Number Concepts

1

Unit Overview

In this unit you will extend your knowledge of numbers as you solve problems using whole numbers, decimals, and fractions. You will also study factorization and exponents.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- annex
- algorithm
- conjecture
- simulate

Math Terms

- visual representation
- prime number
- evaluate
- least common multiple (LCM)
- proper fraction
- mixed number
- factor
- composite number
- greatest common factor (GCF)
- least common denominator
- improper fraction
- reciprocal

ESSENTIAL QUESTIONS

? Why is it important to be able to use whole numbers, fractions, and decimals to solve problems?

? How can you use visualization and estimation to solve problems?

EMBEDDED ASSESSMENTS

These assessments, following Activities 1, 3, and 6, will give you an opportunity to demonstrate how you can use your understanding of number concepts to solve mathematical and real-world problems.

UNIT 1

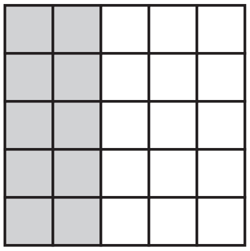
Getting Ready

1. Order the following numbers from least to greatest:

30 303 11 31 1,111 313 333

2. Why is 4×9 equal to 9×4?

3. The grid below represents the number 1. Write the number shown by the shaded part of the grid as a fraction and as a decimal.



4. Use a model to represent the fraction $\frac{5}{8}$. Then explain why your model represents $\frac{5}{8}$.

5. Is $1\frac{3}{4}$ closer to 1 or to 2? Explain your answer.

6. List three numbers that are divisible by:

a. 3 **b.** 4 **c.** 5

7. What is a divisibility rule for 2?

8. Give three numbers that satisfy each set of given conditions.

a. whole numbers that are less than 7

b. decimals that are greater than 5

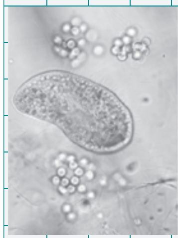
c. fractions that are less than 1

Whole Numbers and Decimals

Science, Shopping, and Society

Lesson 1-1 Comparing and Ordering Whole Numbers and Decimals

My Notes



Learning Targets:

- Locate whole numbers and decimals on a number line.
- Interpret statements of inequality of whole numbers and positive decimals.
- Order a set of positive whole numbers and decimals.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Think-Pair-Share, Sharing and Responding

Paramecia are tiny one-celled organisms commonly found in freshwater environments. They are shaped like a grain of rice. If you have excellent eyesight you might see a paramecium as a tiny moving speck, but to see one in detail you need a microscope. The lengths of four specimens of common types of paramecia were measured and are given in the table.

Type	Length (meters)
Aurelia	0.00156
Bursaria	0.00097
Caudatum	0.00181
Multimicronucleatum	0.002

1. **Model with mathematics.** Complete the table below showing the lengths of each type of paramecium.

	ones		tenths	hundredths	thousandths	ten thousandths	hundred thousandths
	1	.	0.1	0.01	0.001	0.0001	0.00001
Aurelia							
Bursaria							
Caudatum							
Multimicro-nucleatum							

2. The table shows zeros in both the tenths and hundredths place for aurelia. Name the value of the 1, the 5, and the 6.

My Notes

ACADEMIC VOCABULARY

The verb ***annex*** means to attach or to add something.

MATH TIP

If you ***annex*** or add zeros to the right of the last digit in a decimal, you do not change the value of the decimal. Annexing zeros can help you compare decimals.

For example, to compare 1.53 and 1.5342, you can add two 0s to 1.53:

$1.5300 < 1.5342$, so
$1.53 < 1.5342$.

3. **Make use of structure.** Which paramecium was longer, aurelia or bursaria? Explain how you found the answer.

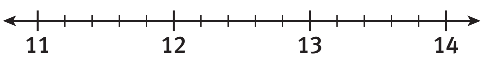
4. Which paramecium was shorter, aurelia or caudatum? Explain how you found the answer.

5. Which paramecium was longest? Explain how you found the answer.

6. **Make use of structure.** Another way to compare decimals is to use a number line. Paramecia have cilia (hairlike structures) that act like oars and propel them through the water. The table below shows the times in which different paramecia swam 10 mm.

Swimmer	Time (sec)
Aurelia	11.6
Bursaria	11.3
Caudatum	13.4
Multimicronucleatum	12.7
Jenningsi	13.0

Plot each paramecium's time on the number line below. Write the first letter of the paramecium's name above the point representing its time.



7. Name the paramecia with times faster than 12 seconds and those with slower times. Explain how you made your decisions.

8. How can you tell which of two numbers plotted on a number line is the greater number?

My Notes

9. How can a number line be used to explain why $11.6 > 11.3$?

10. Insert $>$ or $<$ between each pair of numbers to create a true inequality statement.

a. 11.3 13.4 **b.** 13 12.7 **c.** 11.3 11.6

Check Your Understanding

11. Insert $>$ or $<$ between each pair of numbers to create a true inequality statement. Identify the larger number.

a. 0.7 and 0.652
b. 31 and 31.59
c. 6.700 and 6.8
d. 377.151 and 377.1509

12. Order the numbers in each group from least to greatest.

a. 76, 34, 85.2, 37.5, 34.8
b. 2.31, 0.231, 23.1, 0.23, 3.21
c. 5.78, 5.7001, 5.701, 5.71, 5.7

13. **Make use of structure.** Describe the steps you would follow to compare two decimals. Use an example.

LESSON 1-1 PRACTICE

14. Radon is a gas that occurs in nature. It is considered to be a health hazard. Radon is measured in "pico Curies per liter" or pCi/L. One city considers 3.85 pCi/L of radon to be the maximum safe level of radon in public buildings. The table gives the measured levels in four buildings. Order the radon levels from least to greatest.

Building	Radon Level (pCi/L)
City Hall	3.855
Library	3.8095
Art Museum	3.839
Police Station	3.850
Fire Station	4.08

Which buildings, if any, exceeded the recommended level?

15. All cheeses in Blake's Grocery sell for no less than \$3.95 per pound but less than \$4 per pound. List all possible cheese prices at the store from least to greatest.

16. **Reason quantitatively.** If you were to plot 36.948516 on a number line, would it be closer to 36.9 or to 37? Explain your reasoning.

My Notes

CONNECT TO SCIENCE

Ohms are units used to measure resistance in an electrical circuit. *Resistance* is the measurement of the opposition to the flow of current in a circuit. It limits the amount of current flowing through a wire.

WRITING MATH

Subscripts are small numbers written below and to the right of letters. Subscripts indicate different members of a group. For instance, three different places of resistance can be labeled R_1, R_2, and R_3.

MATH TERMS

A **visual representation** is a model that uses pictures in order to help solve a problem.

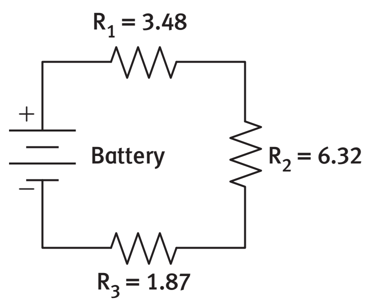
Learning Targets:

- Add and subtract multidigit decimals.
- Solve real-world problems by adding and subtracting decimals.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think-Pair-Share, Discussion Groups, Visualization, Create Representations

George is going to join his father at work on "Bring Your Son to Work Day." To understand his dad's work as an electrician, George needs to know how to add and subtract decimals.

George's dad works on heaters. He gives George the following problem for practice.



The control circuit for the heater circuit (shown above) is not working properly. George needs to troubleshoot to find where the problem is. The heater elements are represented by R_1, R_2, and R_3. If $R_1 = 3.48$ ohms, $R_2 = 6.32$ ohms, and $R_3 = 1.87$ ohms, George must figure out how much total resistance (measured in ohms) the circuit has.

1. The total resistance in the circuit is found using the formula $R_1 + R_2 + R_3$. Using the numbers George's dad gave him, write an expression for the total resistance.

George needs to find the sum so that he can determine which heater element is not working properly.

You can use a ***visual representation*** to find a decimal sum like this one.

My Notes

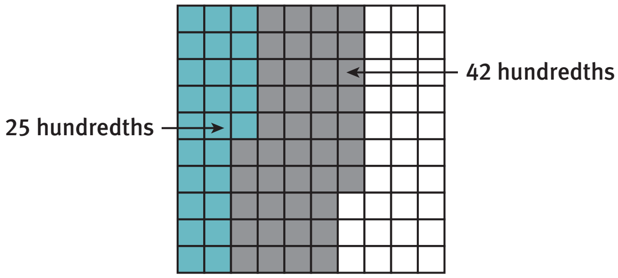
Example A

Find the sum of 4.25 and 9.42.

Step 1: Add the whole numbers to the left of the decimal point.

$$4 + 9 = 13$$

Step 2: Find the sum of the decimal parts of each addend, using a 10-by-10 grid.



Since 67 of the boxes are shaded, the sum is 0.67.

Step 3: Combine your sums from Step 1 and Step 2.

$$13 + 0.67 = 13.67$$

Solution: The sum of 4.25 and 9.42 is 13.67.

Try These A

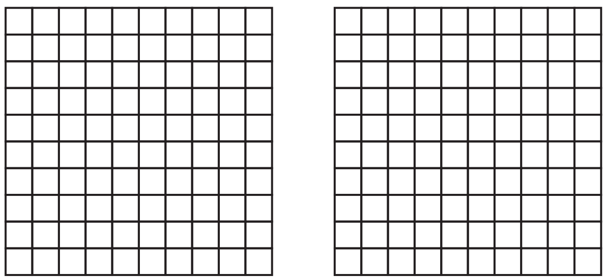
Find each sum. Use the My Notes section to create and shade 10-by-10 grids.

a. 1.45 + 2.15 **b.** 4.2 + 3.25

Now find the sum of the resistances in George's circuit. Remember that $R_1 = 3.48$, $R_2 = 6.32$, and $R_3 = 1.87$.

2. Find the sum of the whole numbers in the three addends.

3. Find the sum of the decimal parts on the 10-by-10 grids below.



sum = ________

4. To find the total resistance in George's circuit, find the sum of the whole numbers and the decimals.

total resistance = ________ ohms

My Notes

ACADEMIC VOCABULARY

An ***algorithm*** is a set of steps or a procedure used to carry out a computation or to solve a problem.

MATH TIP

When you add or subtract decimals, you can add one or more zeros to the end of a decimal number to give all the numbers the same number of decimal places to the right of the decimal point.

Another way to find the sum is to use an ***algorithm.*** As you discuss the following examples be sure to use the academic vocabulary precisely. Make notes to help you remember the meaning of new words.

Example B

Find the sum of 4.8 and 12.75.

Step 1: Write the problem vertically. Be sure to align the decimal points.

$$\begin{array}{r} 12.75 \\ +\ 4.80 \\ \hline \end{array}$$

Step 2: Add the digits from right to left. Keep the decimal points aligned.

$$\begin{array}{r} 12.75 \\ +\ 4.80 \\ \hline 17.55 \end{array}$$

Solution: The sum of 4.8 and 12.75 is 17.55.

Try These B

a. $32.8 + 9.25$ **b.** $9.8 + 12.41 + 4.32$

The algorithm for subtracting decimals is similar to the algorithm for subtracting whole numbers.

Example C

Find the difference: $27.6 - 12.24$

Step 1: Write the problem vertically. Align the decimal points.

$$\begin{array}{r} 27.60 \\ -12.24 \\ \hline \end{array}$$

Step 2: Subtract the digits from right to left.

$$\begin{array}{r} 27.60 \\ -12.24 \\ \hline 15.36 \end{array}$$

Solution: $27.6 - 12.24 = 15.36$

Try These C

Find each difference.

a. $27.16 - 7.52$ **b.** $42.56 - 9.7$ **c.** $36.3 - 13.48$

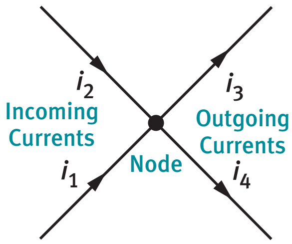
Work with your group on items 5 through 8.

5. In Item 1, the total resistance in George's circuit should be $R_1 + R_2 + R_3 = 3.48 + 6.32 + 1.87$ ohms. However, measurements show that the total resistance is only 9.8 ohms. Predict which of the three heater elements, R_1, R_2, or R_3, is not working. Explain how you made your prediction.

6. Confirm your prediction by showing that the sum of the resistances of the two working heater elements is 9.8.

In order to make sure George is ready for the day tomorrow, George's dad wants to challenge him with one more problem.

The place where multiple electric currents intercept is called a node.



Kirchhoff's Current Law states that the sum of all the currents at any node must equal zero. This means that at any node, the sum of the incoming currents will be equal to the sum of the outgoing currents.

George's dad gives him the following information about an electrical circuit:

$$i_1 = 0.45 \text{ amps} \quad i_2 = 0.67 \text{ amps} \quad i_3 = 0.34 \text{ amps}$$

7. Reason quantitatively. Use the given information and the diagram to find i_4. Show your work.

8. Describe four real-life situations in which you might need to add or subtract decimals.

My Notes

DISCUSSION GROUP TIPS

If you do not understand something in group discussions, ask for help or raise your hand for help. Describe your questions as clearly as possible, using synonyms or other words when you do not know the precise words to use.

CONNECT TO SCIENCE

A *node* is a place on a circuit where two or more wires connect.

My Notes

Check Your Understanding

9. Find the sum or difference. Use any method you like.

a. 3.86 + 0.98 **b.** 76.31 − 48.55
c. 126 + 87.457 **d.** 17 − 8.63
e. 0.55 + 4 + 13.708 **f.** 90.01 − 5.77
g. 0.045 + 0.39 + 0.7 **h.** 2,006 − 29.95

10. The total resistance in an electrical circuit consisting of three heater elements is 14.7 ohms. The resistances of two of the heater elements are 5.57 ohms and 4.91 ohms. Find the resistance of the third heater element.

11. The three incoming currents at a node in an electrical circuit measure 0.38 amps, 0.75 amps, and 0.29 amps. Two of the three outgoing currents measure 0.48 amps and 0.6 amps. Find the measure of the third outgoing current.

LESSON 1-2 PRACTICE

12. Reason quantitatively. Mark and Marcus ate dinner in a restaurant. Mark's meal cost \$34.16. Marcus's meal cost \$3.68 less than Mark's. Sales tax on the two meals was \$4.52. Mark paid with a \$100 bill. How much change did he receive?

13. The map shows the road from Abbott (A) to Baxter (B) to Carlton (C) to Dalton (D). The total distance from Abbott to Dalton is 233 km.

a. How far is it from Carlton to Dalton?
b. The new freeway shortens the distance from Baxter to Dalton by 10.8 km. How far is it from Baxter to Dalton if you take the freeway?

14. Julie's scores in all but her final event at a gymnastics meet are given below. The current leader has a total score of 104.1 for all the events. How many points must Julie score on her final vault to win the meet?

Floor: 8.3, 8.9, 9.2 Beam: 7.2, 7.7, 8.1
Bars: 8.8, 8.8, 8.7 Vault: 9.8, 9.7, $\boxed{?}$

My Notes

Learning Targets:

- Multiply multidigit decimals.
- Estimate products of decimals.
- Solve real-world problems by multiplying decimal numbers.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Predict and Confirm, Paraphrasing, Quickwrite, Simplify the Problem

Amanda is going to a family reunion and is in charge of bringing a fruit salad. She has budgeted $35 for the salad. She plans to buy apples, blueberries, strawberries, kiwis, and yogurt. Since she is not sure how much it will cost, she decides to estimate the price.

She knows from her previous trips to the grocery store that the following prices are what each item should cost.

Item	Price
Apples	$1.99 per pound
Blueberries	$1.25 per pound
Strawberries	$2.99 per pound
Kiwis	$1.75 per pound
Yogurt	$2.75 per container

For her recipe, Amanda needs 3.25 pounds of apples, 1.75 pounds of blueberries, 3.5 pounds of strawberries, 1.2 pounds of kiwis, and 5 containers of yogurt. Work with your group to answer items 1-5. If you need help expressing your ideas to your group, make notes about what you want to say, listen to others, and ask for the meanings of any words they use that you don't understand.

1. Predict the total cost. Does your prediction put Amanda over or under budget?

2. Round the number of pounds of apples that Amanda needs to the nearest pound and the price of apples to the nearest dollar. Then write an expression you can use to estimate the total cost of the apples.

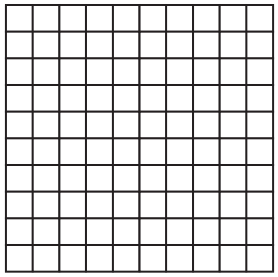
3. Evaluate the expression to estimate the total cost of the apples.

4. Use the method given in item 2 to estimate the total cost of each ingredient. Then find the total cost.
Blueberries
Strawberries
Kiwis
Yogurt

My Notes

5. Based on the estimated costs in Item 4, would you change your prediction from Item 1? Why or why not?

To find the exact costs, Amanda will need to multiply decimal numbers. One way to multiply decimals is to use a decimal model like the one at the right.



