NUMBER THEORY

Rational Numbers and Expressions

GRADES 11-12

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Executive Summary:

Effective number sense strategies are essential for students to acquire when building their understanding of fractions, fraction relationships, and mathematical processes with fractions. With an ever-increasing emphasis on student mastery of mathematical computation, reasoning, conceptual understanding, real-world problems, and connections, teachers must be flexible in their approach to teaching and adapt instructional approaches to maximize student growth in mathematical understanding. Knowing that students have varying background knowledge, readiness, interests, and preferences in learning, we plan to implement strategies that recognize and respond to this variety. Through the development of an understanding of equivalent fractions, the use of visual supports, manipulatives, cooperative learning, and real-world connections, our lessons will help students to be able to make sense of the standard algorithms they have used with fractions.

Minnesota State Mathematics Benchmarks Addressed:

6.1.1.6

Determine greatest common factors and least common multiples. Use common factors and common multiples to calculate with fractions and find equivalent fractions.

6.1.1.7

Convert between equivalent representations of positive rational numbers.

6.1.3.1

Multiply and divide decimals and fractions, using efficient and generalizable procedures, including standard algorithms.

6.1.3.2

Use the meanings of fractions, multiplication, division and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions

6.1.3.4

Solve real-world and mathematical problems requiring arithmetic with decimals, fractions and mixed numbers.

7.1.1.1

Know that every rational number can be written as the ratio of two integers or as a terminating or repeating decimal.

9.2.3.4

Add, subtract, multiply, divide and simplify algebraic fractions.

9.2.4.8

Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.

Minnesota Comprehensive Assessment Question(s):

Complete the equation shown by placing the appropriate number into each box.

Drag a number into each box.

$\frac{2}{5} \div \frac{3}{4} = \frac{\square}{\square}$

1
2
3
4
5
6
8
9
10
15
20

Divide.

$$1\frac{1}{10} \div 1\frac{1}{5}$$

- A. $\frac{11}{12}$
- B. $\frac{25}{33}$
- C. $1\frac{8}{25}$
- D. $1\frac{1}{2}$

What is the prime factorization of 630?

- A. $2 \times 3 \times 5 \times 7$
- B. $2 \times 3^2 \times 5 \times 7$
- C. $2 \times 3^2 \times 35$
- D. $2 \times 5 \times 7 \times 9$

Which numbers are equivalent to $\frac{18}{72}$?

Select the numbers you want to choose.

- 25%
- $\frac{1}{9}$
- 0.14
- 4%
- $\frac{9}{36}$
- 1.4%
- 2.5

Simplify.

$$4\left(\frac{1}{2} + \frac{3}{8}\right) - \frac{5}{8} \bullet 2$$

- A. $1\,\frac{1}{8}$
- B. 2
- C. $2\,\frac{1}{4}$
- D. $5\,\frac{3}{4}$

Which numbers are rational?

Select the numbers you want to choose.

2.0	$1\frac{7}{8}$	π	$\sqrt{5}$	$6.3\overline{9}$

An equation is shown.

$$n = 1 \div 17$$

Which describes n?

- A. Integer
- B. Irrational
- C. Rational
- D. Whole

9.2.3.4	Add, subtract, multiply, divide and simplify algebraic fractions. *For example*: $\frac{1}{1-x}+\frac{x}{1+x}$ is equivalent to $\frac{1+2x-x^2}{1-x^2}$.

Table of Contents

DAY 15: Practice Multiplication and Division using Area of a Rectangle

Rational Numbers and Expressions Post Test

Citations

Pre-Test

Name: ____________

1. Describe n as Natural / Whole / Rational/ Irrational if $n = 3 \div 9$

____

2. Divide

$$1\frac{1}{10} \div 1\frac{1}{5}$$

____

3. Add

$$\frac{1}{1-x} + \frac{x}{1+x}$$

____

4. Multiply

$$\frac{x}{2} \times \frac{x+5}{5}$$

____

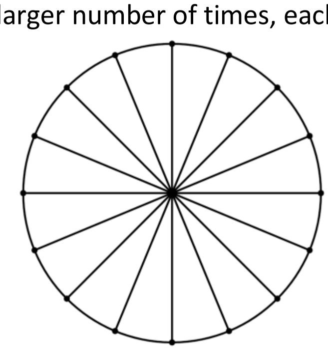
5. Draw a graphical representation of $1\frac{1}{2} + \frac{3}{4}$

6. Is it reasonable to assume that multiplying by $\frac{1}{2}$ or dividing by 2 will give the same result?

____

7. Finish the phrase, as you divide a larger number of times, each fraction of the whole gets __________

8. Color in the equivalent of $\frac{8}{32}$



9. If a person were to say that $\frac{x+5}{5} = x$ what is the mistake that they made? ____________________

Lesson 1

History of the Real Number Systems

Launch:

Today you will explain to students that we are going to talk about the history of the real number system, and focus on the rational numbers.

Natural/Counting Numbers *-Sumerians developed a symbolic system for accounting purposes.*

In your group make a list of 5 things that you can count.

Explore:

Whole Numbers *-India around 650 A.D. to convey the idea of nothing*

<u>Activity 1</u>: From their list ask them to choose one item and describe what none of it would look like.

Describe how we use/see the idea of "nothing" or "none of" in everyday experience.

Have them write down a speculation as to why the "original" Roman Numerals did not include a symbol for "zero".

Then ask them to discuss, if the whole numbers just include zero, or if there are any other numbers in the set of whole number system?

Then ask them to discuss what the function of the zero plays in the number 1023?

Integers *-Chinese used red rods to measure direction and black to measure the opposite direction by 200B.C. but the first use of (-) was not until* 4^{th} *century A.D.*

<u>Activity 2</u>: Ask them to choose from their list an item and describe what the opposite of three of that item would look like or function as.

Then ask them to describe how we use see/use/apply the idea of opposite numbers to our everyday living?

And to discuss, if the integers are the set of just the negatives, or it they include wholes and naturals?

Rational *-Pythagoras*

<u>Activity 3</u>: Ask them to choose one item and discuss what it would mean to divide it, or see how many times something goes into it.

And then ask them to choose from the list of number systems above, which set needs to be used in order to describe how many times something goes into something else?

Pose the Question:

Is it possible for an arrow to travel half way to a target, through the target, and then half the original distance again behind the target (the opposite direction)?

Discuss with examples if any integer can be expressed as a rational.

Ask them if they can write 10.5 as a ratio of two integers?

Ask them to discuss the advantages in commerce of being able to buy and sell using rational numbers.

Ask them to discuss, if the number n is defined as $n = 3 \div 22$ is that a rational number?

Irrational *-Pythagoras*

<u>Activity 4</u>: Give then an example of a known number that cannot be express as a ratio of two integers.

Pythagoras proved that the diagonal of any square is a multiple of the length of a side, but the multiplier was the irrational number $\sqrt{2}$. Ask them to discuss whether or not it is possible to measure the length of a diagonal of a square tile that is 24 inches by 24 inches.

Share:

Have them share what it means for a number to be a rational number.

Summary:

Clarify with them that rational numbers are numbers that can be written as fractions. They do include the natural, whole, and integers.

Lesson 2 (3 Days)

Initial Fraction Ideas and Number Sentences

Launch:

Today we will start by looking at shape, and cutting them out. Give students the lesson 2 launch sheet. (You may want them to cut them out)

How many Blue's cover the Green?

How many Blue's cover Yellow?

How many Yellow's cover Green?

How many Blue's cover Green?

How many Blue's cover Brown?

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

<u>Activity 1</u>: Have students write number sentences. What is a proper number sentence?

<u>Activity 2</u>: Fraction size, relationship between times 4 and 4ths, inverse relationship…If you increase the number of pieces, what happens to the size of the pieces. Draw in $\frac{1}{x}$ for each x, how are they related?

<u>Activity 3</u>: Color in how many times 10 **goes into** 100. Color in 20, how many times does 20 **go into** 100?

Draw a different perfect square, and color in how many times the "root" **goes into** the area. What is the relationship between a root the area?

<u>Activity 4</u>: How many times does x **go into** x^2? What does this mean? What is the root of a perfect square? What is the ratio of a root to the other root of a perfect square? What are the relationships between each of the two roots?

<u>Activity 5</u>: How many $x^{2'}s$ are in the large square? How many times does the $10x^2$ go into the big square?

Ask them to Color in $5x\ times\ 2x$, how many times does that go into the $100x^2$?

<u>Activity 6</u>: What is the relationship between dividing the length of a root, and the number of times its square goes into a "whole" variable?

<u>Activity 7</u>: Write the number sentences, and share. 10 blocks divided by 10 groups, 5 groups, 2 groups, 1 group, 0 groups. What would it mean to divide into zero, or to divide by zero?

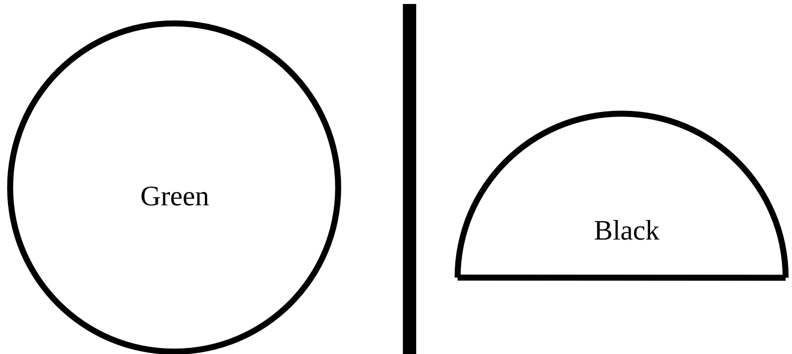
Share:

What makes something a rational number/expression? What is the relationship between the roots and the product of the roots?

