Oxford Cambridge and RSA

Accredited

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \dots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \dots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \dots\ + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \ \dots \qquad \left(|x|<1,\ n \in \mathbb{R}\right)$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}) \}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r \, p^r q^{n-r}$ where $q = 1 - p$

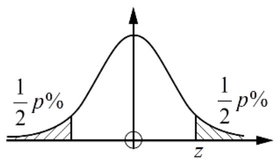
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (23 marks)

1 **Fig. 1** shows a sector of a circle of radius 7 cm. The area of the sector is 5 cm^2.

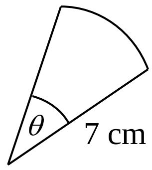


Fig. 1

Find the angle θ in radians. **[2]**

2 A geometric series has first term 3. The sum to infinity of the series is 8.
Find the common ratio. **[3]**

3 Solve the inequality $|2x-1| \geq 4$. **[4]**

4 Differentiate the following.

(a) $\sqrt{1-3x^2}$ **[3]**

(b) $\dfrac{x^2}{3x+2}$ **[3]**

5 A woman is pulling a loaded sledge along horizontal ground. The only resistance to motion of the sledge is due to friction between it and the ground.

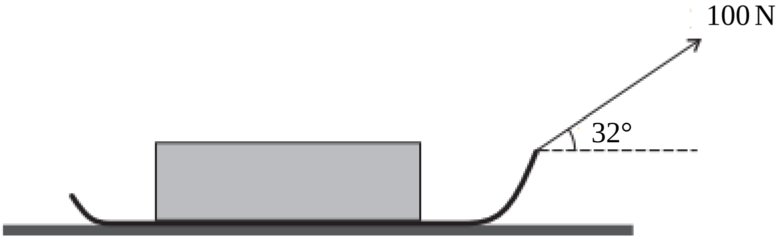


Fig. 5

At first, she pulls with a force of 100 N inclined at 32° to the horizontal, as shown in **Fig.5**, but the sledge does not move.

(a) Determine the frictional force between the ground and the sledge.
Give your answer correct to 3 significant figures. **[2]**

(b) Next she pulls with a force of 100 N inclined at a smaller angle to the horizontal. The sledge still does not move.

Compare the frictional force in this new situation with that in part **(a)**, justifying your answer. **[2]**

6 **Fig. 6** shows a partially completed spreadsheet.

This spreadsheet uses the trapezium rule with four strips to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\, dx$.

	A	**B**	**C**	**D**	**E**
1		x	$\sin x$	y	
2	0	0.0000	0.0000	1.0000	0.5000
3	0.125	0.3927	0.3827	1.1759	1.1759
4	0.25	0.7854	0.7071	1.3066	1.3066
5	0.375	1.1781	0.9239	1.3870	1.3870
6	0.5	1.5708	1.0000	1.4142	0.7071
7					5.0766
8					

Fig. 6

(a) Show how the value in cell B3 is calculated. **[1]**

(b) Show how the values in cells D2 to D6 are used to calculate the value in cell E7. **[1]**

(c) Complete the calculation to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\, dx$.

Give your answer to 3 significant figures. **[2]**

Answer **all** the questions

Section B (77 marks)

7 **In this question take $g = 10$.**

A small stone is projected from a point O with a speed of 26 m s^{-1} at an angle θ above the horizontal. The initial velocity and part of the path of the stone are shown in **Fig. 7**.

You are given that $\sin\theta = \frac{12}{13}$.

After t seconds the horizontal displacement of the stone from O is x metres and the vertical displacement is y metres.

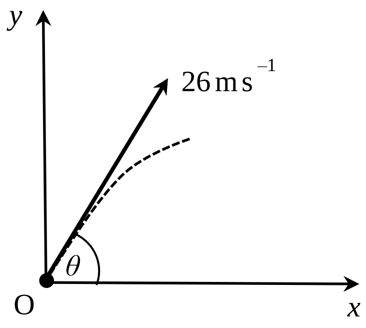


Fig. 7

(a) Using the standard model for projectile motion,

- show that $y = 24t - 5t^2$,
- find an expression for x in terms of t. **[4]**

The stone passes through a point A. Point A is 16 m above the level of O.

(b) Find the two possible horizontal distances of A from O. **[4]**

A toy balloon is projected from O with the same initial velocity as the small stone.