6. The entire 10-by-10 grid represents 1. What does each small square represent?

7. Model with mathematics.

a. To find the product 0.7×0.3, represent 0.7 by shading the first 7 columns of the grid. Then, using a different color, represent 0.3 by shading the first 3 rows of the grid.

b. The number of squares in the region where the shaded columns and rows overlap represents the product. Write a number sentence to show the product of 0.7 and 0.3.

c. How does the total number of decimal places in the product compare with the total number of decimal places in the two factors?

You can also use the multiplication algorithm.

Example A

Find the product: 1.8×2.35

Step 1: Write the factors vertically, placing the number with more digits on the top line

$$\begin{array}{r} 2.35 \\ \times\ 1.8 \\ \hline \end{array}$$

Step 2: Multiply the numbers.

$$\begin{array}{r} 2.35 \\ \times\ \ 1.8 \\ \hline 1880 \\ +2350 \\ \hline 4230 \end{array}$$

Step 3: Count the total number of decimal places in the two factors. Put the decimal point in the product so that it has the total number of decimal places that the factors have.

$$\begin{array}{rl} 2.\boxed{35} & \rightarrow 2 \text{ decimal places} \\ \times\ 1.\boxed{8} & \rightarrow 1 \text{ decimal place} \\ \hline 4.\boxed{230} & \rightarrow 3 \text{ decimal places} \end{array}$$

Solution: The product is 4.230, or 4.23.

My Notes

Try These A

Find the product.

a. 2.3×5 **b.** 4.27×1.3 **c.** 1.473×2.31

When Amanda arrives at the grocery store, she finds that the actual prices of the items she needs are slightly different from those she used in her estimate.

Item	Price
Apples	\$2.25 per pound
Blueberries	\$1.75 per pound
Strawberries	\$2.89 per pound
Kiwis	\$1.85 per pound
Yogurt	\$2.50 per container

Remember that she needs 3.25 pounds of apples, 1.75 pounds of blueberries, 3.5 pounds of strawberries, 1.2 pounds of kiwis, and 5 containers of yogurt.

8. Calculate the total cost of each item. Since your answers will be in dollars, round each product to the nearest cent.

Apples

Blueberries

Strawberries

Kiwis

Yogurt

9. Find the total cost of the ingredients.

10. Will Amanda be within her \$35 budget? How much over or under it will she be?

11. Before going to the store, Amanda estimated the total cost of the salad at \$30. Was her estimate reasonable? Why or why not?

12. Was the prediction you made at the beginning of the lesson correct? If not, how much over or under the actual cost?

My Notes

Check Your Understanding

13. Model with mathematics. Use a decimal model to find the product. Model the first factor by shading columns and the second factor by shading rows.

a. 0.4×0.6 **b.** 0.6×0.4

c. What property of multiplication do your answers to Parts a and b illustrate? Explain.

14. Find the product.

a. 6.7×4.2 **b.** 5.77×0.6 **c.** 9.23×17

d. $0.045 \times 10{,}000$ **e.** 0.0071×0.34 **f.** 12.15×5.2

LESSON 1-3 PRACTICE

15. Damon works 32 hours per week and earns $10.75 per hour.

a. Estimate his weekly earnings. Explain how you determined your estimate.

b. Find the exact amount he earns per week.

c. Is your answer reasonable based on your estimate? Explain.

16. a. Lyla wants to model 3×0.17 on a 10-by-10 grid. Show how she might do this.

b. What is the product?

17. A rectangular soccer field is 70.75 meters wide and 105.25 meters long.

a. Estimate the area of the field.

b. Find the area of the field. Round your answer to the nearest hundredth.

MATH TIP

To find the area of a rectangle, multiply the length and the width.

18. On the morning of a business trip, Maria drove for 3.1 hours at an average rate of 63.8 miles per hour. In the afternoon, she drove for 3.5 hours at an average rate of 59.5 miles per hour.

a. Did she travel farther in the morning or the afternoon?

b. How much farther did she travel?

19. Reason abstractly. Can the product of a whole number and a decimal number less than 1 ever be greater than the whole number? Give examples to support your answer.

20. a. Describe two ways to find the product $4.2(6 + 0.43)$.

b. What property of multiplication does your answer to Part a illustrate?

My Notes

Learning Targets:

- Divide whole numbers by whole numbers.
- Estimate quotients of whole numbers.
- Solve real-world problems by dividing whole numbers.

SUGGESTED LEARNING STRATEGIES: Group Presentation, Quickwrite, Interactive Word Wall, Summarizing

Charley is researching population density for a science project. In order to find the population density of a location, she needs to divide the number of people who live there by the area they occupy.

Population Density = Number of People ÷ Area Occupied

In order for Charley to complete her project, she needs to review the algorithm for long division.

CONNECT TO SOCIAL STUDIES

Population density is the average number of residents per some unit of measure, usually square mile or square meter.

READING MATH

A quotient is calculated by dividing a dividend by a divisor.

$$\text{divisor}\overline{\smash{)}\text{dividend}}^{\;\text{quotient}}$$

Example A

Find the quotient: 3,375 ÷ 15

Step 1: Write the divisor and the dividend.

divisor ↓

$15\overline{\smash{)}3375}$ ← dividend

Step 2: Divide 15 into 33 and subtract.

$$\begin{array}{r} 2 \\ 15\overline{\smash{)}3375} \\ \underline{-30} \\ 3 \end{array}$$

Step 3: Bring down the 7 in the dividend. Divide 15 into 37. Continue subtracting, bringing down the next digit in the dividend, and dividing.

$$\begin{array}{r} 225 \\ 15\overline{\smash{)}3375} \\ \underline{-30} \\ 37 \\ \underline{-30} \\ 75 \\ \underline{-75} \\ 0 \end{array}$$

Solution: The quotient is 225.

Try These A

a. 5,841 ÷ 11 **b.** 2,898 ÷ 126 **c.** 58,650 ÷ 25

My Notes

1. In a recent census, the population of Orlando, Florida, was found to be 238,374. Orlando has an area of 102 square miles. Estimate the population density of Orlando. Explain how you made your estimate.

2. Use the table to find the population density of each city.

City	Population	Area (sq mi)
Dallas, TX	1,197,820	340
Sioux Falls, SD	153,957	73
Orlando, FL	238,374	102
Omaha, NE	409,067	127
Los Angeles, CA	3,792,900	470
New York, NY	8,175,140	302
Seattle, WA	608,664	84
Charlotte, NC	731,700	300

a. Dallas

b. Sioux Falls

c. Orlando

d. Omaha

e. Los Angeles

f. New York City

g. Seattle

h. Charlotte

3. Is your answer for Item 2c reasonable? Explain.

4. Order the cities by population density from greatest to least.

5. **Construct viable arguments.** The towns of Higby and Milton have the same population. Higby's area is greater than Milton's. Which town has the greater population density? Explain your reasoning.

My Notes

When you use the algorithm to find a quotient of two whole numbers, you may need to add a decimal point and zeros to the dividend. This will create a decimal quotient.

Example B

Estimate the quotient $245 \div 20$. Then find the quotient and determine if it is reasonable.

Step 1: Estimate the quotient. Use the compatible numbers 240 and 20:

$$240 \div 20 = 12.$$

So, 12 is a good estimate of the quotient.

Step 2: Use the algorithm to divide.

$$\begin{array}{r} 12 \\ 20\overline{)245} \\ -20 \\ \hline 45 \\ -40 \\ \hline 5 \end{array}$$

Step 3: The remainder is not zero, and there are no more digits in the dividend to bring down. So, write a decimal point after the 5 in the dividend and place a zero to its right. Place the decimal point in the quotient directly above the decimal point in the dividend.

$$\begin{array}{r} 12.2 \\ 20\overline{)245.0} \\ -20 \\ \hline 45 \\ -40 \\ \hline 50 \\ -40 \\ \hline 10 \end{array}$$

Step 4: Continue to divide, adding zeros as necessary until the remainder is zero.

$$\begin{array}{r} 12.25 \\ 20\overline{)245.00} \\ -20 \\ \hline 45 \\ -40 \\ \hline 50 \\ -40 \\ \hline 100 \\ -100 \\ \hline 0 \end{array}$$

Solution: $245 \div 20 = 12.25$. This is reasonable because it is close to the estimate of 12.

Try These B

a. $305 \div 122$ **b.** $10 \div 8$ **c.** $82 \div 16$

My Notes

Check Your Understanding

Find each quotient.

6. $325 \div 13$
7. $2{,}128 \div 28$
8. $48{,}184 \div 152$
9. $221 \div 65$
10. $2{,}052 \div 240$
11. $297 \div 88$

LESSON 1-4 PRACTICE

12. Dexter drove 546 miles on one tank of gasoline. His car's gas tank holds 15 gallons of gas. Find the average number of miles he drove per gallon.

13. Crystal works 6 hours three days a week and 7 hours two days a week. She earns $472 per week. What is her hourly rate of pay?

14. **Reasoning abstractly and quantitatively.** The town of Brighton has a population of 12,096 and an area of 15 square miles. The town of Pauling has the same population density as Brighton and an area of 22.5 square miles. What is the population of Pauling?

15. Great Wilderness Animal Park is divided into four sections. Each section features animals from a single continent. The table shows the number of animals in each section and the area of the section, in square yards.

Continent	Number of Animals	Area (square yards)
North America	228	4,560
South America	684	11,400
Africa	912	18,240
Asia	456	11,400

a. Find the population density of each section of the park, in animals per square yard.
b. Find the population density of the entire park.
c. Explain how you found the population density of the entire park.

16. a. Use the fact that $8 \times 3.5 = 28$ to write two division problems relating 8, 3.5, and 28.
b. Describe the relationship between multiplication and division that allowed you to write the two division problems.

17. **Construct viable arguments.** Do $6 \div 3$ and $600 \div 300$ have the same quotient? Support your answer.

My Notes

Learning Targets:

- Divide decimals by whole numbers.
- Divide whole numbers and decimals by decimals.
- Estimate quotients.
- Solve real-world problems by dividing decimals.

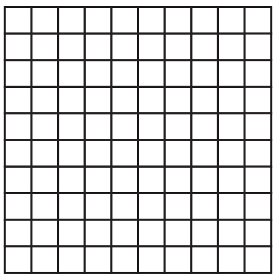
SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Predict and Confirm, Create Representations, Shared Reading

In the last lesson, you learned that population density is the number of people per 1 square mile of area. There are many types of density. Like population density, other measures of density give the amount of one quantity that is contained in one unit of another quantity. At 50 degrees F, the density of water, for example, is about 62.4 pounds per cubic foot.

A chemist wanted to find the density of air, in pounds per cubic foot. You might be surprised to find out that air has weight. It does, but it is much lighter than water! You may also be surprised to know that the density of air changes when the temperature changes.

The chemist found that, at the current temperature, 9 cubic feet of air weighed 0.72 pounds. To write the density in pounds per cubic foot, find the quotient $0.72 \div 9$. Remember, 0.72 means 72 hundredths, or 72 out of 100.

1. **Model with mathematics.** This grid represents $\frac{100}{100}$ or 1 whole. To divide 0.72 by 9, you need to make 9 equal groups. Model 0.72 by shading a rectangle that is 9 units wide and 8 units high. Mark the shaded rectangle to show 9 groups of equal size. How many squares are in each group?



2. 0.72 divided by 9 equals _______.

3. So, at the temperature when the chemist performed the experiment, the density of air was _______ pounds per cubic foot.

Example A

Thirty-two chemistry students raised \$272.64 to purchase a precision electronic balance for their laboratory. What was the average amount of money raised per student?

Step 1: Estimate the quotient.
Use compatible numbers. 272.64 is about 300, and 32 is about 30.
$300 \div 30 = 10$. So, \$10 is a good estimate of the quotient.

Step 2: Use the algorithm for dividing whole numbers.

$$\begin{array}{r} 8.52 \\ 32\overline{)\;272.64} \\ -256 \\ \hline 166 \\ -160 \\ \hline 64 \\ -64 \\ \hline 0 \end{array}$$

Solution: The average amount raised per student was \$8.52. The answer is reasonable because it is close to the estimate of \$10.

Try These A

Estimate each quotient. Then find the quotient.

a. $25\overline{)168.75}$ Estimate: Quotient:

b. $7\overline{)339.5}$ Estimate: Quotient:

All of the division problems you have solved so far have had whole numbers as divisors. When the divisor is not a whole number, it has to be multiplied by 10, 100, 1,000, or some higher power of 10 to create a whole number. Both the divisor and the dividend have to be multiplied by the same number so that the value of the quotient is not affected. This can be done by moving the decimal points of the dividend and divisor an equal number of spaces to the right.

My Notes

Example B

Find the quotient $103.5 \div 0.45$.

Step 1: Estimate the quotient.
Use compatible numbers. 103.5 is about 100; 0.45 is about 0.5.
$100 \div 0.5 = 200$. So, 200 is a good estimate of the quotient.

Step 2: Think: I can rewrite 0.45 as a whole number by multiplying it by 100. I'll do the same thing to the dividend 103.5. I can do this by moving both decimal points two places to the right.
This changes the problem from $103.5 \div 0.45$ to the equivalent problem $10{,}350 \div 45$.

$$\begin{array}{r} 230 \\ 0.45\overline{)\,103.50} \\ -90 \\ 135 \\ -135 \\ 00 \\ -00 \\ 0 \end{array}$$

Solution: The quotient is 230. The answer is reasonable because it is close to the estimate of 200.

Try These B

Estimate each quotient. Then find the quotient.

a. $2.7\overline{)13.041}$ Estimate: Quotient:

b. $0.31\overline{)682}$ Estimate: Quotient:

Check Your Understanding

4. Complete.
  a. $231 \div 5.07 =$ _______ $\div\ 507$
  b. $0.4472 \div 0.315 =$ _______ $\div\ 315$
  c. $61 \div 0.9 =$ _______ $\div\ 9$

5. Imam makes bead bracelets. She can buy 12 beads for \$2.04 or 17 beads for \$3.57. Which deal gives her the lower cost for one bead? Explain.

6. Sam is saving \$5.75 per week to buy a CD player that costs \$46. How many weeks will he have to save before he can buy the player?

7. A 17.5-kilometer racecourse is divided into 2.5-kilometer portions. How many portions are there in the complete course?

My Notes

LESSON 1-5 PRACTICE

8. Write the mixed number $2\frac{5}{16}$ as a decimal. Explain your method.

9. **Attend to precision.** Find the quotient $0.8 \div 0.04 \div 0.002$. Explain how you found the answer.

The table below gives the densities of four gases.

Gas	Density (lb/cu ft)
Carbon Monoxide	0.08
Helium	0.012
Hydrogen Chloride	0.09
Ozone	0.125

10. How many times the density of helium is the density of hydrogen chloride?

11. How many times the density of carbon monoxide is the density of ozone?

ACTIVITY 1 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 1-1

1. A geologist weighed four rocks, with these results: Rock A, 1.147 kg; Rock B, 1.15 kg; Rock C, 1.098 kg; Rock D, 0.884 kg. Order the rocks from heaviest to lightest.

2. Which number is closest to 51.4?
   A. 51.41 **B.** 51.041
   C. 51.39 **D.** 51.402

3. The table gives the heights of five South American mountains. List the mountains from highest to lowest.

Name	Country	Height (ft)
Solo	Argentina	20,492
Palermo	Argentina	20,079
Chimborazo	Ecuador	20,702
Solimana	Peru	20,068
El Condor	Argentina	20,669

4. Which number is greater, 10.6395 or 10.64? Explain how you decided.

5. On a number line, point K is to the left of point H and to the right of point R. Point T is between points R and K. Order the points from least to greatest.

Lesson 1-2

6. The total resistance in an electrical circuit consisting of three heater elements is 12.61 ohms. The resistances of two of the heater elements are 3.9 ohms and 5.04 ohms. Find the resistance of the third heater element.

7. Mike bought three shirts costing \$15.98, \$19, and \$24.50. He paid for his purchase with a \$100 bill. How much change did he receive?

8. The three incoming currents at a node in an electrical circuit measure 0.7 amps, 0.68 amps, and 0.47 amps. Two of the three outgoing currents measure 0.8 amps and 0.55 amps. Find the measure of the third outgoing current.