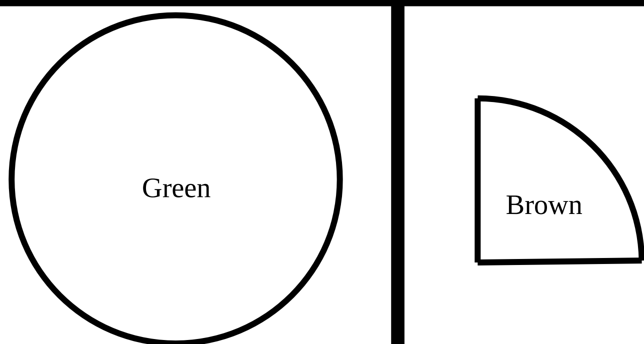
Summarize:

A rational number is the idea of how many times something goes into somethings else, a ratio between to quantities (integers), or a fraction. Rational expressions are the same idea, but there are variables involved. Rational numbers are defined through multiplication.

Lesson 2 Activity 1



Write an English sentence and a number sentence to describe.



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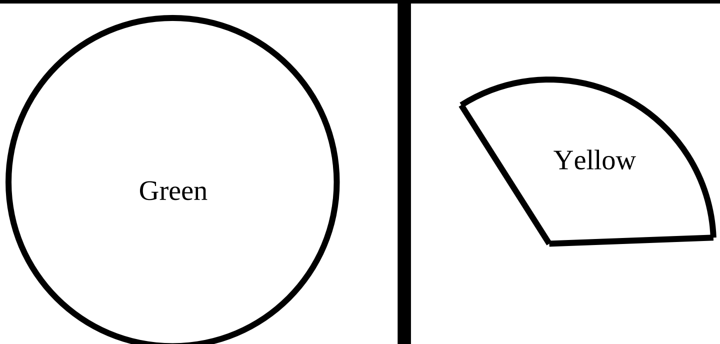

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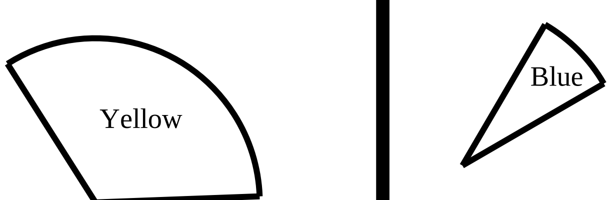
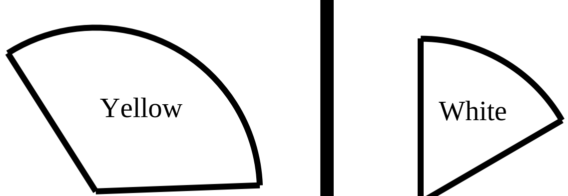


Write an English sentence and a number sentence to describe.