(c) Suggest two ways in which the standard model could be adapted. **[2]**

8 Find $\int x^2 \mathrm{e}^{2x}\,\mathrm{d}x$. **[7]**

9 In an experiment, a small box is hit across a floor. After it has been hit, the box slides without rotation.

The box passes a point A. The distance the box travels after passing A before coming to rest is S metres and the time this takes is T seconds.

The only resistance to the box's motion is friction due to the floor. The mass of the box is m kg and the frictional force is a constant F N.

(a) **(i)** Find the equation of motion for the box while it is sliding.

(ii) Show that $S = kT^2$ where $k = \dfrac{F}{2m}$. **[4]**

(b) Given that $k = 1.4$, find the value of the coefficient of friction between the box and the floor. **[4]**

10 In a certain region, the populations of grey squirrels, P_G and red squirrels P_R, at time t years are modelled by the equations:

$$P_G = 10\,000\left(1 - e^{-kt}\right)$$

$$P_R = 20\,000e^{-kt}$$

where $t \geq 0$ and k is a positive constant.

(a) **(i)** On the axes in your Printed Answer Book, sketch the graphs of P_G and P_R on the same axes.

(ii) Give the equations of any asymptotes. **[4]**

(b) What does the model predict about the long term population of

- grey squirrels
- red squirrels? **[2]**

Grey squirrels and red squirrels compete for food and space. Grey squirrels are larger and more successful than red squirrels.

(c) Comment on the validity of the model given by the equations, giving a reason for your answer. **[1]**

(d) Show that, according to the model, the rate of decrease of the population of red squirrels is always double the rate of increase of the population of grey squirrels. **[4]**

(e) When $t = 3$, the numbers of grey and red squirrels are equal. Find the value of k. **[4]**

11 **Fig. 11** shows the curve with parametric equations

$$x = 2\cos\theta,\ y = \sin\theta,\ 0 \le \theta \le 2\pi.$$

The point P has parameter $\frac{1}{4}\pi$. The tangent at P to the curve meets the axes at A and B.

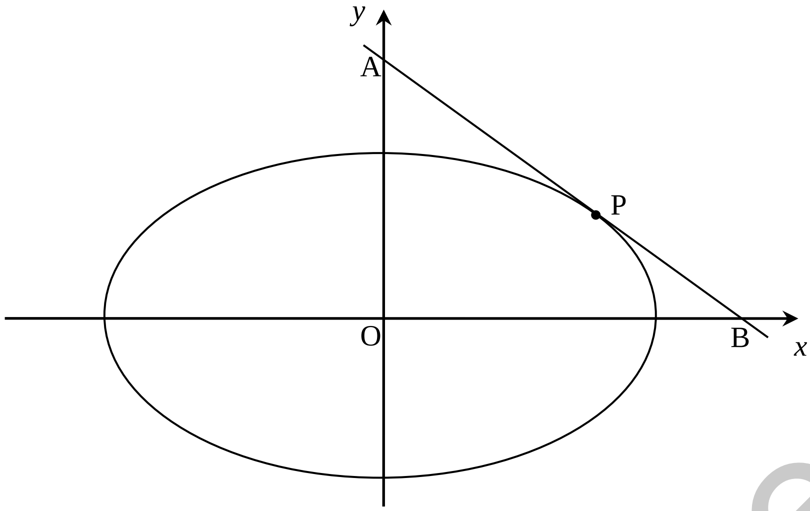


Fig. 11

(a) Show that the equation of the line AB is $x + 2y = 2\sqrt{2}$. **[6]**

(b) Determine the area of the triangle AOB. **[3]**

12 A model boat has velocity $\mathbf{v} = \left((2t-2)\mathbf{i} + (2t+2)\mathbf{j}\right)$ m s^{-1} for $t \ge 0$, where t is the time in seconds.
$\mathbf{i}$ is the unit vector east and $\mathbf{j}$ is the unit vector north.
When $t = 3$, the position vector of the boat is (3**i** + 14**j**) m.

(a) Show that the boat is never instantaneously at rest. **[2]**

(b) Determine any times at which the boat is moving directly northwards. **[2]**

(c) Determine any times at which the boat is north-east of the origin. **[5]**

13 **In this question you must show detailed reasoning.**

Determine the values of k for which part of the graph of $y = x^2 - kx + 2k$ appears below the x-axis.

