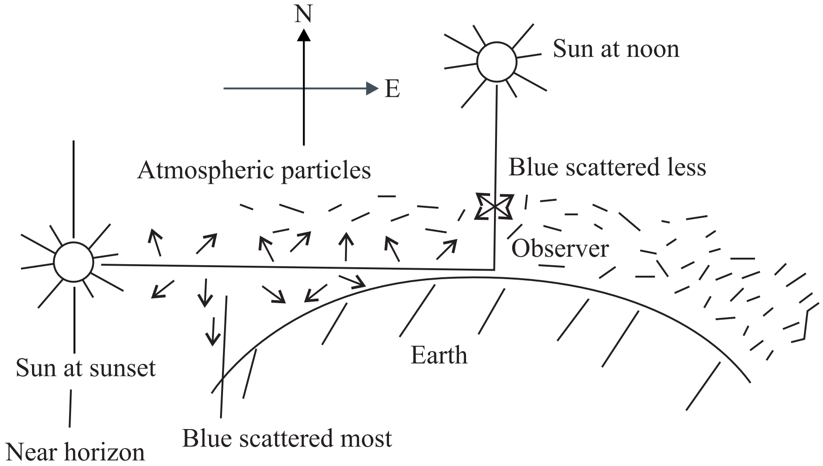


Fig. 21.9 : Red colour of the sun at sunset and sunrise (blue is scattered away).

At noon, the Sun is overhead and its distance from the observer is comparatively less. The blue colour is also scattered less. This results in the Sun appearing white, as a matter of fact, crimson.

21.2.2 Raman effect

When light radiation undergoes scattering from a transparent substance (solid, liquid or gas) then the frequency of the scattered radiation may be greater or less than the frequency of the incident radiation. This phenomenon is known as Raman effect as it was first observed by C. V. Raman in 1926. An analogue of this optical phenomenon was observed earlier by A. H. Compton in connection with the scattering of X-rays. The spectrum of the scattered radiation is known as Raman spectrum. This has lines having frequency greater than the frequency of the incident radiation (known as anti-Stokes' lines) as also lines having frequency less than the frequency of the incident radiation (called Stokes' lines).

A simple explanation of Raman effect can be given as follows. When light radiation interacts with a substance three possibilities may arise. In the first possibility, the light radiation interacting with the substance does not undergo any change of energy. Hence, its frequency remains unchanged. In the second possibility, the light radiation may impart some of its energy to the substance. As a result, the energy of the light radiation decreases. This leads to a decrease in the frequency of the scattered radiation (corresponding to Stokes' lines). In the third possibility, the incident radiation may interact with the substance which is already in the excited state. In the process, the radiation gains energy resulting into increase in its frequency (corresponding to anti-Stokes' lines).

Raman effect has lot of applications in various fields. C. V. Raman was awarded Nobel prize in physics for this discovery in 1930.

INTEXT QUESTIONS 21.2

1. Why dense clouds appear black?
2. Why does the sky appear deep blue after rains on a clear day?
3. Can you suggest an experiment to demonstrate the red colour of the Sun at sunrise and sunset?
4. The photographs taken from a satellite show the sky dark. Why?
5. What are anti stokes' lines?

WHAT YOU HAVE LEARNT

- Light of single wavelength or colour is said to be monochromatic but sunlight, which has several colours or wavelengths, is polychromatic.
- The splitting of light into its constituent wavelengths on entering an optically denser medium is called dispersion.
- A prism is used to produce dispersed light, which when taken on the screen, forms the spectrum.
- The angle of deviation is minimum if the angles of incidence and emergence become equal. In this situation, the beam is most intense for that colour.

- The angle of deviation and refractive index for a small-angled prism are connected by the ralation $\delta = (\mu - 1)A$.
- The rainbow is formed by dispersion of sunlight by raindrops at definite angles for each colour so that the condition of minimum deviation is satisfied.
- Rainbows are of two types : primary and secondary. The outer side of the primary rainbow is red but the inner side is violet. The remaining colours lie in between to follow the order (VIBGYOR). The scheme of colours gets reversed in the secondary rainbow.
- The blue colour of the sky, the white colour of clouds and the reddish colour of the Sun at sunrise and sunset are due to scattering of light. The intensity of scattered light is inversely proportional to the fourth power of the wavelength $\left(I \,\alpha \frac{1}{\lambda^4}\right)$. This is called Rayleigh's law. So the blue colour is scattered more than the red.
- When light radiation undergoes scattering from a transparent substance, then frequency of scatered radiation may be greater or less than frequency of incident ratiation. This phenomenon is known as Raman effect.

TERMINAL EXERCISE

1. For a prism, show that $i + e = A + \delta$.
2. Would you prefer small-angled or a large-angled prism to produce dispersion. Why?
3. Under what condition is the deviation caused by a prism directly proportional to its refractive index?
4. Explain why the sea water appears blue at high seas.
5. The angle of minimum deviation for a 60^0 glass prism is 39^0. Calculate the refractive index of glass.
6. The deviation produced for red, yellow and violet colours by a crown glass are 2.84^o, 3.28^o and 3.72^o respectively. Calculate the dispersive power of the glass material.
7. Calculate the dispersive power for flint glass for the following data : $\mu_C = 1.6444$, $\mu_D = 1.6520$ and $\mu_F = 1.6637$, where C, D & F are the Fraunhofer nomenclatures.
8. A lens can be viewed as a combination of two prisms placed with their bases together. Can we observe dispersion using a lens. Justify your answer.
9. Human eye has a convex lens. Do we observe dispersion with unaided eye?

ANSEWERS TO INTEXT QUESTIONS

21.1

1. The velocity of propagation of waves of different wavelengths of visible light is almost the same in most ordinary gases. Hence, they do not disperse visible light. Their refractive index is also very close to 1.

2. Violet, because $\lambda_r > \lambda_v$ and the velocity of the red light is more than that of the violet light inside an optically denser medium.

3. No

4. $\mu = \dfrac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3} = 1.732$

21.2

1. It absorbs sunlight

2. It becomes clear of dust particles and bigger water molecules. The scattering now takes place strictly according to Rayleigh's law.