9. Find the difference $6 - 0.564$.
   A. 0.546 **B.** 5.546
   C. 5.436 **D.** 5.536

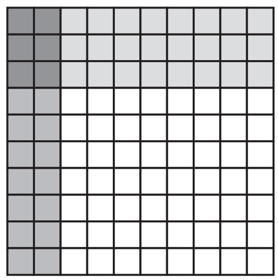
10. Find the sum $98.87 + 9.89$.
    A. 197.77 **B.** 108.76
    C. 107.66 **D.** 97.66

11. Complete.
    a. $4.79 + \_\_\_\_\_\_ = 37$
    b. $100 - \_\_\_\_\_\_ = 52.7$
    c. $\_\_\_\_\_\_ + 8{,}477.6 = 10{,}248$
    d. $\_\_\_\_\_\_ - 0.07 = 0.1$

12. Name the number property illustrated by this equation:
    $4.78 + (2.5 + 12.66) = 4.78 + (12.66 + 2.5)$

Lesson 1-3

13. Which multiplication problem is modeled by the grid?



    A. $0.2 \times 0.3 = 0.6$ **B.** $0.2 \times 0.3 = 0.06$
    C. $0.02 \times 0.03 = 0.6$ **D.** $0.02 \times 0.03 = 0.06$

14. Find the product 3.6×2.05.
    A. 0.738 **B.** 1.845
    C. 7.38 **D.** 18.45

15. Marie bought 2.5 pounds of cheddar cheese, 3.2 pounds of jack cheese, and 1.9 pounds of Swiss cheese. Use the table to find the total cost of her purchase.

Cheese	Price per Pound
Cheddar	\$4.92
Jack	\$3.75
Swiss	\$4.50

16. Margo drove for 3.85 hours at an average speed of 61.4 miles per hour.
a. Estimate the total distance that she drove. Explain your method.
b. Find the exact distance that she drove.
c. Is your answer reasonable? Explain why or why not.

17. Centennial Park in Fernville is a rectangle measuring 87.5 meters by 56.2 meters.
a. What is the perimeter of the park?
b. What is the area of the park?

Lesson 1-4

18. Find the quotient: 10,404 ÷ 51
A. 24 **B.** 200
C. 204 **D.** 240

19. One dozen donuts cost \$6.96. What is the price of one donut?
A. \$0.55 **B.** \$0.58
C. \$0.65 **D.** \$1.16

20. a. A city of 6,688 residents has an area of 16 square miles. What is the population density of the city?
b. A city of 82,080 residents has a population density of 2,160 residents per square mile. What is the area of the city?
c. A city with an area of 87 square miles has a population density of 3,517 residents per square mile. What is the city's population?

21. The table shows the number of magazine subscriptions Craig sold in one week. His total sales were \$2,628. What was the average price of a subscription?

Day	Subscriptions Sold
Monday	36
Tuesday	52
Wednesday	15
Thursday	29
Friday	87

Lesson 1-5

22. The table gives the costs for organizing Field Day at Montgomery Middle School. Thirty-two students will attend the event.

Expense	Total Cost (\$)
Supplies	272.64
Announcer	168.84
Refreshments	113.40

a. What is the cost per student for this event?
b. What is the cost per student for supplies only?

23. Find the quotient 60.146 ÷ 0.58.
A. 1.037 **B.** 10.37
C. 103.7 **D.** 1.037

24. A kangaroo weighs about 0.03 ounces at birth. Fully grown, it can weigh 180 pounds. How many times as heavy as a newborn kangaroo is a fully grown adult?

MATHEMATICAL PRACTICES
Model with Mathematics

25. Describe how to use a 10-by-10 decimal grid to show the quotient 0.48 ÷ 6.

Comparing and Computing with Whole Numbers and Decimals
FOR THE BIRDS

Embedded Assessment 1
Use after Activity 1

Write your answers on notebook paper. Show your work.

1. Ramon's hobby is raising parrots. The table gives the weights of five of his birds.

Parrot	Weight (oz)
Jack	6.102
Tippy	5.98
Fritz	6.058
Danny	6.8
Abe	6.06

a. Order the weights from least to greatest.
b. Explain how you decided which parrot was heavier, Fritz or Abe.
c. Estimate the combined weight of the parrots. Explain how you made your estimate.
d. What is the exact combined weight of all five parrots?
e. Is your answer reasonable? Explain.
f. What is the average weight of the parrots?
g. Jack, Fritz, and a third parrot have a combined weight of 18.22 ounces. What is the name of the third parrot?

2. A female parrot that weighs 13.44 ounces has a chick that weighs 0.56 ounces. How many times the chick's weight is the weight of the mother?

3. Ramon has an African Grey parrot named Curly that weighs 17.4 ounces. How much heavier is Curly than Tippy?

4. Ramon buys five 3-pound bags of natural parrot food for $8.79 per bag and two 5-pound bags for $13.90 per bag.
a. How many pounds of parrot food does he buy?
b. What is the total cost?
c. What is the average cost per pound of parrot food?

Comparing and Computing with Whole Numbers and Decimals

FOR THE BIRDS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a, 1c-d, 1f-g, 2, 3, 4a-c)	• Clear and accurate understanding of operations with decimals. • Effective understanding and accuracy in ordering and comparing decimals.	• Operations with decimals that are usually correct. • Correct comparison of decimals by ordering.	• Operations with decimals that are sometimes correct. • Partially correct comparison or ordering of decimals.	• Incorrect or incomplete computation in operations with decimals. • No comparison or ordering of decimals.
Problem Solving (Items 1c-g, 2, 3, 4a-c)	• An appropriate and efficient strategy that results in a correct answer. • A clear understanding of whether a solution is reasonable.	• A strategy that may include unnecessary steps but results in a correct answer. • Some understanding of whether a solution is reasonable.	• A strategy that results in some incorrect answers. • Uncertainty regarding the reasonableness of a solution.	• No clear strategy when solving problems. • No understanding of whether a solution is reasonable.
Mathematical Modeling / Representations (Items 1a, 1c-d, 1f-g, 2, 3, 4a-c)	• Clear and accurately written expressions for operations with decimals. • Clear and correct ordering and comparison of decimals.	• Some difficulty in writing the best expression for a problem situation, but still shows correct answers. • An understanding of ordering decimals.	• Errors in writing expressions for a given problem situation. • Errors in ordering decimals (for example, orders greatest to least instead of least to greatest).	• Inaccurately written expressions. • Incorrect ordering of decimals.
Reasoning and Communication (Items 1b-c, 1e)	• Precise use of appropriate math terms and language to explain comparing decimals, estimating a sum, and determining reasonableness.	• An adequate explanation of comparing decimals, estimating a sum, and determining reasonableness.	• A misleading or confusing explanation of comparing decimals, estimating a sum, and determining reasonableness.	• An incomplete or inaccurate description of comparing decimals, estimating a sum, and determining reasonableness.

Prime Factorization and Exponents

ACTIVITY 2

The Primes of Your Life
Lesson 2-1 Prime Factorization

My Notes

Learning Targets:

- Determine whether a given whole number is a prime number or a composite number.
- Express a composite number as a product of prime numbers.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note Taking, Think-Pair-Share, Visualization, Sharing and Responding

The ***prime factorization*** of a number shows the number as a product of factors that are all prime numbers. One way to find the prime factorization of a number is to use a visual model called a factor tree.

MATH TERMS

A **prime number** is a natural number greater than 1 that has exactly two factors, itself and 1. For example, 2 is a prime number ($2 = 2 \times 1$), as is 13 ($13 = 13 \times 1$).

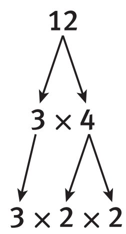
A **factor** is one of the numbers you multiply to get a product.

A **prime factor** is a factor that is prime.

A **composite number** is a natural number that has more than two factors. For example, 15 is a composite number ($15 = 1 \times 3 \times 5$).

Example A

Find the prime factorization of 12.



Step 1: Use divisibility rules to find two factors of the number.

Try 3 and 4. Use branches to show the factors.

Step 2: If both numbers are prime, stop. If not, continue factoring until all factors are prime numbers.

3 is prime. Bring it down to the next branch. Continue by factoring 4.

Step 3: Check again to be sure all factors are now prime numbers.

$3 = 3 \times 1 \qquad 2 = 2 \times 1$

There is no other way to factor these numbers, so 2 and 3 are prime.

Solution: The prime factorization of 12 is $3 \times 2 \times 2$.

Try These A

a. **Reason quantitatively.** Will the prime factorization of 12 be different if you start with the factors 2 and 6? Explain.

Find the prime factorization of each number.

b. 21 **c.** 16 **d.** 18

My Notes

Here is another method for finding prime factors.

Example B

Find the prime factorization of 60.

Step 1: Write the number as the dividend inside a division symbol. Write one of the prime factors as the divisor on the outside.

$$\begin{array}{rrl} & 5 & \leftarrow \text{Step 4} \\ 2\,) & 10 & \\ 3\,) & 30 & \leftarrow \text{Steps 2 \& 3} \\ 2\,) & 60 & \leftarrow \text{Step 1} \end{array}$$

Step 2: Divide as you would if you were using long division.

Step 3: Repeat the steps, this time using the quotient on top of the division symbol as the new dividend.

Step 4: Stop when the quotient is a prime number.

Step 5: Use the divisors and the final quotient to write the prime factorization.

Solution: $60 = 2 \times 2 \times 3 \times 5$

WRITING MATH

Prime factorizations are usually written with the factors ordered from least to greatest:
$2 \times 3 \times 3 \times 7$, not $3 \times 7 \times 3 \times 2$

Try These B

Find each prime factorization using the long division method.

a. 32 **b.** 45 **c.** 56

Even if a number is not divisible by any small natural numbers such as 2, 3, or 5, it may still be a composite number. To find its prime factorization, you may have to use larger numbers to guess and check with.

Example C

At After School Sports Club, 143 students are divided into teams, with the same number on each team. How many teams are there and how many students are on each team?

To solve, use divisibility rules to see if 143 is divisible by any prime numbers. Start with 2 and work upwards.

2? No. 143 is not even.

3? No. $1 + 4 + 3 = 8$, which is not divisible by 3.

5? No. The ones digit of 143 is not 0 or 5.

7? No. When you divide 143 by 7, there is a remainder.

11? Yes. $143 \div 11 = 13$.

Solution: Since $143 \div 11 = 13$, $143 = 11 \times 13$. That means there could by 11 teams with 13 on each team, or 13 teams with 11 on each team.

My Notes

Try These C

a. Last year there were 133 students in After School Sports Club. How many teams were there, and how many students were on each team?

b. There are 221 math books in a closet arranged in equal stacks. How many stacks are there, and how many books are in each stack?

Check Your Understanding

1. Determine the prime factorization of each number.

a. 14	**b.** 30	**c.** 27
d. 38	**e.** 84	**f.** 41
g. 100	**h.** 77	**i.** 180

2. Why is every prime number greater than 2 an odd number?

3. Explain why numbers with a 5 in the ones place are not prime numbers.

4. List all the prime numbers from 1 to 50.

5. Explain why you cannot find the prime factorization of 4.8.

LESSON 2-1 PRACTICE

6. **Construct viable arguments.** $7{,}719 = 83 \times 93$. Explain why 83×93 is not the prime factorization of 7,719.

7. A ***conjecture*** is a statement that appears to be true, but which remains unproven. In 1976, a seventh-grade student named Arthur Hamann made a conjecture that every even number can be expressed as the difference between two prime numbers.

$$6 = 19 - 13 \qquad 20 = 23 - 3$$

No one has ever found an even number for which Arthur Hamann's conjecture is not true. Test the conjecture for these even numbers:

a. 14 **b.** 18 **c.** 22

ACADEMIC VOCABULARY

A ***conjecture*** is a statement that appears to be true but has not been proven.

8. A famous conjecture by the eighteenth-century mathematician Christian Goldbach also remains unproven. Goldbach's conjecture states that every even number greater than 2 can be expressed as the sum of two prime numbers.

$$12 = 5 + 7 \qquad 26 = 13 + 13$$

Test Goldbach's conjecture for these even numbers:

a. 16 **b.** 26 **c.** 34

My Notes

DISCUSSION GROUP TIPS

As you discuss this lesson with your partner or with your group, be sure to use the many new math terms precisely. Make notes to help you remember the meanings of new words and how they are used to describe mathematical topics.

MATH TERMS

A **numeric expression** is a mathematical phrase that uses numbers.

Examples of numerical expressions are $2 + 3$, $35 \div 7$, and 6^2.

You **evaluate** a numeric expression by performing any indicated operation in the expression.

Learning Targets:

- Evaluate a whole number or decimal raised to a whole number exponent.
- Express prime factorizations using exponents when a prime factor occurs more than once.

SUGGESTED LEARNING STRATEGIES: Note Taking, Think-Pair-Share, Vocabulary Organizer, Sharing and Responding, Discussion Group

An **exponent** tells how many times a **base** is to be used as a factor.

exponent

$$2^3 = 2 \times 2 \times 2 = 8$$

base

Powers are numbers expressed using exponents.

Power	Verbal Expression	Expanded Expression	Product
2^6	2 to the sixth power	$2 \times 2 \times 2 \times 2 \times 2 \times 2$	64
4.7^2	4.7 to the second power, or 4.7 **squared**	4.7×4.7	22.09
5^3	5 to the third power, or 5 **cubed**	$5 \times 5 \times 5$	125

A number written without an exponent is said to be in **standard** form. When it is written with an exponent, it is in **exponential** form.

49 $\rightarrow$ standard form $\qquad$ 7^2 $\rightarrow$ exponential form

1. Write each power as an expanded ***numeric expression***. Then ***evaluate*** the expression.

a. 3^4 **b.** 4^5 **c.** 5.3^2

d. eight squared **e.** six cubed

2. Write each expanded expression as a power. Then evaluate the power.

a. $1.7 \times 1.7 \times 1.7$ **b.** $2 \times 2 \times 2 \times 2 \times 2$ **c.** 16×16

3. a. Find 1^{17}.
b. Explain how you can find any power of 1.

4. a. Find 0^{31}.
b. Explain how you can find any power of 0.

5. a. Find 89^1.
b. Explain how you can find the first power of any number.

6. Make use of structure. Complete the following pattern:

$$2^4 = 16$$
$$2^3 = 8$$
$$2^2 = 4$$
$$2^1 = 2$$
$$2^0 = \_\_\_$$

You can use logic to complete the above pattern and will learn more about the numbers with an exponent of zero in later math classes. For now, remember that the zero power of any number is 1.

$8^0 = 1$ $\qquad$ $37^0 = 1$ $\qquad$ $(9.264)^0 = 1$

You can use exponents to write the prime factorization of a number with repeated prime factors. For example:

$$28 = 2 \times 2 \times 7 = 2^2 \times 7$$

7. Describe why the prime factorization of a number is sometimes written with exponents. Use an example in your answer.

8. Model with mathematics. Use exponents to write the prime factorization of each number.
a. 72 $\qquad$ **b.** 144 $\qquad$ **c.** 800

My Notes

Check Your Understanding

9. Evaluate each expression.

a. 2^4 **b.** 5^3 **c.** 8.3^2

d. 1^9 **e.** 13^1 **f.** 56.8^0

g. 10^5 **h.** 100^2 **i.** 0^{12}

j. eleven squared

k. one and three-tenths cubed

l. seven squared times two cubed

m. three to the fifth power

n. four squared cubed

10. Write the prime factorization of each number, using exponents as needed.

a. 20 **b.** 54 **c.** 45

d. 225 **e.** 98 **f.** 729

LESSON 2-2 PRACTICE

11. Evaluate each expression.

a. 3^4 **b.** 7^3 **c.** 3.8^2

d. 1^7 **e.** 31^1 **f.** 86.5^0

g. fifteen squared

h. four to the fourth power

12. Write the prime factorization of each number.

a. 40 **b.** 63 **c.** 120

13. a. Find 10^1. **b.** Find 10^2. **c.** Find 10^3.

d. Make use of structure. Describe a method you can use to find any power of 10 easily.

14. Model with mathematics. Write 64 in exponential form three different ways.

15. a. Write a possible explanation for why 5^2 is read "five *squared*."

b. Write a possible explanation for why 5^3 is read "five *cubed*."

16. Make use of structure. Evaluate 11^2, 111^2, and $1{,}111^2$. Then predict the value of $11{,}111^2$. Explain how you made your prediction.

ACTIVITY 2 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 2-1

1. Which of the numbers below is NOT a prime number?
A. 13 **B.** 37
C. 39 **D.** 47

2. Which is the prime factorization of 81?
A. 1×81 **B.** 3×27
C. 9×9 **D.** $3 \times 3 \times 3 \times 3$

3. Explain the difference between a prime number and a composite number. Include examples in your explanation.

4. List the prime numbers between 80 and 90.

5. List the composite numbers between 39 and 50.

6. To check whether 59 is prime, Claudia divided 59 by every odd number from 3 to 29. All of the quotients had a remainder.
a. Why did she test with only odd numbers?
b. To be sure that 59 is prime, does she need to continue to divide 59 by odd numbers from 31 to 57? Why or why not?

7. A certain natural number is not divisible by 7. Can it be divisible by 14? Why or why not?

8. Determine the prime factorization of each number.
a. 30 **b.** 33
c. 52 **d.** 72
e. 65 **f.** 76

9. Is the number 147 prime or composite? Explain how you know.

10. Is $2 \times 3 \times 4$ the prime factorization of 24? Explain your reasoning.

11. List all of the numbers from 1 to 100 that have 13 as a factor.

12. Desmond has 169 square bricks that he is arranging in rows to make a patio. Every row must have the same number of bricks.
a. How many bricks are in each row?
b. Describe the shape of the patio. Explain your reasoning.

