Part 3

Number and Operations in Base Ten

Number and Operations in Base Ten

Domain Overview

KINDERGARTEN

The work in kindergarten forms the foundation for students to develop an understanding of the base ten system. Special attention is focused on 10 and connections to the meaning of numbers from 11 to 19. Once students understand counts from 1 to 9, they can begin to think of the number 10 as 10 ones and the number 11 as ten ones and one more. This domain comprises the major work of kindergarten and will be developed across the entire school year and linked closely with the Counting and Cardinality and Operations and Algebraic Thinking domains.

GRADE 1

First graders build on kindergarten work and begin to think of ten ones as a unit called a ten. Students continue to form initial understanding using a variety of materials then move to drawings and then to symbolic notation, including <, >, and =. They will find sums to 100 with understanding by using materials and developing strategies. Subtraction with two-digit numbers includes only differences among multiples of 10 in first grade.

GRADE 2

In second grade, students extend place value understanding to working with a new unit comprising 10 tens or a group of 100. Using place value and properties of addition and subtraction, students become fluent with two-digit addition and subtraction including composing and decomposing groups of 10. They extend this work to addition and subtraction to 1,000 with careful scaffolding from concrete representations to pictorial representations and finally to working with abstract representations.

Table 2 illustrates common mental computation strategies that develop across K–2 and extend to more sophisticated addition and subtraction examples.

SUGGESTED MATERIALS FOR THIS DOMAIN

K	1	2	
✓	✓	✓	Objects for counting such as beans, linking cubes, counters, chips, coins, and straws
✓			Five frames (Reproducible 1)
✓	✓	✓	Ten frames (Reproducible 2)
	✓	✓	Double ten frames (Reproducible 3)
✓	✓	✓	Hundreds chart (Reproducible 4)
✓	✓	✓	Dot cards (Reproducible 5)
✓	✓		Numeral cards (Reproducible 6)
	✓	✓	Number line to 20 (Reproducible 7)
	✓	✓	Open number line (Reproducible 8)
✓	✓	✓	Part-Part-Whole chart (Reproducible 9)
	✓	✓	Straws or coffee stirrers and rubber bands for place value
	✓	✓	Linking cubes for place value
✓	✓	✓	Place value chart to tens (Reproducible 10)
		✓	Place value chart to hundreds (Reproducible 11)
	✓	✓	Greater than, Less than = cards (Reproducible 12)
		✓	*Base ten blocks

***** Note that students should use materials such as linking cubes and straws that provide opportunities to physically put together and take apart tens and hundreds. Base ten blocks should only be used after students show competency with making tens and hundreds as they involve exchanging 10 ones for 1 ten or 10 tens for 1 hundred, which requires a higher level of understanding than actually putting together or taking apart objects to make ones, tens, and hundreds.

KEY VOCABULARY

K	1	2	
✓	✓	✓	**add—**to combine or join together *related words: add, and, plus, join, put together, (+)*
	✓	✓	***associative property of addition—**an extension of the commutative property, to change the order and group two addends to find convenient sums (such as 10) in order to make the addition easier. Note that students do not use parenthesis at this level. The focus is on looking for sums of 10. 4 + 8 + 2 = 4 + 10 = 14 or 6 + 8 + 4 = 6 + 4 + 8 = 18
	✓	✓	**bundle—**to put individual units together to make a larger unit, for example, connecting 10 individual linking cubes to make a ten.
	✓	✓	***commutative property of addition—**reversing the order of the addends does not change the total (sum) 8 + 5 = 13 and 5 + 8 = 13 so 8 + 5 = 5 + 8
✓	✓	✓	**compare—**to look for similarities or differences among numbers
	✓	✓	**compose—**put a number together using other numbers 1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5, 1 + 2 + 3 + 4 are ways to compose 10

(Continued)

KEY VOCABULARY (Continued)