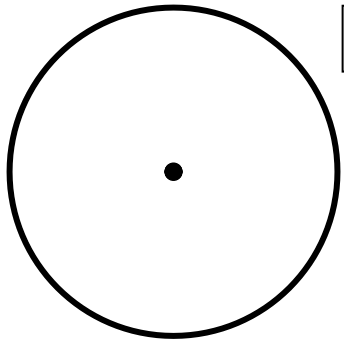


Write an English sentence and a number sentence to describe

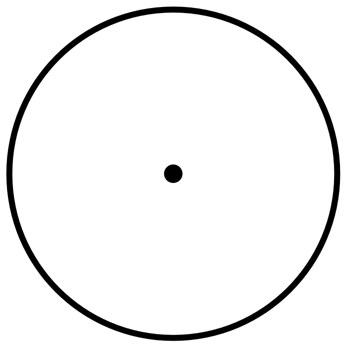
Lesson2 Activity 2

Number of slices

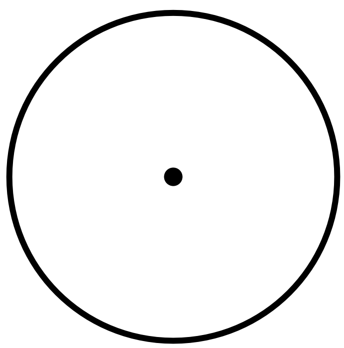
Size of each slice



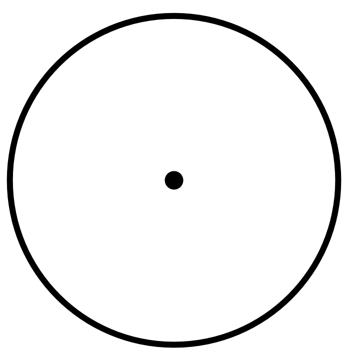
$x = 1$



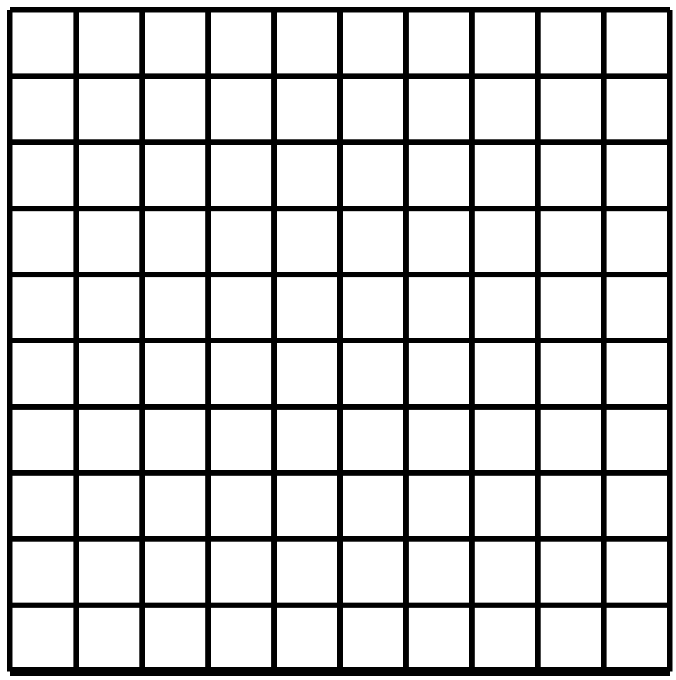
$x = 2$

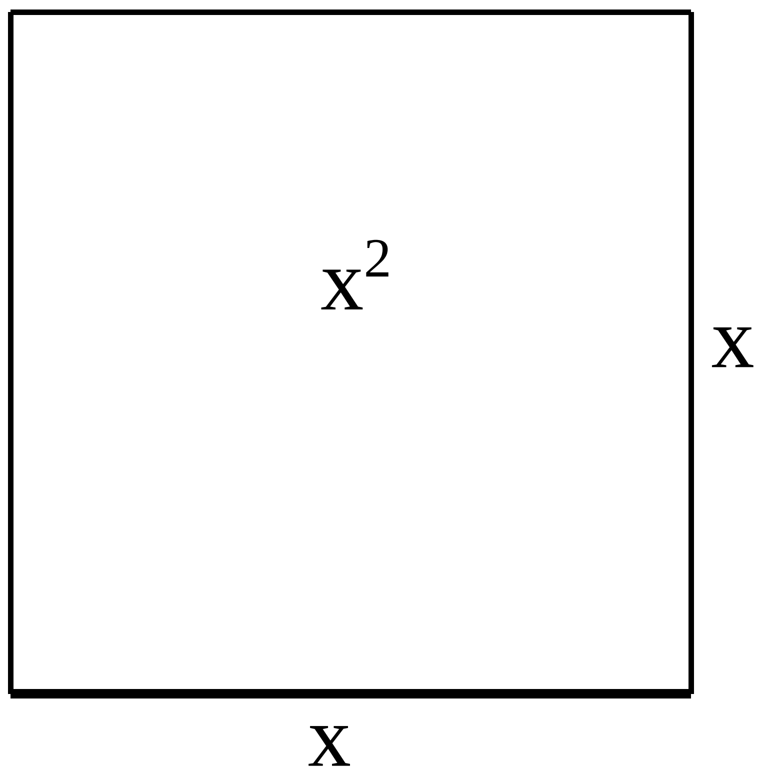


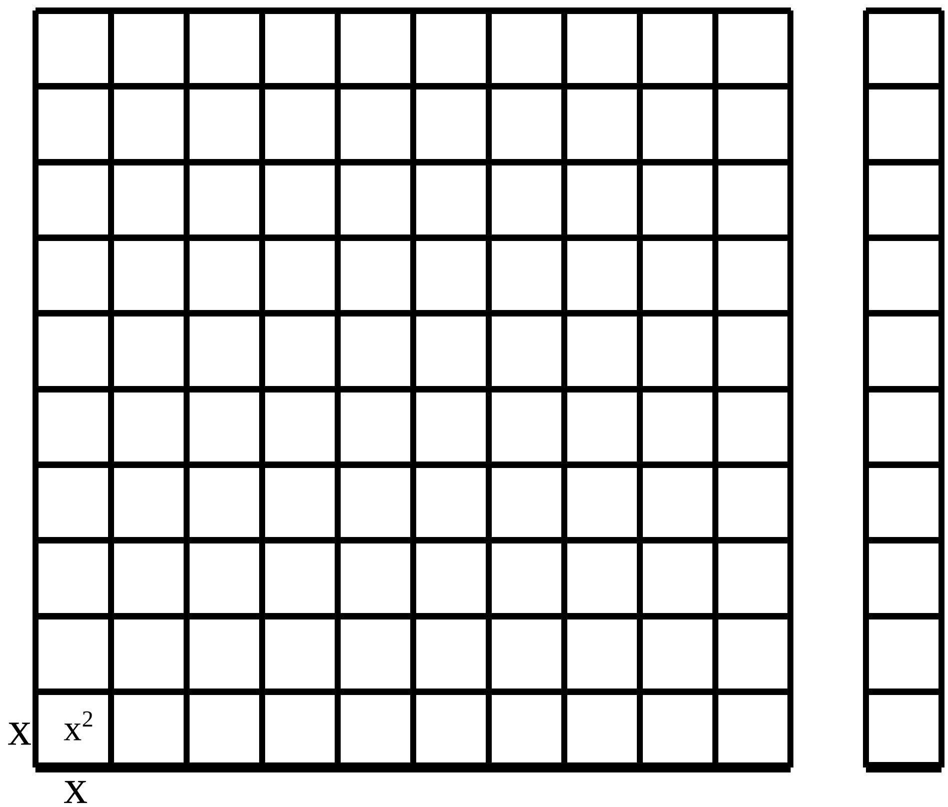
$x = 4$

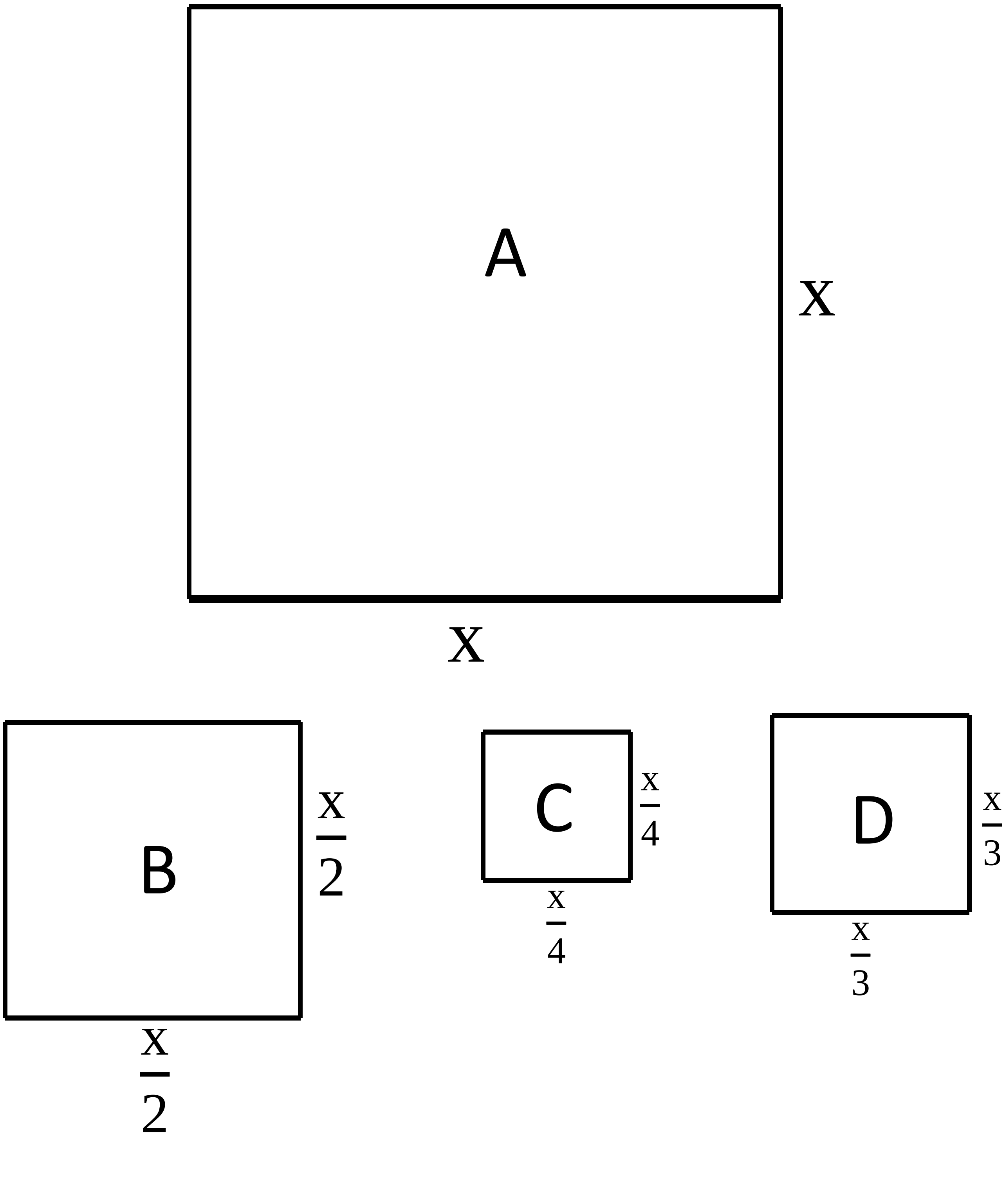


$x = 8$

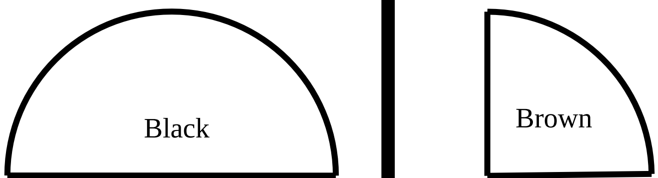



x2
x
x

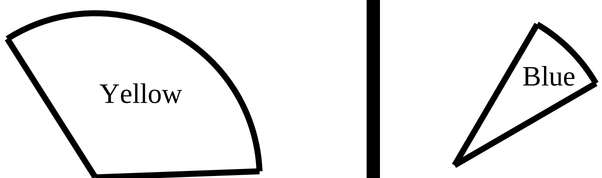

x
x2
x


A
x
x
B
x/2
x/2
C
x/4
x/4
D
x/3
x/3

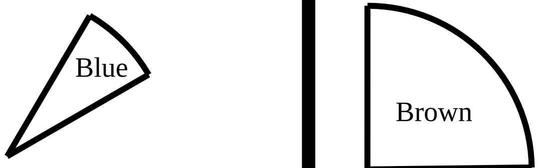
Lesson 2 Activity 6



Write a number sentence to describe



Write a number sentence to describe



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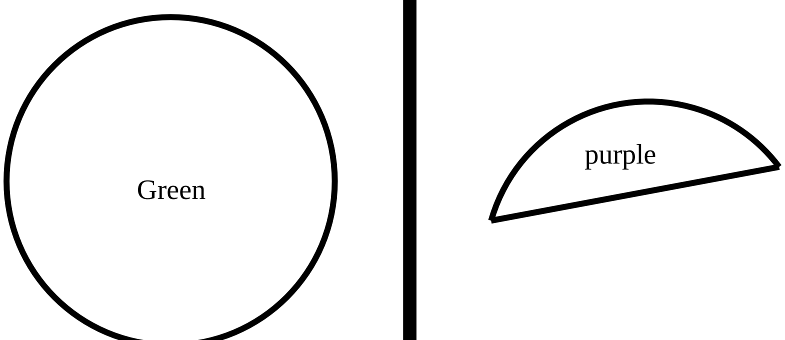
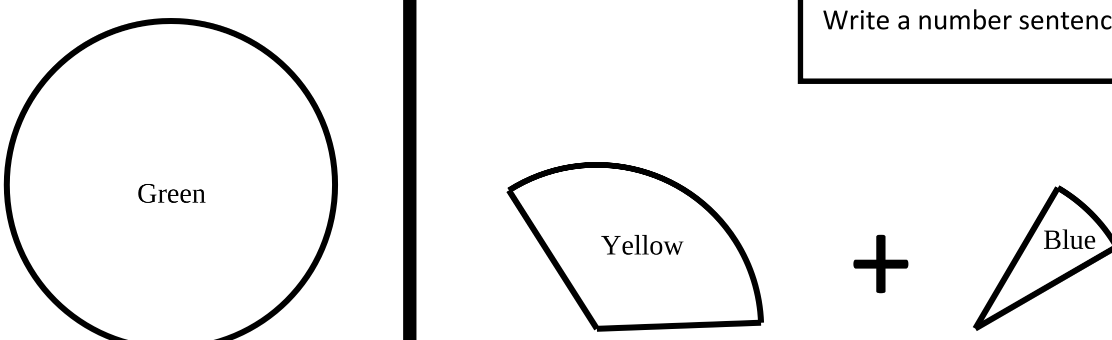
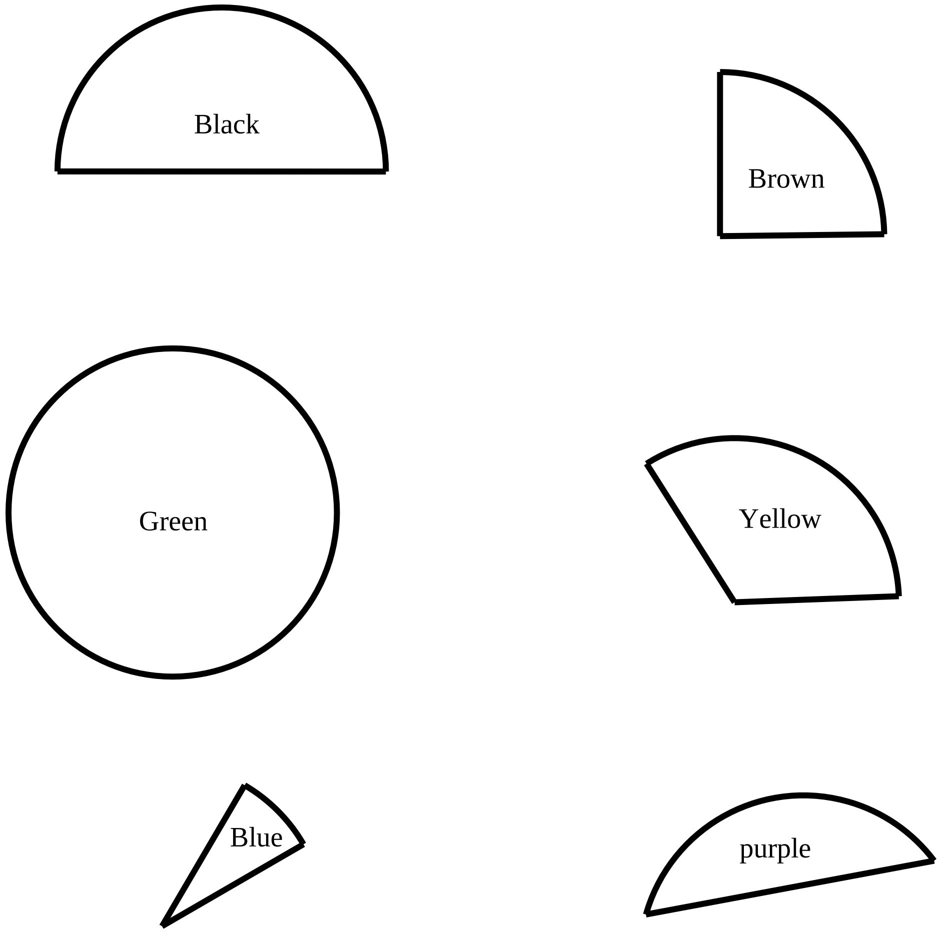


Write a number sentence to describe



Write a number sentence to describe

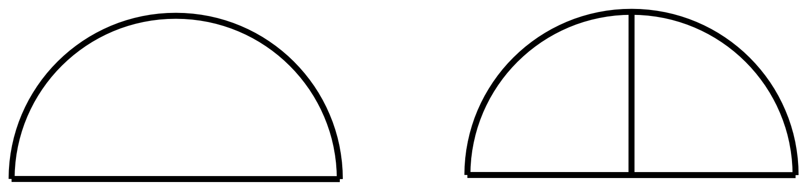

Black
Brown
Green
Yellow
Blue
purple

Lesson 3

Equivalent Fractions

Launch:

Today we will start with a diagram of two equivalent fractions (such as these two), and I will be asking a few questions:



Pose the question – Are they fractions (the first could be a whole)?