13. a. Find the prime factorization of 132.
b. Explain how you found the prime factorization.

14. a. Find the prime factorization of 180.
b. Explain why every natural number with a zero in the ones place is a composite number.

15. Jason said that any number that has 1 as a factor must be a composite number? Do you agree? Explain your reasoning.

16. Express each number as a sum of two prime numbers.
a. 10 **b.** 22
c. 30 **d.** 50

17. Express each number as a difference of two prime numbers.
a. 2 **b.** 8
c. 14 **d.** 16

Lesson 2-2

18. Evaluate each expression.

a. 3^3 **b.** 9^2
c. 1.6^2 **d.** 26^1
e. 1^{15} **f.** 22.91^0
g. 0^{25} **h.** 10^7
i. two and two-tenths squared
j. four to the fourth power
k. seven cubed
l. three cubed squared

19. Write the prime factorization of each number. Use exponents if needed.

a. 32 **b.** 38
c. 175 **d.** 120
e. 108 **f.** 121

20. Write 81 in exponential form two different ways.

21. **a.** Write 25 in exponential form.
b. Identify the base in the expression that you wrote and explain what it means.
c. Identify the exponent in the expression that you wrote and explain what it means.

22. There were two bacteria in a dish at the beginning of an experiment. Every 15 minutes, the number of bacteria in the dish doubled. Write in exponential form the number of bacteria in the dish two hours after the experiment began.

23. Which of the given numbers equals $2^5 \times 3^2$?

A. 90 **B.** 192
C. 240 **D.** 288

24. Which expression equals 5^0?

A. 0^5 **B.** 1^5
C. 5^1 **D.** 5^0

25. How many factors does the number $2^2 \times 3^2$ have?

A. 4 **B.** 6
C. 9 **D.** 36

26. Which equation is true?

A. $1^6 = 6^1$ **B.** $2^3 = 3^2$
C. $2^4 = 4^2$ **D.** $3^4 = 4^3$

27. Each room in a library has 3^4 bookshelves. Each shelf holds 2^7 books.

a. Write an expression for the number of books that are shelved in each room.
b. How many rooms do you need to hold $2^7 \times 3^6$ books?

MATHEMATICAL PRACTICES
Construct Viable Arguments

28. A sixth grade math student made this conjecture: Every prime number can be expressed as the product of two prime numbers. Do you agree with the conjecture? Why or why not?

Greatest Common Factor and Least Common Multiple ACTIVITY 3

Parties and Pups

Lesson 3-1 Greatest Common Factor

Learning Targets:

- Find all the factors of a whole number.
- Find the greatest common factor of two whole numbers.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Paraphrasing, Discussion Groups, Create Representations

Sheila is planning a surprise birthday party for her little sister. She makes a list of all the necessary party items. One of the first items on the list is balloons.

Sheila buys 24 purple balloons and 36 red balloons. For party decorations, she will make balloon arrangements. Each arrangement has will have the same number of purple balloons and the same number of red balloons. What is the greatest number of arrangements she can make?

You can use the ***greatest common factor (GCF)*** to solve problems like the balloon problem. One way to find the GCF is to use prime factorizations.

My Notes

Example A

Find the GCF of 24 and 36.

Step 1: Write the prime factorization of each number.

$$24 = 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$

Step 2: Look for common factors (factors that are the same in both lists).

$$24 = \boxed{2} \times \boxed{2} \times 2 \times \boxed{3}$$
$$36 = \boxed{2} \times \boxed{2} \times 3 \times \boxed{3}$$

Step 3: Find the product of the common factors.

$$2 \times 2 \times 3 = 12$$

Solution: The GCF of 24 and 36 is 12. This means that 12 is the *greatest* number that is a factor of both 24 and 36.

Try These A

Find the greatest common factor of each set of numbers.

a. 18, 24 **b.** 12, 20 **c.** 11, 16

d. 28, 42 **e.** 15, 25, 40 **f.** 16, 24, 48

MATH TERMS

The **greatest common factor (GCF)** for a set of two or more whole numbers is the largest number that is a factor of all the numbers in the set.

MATH TIP

When finding the GCF using prime factorization, it is usually easier to spot the common factors by listing them all, rather than by using exponents:

$2 \times 2 \times 2 \times 3$ rather than $2^3 \times 3$

My Notes

1. You determined in Example A that the greatest number of arrangements that Sheila can make is 12.
   a. How many purple balloons will there be in each arrangement?
   b. How many red balloons will there be in each arrangement?

2. The are 60 girls and 48 boys in the school chorus. They are seated in rows in the auditorium, with the same number of students in each row. Each row has only girls or only boys. What is the greatest number of students who could be in each row?

Another way to find the GCF is to use common factors.

MATH TIP

Use the pattern shown below to check that you have listed all the factors of a number:

24: 1, 2, 3, 4, 6, 8, 12, 24

$1 \times 24 = 24$ **$2 \times 12 = 24$**

$3 \times 8 = 24$ **$4 \times 6 = 24$**

Sometimes there may be a single number in the middle:

16: 1, 2, 4, 8, 16

$1 \times 16 = 16$ **$2 \times 8 = 16$**

$4 \times 4 = 16$

Example B

Find the GCF of 24 and 40.

Step 1: List all the factors (not just the prime factors) of each number.
24: 1, 2, 3, 4, 6, 8, 12, 24
40: 1, 2, 4, 5, 8, 10, 20, 40

Step 2: List the common factors.
The common factors are 1, 2, 4, and 8.

Step 3: Choose the greatest factor from the list of common factors. This is the greatest common factor.

Solution: The GCF is 8.

Try These B

List the factors of each number. Then find the greatest common factor for each set of numbers.

a. 18, 24
b. 12, 20
c. 16, 21

3. **Attend to precision.** Of the 108 students in the school chorus, 48 are sopranos, 24 are altos, and 36 are tenors. What is the greatest number of groups that can be formed with the same number of each type of voice in each group?

My Notes

Check Your Understanding

4. Find the greatest common factor of each set of numbers.
a. 28, 40 **b.** 40, 48 **c.** 63, 135
d. 26, 39, 78 **e.** 12, 18, 26 **f.** 54, 72, 90

5. The are 72 girls and 60 boys waiting to audition for the school play. They are seated in rows the auditorium, with the same number of students in each row. Each row has only girls or only boys.
a. How many rows of girls will there be?
b. How many rows of boys will there be?

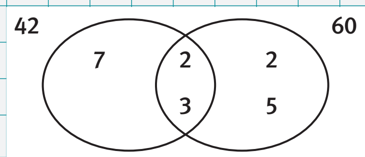
6. Of the 108 students in the school band, 48 are in the woodwind section, 24 are in the percussion section, and 36 are in the brass section. How many woodwind players, percussion players, and brass players will there be in each group?

7. Describe how to find the GCF of three numbers. Use an example with your explanation.

LESSON 3-1 PRACTICE

8. The youth orchestra has 18 violinists and 24 flutists. The music director wants to place these 42 musicians in rows with only violinists or flutists in each row. Each row must have the same number of musicians. What is the greatest number of musicians the director can place in each row?

9. **Model with mathematics.** The Venn diagram at the right shows a way to find the greatest common factor of 42 and 60.



Draw a Venn diagram to find the GCF of 154 and 210. Explain how your diagram shows the GCF.

10. **Attend to precision.** A carpenter has three pieces of wood measuring 210 cm, 245 cm, and 315 cm in length. She wants to cut them into shorter pieces that are all the same length, and with no wood left over.
a. What is the greatest possible length of each shorter piece?
b. How many shorter pieces will there be altogether?

11. The greatest common factor of 24 and another number is 6.
a. Find two possible values of the other number.
b. Describe the process you used to find the other number.

12. Ingrid has 48 roses, 32 daisies, and 24 tulips.
a. How many floral bouquets can she make if each bouquet must have the same number of each flower and every flower is used?
b. How many of each type of flower will there be in each bouquet?

My Notes

Learning Targets:

- Find multiples of a whole number.
- Find the least common multiple of two or more whole numbers.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Paraphrasing, Discussion Groups, Create Representations

Sheila decides to purchase hot dogs to serve at her sister's birthday party. Hot dogs come in packages of 10 and hot dog buns come in packages of 8.

1. It is hard for Sheila to decide how many packages of hot dogs and how many packages of buns to buy. Explain why this may be a hard decision for Sheila.

Sheila's first thought is to buy 3 packages of hot dogs and 4 packages of buns.

2. Will she end up with the same number of hot dogs and buns? Explain.

To solve this problem, Sheila can find the ***least common multiple (LCM)*** of 8 and 10. The LCM will help her to determine the *least* number of packages of hot dogs and buns that she must buy in order to have the same number of each.

MATH TERMS

The **least common multiple (LCM)** for a set of two or more whole numbers is the smallest number that is a multiple of all the numbers in the set.

Example A

Find the least common multiple of 8 and 10. Use your answer to determine the least number of packages of hot dogs and buns Sheila can buy to have the same number of hot dogs and buns.

Step 1: List multiples of 8 and 10.

8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80

10: 10, 20, 30, 40, 50, 60, 70, 80, 90

Step 2: Identify the smallest multiple that is in both lists.

The least common multiple of 8 and 10 is 40, so Sheila needs to determine how many packages of hot dogs will give her exactly 40 hot dogs and how many packages of buns will give her exactly 40 buns.

Step 3: Use the least common multiple, 40, to solve the problem.

There are 10 hot dogs to a package, and $10 \times \mathbf{4} = 40$.
There are 8 buns to a package, and $8 \times \mathbf{5} = 40$.

Solution: 4 packages of hot dogs and 5 packages of buns

MATH TIP

To find the multiples of a number, multiply the number by whole numbers starting with 1. For example:

$5 \times 1 = 5$
$5 \times 2 = 10$
$5 \times 3 = 15$ and so on.

The first three multiples of 5 are 5, 10, and 15.

My Notes

Try These A

Use multiples to find the LCM of each set of numbers.

a. 6, 8 **b.** 5, 10 **c.** 9, 12

3. **Reason abstractly.** What is the next equal number of hot dogs and buns Sheila could buy after 40 to end up with the same number of hot dogs and buns? Explain.

You can also find the least common multiple of two or more numbers using prime factorizations.

Example B

Find the LCM of 12 and 30.

Step 1: List the prime factorizations of the numbers.

$$12: 2 \times 2 \times 3$$
$$30: 2 \times 3 \times 5$$

Step 2: Find the product of the common factors in both lists.

$$12 = \boxed{2} \times 2 \times \boxed{3}$$
$$30 = \boxed{2} \times \boxed{3} \times 5$$

The common factors are 2 and 3, and their product is $2 \times 3 = \mathbf{6}$.

Step 3: Multiply the product of the common factors, 6, by the factors that are not common, 2 and 5.

$$\mathbf{6} \times 2 \times 5 = 60$$

Solution: The least common multiple of 12 and 30 is 60.

Try These B

Use prime factorizations to find the LCM of each set of numbers.

a. 12, 20 **b.** 24, 30 **c.** 15, 25

4. **Construct viable arguments.** Which is greater, the least common multiple of two numbers or their greatest common factor? Explain your reasoning. As you explain your reasoning, remember to use complete sentences, including transitions and words such as *and, or, since, for example, therefore,* and *because of*. Also, remember to provide a concluding statement in your explanation.

My Notes

Check Your Understanding

5. Find the least common multiple of each set of numbers using the prime factorization method.

   a. 8, 12 **b.** 10, 15 **c.** 10, 20
   d. 6, 9 **e.** 14, 21 **f.** 9, 12
   g. 3, 4, 6 **h.** 2, 4, 10 **i.** 4, 15, 30

6. **Construct viable arguments.** This is part of Marcia's homework. Is she finding the GCF or LCM? Explain your choice.

   $$72 = 9 \times 8 = 2 \times 2 \times 2 \times 3 \times 3$$
   $$48 = 6 \times 8 = 2 \times 2 \times 2 \times 2 \times 3$$

   The common factors are 2, 2, 2, and 3.

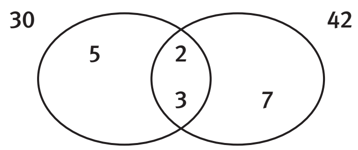
   The answer is 24.

LESSON 3-2 PRACTICE

7. Three dogs barked at exactly 11 P.M. Thereafter, Fritzie barked every 4 minutes, Spike barked every 5 minutes, and Dixie barked every 3 minutes. What was the next time when the dogs barked at the same time?

8. Ben's car gets 24 miles to each gallon of gasoline. Tina's car gets 20 miles to the gallon of gasoline. While driving from Hartdale to Lawton, they each used the same number of gallons of gas. What is the least number of miles that could be the distance between Hartdale and Lawton?

9. **Make sense of problems.** The Venn diagram shows the prime factors of 30 and 42. Explain how you can use the diagram to find the LCM of 30 and 42.



10. Of the methods you have learned for finding the least common multiple, which do you think is the most useful? Explain your reasoning.

11. What is the LCM of the two numbers $2^6 \times 3^4 \times 5$ and $2^5 \times 3^7 \times 5$? Write the solution using exponents.

ACTIVITY 3 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 3-1

1. What is the greatest common factor of 36 and 60?
   A. 3 **B.** 4
   C. 6 **D.** 12
2. Which number is NOT a common factor of 32 and 48?
   A. 2 **B.** 4
   C. 3 **D.** 8
3. The factors of a number are given below:
   1, 2, 4, 5, 8, 10, 16, 20, 40, 80
   What is the number?
4. The factors of two numbers are given below:
   Number 1: 1, 3, 5, 7, 15, 21, 35, 105
   Number 2: 1, 3, 7, 21, 49, 147
   What is the greatest common factor of the numbers?
5. Find the greatest common factor of each set of numbers using either the listing method or the prime factorization method.
   a. 12, 14 **b.** 9, 27
   c. 18, 32 **d.** 15, 35
   e. 24, 60 **f.** 70, 90
   g. 13, 21 **h.** 28, 42
   i. 28, 35, 63 **j.** 12, 16, 21
   k. 24, 56, 64 **l.** 24, 51, 66
6. **a.** Find two numbers that have a greatest common factor of 13.
   b. Explain how you found the two numbers.
7. Rory has 18 granola bars and 12 apples to package for snack bags. He wants each bag to contain the same number of granola bars and the same number of apples.
   a. What is the maximum number of bags he can make?
   b. How many granola bars and how many apples will each bag contain?
8. A courtyard has three walls with widths measuring 27 feet, 18 feet, and 30 feet. The builder plans to lay square tiles at the base of each wall. The dimensions of the tiles must be such that a whole number of them will exactly fit the width of each wall. If all tiles must be the same size what are the greatest dimensions each tile can have?
9. The 30 sixth graders, 70 seventh graders, and 45 eighth graders at science camp are being formed into teams. Each team must have the same numbers of sixth, seventh, and eight graders, and every camper must be on a team.
   a. How many teams will there be?
   b. How many campers from each grade will be on each team?
10. Peggy has three ribbons. The blue ribbon measures 96 inches, the green one 84 inches, and the yellow one 120 inches. She wants to cut them into shorter pieces all of the same length, with no ribbon left over.
    a. What is the greatest possible length of each shorter piece?
    b. What is the total number of shorter pieces she can cut?
    c. How many green pieces will there be?

Lesson 3-2

11. What is the least common multiple of 12 and 24?
A. 4 **B.** 12
C. 24 **D.** 60

12. Which number is NOT a common multiple of 4 and 6?
A. 8 **B.** 24
C. 12 **D.** 60

13. The factors of two numbers are given below:

Number 1: 1, 2, 3, 6, 9, 18

Number 2: 1, 2, 3, 4, 6, 12

What is the least common multiple of the numbers?

14. Find the least common multiple of each set of numbers using either the prime factorization method or the listing method.
a. 4, 8 **b.** 9, 12
c. 3, 5 **d.** 1, 29
e. 2, 4, 6 **f.** 2, 5, 6
g. 8, 12, 36 **h.** 4, 6, 18

15. A warehouse has boxes that are 18 inches tall and other boxes that are 30 inches tall.
a. If 18-inch boxes are stacked next to 30-inch boxes, what is the lowest height at which the two stacks will be the same height?
b. How many boxes will there be in each stack?

16. Pens come in packages of 6. Pencils come in packages of 8.
a. What is the smallest number of packages of pens and of pencils you need to buy in order to have the same number of pens and pencils?
b. How many pens and how many pencils will you have?

17. Baxter waters the lawn every 3 days and mows it every 7 days. He both watered and mowed the lawn on July 2. When will he next water and mow on the same day?

18. A restaurant manager buys eggs in 18-egg cartons and muffins in packages of 20 muffins.
a. What is the smallest number of egg cartons and muffin packages the manager must buy to have the same number of eggs and muffins?
b. How many eggs would the manager have, and how many muffins?