K	1	2	
	✓	✓	**decompose**—separate a number into parts using other numbers 8 can be decomposed into 4 + 4, 3 + 5, 2 + 2 + 2 + 2
✓	✓	✓	**difference**—the amount by which one number is greater or less than another number. The difference can be found by subtracting, comparing, or finding a missing addend.
✓	✓	✓	**equal (=)**—same as in value or size
	✓	✓	**equation**—a mathematical sentence in which one part is the same as, or equal to, the other part 3 + 5 = 8, 12 – 7 = 5, 11 = 8 + 3, 6 = 9 – 3
✓	✓	✓	**fewer**—less than
✓	✓	✓	**greater**—more than
		✓	**hundred**—a group or bundle of 10 tens or 100 ones
✓	✓	✓	**hundreds chart**—10 by 10 grid with the counting numbers from 1 to 100 listed; used to develop and demonstrate patterns and strategies for counting, addition, subtraction, and place value
	✓	✓	*** identity property of addition**—any number plus 0 equals the number 12 + 0 = 12 or 0 + 12 = 12
	✓	✓	*** identity property of subtraction**—any number minus 0 equals the number 15 – 0 = 15 or 15 = 15 – 0
✓	✓	✓	**missing addend**—given an equation in which the total (sum) and one addend is known, the unknown addend 5 + ☐ = 8 In this equation ☐ has a value of 3. It is the missing addend.
	✓	✓	**number line**—a line used to show the position of a number in relation to other numbers (Reproducible 7)
	✓	✓	**open number line (empty number line)**—a number line with no numbers or markers; used as a visual representation for recording and sharing strategies for adding or subtracting numbers (Reproducible 8)
	✓	✓	**part-part-whole model**—a visual model for showing the relationship among numbers in addition and subtraction situations (Reproducible 9)
	✓	✓	**place value**—the value of a digit is determined by its place in a number In 23, the 2 is in the tens place and has a value of 20; the 3 is in the ones place and has a value of 3.
✓	✓	✓	**subtract**—to take one number away from another; to find the difference between two numbers *related words: subtract, minus, take from, take apart (–)*
		✓	**sum**—the answer in an addition problem; total
✓	✓	✓	**ten**—a group or bundle of 10 ones
✓	✓	✓	**ten frame**—a graphic representation that is useful to help students to count, see number relationships, and learn basic facts (Reproducible 2)
		✓	**thousand**—a group or bundle of 10 hundreds, 100 tens, or 1,000 ones
✓	✓	✓	**total (sum)**—the result when two or more numbers are added together

* Students are not responsible for these vocabulary words; however, they should understand the mathematical concept.

Number and Operations in Base Ten
K.NBT.A*

Cluster A

Work with numbers 11–19 to gain foundation for place value.

STANDARD 1

K.NBT.A 1: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

*Major cluster

Number and Operations in Base Ten K.NBT

Cluster A: Work with numbers 11–19 to gain foundations for place value.
Kindergarten Overview

The focus of this cluster is on composing and decomposing numbers from 11 to 19 into 10 ones and some more ones. The idea of ten as a unit will not be introduced until Grade 1.

Standards for Mathematical Practice
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 7. Look for and make use of structure.

As they explore the meaning of place value with numbers from 11 to 19 as 10 ones and some more ones, students are having their first experience with the structure of mathematics. Place value is the foundation for all future work with whole numbers and decimal numbers. The use of concrete and pictorial models and later connecting those models to symbolic notation is fundamental to developing conceptual understanding. Students explain their reasoning and listen to the thinking of others to solidify their understanding.

Related Content Standards
1.NBT.B.2 2.NBT.A.1

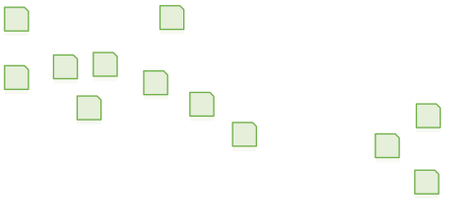
Notes

STANDARD 1 (K.NBT.A)

Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

What the TEACHER does:

- Give students a variety of experiences with counting 10 ones and composing and decomposing ten using concrete materials.
- Progress to giving students a number from 11 to 19 (both orally and showing the numeral) beginning with numbers closer to 10. After students count out that number of objects, the teacher models putting a group of 10 together (composing 10) and counting the leftover objects. Students do the same. After doing several examples together, students continue to work independently.

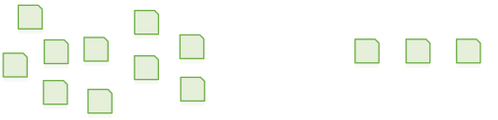


- Model written equations showing a teen number composed (10 + 8 = 18) and decomposed (18 = 10 + 8). Connect the equation with the physical models and drawings. Provide students with number cards or situations in which they match the number with the correct equation. The final expectation is for students to write the equations. Students should have many opportunities to practice this through games and activities.