3. We can take sodium thiosulphate solution in a round bottom flask and add a small quality of sulphuric acid. On illuminating this solution with a high power bulb, we can see a scenario similar to the colour of the sun at sunrise and sunset.

4. At very high altitudes no centres (particles) of scattering of sunlight are present. So the sky appears dark.

5. The spectral lines having frequency greater than the frequency of incident radiation are known as anti stokes' lines.

Answers to Problems in Terminal Exercise

5. 1.5　　6. 0.27

7. 0.03

23

OPTICAL INSTRUMENTS

We get most of the information about the surrounding world through our eyes. But as you know, an unaided eye has limitations; objects which are too far like stars, planets etc. appear so small that we are unable to see their details. Similarly, objects which are too small, e.g. pollen grains, bacteria, viruses etc. remain invisible to the unaided eyes. Moreover, our eyes do not keep a permanent record of what they see, except what is retained by our memory. You may therefore ask the question: How can we see very minute and very distant objects? The special devices meant for this purpose are called **optical instruments.**

In this lesson you will study about two important optical instruments, namely, a microscope and a telescope. As you must be knowing, a microscope magnifies small objects while a telescope is used to see distant objects. The design of these appliances depends on the requirement. (The knowledge of image formation by the mirrors and lenses, which you have acquired in Lesson 20, will help you understand the working of these optical instruments.) The utility of a microscope is determined by its magnifying power and resolving power. For a telescope, the keyword is *resolving power.* You must have read about Hubble's space telescope, which is being used by scientists to get details of far off galaxies and search for a life-sustaining planet beyond our solar system.

OBJECTIVES

After studying this lesson, you should be able to:

- *explain the working principle of simple and compound microscopes;*
- *derive an expression for the magnifying power of a microscope;*
- *distinguish between linear and angular magnifications;*
- *explain the working principle of refracting and reflecting telescopes; and*
- *calculate the resolving powers of an eye, a telescope and a microscope.*

23.1 MICROSCOPE

In Lesson 20 you have learnt about image formation by mirrors and lenses. If you take a convex lens and hold it above this page, you will see images of the alphabets/ words. If you move the lens and bring it closer and closer to the page, the alphabets printed on it will start looking enlarged. This is because their enlarged, virtual and errect image is being formed by the lens. That is, it is essentially acting as a magnifying glass or simple microscope. You may have seen a doctor, examining measels on the body of a child. Watch makers and jewellers use it to magnify small components of watches and fine jewellery work. You can take a convex lens and try to focus sunlight on a small piece of paper. You will see that after some time, the piece of paper start burning. A convex lens can, therefore start a fire. That is why it is dangerous to leave empty glass bottles in the woods. The sunlight falling on the glass bottles may get focused on dry leaves in the woods and set them on fire. Sometimes, these result in wild fires, which destroy large parts of a forest and/or habitation. Such fires are quite common in Australia, Indonesia and U.S.

As a simple microscope, a convex lens is satisfactory for magnifying small nearby objects upto about twenty times their original size. For large magnification, a compound microscope is used, which is a combination of basically two lenses. In a physics laboratory, a magnifying glass is used to read vernier scales attached to a travelling microscope and a spectrometer.

While studying simple and compound microscopes, we come across scientific terms like (i) near point, (ii) least distance of distinct vision, (iii) angular magnification or magnifying power, (iv) normal adjustment etc. Let us first define these.

(i) **Near point** is the distance from the eye for which the image of an object placed there is formed (by eye lens) on the retina. The near point varies from person to person and with the age of an individual. At a young age (say below 10 years), the near point may be as close as 7-8 cm. In the old age, the near point shifts to larger values, say 100-200 cm or even more. That is why young children tend to keep their books so close whereas the aged persons keep a book or newspaper far away from the eye.

(ii) **Least distance of distinct vision** is the distance upto which the human eye can see the object clearly without any strain on it. For a normal human eye, this distance is generally taken to be 25 cm.

(iii) **Angular magnification** is the ratio of the angle subtended by the image at the eye (when the microscope is used) to the angle subtended by the object at the unaided eye when the object is placed at the least distance of distinct vision. It is also called the magnifying power of the microscope.

(iv) **Normal Adjustment:** When the image is formed at infinity, least strain is exerted on the eye for getting it focused on the retina. This is known as normal adjustment.

(v) **Linear magnification** is the ratio of the size of the image to the size of the object.

(vi) **Visual angle** is the angle subtended by the object at human eye.

23.1.1 A Simple Microscope

When a convex lens of short focal length is used to see magnified image of a small object, it is called a simple microscope.

We know that when an object is placed between the optical center and the focus of a convex lens, its image is virtual, erect, and magnified and on the same side as the object. In practice, such a lens is held close to eye and the distance of the object is adjusted till a clear image is formed at the least distance of distinct vision. This is illustrated in Fig. 23.1, which shows an object AB placed between F and O. Its virtual image $A'B'$ is formed on the same side as the object. The position of the object is so adjusted that the image is formed at the least distance of distinct vision (D).

Magnifying power of a simple microscope

Magnifying power of an optical instrument is the ratio of the angle subtended by the image at the eye to the angle subtented by the object seen directly, when both lie at the least distance of distinct vision or the near point. It is also called angular magnification and is denoted by M. Referring to Fig. 23.1(a) and (b), the angular magnification of simple

