Ontario

Number Sense and Numeration, Grades 4 to 6

Volume 5
Fractions

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

Ontario Education excellence for all

2006

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A Guide to Effective Instruction
in Mathematics,
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CONTENTS

INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories behind good instruction that are elaborated in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The guide comprises the following volumes:

- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 5: Fractions – provides:

- a discussion of mathematical models and instructional strategies that support student understanding of fractions;
- sample learning activities dealing with fractions for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 37, 49, and 67).

Relating Mathematics Topics to the Big Ideas

> *The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the "big ideas" of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.*
>
> (*The Ontario Curriculum, Grades 1–8: Mathematics, 2005*, p. 4)

In planning mathematics instruction, teachers generally develop learning activities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- relationships
- representation
- proportional reasoning

Each big idea is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Intellectual development	Generally, students in the junior grades: • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour.	The mathematics program should provide: • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas.
Physical development	Generally, students in the junior grades: • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity.	The mathematics program should provide: • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing.
Psychological development	Generally, students in the junior grades: • are less reliant on praise but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups.	The mathematics program should provide: • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibility for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social development	Generally, students in the junior grades: • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates.	The mathematics program should provide: • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability. *(continued)*

Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Moral and ethical development	Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask “why” questions; • begin to consider others’ points of view.	The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Grades*. Elementary Teachers’ Federation of Ontario, 2004.)

LEARNING ABOUT FRACTIONS IN THE JUNIOR GRADES

Introduction

The development of fraction concepts allows students to extend their understanding of numbers beyond whole numbers, and enables them to comprehend and work with quantities that are less than one. Instruction in the junior grades should emphasize the meaning of fractions by having students represent fractional quantities in various contexts, using a variety of materials. Through these experiences, students learn to see fractions as useful and helpful numbers.

PRIOR LEARNING

In the primary grades, students learn to divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., half, third, fourth). Students use concrete materials and drawings to represent and compare fractions (e.g., use fraction pieces to show that three fourths is greater than one half). Generally, students model fractions as *parts of a whole*, where the parts representing a quantity are less than one.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

As in the primary grades, the exploration of concepts through problem situations, the use of models, and an emphasis on oral language help students in the junior grades to develop their understanding of fractions.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to fractions, listed in the table on p. 12.

Curriculum Expectations Related to Fractions, Grades 4, 5, and 6		
By the end of Grade 4, students will:	**By the end of Grade 5, students will:**	**By the end of Grade 6, students will:**
Overall Expectations • read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100; • demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts. ***Specific Expectations*** • represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered; • compare and order fractions (i.e., halves, thirds, fourths, fifths, tenths) by considering the size and the number of fractional parts; • compare fractions to the benchmarks of 0, 1/2, and 1; • demonstrate and explain the relationship between equivalent fractions, using concrete materials and drawings; • count forward by halves, thirds, fourths, and tenths to beyond one whole, using concrete materials and number lines; • determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools and strategies.	***Overall Expectation*** • read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers. ***Specific Expectations*** • represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools and using standard fractional notation; • demonstrate and explain the concept of equivalent fractions, using concrete materials; • describe multiplicative relationships between quantities by using simple fractions and decimals; • determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship between fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100) and their equivalent decimal forms.	***Overall Expectations*** • read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers; • demonstrate an understanding of relationships involving percent, ratio, and unit rate. ***Specific Expectations*** • represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools and using standard fractional notation; • represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation; • determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)

The following sections explain content knowledge related to fraction concepts in the junior grades, and provide instructional strategies that help students develop an understanding of fractions. Teachers can facilitate this understanding by helping students to:

- model fractions as parts of a whole;
- count fractional parts beyond one whole;
- relate fraction symbols to their meanings;
- relate fractions to division;
- establish part-whole relationships;
- relate fractions to the benchmarks of 0, 1/2, and 1;
- compare and order fractions;
- determine equivalent fractions.

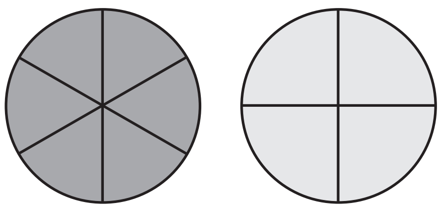
Modelling Fractions as Parts of a Whole

Modelling fractions using concrete materials and drawings allows students to develop a sense of fractional quantity. It is important that students have opportunities to use area models, set models, and linear models, and to experience the usefulness of these models in solving problems.

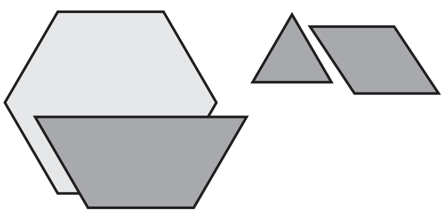
Area Models

In an area model, one shape represents the whole. The whole is divided into fractional parts. Although the fractional parts are equal in area, they are not necessarily congruent (the same size and shape).

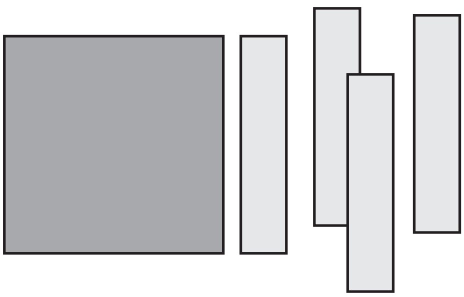
A variety of materials can serve as area models.



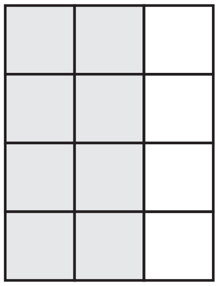
Fraction Circles



Pattern Blocks



Fraction Rectangles

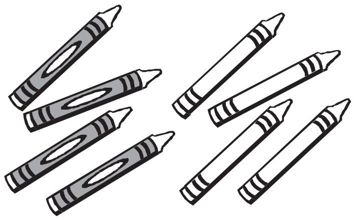


Square Tiles

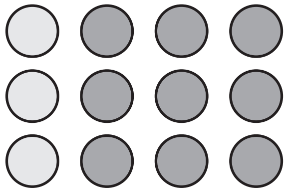
Set Models

In a set model, a collection of objects represents the whole amount. Subsets of the whole make up the fractional parts. Students can use set models to solve problems that involve partitioning a collection of objects into fractional parts.

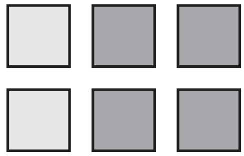
A variety of materials can serve as set models.



Real Objects



Counters

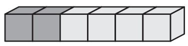


Square Tiles

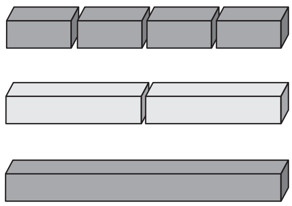
Linear Models

In a linear model, a length is identified as the whole unit and is divided into fractional parts.

Line-segment drawings and a variety of manipulatives can be used as linear models.

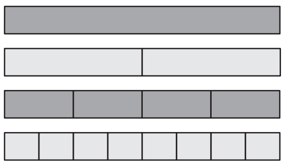


Interlocking Cubes



Cuisenaire Rods

Line Segments



Paper Strips

Modelling fractions using area, set, and linear models helps students develop their understanding of relationships between fractional parts and the whole. It is important for students to understand that:

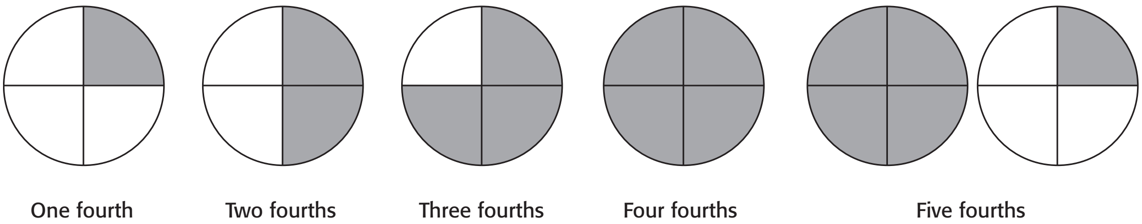
- all the fractional parts that make up the whole are equal in size;
- the number of parts that make up the whole determine the name of the fractional parts (e.g., if five fractional parts make up the whole, each part is a "fifth").

Teachers need to provide experiences in which students explore the usefulness of different models in problem-solving situations:

- Area models are useful for solving problems in which a whole object is divided into equal parts.
- Set models provide a representation of problem situations in which a collection of objects is divided into equal amounts.
- Length models provide a tool for comparing fractions, and for adding and subtracting fractions in later grades.

Counting Fractional Parts Beyond One Whole

Once students understand how fractional parts (e.g., thirds, fourths, fifths) are named, they can count these parts in much the same way as they would count other objects (e.g., "One fourth, two fourths, three fourths, four fourths, five fourths, . . .").



Activities in which students count fractional parts help them develop an understanding of fractional quantities greater than one whole. Such activities give students experience in representing improper fractions concretely and allow them to observe the relationship between improper fractions and the whole (e.g., that five fourths is the same as one whole and one fourth).

Relating Fraction Symbols to Their Meaning

Teachers should introduce standard fractional notation after students have had many opportunities to identify and describe fractional parts orally. The significance of fraction symbols is more meaningful to students if they have developed an understanding of halves, thirds, fourths, and so on, through concrete experiences with area, set, and linear models.

The meaning of standard fractional notation can be connected to the idea that a fraction is part of a whole – the denominator represents the number of equal parts into which the whole is divided, and the numerator represents the number of parts being considered. Teachers should encourage students to read fraction symbols in a way that reflects their meaning (e.g., read 3/5 as "three fifths" rather than "three over five").

Students should also learn to identify proper fractions, improper fractions, and mixed numbers in symbolic notations:

- In **proper fractions**, the fractional part is less than the whole; therefore, the numerator is less than the denominator (e.g., 2/3, 3/5).

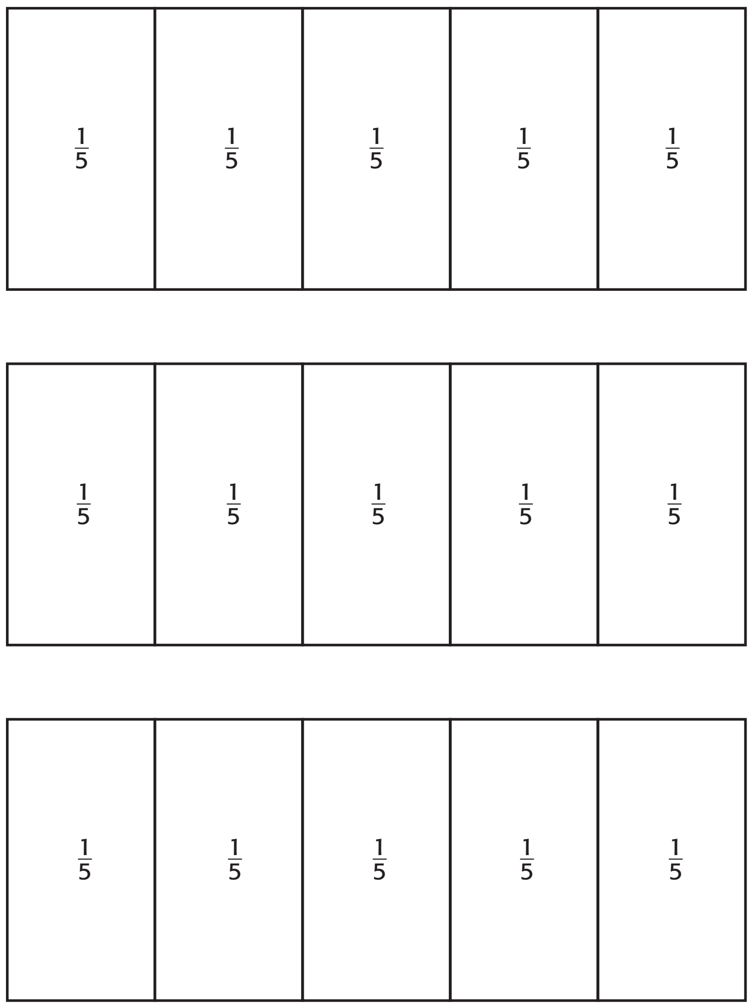
- In **improper fractions**, the combined fractional parts are greater than the whole; therefore, the numerator is greater than the denominator (e.g., 5/2, 8/5).
- In **mixed numbers**, both the number of wholes and the fractional parts are represented (e.g., 4 1/3, 2 2/10).

Relating Fractions to Division

Students should have opportunities to solve problems in which the resulting quotient is a fraction. Such problems often involve sharing a quantity equally, as illustrated below.

> *"Suppose 3 fruit bars were shared equally among 5 children. How much of a fruit bar did each child eat?"*

To solve this problem, students might divide each of the 3 bars into 5 equal pieces. Each piece is 1/5 of a bar.

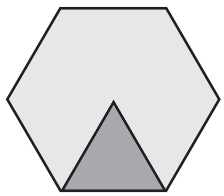


After distributing the fifths to the 5 children, students discover that each child receives 3/5 of a bar. An important learning from this investigation is that the number of objects shared among the number of sets (children, in this case) determines the *fractional amount* in each set (e.g., 4 bars shared among 7 children results in each child getting 4/7 of a bar; 2 bars shared among 3 students results in each child getting 2/3 of a bar). This type of investigation allows students to develop an understanding of fractions as *division*.

When modelling fractions as division, students need to connect fractional notation to what is happening in the problem. In the preceding example, the denominator (5) represents the number of children who are sharing the fruit bars, and the numerator (3) represents the number of objects (fruit bars) being shared.

Establishing Part-Whole Relationships

Fractions are meaningful to students only if they understand the relationship between the fractional parts and the whole. In the following diagram, the hexagon is the *whole*, the triangle is the *part*, and one sixth (1/6) is the *fraction* that represents the relationship between the part and whole.

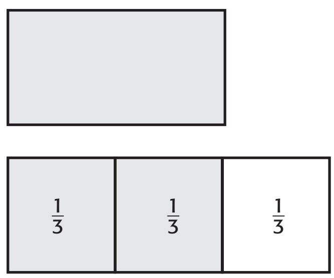


By providing two of these three elements (whole, part, fraction) and having students determine the missing element, teachers can create activities that promote a deeper understanding of part-whole relationships. Using concrete materials and/or drawings, students can determine the unknown whole, part, or fraction. Examples of the three problem types are shown below.

FIND THE WHOLE

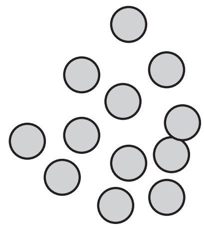
> *"If this rectangle represents 2/3 of the whole, what does the whole look like?"*

To solve this problem, students might divide the rectangle into two parts, recognizing that each part is 1/3. To determine the whole (3/3), students would need to add another part.



FIND THE PART

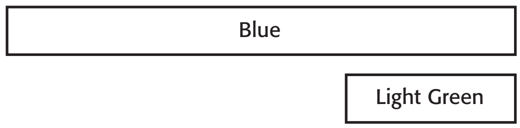
"If 12 counters are the whole set, how many counters are 3/4 of the set?"



