Objective 8

The student will demonstrate an understanding of the concepts and uses of measurement and similarity.

For this objective you should be able to

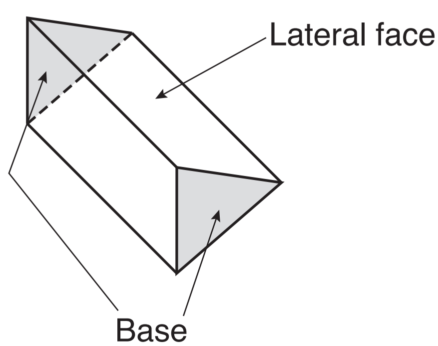
- use procedures to determine measures of three-dimensional figures;
- use indirect measurement to solve problems; and
- describe how changes in dimensions affect linear, area, and volume measurements.

How Do You Find the Surface Area of Three-Dimensional Figures?

You can use models or formulas to find the surface area of prisms, cylinders, and other three-dimensional figures.

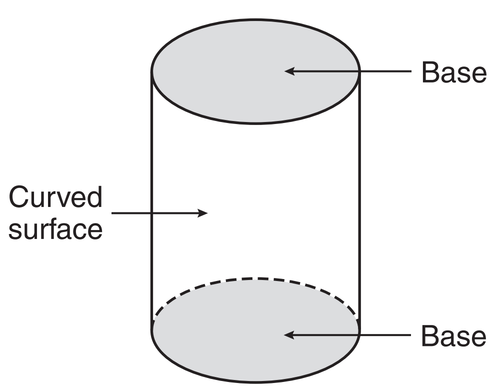
- A **prism** is a three-dimensional figure with two bases. The bases are congruent polygons. The other faces of the prism are rectangular and are called lateral faces. The prism is named by the shape of its bases. For example, a triangular prism has two triangles as its bases.

Triangular Prism



- A **cylinder** is a three-dimensional figure with two congruent circular bases and a curved surface.

Cylinder



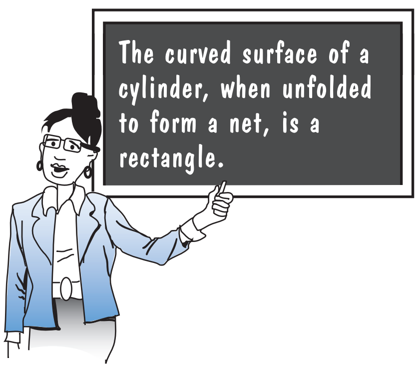
Objective 8

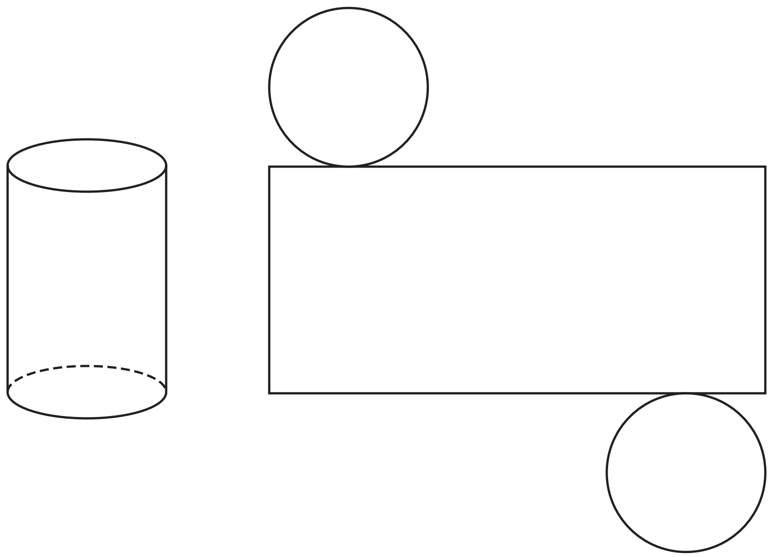
Like the area of a plane figure, the surface area of a three-dimensional figure is measured in square units.

- The **total surface area** of a three-dimensional figure is equal to the sum of the areas of all its surfaces.
- The **lateral surface area** of a three-dimensional figure is equal to the sum of the areas of all its faces and curved surfaces but does not include the area of the figure's base(s).

One way to find the surface area of a three-dimensional figure is to use a net of the figure. A **net** of a three-dimensional figure is a two-dimensional drawing that shows what the figure would look like when opened up and unfolded with all its surfaces laid out flat. Use the net to find the area of each surface.

For example, the net of a cylinder is shown below. It is composed of two circles for the bases and a rectangle for the curved surface.

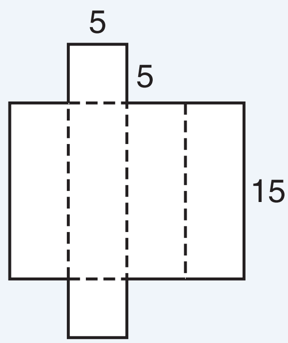




- The **total surface area** of a cylinder is equal to the sum of the areas of the two circular bases and the area of the rectangular curved surface of the cylinder.
- The **lateral surface area** of a cylinder is equal to the area of the rectangular curved surface of the cylinder.

You can also find the surface area of a three-dimensional figure by using a formula. Substitute the appropriate dimensions of the figure into the formula and calculate its surface area. The formulas for the total surface area and lateral surface area of several three-dimensional figures are included in the Mathematics Chart.

The net of a square prism is shown below. Find the total surface area of the prism.



The total surface area of the prism is equal to the sum of the areas of all its faces.

Find the area of a square base.

$$A = s^2$$
$$A = 5^2$$
$$A = 25$$

Each square base has an area of 25 square units.

The prism has 2 square bases. Since $2 \cdot 25 = 50$, the combined area of the squares is 50 square units.

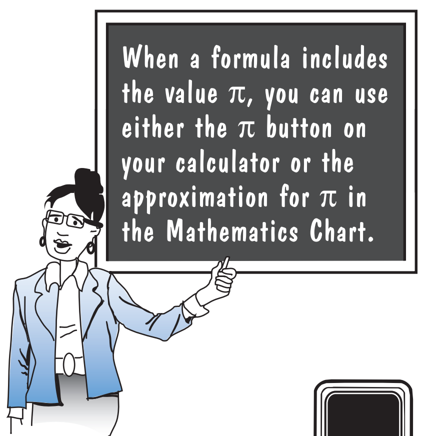
Find the area of a rectangular face.