Pose the question – Are they the same thing?

Give an example of Equal vs Equivalent such as 2+2=2+2 and 2+2=4.

Demonstrate writing a number sentence that shows equivalence. i.e. $\frac{1}{2} = 2\left(\frac{1}{4}\right) = \frac{2}{4}$ or $2\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right)$, then pose the question, "What would the English sentence be?".

Explore: (groups of 3 or 4) (Give appropriate activity sheets as needed)

<u>Activity 1</u>: They need to decide how to make them equivalent, or in some cases, where they cannot be made equivalent, and write an English sentence and a number sentence.

<u>Activity 2</u>: Place the fractions on the edge of the circle in the correct places.

After assigning the correct positions of the 12ths, 6ths, 4ths, 3rds, and halves, have them discuss the role of reducing equivalent fractions to be in the smallest terms.

Have them write out 2 "Real life" examples where they see fractions that need to be put into reduced form, or are presented in reduced form. (tape measure, wrenches, cooking volume measurement…)

<u>Activity 3</u>: How can you color in $5\frac{1}{2}$ using the fewest individual figures? How can you color in $5\frac{1}{2}$ using the most individual figures?

<u>Activity 4</u>: Ask them to discuss if $\frac{10}{16}$ *is equivalent to* $\frac{15}{24}$, and if $\frac{1+6}{6}$ *is equivalent to* $\frac{1}{1}$?

Share:

Ask the students to share what it takes for two expressions to be equivalent, and then share the process of reducing fractions. Ask them also to discuss if there is a wrong way to reduce.

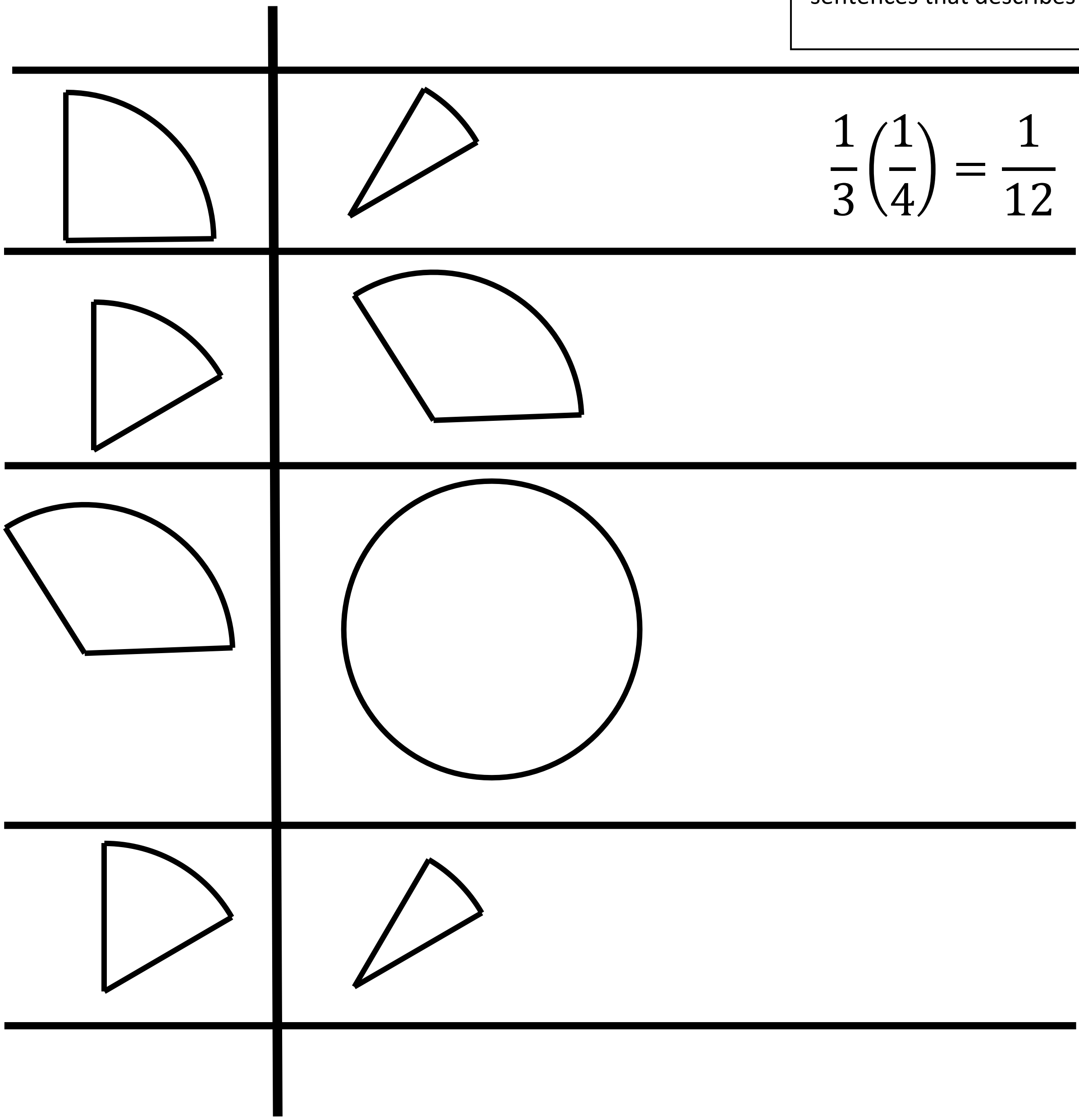
Summarize:

Two rational numbers are equivalent if they can reduce to be the same number.

You need to be careful about how you reduce though, there are incorrect ways.

Lesson 3 Activity 1

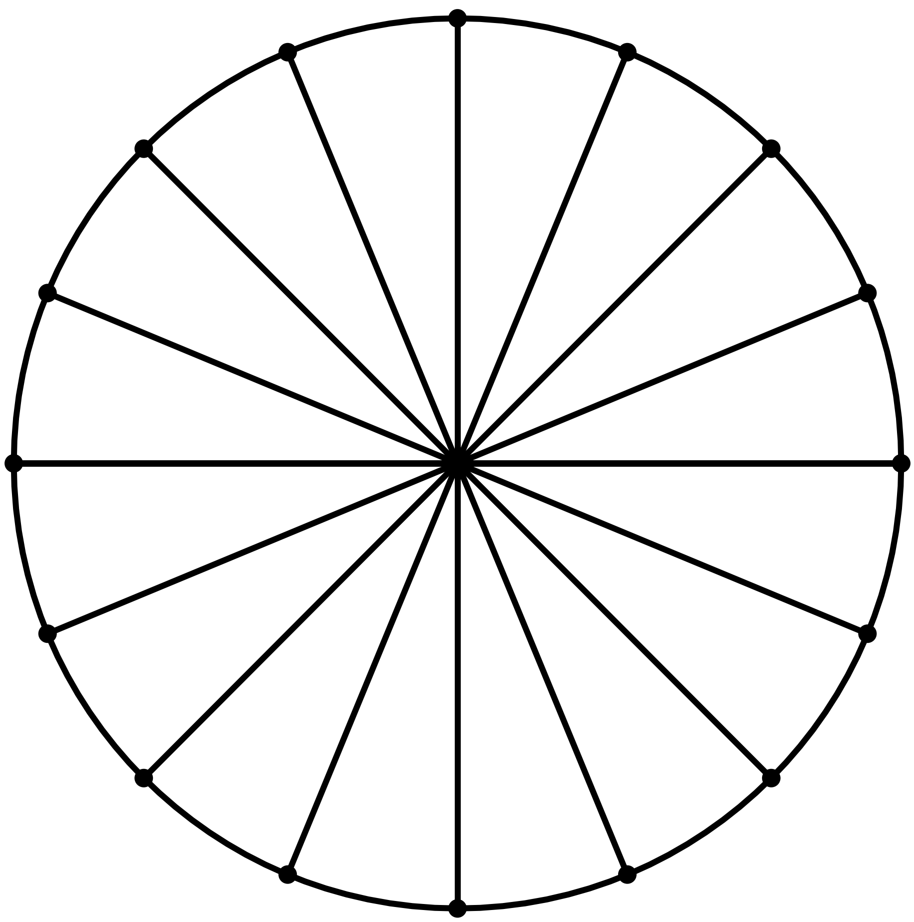
Write an English and a number sentences that describes equivalence.



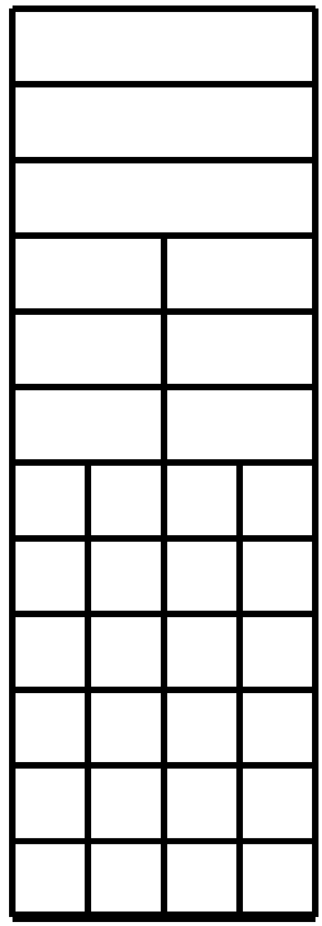
$$\frac{1}{3}\left(\frac{1}{4}\right) = \frac{1}{12}$$

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$

$\frac{1}{12}$ $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$ $\frac{5}{12}$ $\frac{6}{12}$ $\frac{7}{12}$ $\frac{8}{12}$ $\frac{9}{12}$ $\frac{10}{12}$ $\frac{11}{12}$



What would be the length of the rectangle? ____________



Lesson 4 (2 Days)

Equivalent Rational Expressions

Launch:

Today we will start with a review the summary from previous lesson.