To solve this problem, students might divide the counters into four equal groups (fourths), then recognize that 3 counters represent 1/4 of the whole set, and then determine that 9 counters are 3/4 of the whole set.

FIND THE FRACTION

"If the blue Cuisenaire rod is the whole, what fraction of the whole is the light green rod?"



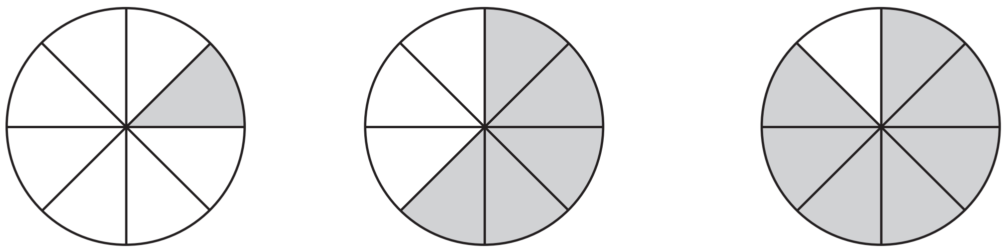
To solve this problem, students might find that 3 light green rods are the same length as the blue rods. A light green rod is 1/3 of the blue rod.

Relating Fractions to Benchmarks

A numerical benchmark refers to a number to which other numbers can be related. For example, 100 is a whole-number benchmark with which students can compare other numbers (e.g., 98 is a little less than 100; 52 is about one half of 100; 205 is a little more than 2 hundreds).

As students explore fractional quantities that are less than 1, they learn to relate them to the benchmarks 0, 1/2, and 1. Using a variety of representations allows students to visualize the relationships of fractions to these benchmarks.

Using Fraction Circles (Area Model)

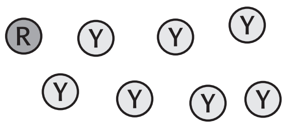


$\frac{1}{8}$ of the fraction circle is covered. That is close to 0.

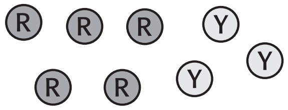
$\frac{5}{8}$ of the fraction circle is covered. That is close to $\frac{1}{2}$.

$\frac{7}{8}$ of the fraction circle is covered. That is close to 1.

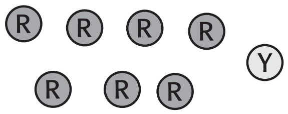
Using Two-Colour Counters (Set Model)



$\frac{1}{8}$ (almost none) of the set of counters is red. That is close to 0.

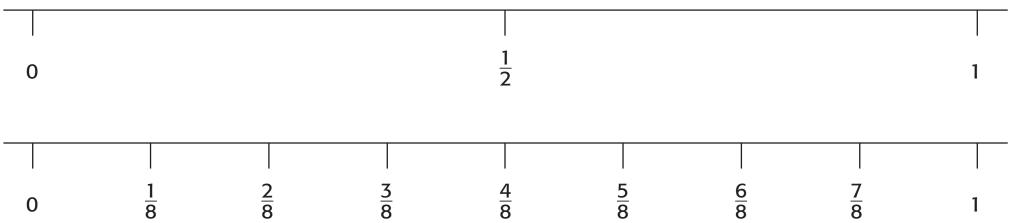


$\frac{5}{8}$ (about half) of the set of counters is red. That is close to $\frac{1}{2}$.



$\frac{7}{8}$ (almost all) of the set of counters is red. That is close to 1 (the whole set).

Using Number Lines (Linear Model)



The number lines for halves and eighths indicate that $\frac{1}{8}$ is close to 0, $\frac{5}{8}$ is close to $\frac{1}{2}$, and $\frac{7}{8}$ is close to 1.

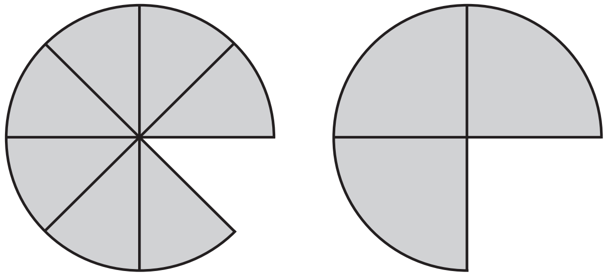
As students develop a sense of fractional quantities, they can use reasoning to determine whether fractions are close to 0, 1/2, or 1.

- In 1/8, there is only 1 of 8 fractional parts. The fraction is close to 0.
- One half of 8 is 4; therefore, 4/8 is equal to 1/2. 5/8 is close to (but greater than) 1/2.
- Eight eighths (8/8) represents one whole (1). 7/8 is close to (but less than) 1.

Comparing and Ordering Fractions

The ability to determine which of two fractions is greater and to order a set of fractions from least to greatest (or vice versa) is an important aspect of quantity and fractional number sense.

Students' early experiences in comparing fractions involve the use of concrete materials (e.g., fraction circles, fraction strips) and drawings to visualize the difference in the quantities of two fractions. For example, as the diagram on p. 20 illustrates, students could use fraction circles to determine that 7/8 of a pizza is greater than 3/4 of a pizza.



As students use concrete materials to compare fractions, they develop an understanding of the relationship between the number of pieces that make the whole and the size of the pieces. Simply telling students that "the bigger the number on the bottom of a fraction, the smaller the pieces are" does little to help them understand this relationship. However, when students have opportunities to represent fractions using materials such as fraction circles and fraction strips, they can observe the relative size of fractional parts (e.g., eighths are smaller parts than fourths). An understanding about the size of fractional parts is critical for students as they develop reasoning strategies for comparing and ordering fractions.

Students can use several strategies to reason about the relative size of fractions.

$\frac{4}{6}$? $\frac{5}{6}$

Same-size parts: The size of the parts (sixths) is the same for both fractions. Therefore, $\frac{4}{6} < \frac{5}{6}$.

$\frac{3}{4}$? $\frac{3}{6}$

Same number of parts but different-sized parts: Fourths are larger parts than sixths. Therefore, $\frac{3}{4} > \frac{3}{6}$.

$\frac{4}{6}$? $\frac{3}{8}$

Nearness to one half: $\frac{4}{6}$ is greater than one half ($\frac{3}{6}$). $\frac{3}{8}$ is less than one half ($\frac{4}{8}$). Therefore, $\frac{4}{6} > \frac{3}{8}$.

$\frac{7}{8}$? $\frac{3}{4}$

Nearness to one whole: Eighths are smaller than fourths, so $\frac{7}{8}$ is closer to one whole than $\frac{3}{4}$ is. Therefore, $\frac{7}{8} > \frac{3}{4}$.

The strategies that students use to compare fractions (i.e., using concrete materials, using reasoning) can be applied to ordering three or more fractions. In a problem situation in which students need to order 3/5, 3/8, and 5/6, students might reason in the following way:

- Since eighths are smaller parts than fifths, 3/8 is less than 3/5.
- Since 5/6 is closer to 1 than 3/5 is, 5/6 is greater than 3/5.
- The fractions ordered from least to greatest are 3/8, 3/5, 5/6.

Determining Equivalent Fractions

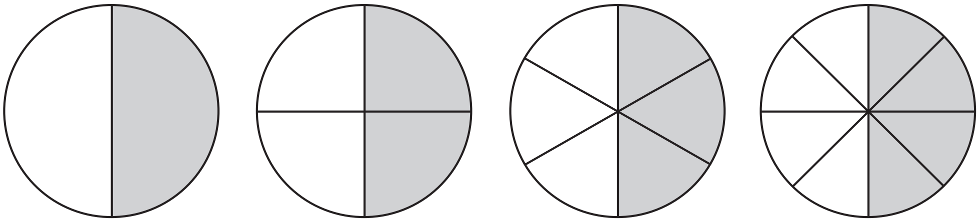
Fractions are equivalent if they represent the same quantity. For example, in a bowl of eight fruits containing two oranges and six bananas, 2/8 or 1/4 of the fruits are oranges; 2/8 and 1/4 are equivalent fractions.

Students' understanding of equivalent fractions should be developed in problem-solving situations rather than procedurally. Simply telling students to "multiply both the numerator and denominator by the same number to get an equivalent fraction" does little to further their understanding of fractions or equivalence.

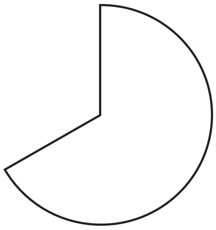
Students can explore fraction equivalencies using area, set, and linear models.

Finding Equivalent Fractions Using Area Models

Area models, such as fraction circles, fraction rectangles, and pattern blocks, can be used to represent equivalent fractions. Students can determine equivalent fractions by investigating which fractional pieces cover a certain portion of a whole. For example, as the following diagram illustrates, fraction pieces covering the same area of a circle demonstrate that 1/2, 2/4, 3/6, and 4/8 are equivalent fractions.



The following investigation involves using an area model to explore equivalent fractions.

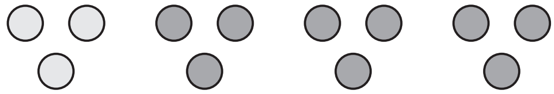


- Cover this shape using one type of fraction piece at a time. Do not combine different types of pieces.
- Which types of fraction pieces cover the shape completely with no leftover pieces?
- Write a fraction for each of the ways you can cover the shape.
- What is true about these fractions?

(continued)

Finding Equivalent Fractions Using Set Models

Students can use counters to determine equivalent fractions in situations that involve sets of objects. In the following diagram, counters show that 1/4 and 3/12 are equivalent fractions.



The following investigation involves using a set model to explore equivalent fractions.

> *"Arrange a set of 12 red counters and 4 yellow counters in equal-sized groups. All the counters within a group must be the same colour. How many different sizes of groups can you make? For each arrangement, record a fraction that represents the part that each colour is of the whole set."*

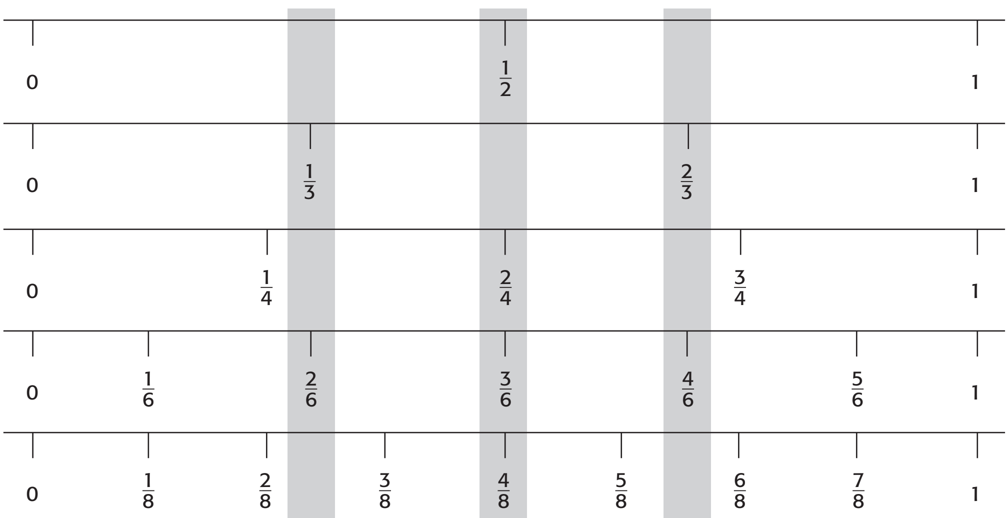
Students might record the results of their investigation in a chart:

Number of Groups	Fraction Red	Fraction Yellow
4	$\frac{3}{4}$	$\frac{1}{4}$
8	$\frac{6}{8}$	$\frac{2}{8}$
16	$\frac{12}{16}$	$\frac{4}{16}$

The arrangement of counters in different-sized groups shows that 3/4, 6/8, and 12/16 are equivalent fractions, as are 1/4, 2/8, and 4/16.

Finding Equivalent Fractions Using Linear Models

Students can use fraction number lines to demonstrate equivalent fractions. All the following number lines show the same line segment from 0 to 1, but each is divided into different fractional segments. Equivalent fractions (indicated by the shaded bands) occupy the same position on the number line.



The following investigation involves using a set model to explore equivalent fractions.

> *"Use paper strips to find equivalent fractions. Create a poster that shows different sets of equivalent fractions."*

A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

- partitioning objects and sets of objects into fractions, and discussing the relationship between fractional parts and the whole object or set;
- providing experiences with representations of fractions using area, set, and linear models;
- counting fraction pieces to beyond one whole using concrete materials and number lines (e.g., use fraction circles to count fourths: "One fourth, two fourths, three fourths, four fourths, five fourths, six fourths, . . ."');
- connecting fractional parts to the symbols for numerators and denominators of proper and improper fractions;
- providing experiences of comparing and ordering fractions using concrete and pictorial representations of fractions;
- discussing reasoning strategies for comparing and ordering fractions;
- investigating the proximity of fractions to the benchmarks of 0, 1/2, and 1;
- determining equivalent fractions using concrete and pictorial models.

The Grades 4–6 Fractions module at www.eworkshop.on.ca provides additional information on developing fraction concepts with students. The module also contains a variety of learning activities and teaching resources.

eworkshop.on.ca

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Learning Activities for Fractions

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to fractions. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts.

HOME CONNECTION: This section is addressed to parents or guardians and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.

Grade 4 Learning Activity
Every Vote Counts!

OVERVIEW

In this learning activity, students examine fractions that represent the results of a vote held by swim team members deciding whether or not to enter a swim meet. Students compare each fraction with the benchmarks of 0, 1/2, and 1 to determine whether at least half the team voted in favour of entering the meet.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: Students explore the "howmuchness" of different fractions by determining whether they are close to 0, 1/2, or 1.

Relationships: Students compare fractions with the benchmarks of 0, 1/2, and 1.

Representation: Students discuss the meaning of the numerator and the denominator in fraction representations.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered;
- compare fractions to the benchmarks of 0, 1/2, and 1 (e.g., 1/8 is closer to 0 than to 1/2; 3/5 is more than 1/2).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100.