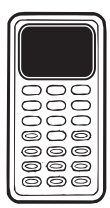
$$A = lw$$
$$A = 5 \cdot 15$$
$$A = 75$$

Each rectangular face has an area of 75 square units.

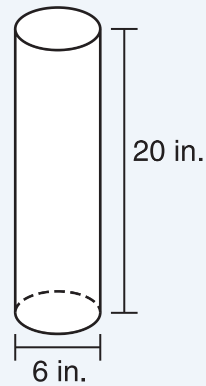
The prism has 4 rectangular faces. Since $4 \cdot 75 = 300$, the combined area of the rectangles is 300 square units.

The total surface area of the square prism is $50 + 300 = 350$ square units.





What is the lateral surface area of the cylinder shown below?



Use the formula for the lateral surface area of a cylinder, $S = 2\pi rh$.

- The diameter of the cylinder is 6 inches.

  To find the radius, r, divide the diameter by 2.

$$r = 6 \div 2$$
$$r = 3 \text{ in.}$$

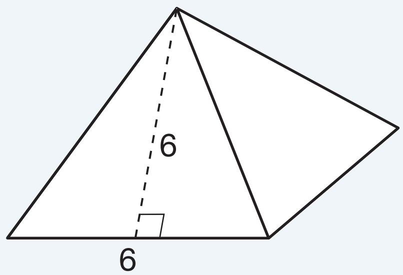
- The height, h, of the cylinder is 20 inches.
- Substitute the known values into the formula and solve for the lateral surface area, S.

$$S = 2\pi rh$$
$$S = 2\pi \cdot 3 \cdot 20$$
$$S = 2\pi \cdot 60$$
$$S = 120\pi$$
$$S \approx 376.99 \text{ in.}^2$$

The lateral surface area of the cylinder is about 377 square inches.

In the square pyramid below, the base edge has a length of 6 feet and the slant height, l, has a length of 6 feet.

What is the total surface area of the square pyramid shown below?



Use the formula for the total surface area of a pyramid, $S = \frac{1}{2}Pl + B$.

$$S = \frac{1}{2}Pl + B$$

The perimeter, P, of the base of the pyramid is $4 \cdot 6 = 24$, and the area of the square base, B, is $6 \cdot 6 = 36$.

$$S = \frac{1}{2}(24)(6) + 36$$

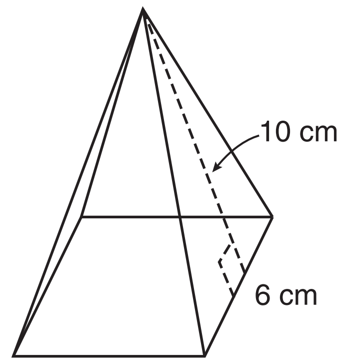
$$S = 72 + 36$$

$$S = 108$$

The total surface area of the pyramid is 108 square feet.

Try It

Quinlan is making a paperweight in the shape shown below. Its base is a square. He wants to cover each of the triangular faces, but not the square base, with gold foil. How many square centimeters of foil will Quinlan need?



The paperweight is shaped like a square ____________________ .

The lateral surface area of the pyramid is equal to the __________ of the areas of its triangular faces; this does not include its square base.

Find the area of a triangular face.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot \_\_\_\_\_ \cdot \_\_\_\_\_$$

$$A = \_\_\_\_\_ \text{ cm}^2$$

Each triangular face has an area of ________ square centimeters.

The pyramid has _____ triangular faces. Since 4 • _____ = _____ , the combined area of the triangular faces is ________ square centimeters. Quinlan will need ________ square centimeters of foil.

The paperweight is shaped like a square pyramid. The lateral surface area of the pyramid is equal to the sum of the areas of its triangular faces; this does not include its square base.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 6 \cdot 10$$

$$A = 30 \text{ cm}^2$$

Each triangular face has an area of 30 square centimeters. The pyramid has 4 triangular faces. Since 4 • 30 = 120, the combined area of the triangular faces is 120 square centimeters. Quinlan will need 120 square centimeters of foil.

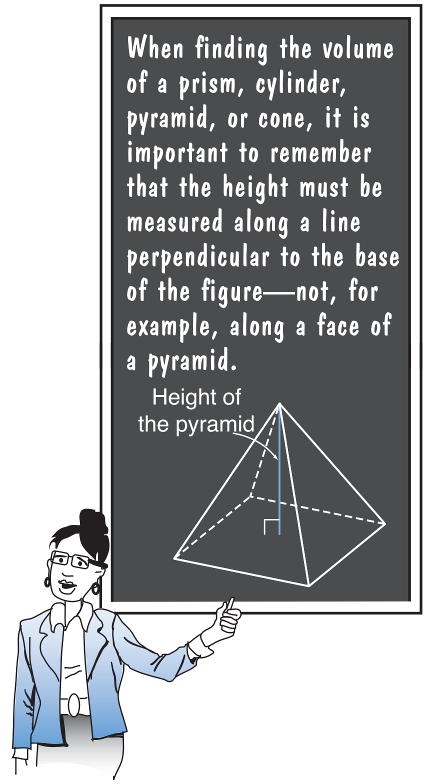
What Is the Volume of a Three-Dimensional Figure?

The **volume** of a three-dimensional figure is a measure of the space it occupies. Volume is measured in cubic units.

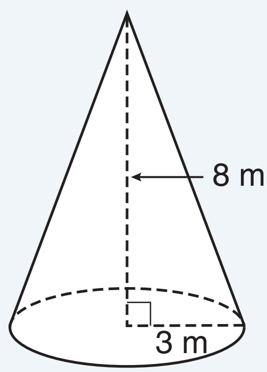
You can use formulas or models to find the volume of three-dimensional figures. The formulas for calculating the volume of several three-dimensional figures are in the Mathematics Chart.

