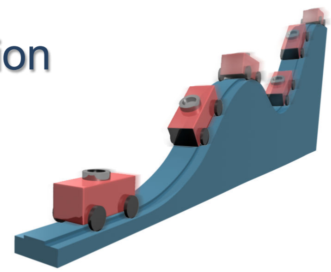
Conservation of energy



Objectives

- Use a variety of approaches and techniques to solve physics problems.
- Describe the advantages and drawbacks of using conservation of energy to solve problems.
- Apply the law of conservation of energy to systems involving kinetic energy, gravitational potential energy, and elastic potential energy.

Assessment

1. List two problem-solving approaches you could use to solve this physics problem:

   A brick falls off a 20 meter high chimney.
   What is its speed when it hits the ground?

2. Many physics problems can be solved in more than one way. Name one advantage and one disadvantage of using the law of conservation of energy to solve problems.

Assessment

3. A cannonball and a bullet are fired straight up. They both leave the ground at a speed of 100 m/s. Which one reaches the highest height? Neglect air resistance.

   A. the cannonball

   B. the bullet

   C. it's a tie

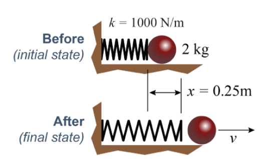
Assessment

4. A cannonball is fired straight up at a speed of 100 m/s. What is the highest height it reaches? Solve this problem using conservation of energy. Neglect air resistance.

5. What is the final velocity of a ball, dropped from rest, that falls 25 meters? Solve this problem using conservation of energy. Neglect air resistance.

Assessment

6. A spring is used to launch a 2.0 kg ball. The spring is compressed 0.25 meters and has a spring constant k = 1000 N/m. What is the maximum speed of the ball?



Physics terms

- mechanical energy
- conservation of energy
- air resistance

Equations

For a closed system that undergoes changes in mechanical energy only:

$$\overbrace{E_k + E_p}^{\text{initial}} = \overbrace{E_k + E_p}^{\text{final}}$$

$E_k = \frac{1}{2}mv^2$ kinetic energy

$E_p = mgh$ gravitational potential energy

$E_p = \frac{1}{2}kx^2$ elastic potential energy

Essential questions

Do heavy objects fall faster than light objects, or do all objects fall at the same rate? How can we explain this?

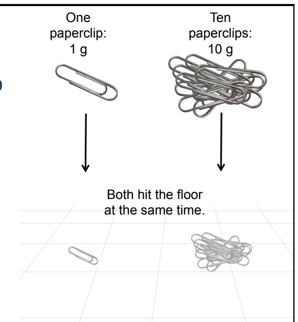
Can physics problems be solved in more than one way, using different approaches?

Free fall

The force of gravity on a group of 10 paperclips is 10 times greater than the force on a single clip.

But if you drop them, they hit the ground at the same time!

Why? Why doesn't the group of paperclips fall 10 times faster??



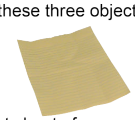
Free fall

What happens when we drop these three objects?

The 1 gram sheet of paper falls slowly, but the crumpled sheet falls almost as fast as the 7 gram marble.

Why? What's the difference?

Consider dropping these three objects



Flat sheet of paper: 1 g

Crumpled sheet of paper: 1 g

3/4" glass marble: 7 g

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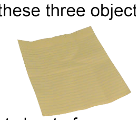


Flat sheet of paper: 1 g

Crumpled sheet of paper: 1 g

3/4" glass marble: 7 g

Free fall

Galileo Galilei was the first person to realize that all objects fall at the SAME rate in the absence of air resistance.

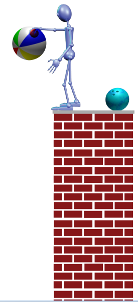
Popular legend holds that he dropped two balls of different mass from the top of the Leaning Tower of Pisa to demonstrate his idea.

Historians dispute this story, but the legend lives on.