ABOUT THE LEARNING ACTIVITY

MATERIALS

- a chart with agree/disagree statements, written on the board or chart paper
- "Should We Enter the Swim Meet?" chart, written on the board or chart paper (for the Working on It part of the learning activity)
- a variety of manipulatives for representing fractions (e.g., fraction circles, counters, square tiles)
- half sheets of chart paper or large sheets of newsprint (1 per pair of students)
- markers (a few per pair of students)
- "Should We Enter the Swim Meet?" chart, written on the board or chart paper (for the Reflecting and Connecting part of the learning activity)
- **Fra4.BLM1: Less Than, Equal to, or Greater Than 1/2** (1 per student)

MATH LANGUAGE

- fractional names (e.g., half, third, fourth)
- numerator
- denominator
- at least

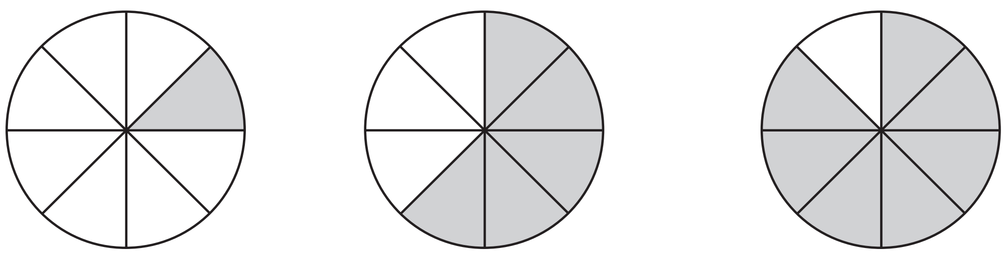
INSTRUCTIONAL GROUPING:
pairs

INSTRUCTIONAL SEQUENCING

Before this learning activity, students should have had many experiences representing fractions using concrete materials (e.g., fraction circles, counters, square tiles) and drawings. This learning activity provides opportunities for students to compare fractions with the benchmarks of 0, 1/2, and 1 – a strategy that students can also apply when they compare fractions with other fractions (e.g., 3/8 is less than 1/2, and 4/5 is greater than 1/2; therefore, 3/8 is less than 4/5).

ABOUT THE MATH

Students develop a sense of fractional quantities by relating them to the benchmarks of 0, 1/2, and 1 (e.g., 1/8 is close to 0; 5/8 is close to 1/2; 7/8 is close to 1). Initially, students use concrete materials and drawings to determine the proximity of fractions to 0, 1/2, and 1. For example, they might use fraction circles as illustrated in the following diagrams.



$\frac{1}{8}$ of the fraction circle is covered. That is close to 0.

$\frac{5}{8}$ of the fraction circle is covered. That is close to $\frac{1}{2}$.

$\frac{7}{8}$ of the fraction circle is covered. That is close to 1.

As students develop a stronger sense of fractional quantities, they can use reasoning strategies, such as the following, to determine whether fractions are close to 0, 1/2, or 1.

- In 1/8, there is only 1 of 8 fractional parts. The fraction is close to 0.
- One half of 8 is 4; therefore, 4/8 is equal to 1/2. 5/8 is close to (but greater than) 1/2.
- Eight eighths (8/8) represents one whole (1). 7/8 is close to (but less than) 1.

In this learning activity, students are asked to determine whether given fractions are less than or greater than 1/2. They are encouraged to use strategies that make sense to them – some students may use manipulatives or drawings to represent fractions, while others may use reasoning skills.

GETTING STARTED

Show the following chart, written on the board or chart paper, to the class.

	Agree	Disagree
1. Taking a vote is the best way for a class to make a group decision.		
2. In a class vote, the teacher should decide who may vote and who may not.		
3. In a class vote, most students vote the same way as their friends.		

Ask eight students to stand. Read the first statement in the chart, and ask the students who are standing to vote with their thumbs – using a thumbs-up gesture if they agree with the statement or using a thumbs-down signal if they disagree. Record the results on the chart (e.g., if 7 students agree and 1 student disagrees, write "7/8" in the Agree column and "1/8" in the Disagree column). Explain to the class that you used fractions to record the results of the vote. Ask the eight students to sit down.

Refer to the fraction in the "Agree" column, and ask:

- "What does the 8 mean?" (the number of students in the whole group)
- "What is the name for this part of the fraction?" (denominator)
- "What does the 7 mean?" (the number of students who agreed – part of the whole group)
- "What is the name for this part of the fraction?" (numerator)
- "How do we read this fraction?" (seven eighths)

Reinforce the meaning of *denominator* and *numerator* by asking similar questions about the fraction in the Disagree column.

Select six other students to stand. Ask these students to indicate using the thumbs-voting technique whether they agree or disagree with the second statement. Record the results on the chart using fractions expressed as sixths. Ask the six students to sit down.

Refer to the fractions recorded beside the second statement in the chart, and pose the following questions:

- "What do these two fractions mean?"
- "What is the denominator? Why is 6 the denominator?"

- "What are the numerators? Why?"
- "What fraction of the group agrees with the statement?"
- "Is this fraction close to none of the group, to half of the group, or to the whole group? How do you know?"
- "What fraction of the group disagrees with the statement?"
- "Is this fraction close to none of the group, to half of the group, or to the whole group? How do you know?"

Finally, ask 10 students to stand. Record the results of their voting for the third statement on the chart.

Use a think-pair-share strategy to have students reflect on and discuss the results of the vote for the third statement. Ask students to think about how the results of the vote could be interpreted using fraction language. Encourage them to think about whether the "agree" and "disagree" votes are close to none of the group, to half of the group, or to the whole group. Provide approximately 30 seconds for students to think individually, and then have them share their thoughts with a partner.

WORKING ON IT

Tell students the following:

> "A swim meet is coming up. Teams may enter the meet if at least 1/2 of their team members agree to participate. Each team holds a vote to decide whether it will enter the meet."

Display a partially completed chart with the names of the swim teams.

Should We Enter the Swim Meet?

Team	Agree
Dolphins	
Marlins	
Goldfish	

Explain that the Dolphins team has 6 members, and that 4 members vote in favour of entering the meet. Ask: "What fraction of the Dolphins team agrees to enter the meet?" Record "4/6" in the Agree column beside Dolphins.

Complete the chart with students by explaining that 3 out of 7 Marlins team members voted to enter the meet (record "3/7" beside Marlins) and that 4 out of 8 Goldfish team members want to enter the meet (record "4/8" beside Goldfish).

Explain to students that their task is to determine which teams may enter the swim meet.

Organize students into pairs. Explain that students will work with a partner to solve the problem. Encourage them to use manipulatives (e.g., fraction circles, counters, square tiles) to help them think about the problem and a solution. Provide each pair of students with markers and a half sheet of chart paper or a large sheet of newsprint. Ask students to show how they solved the problem in a way that can be clearly understood by others.

Circulate around the room and observe students as they are working. Ask them questions such as the following:

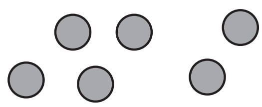
- "What strategy are you using to solve the problem?"
- "How can you figure out whether 4/6 (3/7, 4/8) represents at least 1/2 of the team?"
- "How can you prove that your thinking is right?"

Students might use manipulatives and/or reasoning to determine whether the fractions are greater than 1/2.

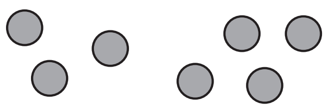
STRATEGIES STUDENTS MIGHT USE

USING MANIPULATIVES

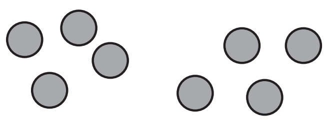
Students might use counters to represent the team members and separate the counters into two groups – one group to represent the "agree" members and the other group, the "disagree" members. For example, students could use 6 counters to represent the Dolphins team members. They might observe that 1/2 of 6 counters is 3 counters, so 4/6 is greater than 1/2.



For the Marlins team, students might separate 7 counters into a group of 3 and a group of 4. They might reason that 3 counters is 1/2 of 6 counters, therefore 3/7 is less than 1/2.



Using counters to model the outcome of the Goldfish team's vote allows students to observe and represent equivalent fractions (4/8 is equal to 1/2).



USING REASONING

Students can use their knowledge of fractions to reason whether 4/6, 3/7, and 4/8 are greater than 1/2.

(continued)

For the Dolphins team, students might determine that 1/2 of 6 is 3 and conclude that 4/6 is greater than 1/2.

For the Marlins team, students might determine that 1/2 of 7 is 3 1/2 and decide, therefore, that 3/7 is less than 1/2. Or they might realize that 3 (the numerator in 3/7) is 1/2 of 6 and determine that 3/7 is less than 1/2.

For the Goldfish team, students might realize that 4 is 1/2 of 8 and recognize that 4/8 and 1/2 are equivalent fractions.

REFLECTING AND CONNECTING

Provide an opportunity for pairs of students to share their work and to explain their solutions to the whole class. Select pairs who used different strategies (e.g., using manipulatives, using reasoning), and allow students to observe various approaches to solving the problem. Make positive comments about students' work, being careful not to infer that some approaches are better than others. Your goal is to have students determine for themselves which strategies are meaningful and efficient.

Post students' work and ask questions such as:

- "What strategies are similar? How are they alike?"
- "Which strategy would you use if you solved another problem like this again?"
- "How would you change any of the strategies that were presented? Why?"
- "Which work clearly explains a solution? Why is the work clear and easy to understand?"

Provide another opportunity for students to relate fractions to the benchmarks of 0, 1/2, and 1 using reasoning strategies. Display the following chart, and explain that it shows the names of five other swim teams and the fraction of team members who voted in favour of entering the swim meet.

Should We Enter the Swim Meet?

Team	Agree
Clownfish	$\frac{7}{8}$
Barracudas	$\frac{4}{9}$
Guppies	$\frac{2}{8}$
Orcas	$\frac{1}{6}$
Sharks	$\frac{4}{10}$

Referring to each team in the chart, ask the following questions:

- "What fraction of the team voted in favour of entering the swim meet?"
- "Is the fraction closer to 0 (none of the team), 1/2, or 1 (the whole team)?"
- "How do you know that the fraction is closer to 0 (or 1/2 or 1)?"

Provide opportunities for several students to explain their thinking. Have them use manipulatives (e.g., fraction circles, counters, square tiles) to demonstrate their reasoning, thereby helping students who may have difficulty following oral explanations.

ADAPTATIONS/EXTENSIONS

Simplify the problem for students who might experience difficulties. For example: "8 out of 10 team members voted in favour of entering the swim meet. Did at least 1/2 of the team members vote in favour of entering the meet?" Encourage students to use manipulatives (e.g., fraction circles, counters, square tiles) to solve the problem.

Extend the problem for students who require a greater challenge:

> "A coach agrees to enter teams in a swim meet if at least 1/2 of the members on each team vote in favour of doing so. Here are the numbers of team members who voted in favour of entering the meet:
>
> - Mackerels: 12 out of 15
> - Snappers: 9 out of 13
> - Angelfish: 8 out of 14
> - Trout: 11 out of 16
>
> Which teams will enter the meet?"

ASSESSMENT

Have students, individually, solve the following problem. Ask students to record their solutions, reminding them to show their ideas in a way that can be clearly understood by others.

> "Twelve members of a team are holding a vote to decide whether their team should enter a weekend competition. 7/12 of the team vote in favour of entering the competition. Is 7/12 closer to 0, 1/2, or 1? Explain your reasoning so that others will understand your thinking."

Observe students' work to assess how well they:

- determine that 7/12 is close to 1/2 (e.g., 6 is 1/2 of 12; therefore, 7/12 is close to 1/2);
- communicate a strategy and solution clearly;
- use appropriate drawings and/or explanations to demonstrate their thinking.

HOME CONNECTION

Send home **Fra4.BLM1: Less Than, Equal to, or Greater Than 1/2**. In this Home Connection activity, students and parents/guardians find examples of fractions at home and compare these fractions with 1/2. In class, encourage students to share their drawings and explain how their fraction examples compare with one half.

LEARNING CONNECTION 1
One Half as a Benchmark

MATERIALS

- a variety of fraction models, including area models (e.g., fraction circles, pattern blocks), set models (e.g., two-colour counters), and linear models (e.g., fraction strips, Cuisenaire rods). See pp. 13–14 for other examples of area, set, and linear models.

Show students different representations of fractions, including area, set, and linear models. For each fraction, ask students to describe what the whole looks like. Next, ask students to determine whether each fraction is less than, equal to, or more than 1/2. Have students explain their reasoning.

LEARNING CONNECTION 2
Between 2/3 and 1

MATERIALS

- a variety of manipulatives for representing fractions (e.g., fraction circles, two-colour counters, Cuisenaire rods, square tiles)
- paper (1 per pair of students)

Have pairs of students find and record fractions that are between 2/3 and 1. Encourage students to use manipulatives (e.g., fraction circles, two-colour counters, Cuisenaire rods, square tiles) and drawings to help them.

Ask a few pairs to share their work with the class. Challenge students to prove that their fractions are between 2/3 and 1.

LEARNING CONNECTION 3
Making the Whole

MATERIALS

- *Math Curse* by Jon Scieszka
- manipulatives for representing fractions (e.g., fraction circles, fraction strips)

Read *Math Curse* by Jon Scieszka (New York: Viking Books, 1995), if available. In this book, the character sees everything in the world as a math problem. Towards the end of the book, the character is trapped in a room with a board that is covered with "a lifetime of problems". The character breaks a stick of chalk in two and then puts the two halves of chalk together to make one whole. With a play on words, "whole" becomes "hole", and the character escapes through a hole in the wall.

Ask students: "What fraction would I need to add to 1/4 to make 1 whole? How do you know?" Encourage students to use drawings (e.g., circles or rectangles divided into parts) and manipulatives (e.g., fraction circles, fraction strips) to explain their thinking.

Provide other fractions (e.g., 2/3, 4/5, 1/6, 5/8), and ask students to determine the fraction that must be added to each to make one whole. Have students explain their thinking.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on fraction concepts. On the home page, click "Toolkit". In the "Numeracy" section, find "Fractions (4 to 6)", and then click the number to the right of it.

eworkshop.on.ca

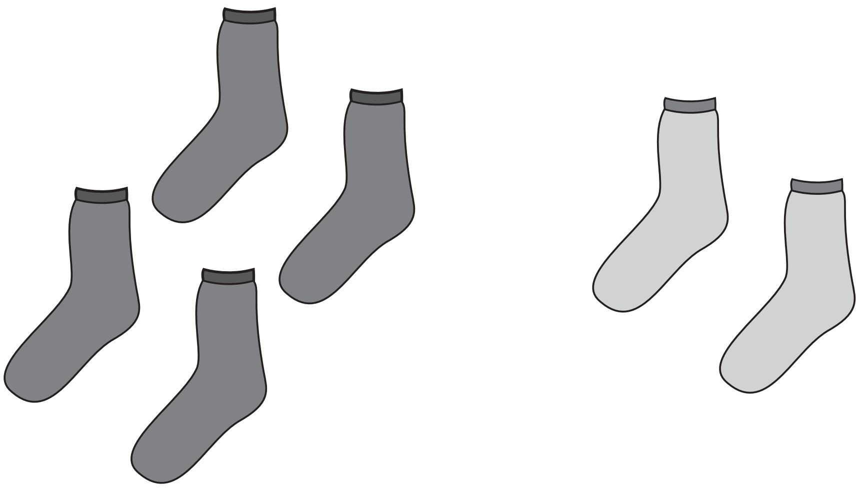
Less Than, Equal to, or Greater Than 1/2

Dear Parent/Guardian:

We have been learning about ways to decide whether a fraction is close to 0, 1/2, or 1.

With your child, find examples of fractions in your home (e.g., the fraction of socks in a drawer that are black; the fraction of doors that have locks; the fraction that describes the amount of water in a bottle). Ask your child to compare these fractions with 1/2 (e.g. more than 1/2 of the socks in the drawer are black; exactly 1/2 of the doors have locks; the water bottle is less than 1/2 full).

Ask your child to make drawings to show how some of the fractions you found compare with 1/2.