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1: Ask the students to look at the square within the square and create equivalent fractions, and write both an English sentence for it and a number sentence. After (round one), ask them to find another set of equivalent fractions (rational expressions).

Activity 2: First ask students to discuss why the squared expressions show up, and how they are arrived at. After students have a good understanding of what the diagram is depicting, have them write a number sentence that depicts equivalent fractions. Discuss the results, and then have them write 2 more that differ from the first and each other.

Have the students discuss what $\frac{12x}{4x} = 3$ would mean in the context of the diagram.

Activity 3: Pose the questions, "What would be the area of each of the rectangle?" Then have them find the ratio of the area of each rectangle to the area of the whole. After that have them write a number sentence that describes a different section(s) which area is equivalent to:

Section A ________________________
Section F ________________________
Section B ________________________

When finished, instruct to create their own rectangle that has variables as units of length, with different ratios. Use theirs to write three number sentences that describe equivalent rational expressions, then for one of them write the English sentence.

Activity 4: Ask students to discuss whether it is OK or not to reduce a fraction like $\frac{5}{10}$, be prepared to explain. Then the same question with $\frac{5}{10+1}$. Then ask if it would be ok reduce $\frac{2x}{3x}$? What about $\frac{2x}{3x+1}$? How about $\frac{2+4}{2}$ *and* $\frac{2x+4x}{x}$? Then have them write down a rule for when it is OK to reduce, and when it is not.

Activity 5: During this activity, students may be working independently or in groups. If working with groups of students, ask the groups to exchange answers to check their work.

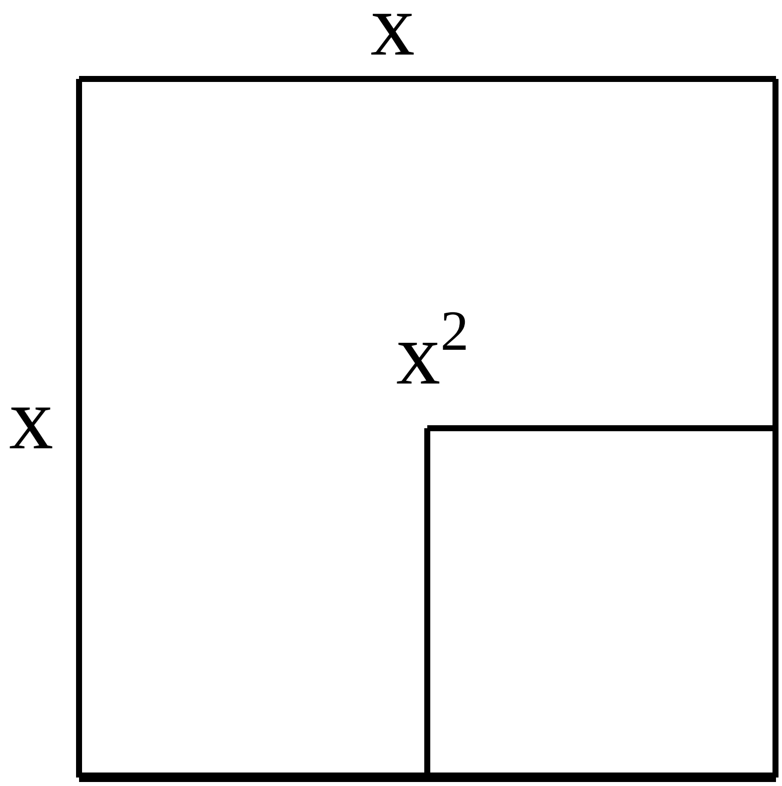
Share:

Ask students to share how one way someone might arrive at the usage for a rational expression.

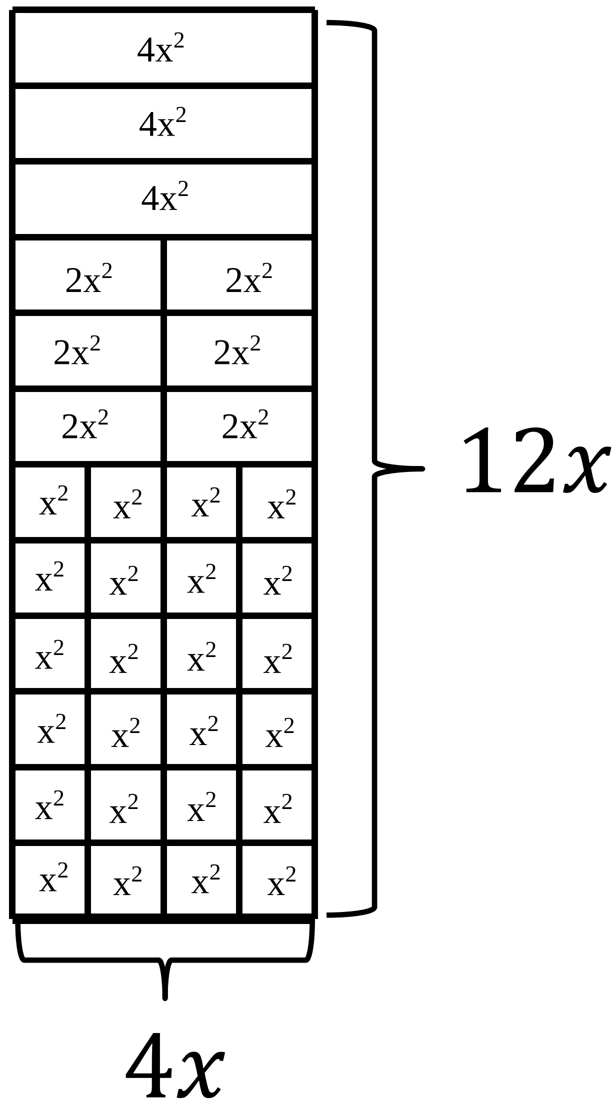
Summarize:

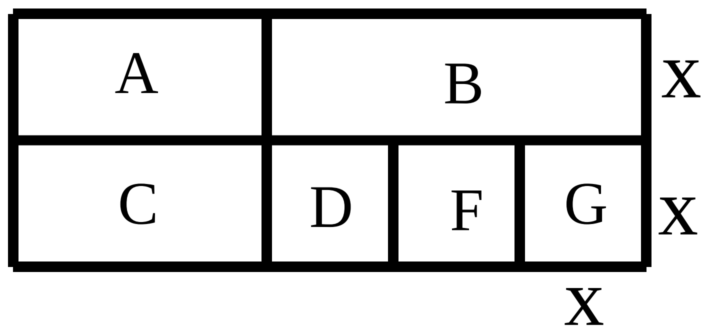
Rational Expressions come up when we are using ratios of part of things that are variable. An example might be a proclamation that “Half of all the proceeds raised at a fundraiser will be split up between 3 charity groups.” Also when looking at the ratio of two quantities, we can also compare rational expressions of their products.

When reducing the ratio of length to width of a rectangle the result tells you how many times the width goes into the length.


x
x
x2

Number Sentences





Section A ________________________
Section F ________________________
Section B ________________________

The rules are simple:

- the answer must contain at least one variable,
- the answer must be in the form of a fraction, and
- the answer must reduce to the given term or expression.
- **(more advanced rule)** the answer must contain binomials.

Examples:

Build an algebraic fraction which will reduce to 2.
Build an algebraic fraction which will reduce to 3x.
Build an algebraic fraction which will reduce to y + 1.
Build an algebraic fraction which will reduce to 2xy.
Build an algebraic fraction which will reduce to 5x/w.

Start out will simple problems and progress to more difficult situations.

Lesson 5

Simplifying Rational Expressions with Numbers and/or Variables

Launch:

Today we will start by passing out student activity 1, and then look at the question

"How can we write a number sentence that describes student activity 1".

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1: Create and color in 5 different reducible fractions, then write out number sentences to accompany each.

Activity 2:

Ask the students to find the ratio of length to width, and simplify.

Describe what it means for 2x to **go into** 2(x+3). "If we cut each in half it goes into it x+3 times" or "if reduced by 2 the ratio becomes x to (x+3)".

Ask the students what how they would find the area of each smaller rectangle.

Ask the students to find the ratio of $x \ to \ x^2$ terms, and then reduce. What does the result show?

Review Lesson 4 Activity 2, and discuss what $\frac{2x+6}{2x}$ would tell us. Then have them discuss why 6 would be in improper result of reducing that fraction.

Ask students to create two rectangles with variables and numeric values for lengths of sides, one that they can reduce the width to length ratio, and one that they cannot.