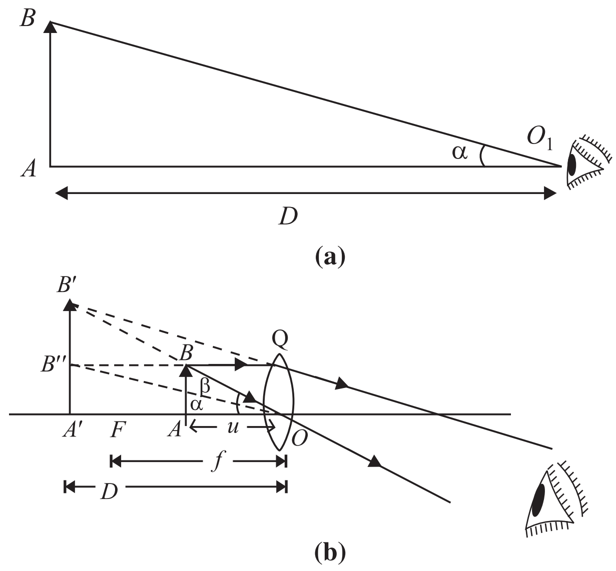


Fig.23.1 : Angular magnification of a magnifying glass

microscope is given by $M = \frac{\angle A'OB'}{AO'B} = \frac{\beta}{\alpha}$. In practice, the angles α and β are small. Therefore, you can replace these by their tangents, i.e. write

$$M = \frac{\tan\beta}{\tan\alpha} \qquad (23.1)$$

From $\Delta s\ A'OB'$ and AOB, we can write $\tan\beta = \frac{A'B'}{A'O} = \frac{A'B'}{D}$ and

$\tan\alpha = \frac{A'B''}{A'O} = \frac{AB}{D}$. On putting these values of $\tan\beta$ and $\tan\alpha$ in Eqn. (23.1), we get

$$M = \frac{A'B'}{D} \Big/ \frac{AB}{D} = \frac{A'B'}{AB}$$

Since $\Delta s\ AOB$ and $A'OB'$ in Fig 23.1(b) are similar, we can write

$$\frac{A'B'}{AB} = \frac{A'O}{AO} \qquad (23.2)$$

Following the standard sign convention, we note that

$$A'O = -D$$

and

$$AO = -u$$

Hence, from Eqn. (23.2), we obtain

$$\frac{A'B'}{AB} = \frac{D}{u} \qquad (23.3)$$

If f is the focal length of the lens acting as a simple microscope, then using the lens formula $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ and noting that $v = -D$, $u = -u$ and $f = f$, we get

$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$$

or

$$-\frac{1}{D} + \frac{1}{u} = \frac{1}{f}$$

Multiplying both the sides by D, and rearranging term, you can write

$$\frac{D}{u} = 1 + \frac{D}{f} \qquad (23.4)$$

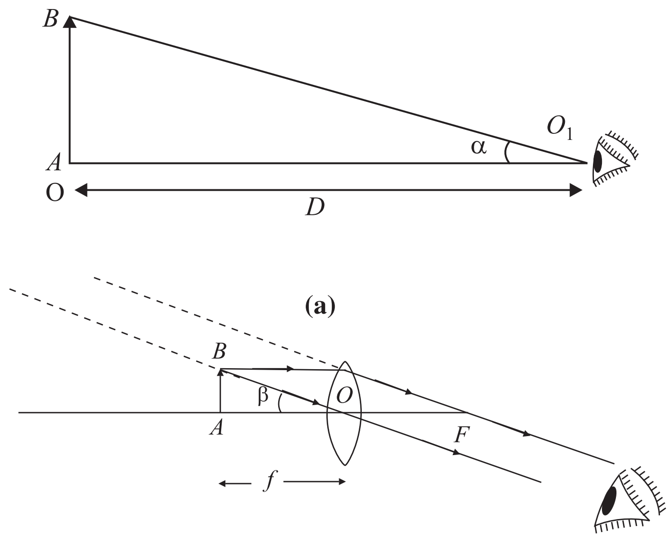
On combining Eqns. (23.3) and (23.4), we get

$$\frac{A'B'}{AB} = 1 + \frac{D}{f}$$

or $$M = 1 + \frac{D}{f} \qquad (23.5)$$

From this result we note that lesser the focal length of the convex lens, greater is the value of the angular magnification or magnifying power of the simple microscope.

Normal Adjustment : In this case, the image is formed at infinity. The magnifying power of the microscope is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the unaided eye when the object is placed at *D*. Fig 23.2(a) shows that the object is placed at the least distance of distinct vision *D*.



(b)

Fig.23.2 : Image formation for normal adjustment

The angles subtended by the object and the image at the unaided eye are α and β, respectively. The magnifying power is defined as

$$M = \frac{\beta}{\alpha}$$

In practice, the angles α and β are small, and, as before, replacing these by their tangents, we get

$$M = \frac{\tan\beta}{\tan\alpha}$$

i.e. $$= \frac{AB}{AO} \Big/ \frac{AB}{AO_1}$$

$$= \frac{AO_1}{AO} = \frac{D}{f}$$

or $$M = \frac{D}{f} \qquad (23.6)$$

You may note that in the normal adjustment, the viewing of the image is more comfortable. To help you fix your ideas, we now give a solved example. Read it carefully.

Example 23.1: Calculate the magnifying power of a simple microscope having a focal length of 2.5 cm.

Solution : For a simple microscope, the magnifying power is given by [Eqn. (23.5)] :

$$M = 1 + \frac{D}{f}$$

Putting D = 25 cm and f = 2.5cm, we get

$$M = 1 + \frac{25}{2.5} = 1 + 10 = 11$$

23.1.2 A Compound Microscope

A compound microscope consists of two convex lenses. A lens of short aperture and short focal length faces the object and is called the **objective**. Another lens of short focal length but large aperture facing the eye is called the **eye piece**. The objective and eye piece are placed coaxially at the two ends of a tube.

When the object is placed between F and $2F$ of the objective, its a real, inverted and magnified image is formed beyond $2F$ on the other side of the objective. This image acts as an object for the eye lens, which then acts as a simple microscope. The eye lens is so adjusted that the image lies between its focus and the optical center so as to form a magnified image at the least distance of distinct vision from the eye lens.