When using a formula to find the volume of a three-dimensional figure, follow these guidelines:

- Identify the three-dimensional figure you are working with. This will help you select the correct volume formula.
- Use models to help visualize the three-dimensional figure and assign the variables in the volume formula. A model can also be used to find the dimensions of a figure.
- Substitute the appropriate dimensions of the figure for the corresponding variables in the volume formula.
- Calculate the volume. State your answer in cubic units.



Find the volume of the cone shown on the right.



Use the formula for the volume of a cone.

$$V = \frac{1}{3}Bh$$

In this formula B represents the area of the cone's base, and h represents the cone's height.

- Find the area of the base.

  The base of a cone is a circle.

  Use the formula for the area of a circle.

  The circle's radius, r, is 3 meters.

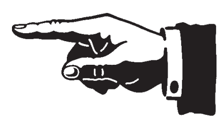
  $$A = \pi r^2 = \pi \cdot 3^2 \approx 28.27 \text{ m}^2$$

  The area of the cone's base, B, is about 28.27 square meters.

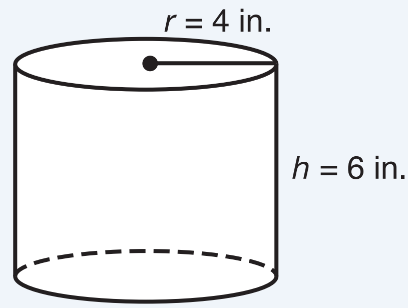
- The cone's height, h, is 8 meters.
- Now substitute the values for B and h into the volume formula.

$$V = \frac{1}{3}Bh \approx \frac{1}{3} \cdot 28.27 \cdot 8 \approx 75.39 \text{ m}^3$$

The volume of the cone is about 75.39 cubic meters.



Find the approximate volume of the cylinder shown below.



Use the formula for the volume of a cylinder.

$$V = Bh \quad \text{or} \quad V = \pi r^2 h$$

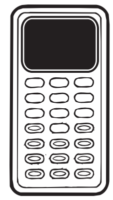
In the formula $V = \pi r^2 h$, r represents the radius of the cylinder, and h represents the height. Substitute 3.14 for π, 4 for r, and 6 for h into the formula.

$$\begin{aligned} V &= \pi r^2 h \\ &\approx (3.14)(4^2)(6) \\ &\approx (3.14)(16)(6) \\ &\approx 301.44 \end{aligned}$$

The volume of the cylinder is approximately 301.44 cubic inches.

Try It

An Internet bookstore is shipping a book to a customer. The dimensions of the book are 7 inches by 4 inches by 1.5 inches. The dimensions of the box in which the book will be shipped are 12 inches by 8 inches by 2.75 inches. How many cubic inches of packing material must be added to completely fill the box?



The volume of packing material to be added is equal to the volume of the ______________ minus the volume of the ______________ .

The box and the book are both shaped like a __________________ .

The formula for the volume of a prism is $V =$ ______ .

The bases of a rectangular prism are ________________ . Therefore, the area of a rectangular prism's base, B, is equal to the area of a rectangle ($A = lw$).

Find the volume of the box.

$$V = Bh$$

$$V = lwh$$

$$V = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ \cdot 2.75 = \_\_\_\_\_\_\_\_\_\_ \text{ in.}^3$$

The volume of the box is ______ cubic inches.

Find the volume of the book.

$$V = Bh$$

$$V = lwh$$

$$V = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ \cdot 1.5 = \_\_\_\_\_\_\_\_\_\_ \text{ in.}^3$$

The volume of the book is ______ cubic inches.

Subtract the volume of the book from the volume of the box.

$$\_\_\_\_\_\_ - \_\_\_\_\_\_ = \_\_\_\_\_\_$$

The volume of packing material needed to fill the box is ______ cubic inches.

The volume of packing material to be added is equal to the volume of the box minus the volume of the book. The box and the book are both shaped like a rectangular prism. The formula for the volume of a prism is $V = Bh$. The bases of a rectangular prism are rectangles.

$$V = Bh$$
$$V = lwh$$
$$V = 12 \cdot 8 \cdot 2.75 = 264 \text{ in.}^3$$

The volume of the box is 264 cubic inches.

$$V = Bh$$
$$V = lwh$$
$$V = 7 \cdot 4 \cdot 1.5 = 42 \text{ in.}^3$$

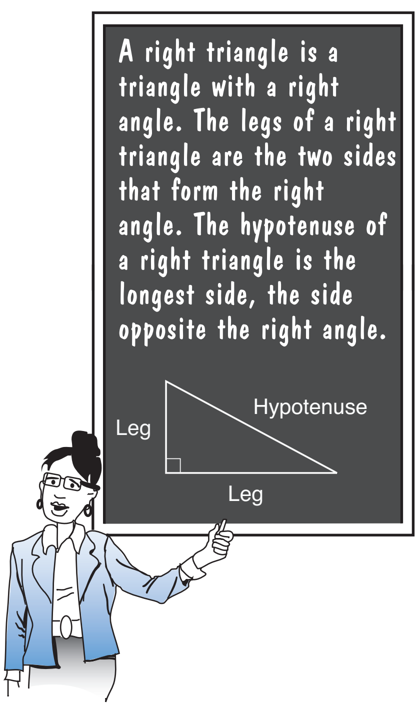
The volume of the book is 42 cubic inches.

Subtract the volume of the book from the volume of the box.

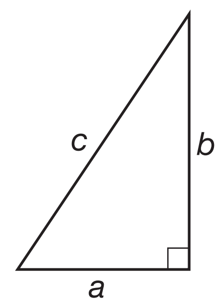
$$264 - 42 = 222$$

The volume of packing material needed to fill the box is 222 cubic inches.

How Can You Solve Problems Using the Pythagorean Theorem?



The **Pythagorean Theorem** is a relationship among the lengths of the three sides of a right triangle. The Pythagorean Theorem applies only to right triangles.



- In any right triangle with leg lengths a and b and hypotenuse length c, $a^2 + b^2 = c^2$.
- If the side lengths of any triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle, and c is its hypotenuse.