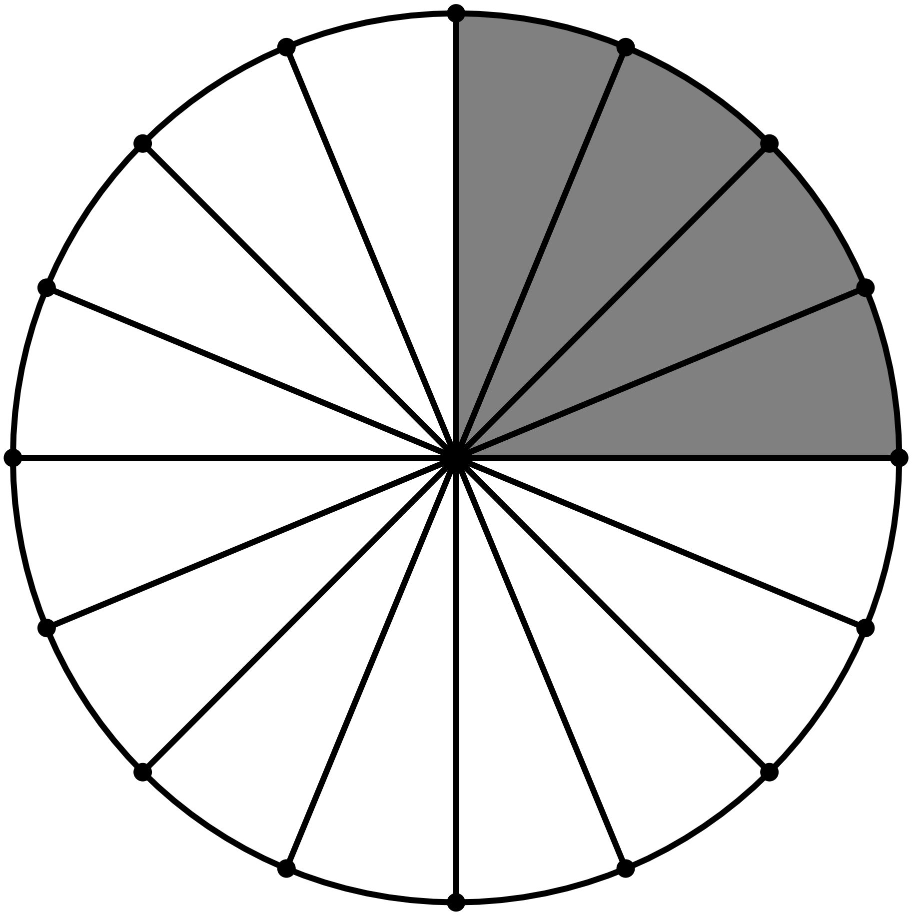
Activity 3: Ask students to reduce the following three fractions: $\frac{x^2+2x^2}{x}$ $\frac{2x}{2}$ $\frac{5x}{5x+7}$

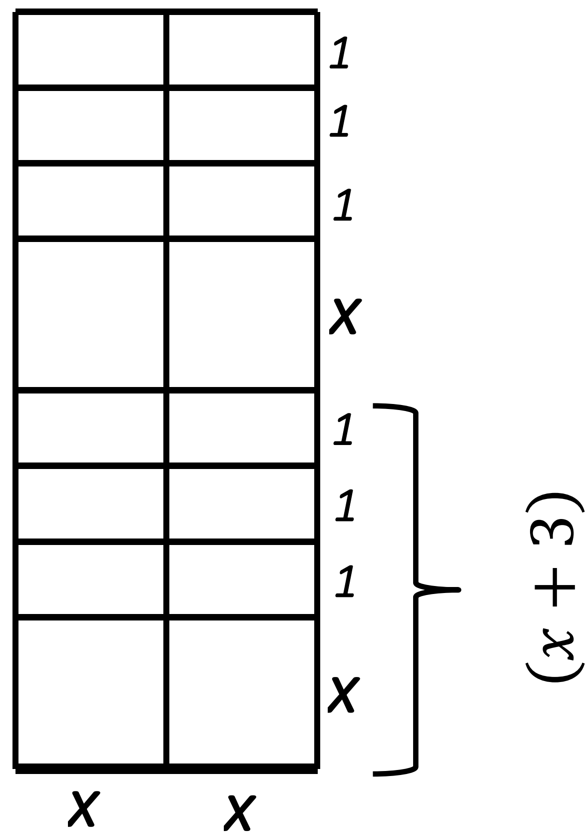
Share:

When is it proper to just cancel out top to bottom, and when is it not.

Summary:

Many times when working with rational expressions we can reduce, but if the numerator or denominator is a sum or difference, we cannot just reduce to one term.




1
1
1
x
1
1
1
x
x
x
(x + 3)

Lesson 6

Addition and Subtraction of Rational Numbers

Launch:

Make the statement that today we are going to look at addition and subtraction of rational numbers to be able to solve problems similar to "$\frac{1}{3}$ of leftover pepperoni pizza was added to a box that already had $\frac{1}{4}$ of a sausage pizza, how much of a pizza was in the box total?"

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1: Instruct the students to write a numeric sentence and an English sentence for each scenario given. Then ask then to find the answer to each.

Then ask them to use the diagrams to explain what a common denominator is and why we need to find common denominators in order to add or subtract rational expressions.

Have them discuss how to find the common denominator when needed, then solve the pictorial problems.

Activity 2: Ask them to complete the two questions, and then write an English sentence for each. Then draw two of their own, one with a common denominator, and one without. Give the solutions to each.

Activity 3: Ask to combine the following fractions: $\frac{3}{8}+\frac{1}{6}=$ $\frac{5}{12}-\frac{2}{3}=$ $\frac{1}{2}+\frac{2}{5}-\frac{1}{10}=$

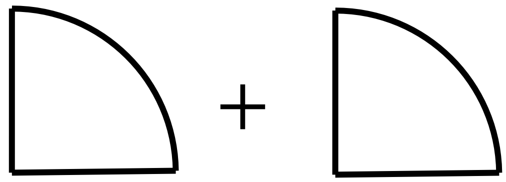
Share:

Ask them why is it necessary for you to find a common denominator before addition or subtraction of rational expressions, how to you get a common denominator?

Ask them, once you have a common denominator, ask then what?

Summarize:

In order to add or subtract rational expressions you must first have a common denominator. To do that you find the factors that each denominator, compare which factors they have in common, multiply the other fraction(s) by any missing factors both top and bottom, then finish the operation by combining numerators according to the sign.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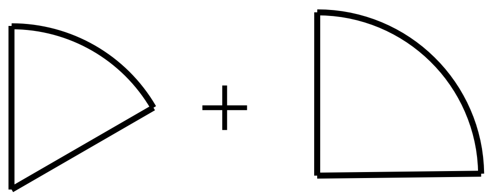
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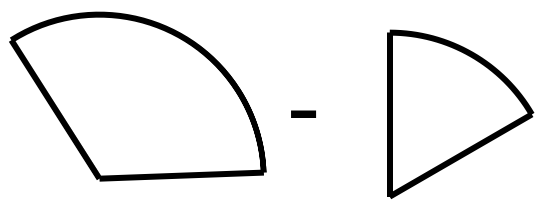
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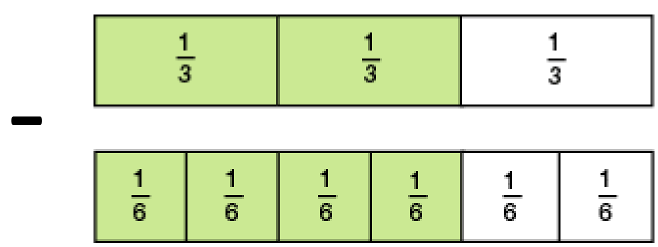
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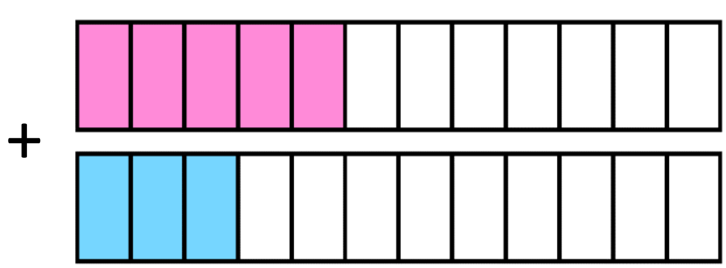
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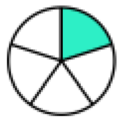 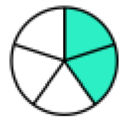 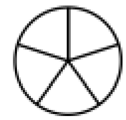

$$\frac{1}{5} + \frac{2}{5} = \_\_\_\_\_\_$$

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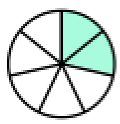
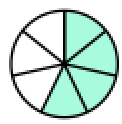
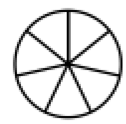
  

$$\frac{2}{7} + \frac{4}{7} = \_\_\_\_\_\_$$

Lesson 7

Adding and Subtracting Rational Expressions (Variables)

Launch:

Yesterday we looked at adding and subtracting rational expressions. Ask "Why did we have to find a common denominator, and how was this accomplished?"

Today we are going to look at adding and subtracting rational expressions.

Review that in lesson 5 we discussed that we could not cancel in this situation $\frac{2x+6}{2x}$.

Explore: (groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1: Give them the activity 1 sheet, and then ask them to explain why the two diagrams are equivalent, and what was accomplished by dividing up the larger square. Then write a number sentence and an English sentence with answer to addition problem.

Activity 2: Ask students to look at the problem $\frac{2x}{8x} + \frac{3}{12x} =$ and explain the process of how to find a common denominator, what happens to the numerators, and which numbers get combined with the plus symbol.

Activity 3: Ask students to solve the following using the process described in activity 2:

$\frac{x+7}{2y} + \frac{5}{3x} =$ $\frac{x^2}{3xy} - \frac{6-1}{xy} =$ $\frac{2}{x+2} + \frac{5x^2}{5x+1} =$

Share:

Have them summarize the difficulties in the above problems.

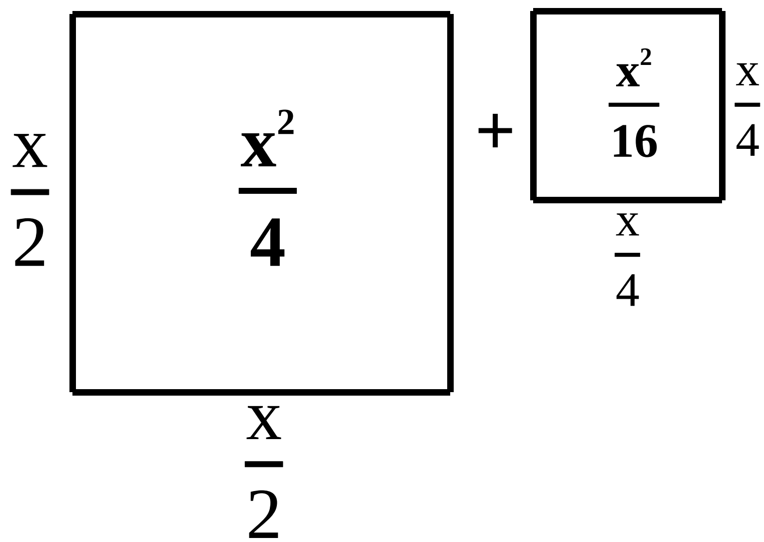
Have them review what they said was the process for finding common denominators and combining.