Test your knowledge

A large beach ball and a bowling ball are dropped from a chimney top.

Which hits the ground first?

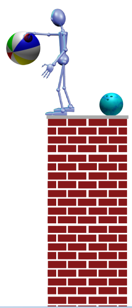


Test your knowledge

A large beach ball and a bowling ball are dropped from a chimney top.

Which hits the ground first?

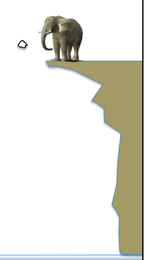
The bowling ball will reach the ground faster because the effect of air resistance on the beach ball is significant.



Test your knowledge

An enormous elephant and a small rock tumble off a cliff.

Which hits the ground first?



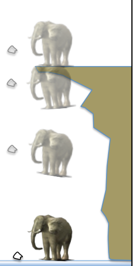
Test your knowledge

An enormous elephant and a small rock tumble off a cliff.

Which hits the ground first?

They will both hit the ground at roughly the same time.

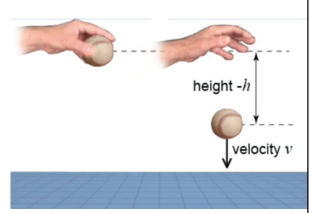
The effect of air resistance will be negligible for these dense objects.



Solving free fall problems

If you drop a ball from height h, what is its speed just before hitting the ground? Assume no air resistance.

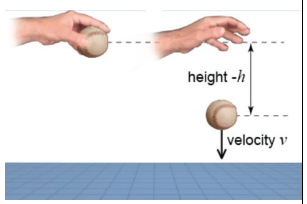
Can you describe two <u>different</u> ways to solve this problem?



Solving free fall problems

Method 1: Use the equations of motion to solve for *v*.

$h = \frac{1}{2}gt^2$ solve for t: $t = \sqrt{\frac{2h}{g}}$



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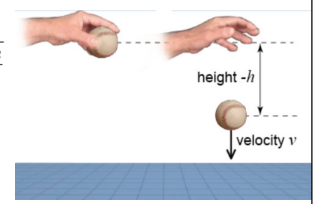
if v_0 is 0, $v = gt$



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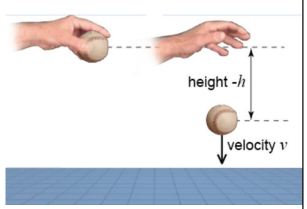
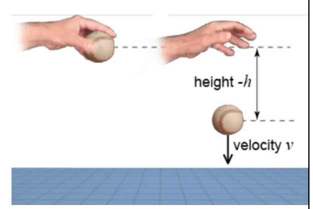


Solving free fall problems

Method 2: Use conservation of energy to solve for *v*.

$$\overbrace{E_k + E_p}^{\text{initial}} = \overbrace{E_k + E_p}^{\text{final}}$$



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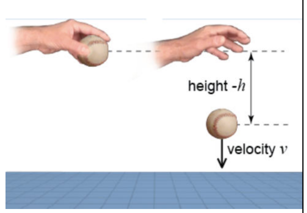

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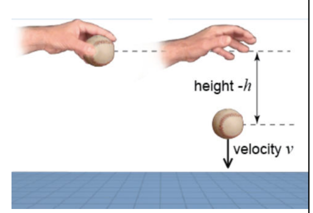

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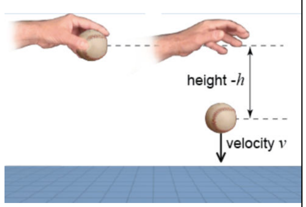

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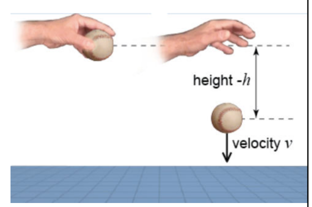

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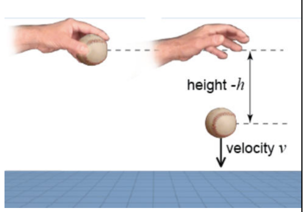
$$v = \sqrt{2gh}$$



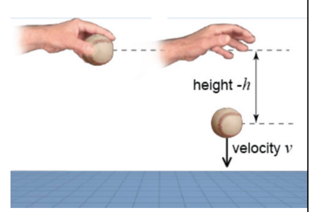
Key ideas

- **The final velocity doesn't depend on mass.**

  We used conservation of energy to show that the *mass cancels out*.