19. Pat and Fran are quality control specialists in a light bulb factory. Pat inspects every 20 cartons of bulbs for breakage. Fran inspects every 36 cartons for incorrect labeling. If they start at the same time, which carton will be the first that both of them inspect?

MATHEMATICAL PRACTICES
Make Sense of Problems

20. a. Find the greatest common factor and least common multiple of each pair of numbers in the table.

Numbers	GCF	LCM
8, 10		
9, 12		
12, 18		
5, 9		

b. For each row in the table, compare the product of the original numbers with the product GCF $\times$ LCM. Then make a conjecture about the relationship between the product of two given numbers and the product of the greatest common factor and the least common multiple of those numbers.
c. Use your conjecture to solve this problem: The GCF of two numbers is 5. The LCM is 60. One of the numbers is 20. What is the other number? Explain how you found the answer.

Prime Factorization, Exponents, GCF, and LCM
WINTER SPORTS

Write your answers on notebook paper. Show your work.

1. Ninety-one students signed up for the skating club. Coach Link wants to form teams with the same number of members on each team, but says that is impossible: the only teams he can make are 91 teams of 1 or 1 team of 91. Is Coach Link correct? Explain why or why not.
2. Coach Link's assistant is trying to figure out how to organize the 35 boys and 56 girls who signed up for the skating club in groups. To do so, she needs to find the greatest common factor and least common multiple of 35 and 56.
   a. Explain two ways she can find the GCF.
   b. Explain two ways she can find the LCM.
3. This year, there are 12 boys and 18 girls in the ski club. Coach Link wants to form teams with the same number of girls and the same number of boys on each team.
   a. What is the greatest number of teams that can be formed? Explain how you found the answer.
   b. How many boys and how many girls will be on each team?
4. There were 1,260 contestants at the State Ski and Skate meet.
   a. Without using a calculator, determine whether 1,260 is divisible by each number. Explain how you found your answers.

Number to Check	Divisible? (yes or no)	Explanation
2		
3		
4		
5		
6		
9		
10		

   b. Write the prime factorization of 1,260, using exponents if needed.

Prime Factorization, Exponents, GCF, and LCM
WINTER SPORTS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2a-b, 3a-b, 4a-b)	• Clear and accurate understanding of finding the LCF and GCM of a pair of numbers. • Clear and accurate understanding of the rules of divisibility and factorization.	• Correctly finding the GCF and LCM of a pair of numbers. • Correct application of the rules of divisibility and factoring a number.	• Finding factors and multiples of a pair of numbers, but not necessarily the GCF and LCM. • Use of some of the rules of divisibility and partial factoring of a number.	• Incorrect or incomplete determination of factors and multiples. • Use of few or none of the rules of divisibility; no factoring of numbers.
Problem Solving (Items 3a-b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps that result in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Item 4b)	• Clear and accurate understanding of representing a number with its prime factorization.	• Some difficulty in writing the prime factorization of a number.	• Errors in writing the prime factorization of a number.	• Inaccurate or incomplete prime factorization of a number.
Reasoning and Communication (Items 1, 2a-b, 3a, 4a)	• Precise use of appropriate math terms and language when explaining GCF, LCM, and divisibility.	• Adequate explanation of GCF, LCM, and divisibility.	• A misleading or confusing explanation of GCF, LCM, and divisibility.	• Incomplete or inaccurate explanation of GCF, LCM, and divisibility.

Fractions and Mixed Numbers

The Choice Is Yours

Lesson 4-1 Meaning of Fractions

Learning Targets:

- Given a proper fraction, find equivalent fractions.
- Express proper fractions in simplest form.
- Locate proper fractions on a number line.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Create Representations, Critique Reasoning, Use Manipulatives

During physical education class, Ms. Pitts let the students vote on what game to play: $\frac{1}{3}$ of the students chose volleyball, $\frac{2}{6}$ chose basketball, and $\frac{3}{9}$ chose dodge ball. Which sport received the most votes?

To help you determine which fraction of the votes is largest, you can use fraction strips. Since the fractions have denominators of 3, 6, and 9, cut out the strips showing thirds, sixths, and ninths.

1. **Use tools strategically.** Use fraction strips to compare the fractions $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$. What do you notice about these fractions?

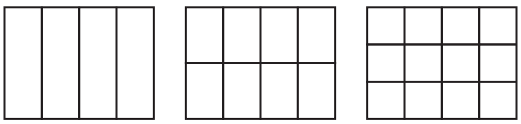
Fractions that name the same part of the whole are called ***equivalent fractions***. One way to determine equivalent fractions is to use the ***Property of One***. To find an equivalent fraction, multiply or divide the numerator and denominator of a fraction by the same number.

$$\frac{3}{4} \cdot \left[\frac{2}{2}\right] = \frac{6}{8} \qquad \frac{3}{4} \cdot \left[\frac{3}{3}\right] = \frac{9}{12} \qquad \frac{3}{4} \cdot \left[\frac{4}{4}\right] = \frac{12}{16}$$

The fractions $\frac{3}{4}$, $\frac{6}{8}$, $\frac{9}{12}$, and $\frac{12}{16}$ are equivalent fractions.

2. Why do you think multiplying by a fraction such as $\frac{2}{2}$ or $\frac{5}{5}$ is called using the Property of One?

3. Shade the figures to show that $\frac{3}{4}$, $\frac{6}{8}$, and $\frac{9}{12}$ are equivalent fractions.



You can use the Property of One with multiplication to find equivalent fractions.

My Notes

MATH TERMS

The **Property of One** states that when a given number is multiplied by 1, the product is the given number. For example, $6 \times 1 = 6$.

Recall that a fraction with the same numerator and denominator is equal to 1. So, multiplying a given number by a fraction equal to 1 is similar to multiplying by 1.

MATH TERMS

A **proper fraction** is a fraction with a numerator that is less than the denominator.

$\frac{5}{8}$ is a proper fraction because $5 < 8$. A proper fraction has a value less than 1.

My Notes

Example A

Complete: $\frac{2}{5} = \frac{?}{15}$

Step 1: Think: To change the denominator 5 to 15, multiply by 3.

$$5 \times 3 = 15$$

Step 2: Use the Property of One and multiply the numerator and denominator of $\frac{2}{5}$ by 3.

$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

Solution: $\frac{2}{5} = \frac{6}{15}$

Try These A

Complete.

a. $\frac{1}{2} = \frac{?}{8}$ **b.** $\frac{3}{5} = \frac{12}{?}$ **c.** $\frac{5}{6} = \frac{?}{30}$

You can use the Property of One with division to find equivalent fractions.

Example B

Complete: $\frac{16}{24} = \frac{2}{?}$

Step 1: Think: To change 16 to 2, divide by 8.

$$16 \div \mathbf{8} = 2$$

Step 2: Use the Property of One and divide the numerator and denominator of $\frac{16}{24}$ by 8.

$$\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$$

Solution: $\frac{16}{24} = \frac{2}{3}$

Try These B

Complete.

a. $\frac{6}{9} = \frac{?}{3}$ **b.** $\frac{6}{36} = \frac{1}{?}$ **c.** $\frac{6}{21} = \frac{?}{7}$

My Notes

A fraction is in **simplest form** when the only factor that the numerator and denominator have in common is 1.

$\frac{5}{12}$ is in simplest form because 1 is the only common factor of 5 and 12.

$\frac{14}{28}$ is NOT in simplest form because 1, 2, and 7 are common factors of 14 and 28.

4. Is $\frac{25}{30}$ in simplest form? Explain your reasoning.

Example C

Physical education class lasts $\frac{24}{30}$ of an hour. Write $\frac{24}{30}$ in simplest form.

Step 1: Find the common factors of 24 and 30.

24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

30: **1**, **2**, **3**, 5, **6**, 10, 15, 30

Step 2: Divide the numerator and denominator by a common factor.

$$\frac{24}{30} = \frac{24 \div \mathbf{2}}{30 \div \mathbf{2}} = \frac{12}{15}$$

Step 3: Continue dividing until the fraction is in simplest form.

$$\frac{12}{15} = \frac{12 \div \mathbf{3}}{15 \div \mathbf{3}} = \frac{4}{5}$$

Solution: $\frac{24}{30}$ in simplest form is $\frac{4}{5}$.

Try These C

Determine the simplest form of each fraction.

a. $\frac{16}{32}$ **b.** $\frac{15}{18}$ **c.** $\frac{8}{12}$ **d.** $\frac{18}{24}$

Notice in Example C that two division steps were used to convert $\frac{24}{30}$ to its simplest form. You can also find the simplest form of a fraction in just one division step by dividing by the greatest common factor.

5. What is the GCF of 24 and 30?

6. Complete Step 2 of Example C by dividing by the GCF.

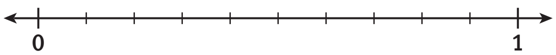
$$\frac{24}{30} = \frac{24 \div \underline{\quad}}{24 \div \underline{\quad}} = \underline{\qquad}$$

My Notes

Each student in the physical education class set a goal of how many chin-ups they want to be able to do by year's end. The table below shows the goals set by four students.

Name	Goal
Carrie	20
Franco	31
Destiny	10
Vince	25

The physical education teacher hung a large number line in the gym where students mark the progress they make toward reaching their goals. This line runs from 0 to 1, where 1 represents the goal of each student.



7. The number of chin-ups each student can do at the beginning of the year is shown below.

Carrie, 16 Franco, 17 Destiny, 7 Vince, 23

Write fractions to represent how far the students are toward reaching their goals.

Carrie: Franco: Destiny: Vince:

8. Draw a point on the number line to indicate Destiny's progress and write "D" over the point. Explain how you determined where to place this point.

9. Explain how equivalent fractions can be used to decide where to put the point showing Carrie's progress and write "C" over it.

10. **Construct viable arguments.** Explain how estimation can be used to determine where to mark Franco's and Vince's progress. Add points for the progress of each boy and write "F" and "V" over them.

My Notes

Check Your Understanding

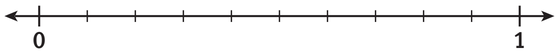
11. Determine the equivalent fraction.

a. $\frac{10}{15} = \frac{?}{3}$ **b.** $\frac{3}{4} = \frac{21}{?}$ **c.** $\frac{?}{8} = \frac{35}{40}$

12. Write each fraction in simplest form.

a. $\frac{18}{90}$ **b.** $\frac{28}{42}$ **c.** $\frac{16}{40}$

13. Draw points on the number line to represent each fraction. Write the appropriate letter over each point.



a. $\frac{2}{5}$ **b.** $\frac{9}{10}$ **c.** $\frac{6}{25}$ **d.** $\frac{19}{40}$

LESSON 4-1 PRACTICE

Write each fraction in simplest form.

14. $\frac{4}{24}$ **15.** $\frac{10}{18}$ **16.** $\frac{24}{30}$

17. Write three fractions equivalent to $\frac{9}{15}$.

18. Is $\frac{27}{51} = \frac{18}{34}$? Explain your reasoning.

19. Explain how fraction strips can be used to find five fractions equal to $\frac{3}{6}$. List these equivalent fractions.

20. Arthur answered 85 out of 95 questions correctly on his math exam. Miguel answered 64 out of 76 questions correctly on his history exam. Which student received the higher score? Explain your reasoning.

21. **Construct viable arguments.** The Property of One permits you to multiply or divide the numerator and denominator of a fraction by the same number without changing the value of the fraction. Is there a Property of One for addition? Support your answer using examples.

My Notes

Learning Targets:

- Interpret statements of inequality of proper fractions in terms of a number line and in terms of real-world contexts.
- Compare proper fractions.
- Order a set of proper fractions.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Sharing and Responding, Create Representations, Look for a Pattern, Create a Plan, Identify a Subtask

Every West Middle School homeroom elects a student council representative at the beginning of each school year. Since Mr. Fare's homeroom students do not know each other yet, he has asked interested students to volunteer. Andy, Betty, Chenetta, and Deon decide to volunteer.

ACADEMIC VOCABULARY

To ***simulate*** means to use an experiment to model a situation that can be represented with numbers such as votes in an election.

To ***simulate*** the election, each of the 23 students in Mr. Fare's homeroom will roll a number cube to vote. A roll of 1 is a vote for Andy, 2 is a vote for Betty, 3 is a vote for Chenetta, and 4 is a vote for Deon. If 5 or 6 is rolled, the student rolls again until either a 1, 2, 3, or 4 is rolled.

1. Simulate the election in your group.

a. Take turns rolling a number cube until you have a total of 23 votes. Organize your data in the table below.

Andy (1)	Betty (2)	Chenetta (3)	Deon (4)	Total Votes

b. Whom did your group elect as the homeroom representative?

2. List the names of the candidates by the number of votes received from greatest to least. Next to each name, write the number of votes the candidate received.

3. Find the fraction of the total number of votes each candidate received. Write the fractions in order from greatest to least.

My Notes

4. In the actual election in Mr. Fare's homeroom, Andy received $\frac{5}{23}$ of the total votes. Betty received $\frac{7}{23}$ of the total, Chenetta received $\frac{8}{23}$ of the total, and Deon received $\frac{3}{23}$ of the total. Which candidate was elected? Explain how you know.

The 300 students at West Middle School held a traditional election for student council officers. Eden, Frank, Gabrielle, and Hernando ran for president. Eden received $\frac{4}{15}$ of the votes. Frank received $\frac{3}{10}$, Gabrielle received $\frac{1}{30}$, and Hernando received $\frac{2}{5}$.

5. Why is it more difficult to decide who won this election than it was for the election in Item 1?

To make it easier to compare the results from this election, you can rewrite these fractions as equivalent fractions with a common denominator.

6. What is the least common denominator of the fractions of votes received in the West Middle School election?

MATH TIP

The Least Common Denominator (LCD) is the smallest number that can be used as the denominator of two or more fractions. It is always the Least Common Multiple (LCM) for a set of denominators.

7. Write the fractions of votes from the West Middle School election as equivalent fractions using this least common denominator.

Eden: $\frac{4}{15} =$ Frank: $\frac{3}{10} =$

Gabrielle: $\frac{1}{30} =$ Hernando: $\frac{2}{5} =$

8. **Reason quantitatively.** Write the fractions in order from least to greatest. Explain your reasoning.

My Notes

MATH TIP

You do not have to find the LCD to write equivalent fractions. You can always use the product of the denominators as a common denominator.

READING MATH

The symbols $<$, $>$, $\leq$ and $\geq$ are *inequality symbols*.
$<$ means "less than"
$>$ means "greater than"
$\leq$ means "less than or equal to"
$\geq$ means "greater than or equal to"

9. Construct viable arguments. Nathaniel ordered the fractions in Item 7 from least to greatest after finding equivalent fractions with denominators of 300. Do you think Nathaniel's method is less difficult or more difficult than the method you used to order the fractions? Explain.

10. List the candidates in order from first place to fourth place.

Example A

Find common denominators to compare $\frac{4}{9}$ and $\frac{7}{18}$. Use inequality symbols to write the comparison.

		Using the LCD	Using the Product of the Denominators
Step 1:	Find a common denominator.	18	$9 \times 18 = 162$
Step 2:	Write equivalent fractions.	$\frac{4}{9} = \frac{\mathbf{8}}{\mathbf{18}}$ $\frac{7}{18} = \frac{\mathbf{7}}{\mathbf{18}}$	$\frac{4}{9} = \frac{\mathbf{72}}{\mathbf{162}}$ $\frac{7}{18} = \frac{\mathbf{63}}{\mathbf{162}}$
Step 3:	Compare the fractions with the common denominator.	$\frac{8}{18} > \frac{7}{18}$	$\frac{72}{162} > \frac{63}{162}$

Step 4: Substitute original fractions in the Step 3 inequalities.

<u>LCD denominators:</u>

$\frac{8}{18} > \frac{7}{18}$ and $\frac{8}{18} = \frac{4}{9}$,

so substitute $\frac{4}{9}$ for $\frac{8}{18}$ to get $\frac{4}{9} > \frac{7}{18}$.

<u>Product of the denominators:</u>

Since $\frac{72}{162} > \frac{63}{162}$ and $\frac{72}{162} = \frac{4}{9}$ and $\frac{63}{162} = \frac{7}{18}$,

substitute $\frac{4}{9}$ for $\frac{72}{162}$ and $\frac{7}{18}$ for $\frac{63}{162}$ to get $\frac{4}{9} > \frac{7}{18}$.

Solution: $\frac{4}{9} > \frac{7}{18}$

Try These A

Use common denominators to compare the fractions. Write $<$, $=$, or $>$ in the circle.

a. $\frac{2}{5} \bigcirc \frac{1}{3}$ **b.** $\frac{7}{9} \bigcirc \frac{5}{6}$ **c.** $\frac{5}{6} \bigcirc \frac{3}{4}$

Lesson 4-2
Comparing and Ordering Fractions

My Notes

You can also compare fractions using cross products.

Example B

Compare $\frac{4}{9}$ and $\frac{5}{11}$ using cross products. Use inequality symbols to write the comparison.

