

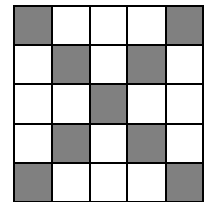
## Introduction to fractions

Fractions are commonly used in everyday language to express part of a whole number. Many recipes use fractions to express the quantity of ingredients required. In general conversation, people will talk about one fifth of a pie rather than the decimal equivalent 0.2 of a pie. People may cut apples into quarters and eat three eighths of a pizza. Footballers get to have a rest at half time and a block of chocolate can easily be broken into fraction amounts.

Fractions are the preferred way to communicate parts of whole numbers, however, decimals are often preferred for calculation because of the use of the metric system of measurement and the use of electronic calculators.

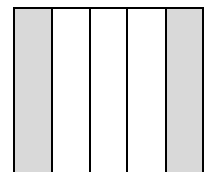
The diagram is divided into 25 equal parts. 9 parts are shaded.

The shaded part can be written as the fraction  $\frac{9}{25}$ , said as nine twenty fifths



The diagram is divided into 5 equal parts. 2 parts are shaded.

The shaded part can be written as the fraction  $\frac{2}{5}$ . The unshaded part is  $\frac{3}{5}$ .



2 ← This number is called the numerator, it is the number of parts present

5 ← This number is called the denominator, it is the number of parts in the whole

The bar between the numerator and the denominator is called the **vinculum**. The vinculum can represent the operation **division**. Division by 0 is not possible (share among 0 people!), it is undefined, it doesn't make sense. Similarly, a fraction that has a denominator of 0 is also undefined. The definition of a fraction includes this.

A fraction is defined as  $\frac{a}{b}$  where  $a$  and  $b$  are integers,  $b \neq 0$  ( $b$  not equal to 0).

Any integer can be written as a fraction. The denominator is always 1. The integer 4 as a fraction is  $\frac{4}{1}$ ,

likewise,  $-2$  can be written as a fraction  $\frac{-2}{1}$

## Naming Fractions

When saying fractions, the format is: numerator denominator, with the denominator usually having the suffix ths. Fractions with denominators 2,3 and 4 are given special names.

For example:

$\frac{1}{2}$  One half       $\frac{1}{3}$  One third       $\frac{1}{4}$  One quarter       $\frac{1}{5}$  One fifth       $\frac{1}{6}$  One sixth

$\frac{3}{8}$  Three eighths       $\frac{5}{12}$  Five twelfths       $\frac{6}{21}$  Six twenty firsts       $\frac{1}{44}$  One forty fourth

An alternative naming system is saying numerator over/on denominator. For example:  $\frac{5}{7}$  can be said as five over seven or five on seven.

## Unit Fractions

Fractions where the numerator is 1 are called unit fractions.

Examples are :  $\frac{1}{3}$ ,  $\frac{1}{6}$ ,  $\frac{1}{15}$

Don't get confused! When unit fractions are arranged in the order below:

$\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$  ...

The size of the denominator may be getting larger; however, the size of the fraction is getting smaller.

## Equity in Fraction Amounts

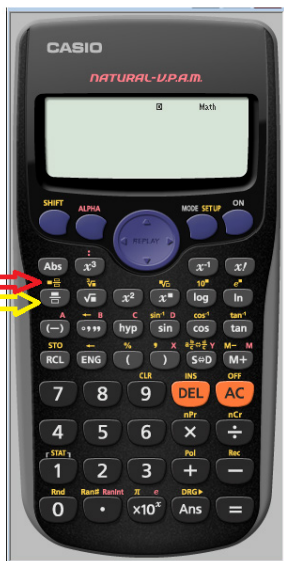
Does the red section of this circle represent half the circle?      No.

When an object is cut into two parts, each part can only represent a half if the two parts are the same size.

The red part is greater than a half and the yellow section is less than a half.

The saying, 'Cut the cake into halves and I'll take the greatest half', is impossible if the cake was originally cut exactly into halves.

## Fractions on a Calculator




Fraction keys






To enter a simple fraction: use the  key.


Example: The fraction three eighths  $\frac{3}{8}$  can be entered as

  .

To enter a mixed number: use the () key.

Example: The mixed number two and five sixths  $2\frac{5}{6}$  can

be entered as  ()   

Sometimes the  button is used to navigate around the fraction so numbers can be entered into the correct part of the fraction. It is also used to move the cursor outside of the fraction in order to enter additional operators, numbers etc.

## Module contents

### Introduction

- Types of fractions
- Equivalent fractions
- Adding and subtracting fractions
- Multiplying and dividing fractions
- Changing between fractions and decimals

### Answers to activity questions

## Outcomes

- Name different types of fractions and convert between improper fractions and mixed numbers.
- Use equivalent fractions to compare the size of fractions.
- Perform all four operations using fractions.
- Convert fractions to decimals.

## Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module.

- Change  $3\frac{1}{4}$  to an improper fraction
  - Change  $\frac{17}{3}$  to a mixed number
- Find the missing number in  $\frac{3}{7} = \frac{12}{?}$
  - Simplify  $\frac{16}{24}$
  - Which is larger?  $\frac{7}{11}$  or  $\frac{3}{4}$
- Perform the operation indicated.
  - $2\frac{1}{4} + \frac{2}{3}$
  - $\frac{3}{8} - \frac{4}{5}$
  - $-\frac{3}{4} \times 2\frac{2}{5}$
  - $\frac{3}{4} \div \frac{2}{7}$
- Convert these fractions to decimals.
  - $\frac{3}{5}$
  - $-\frac{4}{7}$

Check your answers from the answer section at the end of the module.

## Topic 1: Types of fractions

There are two types of fractions: proper (or common) fractions and improper (or top heavy) fractions.

To be a proper fraction, the numerator must be less than the denominator, or the numerator  $<$  denominator (In maths, the symbol  $<$  means less than and the symbol  $>$  means greater than.) A proper fraction is less than a whole.

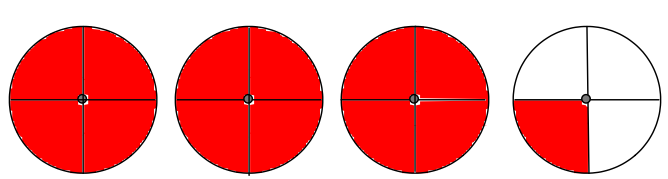
To be an improper fraction, the numerator must be greater than or equal to the denominator, or numerator  $\geq$  denominator. An improper fraction is greater than or equal to one whole.

For example:

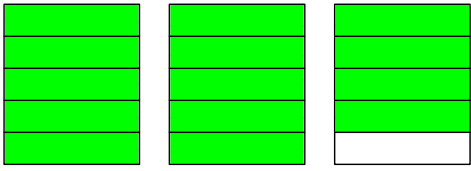
Fraction	Type	Reason
$\frac{1}{7}$	Proper	The numerator $<$ the denominator
$\frac{15}{11}$	Improper	The numerator $\geq$ the denominator
$\frac{23}{23}$	Improper	The numerator $\geq$ the denominator
$\frac{1}{9}$	Proper	The numerator $<$ the denominator
$\frac{105}{99}$	Improper	The numerator $\geq$ the denominator

If numerator = denominator, then the result is 1 whole. It is like a pizza cut into 8 pieces and all 8 pieces are present; this is obviously 1 whole pizza.

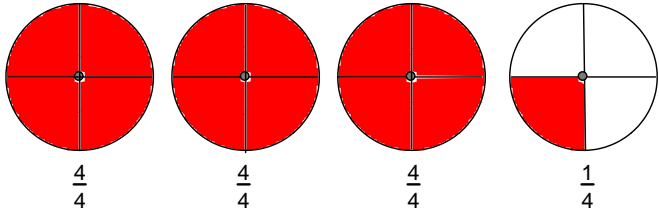
When a whole number and a fraction are combined, the result is a mixed number. A mixed number can represent this story: The boys ate three and a quarter pizzas last night. This is written and drawn as below.

$3\frac{1}{4}$	 <p>One whole    One whole    One whole    <math>\frac{1}{4}</math></p>
----------------	---

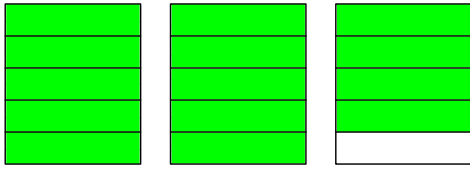
Two and four fifths can be written and drawn as below.

$2\frac{4}{5}$	 <p style="text-align: center;">One whole    One whole    <math>\frac{4}{5}</math></p>
----------------	--

It is important to be able to **change a mixed number to an improper fraction**. Let's look at the two examples above and change the mixed number to an improper fraction. Using the denominator of the fraction part (quarters), one whole is the same as four quarters, there are  $4+4+4+1$  quarters, which is 13 quarters.

Mixed Number	Diagram	Improper Fraction
$3\frac{1}{4}$	 <p style="text-align: center;"><math>\frac{4}{4}</math>    <math>\frac{4}{4}</math>    <math>\frac{4}{4}</math>    <math>\frac{1}{4}</math></p>	$\frac{13}{4}$

Using the denominator of the fraction part (fifths), one whole is the same as five fifths, there are  $5+5+4$  fifths, which is 14 fifths.