[4]

14 Blocks A and B are connected by a light rigid horizontal bar and are sliding on a rough horizontal surface.

A light horizontal string exerts a force of 40 N on B.
This situation is shown in **Fig. 14**, which also shows the direction of motion, the mass of each of the blocks and the resistances to their motion.

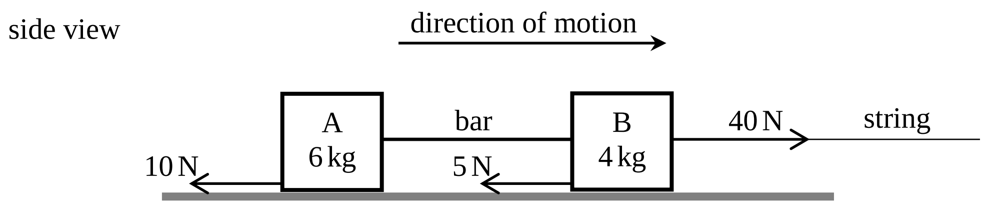


Fig. 14

(a) Calculate the tension in the bar. **[4]**

The string breaks while the blocks are sliding. The resistances to motion are unchanged.

(b) Determine
- the magnitude of the new force in the bar,
- whether the bar is in tension or in compression. **[5]**

15 **Fig. 15** shows a uniform shelf AB of weight W N.
The shelf is 180 cm long and rests on supports at points C and D. Point C is 30 cm from A and point D is 60 cm from B.

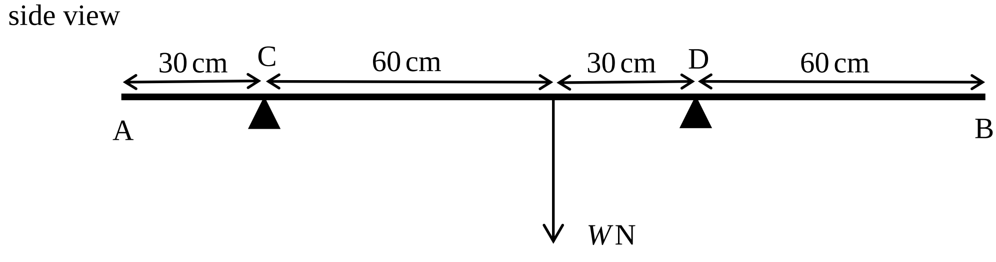


Fig. 15

Determine the range of positions a point load of 3 W could be placed on the shelf without the shelf tipping. **[6]**

END OF QUESTION PAPER

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…day June 20XX – Morning/Afternoon

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 20 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

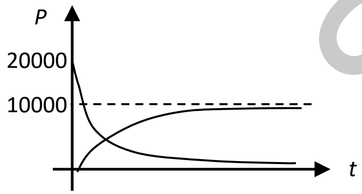
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			Area = $5 = \frac{1}{2} \times 7^2 \times \theta$	M1	3.1a		
			$\theta = \frac{10}{49} [= 0.204]$	A1	1.1		
				[2]			
2			$\frac{3}{1-r} = 8$		1.1		
				M1		Use of correct formula	
			$\Rightarrow 3 = 8(1 - r)$	M1	1.1	Clearing fraction	
			$\Rightarrow r = \frac{5}{8}$	A1	1.1		
				[3]			
3			**Either**				
			$\lvert 2x-1 \rvert \geq 4$				
			$\Rightarrow \quad 2x - 1 \geq 4$	M1	1.1		
			or $\quad 2x - 1 \leq -4$	M1	1.1		
			Or				
			$(2x-1)^2 \geq 16$	M1	1.1	M1 for sketch graph of $y = (2x-1)^2$ and $y = 16$	
			$4x^2 - 4x - 15 \geq 0$ $(2x-5)(2x+3) \geq 0$	M1	1.1	M1 for $x = 2\frac{1}{2}, -1\frac{1}{2}$	
			$\Rightarrow x \geq 2\frac{1}{2}$	A1	1.1		
			$\Rightarrow x \leq -1\frac{1}{2}$				
			$\{x : x \leq -1\frac{1}{2}\} \cup \{x : x \geq 2\frac{1}{2}\}$	A1	2.5	OR $x \geq 2\frac{1}{2}$ or $x \leq -1\frac{1}{2}$ If final ans not in one of these forms then withhold final A1	
				[4]			