To compare two fractions using cross products, multiply the numerator of one fraction by the denominator of the other fraction. Then compare the products.

$\frac{4}{9} \quad \frac{5}{11}$ $\qquad 4 \times 11 = \qquad 5 \times 9 =$

$44 < 45$, so $\frac{4}{9} < \frac{5}{11}$

Solution: $\frac{4}{9} < \frac{5}{11}$

Try These B

Use cross products to compare the fractions. Write an inequality symbol in the circle.

a. $\frac{2}{9} \bigcirc \frac{3}{7}$ **b.** $\frac{5}{9} \bigcirc \frac{7}{13}$ **c.** $\frac{11}{15} \bigcirc \frac{5}{7}$

As president of the student council, Hernando wants to speak with all the student groups about their concerns. The guidance counselor gave Hernando the following data:

- $\frac{8}{15}$ of the students take part in music.
- $\frac{1}{6}$ of the students are in art club.
- $\frac{16}{33}$ of the students participate in sports.
- $\frac{4}{9}$ of the students are in academic clubs.

Hernando decides to speak first with the groups that have the most participants. To do so he must order these fractions. He knows that a common denominator for them would be very large, so he asks his math teacher, Ms. Germain, if there is an easier way to order the fractions.

11. Model with mathematics. Ms. Germain decides to explain the concept using easier fractions. She first asks Hernando to represent each of these unit fractions in different ways to compare them.

a. Shade each rectangle below to show the fraction.

$\frac{1}{3}$

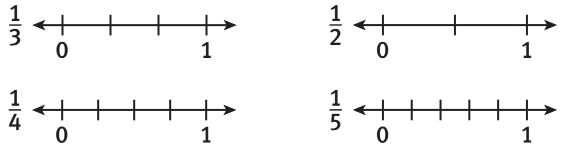
$\frac{1}{4}$

$\frac{1}{2}$

$\frac{1}{5}$

My Notes

b. Graph each fraction on the number lines below.



c. Use the representations you created in Parts a and b to order the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{5}$ from greatest to least. Explain how the representations helped you.

d. Each of the four fractions you just ordered has the same numerator. Explain how to use numerators to order fractions.

e. Order the fractions $\frac{4}{5}$, $\frac{4}{11}$, $\frac{4}{7}$, and $\frac{4}{25}$ from greatest to least.

12. The fractions Hernando wants to order have neither a common numerator nor a common denominator. He already knows it would be hard to find a common denominator.

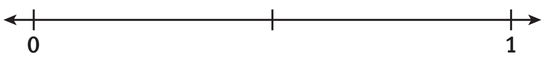
a. What number could Hernando use as a common *numerator* of the fractions $\frac{8}{15}$, $\frac{1}{6}$, $\frac{16}{33}$, and $\frac{4}{9}$? Explain.

b. Rewrite each of the fractions in Part a as an equivalent fraction with the common numerator you found.

c. Use inequality symbols to write the fractions in Part a from least to greatest.

d. If two numbers are graphed on a number line, how can you tell which number is greater?

e. Graph the fractions in Part a on the number line below.



f. In what order will Hernando talk with the student groups?

My Notes

Check Your Understanding

13. Compare the fractions. Use <, =, or >.

a. $\frac{9}{16} \bigcirc \frac{5}{8}$ **b.** $\frac{3}{10} \bigcirc \frac{4}{15}$ **c.** $\frac{5}{21} \bigcirc \frac{3}{14}$

14. **Model with mathematics.** Draw and shade rectangles to model each fraction. Then order the fractions from greatest to least.

$\frac{3}{4}$

$\frac{1}{2}$

$\frac{7}{8}$

15. What is the difference between an LCD and an LCM?

LESSON 4-2 PRACTICE

16. A jar is filled with 70 cubes. There are 15 red cubes, 9 green cubes, 21 yellow cubes, 20 purple cubes, and 5 orange cubes. Write the fraction of the total represented by each color and order them from least to greatest.

17. SB Middle School is holding a mock election for president. Candidate 1 receives $\frac{10}{50}$ of the votes. Candidate 2 receives $\frac{9}{25}$ of the votes. Candidate 3 receives $\frac{4}{10}$ of the votes. Candidate 4 receives $\frac{5}{125}$ of the votes. List the candidates in order from least to greatest number of votes.

18. After planting, a plant grew $\frac{3}{11}$ inch the first week, $\frac{6}{7}$ inch the second week, and $\frac{12}{13}$ inch the third week. In which week did the plant grow the most?

19. **Reason quantitatively.** Students voted for their favorite after-school activity. The table shows the fraction voting for each activity. Use mental math to order the activities from most popular to least popular. Explain your reasoning.

Computer Games	Read	Watch TV	Play Sports
$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$

My Notes

Learning Targets:

- Locate mixed numbers on a number line.
- Convert an improper fraction to a whole number or mixed number.
- Convert a whole number or mixed number to an improper fraction.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Summarizing Visualization, Simplify the Problem, Create Representations

Members of the Science Club are studying fossils. Fossils are the remains of animals and plants that lived long ago. The shells of ancient sea creatures called ammonites are among the most common fossils. Ammonites have spiral-shaped shells as shown in the picture to the left. They have been found in a wide variety of colors and range in width from a fraction of an inch to more than 6 feet.

Measurements in feet and inches are often expressed as **mixed numbers.** A mixed number is the sum of a whole number and a proper fraction. The number $1\frac{3}{4}$ is an example of a mixed number; it is the sum of 1 and $\frac{3}{4}$:

$$1\frac{3}{4} = 1 + \frac{3}{4}.$$

Fraction strips can be used to represent mixed numbers.

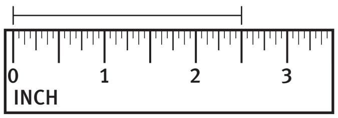
MATH TERMS

An **improper fraction** is a fraction with a numerator that is greater than the denominator.

$\frac{9}{5}$ is an improper fraction because $9 > 5$.

An improper fraction has a value greater than 1.

1. **Attend to precision.** Kelly laid a length of string across the widest part of an ammonite. Then she measured the length of the string.

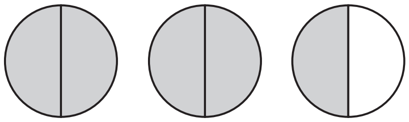


How wide is the ammonite? Write your answer as a mixed number.

2. How many $\frac{1}{2}$ inches are in $2\frac{1}{2}$ inches? Count the number of $\frac{1}{2}$-inch intervals on the ruler and write your answer as an ***improper fraction***.

My Notes

3. You can also use models to represent mixed numbers and improper fractions. Name the shaded parts of this model two ways.

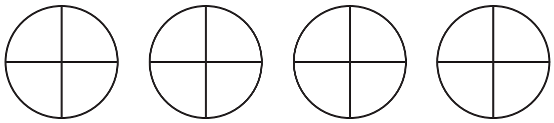


mixed number: improper fraction:

4. Both the mixed number and the improper fraction in Item 3 describe the same number. Write a ***number sentence*** that shows that the mixed number and the improper fraction are equal.

5. Another ammonite had a width of $3\frac{1}{4}$ inches.

   a. Shade the circles to represent the width.



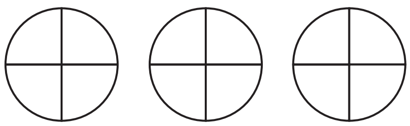
   b. Write $3\frac{1}{4}$ as a sum of whole numbers and a fraction.

$$3\frac{1}{4} = 1 + \_\_ + \_\_ + \frac{1}{4}$$
$$= \frac{4}{4} + \frac{\_\_}{\_\_} + \frac{\_\_}{\_\_} + \frac{1}{4}$$
$$= \frac{\_\_}{4}$$

6. Rewrite $3\frac{1}{4}$ as an improper fraction.

Drawing a model and division are two methods for changing an improper fraction to a mixed number.

7. **a.** Color the model below to show $\frac{9}{4}$.



   b. Write $\frac{9}{4}$ as a mixed number.

MATH TIP

A **number sentence** is an equation or inequality expressed using numbers and symbols. Number sentences can be true or false.

$12 + 3 = 15$	True
$9 - 4 = 1$	False
$2 + 7 \leq 9$	True
$13 > 13$	False

My Notes

c. Reason abstractly. How does the model in Part a help you write the mixed number in Part b?

You can use division to change an improper fraction to a mixed number since the fraction bar indicates division.

$\frac{a}{b}$ represents the same number as $a \div b$, where $b \neq 0$.

$\frac{2}{5} = 2 \div 5$ $\quad$ $\frac{6}{3} = 6 \div 3$ $\quad$ $\frac{27}{15} = 27 \div 15$

Example A

Rewrite $\frac{14}{4}$ as a mixed number in simplest form by dividing. Express the remainder as a fraction.

Step 1: Set up the division.

$$\frac{14}{4} = 14 \div 4 = 4\overline{)14}$$

Step 2: Divide.

$$\begin{array}{r} 3 \\ 4\overline{)14} \\ \underline{12} \\ 2 \end{array} \leftarrow \text{The remainder is 2.}$$

Step 3: Write the remainder over the divisor and simplify, if possible.

$$\frac{2}{4} = \frac{1}{2}$$

Step 4: Use the quotient and the remainder to form a mixed number.

$$3 + \frac{1}{2} = 3\frac{1}{2}$$

Solution: $\frac{14}{4} = 3\frac{1}{2}$

Try These A

Write each improper fraction as a mixed number in simplest form.

a. $\frac{11}{3}$ $\quad$ **b.** $\frac{15}{7}$ $\quad$ **c.** $\frac{16}{6}$

My Notes

Check Your Understanding

8. Write each mixed number as an improper fraction.

a. $4\frac{1}{5}$ **b.** $3\frac{2}{7}$ **c.** $7\frac{2}{3}$

d. $2\frac{3}{4}$ **e.** $1\frac{1}{6}$ **f.** $9\frac{1}{2}$

9. Write each improper fraction as a mixed number in simplest form.

a. $\frac{8}{3}$ **b.** $\frac{3}{2}$ **c.** $\frac{17}{5}$

d. $\frac{10}{9}$ **e.** $\frac{18}{4}$ **f.** $\frac{15}{6}$

10. Explain how you can find the number of sevenths in $5\frac{3}{7}$.

11. Draw a model representing $\frac{7}{3} = 2\frac{1}{3}$.

LESSON 4-3 PRACTICE

12. Write each mixed number as an improper fraction.

a. $4\frac{4}{5}$ **b.** $3\frac{5}{7}$ **c.** $7\frac{1}{7}$

13. Write each improper fraction as a mixed number in simplest form.

a. $\frac{8}{3}$ **b.** $\frac{3}{2}$ **c.** $\frac{17}{5}$

14. The largest ammonite ever found measured $8\frac{1}{2}$ feet in diameter. Write the width in feet as an improper fraction.

15. The largest ammonite weighs $3\frac{1}{2}$ tons. Write $3\frac{1}{2}$ as an improper fraction three different ways.

16. Convert $11\frac{28}{6}$ to an improper fraction and a mixed number, both in simplest form.

17. Describe an improper fraction that simplifies to a whole number.

18. **Reason abstractly.** Can any fraction be written as a mixed number? Explain, giving examples to illustrate your answer.

My Notes

Learning Targets:

- Interpret statements of inequality of mixed numbers in terms of a number line and in terms of real-world contexts.
- Compare mixed numbers.
- Order a set of mixed numbers or fractions.

SUGGESTED LEARNING STRATEGIES: Summarizing, Visualization, Sharing and Responding, Simplify the Problem

Five students are growing sunflowers in the school garden. The table below shows the height of the tallest sunflower each student has grown.

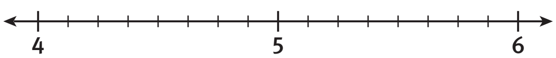
Andy	Bianca	Cody	Deronda	Elyse
$4\frac{1}{4}$ ft	$4\frac{3}{4}$ ft	$\frac{37}{8}$ ft	$5\frac{1}{2}$ ft	$4\frac{7}{8}$ ft

1. **Make use of structure.** Write an inequality using $<$ or $>$ to compare the sunflower heights for the students listed. Explain how you determined which of the heights is greater.
   a. Elyse and Deronda

   b. Andy and Bianca

   c. Bianca, Cody, and Elyse

2. Place the flower heights on the number line below. Write the first letter of each student's name over the appropriate point.



My Notes

3. When two numbers are plotted on a number line, how can you tell which number is greater?

Check Your Understanding

4. Compare the numbers. Use $<$, $=$, or $>$.

a. $3\frac{1}{10} \bigcirc 2\frac{9}{10}$ **b.** $5\frac{2}{7} \bigcirc 5\frac{5}{7}$ **c.** $1\frac{8}{12} \bigcirc 1\frac{6}{9}$

d. $2\frac{1}{3} \bigcirc 2\frac{1}{2}$ **e.** $6\frac{3}{4} \bigcirc 6\frac{8}{10}$ **f.** $1\frac{7}{12} \bigcirc \frac{3}{2}$

g. $7\frac{1}{3} \bigcirc 7\frac{2}{7}$ **h.** $\frac{41}{8} \bigcirc \frac{31}{6}$ **i.** $2\frac{16}{20} \bigcirc 2\frac{12}{15}$

5. Make use of structure. Explain how you can compare the given types of numbers.

a. two mixed numbers with different whole-number parts
b. two improper fractions
c. one mixed number and one improper fraction
d. two mixed numbers with equal whole number parts

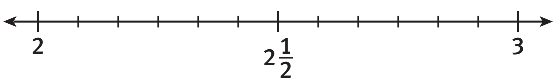
LESSON 4-4 PRACTICE

6. The table gives the heights of five of the tallest players ever to play professional basketball. Order the players from tallest to least tall.

Name	Height (m)
Manute Bol	$2\frac{3}{10}$
Randy Breuer	$2\frac{21}{100}$
Yao Ming	$2\frac{7}{25}$
Chuck Nevitt	$2\frac{13}{50}$
Ralph Sampson	$2\frac{6}{25}$

7. Use appropriate tools strategically. Draw a number line like the one below. Plot points A–E. Write the letter above the point.

A. $2\frac{7}{12}$ **B.** $2\frac{5}{6}$ **C.** $2\frac{2}{3}$ **D.** $2\frac{3}{4}$ **E.** $2\frac{1}{2}$

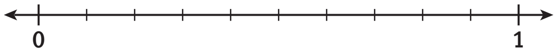


ACTIVITY 4 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 4-1

1. Write each fraction in simplest form.
   a. $\frac{16}{80}$ **b.** $\frac{36}{96}$ **c.** $\frac{34}{51}$
2. Which of the following fractions is NOT equivalent to $\frac{36}{54}$?
   A. $\frac{12}{18}$ **B.** $\frac{8}{12}$
   C. $\frac{24}{30}$ **D.** $\frac{48}{72}$
3. This number line goes from 0 to 1 and is subdivided into tenths. Describe where you would locate the point for $\frac{15}{25}$.



Lesson 4-2

4. Which of the following inequalities is true?
   A. $\frac{3}{4} < \frac{5}{9}$ **B.** $\frac{17}{20} > \frac{4}{5}$
   C. $\frac{5}{6} < \frac{11}{14}$ **D.** $\frac{7}{12} > \frac{16}{25}$
5. There are 60 marbles in a bag. One-fourth of the marbles are red, $\frac{2}{5}$ of the marbles are blue, $\frac{1}{12}$ of the marbles are green, and $\frac{4}{15}$ of the marbles are yellow. List the fractions from greatest to least. Explain how you determined your list.
6. Last month Ellen grew $\frac{7}{8}$ inch, Kelvin grew $\frac{9}{16}$ inch, and Grady grew $\frac{3}{4}$ inch. Which student grew the least?

Lesson 4-3

Write each mixed number as an improper fraction.

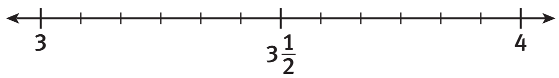
7. $5\frac{1}{5}$ 8. $4\frac{2}{7}$ 9. $1\frac{1}{8}$
10. $6\frac{4}{9}$ 11. $1\frac{5}{12}$ 12. $7\frac{2}{3}$

Write each improper fraction as a mixed number in simplest form.

13. $\frac{4}{3}$ 14. $\frac{19}{4}$ 15. $\frac{18}{8}$
16. $\frac{25}{15}$ 17. $\frac{12}{11}$ 18. $\frac{12}{9}$
19. Convert $8\frac{9}{6}$ to an improper fraction and a mixed number, both in simplest form.
20. Write $6\,\frac{7}{8}$ as an improper fraction three different ways.
21. Explain how you can find the number of ninths in $2\frac{5}{9}$.

Lesson 4-4

22. This number line goes from 3 to 4 and is subdivided into twelfths. Describe where you would locate the point for $3\frac{5}{6}$.



Compare each pair of numbers. Use <, =, or >.