Mixed Number	Diagram	Improper Fraction
$2\frac{4}{5}$	 <p style="text-align: center;"><math>\frac{5}{5}</math>    <math>\frac{5}{5}</math>    <math>\frac{4}{5}</math></p>	$\frac{14}{5}$

The examples above explain how to change a mixed number to an improper fraction using diagrams, however a quicker method is to:

1. Multiply the whole number and the denominator of the fraction.
2. Add this to numerator of the fraction.
3. The improper fraction is the answer from step 2 over the denominator already used.

Examples:

Mixed Number

$$3\frac{1}{4}$$

Mental working

$$\frac{3 \times 4 + 1 = 13}{4}$$

Improper Fraction

$$\frac{13}{4}$$

Mixed Number

$$2\frac{4}{5}$$

Mental working

$$\frac{2 \times 5 + 4 = 14}{5}$$

Improper Fraction

$$\frac{14}{5}$$

More examples:

$$3\frac{2}{7} = \frac{3 \times 7 + 2}{7} = \frac{23}{7}$$


$$-5\frac{1}{3} = \frac{-(5 \times 3 + 1)}{3} = \frac{-16}{3}$$


$$12\frac{5}{6} = \frac{12 \times 6 + 5}{6} = \frac{77}{6}$$



[Video 'Changing Mixed Numbers to Improper Fractions'](#)

Some calculators (Casio series) have keys labelled  and , these are called the fraction keys.

A simple fraction is entered as such: numerator  denominator.

A mixed number is entered as: whole number  numerator  denominator.

A **mixed number** can be changed to a **improper fraction** using your calculator.

For example:

$$2\frac{4}{5}$$

$$\boxed{2} \left( \frac{\square}{\square} \right) \boxed{4} \left( \frac{\square}{\square} \right) \boxed{5} \boxed{=}$$

$$-5\frac{1}{3}$$

$$\boxed{(-)} \boxed{5} \left( \frac{\square}{\square} \right) \boxed{1} \left( \frac{\square}{\square} \right) \boxed{3} \boxed{=}$$

When **changing an improper fraction back to a mixed number**, divide by the denominator. Remember that the denominator tells us how many pieces make up a whole. So when you divide the numerator by the denominator, the whole number part of the answer is the number of whole numbers and the remainder is written as a fraction.

For example:

Improper Fraction

$$\frac{13}{5}$$

Mental working

$$\begin{array}{r} 2R3 \\ 5 \overline{)13} \end{array}$$

Mixed Number

$$2\frac{3}{5}$$

$$\frac{12}{7} \rightarrow 7 \overline{)12} \rightarrow 1\frac{5}{7}$$

$$\frac{-21}{9} \rightarrow 9 \overline{)21} \rightarrow -2\frac{3}{9} \rightarrow -2\frac{1}{3} \text{ Remember } (- \div + = -)$$

$$\frac{33}{3} \rightarrow 3 \overline{)33} \rightarrow 11\frac{0}{3} \rightarrow 11$$



[Video 'Changing Improper Fractions to Mixed Numbers'](#)

Some of these examples, done on a calculator, are below.

$$\frac{13}{5} \quad \boxed{1} \boxed{3} \boxed{\frac{\square}{\square}} \boxed{5} \boxed{=} \quad \frac{13}{5} \quad (a \frac{b}{c} \leftrightarrow \frac{d}{c}) \quad 2\frac{3}{5}$$

$$\frac{-21}{9} \quad \boxed{(-)} \boxed{2} \boxed{1} \boxed{\frac{\square}{\square}} \boxed{9} \boxed{=} \quad -\frac{21}{9} \quad (a \frac{b}{c} \leftrightarrow \frac{d}{c}) \quad -2\frac{1}{3}$$

Note: if a fraction is entered into a calculator that can be simplified, the calculator will automatically simplify it. Topic 2 covers Simplifying Fractions.



## Activity

1. Classify the following fractions as either proper or improper.

- |                     |                    |                     |
|---------------------|--------------------|---------------------|
| (a) $\frac{11}{3}$  | (b) $\frac{4}{7}$  | (c) $\frac{-5}{7}$  |
| (d) $\frac{10}{10}$ | (e) $\frac{16}{9}$ | (f) $\frac{1}{11}$  |
| (g) $\frac{-19}{3}$ | (h) $\frac{4}{9}$  | (i) $\frac{17}{19}$ |

2. Convert each of these mixed numbers to an improper fraction.

- |                     |                     |                      |
|---------------------|---------------------|----------------------|
| (a) $1\frac{1}{3}$  | (b) $-2\frac{5}{7}$ | (c) $3\frac{1}{5}$   |
| (d) $4\frac{7}{10}$ | (e) $2\frac{4}{9}$  | (f) $5\frac{3}{4}$   |
| (g) $-3\frac{3}{8}$ | (h) $7\frac{4}{9}$  | (i) $3\frac{10}{13}$ |

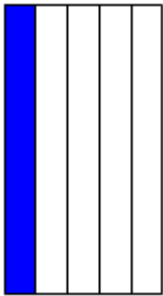
3. Convert each of these improper fractions to mixed numbers.

- |                     |                     |                     |
|---------------------|---------------------|---------------------|
| (a) $\frac{11}{3}$  | (b) $\frac{14}{7}$  | (c) $\frac{-25}{7}$ |
| (d) $\frac{10}{10}$ | (e) $\frac{-16}{9}$ | (f) $\frac{19}{11}$ |
| (g) $\frac{-19}{3}$ | (h) $\frac{44}{9}$  | (i) $\frac{57}{19}$ |

## Topic 2: Equivalent fractions

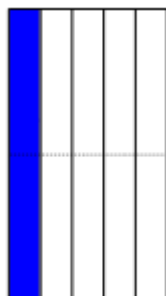
As background skills for addition and subtraction of fractions, the ability to make equivalent fractions is essential. Imagine you have a pizza with two pieces remaining of the original eight. This is two eighths of a pizza, and it is also one quarter!

Consider this diagram representing one fifth.



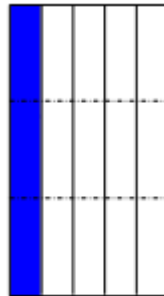
$$\frac{1}{5}$$

Now cut it. What is the equivalent fraction?



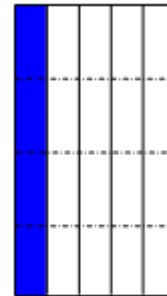
$$\frac{2}{10}$$

Cut it twice. What is the equivalent fraction?



$$\frac{3}{15}$$

Cut it 3 times. What is the equivalent fraction?



$$\frac{4}{20}$$

All of these fractions are equivalent. In each diagram there is the same amount it has just been cut into smaller pieces. Another way of looking at this is below:

The first four equivalent fractions of  $\frac{1}{5}$  are:

$$\frac{1^{1 \times 2}}{5^{1 \times 2}} = \frac{2}{10}, \quad \text{then} \quad \frac{1^{1 \times 3}}{5^{1 \times 3}} = \frac{3}{15}, \quad \text{then} \quad \frac{1^{1 \times 4}}{5^{1 \times 4}} = \frac{4}{20}, \quad \text{then} \quad \frac{1^{1 \times 5}}{5^{1 \times 5}} = \frac{5}{25}, \quad \text{and so on.}$$

Equivalent fractions of  $\frac{3}{8}$  are:

$$\left( \frac{3^{1 \times 2}}{8^{1 \times 2}} \right) = \frac{6}{16} \quad \left( \frac{3^{1 \times 3}}{8^{1 \times 3}} \right) = \frac{9}{24} \quad \left( \frac{3^{1 \times 4}}{8^{1 \times 4}} \right) = \frac{12}{32} \quad \left( \frac{3^{1 \times 5}}{8^{1 \times 5}} \right) = \frac{15}{40} \quad \left( \frac{3^{1 \times 6}}{8^{1 \times 6}} \right) = \frac{18}{48}$$

Sometimes, a number ( $n$ ) in an equivalent fraction is missing.

$\frac{1}{4} = \frac{n}{20}$ , looking at the denominator, we notice  $4 \times 5 = 20$ , so the numerator also needs to be multiplied by

5.  $\frac{1^{ \times 5}}{4^{ \times 5}} = \frac{5}{20}$ ,  $n$  is 5.

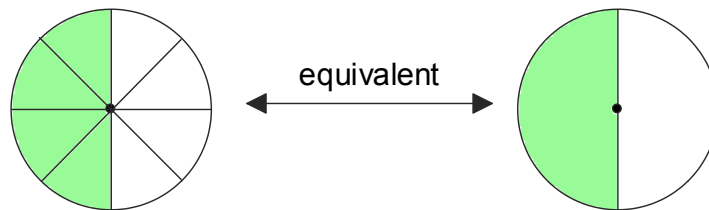
$\frac{3}{7} = \frac{12}{n}$ , looking at the numerator, we notice  $3 \times 4 = 12$ , so the denominator also needs to be multiplied by

4.  $\frac{3^{ \times 4}}{7^{ \times 4}} = \frac{12}{28}$ ,  $n$  is 28.