Have them explain why in the third problem of activity 3 they could not just cancel.

Have then describe how to handle the distribution in Activity 3.

Summary:

To add or subtract rational expressions they need to have like terms. To get like terms, you need to factor, find out which factors each fraction needs that the other has, multiply both top and bottom by missing factors. Then once a common denominator has been achieved, only combine the numerators with the addition or subtraction operation.



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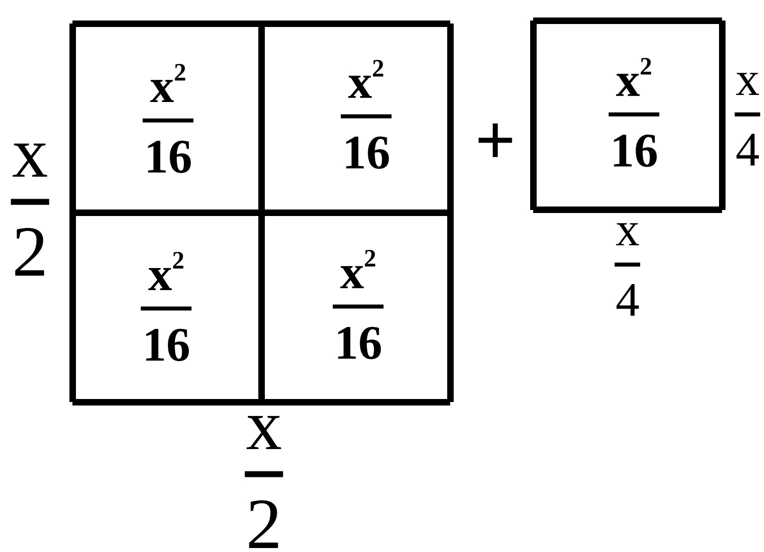


Lesson 8

Multiplication of Rational Numbers

Launch:

Today we are going to look at multiplication of rational numbers. Ask them what it would mean to multiply two rational numbers such as $\frac{2}{3} \times \frac{4}{5}$. If direction is needed, point out that $\frac{1}{2} \times 4$ results in a smaller number "half of 4".

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1: Ask them to look at activity 1 diagram's, and decide if they fits their idea of what they said in the launch. Then write an English sentence to go with each diagram.

Ask them to decide of a common denominator is required for multiplication.

Activity 2: Create 2 of your own rational number multiplication problems, and a diagram to go with each one.

Activity 3: Give them activity 3 sheet, and ask them to write a number sentence of what it represents. Have them discuss mixed numbers and how to deal with mixed numbers.

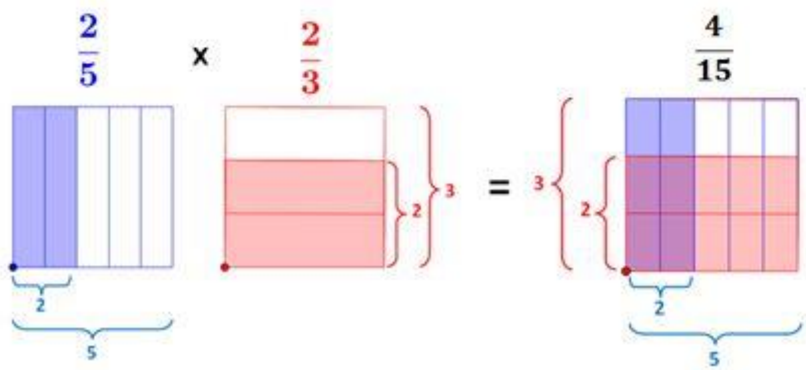
Ask them to create a visual representation of $\frac{2}{9} \times 2\frac{1}{3}$ then give the solution.

Share:

Ask them to discuss the process of multiplication of rational numbers, and how to multiply mixed numbers.

Summarize:

When multiplying rational numbers you do not have to get a common denominator, just simply multiply numerator to numerator and denominator to denominator. If a number is a mixed number then it must be converted to an improper fraction first.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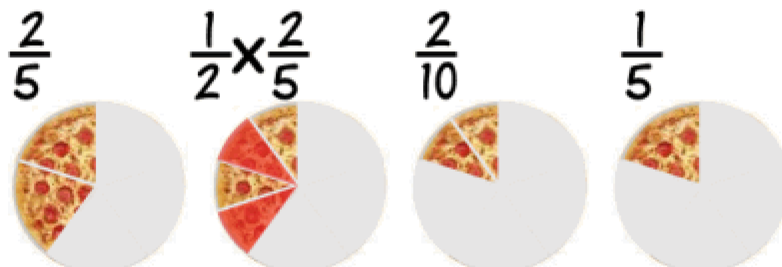
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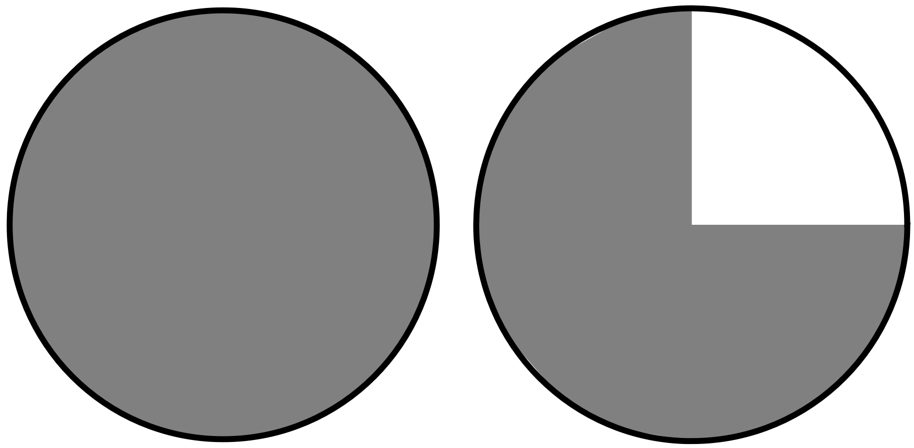
$\frac{1}{7}$ x 

Lesson 9

Multiplication of Rational Expressions

Launch:

Today we are going to look at multiplication of rational expressions. Review the pizza example from lesson 8, and the idea of multiplying rational numbers. Ask what the process was for multiplying rational numbers.

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

<u>Activity 1</u>: Ask students to write a number sentence and an English sentence to go with the diagram in on activity 1, ask that the English sentence include the phrase **out of**.

After they have finished, ask them what the units are of the composite "square". It's dimensions are $x\ by\ y$, so the result of the multiplication should be $4\ out\ of\ xy$.

<u>Activity 2</u>: Have the students create two other figures to represent the multiplication of two rational expressions.

<u>Activity 3</u>: Ask the students to solve the following multiplication of rational expressions.

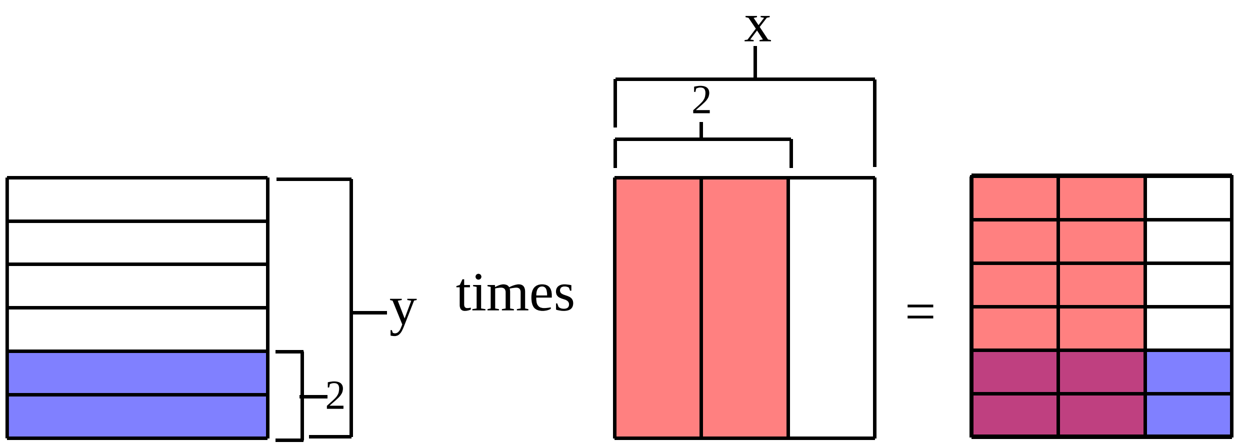
$$\frac{2x}{3x} \times \frac{5}{2y} = \qquad \frac{x+1}{y} \times \frac{3}{4y} = \qquad \frac{5+x}{x} \times \frac{3}{x-3} =$$

Share:

Ask students to share their results of the three problems. Ask if it mattered in the first one whether they reduced before they multiplied or after. Then ask them to share the potential mistake in the third.

Summarize:

When multiplying rational expressions you do not have to get a common denominator, just simply multiply numerator to numerator and denominator to denominator. Remember our rule from lesson 4 about when you can reduce and when you cannot.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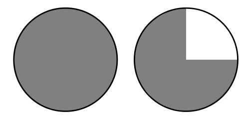
Lesson 10

Division of Rational Numbers

Launch:

Today we are going to look at division of rational numbers.



Ask: If I wanted to get 7 pieces out of this diagram how many pieces would I have? Ask where have we seen this idea before?

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1: Have them in there groups discuss the question from the launch, and ask them to solve it like they did before. Then have them discuss how else they could solve it.

Activity 2: Have them discuss whether multiplying by $1\frac{3}{4} \times \frac{1}{7}$ is the same as $1\frac{3}{4} \div 7$. Then draw out another scenario that can be solved by multiplication or division.

Activity 3: Have the students come up with a rule that will works for a fraction divided by a fraction such as $\frac{1}{2} \div \frac{1}{4} =$. Then have the groups exchange their rules and see if it works for $\frac{2}{16} \div \frac{6}{16} =$.