What the STUDENTS do:

- Experience putting together and taking apart numbers from 11 to 19 by forming a group of 10 ones and some more ones, using objects such as linking cubes, straws, double tens frames, or a cup of 10 and some ones.

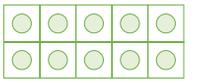
  Ways to represent 13:



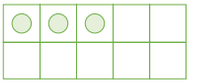
A group of 10 ones and 3 more ones

10 ones

3 more ones



10 ones



3 more ones

- After many experiences with concrete materials, students use drawings to represent and explain their thinking.
- Students match representations (either concrete or pictorial) with numerals.
- Students continue to work with a variety of representations, including concrete models, drawings, and numerals, to develop a deep understanding of a teen number being composed of a ten ones and some more ones. Since this concept is the foundation for future work developing place value in Grades 1 and 2, the concept should be developed over the entire year.

Addressing Student Misconceptions and Common Errors

Kindergarten students have several new concepts with which to grapple as part of this standard including the notion of 10 ones being grouped together. Watch for those who struggle with this important place value concept. The concept that 1 group of 10 ones and some more ones can represent the same idea as the number they originally counted will be a stretch for some students, and they will need many opportunities to compose groups of 10 with concrete materials. The other concept that may present a challenge is the teen number names. A group of 10 and one more has the name "eleven"; a group of 10 and two more is called "twelve"; a group of 10 and three more is called "thirteen." Students entering kindergarten with little number experience may need much more practice with the representations and connecting representations to the number names.

Notes

Sample PLANNING PAGE

K.NBT.A

Number and Operations in Base Ten
Cluster A: Work with numbers 11–19 to gain foundations for place value.

Standard: K.NBT.A.1. *Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.*

Standards for Mathematical Practice:

SFMP 4. Model with mathematics.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
Students use a variety of concrete materials to model the numbers between 11 and 19. They put 10 ones together to make a set of 10 and will have some leftover ones. After making physical models, students describe their representations with words and equations.

Goal:

This is students' first experience with place value, a fundamental structure of the whole number system. Students develop the concept of decomposing a number into a group of 10 ones and some more ones.

Planning:

Materials: Small cups, plastic sandwich bags, counters of all types (such as beans, chips, shells, buttons, teddy bear counters, etc.), numeral cards from 8 to 19. Put each type of counter and a set of numeral cards at workstations around the classroom.

Sample Activity:
Introduce the activity demonstrating that students will turn over a numeral card; count out that number of counters. If there are more than 9 counters, they take 10 counters and put them in a cup or in a plastic bag. They describe their representation and write an equation to represent the number. For example, if they draw 13, they would count out 13 chips. Count 10 of the chips and put them in a cup. They would describe their representation as 10 ones and 3 ones. They would write the equation as 13 =10 +3

Questions/Prompts:

How many counters do you have?
(13)

Are there more than 10? (If yes) What should you do with 10 of the counters?
(Put them in the cup.)

Describe what you have using words.
(10 ones and 3 more ones)

How can you write that using numbers?
(13 = 10 + 3)

Differentiating Instruction:

Struggling Students: Prerequisite skills to be successful with this activity include counting to 19 and using physical materials to represent each number. If students are unable to successfully represent numbers to 19, they will need more practice with that skill before they are ready for this standard. Scaffold the range of numbers beginning with a range of 8–13. Provide extra support in describing and writing the equation.

Extension: Students who are successful with this activity can decompose the teen numbers with drawings and then just by writing the equations. They might also begin to work with numbers to 25 filling two cups or plastic bags each with 10 ones for numbers from 20 to 25.

PLANNING PAGE

K.NBT.A

Cluster A: Work with numbers 11–19 to gain foundation for place value.

Standard:

Standards for Mathematical Practice:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Number and Operations in Base Ten
1.NBT.A*

Cluster A

Extend the counting sequence.

STANDARD 1 **1.NBT.A.1:** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

*Major cluster

Number and Operations in Base Ten 1.NBT.A

Cluster A: Extend the counting sequence.
Grade 1 Overview

This cluster extends earlier work with standard K.CC.A.1 in which students rote counted to 100. Building on previous counting experiences, students count on from any number less than 120. They read, write, and represent any number to 120.

Standard for Mathematical Practice
SFMP 6. Attend to precision.

Students first learn numbers by rote counting. They build vocabulary with numbers to 120.