[Video 'Finding Equivalent Fractions'](#)

Fractions can also be **reduced** (or simplified) to find an equivalent fraction. Most pizza eaters will confirm that four eighths of a pizza is the same as one half.



Looking at the fraction  $\frac{10}{25}$  we notice that 5 goes into both 10 and 25, so 5 is a common factor to 10 and 25.

Now divide both the numerator and the denominator by the common factor 5,  $\frac{10^{ \div 5}}{25^{ \div 5}} = \frac{2}{5}$ , now the fraction has been expressed in lowest form.

Express  $\frac{24}{30}$  in its lowest form.

We need to find a common factor (preferably the highest) of 24 and 30. While both 2 and 3 are common factors, 6 is the highest common factor, so divide top and bottom by 6,  $\frac{24^{ \div 6}}{30^{ \div 6}} = \frac{4}{5}$ , the lowest form of  $\frac{24}{30}$  is

$$\frac{4}{5}$$

Simplify  $\frac{18}{27}$ .

We need to find a common factor (preferably the highest) of 18 and 27. While 3 is a common factor, 9 is the highest common factor, so divide top and bottom by 9,  $\frac{18^{ \div 9}}{27^{ \div 9}} = \frac{2}{3}$ , so  $\frac{18}{27}$  simplifies to  $\frac{2}{3}$ .

Simplifying fractions using a calculator is shown below:

Simplify $\frac{10}{25}$	$\boxed{1} \boxed{0} \boxed{\div} \boxed{2} \boxed{5} \boxed{=}$	$\frac{2}{5}$
Express $\frac{24}{30}$ in its lowest form.	$\boxed{2} \boxed{4} \boxed{\div} \boxed{3} \boxed{0} \boxed{=}$	$\frac{4}{5}$
Simplify $\frac{18}{27}$	$\boxed{1} \boxed{8} \boxed{\div} \boxed{2} \boxed{7} \boxed{=}$	$\frac{2}{3}$



[Video 'Simplifying Fractions'](#)

Fractions can be compared for size using equivalent fractions. Consider the following problem.

If John has  $\frac{4}{5}$  of a cake and Julie has  $\frac{7}{9}$  of the same type of cake, who has the most cake?

The only way to be sure is to change both fractions to equivalent fractions with the same denominator, using the lowest common denominator. With denominators 5 and 9, the lowest number that 5 and 9 goes into is 45. Be aware that multiplying the two denominators together will always give a common denominator, but it may not be the lowest!

Finishing the problem above:

$\frac{4}{5} = \frac{n}{45}$ , looking at the denominator, we notice  $5 \times 9 = 45$ , so the numerator also needs to be multiplied by

$$9. \frac{4 \times 9}{5 \times 9} = \frac{36}{45}.$$

$\frac{7}{9} = \frac{n}{45}$ , looking at the numerator, we notice  $9 \times 5 = 45$ , so the denominator also needs to be multiplied by

$$5. \frac{7 \times 5}{9 \times 5} = \frac{35}{45}.$$

$$\text{As } \frac{36}{45} > \frac{35}{45} \text{ then } \frac{4}{5} > \frac{7}{9}.$$

Example:

Is  $\frac{3}{4} > \frac{13}{16}$ ? The lowest number that 4 and 16 go into is 16, so the lowest common denominator is 16.

$\frac{3}{4} = \frac{n}{16}$ , looking at the denominator, we notice  $4 \times 4 = 16$ , so the numerator also needs to be multiplied by

$$4. \frac{3 \times 4}{4 \times 4} = \frac{12}{16}. \quad \frac{13}{16} \text{ is already written in sixteenths.}$$

The answer is False,  $\frac{3}{4} < \frac{13}{16}$  because  $\frac{12}{16} < \frac{13}{16}$ .



## Activity

1. Write five equivalent fractions for

(a)  $\frac{1}{3}$

(b)  $\frac{2}{5}$

(c)  $\frac{-5}{7}$

2. Find the value on  $n$  in these equivalent fractions.

(a)  $\frac{1}{3} = \frac{n}{15}$

(b)  $\frac{3}{4} = \frac{n}{24}$

(c)  $\frac{2}{7} = \frac{10}{n}$

(d)  $\frac{-5}{6} = \frac{-25}{n}$

(e)  $\frac{7}{3} = \frac{49}{n}$

(f)  $\frac{-1}{5} = \frac{n}{30}$

3. Simplify these fractions (write in lowest form). Check with your calculator.

(a)  $\frac{14}{21}$

(b)  $\frac{5}{25}$

(c)  $\frac{-10}{12}$

(d)  $\frac{30}{80}$

(e)  $\frac{7}{20}$

(f)  $\frac{-16}{32}$

(g)  $\frac{36}{45}$

(h)  $\frac{16}{24}$

(i)  $\frac{-12}{9}$

4. (a) Which is the larger fraction:  $\frac{4}{7}$  or  $\frac{5}{9}$ ?

(b) Which is the smaller fraction:  $\frac{3}{4}$  or  $\frac{2}{3}$ ?

(c) Is  $\frac{4}{11} > \frac{1}{3}$ ?

(d) Is  $\frac{3}{5} < \frac{5}{9}$ ?

5. What fraction of an hour is 35 minutes?

6. What fraction of a year are the months having 30 days?

7. Tom ate  $\frac{2}{5}$  of a pie, Amy ate  $\frac{1}{4}$  of a similar pie, who ate the most?

## Topic 3: Adding and subtracting fractions

When adding fractions together, such as 2 sevenths plus 3 sevenths, the answer is equal to 5 sevenths or in number form:  $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$ .

However, when adding one third and one quarter, you might have ‘a bit more than a half’ but that answer is not precise enough. It is clear from the first example that it is easy to add fractions when the denominators are the same. If the denominators are not the same, then using equivalent fractions, the original fraction(s) can be changed to achieve equivalent fractions with equal denominators.

<u>Nature of Denominators</u>	<u>Example</u>	<u>Solution</u>
<ul style="list-style-type: none"> <li>Denominators the same</li> </ul>	$\frac{2}{9} + \frac{5}{9}$	Just add the numerators. $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$
<ul style="list-style-type: none"> <li>Similar denominators (change 1 denominator)</li> </ul>	$\frac{2}{3} + \frac{1}{6}$	Because $\frac{2}{3}$ can easily be written as the equivalent fraction $\frac{4}{6}$ , the denominators both will be 6, so the question becomes $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$ .
<ul style="list-style-type: none"> <li>Unlike denominators (change both denominators)</li> </ul>	$\frac{1}{3} + \frac{1}{4}$	<p>To choose a new denominator, think ‘what is the smallest number that both 3 and 4 go into?’</p> <p>Sometimes the answer is obtained by just multiplying 3 and 4 but it is worth checking to see if there is a lower number. The new denominator is 12.</p> <p><math>\frac{1 \times 4}{3 \times 4} = \frac{?}{12}</math> 3 was multiplied by 4 to give 12, so 1 has to be multiplied by 4 also to give 4 so <math>\frac{1}{3} = \frac{4}{12}</math></p> <p><math>\frac{1 \times 3}{4 \times 3} = \frac{?}{12}</math> 4 was multiplied by 3 to give 12, so 1 has to be multiplied by 3 also to give 3 so <math>\frac{1}{4} = \frac{3}{12}</math></p> <p>The question becomes <math>\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}</math></p>

This example has denominators the **same**

$$\frac{4}{9} + \frac{2}{9}$$

The denominators are both the same, just add the numerators.

$$= \frac{6+3}{9+3}$$

The answer should always be checked to see if it will simplify, 3 is a common factor of both 6 and 9.

$$= \frac{2}{3}$$

The simplified answer

This example has **similar** denominators

$$\frac{1}{2} + \frac{3}{8}$$

The denominators are similar, because 2 will go into 8, new denominator is 8.

$$= \frac{4}{8} + \frac{3}{8}$$

$\frac{1}{2} = \frac{?}{8}$  is multiplied by 4 to give 8, so 1 should be multiplied by 4 also.  $\frac{1}{2} = \frac{4}{8}$

$$= \frac{7}{8}$$

This answer cannot be simplified.

This example has **unlike** denominators

$$\frac{1}{4} + \frac{3}{5}$$

The denominators are unlike, the lowest number that 4 and 5 go into is 20 (4x5).

$$= \frac{5}{20} + \frac{12}{20}$$

$\frac{1}{4} = \frac{?}{20}$  4 is multiplied by 5 to give 20, so 1 should be multiplied by 5.  $\frac{1}{4} = \frac{5}{20}$

$\frac{3}{5} = \frac{?}{20}$  5 is multiplied by 4 to give 20, so 3 should be multiplied by 4.  $\frac{3}{5} = \frac{12}{20}$

$$= \frac{17}{20}$$

This answer cannot be simplified.

This example has **unlike** denominators, however, this time the lowest common denominator is not the product of the two individual denominators.

$$\frac{7}{10} + \frac{11}{15}$$

The denominators are unlike, the lowest number that 10 and 15 go into is 30.

