# Number Sense \& Numeration Unit 2 

Ratios, Rates and Fractions

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2012-13


## Fractions to Decimals to Percentages

So, we are already aware of the relationship between numerators and denominators, but let's refresh to be sure, ok?

gets divided by the bottom number a division problem, which will give us a

Therefore, if we take $3 / 4=3 \div 4$, then we get 0.75 as an equivalent, or equal, value. That allows us to create a percentage by taking the decimal and multiplying it by 100 . So, if we take $0.75 \times 100=75 \%$.

What changes now, is that we must begin to recognize equivalent fractions as also be equal to the same percentage. Sounds tricky, but really, it's not that hard.

Let's take $6 / 8$ as our fraction. If we apply the same rules as above, we can see if it is an equivalent fraction to $3 / 4$.

| Original Fraction | Operational View | Decimal | Percentage |
| :--- | :--- | :--- | :--- |
| $3 / 4$ | $3 \div 4$ | 0.75 | $75 \%$ |
| $6 / 8$ | $6 \div 8$ | 0.75 | $75 \%$ |

The trick is to do the fraction as an operation, in this case, division and see if the decimal is the same. If it is, then you have yourself a pair of equivalent fractions, or a matched set. Complete the chart below to see if you can identify the equivalent fractions. Put a star next to all the equivalent fractions.

| Original <br> Fraction | Operational <br> View | Decimal | Percentage |
| :--- | :--- | :--- | :--- |
| $2 / 3$ |  |  |  |
| $16 / 24$ |  |  |  |
| $5 / 8$ |  |  |  |
| $9 / 21$ |  |  |  |
| $3 / 5$ |  |  |  |
| $1600 / 2400$ |  |  |  |
| $7 / 10$ |  |  |  |

$\qquad$

## Lesson 1 Proportional Relationship:

a) The price of oranges at store can be determined by the equation:
$P=\$ 0.35 n$, where $P$ is the price and $n$ is the number of pounds of oranges.
What is the constant of proportionality (unit rate)?


## Explanation:

Step 1) Proportional quantities have a constant ratio and can be described by the equation $y=k x$, where $k$ is a constant ratio. The constant of proportionality is the coefficient of $x$ or the independent variable.

Step 2) We have the constant of proportionality as
$p=.35 n$
We can conclude that . 35 is the coefficient of variable ' $n$ '

## Ratio Worksheet

Compare the following ratios: (Use >, < or = sign in the place holders) Work Space

2:3 $\quad$ 4:1

1:2 $\square$ 3:6

3:5 $\square$ 2:4

6:7 $\square 4: 8$

5:1 7:9

4:6 7:6
$1: 8 \square 4: 11$
$5: 2 \square 10: 4$
$4: 3 \square 2: 4$

7:2 $\square$ 10:3

## Write the Fractions as Ratio

Example:

| $\frac{4}{5}=4: 5$ |
| :--- |
| $\frac{3}{4}=\square$ |
| $\frac{5}{9}=\square$ |
| $\frac{11}{15}=\square$ |
| $\frac{7}{12}=\square$ |
| $\frac{7}{10}=\square$ |
| $\frac{8}{5}=$ |
| $\frac{9}{11}=$ |
| $\frac{1}{3}=$ |
| $\frac{2}{5}=$ |
| $\frac{4}{7}=$ |

Equivalent Ratios

1) $1: 3=2: \__{-}=\ldots: 9=\ldots: 12=\ldots: 15=6:$
2) $1: 6=\ldots: 12=\ldots: 18=\ldots: 24=5:]_{=}: 36$
3) $5: 6=10: Z_{=} 15: Z_{=} 20: Z_{=}: 30=\ldots: 36$