$\frac{4}{6}$ of the socks are black.

More than $\frac{1}{2}$ of the socks are black.

In class, students will share their drawings and explain how their fraction examples compare with 1/2.

Thank you for doing this activity with your child.

Grade 5 Learning Activity Investigating Fractions Using Tangrams

OVERVIEW

In this learning activity, students explore the fractional relationships between tangram pieces, and between each piece and the whole tangram square. Students also investigate equivalent fractions using different tangram pieces to represent 1/4, 1/2, and 3/4 of the whole tangram square.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: Students explore fractional quantities by identifying the fraction that each tangram piece represents of the whole tangram square, by comparing the sizes of different fractions, and by representing equivalent fractions.

Relationships: Students explore the relationships between equivalent fractions (e.g., multiplying the numerator and denominator of a fraction by the same number produces an equivalent fraction).

Representation: Students observe that a fractional amount can be represented by equivalent fractions (e.g., 1/4, 2/8, and 4/16 represent the same area of the whole tangram square).

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, number lines) and using standard fractional notation;
- demonstrate and explain the concept of equivalent fractions, using concrete materials (e.g., use fraction strips to show that 3/4 is equal to 9/12).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers.

TIME: approximately two 60-minute periods

ABOUT THE LEARNING ACTIVITY

MATERIALS

- overhead projector
- overhead transparency of tangram pieces
- sets of tangrams, either commercially produced or cut out of stiff paper using **Fra5.BLM1: Tangram Puzzle** as a template (1 set per student)
- **Fra5.BLM2: Tangram Square** (1 per pair of students)
- **Fra5.BLM3: Tangram Fractions** (1 copy)
- instructions for Working on It Activity 1 posted on the board or chart paper
- **Fra5.BLM4: 1/4 Tangram Square** (1 per pair of students)
- sheets of paper (several per pair of students)
- scissors (1 pair per pair of students)
- **Fra5.BLM5: 1/2 Tangram Square** (1 per pair of students)
- **Fra5.BLM6: 3/4 Tangram Square** (1 per pair of students)
- **Fra5.BLM7: Finding Equivalent Fractions** (1 per student)
- sheets of paper or math journals (1 per student)

MATH LANGUAGE

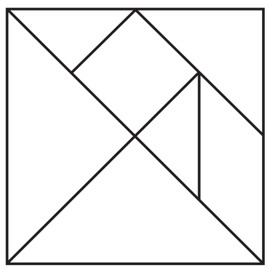
- fractional names (e.g., half, fourth, eighth, sixteenth)
- numerator
- denominator
- equal/equivalent

INSTRUCTIONAL GROUPING: pairs

INSTRUCTIONAL SEQUENCING

Before this learning activity, students should have had experiences dividing whole objects into equal parts and identifying the parts using fractional language (e.g., "three fourths") and standard fractional notation (e.g., 3/4). The following activity provides an opportunity for students to represent fractions concretely and to investigate equivalent fractions.

ABOUT THE MATH



In this learning activity, students explore the fractional relationship between each tangram piece and the whole tangram square (e.g., the large triangles each represent 1/4 of the tangram square; the medium-sized triangle represents 1/8 of the square; the small triangles each represent 1/16 of the square).

Students also find different ways to represent 1/4, 1/2, and 3/4 of the whole tangram square using different tangram pieces. For example, they discover that 1/4 of the whole tangram square can be covered using one of the large triangles (1/4 of the whole tangram square) or two medium-sized triangles (2/8 of the square) or four small triangles (4/16 of the square). This discovery leads to a discussion about equivalent fractions as different fractions that represent the same quantity (same area). Students investigate the notion that multiplying the numerator and the denominator of a fraction by the same number produces an equivalent fraction.

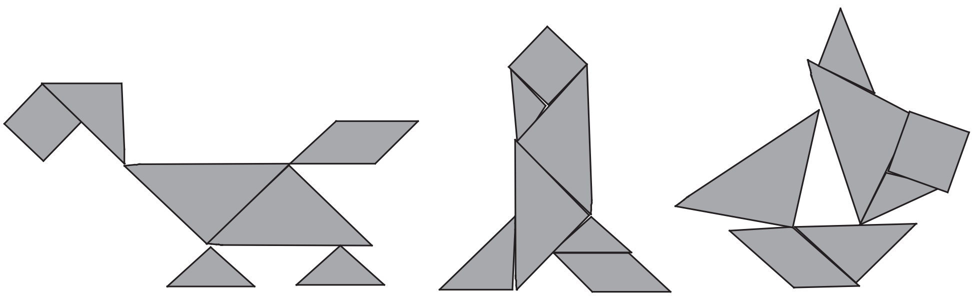
Note: In this activity, fractions represent the *areas* of the tangram pieces compared with a whole tangram (e.g., the area of the large triangle represents 1/4 of the area of the whole tangram square). Be aware that some students might think about the fractions in terms of the *number* of tangram pieces compared with the whole (e.g., since there are 7 tangram pieces, each piece is 1/7 of the whole tangram). Focus these students' attention on the relative areas of the tangram pieces. For example, you might have students compare the areas of the tangram pieces by asking them to decide which piece has a bigger or smaller area. You might also have students superimpose tangram pieces to find equal areas (e.g., two medium-sized triangles have the same area as one large triangle).

GETTING STARTED

Tell the following story about the origin of the tangram puzzle.

> "Many years ago, in China, there lived a man called Mr. Tan. Of all his possessions, he most treasured an exquisite porcelain square tile. One day, he heard that the Emperor of China was coming to his village. To show his great admiration for and loyalty to the Emperor, Mr. Tan decided to offer his very precious tile to the Emperor as a gift. In great excitement, he began to polish his tile so that it would shine. As he handled the tile in different ways, to polish every surface, he dropped it. The porcelain tile broke into the seven pieces of the tangram puzzle. Mr. Tan was so very unhappy. As he wiped away his tears, he thought that if he could put the pieces back together, he would have the square tile again. Mr. Tan thought it would be easy to do, but it took him a very long time. While he was trying to form the square, he discovered lots of interesting two-dimensional shapes."

Using an overhead projector and an overhead transparency of tangram pieces (or pieces cut out from **Fra5.BLM1: Tangram Puzzle**), show the following shapes of objects made using the seven tangram pieces. Ask students to try to identify the objects.



Provide each student with a set of tangram pieces. (If you don't have enough sets of commercially produced tangrams, distribute copies of **Fra5.BLM1: Tangram Puzzle**, and have students cut out their own sets of tangram pieces.) Challenge students to construct a square using all seven tangram pieces. Invite students who finish before others to find different ways to create the square.

Have students compare their square arrangement with a partner's. Some students may observe that all the different arrangements are congruent and that some are rotated (turned) or reflected (flipped) versions of others.

Provide each pair of students with a copy of **Fra5.BLM2: Tangram Square**, and explain that the square outline on the page is the same size as the tangram square. (If the tangrams in your classroom form a different size of square from the one on **Fra5.BLM2**, you will need to resize the blackline master.) Ask: "How many large triangles would you need to cover the square?" Provide an opportunity for students to manipulate the tangram pieces (students may combine their sets of tangram pieces) and to determine that four large triangles would cover the square. Discuss how the large triangle represents 1/4 of the square. Post a copy of **Fra5.BLM3: Tangram Fractions**, and label "1/4" on each of both large triangles.

Next, challenge students to find the fractional relationship between each of the other tangram pieces and the whole tangram square.

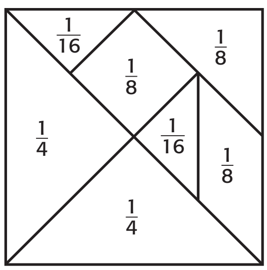
After students have had sufficient time to conduct their investigations, ask the following questions:
- "How many medium-sized triangles do you need to cover the whole square?" (8)
- "What fraction of the whole square is the medium-sized triangle?" (one eighth)
- "How many small triangles would you need to cover the whole square?" (16)
- "What fraction of the whole square is one small triangle?" (one sixteenth)

On the posted copy of **Fra5.BLM3: Tangram Fractions**, label "1/8" on the medium-sized triangle and "1/16" on each of both small triangles.

Next, have pairs of students explain how they determined the fractional relationship between the square tangram piece and the whole tangram square. They might have discovered that the area of the square tangram piece is equal to the area of the two small triangles; therefore, its area is equal to that of the medium-sized triangle, which is 1/8 of the whole tangram square. Label "1/8" on the square tangram piece on the posted copy of **Fra5.BLM3: Tangram Fractions**.

Finally, discuss how the parallelogram has the same area as the medium-sized triangle and as the square tangram piece; therefore, it is 1/8 of the whole square. Label "1/8" on the parallelogram tangram piece on the posted copy of **Fra5.BLM3: Tangram Fractions**.

Ask students to examine the labelled copy of **Fra5.BLM3: Tangram Fractions**, and invite them to make observations. For example, students might observe that the square, the medium-sized triangle, and the parallelogram each represent 1/8 of the whole square even though they are different shapes. Students might also describe the relationships between shapes (e.g., that the parallelogram piece could be composed by combining two small triangles).



WORKING ON IT

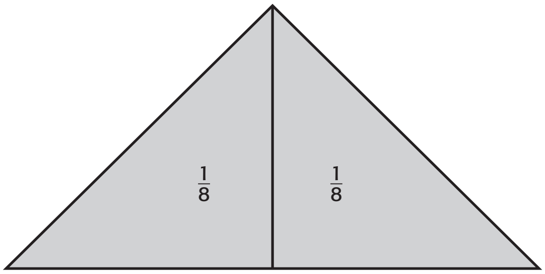
ACTIVITY 1: FINDING EQUIVALENT REPRESENTATIONS OF 1/4

Provide each pair of students with a copy of **Fra5.BLM4: 1/4 Tangram Square**. Explain to students that they are to find different ways to cover 1/4 of the whole square (the shaded area of the square). Ask students to use one kind of tangram piece (e.g., only small triangles) each time. Tell students that they may find arrangements that involve more tangram pieces than those in their set. For example, they may come up with an arrangement that involves four small triangles even though the tangram set contains only two small triangles.

Provide the following instructions (posted on the board or chart paper), and ask students to follow the process for each different arrangement:

- Cover 1/4 of the large square with identical tangram pieces.
- Arrange the identical tangram pieces on another sheet of paper, and trace around the arrangement.
- Cut out the arrangement (along only the outside lines to make one piece).
- Label each tangram piece within the cut-out with its fraction name.

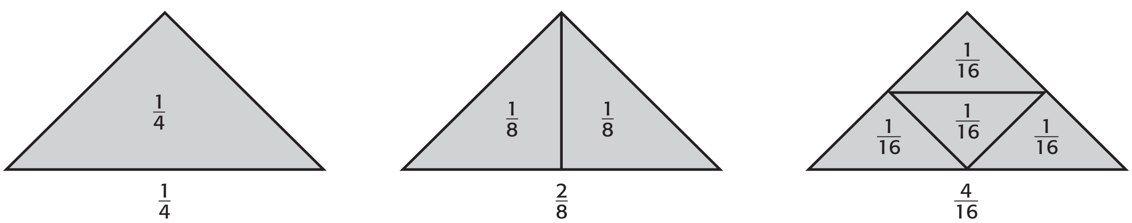
Show students an example of a completed cut-out arrangement:



Gather the class after students have completed the activity. Have students show different cut-outs that cover 1/4 of the tangram square, and ask them to explain the fractions represented by the tangram pieces within each cut-out.

Note: If students propose arrangements other than those involving one large triangle, two medium-sized triangles, or four small triangles, ask them to flip or rotate their cut-outs to see if they are congruent to those already discussed.

Borrow the following cut-outs from students, and post them on the board or chart paper. Record the corresponding fraction below each cut-out.



Help students to recognize the equivalence of 1/4, 2/8, and 4/16 by asking the following questions:

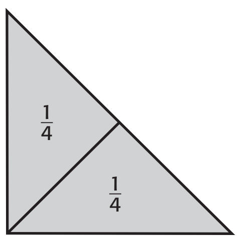
- "What fraction of the large tangram square do these cut-outs represent? How do you know?"
- "How many eighths are equal to 1/4?"
- "How many sixteenths are equal to 1/4?"
- "How do you know that 1/4, 2/8, and 4/16 are equivalent fractions?" (They all represent the same quantity/area.)

ACTIVITY 2: FINDING EQUIVALENT REPRESENTATIONS OF 1/2 AND 3/4

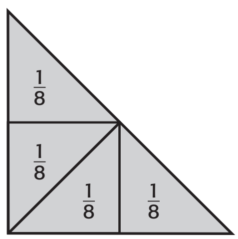
Provide each pair of students with a copy of **Fra5.BLM5: 1/2 Tangram Square** and **Fra5.BLM6: 3/4 Tangram Square**. Explain that the activity is similar to the previous one but that this time students need to find different ways to cover 1/2 and 3/4 of the large square. Review the process of covering the shaded part of the whole tangram square with identical tangram pieces (one kind of tangram piece each time), tracing around the pieces on paper, cutting out the arrangement, and labelling the fractions.

After students have had sufficient time to find different representations for 1/2 and 3/4, ask each pair of students to join another pair. Have the groups discuss how they know their cut-outs represent 1/2 or 3/4.

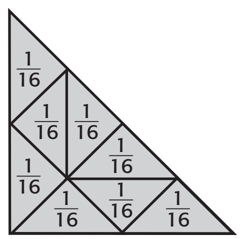
Borrow the following cut-outs from students, and post them on the board or chart paper. Record the corresponding fraction below each cut-out.



$\frac{2}{4}$



$\frac{4}{8}$

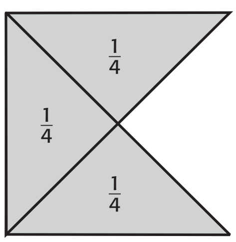


$\frac{8}{16}$

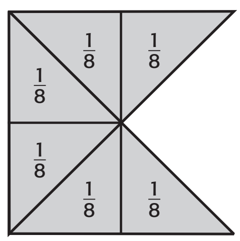
Ask the following questions:

- "How do you know that all these cut-outs represent 1/2 of the large square?"
- "What are some equivalent fractions for 1/2?"

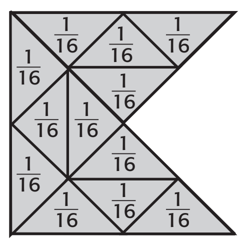
Next, post the following cut-outs for 3/4, and record the corresponding fraction below each cut-out.



$\frac{3}{4}$



$\frac{6}{8}$



$\frac{12}{16}$

Ask students to explain how they know that all the cut-outs represent 3/4 of the whole tangram square. Discuss the equivalence of 3/4, 6/8, and 12/16.

REFLECTING AND CONNECTING

Review the equivalent fractions for 1/2 that the class found using tangram pieces, and record the following on the board or chart paper:

$$\frac{1}{2} = \frac{2}{4} \qquad \frac{1}{2} = \frac{4}{8} \qquad \frac{1}{2} = \frac{8}{16}$$

Ask students to describe any patterns they observe. Elicit the idea that when the numerator and denominator of 1/2 are both multiplied by 2 or 4 or 8, this produces an equivalent fraction.