Magnifying Power of a compound microscope

Magnifying power of a compound microscope is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object at unaided eye, when both are placed at the least distance of distinct vision. It is denoted by M. By referring to Fig. 23.3, we can write

$$M = \frac{\beta}{\alpha}$$

Since the angles α and β are small, these can be replaced by their tangents, so that

$$M = \frac{\tan\beta}{\tan\alpha}$$

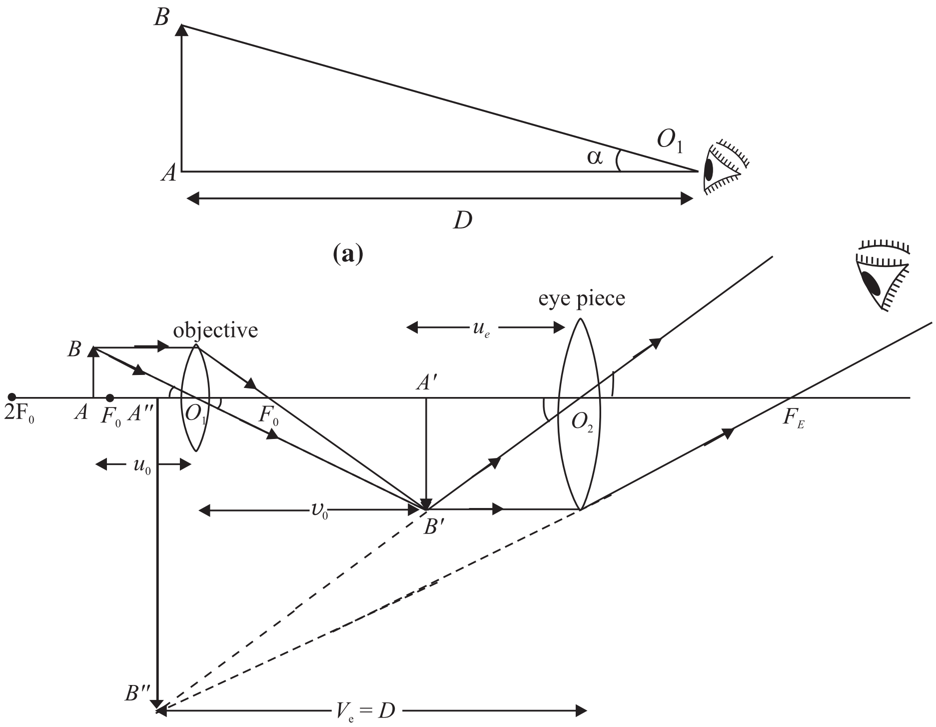


Fig.23.3 : Image formation by a compound microscope when the final image is formed at the least distance of distinct vision.

$$M = \frac{A''B''}{D} \Big/ \frac{AB}{D}$$

$$\Rightarrow \qquad M = \frac{A''B''}{AB} = \frac{A''B''}{A'B'} \cdot \frac{A'B'}{AB}$$

From similar $\Delta s\ A''B''O_2$ and $A'B'O_2$, we can write

$$\frac{A''B''}{A'B'} = \frac{A''O_2}{A'O_2} = \frac{D}{u_e}$$

Also from similar $\Delta s A'B'O_1$ and ABO, we have

$$\frac{A'B'}{AB} = \frac{v_o}{u_o}$$

Note that $m_e = \dfrac{A''B''}{A'B'}$ defines magnification produced by eye lens and $m_o = \dfrac{A'B'}{AB}$ denotes magnification produced by the objective lens. Hence

$$M = \frac{D}{u_e} \cdot \frac{v_o}{u_o} = m_e \times m_o \qquad (23.7)$$

From Lesson 20, you may recall the lens formula. For eye lens, we can write

$$\frac{1}{\upsilon_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

Multiply on both sides by υ_e to get

$$\frac{\upsilon_e}{\upsilon_e} - \frac{\upsilon_e}{u_e} = \frac{\upsilon_e}{f_e}$$

$$\Rightarrow \qquad \frac{\upsilon_e}{u_e} = 1 - \frac{\upsilon_e}{f_e}$$

Since f_e is positive and $\upsilon_e = -D$ as per sign convention, we can write

$$m_e = \frac{\upsilon_e}{u_e} = 1 + \frac{D}{f_e} \qquad (23.8)$$

On combining Eqns. (23.7) and (23.8), we get

$$M = \frac{\upsilon_o}{u_o} \times \left(1 + \frac{D}{f_e}\right)$$

In practice, the focal length of an objective of a microscope is very small and object *AB* is placed just outside the focus of objective. That is

$$\therefore \qquad u_o \approx f_0$$

Since the focal length of the eye lens is also small, the distance of the image *A′B′* from the object lens is nearly equal to the length of the microscope tube i.e.

$$\upsilon_o \approx L$$

Hence, the relation for the magnifying power in terms of parameters related to the microscope may be written as

$$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \qquad (23.10)$$

Magnifying power in normal adjustment : In this case the image is formed at infinity. As discussed earlier, the magnifying power of the compound microscope may be written as

$$M = m_o \times m_e$$

$$= \frac{\upsilon_o}{u_o}\left(\frac{D}{f_e}\right)$$

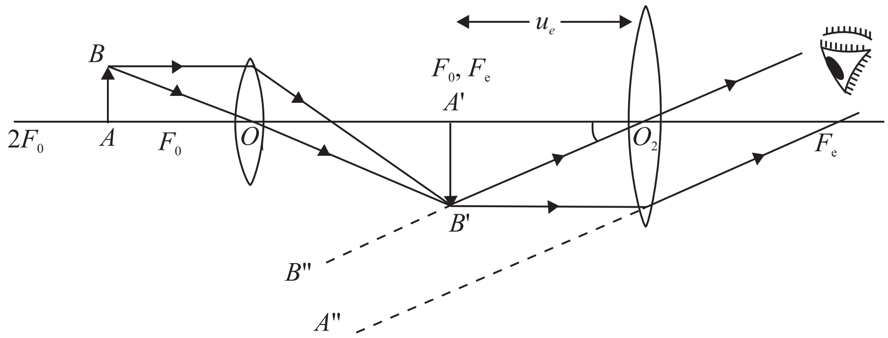


Fig. 23.4 : Compound microscope in normal adjustment

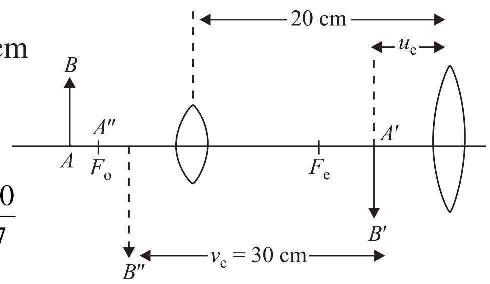
You may now like to go through a numerical example.