4) $3: 4=\ldots: 8=9: \ldots=12: \ldots=\ldots: 20=\ldots: 24$
5) $\left.2: 5=4: Z_{=} 6: \neq{ }_{C}: 20=10:\right]_{=}: 30$
6) $\left.1: 2=2: \_=\ldots: 6=\ldots: 8=5:\right]_{=}: 12$
7) $8: 9=\ldots: 18=24:]_{=}: 36=\ldots: 45=\ldots: 54$
8) $3: 5=6: Z_{-}=9: Z_{=} 12: \chi_{工}: 25=18:$
9) $5: 7=\ldots: 14=15: \ldots=\ldots: 28=25: \ldots=\ldots: 42$
Take your time and think it through. Good Luck
$\qquad$

Recognizing Proportional Relationships - Step-by-Step Lesson

The table below gives the shipping price for different numbers of soccer balls. Do the numbers in the table represent a proportional relationship?


| Number of Balls | Price (In Dollars) |
| :---: | :---: |
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 7 |

## Explanation:

Step 1) We could do this in two ways:
a) by testing for an equivalent ratios in the table
b) by graphing the table to see if it creates a straight line through the origin.

Step 2 a) we can examine the numbers to determine that the price is the number of balls multiplied by 2 , except for 4 balls. The row with 4 balls costs $\$ 7$. This is not proportional to the other amounts in the table; therefore, the table does not represent a proportional relationship.

Step 2 b) we can graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of balls, price), the pairs (1, 2), $(2,4)$, and $(3,6)$ will form a straight line through the origin ( 0 balls, 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair $(3,6)$ means that 3 balls cost $\$ 6$. However, the ordered pair $(4,7)$ would not be on the line, indicating that it is not proportional to the other pairs.

## Ratio Worksheet

Create proportion from the given set of numbers:
Example: 1, 6, 2, 3
Proportion is 1:2=3:6
$3,6,7,14$
Proportion is $\qquad$
$2,6,5,15$
Proportion is $\qquad$
$7,2,14,4$
Proportion is $\qquad$
$2,2,1,4$
Proportion is $\qquad$
$10,5,8,4$
Proportion is $\qquad$
$3,12,8,2$
Proportion is $\qquad$

## Proportion Worksheet

State whether the following ratios are proportional: Work Space

2:3 and 4:6

1:4 and 6:18

3:5 and 1:3

2:5 and 8:20

4:9 and 12:27
$\qquad$

6:15 and 2:5

8 apples to 4 apples
$\qquad$

3 chocolates to 9 chocolates

6 ml to 9 ml

70 m to 100 m
$\qquad$

16 gallons to 28 gallons
$\qquad$

26 yards to 13 yards
$\qquad$

## Write the Ratios as Fraction

Example:
$2: 3=\frac{2}{3}$

5:7 = $\qquad$

9:10 = $\qquad$
$4: 9=$ $\qquad$

15:13 = $\qquad$

6:11 = $\qquad$

1:5 = $\qquad$

3:8 = $\qquad$

2:9 = $\qquad$
$\qquad$

Unit Rates and Ratios of Fractions - Step-by-Step Lesson

## Lesson 1 Fractions Problem:

If $1 / 3$ of a gallon of paint covers $1 / 9$ of a gate, then how many gallons of paint is needed to cover the entire gate?


## Explanation:

Divide the total amount of amount of paint given ( $1 / 3$ of a gallon) by the amount gate that was covered (1/9).
$\frac{1}{3} \div \frac{1}{9}=$
To complete a fractional division problem, we will write $1 / 9$ as an improper fraction. Turn this from a division problem into a multiplication problem by multiply by the reciprocal.

$$
\frac{1}{3} \div \frac{1}{9}=\frac{1}{3} \times \frac{9}{1}
$$

Now we will multiply:
$=\frac{1 \times 9}{3 \times 1}=\frac{9}{3}=3$
So the answer is 3 gallons of paint.
$\qquad$

## Multistep Ratio and Percent Word problems- Step-by-Step Lesson

1. Andrea has a recipe that needs $\frac{5}{4}$ teaspoon of cream for every 2 cups of milk. If Andrea increases the amount of milk to 8 cups of milk, how many teaspoons of cream are needed?