  Heavy objects and light objects all fall at the same rate.

$$v = \sqrt{2gh}$$



Key ideas

- **There is often more than one way to solve a physics problem.**

  Two *different* solution methods gave us the same answer for *v*.

$$v = \sqrt{2gh}$$

Method 1: Use the equations of motion to solve for *t*.

$h = \frac{1}{2}gt^2$ solve for t: $t = \sqrt{\frac{2h}{g}}$

if v_0 is 0, $v = gt$

substituting for t gives:

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Method 2: Use conservation of energy to solve for *v*.

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$$gh_i = \frac{1}{2}v_f^2$$

$$v = \sqrt{2gh}$$

Key ideas

- **Science must be *self-consistent*.**

  Different solution methods must give the same results.

  Every physics theory and equation must agree with all other theories and equations when applied to the same situation.

$$v = \sqrt{2gh}$$

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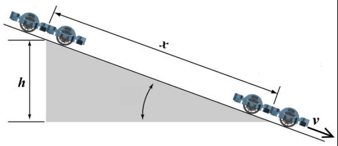
Motion on an incline plane

Let's look at a different problem.

A cart is released from rest at the top of a frictionless ramp.

What is the final velocity of the cart?

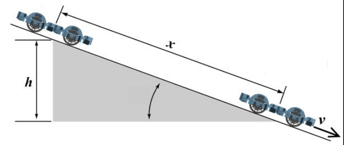


Motion on an incline plane

Use conservation of energy to solve for *v*.

$$\overbrace{E_k + E_p}^{\text{initial}} = \overbrace{E_k + E_p}^{\text{final}}$$

Which terms can we get rid of because they equal zero?



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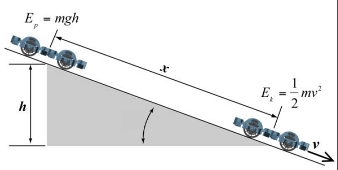

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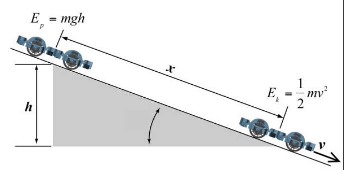

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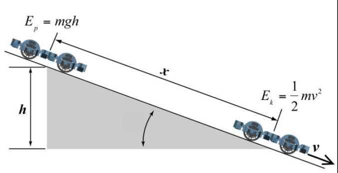


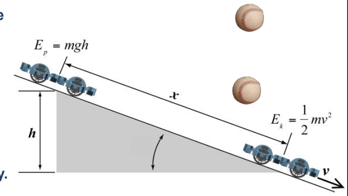
Key idea

The cart on the ramp and the dropped ball have the same final velocity!

In both cases, ALL the potential energy becomes kinetic energy.

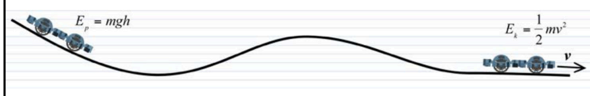
It doesn't matter HOW the object loses potential energy.

$$v = \sqrt{2gh}$$



Applying conservation of energy

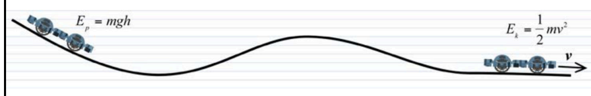
Conservation of energy works the same way for a falling ball, a straight ramp or a frictionless roller coaster.