Any set of three whole numbers that satisfy the Pythagorean Theorem is called a **Pythagorean triple**. The set of numbers {5, 12, 13} forms a Pythagorean triple because these numbers satisfy the Pythagorean Theorem. To show this, substitute 13 for c in the formula—since 13 is the greatest number—and substitute 5 and 12 for a and b.

$$a^2 + b^2 = c^2$$
$$5^2 + 12^2 = 13^2$$
$$25 + 144 = 169$$
$$169 = 169$$

A triangle with side lengths 5, 12, and 13 units is a right triangle.

Any multiple of a Pythagorean triple is also a Pythagorean triple. Since the set of numbers {5, 12, 13} is a Pythagorean triple, the triple formed by multiplying each number in the set by 2, {10, 24, 26}, is also a Pythagorean triple. A triangle with side lengths 10, 24, and 26 units is also a right triangle.

Is a triangle with side lengths of 32 millimeters, 44 millimeters, and 56 millimeters a right triangle?

Determine whether the side lengths satisfy the Pythagorean Theorem. Since 56 is the greatest length, it would be the length of the triangle's hypotenuse. Substitute 56 for *c* in the formula. Substitute 32 and 44 for *a* and *b*, the two legs.

$$a^2 + b^2 = c^2$$

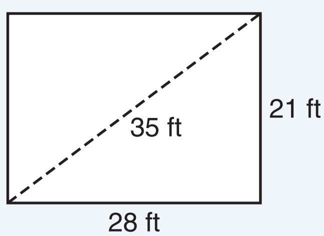
$$32^2 + 44^2 \stackrel{?}{=} 56^2$$

$$1024 + 1936 \stackrel{?}{=} 3136$$

$$2960 \neq 3136$$

Since the side lengths do not satisfy the Pythagorean Theorem, the triangle is not a right triangle.

Allan wanted to enclose an area in his yard with a fence. He marked the widths at 21 feet each and the lengths at 28 feet each. To determine whether his enclosure will form a rectangle, he measured a diagonal and found it to be 35 feet. Is his enclosure a rectangle?



The area Allan enclosed will be a rectangle if its angles are all right angles. The angles are right angles if the two triangles formed by the diagonal are right triangles. Determine whether the dimensions 21 feet, 28 feet, and 35 feet form a right triangle. Substitute 35, the largest value, for *c*.

$$a^2 + b^2 = c^2$$

$$21^2 + 28^2 \stackrel{?}{=} 35^2$$

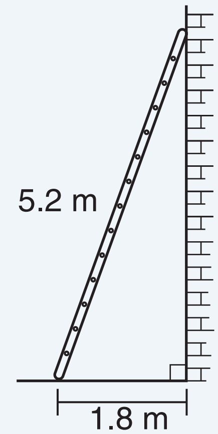
$$441 + 784 \stackrel{?}{=} 1225$$

$$1225 = 1225$$

Since the side lengths satisfy the Pythagorean Theorem, the triangles formed by the diagonal are right triangles. Allan's enclosure will be a rectangle.

Objective 8

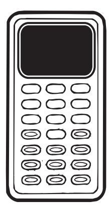
A ladder 5.2 meters in length rests against a wall. If the ladder is 1.8 meters from the base of the wall, how high up the wall does the ladder reach?



- The ladder forms the hypotenuse of the triangle, so let $c = 5.2$ meters.
- The distance from the base of the wall to the ladder forms one leg of the triangle, so let $a = 1.8$ meters.
- The height of the wall from the ground to the top of the ladder is the length of the other leg of the triangle. Let b represent this leg.

Use the Pythagorean Theorem to solve for the length of leg b.

$$a^2 + b^2 = c^2$$
$$(1.8)^2 + b^2 = (5.2)^2$$
$$3.24 + b^2 = 27.04$$
$$b^2 = 23.8$$
$$b = \sqrt{23.8}$$



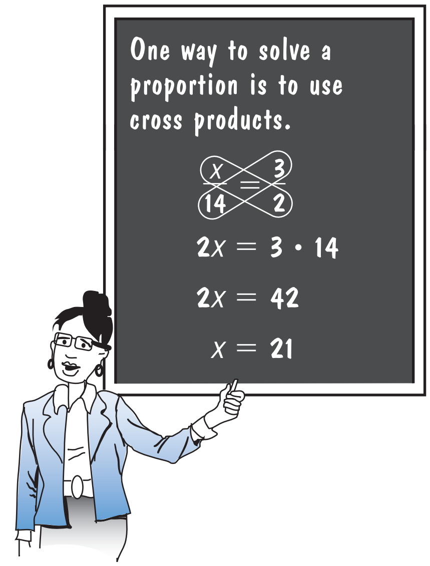
Find a decimal approximation of $\sqrt{23.8}$.

$$\sqrt{23.8} \approx 4.9$$

The ladder reaches about 4.9 meters up the wall.

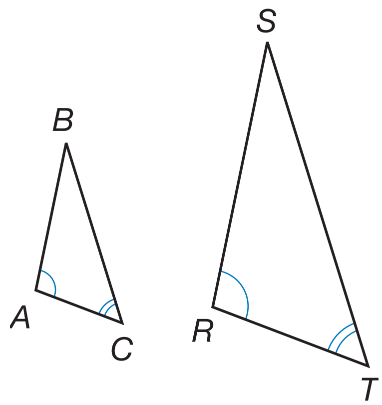
How Can You Use Proportional Relationships to Solve Problems?

You can use proportional relationships to find missing side lengths in similar figures. To solve problems that involve similar figures, follow these guidelines:



- Identify which figures are similar and which sides correspond. Similar figures have the same shape, but not necessarily the same size. The lengths of the corresponding sides of similar figures are proportional.

  Triangle *ABC* is similar to triangle *RST*.

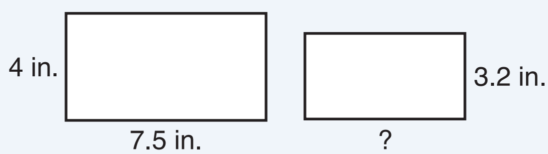


$$\triangle ABC \sim \triangle RST$$

$$\frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}$$

- Write a proportion and solve it.
- Answer the question asked.

The rectangles shown below are similar. Find the length of the smaller rectangle.