$\frac{7}{10} = \frac{?}{30}$  10 is multiplied by 3 to give 30, so 7 should be multiplied by 3.

$$\frac{7}{10} = \frac{21}{30}$$

$$= \frac{21}{30} + \frac{22}{30}$$

$\frac{11}{15} = \frac{?}{30}$  15 is multiplied by 2 to give 30, so 11 should be multiplied by 2.

$$\frac{11}{15} = \frac{22}{30}$$

$$= \frac{43}{30} = 1\frac{13}{30}$$

This answer is an improper fraction, change it to a mixed number.

This example has a whole number added to two fractions with **unlike** denominators.

$$\frac{1}{3} + \frac{2}{7} + 2$$

2 must be written with a denominator. Any whole number has a denominator of 1.

$$= \frac{1}{3} + \frac{2}{7} + \frac{2}{1}$$

The denominators are unlike, the lowest number that 1, 3 and 7 goes into is 21.

$$= \frac{7}{21} + \frac{6}{21} + \frac{42}{21}$$

$\frac{1}{3} = \frac{?}{21}$  3 is multiplied by 7 to give 21, so 1 should be multiplied by 7.  $\frac{1^{x7}}{3^{x7}} = \frac{7}{21}$

$\frac{2}{7} = \frac{?}{21}$  7 is multiplied by 3 to give 21, so 2 should be multiplied by 3.  $\frac{2^{x3}}{7^{x3}} = \frac{6}{21}$

$\frac{2}{1} = \frac{?}{21}$  1 is multiplied by 21 to give 21, so 2 should be multiplied by 21.  $\frac{2^{x21}}{1^{x21}} = \frac{42}{21}$

$$= \frac{55}{21} = 2\frac{13}{21}$$

This answer is an improper fraction, change it to a mixed number.

If the question involves mixed numbers, change them to improper fractions first.

$$1\frac{1}{4} + 2\frac{7}{8}$$

Change the mixed numbers to improper fractions.

$$= \frac{5}{4} + \frac{23}{8}$$

The denominators are **similar** because 4 goes into 8, the new denominator is 8.

$$= \frac{10}{8} + \frac{23}{8}$$

$\frac{5}{4} = \frac{?}{8}$  4 is multiplied by 2 to give 8, so 5 should be multiplied by 2.  $\frac{5}{4} = \frac{10}{8}$

$$= \frac{33}{8} = 4\frac{1}{8}$$

This answer is an improper fraction, change it to a mixed number.

Subtraction questions are performed by the same method except subtraction replaces addition.

$$1\frac{1}{4} - \frac{3}{7}$$

Change the mixed number to an improper fraction. The denominators are unlike, the lowest number that 4 and 7 go into is 28.

$$= \frac{5}{4} - \frac{3}{7}$$

$\frac{5}{4} = \frac{?}{28}$  4 is multiplied by 7 to give 28, so 5 should be multiplied by 7.  $\frac{5}{4} = \frac{35}{28}$

$$= \frac{35}{28} - \frac{12}{28}$$

$\frac{3}{7} = \frac{?}{28}$  7 is multiplied by 4 to give 28, so 3 should be multiplied by 4.  $\frac{3}{7} = \frac{12}{28}$

$$= \frac{23}{28}$$

This answer cannot be simplified.



The fractions may be positive or negative.

$$\frac{-7}{8} + \frac{-3}{5}$$

Reduce the double sign + - to -.

$$\frac{-7}{8} - \frac{3}{5}$$

The denominators are unlike, the lowest number that 8 and 5 go into is 40.

$$\frac{-7}{8} = \frac{?}{40} \text{ 8 is multiplied by 5 to give 40, so } -7 \text{ should be multiplied by 5.}$$

$$\frac{-7}{8} = \frac{-35}{40}$$

$$= \frac{-35}{40} - \frac{24}{40}$$

$$\frac{3}{5} = \frac{?}{40} \text{ 5 is multiplied by 8 to give 40, so 3 should be multiplied by 8.}$$

$$\frac{3}{5} = \frac{24}{40}$$

$$= \frac{-59}{40}$$




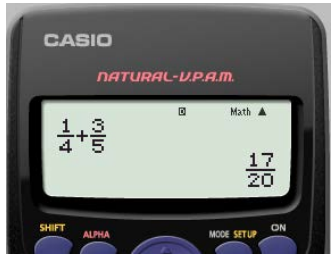

This answer is an improper fraction, change it to a mixed number.

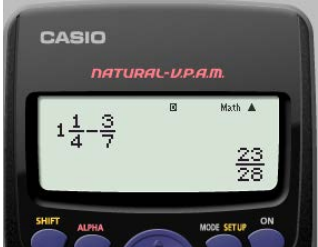

$$= -1\frac{19}{40}$$



[Video 'Adding and Subtracting Fractions'](#)

Some of these examples, done on a calculator, are below.

$\frac{4}{9} + \frac{2}{9}$	 <p>Note: Adds and simplifies</p>	
$\frac{1}{4} + \frac{3}{5}$		
$1\frac{1}{4} + 2\frac{7}{8}$	 <p>To change to a mixed number, press <math>(\frac{a}{c} + \frac{d}{c})</math> the display will show <math>4\frac{1}{8}</math></p>	

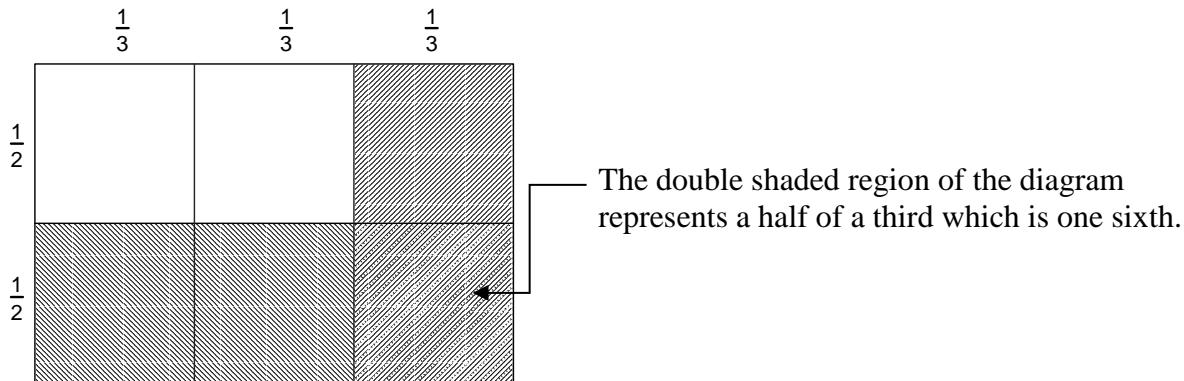
$1\frac{1}{4} - \frac{3}{7}$	$\boxed{1} \boxed{=} \boxed{\frac{1}{4}} \boxed{1} \boxed{=} \boxed{4} \boxed{=} \boxed{-} \boxed{3} \boxed{=} \boxed{\frac{3}{7}} \boxed{=} \frac{23}{28}$	
$-\frac{7}{8} + \frac{-3}{5}$	$\boxed{(-)} \boxed{7} \boxed{=} \boxed{\frac{7}{8}} \boxed{+} \boxed{(-)} \boxed{3} \boxed{=} \boxed{\frac{3}{5}} \boxed{=} -\frac{59}{40}$ To change to a mixed number, press $(\frac{a}{b} + \frac{d}{c})$ the display will show $-1\frac{9}{40}$	

## Activity

- Perform the operation indicated in the question (check with a calculator).
  - $\frac{3}{8} + \frac{1}{8}$
  - $\frac{3}{4} + \frac{7}{12}$
  - $\frac{5}{6} - \frac{1}{3}$
  - $\frac{3}{4} - 1\frac{1}{3}$
  - $\frac{7}{12} - \frac{4}{15}$
  - $\frac{-4}{7} - \frac{1}{3}$
  - $4\frac{1}{2} - 6\frac{11}{14}$
  - $-1\frac{1}{4} - -3\frac{2}{3}$
- A nurse screened  $\frac{7}{16}$  of her patients in the first hour and  $\frac{5}{16}$  in the second hour.
  - What fraction of her patients was screened in the two hours?
  - What fraction was left to screen?
- Inside a house,  $\frac{1}{10}$  of water is used in the kitchen,  $\frac{1}{8}$  in the laundry,  $\frac{1}{4}$  in the toilet and  $\frac{3}{8}$  in the bathroom.
  - What is the fraction total of water that is used in the house?
  - What is the fraction of water is used outside the house?
- James is about to buy sugar for two cakes and a slice. The chocolate cake requires  $2\frac{1}{2}$  cups, the sultana cake requires  $1\frac{1}{3}$  cups and the caramel slice requires  $1\frac{1}{4}$  cups.
  - How many cups of sugar will he need altogether?
  - If each cup contains 230g (0.23 kg), will a 2kg pack of sugar be enough?

## Topic 4: Multiplying and dividing fractions

To help understand the concept of multiplying fraction, let's consider a question such as a half of a third. A diagram that represents this is:



The diagram above can be represented in number form as  $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ .

Using this example, an observation is possible.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \quad \begin{array}{l} \text{Multiply numerators} \rightarrow \text{Numerator of answer} \\ \text{Multiply denominators} \rightarrow \text{Denominator of answer} \end{array}$$

This observation works with all possible questions, so the rule for multiplying fractions is: Multiply two fractions by multiplying the numerators and multiplying the denominators.