Example 23.2 : A microscope has an objective of focal length 2 cm, an eye piece of focal length 5 cm and the distance between the centers of two lens is 20 cm. If the image is formed 30 cm away from the eye piece, find the magnification of the microscope.

Solution : For the objective, f_o= 2 cm and f_e = 5 cm. For the eyepiece, υ_e = –30 cm and f_e = 5 cm. We can calculate υ_e using the relation

$$\frac{1}{\upsilon_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

On solving, you will easily obtain $u_e = -\frac{30}{7}$ cm



For the objective lens

$$\upsilon_o = 20 - \frac{30}{7}$$

$$= \frac{110}{7} \text{ cm}$$

Using the formula

$$\frac{1}{\upsilon_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

we have

$$\frac{1}{110/7} - \frac{1}{u_o} = \frac{1}{2}$$

or

$$u_o = -\frac{110}{48} \text{ cm}$$

The magnifying power of the objective

$$m_o = \frac{v_o}{u_o} = \frac{110/7}{-110/48} = -\frac{48}{7}$$

The magnification due to the eyepiece is

$$m_o = \frac{v_e}{u_e} = \frac{-30/1}{-30/7} = 7$$

Therefore, the magnification of the microscope is given by

$$M = (m_o)\,(m_e)$$

$$= \left(-\frac{48}{7}\right)(7) = -48$$

INTEXT QUESTIONS 23.1

1. What is the nature of images formed by a (i) simple microscope (ii) Compound microscope?
2. Differentiate between the magnifying power and magnification?
3. The magnifying power of a simple microscope is 11. What is its focal length?
4. Suppose you have two lenses of focal lengths 100 cm and 4 cm respectively. Which one would you choose as the eyepiece of your compound microscope and why?
5. Why should both the objective and the eyepiece of a compound microscope have short focal lengths?

23.2 TELESCOPES

Telescopes are used to see distant objects such as celestial and terrestrial bodies. Some of these objects may not be visible to the unaided eye. The visual angle subtended by the distant objects at the eye is so small that the object cannot be perceived. The use of a telescope increases the visual angle and brings the image nearer to the eye. Mainly two types of telescopes are in common use : refracting telescope and reflecting telescope. We now discuss these.

23.2.1 Refracting Telescope

The refracting telescopes are also of two types :

Notes

- **Astronomical telescopes** are used to observe heavenly or astronomical bodies.
- **Terrestrial telescopes** are used to see distant objects on the earth. So it is necessary to see an erect image. Even Galilean telescope is used to see objects distinctly on the surface of earth.

An astronomical telescope produces a virtual and erect image. As heavenly bodies are round, the inverted image does not affect the observation. This telescope consists of a two lens system. The lens facing the object has a large aperture and large focal length (f_o). It is called the *objective*. The other lens, which is towards the eye, is called the *eye lens*. It has a small aperture and short focal length (f_e). The objective and eye-piece are mounted coaxially in two metallic tubes.

The objective forms a real and inverted image of the distant object in its focal plane. The position of the lens is so adjusted that the final image is formed at infinity. (This adjustment is called normal adjustment.) The position of the eyepiece can also be adjusted so that the final image is formed at the least distance of distinct vision.

(a) When the final image is formed at infinity (Normal adjustment), the paraxial rays coming from a heavenly object are parallel to each other and they make an angle α with the principal axis. These rays after passing through the objective, form a real and inverted image in the focal plane of objective. In this case, the position of the eyepiece is so adjusted that the final image is formed at infinity.

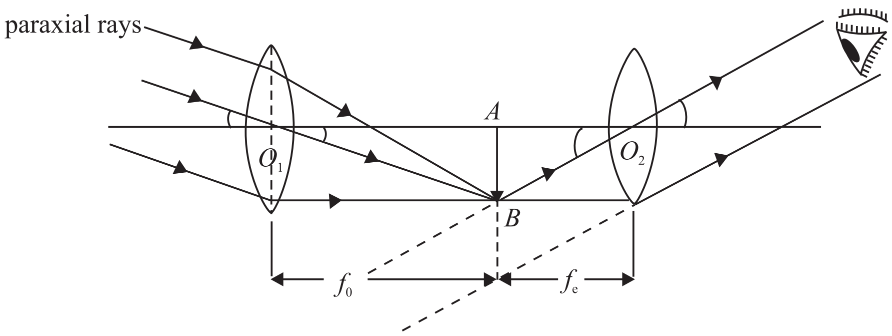


Fig 23.6 : Working principle of an astronomical telescope

Magnifying power of a telescope is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object at objective when both the object and the image lie at infinity. It is also called **angular magnification** and is denoted by M. By definition,

$$M = \frac{\beta}{\alpha}$$

Since α and β are small, they can be replaced by their tangents. Therefore,

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$= \frac{AB/AO_2}{AB/AO_1} = \frac{AO_1}{AO_2}$$

$$= \frac{f_o}{f_e} \qquad (23.11)$$

It follows that the magnifying power of a telescope in normal adjustment will be large if the objective is of large focal length and the eyepiece is of short focal length. The length of telescope in normal adjustment is $(f_o + f_e)$

(b) When the final image is formed at the least distance of distinct vision, the paraxial rays coming from a heavenly object make an angle α with the principal axis. After passing through the objective, they meet on the other side of it and form a real and inverted image *AB*. The position of the eyepiece is so adjusted that it finally forms the image at the least distance of distinct vision.