Next, record the equivalent fractions for 1/4 and 3/4.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{1}{4} = \frac{4}{16}$$

$$\frac{3}{4} = \frac{6}{8} \qquad \frac{3}{4} = \frac{12}{16}$$

Ask students whether the same idea of multiplying the numerator and the denominator by the same number applies to equivalent fractions for 1/4 and 3/4. Have students explain their thinking to a partner. Invite students to apply the rule to determine other equivalent fractions for 1/4, 1/2, and 3/4.

Ask students to record their thoughts about the following question on a sheet of paper or in their math journals: "What do you know about equivalent fractions?"

ADAPTATIONS/EXTENSIONS

Some students may find it difficult to work with different tangram pieces at the same time. Give these students only one type of tangram piece at a time (e.g., only small triangles to represent sixteenths; only medium-sized triangles to represent eighths).

For students who require a greater challenge, ask them to use tangram pieces to find as many different examples of equivalent fractions as possible. For example, students might use 6 small triangles and 3 medium-sized triangles to show that 6/16 and 3/8 are equivalent fractions.

ASSESSMENT

As students are working with tangram pieces, ask the following questions to assess their understanding of fraction representations and relationships:

- "What fraction of the whole square is this tangram piece? How do you know?"
- "How could you show 1/2 (or 1/4 or 3/4) of the whole square? How can you show this fraction in a different way?"

- "What fractions are equivalent to 1/4? To 1/2? To 3/4? How do you know that these fractions are equivalent?"

Examine students' journal entries (completed in the Reflecting and Connecting portion of the learning activity) to assess how well they understand that equivalent fractions represent the same quantity using different-sized fractional parts.

HOME CONNECTION

Send home copies of **Fra5.BLM7: Finding Equivalent Fractions**. In this Home Connection activity, students and their parents/guardians find examples of equivalent fractions in their home. In class, encourage students to share and explain their diagrams.

LEARNING CONNECTION 1
Equivalent Fractions With Set Models

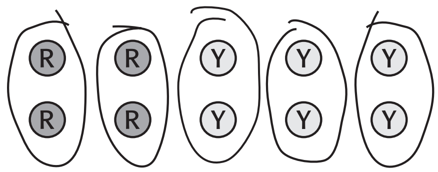
MATERIALS

- two-colour (red, yellow) counters (14 per pair of students)

In this learning activity, students have explored equivalent fractions using an area model (tangrams). Students can also investigate equivalent fractions using a set model.

Provide pairs of students with two-colour counters. Have the students display 4 red counters and 6 yellow counters. Establish the idea that the entire set (the whole) is composed of all 10 counters and that 4/10 of the counters are red and 6/10 are yellow.

Next, have students rearrange the counters into equal-sized groups, where the counters within a group are the same colour, to find equivalent fractions.



Five groups showing $\frac{2}{5}$ red and $\frac{3}{5}$ yellow

Repeat the activity using:
- 3 red and 6 yellow;
- 6 red and 2 yellow;
- 4 red and 8 yellow.

LEARNING CONNECTION 2
Equivalent Fractions Game

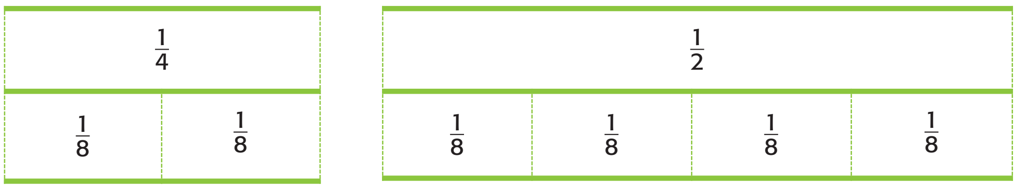
MATERIALS

- copies of **Fra5.BLM8: Fraction Strips** (1 per student)
- scissors (1 pair per student)

Provide students with a copy of **Fra5.BLM8: Fraction Strips**, and instruct them to cut out the fraction strips and cut the pieces apart.

Arrange students in pairs, and explain the game:

- Players combine their fraction pieces in a common pile.
- Players take turns selecting fraction pieces until all pieces have been drawn.
- Each player arranges his or her fraction pieces to create pairs of equivalent fractions, for example:



Each player identifies the equivalent fractions that he or she created (e.g., 1/4 = 2/8 and 1/2 = 4/8) and earns a point for each pair correctly identified. The player with more pairs of equivalent fractions wins the game.

LEARNING CONNECTION 3
Introducing Mixed Numbers

MATERIALS

- overhead transparency of pattern blocks, if available, or regular pattern blocks
- overhead projector

Mixed numbers represent quantities that comprise one or more wholes and fractional parts. The following activity allows students to see that quantities greater than 1 can be written as a fraction and as a mixed number.

Using overhead transparency pattern blocks on an overhead projector, display a yellow hexagon and explain that it represents one whole unit. Next, show a collection of 17 sixths (green triangles). Have students count the sixths orally as you point to each block ("One sixth, two sixths, three sixths, . . ., seventeen sixths"). Ask students how they might record the number of sixths using fractional notation (17/6).

Next, have a student regroup the sixths on the overhead projector into whole hexagon shapes. Again, ask students to explain how they might record the quantity shown on the overhead projector. For example, they might suggest the following forms:

- 2 wholes and 5/6
- 2 and 5/6
- 2 + 5/6

Emphasize that their suggestions are possible ways to record the amount and that 2 5/6 is the standard way to represent 2 wholes and 5/6.

Repeat the activity using other collections of fractional parts (e.g., red trapezoids for halves, blue rhombuses for thirds).

LEARNING CONNECTION 4
Improper Fractions and Mixed Numbers

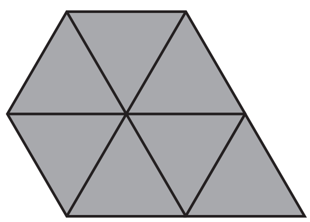
MATERIALS

- pattern blocks, including yellow hexagons, red trapezoids, blue rhombuses, and green triangles (several per pair of students)
- sheets of paper (1 per pair of students)

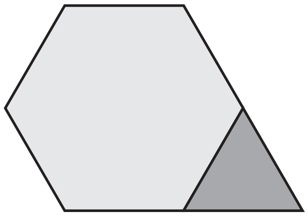
Provide each pair of students with several pattern blocks. Explain that the yellow hexagon pattern block represents one whole. Invite students to cover the surface of the yellow hexagons with red trapezoids, blue rhombuses, and green triangles. Establish that the red trapezoid is 1/2 of the yellow hexagon, the blue rhombus is 1/3 of the yellow hexagon, and the green triangle is 1/6 of the yellow hexagon.

Ask students to work with their partner to create an arrangement using one kind of pattern block (e.g., only green triangles) that represents a quantity greater than one whole. Each arrangement should consist of fewer than 10 pattern blocks.

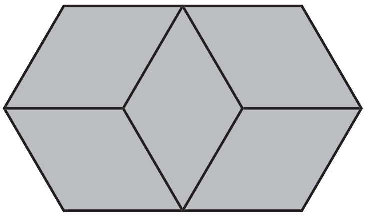
Ask a few pairs to show their arrangement to the class and to identify the fraction represented (e.g., an arrangement with 7 green triangles represents 7/6).



Next, instruct students to substitute a yellow hexagon for the pattern blocks that equal the whole (e.g., in an arrangement of 7 green triangles, students would take out 6 triangles and substitute a yellow hexagon, leaving a yellow hexagon and a green triangle). Have students rename their arrangement using a mixed number.



Have pairs of students create different pattern-block arrangements that are greater than 1. Ask them to record each arrangement by tracing around the pattern blocks on a sheet of paper and to label the arrangement using both an improper fraction and a mixed number, for example:



$\frac{5}{3} = 1\frac{2}{3}$

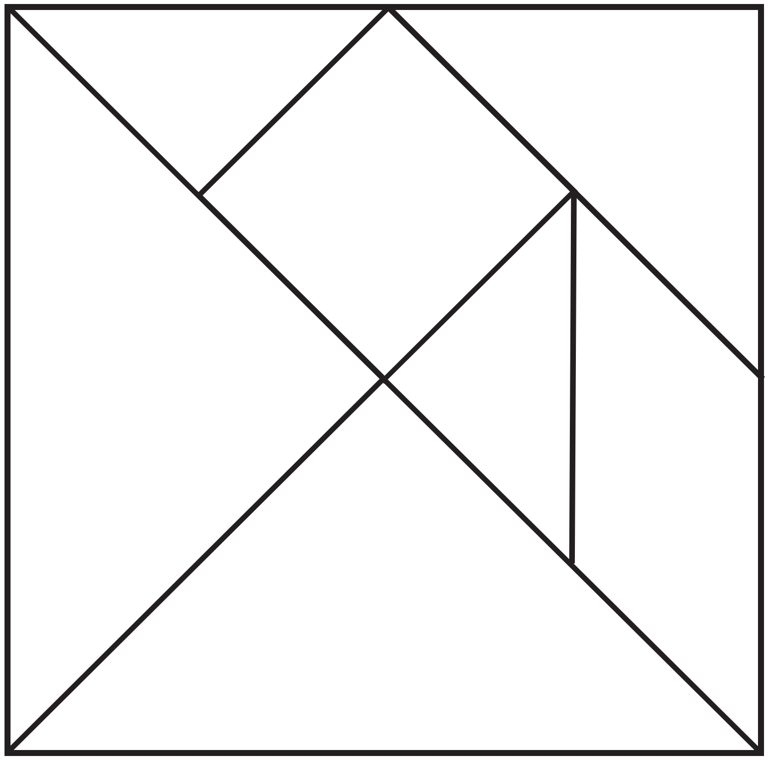
Observe students' work to determine how well they are able to identify improper fractions and corresponding mixed numbers.

eWORKSHOP CONNECTION

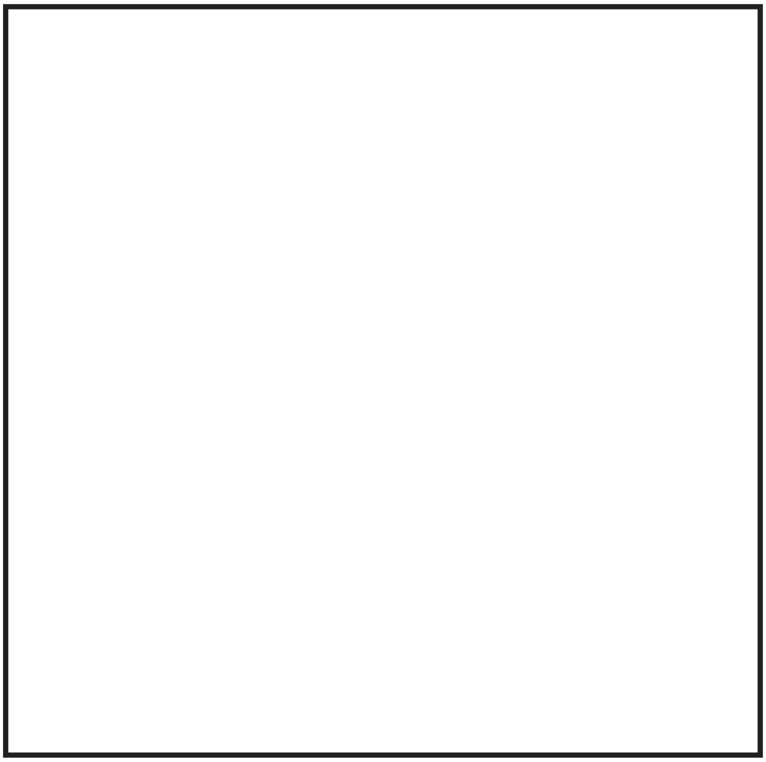
Visit www.eworkshop.on.ca for other instructional activities that focus on fraction concepts. On the home, click "Toolkit". In the "Numeracy" section, find "Fractions (4 to 6)", and then click the number to the right of it.