Examples:

$$\frac{3}{4} \times \frac{1}{5}$$

$$= \frac{3 \times 1}{4 \times 5}$$

$$= \frac{3}{20}$$

Multiply numerators, multiply denominators.

$$\frac{1}{4} \times \frac{8}{9} \times \frac{2}{7}$$

$$= \frac{1 \times 8 \times 2}{4 \times 9 \times 7}$$

$$= \frac{16^{+4}}{252^{+4}}$$

$$= \frac{4}{63}$$

Multiply numerators, multiply denominators.

This answer can be simplified

In the second question (above), the answer had to be simplified, which meant dividing the numerator and the denominator by a common factor. There is shortcut that allows this division to be performed before multiplying the fractions.

For example:

$$\frac{3}{4} \times \frac{8}{11}$$

No short cut

$$= \frac{24}{44}$$

Simplify

$$= \frac{6}{11}$$

$$\frac{3}{4} \times \frac{8}{11}$$

Shortcut method

$$= \frac{3}{\cancel{4}^1} \times \frac{\cancel{8}^2}{11}$$

4 goes into 8, 2 times  
4 goes into 4, 1 time

$$= \frac{6}{11}$$

For this method to work, one numerator and one denominator is divided by a common factor. More examples are below.

$$\frac{4}{3} \times \frac{9}{16}$$

$$= \frac{\cancel{4}^1}{\cancel{3}^1} \times \frac{\cancel{9}^3}{\cancel{16}^4}$$

4 goes into 4, 1 time  
4 goes into 16, 4 times  
3 goes into 9, 3 times  
3 goes into 3, 1 time

$$= \frac{3}{4}$$



$$\frac{1}{8} \times \frac{4}{5} \times \frac{10}{11}$$

$$= \frac{1}{\cancel{8}^{2^1}} \times \frac{\cancel{4}^1}{\cancel{5}^1} \times \frac{\cancel{10}^{2^1}}{11}$$

4 goes into 4, 1 time  
4 goes into 8, 2 times  
5 goes into 10, 2 times  
5 goes into 5, 1 time  
Looking at the 2 in the numerator and the 2 in the denominator, divide these by 2  
2 goes into 2, 1 time  
2 goes into 2, 1 time

$$= \frac{1}{11}$$

If the question contains mixed numbers, these must be made improper before multiplying using either method. Some examples containing mixed numbers are below:

$2\frac{2}{3} \times \frac{6}{7}$ <p>Change mixed numbers to improper fractions</p> $= \frac{8}{\cancel{3}^1} \times \frac{\cancel{6}^2}{7}$ <p>3 goes into 6, 2 times 3 goes into 3, 1 time</p> $= \frac{16}{7}$ <p>Answer is improper, change to a mixed number.</p> $= 2\frac{2}{7}$	$1\frac{3}{4} \times 2\frac{3}{8}$ <p>Change mixed numbers to improper fractions</p> $= \frac{7}{4} \times \frac{19}{8}$ <p>There is no common factor.</p> $= \frac{133}{32}$ <p>Answer is improper, change to a mixed number.</p> $= 4\frac{5}{32}$
<p>Using a calculator:</p>  <p>To change to a mixed number, press <math>(a\frac{b}{c}+d)</math> gives <math>2\frac{2}{7}</math></p>	<p>Using a calculator:</p>  <p>To change to a mixed number, press <math>(a\frac{b}{c}+d)</math> gives <math>4\frac{5}{32}</math></p>

If the question contains an integer, remember any integer can be written over 1.


$$\frac{2}{7} \times 1400$$

Write 1400 with a denominator of 1.

$$\frac{2}{7^1} \times \frac{1400^{200}}{1} \quad \begin{array}{l} 7 \text{ goes into } 1400, 200 \text{ times} \\ 7 \text{ goes into } 7, 1 \text{ time} \end{array}$$

$$= \frac{400}{1} = 400$$

Using a calculator:





[Video 'Multiplying Fractions'](#)

## Dividing Fractions

To introduce this section, consider this scenario: Dividing a pizza into 2 is the same as taking half of a pizza?

The operation  $\div 2$  (or  $\div \frac{2}{1}$ ) is the same as  $\times \frac{1}{2}$

When the operation changed from  $\div$  to  $\times$ , the number changed from  $\frac{2}{1}$  to  $\frac{1}{2}$ .

When a fraction 'flips' like this, it is called taking the **reciprocal** of a number.

The reciprocal of  $\frac{2}{3}$  is  $\frac{3}{2}$ .

The reciprocal of  $2\frac{2}{5}$  (which as an improper fraction is  $\frac{12}{5}$ ) is  $\frac{5}{12}$ .

The reciprocal of 8 (which as a fraction is  $\frac{8}{1}$ ) is  $\frac{1}{8}$ .

To solve division of fraction questions, the idea above can be used. The division can be changed into a multiplication, multiplying by the reciprocal.

For example:

$$\frac{3}{4} \div \frac{5}{7}$$

Change the operation from  $\div$  to  $\times$ , multiply by the reciprocal.

$$= \frac{3}{4} \times \frac{7}{5}$$

$$= \frac{21}{20} = 1\frac{1}{20}$$

$$\frac{-5}{8} \div \frac{3}{4}$$

$$= \frac{-5}{8} \times \frac{4^1}{3}$$

$$= \frac{-5}{6}$$

Change the operation from  $\div$  to  $\times$ , multiply by the reciprocal.

4 goes into 4, 1 time  
4 goes into 8, 2 times

$$^- \times ^+ = ^-$$

$$2\frac{2}{5} \div \frac{6}{7}$$

Change the mixed number to an improper fraction

$$5 \div 1\frac{7}{8}$$

Change the mixed numbers to an improper fractions

$$= \frac{12}{5} \div \frac{6}{7}$$

Change the operation from  $\div$  to  $\times$ , multiply by the reciprocal.

$$= \frac{5}{1} \div \frac{15}{8}$$

Change the operation from  $\div$  to  $\times$ , multiply by the reciprocal.

$$= \frac{\cancel{12}^2}{5} \times \frac{7}{\cancel{6}^1}$$

6 goes into 12, 2 times  
6 goes into 6, 1 time

$$= \frac{\cancel{5}^1}{1} \times \frac{8}{\cancel{15}^3}$$

5 goes into 5, 1 time  
5 goes into 15, 3 times

$$= \frac{14}{5} = 2\frac{4}{5}$$

Change the improper fraction back to a mixed number

$$\frac{8}{3} = 2\frac{2}{3}$$

Change the improper fraction back to a mixed number

$$1\frac{4}{7} \div \frac{-3}{5}$$

Change the mixed number to an improper fraction

$$\frac{11}{7} \div \frac{-3}{5}$$

Change the operation from  $\div$  to  $\times$ , multiply by the reciprocal.

$$\frac{11}{7} \times \frac{-5}{3}$$

$$= \frac{-55}{21} = -2\frac{13}{21}$$

Change the improper fraction back to a mixed number

The three fractions below are equal. Keep the negative sign either with the numerator or in front of the entire fraction.


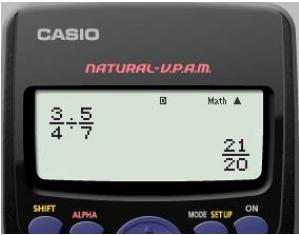


$$\frac{-3}{5} \text{ or } -\frac{3}{5} \text{ not } \frac{3}{-5}$$

Never leave the negative sign with the denominator



### [Video 'Dividing Fractions'](#)

Some of these, done on a calculator, are below.

$\frac{3}{4} \div \frac{5}{7}$		
$-\frac{5}{8} \div \frac{3}{4}$		

$5 \div 1\frac{7}{8}$		
-----------------------	--	--

## Mixed operation fraction operations

When doing mixed operation questions, the order of operations rules must be followed.

$$\frac{3}{8} \div \frac{6}{11} \times \frac{4}{5}$$

Change the operation from  $\div$  to  $\times$ , multiply by the reciprocal.

$$1\frac{7}{8} + \frac{2}{5} \times 1\frac{2}{3}$$

Change the mixed numbers to an improper fractions

$$= \frac{\cancel{3}^1}{\cancel{8}^2} \times \frac{11}{\cancel{6}^2} \times \frac{4}{5}$$

Use shortcut method

$$\frac{15}{8} + \frac{2}{\cancel{5}^1} \times \frac{\cancel{5}^1}{3}$$

Do multiplication first

$$= \frac{11}{20}$$

$$= \frac{15^{\times 3}}{8^{\times 3}} + \frac{2^{\times 8}}{3^{\times 8}}$$

Add using a common denominator of 24

$$= \frac{45}{24} + \frac{16}{24}$$

Change the improper fraction back to a mixed number

$$= \frac{61}{24} = 2\frac{13}{24}$$



[Video 'Mixed Operations with Fractions'](#)

Some of these examples, done on a calculator, are below.

$$\frac{3}{8} \div \frac{6}{11} \times \frac{4}{5}$$

$$1\frac{7}{8} + \frac{2}{5} \times 1\frac{2}{3}$$

## Activity

1. Perform the operation indicated in the question, check with you calculator.

(a)  $\frac{4}{5} \times \frac{3}{8}$

(b)  $\frac{3}{4} \times \frac{7}{12}$

(c)  $1\frac{3}{7} \times 2\frac{9}{20}$

(d)  $\frac{2}{5} \times 800$

(e)  $\frac{2}{9} \times \frac{3}{10} \times 1\frac{4}{11}$

(f)  $2\frac{1}{3} \times 1\frac{2}{7} \times \frac{3}{4}$

2. Perform the operation indicated in the question, check with you calculator.

(a)  $\frac{-1}{2} \div \frac{-1}{3}$

(b)  $\frac{3}{4} \div \frac{9}{11}$

(c)  $3\frac{4}{7} \div \frac{5}{9}$

(d)  $\frac{3}{4} \div 4$

(e)  $8 \div \frac{2}{9}$

(f)  $\frac{3}{4} \times \frac{1}{7} \div \frac{3}{8}$

3. A recent survey showed that  $\frac{2}{5}$  of students have hearing problems. In a school of 450 students, how many would you expect to have hearing problems?