Question			Answer	Marks	AOs	Guidance	
4	(a)		$\frac{dy}{dx} = \frac{1}{2}\left(1 - 3x^2\right)^{-½}.(-6x)$ $= \frac{-3x}{\sqrt{\left(1 - 3x^2\right)}}$	B1 M1 A1 [3]	1.1 1.1 1.1	$\frac{1}{2}u^{-\frac{1}{2}}$ soi Chain rule oe, but must simplify $\frac{1}{2} \times 6$	
4	(b)		$\frac{dy}{dx} = \frac{(3x+2).2x - x^2.3}{(3x+2)^2}$ $= \frac{3x^2+4x}{(3x+2)^2}$	M1 A1 A1 [3]	1.1 1.1 1.1	Quotient rule or product rule oe, but must simplify numerator	
5	(a)		Horizontal component = $100 \times \cos 32^{\circ}$ 84.8 N	M1 A1 [2]	3.3 1.1		
5	(b)		Frictional force is 100 cos θ [with θ < 32°] So frictional force increases. (oe)	M1 E1 [2]	3.4 2.2a		

Question			Answer	Marks	AOs	Guidance	
6	(a)		A3*π oe	B1 [1]	2.2a	Or $0.125\times\pi$ oe	
6	(b)		$\frac{1}{2}$ D2 + D3 + D4 + D5 + $\frac{1}{2}$ D6	B1 [1]	2.2a	Or equivalent expressed in words.	
6	(c)		5.0766 × 0.3927 = 1.9935... 1.99 (units^2) (to 3sf)	M1 A1 [2]	1.1 1.1	Or $5.0766 \times \frac{\pi}{8}$	
7	(a)		$y = ut\sin\theta - \frac{1}{2}gt^2$ stated and used $y = 26\times\frac{12}{13}t - 5t^2$ $= 24t - 5t^2$ $x = 26\times\frac{5}{13}t$ $= 10t$	M1 E1 M1 A1 [4]	3.3 2.1 3.4 1.1	AG Use of $\frac{5}{13}$ Accept any form	Given answer must be seen to score E1
7	(b)		We require $16 = 24t - 5t^2$ Solving $5t^2 - 24t + 16 = 0$ $((5t-4)(t-4) = 0$ or ...) $t = 0.8$ or 4 Distances are 10 × 0.8 = 8 m and 10 × 4 = 40 m.	M1 M1 A1 B1FT [4]	3.4 1.1 1.1 3.2a	Equating their y expression to 16 Method that could give 2 correct roots for their quadratic. Implied by 2 correct roots for their quadratic Cao FT only their t	

Question			Answer	Marks	AOs	Guidance	
7	(c)		E.g. Air resistance should be included E.g. The balloon should not be treated as a particle E.g. Horizontal force due to wind should be considered	**B1** **B1** **[2]**	**3.5c** **3.5c**	Any two appropriate factors that would have an impact on the model.	
8			let $u = x^2$, $u' = 2x$, $v' = e^{2x}$, $v = \frac{1}{2}e^{2x}$ $\int x^2e^{2x}dx = \frac{1}{2}x^2e^{2x} - \int 2x.\frac{1}{2}e^{2x}dx = \frac{1}{2}x^2e^{2x} - \int xe^{2x}dx$ let $u = x$, $u' = 1$, $v' = e^{2x}$, $v = \frac{1}{2}e^{2x}$ $\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$ $= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}(+c)$ so $\int x^2e^{2x}dx = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + c$	**M1A1** **A1** **M1** **A1** **A1** **A1** **[7]**	**1.1a** **1.2** **1.1** **1.1a** **1.1** **1.1** **2.5**	 Do not award if no ‘+ c’	