## Explanation \#1:

Step 1) First we need to know what is being asked of us.
"How many teaspoons of cream are needed and what is the projected
 cost of a gallon of gas for May 2013?

Step 2 a) We can use these numbers to find the unit rate.
For every 2 cups of milk, teaspoon of cream required $=\frac{5}{4}$
For every 1 cup of milk, teaspoon of cream required $=\frac{5}{4} \div \frac{2}{1}$
For every 8 cup of milk, teaspoon of cream required $=\frac{5}{4} \times \frac{1}{2} \times 8$
So 5 teaspoons of cream are needed. $=5$
2. Gas prices are projected to increase by $115 \%$ by the end of next year. A gallon of gas currently costs $\$ 2.50$. What is the projected cost of a gallon of gas for May 2013?

## Explanation \#2:

The original cost of a gallon of gas is $\$ 2.50$. An increase of $100 \%$ means that the cost will double. Another $15 \%$ will need to be added to figure out the final projected cost of a gallon of gas.
$\$ 2.50+2.50+(0.15 \times 2.50)=5+(0.15 \times 2.50)=\$ 5.375$

| $100 \%$ | $100 \%$ | $15 \%$ |
| :---: | :---: | :---: |
| $\$ 2.50$ | $\$ 2.50$ | $\$ 0.375$ |

So the projected cost of a gallon of gas for the end of next year is $\$ 5.375$

## Rates and Ratios 1

1. The ratio of goldfish to gallons of water is 7 to 2 .
a. For every $\qquad$ goldfish, there are $\qquad$ gallons of water.
b. How many goldfish can you get if you have 16 gallons of water?
c. How many gallons of water do you need to keep 28 fish?
2. The price of apples is 3 for $\$ 2.00$.
a. For every $\qquad$ apples, you pay $\qquad$ .
b. How much do you pay for 30 apples?
c. How many apples can you buy for $\$ 10$ ?
3. You are taking a test in science. There are 20 questions and 45 minutes.
a. For every $\qquad$ questions, you have $\qquad$ minutes.
b. How much time do you have for 10 questions?
c. How much time do you have for 5 questions?
d. How much time do you have for 1 question?
4. All of the following rectangle's width to length ratio is 2 to 3 . Fill in the missing lengths. (Hint: The width is the shorter side; the length is the longer side.)
a. For every $\qquad$ inches in the width, there are $\qquad$ inches in the length.


2 inches


4 inches


6 inches


8 inches

## Rates and Ratios 2

1. The ratio of goldfish to gallons of water is 9 to 4 .
a. For every $\qquad$ goldfish, there are $\qquad$ gallons of water.
b. Fill in the table.

| Fish | Gallons of Water |
| :---: | :---: |
| 9 | 4 |
| 18 |  |
|  |  |
|  |  |
|  |  |
|  |  |

c. How many goldfish can you get if you have 12 gallons of water?
d. How many gallons of water do you need to keep 36 fish?
2. The price of oranges is 4 for $\$ 3.00$.
a. For every $\qquad$ oranges, you pay $\qquad$ .
b. Fill in the table.

| Oranges | $\$$ |
| :---: | :---: |
| 4 | 3 |
| 8 |  |
|  |  |
|  |  |
|  |  |
|  |  |

c. How much do you pay for 24 oranges?
d. How many apples can you buy for $\$ 15$ ?
3. You are taking a test in science. For every 3 questions, you have 5 minutes.
a. Fill in the table.

| Questions | Time (minutes) |
| :---: | :---: |
| 3 | 5 |
| 6 |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

b. If you have 30 questions, how much time do you have to take the test?
c. If you have 21 questions, how much time do you have to take the test?
d. If the test is 20 minutes long, how many questions do you have?
e. If the test is 30 minutes long, how many questions do you have?
4. You are taking a test for your math class. You have 10 questions and 10 minutes, a. How much time do you have for 1 question?
b. How much time do you have for 2 questions?
c. How much time do you have for 7 questions?
5. You are taking a test for your English class. You have 20 minutes and 10 questions. a. Fill in the table.