4. How many  $\frac{1}{8}$  are there in  $\frac{7}{9}$ ?

5. Joanne has a  $1\frac{1}{3}$  km circuit that she jogs around. How far will she cover if she jogs 5 circuits?

6. Jim has a  $5\frac{1}{4}$  Ha block of land, a large dam covers  $\frac{2}{11}$  of the property. What area does the dam cover?



## Topic 5: Changing between fractions and decimals

### Fractions to decimals

If your knowledge of decimals is limited, it is suggested that you read the module “Decimals” to have a better understanding of the material below.

There are 2 methods of changing fractions to decimals.

The first method uses the place value of decimals ie: tenths, hundredths, thousandths etc. This method is a quick way of changing from a fraction to a decimal but is limited in what fractions it works for.

The second method involves division. The vinculum (bar) in a fraction means division so changing to a decimal involves dividing the numerator by the denominator.

#### Method One:

Look at the fraction. If the fraction has a denominator of 10, 100, 1000 etc. or if an equivalent fraction has a denominator of 10, 100, 1000 etc. it can just be written as the decimal equivalent.

For example:

$$\frac{3}{10}$$

$$= 0.3$$

This is three tenths, as a decimal this is a 3 in the tenths place value.

$$\frac{2^{\times 2}}{5^{\times 2}}$$

$$= \frac{4}{10}$$

$$= 0.4$$

Two fifths is equivalent to four tenths, as a decimal this is a 4 in the tenths place value.

$$\frac{3^{\times 25}}{4^{\times 25}}$$

$$= \frac{75}{100}$$

$$= 0.75$$

Three quarters is equivalent to seventy five hundredths, as a decimal this is 70 hundredths or 7 tenths and 5 hundredths.

$$\frac{3^{\times 5}}{20^{\times 5}}$$

$$= \frac{15}{100}$$

$$= 0.15$$

Three twentieths is equivalent to fifteen hundredths, as a decimal this is 10 hundredths or 1 tenth and 5 hundredths.

$$\frac{7}{100}$$

$$= 0.07$$

This is seven hundredths, as a decimal this is 7 in the hundredths place value.

$$\frac{3^{\times 125}}{8^{\times 125}}$$

$$= \frac{375}{1000}$$

$$= 0.375$$

Three eights is equivalent to three hundred and seventy five thousandths, as a decimal this is 300 thousandths or 3 tenths, 70 thousandths or 7 hundredths and 5 thousandths

### Method Two:

Remember this method always works and is suitable for calculator use. An alternative way of writing a fraction is numerator  $\div$  denominator, performing this division results in a decimal answer.

For example:

$$\frac{3}{5} \quad \text{This is } 3 \div 5 \text{ calculated by } 5 \overline{)3.0} \quad \begin{array}{r} 0.6 \\ 5 \overline{)3.0} \end{array}$$

To be able to divide, .0 is added to the three, the number 3 is not changed, and this enables tenths to be shared.

$$\frac{4}{9} \quad \text{This is } 4 \div 9 \text{ calculated by } 9 \overline{)4.0^40^40} \quad \begin{array}{r} 0.44\dot{4} \\ 9 \overline{)4.0^40^40} \end{array}$$

In this question, a decimal point and many zeros were added. The answer is a recurring decimal.

$$\frac{2}{7} \quad \text{This is } 2 \div 7 \text{ calculated by } 7 \overline{)2.0^60^40^50^10} \quad \begin{array}{r} 0.28571 \\ 7 \overline{)2.0^60^40^50^10} \end{array}$$

Continuing to divide by 7 will result in a recurring decimal. Usually answering to three or four decimal places is enough. This is 0.2857 to 4 decimal places.

$$\frac{9}{4} \quad \text{This is } 9 \div 4 \text{ calculated by } 4 \overline{)9.10^20} \quad \begin{array}{r} 2.25 \\ 4 \overline{)9.10^20} \end{array}$$

Fractions both proper and improper can be changed to a decimal this way.

$$\frac{-7}{8} \quad \text{This is } 7 \div 8 \text{ calculated by } 8 \overline{)7.0^60^40} \quad \begin{array}{r} 0.875 \\ 8 \overline{)7.0^60^40} \end{array}$$

As a  $\frac{-}{+} = \frac{-}{-}$ , the answer is  $\frac{-}{-}0.875$



[Video 'Changing Fractions to Decimals'](#)

These examples done on a calculator are below.

	Using the $\div$ key	Using the fraction $\frac{\square}{\square}$ key
$\frac{3}{5}$	$3 \div 5 = 0.6$	$\frac{3}{5} = 0.6$
$\frac{4}{9}$	$4 \div 9 = 0.4444444444$	$\frac{4}{9} = 0.4444444444$
$\frac{2}{7}$	$2 \div 7 = 0.2857142857$	$\frac{2}{7} = 0.2857142857$
$\frac{9}{4}$	$9 \div 4 = 2.25$	$\frac{9}{4} = 2.25$
$\frac{-7}{8}$	$(-)\ 7 \div 8 = -0.875$	$(-)\ \frac{7}{8} = -0.875$

## Decimals to fractions

Using the place value of decimals, it is easy to change these to fractions.

Already we know that  $0.3 = \frac{3}{10}$ . The final thing to consider is whether the fraction will simplify. In this example the fraction does not simplify.

Decimal	written as a fraction is ....
0.05	$0.05 = \frac{5^{-5}}{100^{-5}} = \frac{1}{20}$
0.001	$0.001 = \frac{1}{1000}$
0.49	$0.49 = \frac{49}{100}$
0.65	$0.65 = \frac{65^{-5}}{100^{-5}} = \frac{13}{20}$
$^{-}0.44$	$^{-}0.44 = \frac{^{-}44^{-4}}{100^{-4}} = \frac{^{-}11}{25}$
2.75	$2.75 = 2\frac{75^{-25}}{100^{-25}} = 2\frac{3}{4}$
3.87	$3.87 = 3\frac{87}{100}$



[Video 'Changing Decimals to Fractions'](#)

### Activity

1. Change to following fractions to decimals, check with you calculator.

- |     |                   |     |                   |     |                |
|-----|-------------------|-----|-------------------|-----|----------------|
| (a) | $\frac{7}{10}$    | (b) | $\frac{4}{5}$     | (c) | $\frac{3}{11}$ |
| (d) | $\frac{^{-}2}{3}$ | (e) | $\frac{^{-}7}{4}$ | (f) | $\frac{13}{9}$ |

2. Write the following decimals as simplified fractions.

- |     |      |     |       |     |       |
|-----|------|-----|-------|-----|-------|
| (a) | 0.08 | (b) | 0.6   | (c) | 0.14  |
| (d) | 0.64 | (e) | 4.25  | (f) | 6.2   |
| (g) | 12.5 | (h) | 3.125 | (i) | 0.075 |

## Answers to activity questions

### Check your skills

1. (a)

$$3\frac{1}{4} = \frac{3 \times 4 + 1}{4} = \frac{13}{4}$$

(b)

$$\frac{17}{3} = 5\frac{2}{3} \quad \left( \begin{array}{l} 5R2 \\ 3 \overline{)17} \end{array} \right)$$

2. (a)

$$\frac{3}{7} = \frac{12}{?}$$

$$\frac{3^{\times 4}}{7^{\times 4}} = \frac{12}{?}$$

The missing number is 28.

(b)

$$\frac{16^{-8}}{24^{-8}} = \frac{2}{3}$$

(c)

Use denominator of 44.

$$\frac{7^{\times 4}}{11^{\times 4}} = \frac{28}{44} \quad \frac{3^{\times 11}}{4^{\times 11}} = \frac{33}{44}$$

$\frac{3}{4}$  is larger.