eworkshop.on.ca

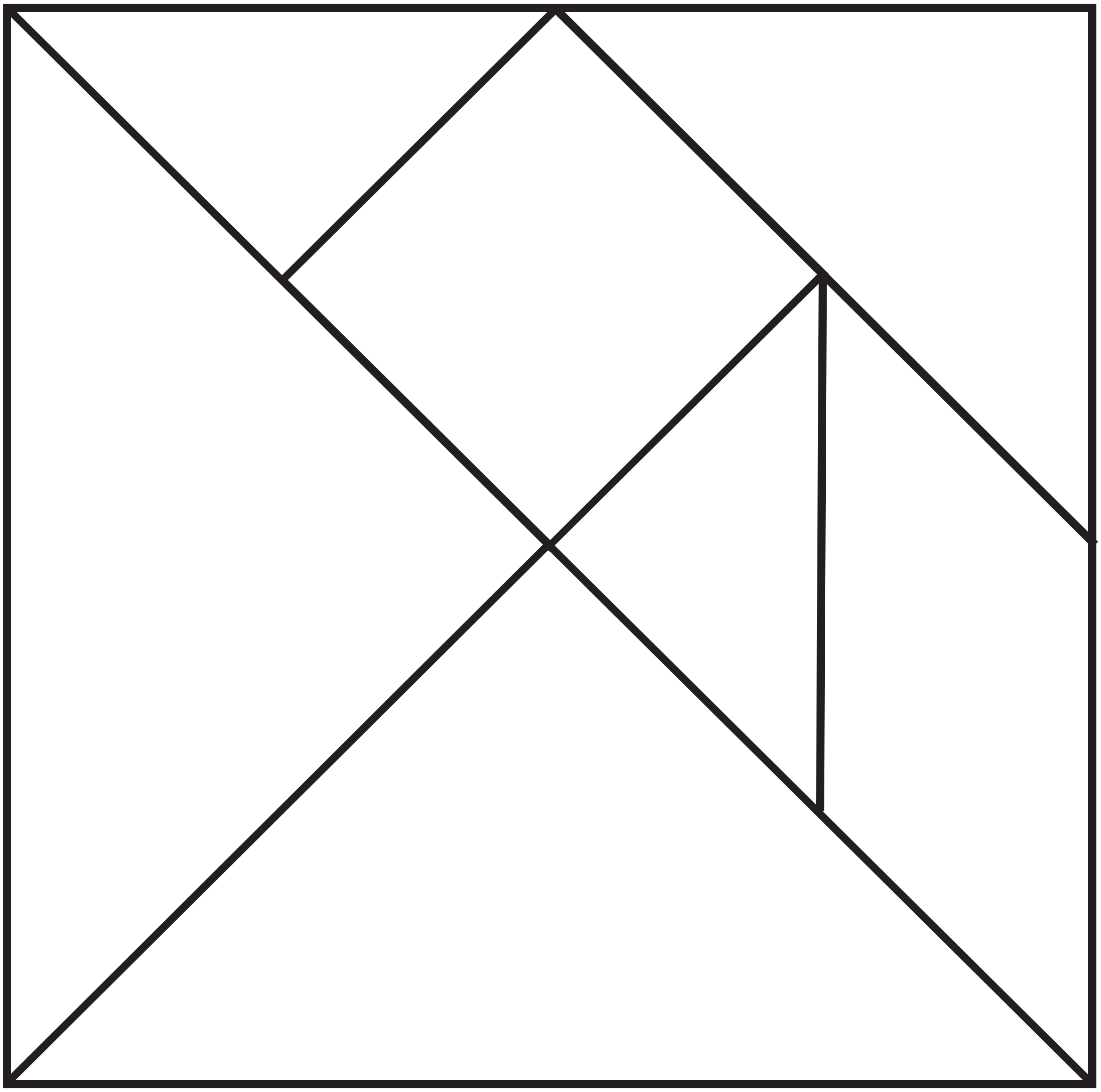
Tangram Puzzle



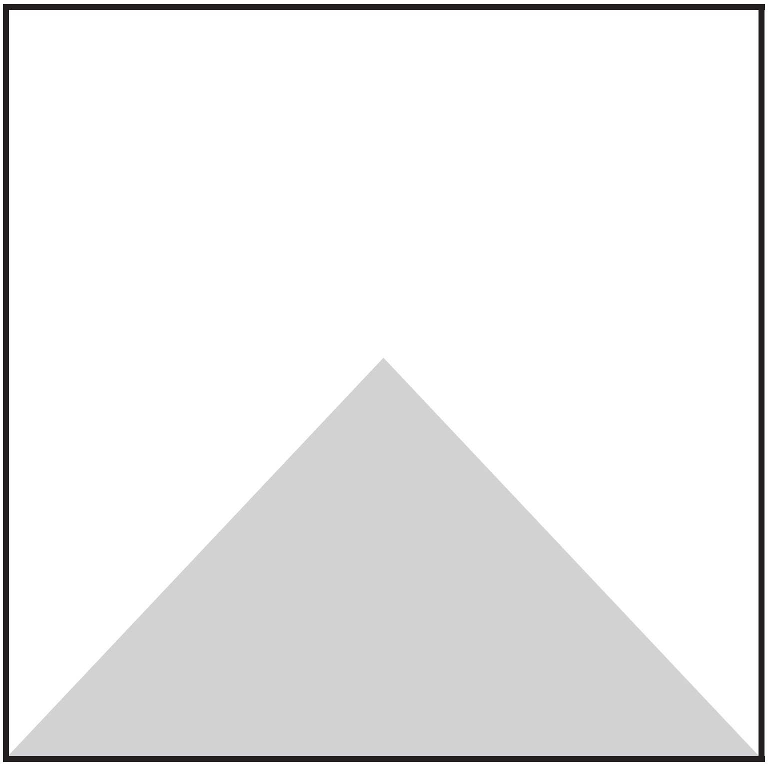
Tangram Square



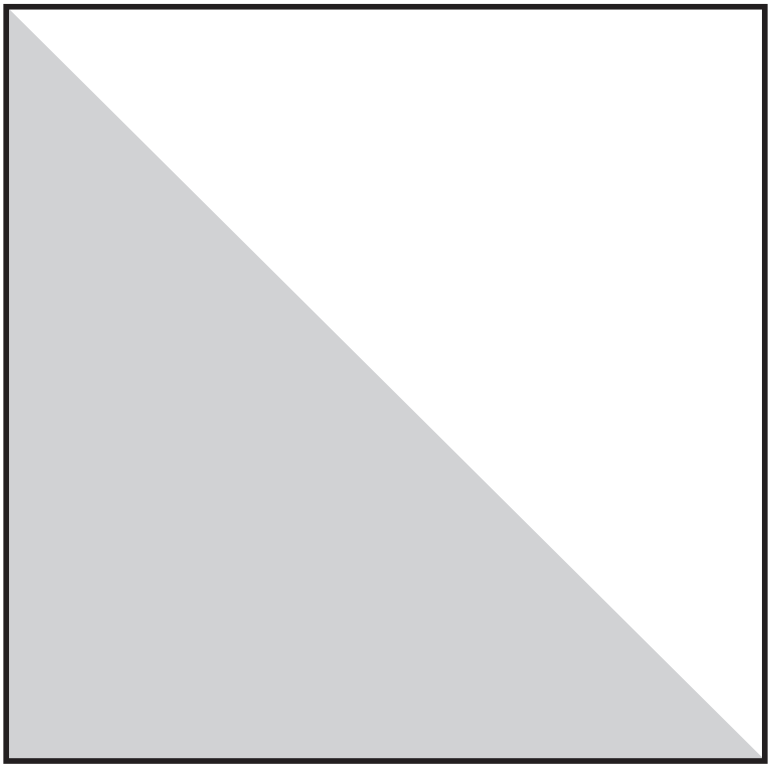
Tangram Fractions



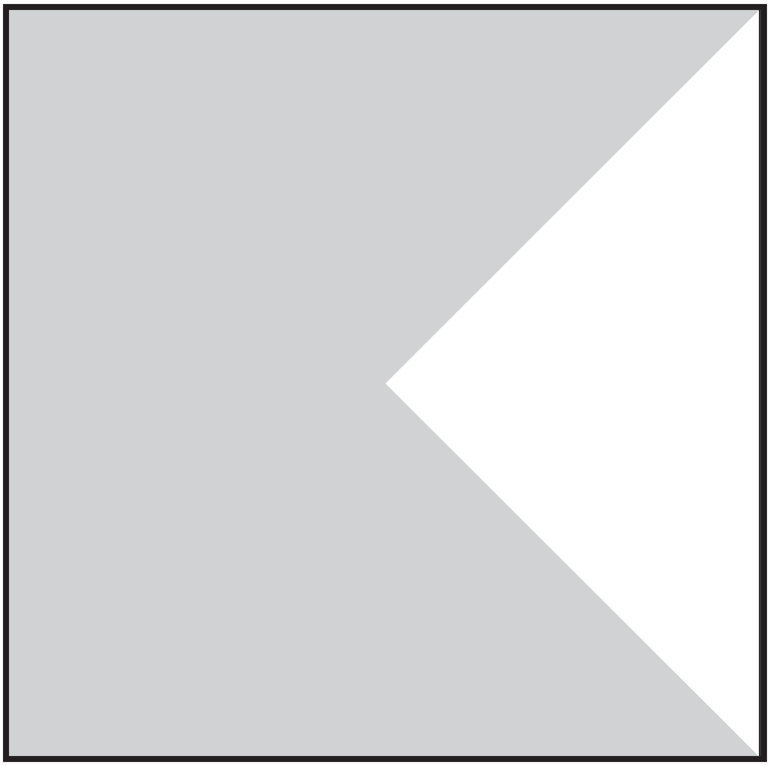
1/4 Tangram Square



1/2 Tangram Square



3/4 Tangram Square



Finding Equivalent Fractions

Dear Parent/Guardian:

We are learning about equivalent fractions. Fractions are equivalent if they represent the same amount. For example, in the following diagram, 6 of the 12 fruits are apples. We could say that 6/12 or 1/2 of the fruits are apples; 6/12 and 1/2 are equivalent fractions.

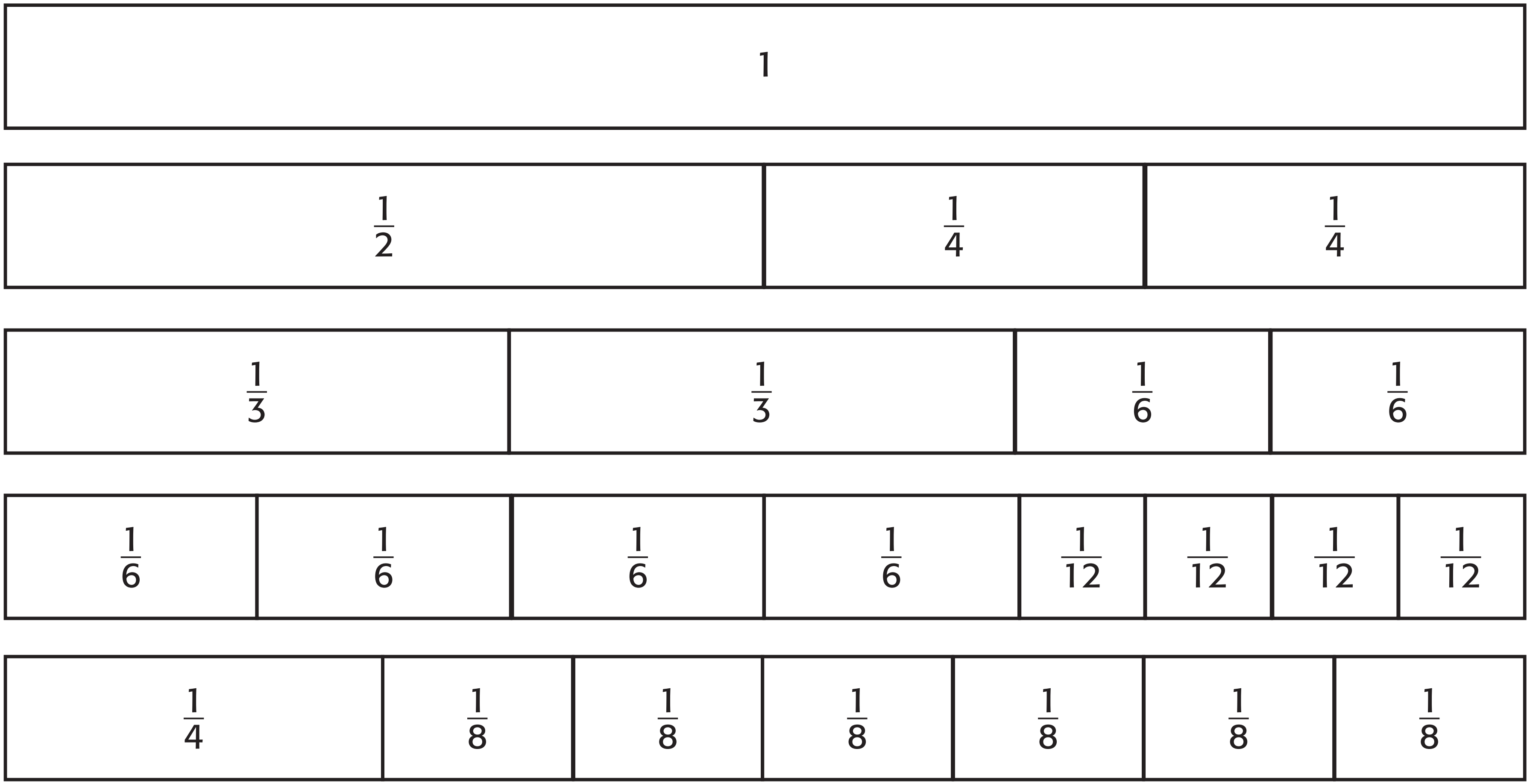
Help your child find examples of equivalent fractions in your home. For instance, you might use a dozen eggs to show that 8/12 of the eggs is the same as 2/3 and 4/6 of the eggs.

Ask your child to draw a diagram that shows an example of equivalent fractions that you found in your home. Have your child label the equivalent fractions in the diagram, and ask him or her to explain why the fractions are equivalent.

In class, students will share their diagrams with their classmates.

Thank you for doing this activity with your child.

Fraction Strips



Grade 6 Learning Activity Fraction Line-Up

OVERVIEW

In the following learning activity, students create proper fractions and improper fractions using numerals obtained by rolling a pair of number cubes. They compare and order the fractions by placing them on a number line.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: Students explore fractional quantity by ordering proper fractions and improper fractions on a number line.

Relationships: Students compare and order proper fractions and improper fractions, and relate them to whole numbers.

Representation: Modelling fractions concretely and pictorially and locating fractions on a number line help students to understand the quantity represented by fraction symbols.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, drawings, number lines, calculators) and using standard fractional notation;
- determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents (e.g., use a 10 × 10 grid to show that 1/4 = 0.25 or 25%).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers.

ABOUT THE LEARNING ACTIVITY

TIME:
60 minutes

MATERIALS

- sheets of paper (1 per group of 3 or 4 students)
- overhead transparency of **Fra6.BLM1: 0–6 Number Line**
- overhead transparency marker
- number cubes (2 per group of 3 or 4 students)
- number lines made with **Fra6.BLM2: Number-Line Strip** (1 per group of 3 or 4 students)
- small pieces of paper – approximately 4 cm × 6 cm (several per group of 3 or 4 students)
- fraction models (e.g., fraction circles)
- **Fra6.BLM3: Fraction Number Line** (1 per group of 3 or 4 students)
- large sheets of paper (2 per pair of students)
- **Fra6.BLM4: Fraction Circles** (a few copies)
- **Fra6.BLM5: Ask Me About Fractions** (1 per student)

MATH LANGUAGE

- fractional names (e.g., halves, thirds, fourths, . . .)
- fractional part
- proper fraction
- improper fraction
- compare
- order
- equal/equivalent

INSTRUCTIONAL GROUPING:
groups of
3 or 4

INSTRUCTIONAL SEQUENCING

Before Grade 6, students represent, compare, and order proper and improper fractions with like denominators. The following learning activity allows students to explore quantities associated with proper and improper fractions, and to order them on a number line.

ABOUT THE MATH

In Grade 6, students are expected to represent, compare, and order proper and improper fractions. They need to understand the meaning of these two kinds of fractions, not merely learn abstract rules (e.g., in proper fractions, the numerator is less than the denominator; in improper fractions, the numerator is greater than the denominator) that contribute little to students' development of fraction sense. In particular, students should be able to:

- compare the fractional part of a proper or an improper fraction with the whole (e.g., in both 2/3 and 4/3, three thirds make a whole);
- recognize whether a fraction is less than, equal to, or greater than 1 (e.g., 2/3 is less than 1; 3/3 is equal to 1; 4/3 is greater than 1);
- express improper fractions as mixed numbers (e.g., 4/3 = 1 1/3).

In the following learning activity, students need to consider the size of proper and improper fractions in order to locate their approximate positions on a number line. The learning activity requires students to think about the proximity of fractions to the whole numbers (0 to 6) that are given on the number line. The task of locating fractions on a number line helps to develop students' understanding of fractional quantities, relationships between fractions, and relationships between whole numbers and fractions.

GETTING STARTED

Relate the following anecdote to the class:

> Owen and Teresa were playing a game with two regular number cubes. Each player took a turn rolling the number cubes, one at a time. The first number rolled was the number of tens, and the second number rolled was the number of ones. At the end of each round, the player who rolled the greatest number won a point. Teresa and Owen always hoped to roll 66 because they knew that was the greatest possible number they could obtain.
>
> After a while, Owen proposed a new game. "What if we played the game and created fractions with the numbers we roll? The number on the first number cube would be the numerator, and the number on the second number cube would be the denominator."
>
> "Sounds like a good game," said Teresa. "So, I wonder . . . what is the greatest possible fraction we could get?"
>
> Owen responded, "Well, I know that three fourths is a big fraction, so maybe we would have to roll a 3 and a 4."
>
> Teresa wasn't sure. "But what if we rolled a 6 both times? Wouldn't six sixths be the greatest possible fraction?"

Pose the problem: "What is the greatest possible fraction you could create if you rolled two number cubes?"

Divide the class into groups of three or four. Provide each group with a sheet of paper and pencils. Ask students to discuss the problem and to record a solution. Encourage students to use diagrams in their solutions to help to explain their thinking.