- The length of the larger rectangle corresponds to the length of the smaller rectangle, and the width of the larger rectangle corresponds to the width of the smaller rectangle. The ratios of these corresponding sides are equal.

$$\frac{\text{length}_{\text{larger}}}{\text{length}_{\text{smaller}}} = \frac{\text{width}_{\text{larger}}}{\text{width}_{\text{smaller}}}$$

- Substitute the measurements given in the diagram. Let l represent the length of the smaller rectangle.

$$\frac{7.5}{l} = \frac{4}{3.2}$$

- Use cross products to solve for the length of the smaller rectangle.

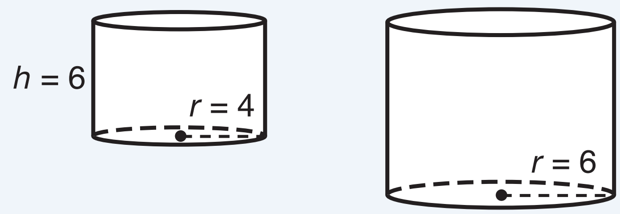
$$4l = 7.5 \cdot 3.2$$

$$4l = 24$$

$$l = 6$$

The length of the smaller rectangle is 6 inches.

The two cylindrical containers shown below are similar.



What is the volume of the larger cylinder in terms of π?

The radius of the smaller cylinder corresponds to the radius of the larger cylinder, and the height of the smaller cylinder corresponds to the height of the larger cylinder. The ratios of these corresponding dimensions are equal.

$$\frac{\text{radius}_{\text{smaller}}}{\text{radius}_{\text{larger}}} = \frac{\text{height}_{\text{smaller}}}{\text{height}_{\text{larger}}}$$

- Substitute the measurements given in the diagram. Let h represent the height of the larger cylinder.

$$\frac{4}{6} = \frac{6}{h}$$

- Use cross products to solve for the height of the larger cylinder.

$$4h = 6 \cdot 6$$

$$4h = 36$$

$$h = 9$$

The height of the larger cylinder is 9 units. To find the volume of the cylinder, we now use $V = Bh$.

$$V = \pi r^2 h$$

$$V = \pi(6)^2(9)$$

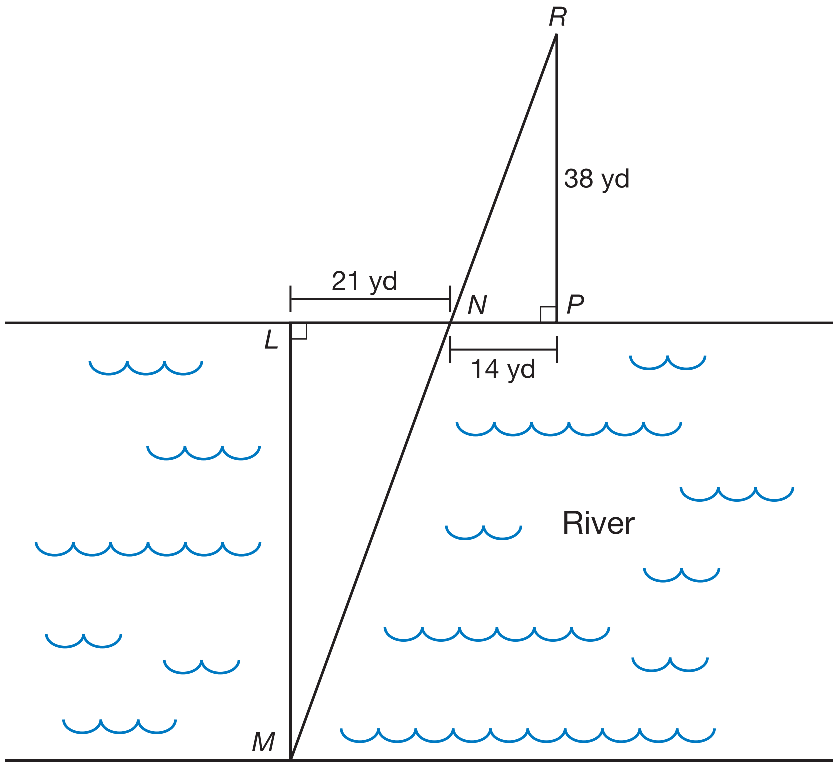
$$V = \pi(36)(9)$$

$$V = 324\pi$$

The volume of the larger cylinder is 324π cubic units.

Try It

A surveyor uses similar triangles to find the distance across a river. He makes the measurements shown in the diagram below.



If $\triangle LMN$ is similar to $\triangle PRN$, what is the width of the river?

In $\triangle LMN$ the length of ______ represents the width of the river.

$\overline{LM}$ corresponds to ______ and $\overline{LN}$ corresponds to ______ .

Since the triangles are similar, the ratios of these corresponding sides are equal.

$$\frac{LM}{\square} = \frac{LN}{\square}$$

Substitute the measurements given in the diagram.

$$\frac{LM}{\square} = \frac{21}{\square}$$

Use cross products to solve for the length of $\overline{LM}$.

$$14 \cdot LM = 38 \cdot 21$$

$$14 \cdot LM = \underline{\qquad\qquad}$$

$$LM = \underline{\qquad\qquad} \text{ yd}$$

The width of the river is ______ yards.

In $\triangle LMN$ the length of $\overline{LM}$ represents the width of the river.

$\overline{LM}$ corresponds to $\overline{PR}$ and $\overline{LN}$ corresponds to $\overline{PN}$.

$$\frac{LM}{PR} = \frac{LN}{PN}$$

$$\frac{LM}{38} = \frac{21}{14}$$

$$14 \cdot LM = 38 \cdot 21$$

$$14 \cdot LM = 798$$

$$LM = 57 \text{ yd}$$

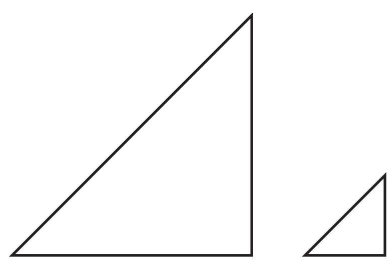
The width of the river is 57 yards.

How Is the Perimeter of a Figure Affected When Its Dimensions Are Changed Proportionally?