Question			Answer	Marks	AOs	Guidance	
9	**(a)**	**(i)**	Let acceleration be a in the direction of motion. N2L in direction of motion gives $-F = ma$ so $a = -\frac{F}{m}$, which is constant.	**M1** **A1**	**3.3** **1.1**	Decide to use N2L to find acceleration No need to say 'constant'	
		(ii)	(As a constant) use *suvat* , giving $S = 0 \times T - \frac{1}{2} \times \left(-\frac{F}{m}\right)T^2$ so $S = \left(\frac{F}{2m}\right)T^2$ and $k = \frac{F}{2m}$	**B1** **E1** **[4]**	**2.1** **2.4**	Use appropriate (sequence of) *suvat*	
9	**(b)**		As sliding, friction is limiting and $F = \mu R$ $R = mg$ $k = \frac{F}{2m}$ so $k = \frac{\mu mg}{2m}$ Hence $\mu = \frac{2k}{g} = \frac{2 \times 1.4}{9.8} = \frac{2}{7}$	**M1** **A1** **M1** **A1** **[4]**	**3.3** **3.4** **1.1** **2.2a**	 In $F = \mu R$, substitute for F & R in terms of m and g Or 0.286 (3s.f.)	
10	**(a)**	**(i)**	 	**M1** **M1**	**1.1** **1.1**	P_G shape through O P_R shape through (0, 20000), [condone graphs for –ve t]	
		(ii)	asymptote for P_G = 10 000 Asymptote for P_R = 0	**A1** **A1** **[4]**	**2.2a** **2.2a**	Or p = 10 000 Or p = 0	

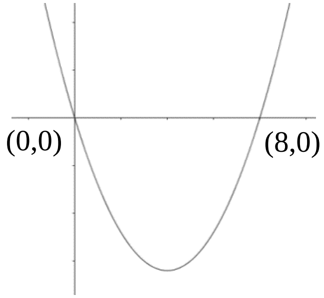
Question			Answer	Marks	AOs	Guidance	
10	**(b)**		Red squirrels zero Grey 10 000	**B1** **B1** **[2]**	**3.4** **3.4**		
10	**(c)**		One relevant comment evaluating the validity of the model	**B1** **[1]**	**3.5a**	E.g. One of • Grey population increases as would be expected [since grey squirrels are larger and more successful] • Red population decreases as would be expected [since red squirrels have to compete with the larger grey squirrels for food] • Number of squirrels tends to a limit as would be expected [since there is limited food and space] • Would expect grey population to grow slower at first • Would expect red population to fall slower at first	

Question			Answer	Marks	AOs	Guidance	
10	(d)			M1	3.1b	Attempts to differentiate either or both	
			$\frac{dP_G}{dt} = 10\ 000ke^{-kt}$	A1	1.1		
			$\frac{dP_R}{dt} = -20\ 000ke^{-kt}$	A1	1.1		
			so $\frac{dP_R}{dt} = -2\frac{dP_G}{dt}$	E1	2.1	Or in words	
				[4]			
10	(e)		$10\ 000(1 - e^{-3k}) = 20\ 000e^{-3k}$ $\Rightarrow 1 - e^{-3k} = 2e^{-3k}$	M1	1.1a		
			$\Rightarrow e^{-3k} = \frac{1}{3}$	A1	1.1		
			$\Rightarrow -3k = \ln\left(\frac{1}{3}\right)$	M1	1.1	Taking natural logs of both sides	
			$\Rightarrow k = -\frac{1}{3}\ln\left(\frac{1}{3}\right) = 0.366$ or $\frac{1}{3}\ln 3$	A1	2.1	cao	
				[4]			

Question			Answer	Marks	AOs	Guidance	
11	(a)		P is $(\sqrt{2}, \frac{\sqrt{2}}{2})$	B1	1.1	oe	
			$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	3.1a		
			$= \frac{\cos\theta}{-2\sin\theta}$	A1	1.1		
			When $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{1}{2}$	A1	1.1		
			Equation of tangent is $(y - \frac{\sqrt{2}}{2}) = -\frac{1}{2}(x - \sqrt{2})$	B1	2.1		
			$\Rightarrow \quad y = -\frac{1}{2}x + \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}$	E1	1.1		
			$\Rightarrow \quad x + 2y = 2\sqrt{2}$			AG	
				[6]			
11	(b)		When $x = 0$, $y = \sqrt{2}$ so A is $(0, \sqrt{2})$	B1	1.1		
			When $y = 0$, $x = 2\sqrt{2}$ so B is $(2\sqrt{2}, 0)$	B1	1.1		
			Area of triangle = $\frac{1}{2}\sqrt{2} \times 2\sqrt{2} = 2$ units^2	B1	1.1		
				[3]			