23. $4\frac{3}{10}$ ◯ $3\frac{9}{10}$ 24. $1\frac{8}{12}$ ◯ $1\frac{12}{18}$
25. $5\frac{1}{2}$ ◯ $5\frac{1}{3}$ 26. $1\frac{5}{12}$ ◯ $\frac{3}{2}$
27. $7\frac{3}{7}$ ◯ $7\frac{1}{2}$ 28. $\frac{51}{8}$ ◯ $\frac{41}{6}$

MATHEMATICAL PRACTICES
Make Use of Structure

29. Explain how you can compare two different mixed numbers that have the same whole number part.

Multiplying Fractions and Mixed Numbers

ACTIVITY 5

Skateboarding Fun!

Lesson 5-1 Multiplying by Fractions

My Notes

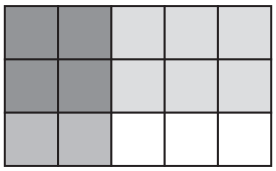
Learning Targets:

- Multiply a whole number by a fraction less than 1.
- Multiply two fractions less than 1.
- Estimate the product of a fraction and a whole number.

SUGGESTED LEARNING STRATEGIES: Paraphrasing, Visualization, Sharing and Responding, Note Taking, Create Representations, Identify a Subtask

At Jefferson Middle School, two-thirds of the students are in the After-School Program. Of the students in the program, two-fifths are skateboarders. To find the fraction of the entire student body that are both in the After-School Program and skateboarders, you need to find the product $\frac{2}{5} \times \frac{2}{3}$. You can use a grid to model the product.

Model with mathematics. The grid represents the whole number 1. The first fraction is $\frac{2}{5}$, so divide the length of the rectangle into five equal columns and shade two of the columns. The second fraction is $\frac{2}{3}$, so divide the height of the rectangle into three equal rows and shade two of the rows.



1. The portion of the grid where the two shadings overlap represents the product $\frac{2}{5} \times \frac{2}{3}$.
   a. How many squares are in the entire rectangle?
   b. How many squares show overlapping shading?
   c. What is the product $\frac{2}{5} \times \frac{2}{3}$?

You can also use an algorithm to find the product $\frac{2}{5} \times \frac{2}{3}$.

2. What is the product of the numerators?
3. What is the product of the denominators?
4. Compare your answers to Items 2 and 3 with your answer for Item 1c. Then write an algorithm for finding the product of two fractions in the My Notes section to the right.

My Notes

Sometimes, the product of two fractions can be simplified.

5. Multiply and write the product in simplest form.

a. $\frac{3}{4} \times \frac{1}{3}$ **b.** $\frac{3}{8} \times \frac{4}{9}$ **c.** $\frac{2}{5} \times \frac{4}{7}$

6. Explain why the product is always less than 1 when you multiply a fraction by another fraction.

MATH TIP

To estimate products of fractions, round each fraction to the nearest one-half or whole to help you make an estimate.

A fraction such as $\frac{7}{8}$ is close to 1 because the numerator and denominator are close in value.

A fraction such as $\frac{7}{12}$ is close to $\frac{1}{2}$ because the numerator is about half of the denominator.

When you added and subtracted fractions, you estimated the sums and differences so you could be sure your answers were reasonable. You can also estimate the products of fractions, and of fractions and whole numbers.

7. Estimate the product $\frac{5}{11} \times \frac{8}{9}$ to the nearest half. Explain how you made your estimate.

8. Estimate the product $\frac{4}{9} \times 15$. Explain how you made your estimate.

When the numerator of one fraction and the denominator of the other fraction being multiplied have common factors, you can simplify before multiplying.

Example A

At Northside Middle School, $\frac{8}{21}$ of the students participate in the After-School Program. Of those in the program, $\frac{7}{12}$ are skateboarders. What fraction of the entire student body are skateboarders?

Step 1: Determine the operation to solve.

Multiply: $\frac{8}{21} \times \frac{7}{12}$

My Notes

Step 2: Look for common factors in the numerator and the denominator. 8 and 12 are divisible by 4; 7 and 21 are divisible by 7.

$$\frac{8}{21} \times \frac{7}{12}$$

Step 3: Divide 8 and 12 by 4. Cross out 8 and 4 and write the quotients, 2 and 3.
Divide 7 and 21 by 7. Cross out 7 and 21 and write the quotients, 1 and 3.

$$\frac{{}^{2}\cancel{8}}{\cancel{21}_{3}} \times \frac{{}^{1}\cancel{7}}{\cancel{12}_{3}}$$

Step 4: After simplifying, multiply the numerators and multiply the denominators.

$$\frac{2}{3} \times \frac{1}{3} = \frac{2 \times 1}{3 \times 3} = \frac{2}{9}$$

Solution: $\frac{2}{9}$ of the students are skateboarders.

Try These A

Determine the product in simplest form by dividing by common factors and then multiplying.

a. $\frac{3}{4} \times \frac{1}{3}$ **b.** $\frac{3}{8} \times \frac{4}{9}$ **c.** $\frac{2}{5} \times \frac{4}{7}$

When multiplying a fraction by a whole number, first write the whole number as a fraction. Then use the algorithm for multiplying two fractions.

9. Estimate the product $\frac{7}{8} \times 24$.

10. Rewrite $\frac{7}{8} \times 24$ with both factors as fractions.

11. Multiply. Simplify first if possible. $\frac{7}{8} \times 24 = ?$

12. Is your answer reasonable? Explain.

My Notes

Check Your Understanding

13. Determine each product in simplest form.

a. $\frac{1}{2}\times\frac{1}{4}$ **b.** $\frac{4}{5}\times\frac{1}{4}$ **c.** $\frac{4}{7}\times\frac{7}{8}$

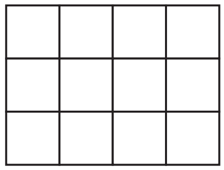
d. $\frac{8}{15}\times\frac{3}{4}$ **e.** $\frac{9}{20}\times\frac{2}{3}$ **f.** $\frac{8}{9}\times\frac{5}{11}$

g. $\frac{4}{5}\times 20$ **h.** $\frac{2}{11}\times 77$ **i.** $\frac{3}{5}\times 24$

14. Model with mathematics.

a. Draw a rectangle like the one shown. Shade it to find $\frac{2}{3}\times\frac{3}{4}$

b. Explain how you would make a model to find $\frac{5}{8}\times\frac{7}{9}$



15. Explain how, without multiplying, you can determine whether to put $<$ or $>$ in the circle below.

$$\frac{2}{7}\times\frac{3}{10}\bigcirc\frac{2}{7}$$

LESSON 5-1 PRACTICE

16. Melissa is making a batch of cookies. The recipe calls for $\frac{2}{3}$ cup of chocolate. Melissa is making only one-fourth of a batch. How much chocolate will she need in order to make her cookies?

17. Yesterday the town of Harrington received $\frac{7}{8}$ inch of snow. Today the town received $\frac{2}{3}$ of yesterday's amount. How much snow fell in Harrington today?

18. Of the 245 shoppers at Shop 'n' Save yesterday, $\frac{2}{7}$ completed the customer survey. How many shoppers did NOT complete the survey?

19. Make sense of problems. Two fifteenths of the 630 students at Dillon Middle School are on a sports team. Of those on a sports team, $\frac{1}{4}$ play basketball. Three-sevenths of the basketball players are girls.

a. How many girls play on the Dillon basketball team?

b. How many boys play on the Dillon basketball team?

My Notes

Learning Targets:

- Multiply mixed numbers by fractions, whole numbers, and other mixed numbers.
- Estimate products involving mixed numbers.

SUGGESTED LEARNING STRATEGIES: Paraphrasing, Visualization, Think-Pair-Share, Sharing and Responding, Identify a Subtask

The method for multiplying mixed numbers is similar to the method for multiplying fractions.

Example A

For his birthday, Buddy wants a skateboard $9\frac{3}{5}$ times the length of his $2\frac{1}{2}$-inch-long model skateboard. How long is the skateboard he wants?

Step 1: Determine the operation to use. Multiply: $9\frac{3}{5} \times 2\frac{1}{2}$

Step 2: Estimate the product to the nearest whole number. $10 \times 3 = 30$ inches

Step 3: Write the mixed numbers as improper fractions. $9\frac{3}{5} \times 2\frac{1}{2} = \frac{48}{5} \times \frac{5}{2}$

Step 4: Divide the numerators and denominators by any common factors. $= \frac{\overset{24}{\cancel{48}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}}$

Step 5: Multiply. $= \frac{24}{1} \times \frac{1}{1} = 24$

Solution: The skateboard he wants is 24 inches long. The answer is reasonable because it is close to the estimate of 30 inches.

Try These A

Determine each product in simplest form.

a. $2\frac{3}{5} \times 1\frac{1}{2}$ **b.** $6\frac{3}{8} \times \frac{1}{4}$ **c.** $3\frac{2}{3} \times 33$

1. **Construct viable arguments.** If you multiply a number by a proper fraction, will the product be greater than or less than the number? Explain. Remember to provide a concluding statement in your written explanation.

2. If you multiply a number by an improper fraction, will the product be greater than or less than the number? Explain. Remember to provide a concluding statement in your written explanation.

My Notes

Check Your Understanding

3. Determine each product in simplest form

a. $3\frac{1}{8}\times3\frac{1}{5}$ **b.** $2\frac{2}{7}\times1\frac{1}{6}$ **c.** $3\frac{5}{6}\times1\frac{5}{7}$

d. $8\frac{1}{3}\times6$ **e.** $1\frac{1}{6}\times\frac{2}{3}$ **f.** $1\frac{2}{3}\times3\frac{2}{5}$

g. $18\times4\frac{2}{3}$ **h.** $2\frac{5}{8}\times\frac{4}{7}$ **i.** $4\frac{2}{11}\times1\frac{10}{23}$

4. Justify each step in the following problem.

Step 1: $7\times3\frac{2}{7}=7\times\left(3+\frac{2}{7}\right)$

Step 2: $=(7\times3)+\left(7\times\frac{2}{7}\right)$

Step 3: $=21+2$, or 23

LESSON 5-2 PRACTICE

5. Madeline is making a quilt. She found some pieces of fabric on sale that each measured $\frac{9}{10}$ of a yard. She is going to use $2\frac{1}{2}$ of the pieces for the quilt. What is the total length of the fabric she will use?

6. Meagan wants to make $2\frac{1}{3}$ batches of muffins. The recipe calls for $1\frac{1}{2}$ cups of sugar per batch. How much sugar will she need?

7. Max has a model of a sports car that is $2\frac{1}{5}$ inches long. He is making a larger model that will be $5\frac{1}{2}$ times the length of the smaller car. How long will the larger model be?

8. **Critique the reasoning of others.** Chase says that an easy way to multiply mixed numbers is simply to multiply the whole numbers, then multiply the fractions, and then add the two. Is he right? Include an example in your answer.

9. **Attend to precision.** Each day for 12 consecutive days, Ricardo jogged for $1\frac{2}{3}$ hours at an average rate of $6\frac{1}{4}$ miles per hour. Find the total distance that he jogged.

10. Stack A consists of three cartons, each $2\frac{2}{3}$ feet in height. Stack B consists of four cartons, each $1\frac{5}{6}$ feet in height.

a. Which stack is taller?
b. How many inches taller is it?

ACTIVITY 5 PRACTICE

Write your answers on notebook paper. Show your work.

Express your answers in simplest form.

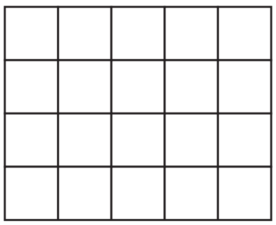
Lesson 5-1

1. Find the product: $\frac{5}{6}\times\frac{3}{10}$
   - a. $\frac{1}{8}$
   - b. $\frac{2}{15}$
   - c. $\frac{1}{2}$
   - d. $\frac{1}{4}$

2. Which is the most accurate statement about the product of a fraction and a whole number?
   - A. It is less than 1.
   - B. It is less than or equal to 1.
   - C. It is less than the whole number.
   - D. It is equal to or greater than the whole number.

3. Which is the best estimate of the product $\frac{17}{33}\times\frac{11}{20}$?
   - A. $\frac{1}{4}$
   - B. $\frac{1}{2}$
   - C. 1
   - D. 2

4. a. Copy the model below. Then shade it to find the product $\frac{3}{4}\times\frac{2}{5}$.



   b. Find the product $\frac{3}{4}\times\frac{2}{5}$ and express the answer in simplest form.

5. Three of the eight rows of a rectangular model are shaded. Five of the seven columns are shaded. What product can be found using the model?

6. The normal amount of rainfall for July in Apple Valley is $\frac{11}{24}$ feet. So far this July, only $\frac{3}{10}$ of the normal amount has fallen. Find the amount of rain that has fallen so far this month.

7. Complete: $\frac{3}{4}\times 20=\frac{5}{6}\times ?$

8. A cookie recipe calls for $\frac{3}{4}$ tablespoon of salt. Hugo plans to make 10 batches of cookies. How much salt will he need?

9. Of the students in Marci's school, $\frac{5}{9}$ have a pet. Of the students who have a pet, $\frac{12}{25}$ have a dog. What fraction of the students in the school have a dog?

10. Each of the elephants in the Metro Zoo eats an average of 128 pounds of food per day. Each of the giraffes eats $\frac{7}{8}$ of the elephants' average. How many pounds of food does each giraffe eat per day?

11. Two-ninths of the 540 students at East Side Middle School are in the chorus. Three-fourths of the chorus members are girls. Two-fifths of the girls in the chorus are sopranos. What fraction of the students in the school are girl sopranos in the chorus?

12. The volume of a box is the product of its length, its width, and its height. What is the volume of a box that measures $\frac{8}{9}$ foot by $\frac{5}{12}$ foot by $\frac{3}{10}$ foot? Express your answer in cubic feet.

Remember to express your answers in simplest form.

Lesson 5-2

13. Find the product $1\frac{2}{3} \times 2\frac{1}{10}$.

a. $2\frac{1}{15}$ **b.** $3\frac{3}{13}$

c. $3\frac{1}{2}$ **d.** $3\frac{23}{30}$

14. Estimate the product by rounding to the nearest whole number: $8\frac{5}{12} \times 3\frac{6}{13}$.

a. 24 **b.** 27
c. 32 **d.** 36

15. Which is the most accurate statement about the product of a number and an improper fraction?
A. It is less than 1.
B. It is less than or equal to 1.
C. It is greater than 1 but less than the number.
D. It is greater than the number.

16. Explain why

$$12 \times \left(5 + \frac{4}{9}\right) = (12 \times 5) + \left(12 \times \frac{4}{9}\right)$$

17. Colette is finding the product $\frac{41}{36} \times \frac{60}{19}$. Explain how she can divide out the common factors from the numerator and the denominator in one step.

18. Twelve square tiles, each measuring $5\frac{3}{8}$ inches on a side, are laid side by side. What is the total width of the tiles?

19. The following shows a student's calculation of the product $1\frac{1}{3} \times 1\frac{2}{3}$. Explain the error in the student's work.

$$\begin{aligned} 1\frac{1}{3} \times 1\frac{2}{3} &= \frac{4}{3} \times \frac{5}{3} \\ &= \frac{20}{3} \\ &= 6\frac{1}{3} \end{aligned}$$

20. The table shows the number of hours Monty spent doing homework last week.

Mon	Tue	Wed	Thu	Fri
$1\frac{1}{2}$	$\frac{3}{4}$	$2\frac{1}{4}$	$1\frac{5}{6}$	1

This week Monty spent $1\frac{1}{2}$ times as much time doing homework as he did last week. How many hours did he spend this week?

21. On the first day of his backpacking trip, Andy left camp at 9:15 A.M. and arrived at his new camp at 4:45 P.M. His average rate of speed was $2\frac{1}{2}$ miles per hour. On the second day he left the new camp at 8:45 A.M. and arrived at his final camp at 3:30 P.M. His average rate of speed was 3 miles per hour. Find the total distance that he hiked over the two days.

MATHEMATICAL PRACTICES
Make Sense of Problems

22. Lainie earns $13.50 per hour as a grocery clerk. This week she worked $4\frac{1}{2}$ days for an average of $5\frac{1}{3}$ hours per day. Find her total earnings.

Dividing Fractions and Mixed Numbers

How Many Sandwiches?

Lesson 6-1 Dividing by Fractions

My Notes

Learning Targets:

- Divide a whole number by a fraction less than 1.
- Divide a fraction by a whole number or fraction.
- Solve real-world problems by dividing such numbers.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Predict and Confirm, Think Aloud, Create Representations, Identify a Subtask, Look for a Pattern

Brenna is in charge of ordering lunch for the student council members at Jackson Middle School. She buys a 6-foot-long sandwich at Significant Subs and slices it into $\frac{1}{2}$-foot sections.

1. a. Draw a pictorial model you can use to find the number of sections Brenda slices. Show the model divided into $\frac{1}{2}$-foot pieces.

b. $6 \div \frac{1}{2} =$ ______

For drinks, Brenna bought 4 quarts of milk. She figured that students would drink an average of $\frac{1}{3}$ quart apiece.