| Questions | Time (minutes) |
| :---: | :---: |
| 10 | 20 |
| 9 |  |
| 8 |  |
| 7 |  |
| 6 |  |
| 5 |  |
| 4 |  |
| 3 |  |
| 2 |  |
| 1 |  |

b. How much time do you have for 10 questions?
c. How much time do you have for 5 questions?
d. How much time do you have for 2 questions?
e. How much time do you have for 1 question?
6. 15 cars enter the mall parking lot every 5 minutes.
a. How many cars enter the parking lot in 10 minutes?
b. How many cars enter the parking lot in 25 minutes?
c. Circle the correct answer. What is the fast way to get the answers?

Multiply
Divide
7. 15 cars enter the mall parking lot every 5 minutes.
a. How many cars enter the parking lot in 1 minute?
b. How many cars enter the parking lot in 2 minutes?
c. How many cars enter the parking lot in 3 minutes?
d. How many cars enter the parking lot in 4 minutes?
e. Circle the correct answer. What is the fast way to get the answers?

Multiply Divide

## Rates and Ratios 3

1. For every 5 yards you run, Jill runs 6 yards.
a. If you have run 30 yards, then how far has Jill run?
b. If Jill has run 54 yards, then how far have you run?
2. 570 people die from smoking related diseases every day. (www.thetruth.com)
a. How many people die from smoking related diseases every hour?
b. How many people die from smoking related diseases every week?
c. How many people die from smoking related diseases every year?
(Hint: There are 365 days in a year.)
3. You type 200 words in 5 minutes. Brenda types 240 words in 6 minutes.
a. How many words you type in 1 minute?
b. How many words does Brenda type in a minute?
c. Who types the fastest?
4. The width to length ratio of the rectangles is 3 to 5 . Fill in the missing lengths. (Hint: The width is the shorter side; the length is the longer side.)


6 inches


30 inches

## Percent Equations

In this next section, you are to use the following equation to help you set up and solve the missing equations.
"a is \% (p) of b"
$\frac{a}{b}=\frac{p}{100}$
For problems 1-3, SET UP the problems using the percent equation. Please do not solve.

1) 20 is what \% of 140 ?
2) What is $30 \%$ of 60 ? 3 ) 40 is $10 \%$ of what?

For problems 4-12, set up the problems using the percent equation, and then solve.
4) 34 is $50 \%$ of what? 5$) 5$ is $20 \%$ of what?
6) What is $15 \%$ of 90 ?
7) What is $25 \%$ of 120 ?
8) 150 is $300 \%$ of what?
9) 30 is $5 \%$ of what?
10) What is $12 \%$ of 130 ?
11) 40 is what $\%$ of 120 ?
12) 12 is $15 \%$ of what?

## Taxes, Tips, and Sales

1) A store has a sale for $25 \%$ off. You decide to buy a sweater that usually costs $\$ 30$.
a) How much money will you save?
b) How much money will you spend on the sweater?
2) A store has a sale for $15 \%$ off. You decide to buy a $\$ 45$ jacket.
a) How much money will you save?
b) How much money will you spend on the sweater?
3) A restaurant requires customers to pay a $15 \%$ tip for the server. Your family spends $\$ 65$ on the meal.
a) How much money do you need to pay the server?
b) How much money will you spend total?
4) Sales tax is $6 \%$ and you spend $\$ 42$.
a) How much money will you spend on tax?
b) How much money will you spend total?
5) Sales tax is 6.5 \% and you spend $\$ 58.50$.
a) How much money will you spend on tax?
b) How much money will you spend total?
6) A restaurant requires customers to pay a $15 \%$ tip for the server. Your family spends $\$ 45$ on the meal.
a) How much money do you need to pay the server?
b) How much money will you spend total?
7) A store has a sale for $20 \%$ off. You decide to buy a $\$ 15$ t-shirt.
a) How much money will you save?
b) How much money will you spend on the t-shirt?
8) A store has a sale for $10 \%$ off. You decide to buy a $\$ 17$ DVD.
a) How much money will you save?
b) How much money will you spend on the DVD?