3. (a)

$$2\frac{1}{4} + \frac{2}{3}$$

$$= \frac{9^{\times 3}}{4^{\times 3}} + \frac{2^{\times 4}}{3^{\times 4}}$$

$$= \frac{27}{12} + \frac{8}{12}$$

$$= \frac{35}{12} = 2\frac{11}{12}$$

(b)

$$\frac{3^{\times 5}}{8^{\times 5}} - \frac{4^{\times 8}}{5^{\times 8}}$$

$$= \frac{15}{40} - \frac{32}{40}$$

$$= \frac{-17}{40}$$

(c)

$$\frac{-3}{4} \times 2\frac{2}{5}$$

$$= \frac{-3}{4^1} \times \frac{12^{\times 3}}{5}$$

$$= \frac{-9}{5} = -1\frac{4}{5}$$

(d)

$$\frac{3}{4} \div \frac{2}{7}$$

$$= \frac{3}{4} \times \frac{7}{2}$$

$$= \frac{21}{8} = 2\frac{5}{8}$$

4. (a)

$$\frac{3^{\times 2}}{5^{\times 2}} = \frac{6}{10} = 0.6$$

(b)

$$\frac{-4}{7} = -0.5714 \text{ to 4 decimal places}$$

$$\begin{array}{r} 0.57142 \\ 7 \overline{)4.050103020} \end{array}$$

## Types of fractions

1. (a)  $\frac{11}{3}$   
improper  
(d)  $\frac{10}{10}$   
improper  
(g)  $\frac{-19}{3}$   
improper
- (b)  $\frac{4}{7}$   
proper  
(e)  $\frac{16}{9}$   
improper  
(h)  $\frac{4}{9}$   
proper
- (c)  $\frac{-5}{7}$   
proper  
(f)  $\frac{1}{11}$   
proper  
(i)  $\frac{17}{19}$   
proper
2. (a)  $1\frac{1}{3} = \frac{1 \times 3 + 1}{3} = \frac{4}{3}$  (b)  $-2\frac{5}{7} = \frac{-(2 \times 7 + 5)}{7} = \frac{-19}{7}$  (c)  $3\frac{1}{5} = \frac{3 \times 5 + 1}{5} = \frac{16}{5}$   
(d)  $4\frac{7}{10} = \frac{4 \times 10 + 7}{10} = \frac{47}{10}$  (e)  $2\frac{4}{9} = \frac{2 \times 9 + 4}{9} = \frac{22}{9}$  (f)  $5\frac{3}{4} = \frac{5 \times 4 + 3}{4} = \frac{23}{4}$   
(g)  $-3\frac{3}{8} = \frac{-(3 \times 8 + 3)}{8} = \frac{-27}{8}$  (h)  $7\frac{4}{9} = \frac{7 \times 9 + 4}{9} = \frac{67}{9}$  (i)  $3\frac{10}{13} = \frac{3 \times 13 + 10}{13} = \frac{49}{13}$
3. (a)  $\frac{11}{3} = 3\frac{2}{3}$  (b)  $\frac{14}{7} = 2$  (c)  $\frac{-25}{7} = -3\frac{4}{7}$   
(d)  $\frac{10}{10} = 1$  (e)  $\frac{-16}{9} = -1\frac{7}{9}$  (f)  $\frac{19}{11} = 1\frac{8}{11}$   
(g)  $\frac{-19}{3} = -6\frac{1}{3}$  (h)  $\frac{44}{9} = 4\frac{8}{9}$  (i)  $\frac{57}{19} = 3$

## Equivalent fractions

1. (a)  $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$   
(b)  $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30}$   
(c)  $\frac{-5}{7} = \frac{-10}{14} = \frac{-15}{21} = \frac{-20}{28} = \frac{-25}{35} = \frac{-30}{42}$
2. (a)  $\frac{1}{3} = \frac{n}{15}$   
as  $3 \times 5 = 15$   
 $n = 1 \times 5 = 5$
- (b)  $\frac{3}{4} = \frac{n}{24}$   
as  $4 \times 6 = 24$   
 $n = 3 \times 6 = 18$
- (c)  $\frac{2}{7} = \frac{10}{n}$   
as  $2 \times 5 = 10$   
 $n = 7 \times 5 = 35$
- (d)  $\frac{-5}{6} = \frac{-25}{n}$   
as  $-5 \times 5 = -25$   
 $n = 6 \times 5 = 30$
- (e)  $\frac{7}{3} = \frac{49}{n}$   
as  $7 \times 7 = 49$   
 $n = 3 \times 7 = 21$
- (f)  $\frac{-1}{5} = \frac{n}{30}$   
as  $5 \times 6 = 30$   
 $n = -1 \times 6 = -6$

3. (a)  $\frac{14^{-7}}{21^{-7}} = \frac{2}{3}$       (b)  $\frac{5^{-5}}{25^{-5}} = \frac{1}{5}$       (c)  $\frac{-10^{-2}}{12^{-2}} = \frac{-5}{6}$   
 (d)  $\frac{30^{-10}}{80^{-10}} = \frac{3}{8}$       (e)  $\frac{7}{20}$       (f)  $\frac{-16^{-16}}{32^{-16}} = \frac{-1}{2}$   
 (g)  $\frac{36^{-9}}{45^{-9}} = \frac{4}{5}$       (h)  $\frac{16^{-8}}{24^{-8}} = \frac{2}{3}$       (i)  $\frac{-12^{-3}}{9^{-3}} = \frac{-4}{3}$  or  $-1\frac{1}{3}$

Will not simplify

4. (a) Choose the lowest common denominator: 63 (from 7x9)  
 $\frac{4^9}{7^9} = \frac{36}{63}$      $\frac{5^7}{9^7} = \frac{35}{63}$     as  $\frac{36}{63} > \frac{35}{63}$ ,  $\frac{4}{7}$  is larger  
 (b) Choose the lowest common denominator: 12 (from 3x4)  
 $\frac{3^3}{4^3} = \frac{9}{12}$      $\frac{2^4}{3^4} = \frac{8}{12}$     as  $\frac{9}{12} > \frac{8}{12}$ ,  $\frac{2}{3}$  is the smaller.  
 (c) Choose the lowest common denominator: 33 (from 3x11)  
 $\frac{4^3}{11^3} = \frac{12}{33}$      $\frac{1^{11}}{3^{11}} = \frac{11}{33}$     as  $\frac{12}{33} > \frac{11}{33}$ , then  $\frac{4}{11} > \frac{1}{3}$  TRUE.  
 (d) Choose the lowest common denominator: 45 (from 5x9)  
 $\frac{3^9}{5^9} = \frac{27}{45}$      $\frac{5^5}{9^5} = \frac{25}{45}$     as  $\frac{27}{45} > \frac{25}{45}$ , then  $\frac{3}{5} < \frac{5}{9}$  FALSE.

5. There are 60 minutes in an hour, so 35 minutes is  $\frac{35}{60}$  of an hour.

This simplifies to  $\frac{35^{-5}}{60^{-5}} = \frac{7}{12}$  of an hour.

6. The months (according to the saying) '30 days has September, April, June and November'. The fraction of the year is  $\frac{4}{12}$  which simplifies to  $\frac{4^{-4}}{12^{-4}} = \frac{1}{3}$ .

7. Choose the lowest common denominator: 20 (from 5x4)  
 $\frac{2^4}{5^4} = \frac{8}{20}$      $\frac{1^5}{4^5} = \frac{5}{20}$     as  $\frac{8}{20} > \frac{5}{20}$ , Tom ate the most.

## Adding and subtracting fractions

1. (a)  $\frac{3}{8} + \frac{1}{8}$   
 $= \frac{4^{-4}}{8^{-4}}$   
 $= \frac{1}{2}$

(b)  $\frac{3^3}{4^3} + \frac{7}{12}$   
 $= \frac{9}{12} + \frac{7}{12}$   
 $= \frac{16^{-4}}{12^{-4}} = \frac{4}{3} = 1\frac{1}{3}$

(c)  $\frac{5}{6} - \frac{1^2}{3^2}$   
 $= \frac{5}{6} - \frac{2}{6}$   
 $= \frac{3^{-3}}{6^{-3}} = \frac{1}{2}$

$$\begin{aligned}
 \text{(d)} \quad & \frac{3}{4} - 1\frac{1}{3} \\
 &= \frac{3^{\times 3}}{4^{\times 3}} - \frac{4^{\times 4}}{3^{\times 4}} \\
 &= \frac{9}{12} - \frac{16}{12} \\
 &= \frac{-7}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & \text{Lowest Common} \\
 & \text{Denominator is} \\
 & \frac{7^{\times 5}}{12^{\times 5}} - \frac{4^{\times 4}}{15^{\times 4}} \\
 &= \frac{35}{60} - \frac{16}{60} \\
 &= \frac{19}{60}
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & \frac{-4^{\times 3}}{7^{\times 3}} - \frac{1^{\times 7}}{3^{\times 7}} \\
 &= \frac{-12}{21} - \frac{7}{21} \\
 &= \frac{-19}{21}
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad & \text{Lowest common} \\
 & \text{denominator is 14} \\
 & 4\frac{1}{2} - 6\frac{11}{14} \\
 &= \frac{9}{2} - \frac{95}{14} \\
 &= \frac{63}{14} - \frac{95}{14} \\
 &= \frac{-32}{14} = -2\frac{4}{14} = -2\frac{2}{7}
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad & -1\frac{1}{4} - -3\frac{2}{3} \\
 &= \frac{-5^{\times 3}}{4^{\times 3}} + \frac{11^{\times 4}}{3^{\times 4}} \\
 &= \frac{-15}{12} + \frac{44}{12} \\
 &= \frac{29}{12} = 2\frac{5}{12}
 \end{aligned}$$

2. (a) What fraction of her patients was screened in the two hours?

$$\begin{aligned}
 & \frac{7}{16} + \frac{5}{16} \\
 &= \frac{12^{\div 4}}{16^{\div 4}} = \frac{3}{4}
 \end{aligned}$$

In the first two hours  $\frac{3}{4}$  of her patients were screened.