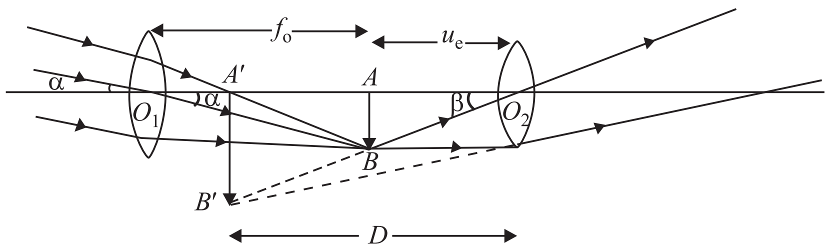


Fig 23.7 : Image formed by a telescope at *D*

Magnifying power: It is defined as the ratio of the angle subtended at the eye by the image formed at *D* to the angle subtended by the object lying at infinity:

$$M = \frac{\beta}{\alpha}$$

$$\approx \frac{\tan\beta}{\tan\alpha}$$

$$= \frac{AB/AO_2}{AB/AO_1} = \frac{AO_1}{AO_2}$$

$$= \frac{f_o}{u_e} \qquad (23.12)$$

Since $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$ for the eyepiece, we can write

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= -\frac{1}{f_e}\left(1 - \frac{f_e}{v_e}\right)$$

or $$M = \frac{f_o}{u_e} = -\frac{f_o}{f_e}\left(1 - \frac{f_e}{v_e}\right) \qquad (23.13)$$

Applying the new cartesian sign convention $f_o = +f_o$, $v_e = -D$, $f_e = +f_e$, we can write

$$M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) \qquad (23.14)$$

The negative sign of magnifying power of the telescope suggests that the final image is inverted and real. The above expression tells that the magnifying power of a telescope is larger when adjusted at the least distance of distinct vision to the telescope when focused for normal adjustment.

Example 23.3: The focal length of the objective of an astronomical telescope is 75 cm and that of the eyepiece is 5 cm. If the final image is formed at the least distance of distinct vision from the eye, calculate the magnifying power of the telescope.

Solution:

Here f_o = 75 cm, f_e = 5 cm, D = 25 cm

$$M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = \frac{-75}{5}\left(1 + \frac{5}{25}\right) = -18$$

23.2.2 Reflecting telescope

A reflecting telescope is used to see distant stars and possesses large light-gathering power in order to obtain a bright image of even a faint star deep in space. The objective is made of a concave mirror, having large aperture and large focal length. This concave mirror, being parabolic in shape, is free from spherical aberration.

Before the reflected rays of light meet to form a real, inverted and diminished image of a distant star at the focal plane of concave mirror, they are intercepted and reflected by a plane Mirror M_1M_2 inclined at an angle of 45 to the principal axis of the concave mirror. This plane mirror deviates the rays and the real image is formed in front of the eye piece, which is at right angle to the principal axis of concave mirror. The function of the eye- piece is to form a magnified, virtual image of the star enabling eye to see it distinctly.

If f_o is the focal length of the concave mirror and f_e is the focal length of eye piece, the magnifying power of the reflecting telescope is given by

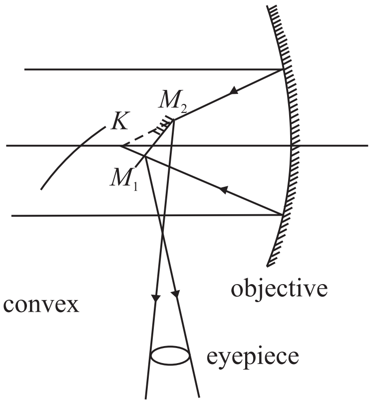


Fig 23.8 : Newtonion Reflector

$$M = \frac{f_o}{f_e}$$

Further, if D is the diameter of the objective and d is the diameter of the pupil of the eye, the brightness ratio is given by

$$B = D^2/d^2$$

The other form of the reflecting telescope is shown in Fig 23.9. It was designed by **Cassegrain**. In this case the objective has a small opening at its center. The rays from the distant star, after striking the concave mirror, are made to intercept at A_2 and the final image is viewed through the eyepiece.

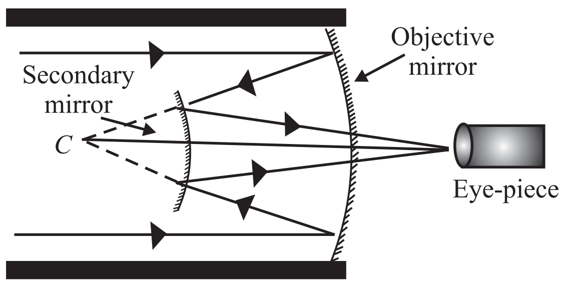


Fig 23.9 : Cassegrain reflector

There are several advantages of a reflecting telescope over a refracting telescope.

- Since the objective is not a lens, the reflecting telescopes are free from chromatic aberration. Thus rays of different colours reaching the objective from distant stars are focussed at the same point.
- Since the spherical mirrors are parabolic mirrors, free from spherical aberration, they produce a very sharp and distinct image.
- Even a very faint star can be seen through the reflecting telescope because they have large aperture and have large light-gathering power. The brightness of the image is directly proportional to the area of the objective :

$$B \ \alpha \ \frac{\pi D^2}{4}$$

where D is the diameter of the objective of the telescope. If d is the diameter of the pupil of the eye then brightness of the telescope B is defined as the ratio of light gathered by the telescope to that gathered by the unaided eye from the distant object

$$B = \frac{\pi D^2/4}{\pi d^2/4} = \frac{D^2}{d^2}$$

- In reflecting type of telescopes, there is negligible absorption of light.
- Large apertures of reflecting telescope enable us to see minute details of distant stars and explore deeper into space. That is why in recent years, astronomers have discovered new stars and stellar systems. You should look out for such details in science magazines and news dailies.

INTEXT QUESTIONS 23.2

1. How would the magnification of a telescope be affected by increasing the focal length of:

   (a) the objective ____

   ____

   (b) the eye piece ____

   ____

2. If the focal length of the objective of a telescope is 50 cm and that of the eyepiece is 2 cm. What is the magnification?
3. State one difference between the refracting and reflecting telescope.
4. What is normal adjustment?
5. If the telescope is inverted, will it serve as a microscope?