2. a. Model with mathematics. Draw a pictorial model you can use to find the number of servings of milk 4 quarts can provide. Show the model divided into $\frac{1}{3}$-quart sections.

b. $4 \div \frac{1}{3} =$ ______

3. Look at the number sentences in Items 1b and 2b.

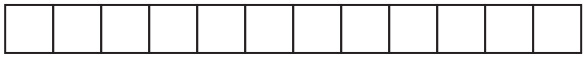
a. Describe patterns that you see.

My Notes

b. Reason abstractly. Explain how you could have solved the problems without using the pictorial models.

The student council meeting lasted $\frac{5}{6}$ of an hour. The president decided that $\frac{1}{12}$ of an hour was to be allotted to discuss each item of business.

4. a. The model represents 1 hour. Shade the correct number of squares to represent $\frac{5}{6}$ of an hour, the length of the meeting.



b. How many $\frac{1}{12}$ hours are in $\frac{5}{6}$ of an hour?

c. Make use of structure. Use the pattern you found in Item 3 to find the answer to this division: $\frac{5}{6} \div \frac{1}{12} =$ ______

d. Does the pattern you described in Item 3 hold when you divide a fraction by another fraction? Explain.

MATH TERMS

Two numbers whose product is 1 are called **reciprocals**.

The patterns you have investigated can be summarized in this algorithm:

- To divide by a fraction, *multiply* by the ***reciprocal*** of the fraction.

The reciprocal of a fraction is the fraction you obtain by interchanging the numerator and the denominator.

fraction: $\frac{3}{7}$ reciprocal: $\frac{7}{3}$

The product of a number and its reciprocal is always 1.

$$\frac{{}^{1}\cancel{3}}{\cancel{7}_{1}} \times \frac{{}^{1}\cancel{7}}{\cancel{3}_{1}} = 1$$

5. Find the reciprocal of each number.

a. $\frac{4}{5}$ **b.** 6 **c.** $\frac{8}{5}$

My Notes

Example A

Divide: $\frac{3}{11} \div \frac{3}{8}$.

Step 1: Multiply by the reciprocal of the divisor.

Step 2: Simplify.

Step 3: Write the quotient in simplest form.

$$\frac{3}{11} \div \frac{3}{8} = \frac{3}{11} \times \frac{8}{3}$$
$$= \frac{{}^{1}\cancel{3}}{11} \times \frac{8}{\cancel{3}_{1}}$$
$$= \frac{8}{11}$$

Solution: $\frac{3}{11} \div \frac{3}{8} = \frac{8}{11}$

Try These A

Determine each quotient in simplest form.

a. $\frac{1}{13} \div \frac{1}{2}$ **b.** $\frac{7}{8} \div \frac{3}{4}$ **c.** $\frac{2}{5} \div \frac{7}{10}$

When you divide by a fraction, the dividend may be a whole number or a mixed number. When this happens, rewrite the dividend as an improper fraction. Then use the algorithm for dividing fractions.

Example B

Divide: $1\frac{2}{3} \div \frac{5}{6}$.

Step 1: Write the mixed number as an improper fraction.

$$1\frac{2}{3} \div \frac{5}{6} = \frac{5}{3} \div \frac{5}{6}$$

Step 2: Multiply by the reciprocal of the divisor.

$$= \frac{5}{3} \times \frac{6}{5}$$

Step 3: Simplify.

$$= \frac{{}^{1}\cancel{5}}{3} \times \frac{6}{\cancel{5}_{1}}$$

Step 4: Write the quotient in simplest form.

$$= \frac{6}{3}, \text{ or } 2$$

Solution: $1\frac{2}{3} \div \frac{5}{6} = 2$

Try These B

Determine each quotient in simplest form.

a. $2\frac{5}{6} \div \frac{5}{12}$ **b.** $4 \div \frac{2}{7}$ **c.** $4\frac{5}{8} \div \frac{5}{8}$

My Notes

6. Complete: The number of fourths in 20 equals the number of ______ in 10 which equals the number of fifths in ______.

7. Find the value of n: $\frac{4}{5} \div \frac{n}{6} = \frac{4}{5} \times \frac{n}{6}$

Check Your Understanding

8. Determine the reciprocal of the fraction.
   a. $\frac{5}{9}$ **b.** $\frac{12}{11}$ **c.** $\frac{1}{15}$
9. Determine each quotient in simplest form.
   a. $\frac{1}{2} \div \frac{1}{4}$ **b.** $\frac{4}{5} \div \frac{2}{3}$ **c.** $7 \div \frac{1}{4}$
   d. $\frac{5}{12} \div \frac{5}{6}$ **e.** $3\frac{1}{3} \div \frac{3}{4}$ **f.** $\frac{18}{25} \div \frac{3}{5}$
   g. $1 \div \frac{9}{13}$ **h.** $1\frac{3}{5} \div \frac{5}{6}$ **i.** $\frac{7}{8} \div \frac{7}{16}$

LESSON 6-1 PRACTICE

10. Hannah bought $9\frac{1}{2}$ pounds of ground beef to make hamburgers for a party. How many $\frac{1}{4}$-pound burgers can she make?
11. A bookshelf has 7 shelves, each measuring 4 feet 3 inches in length. The average width of a book is $\frac{3}{4}$ inch. How many books will the shelves hold?
12. A snail sets out on a $3\frac{3}{5}$-mile journey, traveling at a speed of $\frac{2}{65}$ miles per hour.
    a. How many hours will the journey take?
    b. How many days will the journey take?
13. **Make sense of problems.** A 4-foot-long submarine sandwich costs \$24. A 6-inch submarine sandwich costs \$3.95. How much money can you save by buying the 4-footer and slicing it into 6-inch sandwiches?
14. Jake has \$3 in quarters in his pocket. Explain how to use reciprocals to find the number of quarters that he has.
15. **Reason abstractly.** State a simple rule you can use to find the quotient of two fractions that have the same denominator.
16. Write a problem that you can solve by finding the quotient $\frac{1}{2} \div \frac{2}{3}$.
17. The Baxter Freeway is 24 miles long. There is a telephone at the beginning, the end, and every $\frac{3}{4}$ mile along the freeway. How many telephones are there altogether?

My Notes

Learning Targets:

- Divide a mixed number, whole number, or fraction by a mixed number.
- Estimate such quotients.
- Solve real-world problems by dividing such numbers.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think Aloud, Sharing and Responding, Identify a Subtask

When you divide by a mixed number, rewrite the mixed number as an improper fraction. Then use the algorithm for dividing fractions.

Example A

The school day at Jackson Middle School is $7\frac{1}{2}$ hours long. It is divided into double periods, each lasting $1\frac{1}{4}$ hours. How many double periods are there in a day?

Step 1: Determine the operation:
Divide: $7\frac{1}{2} \div 1\frac{1}{4}$

Step 2: Estimate the number of double periods in a day.
Round $7\frac{1}{2}$ to 8 and $1\frac{1}{4}$ to 1: $8 \div 1 = 8$, so there are about 8 double periods.

Step 3: Write the mixed numbers as improper fractions. $7\frac{1}{2} \div 1\frac{1}{4} = \frac{15}{2} \div \frac{5}{4}$

Step 4: Multiply by the reciprocal of the divisor. $= \frac{15}{2} \times \frac{4}{5}$

Step 5: Multiply. $= \frac{15}{2} \times \frac{4}{5}$

Solution: $= 6$

There are 6 double periods in a day. The answer is reasonable because it is close to the estimate.

Try These A

Divide. Write the quotient in simplest form.

a. $1\frac{2}{5} \div 1\frac{13}{15}$ **b.** $3\frac{3}{5} \div 2\frac{1}{4}$ **c.** $4\frac{1}{8} \div 1\frac{5}{6}$

My Notes

Check Your Understanding

1. Give the reciprocal of the number.
   a. $2\frac{2}{3}$ b. $5\frac{5}{11}$ c. $9\frac{9}{10}$
2. Estimate the quotient.
   a. $14\frac{2}{5} \div 1\frac{3}{4}$ b. $3\frac{3}{7} \div 7\frac{1}{5}$ c. $12\frac{2}{11} \div 2\frac{6}{11}$
3. Divide.
   a. $1\frac{5}{7} \div 1\frac{3}{7}$ b. $5 \div 3\frac{3}{4}$ c. $1\frac{5}{6} \div 1\frac{2}{9}$
   d. $5\frac{1}{4} \div 4\frac{1}{5}$ e. $6\frac{2}{5} \div 1\frac{2}{5}$ f. $1 \div 3\frac{6}{7}$
   g. $8\frac{1}{8} \div 1\frac{5}{8}$ h. $3\frac{5}{8} \div 1\frac{1}{6}$ i. $6\frac{1}{2} \div 3\frac{3}{4}$

LESSON 6-2 PRACTICE

4. A candle burns for $3\frac{2}{15}$ hours. Estimate the number of candles you would need to give light for 24 hours.
5. A machine can make a box in $3\frac{17}{20}$ seconds. Estimate the number of boxes it can make in $33\frac{1}{2}$ seconds.
6. A recipe for onion soup calls for $1\frac{1}{4}$ cups of onions. Amy has 5 cups of onions.
   a. **Model with mathematics.** Draw a model you can use to find the number of batches of soup Amy can make.
   b. $5 \div 1\frac{1}{4} = ?$
7. Ted has 18 feet of lumber. How many $2\frac{1}{2}$-foot shelves can he make? Explain your answer.
8. Claire has 38 yards of material to make costumes for the school play. She needs $2\frac{3}{8}$ yards for each costume. How many costumes can she make?
9. Val talked for $3\frac{1}{2}$ hours on her cell phone this week. That is $1\frac{1}{2}$ times as long as she talked last week. How long did she talk last week?
10. a. $4\frac{2}{11} \times \text{some number} = 6$. What is the number?
    b. **Reason quantitatively.** Explain how you found the number.

ACTIVITY 6 PRACTICE

Write your answers on notebook paper. Show your work.

Express your answers in simplest form.

Lesson 6-1

1. Find the quotient $8 \div \frac{1}{5}$.

A. $\frac{1}{40}$ **B.** $\frac{5}{8}$

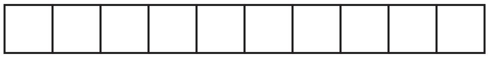
C. $\frac{8}{5}$ **D.** 40

2. What is the reciprocal of $\frac{7}{11}$?

A. $1\frac{4}{7}$ **B.** 7

C. 11 **D.** 77

3. a. Copy the model below. Then shade it to show the fraction $\frac{4}{5}$.



b. Mark X's in the square to show the quotient $\frac{4}{5} \div \frac{1}{10}$.

c. $\frac{4}{5} \div \frac{1}{10} = ?$

4. One event at a debate tournament lasted $2\frac{4}{5}$ hours. Each contestant spoke for $\frac{2}{15}$ of an hour. How many contestants were there?

5. Each side of Jose's patio is $21\frac{1}{4}$ feet long and is lined with bricks measuring $\frac{17}{24}$ of a foot in length. How many bricks line each side of the patio?

6. Use the algorithm for dividing by a fraction to show that when you divide a fraction by itself, the quotient is 1. Use an example to illustrate your answer.

7. Jonas's vacuum cleaner uses $\frac{5}{8}$ kilowatt of electricity per hour. His clothes dryer uses $4\frac{11}{16}$ kilowatts of electricity per hour. How many hours can he operate his vacuum cleaner on the same amount of energy it takes to power his clothes dryer for 1 hour?

8. Denise sells pizza for \$0.89 a slice. Each slice of her pepperoni pizza is $\frac{1}{8}$ of a pizza. Each slice of her mushroom pizza is $\frac{1}{10}$ of a pizza. Today she sold all of the slices of 7 pepperoni pizzas and 6 mushroom pizzas. How much money did she make?

9. A turtle walking at an average speed of $\frac{3}{10}$ mile per hour takes $\frac{13}{20}$ hour to walk from its home to the seashore. How long will the journey take if the turtle doubles its average speed? Explain your reasoning.

Remember to express your answers in simplest form.

Lesson 6-2

10. What is the reciprocal of $2\frac{3}{8}$?

A. $\frac{1}{2} \times \frac{8}{3}$ **B.** $\frac{8}{19}$

C. $\frac{19}{8}$ **D.** $2\frac{8}{3}$

11. Estimate the quotient $24\frac{3}{7} \div 3\frac{7}{13}$ by rounding to the nearest whole number.

A. 6 **B.** $6\frac{1}{4}$

C. 8 **D.** $8\frac{1}{3}$

12. Divide: $1\frac{4}{5} \div 2\frac{1}{4}$.

A. $\frac{20}{81}$ **B.** $\frac{4}{5}$

C. $1\frac{1}{4}$ **D.** $4\frac{1}{20}$

13. In the country of Remish, 1 dollar is worth $1\frac{7}{20}$ rems. Find the value, in dollars, of $33\frac{3}{4}$ rems.

14. During the days leading up to her piano recital, Lana practiced an average of $1\frac{5}{12}$ hours per day. Her total practice time was $11\frac{1}{3}$ hours. How many days did she practice?

15. A rectangular garden has an area of $9\frac{5}{24}$ square yards. The garden is $2\frac{1}{6}$ yards wide. How long is it?

(area of rectangle = length × width)

16. A shoot of bamboo, one of the world's fastest-growing plants, grew at a rate of $2\frac{3}{10}$ inches per hour. How long did it take for the shoot to grow $14\frac{3}{8}$ inches?

17. One rod equals $16\frac{1}{2}$ feet. The One Island East Centre Building in Hong Kong, China, is 979 feet tall. Find the height of the building in rods.

18. The seating area of a school bus is $35\frac{3}{4}$ feet long. Each row uses $3\frac{1}{4}$ feet of space. On the left side of the bus, 3 students sit in each row. On the right side, 2 students sit in each row. How many students can ride the bus?

MATHEMATICAL PRACTICES
Make Use of Structure

19. Explain each step of dividing $13\frac{1}{2}$ by $1\frac{1}{2}$:

a. $13\frac{1}{2} \div 1\frac{1}{2} = \frac{27}{2} \div \frac{3}{2}$

b. $= \frac{27}{2} \times \frac{2}{3}$

c. $= \frac{^{9}\cancel{27}}{\cancel{2}_{1}} \times \frac{^{1}\cancel{2}}{\cancel{3}_{1}}$

d. $= \frac{9}{1}$

e. $= 9$

Multiplying and Dividing Fractions and Mixed Numbers

JUAN'S BOOKCASE

Write your answers on notebook paper. Show your work.

Juan plans to build a bookcase to store his paperback books, DVDs, and CDs. He has lumber that he will use for the sides and back of the bookcase. The bookcase will have five shelves, and each shelf will be $2\frac{1}{2}$ feet long.

1. Juan bought a piece of lumber that is 18 feet long.

   a. Does he have enough lumber to make the five shelves? If not, how much more does he need? If so, how much will be left over?

   b. Describe two ways you could find the answer to Part a.

2. DVD cases are $\frac{9}{16}$ inch wide.

   a. If Juan has 60 DVDs, how many of them will fit on one shelf?

   b. How wide would a DVD case have to be in order for 60 of them to fit on one shelf? Give the answer in feet.

3. Juan has 28 paperback books. Each book is $1\frac{1}{4}$ inches wide. Will all his books fit on one shelf? If not, how many will fit and how many will have to go on another shelf? If yes, how many more paperback books, if any, will fit on the same shelf? Explain.

4. Juan measured the location for the bookcase and realized that his shelves can be no more than $1\frac{3}{4}$ feet wide.

   a. What is the maximum number of shelves Juan could build for this new bookcase using the lumber he bought?

   b. How many DVD cases will he be able to store on each of the shorter shelves?

   c. How many paperback books will he be able to store on each of the shorter shelves?

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a, 2a-b, 3, 4a-c)	• Clear and accurate understanding of multiplying and dividing mixed numbers and fractions.	• Correct multiplication and division of mixed numbers and fractions.	• Errors in multiplying and dividing mixed numbers and fractions.	• Incorrect or incomplete multiplication and division of mixed numbers and fractions.
Problem Solving (Items 1a, 2a-b, 3, 4a-c)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps that result in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a, 2a-b, 3, 4a-c)	• Clear and accurate representation of a problem situation as a multiplication or division expression.	• Some difficulty in representing a problem situation as a multiplication or division expression.	• Errors in representing a problem situation as a multiplication or division expression.	• Little or no understanding of representing a problem situation as a multiplication or division expression.
Reasoning and Communication (Items 1a-b, 2a-b, 3, 4a-c)	• Precise use of appropriate math terms and language when explaining a solution method. • A precise and accurate relating of a mathematical answer to a real-world situation.	• An adequate explanation of a solution method. • A relating of a mathematical answer to a real-world situation.	• A misleading or confusing explanation of a solution method. • Difficulty in the relating of a mathematical answer to a real-world situation.	• An incomplete or inaccurate explanation of a solution method • Little understanding of relating a mathematical answer to a real-world situation.