After students have had an opportunity to discuss the problem and to record their solutions, gather the class and invite some groups to explain their thinking. As students present their solutions, assess how well they explain fractional quantities.

Some students may suggest that the best possible roll would be a 6 and a 1, giving a fraction of 6/1. Record "6/1" on the board, and ask:
- "What is the denominator in this fraction?" (1)
- "What does 1, as a denominator, represent?" (the whole)
- "What does 6/1 mean?" (6 wholes)
- "What does 6/1 look like using fraction circles?" (six whole circles)

On the board, draw 6 circles. Connect the diagram to the fraction symbol by reinforcing the ideas that the denominator represents 1 whole and that the numerator (6) tells the number of wholes.

Invite students to determine whether 6/1 is the greatest possible fraction that can be created using two number cubes. If students propose other possibilities, encourage them to use diagrams or manipulatives (e.g., fraction circles) to support their argument.

Display an overhead transparency of **Fra6.BLM1: 0–6 Number Line**, and emphasize the idea that the number line contains the whole numbers from 0 to 6. Explain to students that the number line can be used to order fractions. As an example, have students indicate the position of 1/2. If students suggest that 1/2 is located halfway along the line (i.e., where 3 is located), ask them to consider whether 1/2 and 3 are equivalent numbers. Emphasize that 1/2 is midway between 0 and 1, and record "1/2" on the number line using an overhead marker.

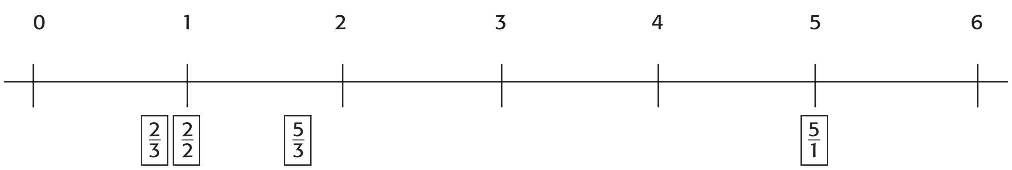
Next, have students consider where 3/4 (Owen's fraction) would be located. Again, emphasize that the location of the fraction is determined by considering its proximity to the whole numbers on the line – not by considering the complete line as one whole. After students determine that 3/4 is located midway between 1/2 and 1, record "3/4" on the number line.

Continue by having students determine where 6/6 (Teresa's fraction) and 6/1 would be located, and record these fractions on the number line.

WORKING ON IT

Explain the following activity:

- Students work in groups of three or four. Each group requires two number cubes and a number line made from **Fra6.BLM2: Number-Line Strip**.
- Each student in the group takes a turn rolling the number cubes, one at a time. The number on the first cube indicates the numerator of a fraction; the second cube indicates the denominator. For example, if the first cube shows 5 and the second cube shows 4, the fraction is 5/4. The student creates a fraction card by recording the fraction on a small piece of paper.
- After all students in the group have each created and recorded a fraction, they decide, collaboratively, where each fraction card should be placed on the number line. Students need to consider the proximity of each fraction to the whole numbers on the number line and to each other. Students should attempt to place the fraction cards, as accurately as possible, in their positions on the number line. For example, if students create fraction cards with 5/3, 2/3, 5/1, and 2/2, they place the cards in the following positions:

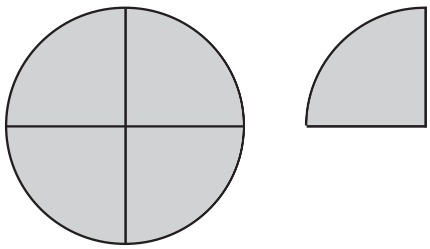


- Students may use fraction models (e.g., fraction circles, diagrams), their knowledge of equivalent fractions and whole numbers, and reasoning to help them place fraction cards in their correct positions on the number line.

STRATEGIES STUDENTS MIGHT USE

USING FRACTION MODELS

Students might use manipulatives (e.g., fraction circles) and diagrams to represent fractions and to determine their proximity to whole numbers. For each fraction, students need to consider the size of the fractional part (the denominator) and the number of parts (the numerator). For example, 5/4 could be represented using a diagram or fraction circles. Students could use the representation to determine that 5/4 is 1/4 more than 1 and, accordingly, locate 5/4 on the number line.



USING KNOWLEDGE OF FRACTION-WHOLE NUMBER EQUIVALENCIES

Students learn to recognize number patterns that help them identify equivalent numbers:

- 1/1, 2/2, 3/3, . . ., all equal 1.
- 1/1, 2/1, 3/1, 4/1 are equivalent to 1, 2, 3, and 4, respectively
- 2/1, 4/2, 6/3 all equal 2.

Understanding these patterns helps students locate fractions on the number line.

USING REASONING

Students might use their knowledge about the number-line position of some fractions to help them reason about the position of other fractions. For example, they might know that 4/2 equals 2. Using this knowledge, students can reason that 5/2 is equal to 2 1/2.

After students have placed the fraction cards on the number line, they may invite another group to check their work, or they can use **Fra6.BLM3: Fraction Number Line** to verify that fraction cards have been placed in their correct positions.

As students do the activity, ask them questions to help them to think about the methods they use to place the fractions on the number line:

- “What fraction did you create using the numbers on the number cubes?”
- “Where would this fraction be located on the number line?”
- “How do you know that this is the location of this fraction?”
- “How do you know that this fraction is greater than 1?”

Have students repeat the activity a few times. The last time they do the activity, ask students to print the fractions directly on the number line.

REFLECTING AND CONNECTING

Bring the class together for a discussion about the activity. Ask some general questions:

- "Which fractions were easy to locate on the number line? Why?"
- "Which fractions were difficult to locate? What was difficult about deciding where the fractions would be located?"
- "What strategies did you use to determine the location of fractions on the number line?"

Provide an opportunity for a few groups to show number lines on which they recorded the fractions. Ask students to explain the strategies they used to determine the position of each fraction on the number line.

Help students to make some generalizations about fractions by asking the following questions:

- "Which fractions are equal to 1?" (1/1, 2/2, 3/3, . . .)
- "Why are these fractions equal to 1?" (There are enough fractional parts to make the whole – for example, 2 halves make 1.)
- "What do you observe about the numerator and the denominator in these fractions?" (They are the same number.)
- "What other fractions would be equal to 1?"

Continue the discussion by asking questions about fractions equivalent to 2 and 3:

- "Which fractions are equal to 2?" (2/1, 4/2, 6/3, . . .)
- "Why are these fractions equal to 2?" (There are enough fractional parts to make 2 wholes – for example, 4 halves make 2.)
- "What do you observe about the numerator and the denominator in these fractions?" (The numerator is double the denominator.)
- "What other fractions would be equal to 2?"
- "Why are 3/1 and 6/2 equal to 3?"
- "What other fractions would be equal to 3?"

Pose similar questions about fractions that are equal to 4 to reinforce the patterns students observe.

Discuss proper and improper fractions. Ask:

- "How can you tell, just by looking at a fraction, that it is greater than 1?" (The numerator is greater than the denominator.)
- "What term is used to name a fraction in which the numerator is greater than the denominator?" (improper fraction)
- "Why is an improper fraction greater than 1?" (There are more fractional parts than 1 whole – for example, in 4/3, there are 3 thirds, or 1 whole, and another 1/3.)
- "How can you tell, just by looking at a fraction, that it is less than 1?" (The numerator is less than the denominator.)
- "What term is used to name a fraction in which the numerator is less than the denominator?" (proper fraction)
- "Why is a proper fraction less than 1?" (There are not enough fractional parts to make a whole – for example, in 2/3, another third would be needed to make a whole.)

Provide pairs of students with two large sheets of paper. Instruct pairs to work together to create two posters, entitled "Proper Fractions" and "Improper Fractions", that explain the meaning of each type of fraction. Encourage students to use diagrams and words to clarify the terms.

Post the completed posters. Discuss and compare the ways in which students presented their ideas.

ADAPTATIONS/EXTENSIONS

Encourage students who have difficulty locating the position of fractions on the number line to use fraction models (e.g., fraction circles, diagrams, **Fra6.BLM4: Fraction Circles**) to help them think about the size of the fractions and their proximity to whole numbers. Some students may benefit from a version of the activity in which they consider the position of fewer fractions on a shorter number line. For this version of the activity, create number cubes with only the numbers 1, 2, 3 (each number printed twice on a cube) and have students work with a 0–3 number line.

Ask students who require a challenge to examine **Fra6.BLM3: Fraction Number Line** and to explain the fraction arrangements on the number line (e.g., the arrangement of fraction symbols in lower rows is more condensed than in higher rows). Students might determine that the fractional parts are increasingly smaller as they move from the top to the bottom row (e.g., fifths are smaller than fourths). Since the fractional parts in lower rows are smaller, more of them are required to make a whole.

Students could also use **Fra6.BLM3: Fraction Number Line** to find equivalent fractions (i.e., fractions that occupy the same position on the number line) and to extrapolate other equivalent fractions.

HOME CONNECTION

The letter on **Fra6.BLM5: Ask Me About Fractions** encourages parents/guardians to ask their child about the fraction concepts being learned in class. Before sending home the letter, conduct a think-pair-share activity to help students prepare for the discussion about fractions with their parents/guardians. Pose the following questions, one at a time, and provide time for students to think about their answers. Then ask students to share their ideas with a partner:

- "What are proper fractions?"
- "What are improper fractions?"
- "What are equivalent fractions?"
- "Which fractions are equal to 1?"

LEARNING CONNECTION 1
Fractions Between Fractions

MATERIALS

- a variety of manipulatives for representing fractions (e.g., fraction circles, Cuisenaire rods, counters, square tiles)

Arrange students in pairs. Challenge students to identify fractions that are between 1/2 and 3/4. Encourage students to use manipulatives and drawings.

As a whole class, discuss the fractions that were found, and ask students to explain how they know that the fractions are between 1/2 and 3/4.

Repeat by having students identify fractions that are between:
- 1/4 and 1/2;
- 1/8 and 1/2;
- 1/3 and 7/8.

LEARNING CONNECTION 2
From Least to Greatest

MATERIALS
- sheets of paper (1 per pair of students)

Arrange students in pairs. Challenge pairs to record all possible fractions (proper and improper) using only 6, 7, 8, and 9 as numerators and denominators. Next, have students arrange the fractions from least to greatest.

Combine pairs of students to form groups of four. Ask the groups to compare their ordered lists and to explain the strategies they used to order the fractions.

LEARNING CONNECTION 3
What's the Whole?

MATERIALS
- pattern blocks (several per pair of students)
- overhead transparency of **Fra6.BLM6: Finding the Part/Finding the Whole**
- overhead projector

Provide opportunities for students to reflect on the relationships between fractional parts and the whole. Give each pair of students several pattern blocks. Display an overhead transparency of **Fra6.BLM6: Finding the Part/Finding the Whole**, and instruct students to work with their partners to solve the two problems.

After they have solved the problems, ask a few students to share their solutions with the class and to explain their thinking.

LEARNING CONNECTION 4
Fractions in a Venn Diagram

MATERIALS

- **Fra6.BLM7: Organizing Fractions in a Venn Diagram** (1 per pair of students)
- a variety of manipulatives for representing fractions (e.g., fraction circles, Cuisenaire rods, counters, square tiles)

Provide each pair of students with a copy of **Fra6.BLM7: Organizing Fractions in a Venn Diagram**. Instruct students to make a list of all proper fractions that have a denominator of 2, 3, 4, and 5. Have them record the fractions in the appropriate sections of the Venn diagram. Encourage students to use manipulatives to model fractions, if necessary.

Have students explain how they identified the fractions for each section of the Venn diagram.

LEARNING CONNECTION 5
Whose Fraction Is Greater?

MATERIALS

- number cubes (1 number cube per pair of students)
- sheets of paper (1 per student)
- a variety of manipulatives for representing fractions (e.g., fraction circles, fraction rectangles, two-colour counters)

Arrange students in pairs. Have students prepare a game sheet by drawing the following structure on their paper:

Explain the game:

- The goal of the game is to create a fraction that is greater than the fraction created by the other player.
- Players take turns rolling a number cube and recording the number shown on the number cube in one of the boxes on their game sheet.
- Players may use the number from the roll of the number cube to create the numerator or denominator of a fraction, or they may record it in one of the reject boxes.
- After players have filled all the boxes on their game sheet, they compare their fractions to determine which player created the greater fraction.

Have a variety of manipulatives (e.g., fraction circles, fraction rectangles, two-colour counters) available, and encourage students to use them to compare the fractions that they created.

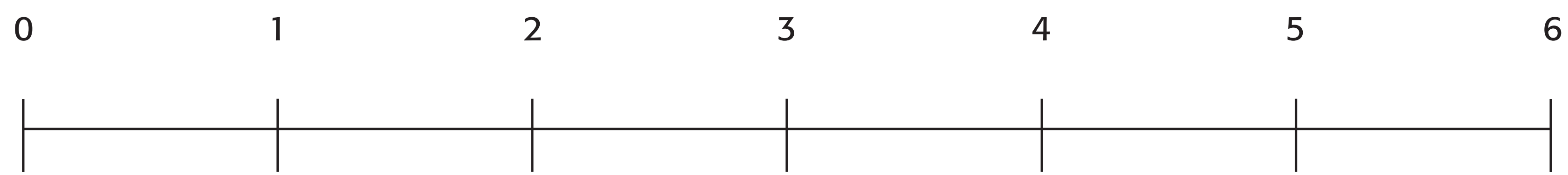
After students have played the game a few times, discuss strategies that they used to create the greatest fraction possible.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on fraction concepts. On the home, click "Toolkit". In the "Numeracy" section, find "Fractions (4 to 6)", and then click the number to the right of it.