23.3 RESOLVING POWER : THE RAYLEIGH'S CRITERION

In earlier lessons, you have seen that the image of a point source is not a point, but has a definite size and is surrounded by a diffraction pattern. Similarly, if there are two point sources very close to each other, the two diffraction patterns formed by the two sources may overlap and hence it may be difficult to distinguish them as separate by the unaided eye. The resolving power of an optical instrument is its ability to resolve (or separate) the images of two point objects lying close to each other. Rayleigh suggested that two images can be seen as distinct when the first minimum of the diffraction pattern due to one object falls on the central maximum of the other. This is called *Rayleigh's criterion.*

If we assume that the pupil of our eye is about 2 mm in diameter, two points can be seen distinctly separate if they subtend an angle equal to about one minute of arc at the eye. **The reciprocal of this angle is known as the resolving power of the eye.**

Now let us calculate the resolving power of common optical instruments. We begin our discussion with a telescope.

23.3.1 Resolving Power of a Telescope

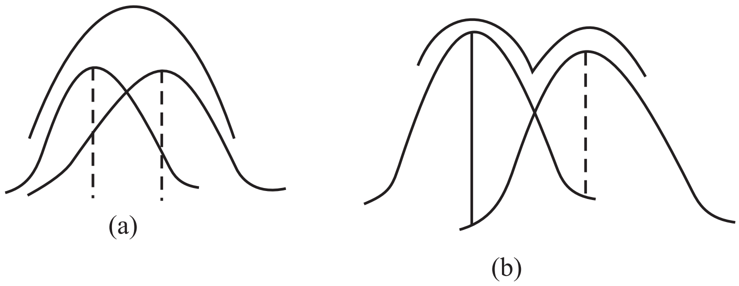


Fig. 23.10 : Rayleigh's criterion for resolution **a)** when the angular separation is less than θ, the two points are seen as one, and **b)** when the angular seperation is more than θ, the two points are distinctly visible.

The resolving power of a telescope is its ability to form separate images of two distant point objects situated close to each other. It is measured in terms of the angle subtended at its objective by two close but distinct objects whose images are just seen in the telescope as separate. This angle is called the **limit of resolution** of the telescope. If the angle subtended by two distinct objects is less than this angle, the images of the objects can not be resolved by the telescope. The smaller the value of this angle, higher will be the resolving power of the telescope. Thus, the reciprocal of the limit of resolution gives the resolving power of the telescope.

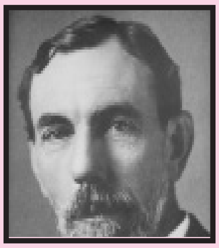
If λ is the wavelength of light, D the diameter of the telescope objective, and θ the angle subtended by the point object at the objective, the limit of resolution of the telescope is given by (Rayleigh's criterion)

$$\theta = \frac{1.22\lambda}{D}$$

Hence, the resolving power of the telescope.

$$(\text{R.P})_T = \frac{1}{\theta} = \frac{D}{1.22\lambda} \qquad (23.15)$$

From Eqn. (23.15) it is clear that to get a high resolving power, a telescope with large aperture objective or light of lower wavelength has to be used.



Lord Rayleigh
(1842 – 1919)

Born to the second Baron Rayleigh of Terling place, Witham in the country of Essex, England, John strutt had a very poor health in his childhood. Due to this he had a disrupted schooling. But he had the good luck of having Edward Rath and Stokes as his teachers. As a result, he passed his tripos examination in 1865 as senior Wrangler and become the first recipient of Smiths prize.

In addition to the discovery of Argon, for which he was awarded Nobel prize (1904), Rayleigh did extensive work in the fields of hydrodynamics, thermodynamics, optics and mathematics. His travelling wave theory, which suggested that elastic waves can be guided by a surface, paved way for researches in seismology and electronic signal processing. During the later years of his life, he also showed interest in psychiatry research. Lunar feature-crater Rayleigh and planetary feature crater Rayleigh on Mars are a tribute to his contributions.

Example 23.4: A telescope of aperture 3 cm is focussed on a window at 80 metre distance fitted with a wiremesh of spacing 2 mm. Will the telescope be able to observe the wire mesh? Mean wavelength of light $\lambda = 5.5 \times 10^{-7}$ m.

Solution: Given $\lambda = 5.5 \times 10^{-7}$ m and $D = 3$ cm $= 3 \times 10^{-2}$ m

Therefore, the limit of resolution

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5.5 \times 10^{-7}\,\text{m}}{3 \times 10^{-2}\,\text{m}} = 2.236 \times 10^{-5}\ \text{rad}$$

The telescope will be able to resolve the wiremesh, if the angle subtended by it on the objective is equal to or greater than θ, the limit of resolution. The angle subtended by the wiremesh on the objective

$$\alpha = \frac{\text{spacing of wiremesh}}{\text{distance of the objective from the wiremesh}}$$

$$= \frac{2\ \text{mm}}{80\ \text{m}} = \frac{2 \times 10^{-3}}{80\ \text{m}} = 2.5 \times 10^{-5}\ \text{rad.}$$

As the angle 2.5×10^{-5} radian exceeds the limit of a resolution ($= 2.236 \times 10^{-5}$ radian), the telescope will be able to observe the wire mesh.

23.3.2 Resolving Power of a Microscope

The resolving power of a microscope represents its ability to form separate images of two objects situated very close to each other. The resolving power of a microscope is measured in terms of the smallest linear separation between the two objects which can just be seen through the microscope as separate. This smallest linear separation between two objects is called **the limit of resolution of the microscope.**

The smaller the value of linear separation, the higher will be the resolving power of the microscope. Thus, **the reciprocal of the limit of resolution gives the resolving power of the microscope.**

If λ is the wavelength of light used to illuminate the object, θ is the half angle of the cone of light from the point object at the eye and n is the refractive index of the medium between the object and the objective, the limit of resolution of the microscope is given by

$$d = \frac{\lambda}{2n\sin\theta} \qquad (23.16)$$

Thus the resolving power of microscope will be

$$(R.P)_m = \frac{2n\sin\theta}{\lambda}$$

(23.17)The expression $2n\sin\theta$ is called numerical aperture (N.A) The highest value of N.A of the objective obtainable in practice is 1.6, and for the eye, N.A is 0.004.