## Ratio Worksheet

Fill in the blanks from the given statements:

There are $\mathbf{7}$ roses and $\mathbf{8}$ sunflowers in the garden.
Ratio of roses to sunflower is $\qquad$
Ratio of sunflowers to total flowers in the garden is $\qquad$
Ratio of roses to total flowers in the garden is $\qquad$

There are 19 girls and 14 boys in the bus.
Ratio of girls to boys is $\qquad$
Ratio of girls to total number of members in the bus is $\qquad$
Ratio of boys to total number of members in the bus is $\qquad$

There are $\mathbf{1 2}$ comic books and $\mathbf{5}$ novels in the shelf.
Ratio of novels to comic books is $\qquad$
Ratio of novels to total books in the shelf is $\qquad$
Ratio of comic books to total books in the shelf is $\qquad$
$\qquad$

## Simplify Ratio: Word Problems

| Questions |  |
| :--- | :--- |
| In a live concert, 425 boys and 300 girls <br> participated. Find the ratio of boys to girls. |  |
| Answer: |  |
| An egg seller finds 432 eggs are either spoiled |  |
| or broken and 1080 eggs are in good |  |
| condition. Find the ratio of bad eggs to good |  |
| eggs. |  |
| Answer: |  |
| Kathy scored 360 in first quarter and 420 in |  |
| second quarter. Find the ratio of scores in the |  |
| first quarter to the second quarter. |  |
| Answer: |  |
| Answer: |  |
| 320 grams Copper. Find the ratio of nickel to |  |
| copper. |  |

$\qquad$

## Simplify Ratio: Word Problems

| Questions |  |
| :--- | :--- |
| The ratio of boys to girls in a kindergarten is |  |
| 2:3. If the number of boys is 12, find the |  |
| number of girls. |  |
| Answer: |  |
| In a basket, there are 24 good oranges and |  |
| rest are bad oranges. The ratio of good |  |
| oranges to the bad oranges is 3: 1, find the |  |
| number of bad oranges. |  |
| Answer: |  |
| In a play school, there are 16 red balls and the |  |
| rest are white balls. The ratio of red balls to |  |
| white balls is 4:5. Find the number of white |  |
| balls. |  |
| Answer: |  |
| Answer: |  |
| The distance travelled by Ben and Tim is in the |  |
| ratio 3 : 4. The distance travelled by Ben is 18 |  |
| miles. Find the distance travelled by Tim. |  |

## Rates, Ratios and Fraction Workbook

Teacher Name: Mr. Leavitt
Student Name:

| CATEGORY | $\mathbf{4}$ | $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{1}$ |
| :--- | :--- | :--- | :--- | :--- |
| Mathematical Errors | 90-100\% of the steps <br> and solutions have no <br> mathematical errors. | Almost all (85-89\%) of <br> the steps and <br> solutions have no <br> mathematical errors. | Most (75-84\%) of the <br> steps and solutions <br> have no mathematical <br> errors. | More than $75 \%$ of the <br> steps and solutions <br> have mathematical <br> errors. |
| Strategy/Procedures | Typically, uses an <br> efficient and effective <br> strategy to solve the <br> problem(s). | Typically, uses an <br> effective strategy to <br> solve the problem(s). | Sometimes uses an <br> effective strategy to <br> solve problems, but <br> does not do it <br> consistently. | Rarely uses an <br> effective strategy to <br> solve problems. |
| Completion | All problems are <br> completed. | All but one of the <br> problems are <br> completed. | All but two of the <br> problems are <br> completed. | Several of the <br> problems are not <br> completed. |
| Neatness and <br> Organization | The work is presented <br> in a neat, clear, <br> organized fashion that <br> is easy to read. | The work is presented <br> in a neat and <br> organized fashion that <br> is usually easy to read. | The work is presented <br> in an organized <br> fashion but may be <br> hard to read at times. | The work appears <br> sloppy and <br> unorganized. It is hard <br> to know what <br> information goes |