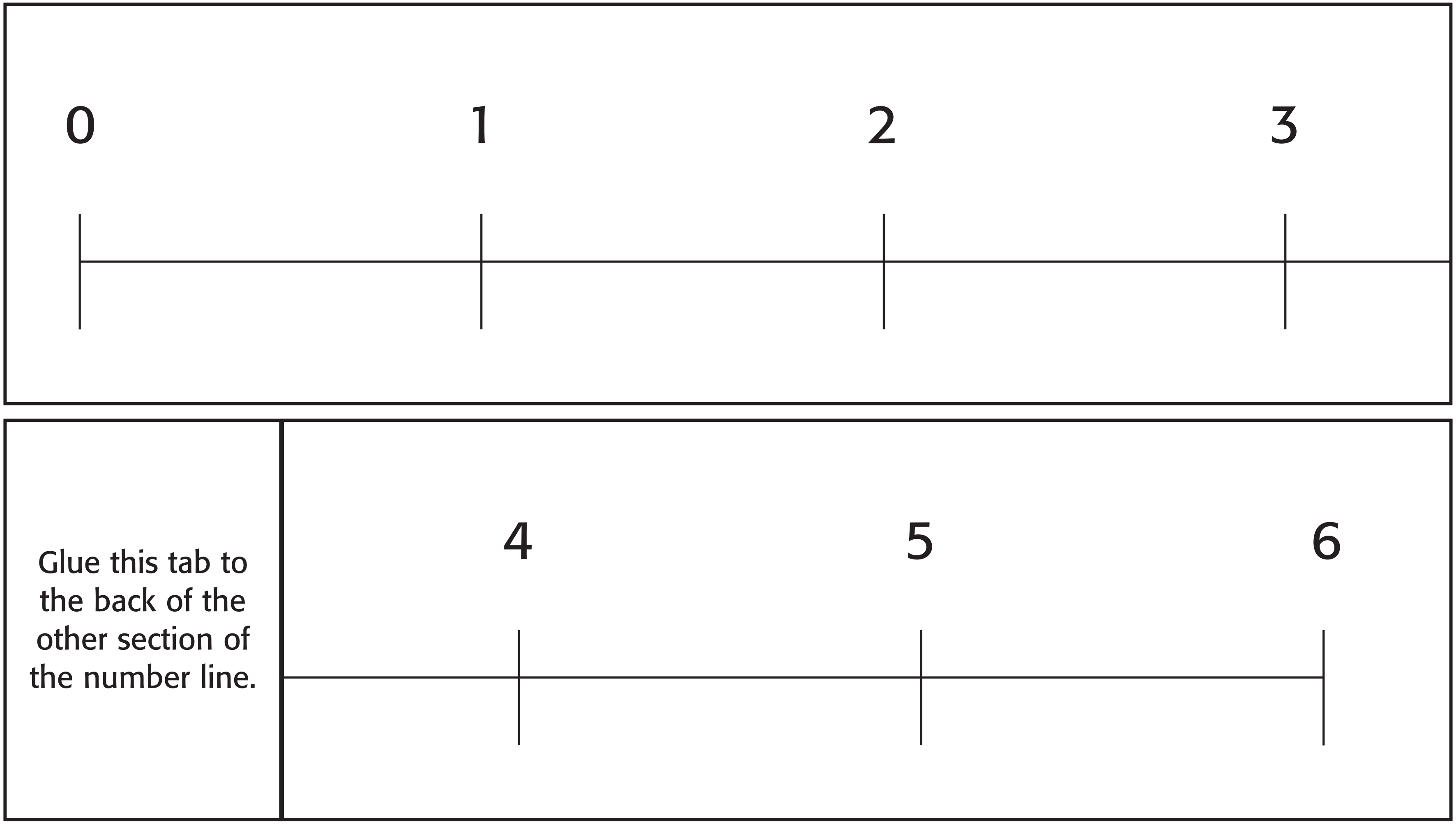
eworkshop.on.ca

0–6 Number Line

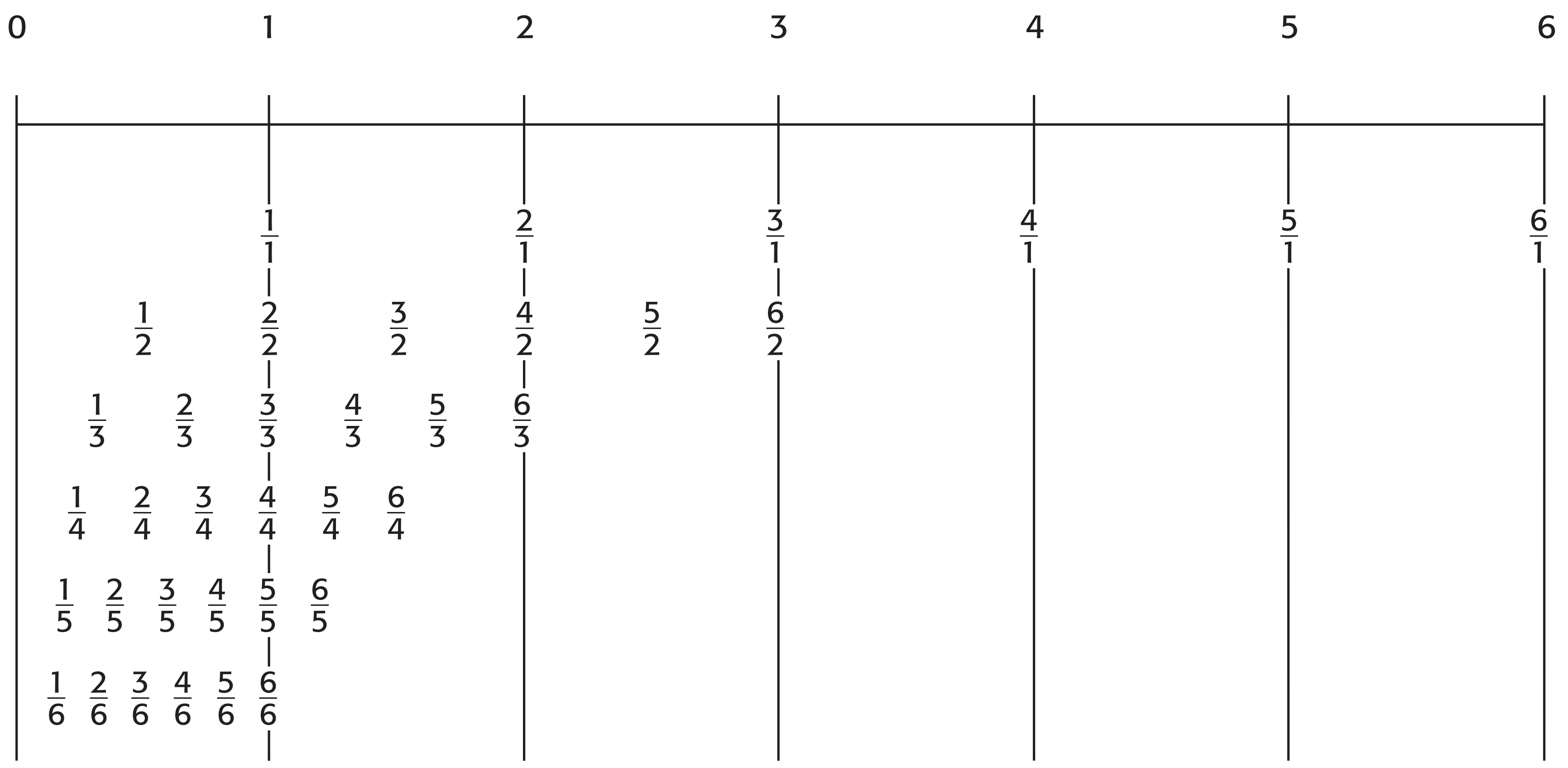


Number-Line Strip

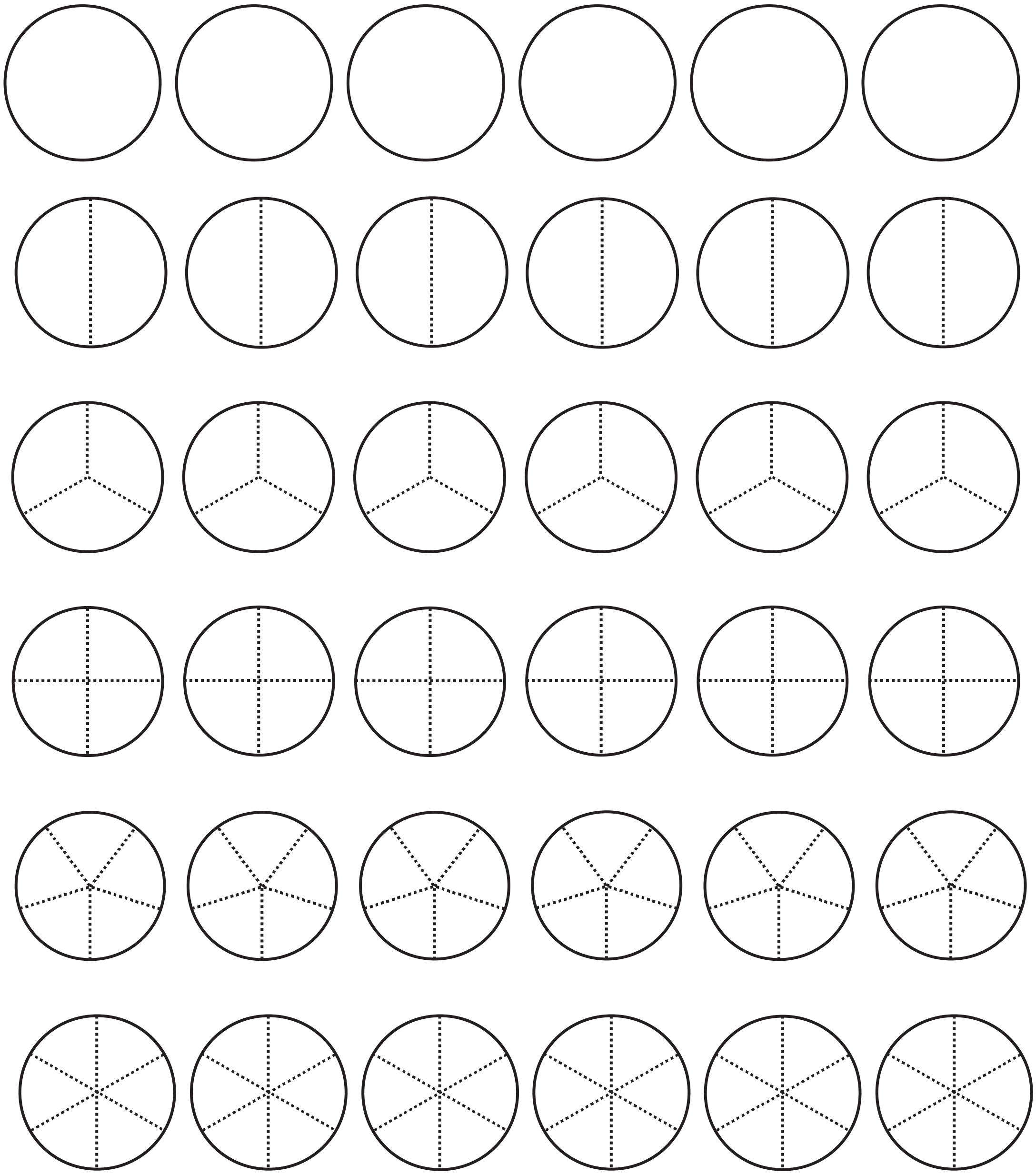
Make a 1 to 6 number line. Cut out both sections. Glue the tab on the second section to the back of the first section.



Fraction Number Line



Fraction Circles



Ask Me About Fractions

Dear ________________,

I have been learning about fractions. I can tell you what I know about fractions, so ask me about:

- proper fractions;
- improper fractions;
- equivalent fractions;
- fractions that are equal to 1.

Thanks for letting me tell you what I know about fractions.

Sincerely,

____

Note to parent/guardian: The discussion about fractions provides an opportunity for your child to review what he or she has learned in math class. During the discussion, ask questions such as the following:

- How did you learn that idea in class?
- Can you draw a diagram that shows that idea?
- Is that idea easy or difficult for you to understand? Why?

Please sign this sheet if you were able to have a discussion about fractions with your child, and have him or her return it to class.

Thank you for discussing fractions with your child.

____

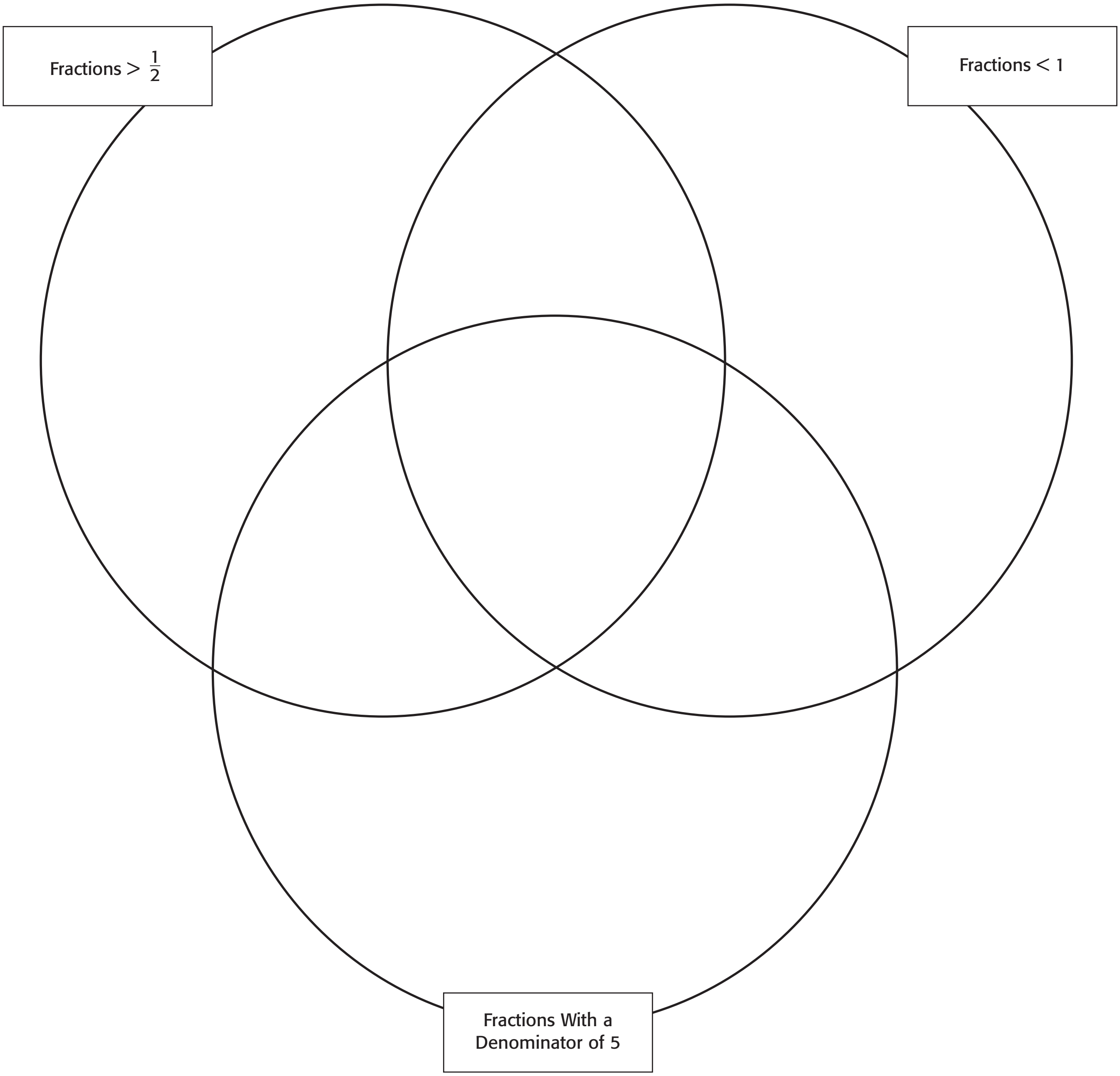
Signature of parent/guardian

Finding the Part/Finding the Whole

1. If the yellow hexagon pattern block is 1 whole, find:
   a) one half.
   b) four sixths.
   c) seven sixths.

2. What would the whole look like if the red trapezoid is:
   a) one third?
   b) three fourths?
   c) three halves?

Organizing Fractions in a Venn Diagram



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