Related Content Standards

K.CC.A.1; 2. NBT.A.2

Notes

STANDARD 1 (1.NBT.A.1)

Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

The main difference between this standard and previous experiences with rote counting is that as students extend the range of counting numbers, focusing on the patterns evident in written numerals. This is the foundation for thinking about place value and the meaning of the digits in a numeral. Students are also expected to read and write numerals to 120.

What the TEACHER does:

- Begin by having students continue to count with objects and write the numeral for each count of objects.
- Give students a number to count on to within a range of numbers. For example, students should be able to count on from 25 to 50 starting with 25, 26, 27, 28 . . .
- Use the hundreds chart (Reproducible 4) for activities that provide opportunities for students to recognize written numerals and begin to recognize patterns on the hundreds chart. For example,
  - All of the numbers in a column have the same digit in the ones place
  - All of the numbers in a row have the same digit in the tens place
  - The number that follows a given number is one more than the number (42 is one more than 41)
  - The number that precedes a given number is one less than the number (40 is one less than 41)
- Provide activities and tasks that explicitly expect students to see the difference between reversed numbers, such as 25 and 52.

What the STUDENTS do:

- Count on from a number ending at any number up to 120.
- Recognize and explain patterns with numerals on a hundreds chart.
- Understand that the place of a digit determines its value. For example, students recognize that 24 is different from and less than 42.
- Explain their thinking with a variety of examples.
- Read and write numerals to 120.

Addressing Student Misconceptions and Common Errors

It is not expected that students develop an understanding of place value with this standard. However, watch for students who reverse digits in writing the numeral or do not demonstrate an understanding that 21 does not have the same value as 12. When reversals occur, have students model each number, using straws or linking cubes to reinforce the place value of digits and to help students differentiate between the numbers.

Notes

Number and Operations in Base Ten
1.NBT.B*

Cluster B

Understand place value.

STANDARD 2 **1.NBT.A.2:** Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones—called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

STANDARD 3 **1.NBT.A.3:** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

*Major cluster

Number and Operations in Base Ten 1.NBT.B

Cluster B: Understand place value.
Grade 1 Overview

This cluster underlies the critical foundation of understanding place value in preparation for understanding addition and subtraction beyond basic facts. Students think of whole numbers in terms of the value of the digits (tens and ones) and recognize that the digit in the tens place represents that many groups of 10, and a digit in the ones place represents that many ones.

Standards for Mathematical Practice
SFMP 2. Reason abstractly and quantitatively.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.

As they explore the meaning of place value with numbers from 11 to 19 as a group of 10 and some ones, students experience the structure of mathematics. Place value is the foundation for all future work with whole numbers and decimal numbers. The use of concrete models and pictorial models and explicitly connecting them to symbolic notation is fundamental to developing conceptual understanding. Students explain their reasoning and listen to the thinking of others to solidify their understanding.

Related Content Standards

K.NBT.A.1 KCC.C.7 2.NBT.A.1 2NBT A.4

Notes

STANDARD 2 (1.NBT.B.2)

Understand that the two digits of a two-digit number represent amounts of tens and ones.

This standard outlines a helpful progression for developing an initial understanding of place value, the concept that the location or place of a digit within a two-digit or three-digit numeral determines the value of that digit.

Understand the following as special cases:

a. *10 can be thought of as a bundle of ten ones—called a "ten."*

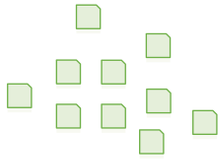
Students begin to unitize or consider 10 ones as a group or unit called a ten. Rather than seeing 10 individual cubes, they can link those cubes and make a group of 1 ten.

What the TEACHER does:

- Use activities with ten frames, bundling straws, linking cubes, and counters that expect students to bundle or group 10 ones to make a ten.
- Develop student vocabulary to see a ten as a unit composed of 10 ones.
- Introduce students to the place value chart with ones and tens (Reproducible 10). A unit or group of 10 always belongs in the tens place. Individual ones always belong in the ones place.

What the STUDENTS do:

- Given objects such as counters, linking cubes, or ten frames, students bundle or group 10 ones to make a ten.
- Develop vocabulary to refer to a group of 10 as 1 ten.
- Differentiate between 1 ten (a bundle) and 10 ones.



