

THIRD EDITION

**Numerical Techniques in
ELECTROMAGNETICS
with MATLAB®**

MATTHEW N. O. SADIKU

Prairie View A&M University

Texas, U.S.A.



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

Contents

Preface	xiii
Author	xvii
1 Fundamental Concepts	1
1.1 Introduction	1
1.2 Review of Electromagnetic Theory	2
1.2.1 Electrostatic Fields	3
1.2.2 Magnetostatic Fields	4
1.2.3 Time-Varying Fields	5
1.2.4 Boundary Conditions	7
1.2.5 Wave Equations	7
1.2.6 Time-Varying Potentials	9
1.2.7 Time-Harmonic Fields	10
1.3 Classification of EM Problems	14
1.3.1 Classification of Solution Regions	14
1.3.2 Classification of Differential Equations	15
1.3.3 Classification of Boundary Conditions	18
1.4 Some Important Theorems	20
1.4.1 Superposition Principle	20
1.4.2 Uniqueness Theorem	21
References	23
Problems	23
2 Analytical Methods	27
2.1 Introduction	27
2.2 Separation of Variables	28
2.3 Separation of Variables in Rectangular Coordinates	30
2.3.1 Laplace's Equation	30
2.3.2 Wave Equation	34
2.4 Separation of Variables in Cylindrical Coordinates	39
2.4.1 Laplace's Equation	40
2.4.2 Wave Equation	42
2.5 Separation of Variables in Spherical Coordinates	53
2.5.1 Laplace's Equation	54
2.5.2 Wave Equation	59

2.6	Some Useful Orthogonal Functions	68
2.7	Series Expansion	78
2.7.1	Poisson's Equation in a Cube	78
2.7.2	Poisson's Equation in a Cylinder	79
2.7.3	Strip Transmission Line	82
2.8	Practical Applications	87
2.8.1	Scattering by Dielectric Sphere	87
2.8.2	Scattering Cross Sections	91
2.9	Attenuation Due to Raindrops	94
2.10	Concluding Remarks	102
	References	102
	Problems	103
3	Finite Difference Methods	119
3.1	Introduction	119
3.2	Finite Difference Schemes	120
3.3	Finite Differencing of Parabolic PDEs	123
3.4	Finite Differencing of Hyperbolic PDEs	129
3.5	Finite Differencing of Elliptic PDEs	132
3.5.1	Band Matrix Method	135
3.5.2	Iterative Methods	135
3.6	Accuracy and Stability of FD Solutions	142
3.7	Practical Applications I — Guided Structures	146
3.7.1	Transmission Lines	146
3.7.2	Waveguides	153
3.8	Practical Applications II — Wave Scattering (FDTD)	159
3.8.1	Yee's Finite Difference Algorithm	159
3.8.2	Accuracy and Stability	163
3.8.3	Lattice Truncation Conditions	163
3.8.4	Initial Fields	166
3.8.5	Programming Aspects	167
3.9	Absorbing Boundary Conditions for FDTD	181
3.10	Finite Differencing for Nonrectangular Systems	185
3.10.1	Cylindrical Coordinates	185
3.10.2	Spherical Coordinates	189
3.11	Numerical Integration	194
3.11.1	Euler's Rule	195
3.11.2	Trapezoidal Rule	196
3.11.3	Simpson's Rule	197
3.11.4	Newton–Cotes Rules	197
3.11.5	Gaussian Rules	199
3.11.6	Multiple Integration	202
3.12	Concluding Remarks	208
	References	210
	Problems	219

4	Variational Methods	235
4.1	Introduction	235
4.2	Operators in Linear Spaces	236
4.3	Calculus of Variations	238
4.4	Construction of Functionals from PDEs	242
4.5	Rayleigh–Ritz Method	245
4.6	Weighted Residual Method	252
4.6.1	Collocation Method	253
4.6.2	Subdomain Method	254
4.6.3	Galerkin Method	254
4.6.4	Least Squares Method	255
4.7	Eigenvalue Problems	261
4.8	Practical Applications	268
4.9	Concluding Remarks	275
	References	276
	Problems	280
5	Moment Methods	285
5.1	Introduction	285
5.2	Integral Equations	286
5.2.1	Classification of Integral Equations	286
5.2.2	Connection Between Differential and Integral Equations	287
5.3	Green’s Functions	290
5.3.1	For Free Space	292
5.3.2	For Domain with Conducting Boundaries	295
5.4	Applications I — Quasi-Static Problems	308
5.5	Applications II — Scattering Problems	314
5.5.1	Scattering by Conducting Cylinder	314
5.5.2	Scattering by an Arbitrary Array of Parallel Wires	317
5.6	Applications III — Radiation Problems	323
5.6.1	Hallen’s Integral Equation	325
5.6.2	Pocklington’s Integral Equation	326
5.6.3	Expansion and Weighting Functions	326
5.7	Applications IV — EM Absorption in the Human Body	337
5.7.1	Derivation of Integral Equations	338
5.7.2	Transformation to Matrix Equation (Discretization)	341
5.7.3	Evaluation of Matrix Elements	342
5.7.4	Solution of the Matrix Equation	344
5.8	Concluding Remarks	355
	References	356
	Problems	362