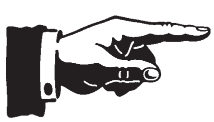
When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The perimeter of the dilated figure will change by the same scale factor.

See Objective 6, page 121, for more information about dilations.

To **dilate** a figure means to enlarge or reduce it by a given scale factor. For example, the triangle on the left has been dilated (reduced) by a scale factor of $\frac{1}{3}$ to form the triangle on the right.

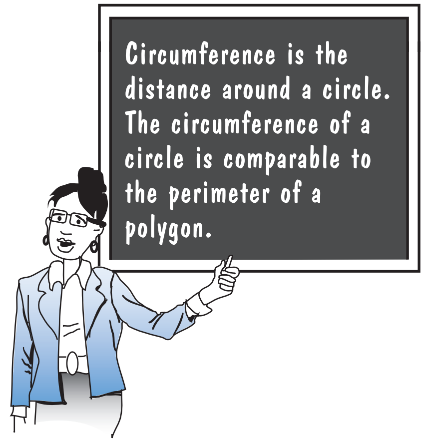


Do you see that . . .

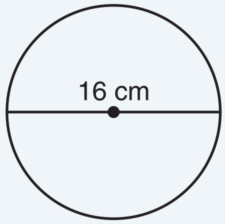


The perimeter of the smaller triangle is $\frac{1}{3}$ the perimeter of the larger triangle.

If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their perimeters will be in the ratio $\frac{a}{b}$.



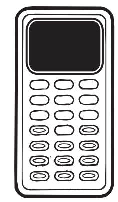
If the circle shown below is dilated by a scale factor of 2, what will be the effect on the circumference of the circle?



The circumference should change by the same scale factor as the dilation. Because the scale factor is 2, the circumference of the circle should increase by a factor of 2.

- To prove this, first find the circumference of the original circle.

$$C = \pi d$$
$$C = \pi \cdot 16$$
$$C \approx 50.27 \text{ cm}$$



- Use the scale factor to find the circumference of the dilated circle.

The circumference of the original circle . . . → $50.27 \cdot 2 = 100.54$ ← . . . should be the circumference of the dilated circle.

. . . times the scale factor . . .

- Use the formula to find the circumference of the dilated circle. The diameter of the dilated circle will be the diameter of the original circle multiplied by the scale factor, 2. The diameter of the dilated circle will therefore be $16 \cdot 2 = 32$ centimeters.

$$C = \pi d$$
$$C = \pi \cdot 32$$
$$C \approx 100.54 \text{ cm}$$

- Compare the two circumferences.

$$\frac{100.54}{50.27} = \frac{2}{1}$$

The circumference of the circle will increase by a factor of 2.

Objective 8

Try It

The perimeter of a rectangle is 36 inches. If the dimensions of the rectangle are dilated by a scale factor of $\frac{1}{4}$, what will be the perimeter of the smaller rectangle?

The perimeter of the original rectangle should decrease by a scale factor of $\frac{\square}{\square}$.

perimeter of smaller rectangle = (perimeter of __________ rectangle) • $\frac{\square}{\square}$

P_{smaller} = ________ • $\frac{\square}{\square}$

P_{smaller} = ________ in.

The perimeter of the smaller rectangle will be ________ inches.

The perimeter of the original rectangle should decrease by a scale factor of $\frac{1}{4}$.

perimeter of smaller rectangle = (perimeter of original rectangle) • $\frac{1}{4}$

$P_{\text{smaller}} = 36 \cdot \frac{1}{4}$

$P_{\text{smaller}} = 9$ in.

The perimeter of the smaller rectangle will be 9 inches.

How Is the Area of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The area of the dilated figure will change by the square of the scale factor.

Do you see that . . .

If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their areas will be in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$.

A triangle has a base of 12 inches and a height of 10 inches. If these dimensions are dilated by a scale factor of 3, how is the area of the triangle affected?

The area of the triangle should increase by the square of the scale factor: $3^2 = 9$. The area should increase by a factor of 9.

- To prove this, first find the area of the original triangle.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 12 \cdot 10$$

$$A = 60 \text{ in.}^2$$

- Use the scale factor to find the area of the larger triangle.

The area of the original triangle . . .

$$60 \cdot 3^2 = 60 \cdot 9 = 540$$

. . . times the square of the scale factor . . .

. . . should be the area of the larger triangle.

- Use the formula to find the area of the larger triangle. The dimensions of the larger triangle are the dimensions of the original triangle multiplied by the scale factor 3.

Base: $12 \cdot 3 = 36$ in. Height: $10 \cdot 3 = 30$ in.

Area of the larger triangle:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 36 \cdot 30$$

$$A = 540 \text{ in.}^2$$

- Compare the two areas.

$$\frac{540}{60} = \frac{9}{1} = \left(\frac{3}{1}\right)^2$$

The area of the original triangle increased by a factor of 3^2, or 9.

Objective 8

Try It

An architect draws a floor plan of a building. He uses a scale of 1 inch = 4 feet. The drawing has an area of 150 square inches. What will be the area of the actual building?

Since the dimensions of these two similar figures are in the ratio $\frac{a}{b}$, the areas of the figures are in the ratio $\left(\frac{a}{b}\right)^2 = \frac{\square^2}{\square^2}$.

The dimensions are in the ratio $\frac{\square}{1}$.

The areas should be in the ratio $\left(\frac{\square}{1}\right)^2 = \frac{\square^2}{1^2} = \frac{\square}{1}$.

The area of the drawing is ________ square inches.

$$150 \cdot \text{\_\_\_\_\_\_\_\_} = \text{\_\_\_\_\_\_\_\_}$$

The area of the actual building should be ________ square feet.

Since the dimensions of these two similar figures are in the ratio $\frac{a}{b}$, the areas of the figures are in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$. The dimensions are in the ratio $\frac{4}{1}$. The areas should be in the ratio $\left(\frac{4}{1}\right)^2 = \frac{4^2}{1^2} = \frac{16}{1}$. The area of the drawing is 150 square inches.

$$150 \cdot 16 = 2400$$

The area of the actual building should be 2400 square feet.