Ten ones

One ten

b. *The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.*

Students build on previous work in kindergarten (K.NBT.A.1) where they composed and decomposed numbers from 11 to 19 into 10 ones and some more ones. This standard expects that students will understand that the 10 ones, from their previous experiences, are now thought of as a bundle or group of 10. It is a unit, that is, *1* ten. Through experiences using a variety of materials such as ten frames, bundling straws, and linking cubes, students see a teen number, such as 16, as 1 ten plus 6 ones.

What the TEACHER does:

- Give students between 11 and 19 concrete objects, which they represent as 1 ten and some ones. Tasks should require putting the 10 ones together, such as bundling straws or linking cubes, and placing the bundle of 10 in the tens place on a place value chart.
- Pose questions that reinforce the concept that when 10 ones are grouped or bundled, they now make up a new unit called 1 ten.

What the STUDENTS do:

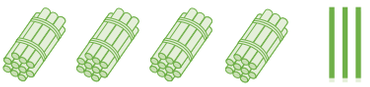
- Given a number of objects between 11 and 19, group them into 1 group of ten and some ones.
- Describe the grouping, using place value language. For example, 17 is 1 ten and 7 ones.

c. *The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).*

Once students have a firm grasp of the concept of teen numbers being made up of 1 ten and some ones, they continue to explore with multiples of ten (10, 20, 30, 40, 50, 60, 70, 80, 90) as groups of ten with no ones leftover. For example, 40 is 4 groups or bundles of ten and no ones. This prepares students for understanding place value with numbers greater than 20. Although not explicit in this standard, there is an expectation that once students understand multiples of 10 through 90 as a specific number of tens and no ones, they begin to work with other numbers, describing them in terms of place value. For example, 57 is 5 tens and 7 ones or 57 ones.

What the TEACHER does:

- Build on experiences with groups of ten by giving students tasks in which they bundle 20, 30, 40 . . . objects into groups of ten. Placing bundles of ten on a place value chart reinforces the concept that these numbers represent bundles of tens and no ones. For example, 30 is represented by 3 tens and 0 ones.
- Provide multiple experiences with a variety of concrete objects including ten frames, linking cubes and counters that are placed in cups or baggies labeled ten, or bundling straws into groups of ten.
- Focus discussion on describing concrete representations using words. For example, 3 bundles of straws would be described as 3 tens and 0 ones.
- Use formative assessment protocols, including tasks, student explanations, and worksheets, to determine if students understand unitizing and can describe multiples of 10 up to 90 as a number of tens and no ones.
- Begin to provide experiences with other numbers up to 100. Give students a number of objects such as linking cubes, which are placed in the ones place on a place value chart (Reproducible 10). Ask students to connect as many groups of 10 as possible and move the rod of 10 cubes to the tens place. Any leftover cubes would remain in the ones place.
- Ask students to describe what they have represented on the place value chart. For example: 35 cubes would be represented as 3 tens and 5 ones.
- Guide students to make explicit connections between concrete materials and pictorial representations for place value. As students orally describe a number in terms of place value, they begin to connect that understanding to the written numeral.



4 tens 3 ones 43

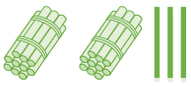
What the STUDENTS do:

- Use concrete materials to represent numbers including 10, 20 30, . . . 90 as groups of ten with no ones.
- Place concrete representations on a place value chart to reinforce that the number is made up of tens and no ones.
- Describe the decade numbers using words that include the number of groups of ten to reinforce understanding ten as a unit that is different from ones. For example, 20 is 2 tens; 30 is 3 tens.
- Use concrete materials to represent any number from 10 to 99 by making as many groups of ten as possible and placing those groups in the tens place on a place value chart. Place any leftover single items in the ones place.
- Describe the representation using words.
- Write the numerals with emphasis on identifying how the written number shows the number of tens and the number of ones.
- Connect words to the written numeral. For example, 3 tens and 2 ones is written as 32.

Addressing Student Misconceptions and Common Errors

Continue to watch for students who reverse digits. These students need more opportunities to decompose numbers into groups of ten and ones using concrete materials and then to put the items in the correct places on a place value chart. They describe the number in terms of tens and ones and then write the numeral below the concrete representation.

Observe students counting tens and ones separately. For example,



Students who count this as 10, 20, 1, 2, 3 rather than 10, 20, 21, 22, 23 need more practice with counting.

Some students may have difficulty differentiating number words that sound alike, for example, *fifty* and *fifteen*. These number words can be spelled out and added to a word wall showing pictures, numbers, and words.

40 forty

14 fourteen

Notes