Activity 4: Have them use their groups rule to solve: $\frac{5}{3} \div \frac{6}{7} = \quad and \quad \frac{3+2}{3} \div \frac{6}{6+1} =$

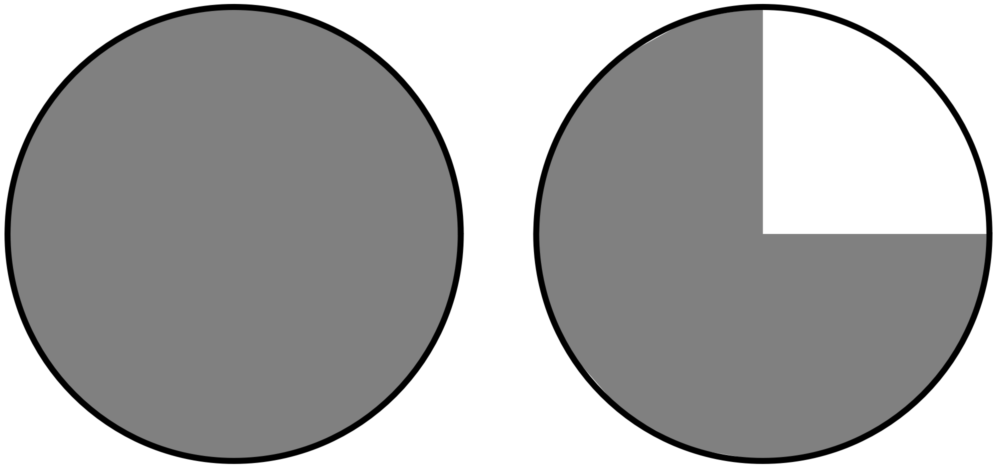
Activity 5: Ask them to consider the problem $\frac{8}{10} \div \frac{4}{2}$ using their rule. After they have come up with $\frac{16}{40}$ or $\frac{2}{5}$ ask them to consider the algorithm $\frac{top\ left \div top\ right}{bottom\ left \div bottom\ right}$. Ask them to create their own problems and test it to see if it is just an anomaly.

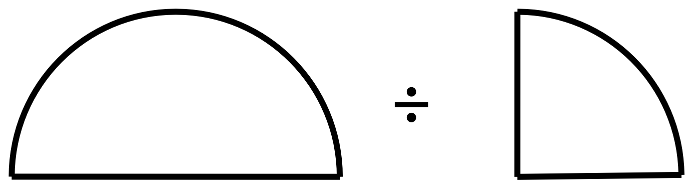
Share:

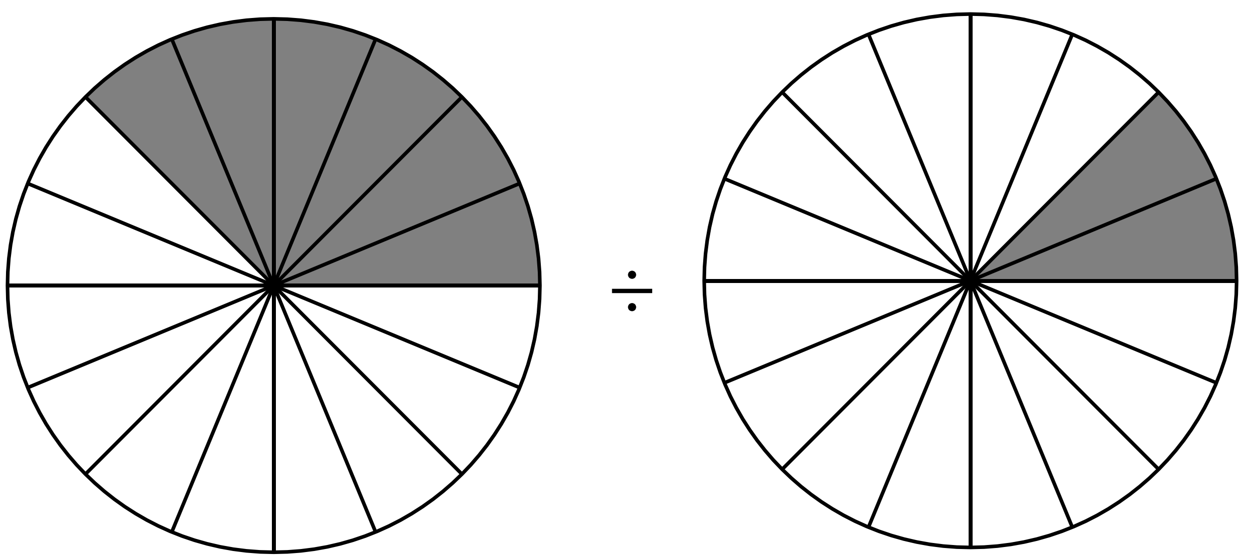
Ask students to explain how their rules are used, and how inverse relationship of multiplication and division affect the way we divide.

Summarize:

To divide a rational number by another rational number we can switch it to a multiplication problem if we use the inverse (reciprocal) of the divisor. We still cannot forget what we learned about improperly canceling back in lesson 4.







Lesson 11

Division of Rational Expressions

Launch:

Today we are going to look at division of rational expressions. Ask the students to restate what they know about division of rational numbers.

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1**:** Ask the students to use their rule for dividing rational numbers on the problems

$$\frac{x}{2} \div \frac{x}{4} \quad \text{and} \quad \frac{5x+15}{7x} \div \frac{5x+15}{7x}$$

Then have the students describe in each case how they can know if their method did it correctly.

Activity 2: Ask the students to explain the division of rational expressions of squares in three ways. Write a number sentence and solve, show graphically, and write and English sentence.

Activity 3: Ask students to perform $\frac{3x}{4} \div \frac{4x}{5} =$ then ask them if we would get the same answer if we reduced the 4's before we multiply by the reciprocal. Then create an example where things can cancel after you "flip" the fraction.

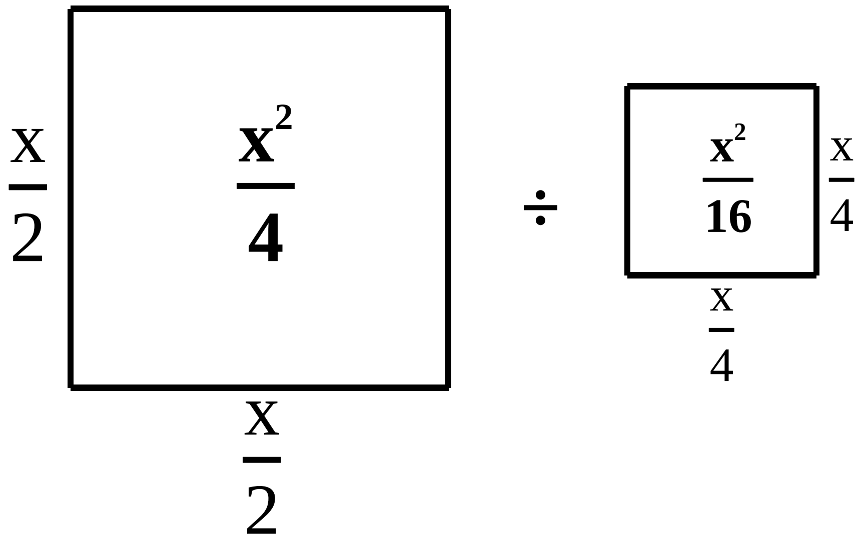
Share:

Ask students to explain how to divide rational expressions.

Ask the students to share whether it is ok to cancel across the division sign before we multiply.

Summarize:

You divide rational expressions by multiplying by the reciprocal, but you cannot reduce across the division sign only the multiplication.


$\frac{x}{2}$
$\frac{x^2}{4}$
÷
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Lesson / Practice 12

Launch:

Today we are going to practice some of the skills we have learned. Ask what the formula for the area of a rectangle is, and formula for perimeter of a rectangle

Explore: (Groups of 3 or 4) (Give appropriate activity sheets as needed)

Activity 1: Ask them to as a group put together a plan of how they will solve for any of the three variables in the area of a rectangle equation given the other two. Or how to find the missing variable if given perimeter or length or width.

Activity 2: Ask them to find the missing third variables on the practice sheet.

Share:

If there are any aspects of multiplication or division that are still not clear.

Summarize:

Confirm that $A = lw \quad l = \frac{A}{w} \quad w = \frac{A}{l}, and\ P = 2l + 2w \qquad \frac{P-2l}{2} = w \qquad \frac{P-2w}{2} = l$

Confirm what the answers were for the practice.

$L = \frac{x-3}{3}$

$A = \frac{x^2-x-12}{3x+12}$

W =

$L = \frac{3x^2y}{6x^2}$

A =

$W = \frac{2x}{3x^2y^4}$

$L = 2x^2-10$

A =

$W = \frac{21}{x}$

W =

$A = \frac{5a + 25b}{10}$

$L = \frac{7a}{5}$

$L = \frac{2x^2+5}{3x}$

P =

$W = \frac{x^2+1}{2x}$

Post-Test

Name: _____________

1. Describe n as Natural / Whole / Rational/ Irrational if $n = 1 \div 17$

____

2. Divide

$$2\frac{1}{10} \div 1\frac{1}{2}$$

____

3. Add

$$\frac{1}{1-x} + \frac{x}{1+x}$$

____

4. Multiply

$$\frac{2x}{2} \times \frac{x+5}{5}$$

____

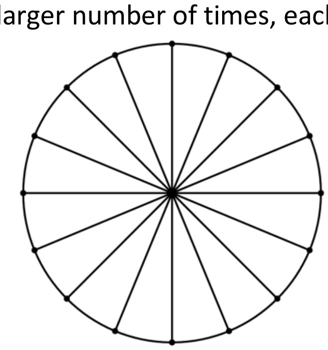
5. Draw a graphical representation of $1\frac{3}{4} + \frac{1}{3}$

6. Is it reasonable to assume that multiplying by $\frac{1}{3}$ or dividing by 3 will give the same result?

____

7. Finish the phrase, as you divide a larger number of times, each fraction of the whole gets _________

8. Color in the equivalent of $\frac{8}{32}$



9. If a person were to say that $\frac{x+7}{7} = x$ what is the mistake that they made? __________________

Citations

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