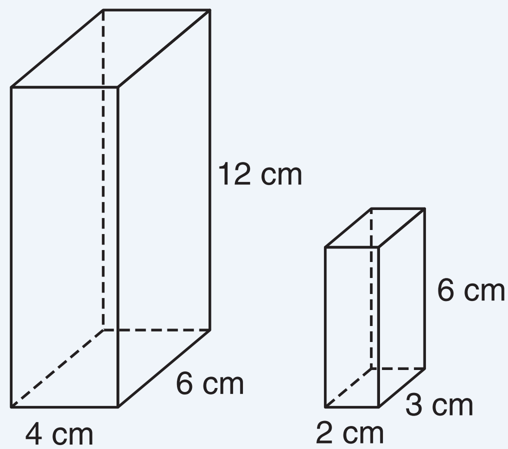
How Is the Volume of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The volume of the dilated figure will change by the cube of the scale factor.

If the dimensions of two similar solid figures are in the ratio $\frac{a}{b}$, then their volumes will be in the ratio $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$.

Do you see that . . .

The dimensions of the prism on the left were reduced by a scale factor of $\frac{1}{2}$ to produce the prism on the right.



By what factor did the volume of the original prism decrease?

The volume of the prism decreased by the cube of the scale factor.

$$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$$

The volume decreased by a factor of $\frac{1}{8}$.

- To prove this, first find the volume of the original prism.

$$V = Bh$$
$$V = (6 \cdot 4) \cdot 12$$
$$V = 288 \text{ cm}^3$$

- Use the scale factor to find the volume of the dilated prism.

The volume of the original prism . . . → $288 \cdot \left(\frac{1}{2}\right)^3 = 288 \cdot \frac{1}{8} = 36$ ← . . . should be the volume of the dilated prism.

. . . times the cube of the scale factor . . .

- Use the formula to find the volume of the dilated prism.

$$V = Bh$$
$$V = (3 \cdot 2) \cdot 6$$
$$V = 36 \text{ cm}^3$$

The volume of the original prism decreased by a factor of $\frac{36}{288} = \frac{1}{8}$.

Try It

A spherical balloon has a volume of 54 cubic inches. By how many cubic inches would its volume increase if it were further inflated to triple its radius?

If the dimensions of the balloon were increased by a scale factor of ________ , then the volume of the balloon would increase by the ___________ of that scale factor.

The volume of the balloon initially is ________ cubic inches.

Its dimensions would increase by a scale factor of ________ .

Its volume would increase by a factor of ________ .

Find the volume of the enlarged balloon.

$$V = \_\_\_\_\_\_\_\_ \bullet (\_\_\_\_\_\_\_\_)^3$$

$$V = \_\_\_\_\_\_\_\_ \bullet \_\_\_\_\_\_\_\_$$

$$V = \_\_\_\_\_\_\_\_ \text{ in.}^3$$

The volume of the enlarged balloon would be ________ cubic inches.

The difference between the volumes is 1458 – 54 = ________ cubic inches.

The volume of the balloon would increase by ________ cubic inches.

If the dimensions of the balloon were increased by a scale factor of 3, then the volume of the balloon would increase by the cube of that scale factor. The volume of the balloon initially is 54 cubic inches. Its dimensions would increase by a scale factor of 3. Its volume would increase by a factor of $(3)^3$.

$$V = 54 \bullet (3)^3$$

$$V = 54 \bullet 27$$

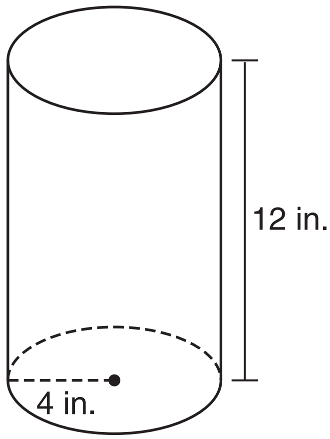
$$V = 1458 \text{ in.}^3$$

The volume of the enlarged balloon would be 1458 cubic inches. The difference between the volumes is 1458 − 54 = 1404 cubic inches. The volume of the balloon would increase by 1404 cubic inches.

Now practice what you've learned.

Question 60

What is the total surface area in square inches of the cylinder shown below?



A 96π in.^{2}

B 128π in.^{2}

C 104π in.^{2}

D 384π in.^{2}

Answer Key: page 226

Question 61

Sterling Coffee is sold in two different cans: regular and large. The dimensions of the large can are twice the dimensions of the regular can. If the regular can has a volume of 72 cubic inches, what is the volume of the large can?

A 144 in.^{3}

B 288 in.^{3}

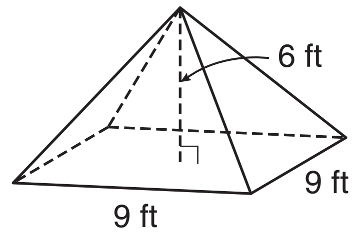
C 576 in.^{3}

D 432 in.^{3}

Answer Key: page 226

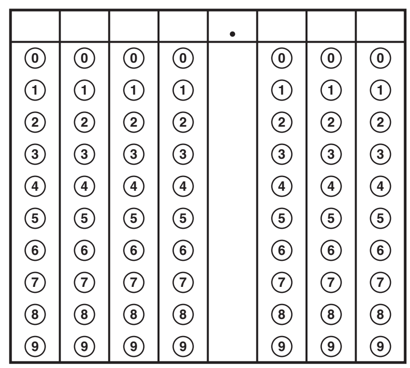
Question 62

A square pyramid has the dimensions shown below.



What is the volume of the pyramid in cubic feet?

Record your answer and fill in the bubbles. Be sure to use the correct place value.



Answer Key: page 226

Question 63

A rectangular picture frame has a length of 14 inches and a width of 11 inches. A larger rectangular frame that is similar has a width of 24 inches. Which measurement is closest to the length of the larger frame?

A 18.9 in.

B 25.0 in.

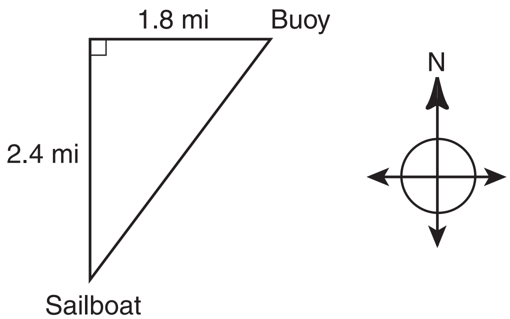
C 30.5 in.