It is clear from Eqn. (23.17) that the resolving power of a microscope can be increased by increasing the numerical aperture and decreasing the wavelength of the light used to illuminate the object. That is why ultraviolet microscopes and an electron microscope have a very high resolving power.

Applications in Astronomy

The astronomical (or optical) telescope can be used for observing stars, planets and other astronomical objects. For better resolving power, the optical telescopes are made of objectives having a large aperture (objective diameter). However, such big lenses are difficult to be made and support. Therefore, most astronomical telescopes use reflecting mirrors instead of lenses. These can be easily supported as a mirror weighs less as compared to a lens of equivalent optical quality.

The astronomical telescopes, which are ground-based, suffer from blurring of images. Also, ultraviolet, x-ray, gamma-ray etc. are absorbed by the earth's surface. They cannot be studied by ground-based telescopes. In order to study these rays coming from astronomical objects, telescopes are mounted in satellites above the Earth's atmosphere. NASA's Hubble space telescope is an example of such telescope. Chandra X-ray observation, Compton x-ray observation and Infrared telescopes have recently been set up in space.

INTEXT QUESTIONS 23.3

1. How can the resolving power of a telescope be improved?
2. What is the relationship between the limit of resolution and the resolving power of the eye?
3. If the wavelength of the light used to illuminate the object is increased, what will be the effect on the limit of resolution of the microscope?
4. If in a telescope objective is made of larger diameter and light of shorter wavelength is used, how would the resolving power change?

WHAT YOU HAVE LEARNT

- The angle subtended by an object at the human eye is called as the visual angle.
- The angular magnification or magnifying power of a microscope is the ratio of the angle subtended by the image at the eye to the angle subtended by the object when both are placed at the near point.
- Linear magnification is defined as the ratio of the size of the image to the size of the object.
- The magnifying power of a simple microscope is $M = 1+\frac{D}{f}$, where D is least distance of distinct vision and f is focal length of the lens.
- In a compound microscope, unlike the simple microscope, magnification takes place at two stages. There is an eye piece and an objective both having short focal lengths. But the focal length of the objective is comparatively shorter than that of the eye piece.
- The magnifying power of a compound microscope is given as

$$M = m_0 \times m_e$$

But $m_e = 1+\frac{D}{f}$. Therefore

$$M = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

where v_0 is distance between the image and the objective, u_0 is object distance from the objective, D is the least distance of distinct vision (= 25cm) and f_e is focal length of the eye-piece.

- Telescope is used to see the distant objects which subtend very small visual angle at the eye. The use of a telescope increases the visual angle at the eye. The far-off object appears to be closer to the eye and can be seen easily.
- Two types of telescopes are used (i) Refracting (ii) Reflecting.
- The objective of the refracting telescope is a converging lens. But the objective in a reflecting telescope is a spherical mirror of large focal length. There are several advantages of reflecting telescope over a refracting telescope.

  The magnifying power of a telescope is

$$M = f_o/f_e$$

where f_0 is focal length of the objective and f_e is focal length of the eyepiece.

TERMINAL EXERCISE

1. What is the difference between simple and compound microscopes? Derive an expression for the magnification of a compound microscope.
2. Distinguish between the refracting and reflecting telescope. Draw a ray diagram for the Newton's telescope.
3. Derive an equation for the magnifying power of a refracting telescope.
4. What do you mean by the least distance of distinct vision? What is its value for a normal eye?
5. Can we photograph the image formed by a compound microscope? Explain your answer.
6. Define the resolving power of an optical instrument. What is the value of limit of resolution for a normal eye?
7. What are the main differences in the design of a compound microscope and a terrestrial telescope?
8. The eyepiece of a telescope has a focal length of 10 cm. The distance between the objective and eye piece is 2.1 m. What is the angular magnification of the telescope?

9. The image formed by a microscope objective of focal length 4 mm is 18 cm from its second focal point. The eyepiece has a focal length of 3.125 cm. What is the magnification of the microscope?

10. The objective of a telescope has a diameter three times that of a second telescope. How much more amount of light is gathered by the first telescope as compared to the second?

ANSWERS TO INTEXT QUESTIONS

23.1

1. Image formed by a simple microscope is virtual erect and magnified. whereas the image formed by a compound microscope is real, inverted and magnified.

2. Magnifying power is the ratio of the angle subtended by the image at eye piece to the angle subtended by the object placed at the near point. Magnification is the ratio of the size of image to the size of object.

3. $M = 11$, $m = 1 + \frac{D}{f}$. Putting D =25 cm, we get $f = 2.5$ cm

4. If you choose the lens with 4 cm focal length, the magnifying power will be high because $m = \frac{f_o}{f_e}$

5. The magnifying power of a compound microscope is given by $M = \frac{-L}{f_o}\left(1+\frac{D}{f_e}\right)$

   Obviously, M will have a large value, if both f_o and f_e are small.

23.2

1. (a) Objective of large focal length increases the magnifying power of the telescope.

   (b) Magnification is reduced by increasing the focal length of eyepiece.

2. Magnification $m = \frac{f_o}{f_e} = \frac{50 \text{ cm}}{2 \text{ cm}} = 25$

3. The objectives of a telescope is a spherical mirror of large focal length instead of converging lens as in a refracting telescope.

4. A telescope is said to be in normal adjustment, if the final image is formed at infinity.

5. No

23.3

1. By taking a large aperture or by using a light of lower wavelength.
2. The limit of resolution of an eye is inversely proportional to its resolving power. Limit of resolution will also be increased.
3. Since resolving power of telescope is given by R.P = $\frac{D}{1.22\lambda}$, it would increase.

Answers To Problems in Terminal Exercise

8. 21   9. 400

10. 9 times.