Pros and Cons

What are the advantages of using conservation of energy to solve problems?

What are the drawbacks?

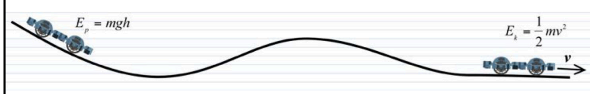


Pros

- One equation can be adapted to solve many problems:

$$\overbrace{E_k + E_p}^{\text{initial}} = \overbrace{E_k + E_p}^{\text{final}}$$

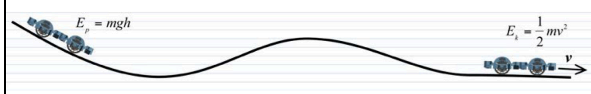
- This law applies even if forces and accelerations are <u>not constant</u>.
- You don't need to know the details of what happened between your initial and final points.



Cons

We lose track of an important piece of information when we use conservation of energy. What is it?

Hint: What variable does NOT appear in ANY of the energy equations?

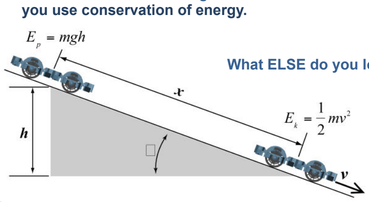


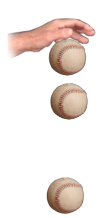
Cons

Time!

You lose track of *how long* an event lasts when you use conservation of energy.

What ELSE do you lose?

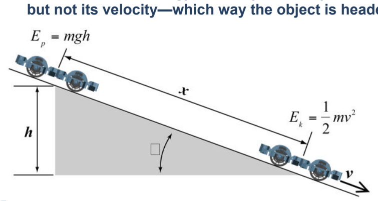




Cons

Direction!

Conservation of energy can tell you an object's speed, but not its velocity—which way the object is headed.



Problems involving springs

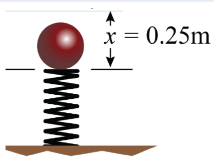
Spring involve a third form of energy: elastic potential energy.

In problems with springs the total energy may have three terms:

- gravitational potential energy
- kinetic energy
- elastic potential energy

Total energy

$$E_{total} = mgh + \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$



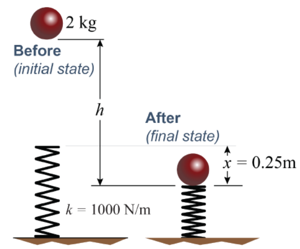
Essential Physics

Problems involving springs

Consider a 2.0 kg ball that drops onto a spring and compresses the spring 25 cm.

How far did the ball drop? The spring constant k is 1000 N/m.

Start by identifying the *Before* and *After* states.



Essential Physics

Problems solving steps

Consider a 2.0 kg ball that drops onto a spring and compresses the spring 25 cm. How far did the ball drop? The spring constant k is 1000 N/m.

	Initial state		Final state
1 Write down relevant forms of energy.	$mgh + \frac{1}{2}mv^2 + \frac{1}{2}kx^2$	=	$mgh + \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

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2 Eliminate any terms that are zero.	$mgh + \frac{1}{2}mv^2_{0} + \frac{1}{2}kx^2_{0}$	=	$mgh_{0} + \frac{1}{2}mv^2_{0} + \frac{1}{2}kx^2$

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2 Eliminate any terms that are zero.	$mgh + \frac{1}{2}mv^2_{0} + \frac{1}{2}kx^2_{0}$	=	$mgh_{0} + \frac{1}{2}mv^2_{0} + \frac{1}{2}kx^2$
3 Solve for the variable you want.	mgh	=	$\frac{1}{2}kx^2$

Solution: $h = \dfrac{kx^2}{2mg}$

$$h = \frac{kx^2}{2mg} = \frac{(1000\ \text{N/m})(0.25\ \text{m})^2}{2(2.0\ \text{kg})(9.8\ \text{m/s}^2)} = 1.6\ \text{m}$$

Essential Physics

Assessment

1. List two problem-solving approaches you could use to solve this physics problem:

   A brick falls off a 20 meter high chimney. What is its speed when it hits the ground?