D 27.0 in.

Answer Key: page 227

Question 64

A sailboat is anchored 1.8 miles west and 2.4 miles south of a buoy.



What is the distance between the sailboat and the buoy?

A 9.0 mi

B 4.5 mi

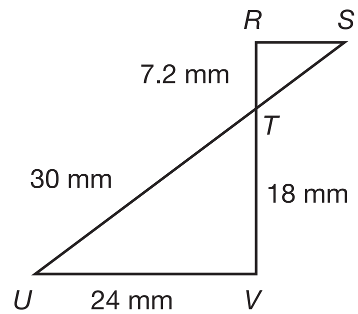
C 4.2 mi

D 3.0 mi

Answer Key: page 227

Question 65

$\triangle RST$ is similar to $\triangle VUT$.



What is the length of $\overline{RS}$?

A 3.3 millimeters

B 9.6 millimeters

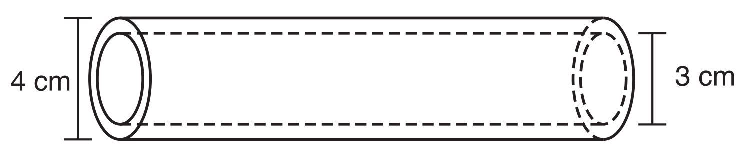
C 9.0 millimeters

D 5.8 millimeters

Answer Key: page 227

Question 66

A water pipe has an outer diameter of 4 centimeters. Its inner diameter is 3 centimeters.



Approximately how many cubic centimeters of water can 50 centimeters of the pipe hold?

A 353 cm^3

B 628 cm^3

C 1413 cm^3

D 2512 cm^3

Answer Key: page 227

Question 67

Carol and Neil both have rectangular vegetable gardens. The rectangles are similar. The area of Carol's garden is 4 times the area of Neil's. Which scale factor represents the ratio of the dimensions of the smaller garden to the dimensions of the larger garden?

A $\frac{1}{2}$

B $\frac{1}{8}$

C $\frac{1}{16}$

D $\frac{1}{4}$

Answer Key: page 228

Question 68

If the lengths of the sides of a square are tripled, how will the perimeter of the new square compare to that of the original square?

A The perimeter will be 3 times the perimeter of the original square.

B The perimeter will be 9 times the perimeter of the original square.

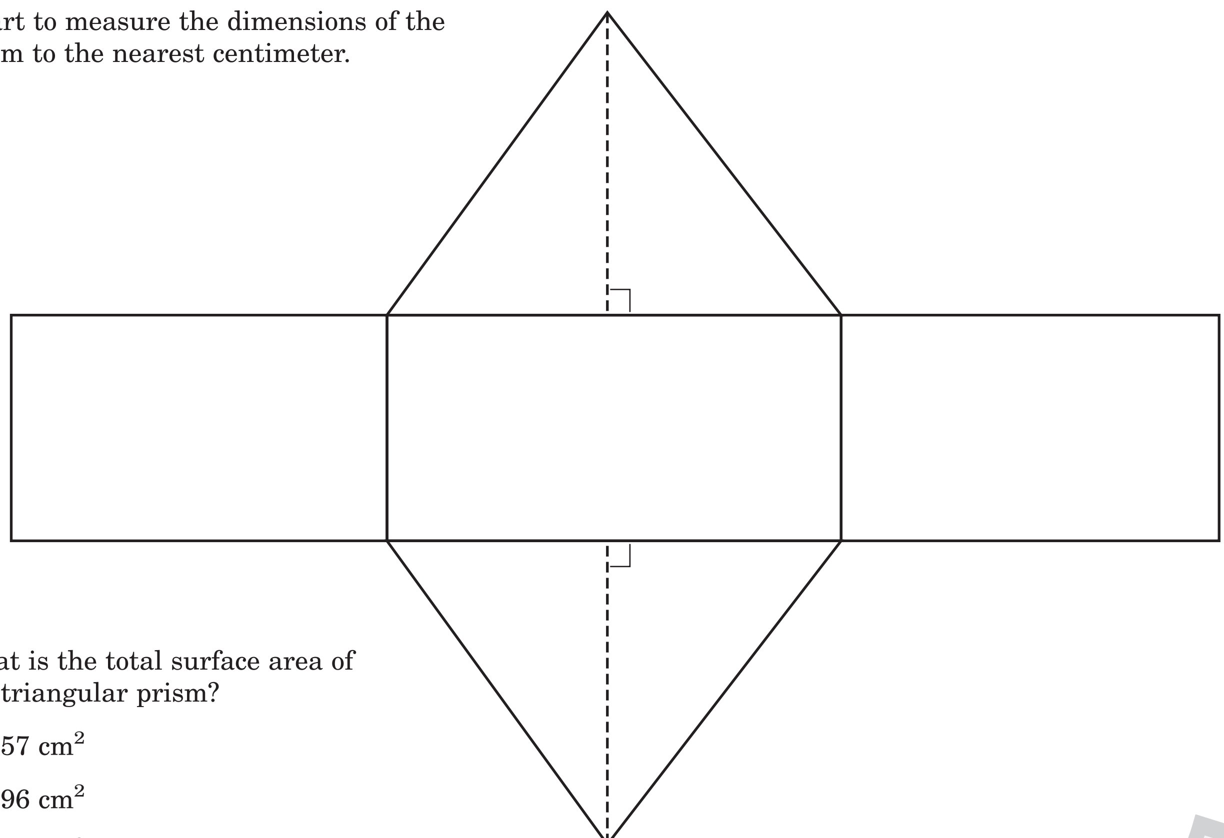
C The perimeter will be 27 times the perimeter of the original square.

D The perimeter will be 6 times the perimeter of the original square.

Answer Key: page 228

Question 69

The net of a triangular prism is shown below. Use the ruler on the Mathematics Chart to measure the dimensions of the prism to the nearest centimeter.



What is the total surface area of the triangular prism?

A 57 cm^2

B 96 cm^2

C 80 cm^2

D 72 cm^2

Answer Key: page 228