6	Finite Element Method	379
6.1	Introduction	379
6.2	Solution of Laplace's Equation	380
6.2.1	Finite Element Discretization	380
6.2.2	Element Governing Equations	382
6.2.3	Assembling of All Elements	385
6.2.4	Solving the Resulting Equations	388
6.3	Solution of Poisson's Equation	398
6.3.1	Deriving Element-Governing Equations	399
6.3.2	Solving the Resulting Equations	400
6.4	Solution of the Wave Equation	402
6.5	Automatic Mesh Generation I — Rectangular Domains	407
6.6	Automatic Mesh Generation II — Arbitrary Domains	409
6.6.1	Definition of Blocks	409
6.6.2	Subdivision of Each Block	412
6.6.3	Connection of Individual Blocks	413
6.7	Bandwidth Reduction	413
6.8	Higher Order Elements	419
6.8.1	Pascal Triangle	420
6.8.2	Local Coordinates	421
6.8.3	Shape Functions	423
6.8.4	Fundamental Matrices	425
6.9	Three-Dimensional Elements	435
6.10	Finite Element Methods for Exterior Problems	440
6.10.1	Infinite Element Method	440
6.10.2	Boundary Element Method	442
6.10.3	Absorbing Boundary Condition	442
6.11	Finite-Element Time-Domain Method	444
6.12	Concluding Remarks	446
	References	446
	Problems	456
7	Transmission-Line-Matrix Method	465
7.1	Introduction	465
7.2	Transmission-Line Equations	467
7.3	Solution of Diffusion Equation	471
7.4	Solution of Wave Equations	476
7.4.1	Equivalence Between Network and Field Parameters	476
7.4.2	Dispersion Relation of Propagation Velocity	479
7.4.3	Scattering Matrix	481
7.4.4	Boundary Representation	484
7.4.5	Computation of Fields and Frequency Response	485
7.4.6	Output Response and Accuracy of Results	485
7.5	Inhomogeneous and Lossy Media in TLM	490
7.5.1	General Two-Dimensional Shunt Node	492

7.5.2	Scattering Matrix	494
7.5.3	Representation of Lossy Boundaries	495
7.6	Three-Dimensional TLM Mesh	499
7.6.1	Series Nodes	500
7.6.2	Three-Dimensional Node	502
7.6.3	Boundary Conditions	507
7.7	Error Sources and Correction	515
7.7.1	Truncation Error	516
7.7.2	Coarseness Error	516
7.7.3	Velocity Error	517
7.7.4	Misalignment Error	517
7.8	Absorbing Boundary Conditions	517
7.9	Concluding Remarks	519
	References	521
	Problems	527
8	Monte Carlo Methods	535
8.1	Introduction	535
8.2	Generation of Random Numbers and Variables	536
8.3	Evaluation of Error	539
8.4	Numerical Integration	544
8.4.1	Crude Monte Carlo Integration	544
8.4.2	Monte Carlo Integration with Antithetic Variates	546
8.4.3	Improper Integrals	547
8.5	Solution of Potential Problems	549
8.5.1	Fixed Random Walk	549
8.5.2	Floating Random Walk	554
8.5.3	Exodus Method	557
8.6	Regional Monte Carlo Methods	571
8.7	Time-Dependent Problems	579
8.8	Concluding Remarks	585
	References	586
	Problems	592
9	Method of Lines	603
9.1	Introduction	603
9.2	Solution of Laplace's Equation	604
9.2.1	Rectangular Coordinates	604
9.2.2	Cylindrical Coordinates	611
9.3	Solution of Wave Equation	615
9.3.1	Planar Microstrip Structures	618
9.3.2	Cylindrical Microstrip Structures	627
9.4	Time-Domain Solution	632
9.5	Concluding Remarks	634
	References	635

Problems	640
A Vector Relations	645
A.1 Vector Identities	645
A.2 Vector Theorems	645
A.3 Orthogonal Coordinates	646
B Programming in MATLAB	649
B.1 MATLAB Fundamentals	649
B.2 Using MATLAB to Plot	653
B.3 Programming with MATLAB	656
B.4 Functions	660
B.5 Solving Equations	661
B.6 Programming Hints	663
B.7 Other Useful MATLAB Commands	664
C Solution of Simultaneous Equations	665
C.1 Elimination Methods	665
C.1.1 Gauss's Method	666
C.1.2 Cholesky's Method	667
C.2 Iterative Methods	670
C.2.1 Jacobi's Method	670
C.2.2 Gauss–Seidel Method	672
C.2.3 Relaxation Method	673
C.2.4 Gradient Methods	674
C.3 Matrix Inversion	677
C.4 Eigenvalue Problems	678
C.4.1 Iteration (or Power) Method	680
C.4.2 Jacobi's Method	681
References	686
D Answers to Odd-Numbered Problems	687
Index	707