Essential Physics

Assessment

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   A brick falls off a 20 meter high chimney.
   What is its speed when it hits the ground?

Method #1: Use the equations of motion: $x = \frac{1}{2}gt^2$ and $v = gt$

Method #2: Apply the law of conservation of energy: $mgh = \frac{1}{2}mv^2$

Assessment

2. Many physics problems can be solved in more than one way. Name one advantage and one disadvantage of using the law of conservation of energy to solve problems.

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Possible advantages:
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- There is one starting equation for all problems.
- You don't need to know every detail of the motion during the event.

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- Conservation of energy works even if the forces are not constant.
- There is one starting equation for all problems.
- You don't need to know every detail of the motion during the event.

Possible disadvantages:
- You don't know how long the event lasted.
- You lose information on direction.

Assessment

3. A cannonball and a bullet are fired straight up. They both leave the ground at a speed of 100 m/s. Which one reaches the highest height? Neglect air resistance.

   A. the cannonball

   B. the bullet

   C. it's a tie

Assessment

3. A cannonball and a bullet are fired straight up. They both leave the ground at a speed of 100 m/s. Which one reaches the highest height? Neglect air resistance.

   A. the cannonball

   B. the bullet

   C. it's a tie

All objects fall at the same rate. They will also slow down at the same rate if they are fired upward. In both cases, kinetic energy is transformed into potential energy. The mass cancels out of the conservation of energy equation.

Assessment

4. A cannonball is fired straight up at a speed of 100 m/s. What is the highest height it reaches? Solve this problem using conservation of energy. (Neglect air resistance.)

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$$\overbrace{E_k + E_p}^{\text{initial}} = \overbrace{E_k + E_p}^{\text{final}}$$

$$\frac{1}{2}mv_i^2 = mgh_f \implies \frac{1}{2}v_i^2 = gh_f$$

$$h = \frac{v_i^2}{2g} = \frac{(100 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 510 \text{ meters}$$

Assessment

5. What is the final velocity of a ball, dropped from rest, that falls 25 meters? Solve this problem using conservation of energy. Neglect air resistance.

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The loss of potential energy equals the gain in kinetic energy:

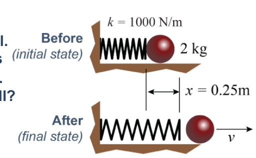
$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(9.8 \text{ m/s}^2)(25 \text{ m})} = 22 \text{ m/s}$$

Assessment

6. A spring is used to launch a 2.0 kg ball. The spring is compressed 0.25 meters and has spring constant k = 1000 N/m. What is the maximum speed of the ball?



Asked:

Given:

Relationships:

Solution:

Assessment

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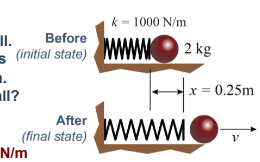

Asked: speed, v

Given: m = 2 kg, x = 0.25 m, k = 1000 N/m

Relationships: $ME = \frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2$

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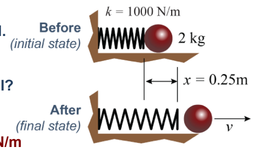

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Given: m = 2 kg, x = 0.25 m, k = 1000 N/m

Relationships: $ME = \frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2$

Solution: $\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 \Rightarrow kx_i^2 = mv_f^2 \Rightarrow v_f = x_i\sqrt{\frac{k}{m}}$

$$v_f = 0.25\text{ m}\sqrt{\frac{1000\text{ N/m}}{2.0\text{ kg}}} = 5.6\text{ m/s}$$

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