- (b) What fraction was left to screen?

$$\begin{aligned}
 & 1 - \frac{3}{4} \\
 &= \frac{4}{4} - \frac{3}{4} \\
 &= \frac{1}{4}
 \end{aligned}$$

There was  $\frac{1}{4}$  of her patients left to screen.

3. (a) What is the fraction total of water that is used in the house?

$$\begin{aligned}
 & \frac{1^{\times 4}}{10^{\times 4}} + \frac{1^{\times 5}}{8^{\times 5}} + \frac{1^{\times 10}}{4^{\times 10}} + \frac{3^{\times 5}}{8^{\times 5}} \\
 &= \frac{4}{40} + \frac{5}{40} + \frac{10}{40} + \frac{15}{40} \\
 &= \frac{34}{40} = \frac{17}{20}
 \end{aligned}$$

- (b) What is the fraction of water is used outside the house?

$$\begin{aligned}
 & 1 - \frac{17}{20} \\
 &= \frac{20}{20} - \frac{17}{20} \\
 &= \frac{3}{20}
 \end{aligned}$$

4. (a) How many cups of sugar will he need altogether?

$$\begin{aligned}
 & 2\frac{1}{2} + 1\frac{1}{3} + 1\frac{1}{4} \\
 &= \frac{5 \times 6}{2 \times 6} + \frac{4 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3} \\
 &= \frac{30}{12} + \frac{16}{12} + \frac{15}{12} \\
 &= \frac{61}{12} = 5\frac{1}{12}
 \end{aligned}$$

James needs  $5\frac{1}{12}$  cups of sugar.

- (b) If each cup contains 230g (0.23 kg), will a 2kg pack of sugar be enough?

Because multiplication of fractions has not been covered, estimation may give a suitable answer.

As James needs about 5 cups, and each cup of sugar weighs 230 g, this is  $5 \times 230 = 1150\text{g}$  or  $1.15\text{kg}$ . So a 2kg bag of sugar will be plenty.

## Multiplying and dividing fractions

1. (a) 
$$\begin{aligned}
 & \frac{\cancel{4}^1}{5} \times \frac{3}{\cancel{8}^2} \\
 &= \frac{3}{10}
 \end{aligned}$$

(b) 
$$\begin{aligned}
 & \frac{\cancel{3}^1}{4} \times \frac{7}{\cancel{12}^4} \\
 &= \frac{7}{16}
 \end{aligned}$$

(c) 
$$\begin{aligned}
 & 1\frac{3}{7} \times 2\frac{9}{20} \\
 &= \frac{\cancel{10}^1}{7^1} \times \frac{\cancel{49}^7}{\cancel{20}^2} \\
 &= \frac{7}{2} = 3\frac{1}{2}
 \end{aligned}$$

(d) 
$$\begin{aligned}
 & \frac{2}{5} \times 800 \\
 &= \frac{2}{\cancel{5}^1} \times \frac{\cancel{800}^{160}}{1} \\
 &= \frac{320}{1} = 320
 \end{aligned}$$

(e) 
$$\begin{aligned}
 & \frac{2}{9} \times \frac{3}{10} \times 1\frac{4}{11} \\
 &= \frac{\cancel{2}^1}{\cancel{9}^3} \times \frac{\cancel{3}^1}{\cancel{10}^2} \times \frac{\cancel{15}^{5^1}}{11} \\
 &= \frac{1}{11}
 \end{aligned}$$

(f) 
$$\begin{aligned}
 & 2\frac{1}{3} \times 1\frac{2}{7} \times \frac{3}{4} \\
 &= \frac{\cancel{7}^1}{\cancel{3}^1} \times \frac{\cancel{9}^3}{\cancel{7}^1} \times \frac{3}{4} \\
 &= \frac{9}{4} = 2\frac{1}{4}
 \end{aligned}$$



$$\begin{array}{lll}
 2. \quad (a) & \frac{-1}{2} \div \frac{-1}{3} & (b) \quad \frac{3}{4} \div \frac{9}{11} \\
 & = \frac{-1}{2} \times \frac{-3}{1} & = \frac{\cancel{3}^1}{4} \times \frac{11}{\cancel{9}^3} \\
 & = \frac{3}{2} = 1\frac{1}{2} & = \frac{11}{12} \\
 (d) & \frac{3}{4} \div 4 & (e) \quad 8 \div \frac{2}{9} \\
 & = \frac{3}{4} \div \frac{4}{1} & = \frac{8}{1} \div \frac{2}{9} \\
 & = \frac{3}{4} \times \frac{1}{4} & = \frac{\cancel{8}^4}{1} \times \frac{9}{\cancel{2}^1} \\
 & = \frac{3}{16} & = \frac{36}{1} = 36 \\
 & & (f) \quad \frac{3}{4} \times \frac{1}{7} \div \frac{3}{8} \\
 & & = \frac{\cancel{3}^1}{4} \times \frac{1}{7} \times \frac{\cancel{8}^2}{\cancel{3}^1} \\
 & & = \frac{2}{7}
 \end{array}$$

3. How many would you expect to have hearing problems?

$$\begin{aligned}
 & \frac{2}{5} \times 450 \\
 & = \frac{2}{\cancel{5}^1} \times \frac{450^{\cancel{90}}}{1} \\
 & = \frac{180}{1} = 180
 \end{aligned}$$

You would expect about 180 students to have hearing problems.

4. How many  $\frac{1}{8}$  are there in  $\frac{7}{9}$ ?

$$\begin{aligned}
 & \frac{7}{9} \div \frac{1}{8} \\
 & = \frac{7}{9} \times \frac{8}{1} \\
 & = \frac{56}{9} = 6\frac{2}{9}
 \end{aligned}$$

5. How far will she cover if she jogs 5 circuits of  $1\frac{1}{3}$  km.

$$\begin{aligned}
 & 5 \times 1\frac{1}{3} \\
 & = \frac{5}{1} \times \frac{4}{3} \\
 & = \frac{20}{3} = 6\frac{2}{3}
 \end{aligned}$$

Joanne will jog  $6\frac{2}{3}$  km.

6. What area does the dam cover?

$$\begin{aligned} & \frac{2}{11} \times 5\frac{1}{4} \\ &= \frac{2^1}{11} \times \frac{21}{4^2} \\ &= \frac{21}{22} \end{aligned}$$

The dam covers  $\frac{21}{22}$  of a hectare (just under 1 Ha)

## Changing between fractions and decimals

1. (a)  $\frac{7}{10} = 0.7$

(b)  $\frac{4}{5} = \frac{8}{10} = 0.8$

(c)  $\frac{3}{11} = 0.273$  (to 3 d.p.)

$$\begin{array}{r} 0.2727 \\ 11 \overline{)3.0^8 0^3 0^8 0} \end{array}$$

(d)  $\frac{-2}{3} = -0.\dot{6}$   
or  $-0.6667$  (to 4 d.p.)

$$\begin{array}{r} 0.6666 \\ 3 \overline{)2.0^2 0^2 0^2 0} \end{array}$$

(e)  $\frac{-7}{4} = -1.75$

$$\begin{array}{r} 1.75 \\ 4 \overline{)7.30^2 0} \end{array}$$

(f)  $\frac{13}{9} = 1.\dot{4} = 1.444$  (to 3d.p.)

$$\begin{array}{r} 1.444 \\ 9 \overline{)13.40^4 0^4 0} \end{array}$$

2. (a)  $0.08$   
 $= \frac{8^{+4}}{100^{+4}}$   
 $= \frac{2}{25}$

(b)  $0.6$   
 $= \frac{6^{+2}}{10^{+2}}$   
 $= \frac{3}{5}$

(c)  $0.14$   
 $= \frac{14^{+2}}{100^{+2}}$   
 $= \frac{7}{50}$

(d)  $0.64$   
 $= \frac{64^{+4}}{100^{+4}}$   
 $= \frac{16}{25}$

(e)  $4.25$   
 $= 4 \frac{25^{+25}}{100^{+25}}$   
 $= 4 \frac{1}{4}$

(f)  $6.2$   
 $= 6 \frac{2^{+2}}{10^{+2}}$   
 $= 6 \frac{1}{5}$

(g)  $12.5$   
 $= 12 \frac{5^{+5}}{10^{+5}}$   
 $= 12 \frac{1}{2}$

(h)  $3.125$   
 $= 3 \frac{125^{+125}}{1000^{+125}}$   
 $= 3 \frac{1}{8}$

(i)  $0.075$   
 $= \frac{75^{+25}}{1000^{+25}}$   
 $= \frac{3}{40}$