Question			Answer	Marks	AOs	Guidance	
12	(a)		Require both components zero at the same *time* **i** component zero only when $t = 1$ and **j** component only when $t = -1$ so there are no such times	M1 A1 [2]	3.1b 2.4	May be implied but must be clear Or say **j** component ≥ 2 since $t \geq 0$	
12	(b)		This requires use of the velocity vector Travelling due north means that the **i** component is zero and the **j** component +ve So we need $2t - 2 = 0$ for **i** component, giving $t = 1$. This gives **j** component $4 > 0$ so yes at $t = 1$.	M1 A1 [2]	3.3 2.4	Recognise velocity vector required Must test **j** component	

Question			Answer	Marks	AOs	Guidance	
12	(c)		This requires use of the position vector **either**	M1	3.1b	Recognise position vector required	
			$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$ so $\mathbf{r} = \int \left(\left(2t-2\right)\mathbf{i} + \left(2t+2\right)\mathbf{j}\right)\mathrm{d}t$ = $\left(t^2 - 2t + C\right)\mathbf{i} + \left(t^2 + 2t + D\right)\mathbf{j}$	M1	1.1	May use + **C** instead	
			$\mathbf{r} = 3\mathbf{i} + 14\mathbf{j}$ when $t = 3$ so $C = 0$ and $D = -1$ so $\mathbf{r} = \left(t^2 - 2t\right)\mathbf{i} + \left(t^2 + 2t - 1\right)\mathbf{j}$	A1	1.1		
			or $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j}$ when $t = 3$ $\mathbf{v} = 4\mathbf{i} + 8\mathbf{j}$, $\mathbf{r} = (4\mathbf{i} + 8\mathbf{j})(t-3) + \frac{1}{2}(2\mathbf{i} + 2\mathbf{j})(t-3)^2 + 3\mathbf{i} + 14\mathbf{j}$	M1	1.1	Must find **a** but may omit 3**i** + 14**j**	
			and so $\mathbf{r} = \left(t^2 - 2t\right)\mathbf{i} + \left(t^2 + 2t - 1\right)\mathbf{j}$	A1	1.1		
			the boat is NE of O when the **i** and **j** components are equal and +ve	M1	3.2b	Award even if +ve not mentioned	
			we require $t^2 - 2t = t^2 + 2t - 1$ so $t = 0.25$ this gives components of -0.4375 so no.	A1 [5]	2.1	Must be complete argument	

Question			Answer	Marks	AOs	Guidance	
13			**DR**				
			discriminant = $k^2 - 8k$	**B1**	**1.2**		
			$\Rightarrow k^2 - 8k > 0$	**M1**	**1.1**		
			 	**E1**	**2.4**	Or give table of values, oe	
			$\Rightarrow k < 0$ or $k > 8$	**A1**	**2.5**		or $(\infty,0)\cup(8,\infty)$ or $\{k : k < 0\}\cup\{k : k > 8\}$
				[4]			
14	(a)		Using N2L in direction of motion, acceleration a m s^{-1}	**M1**	**3.3**	$F = ma$ and all forces present	
			$(40 - 10 - 5) = 10a$				
			so $a = 2.5$	**A1**	**1.1**	Not required, may be implied	
			Mark force in bar as tension T [N]				
			Either				
			For A $T - 10 = 6a$	**M1**	**3.4**	Allow their a	
			or				
			For B $40 - 5 - T = 4a$	**M1**	**3.4**	Allow their a	
			so tension is 25 [N]	**A1**	**1.1**	Or $T = 25$	
				[4]			

Question			Answer	Marks	AOs	Guidance	
14	(b)		Without the force from the string, N2L in direction of motion	M1	3.3		
			$-15 = 10a^*$				
			so $a^* = -1.5$	A1	1.1	Not required, may be implied	
			Mark force in bar as tension T^* [N]				
			either				
			For A $T^* - 10 = 6a^* = -9$	M1	3.4	Allow their a	
			or				
			For B $-5 - T^* = 4a^* = -6$	M1	3.4	Allow their a	
			so $T^* = 1$	A1	1.1		
			giving a force of 1 [N] which is a tension	A1	2.2a	Must be made clear in some way	
				[5]			
15			Let the reactions of the supports on the shelf be U N at C and V N at D				
			Neither U nor V can be negative if the shelf does not tip	M1	2.2a	May be implied	
			Any position between C and D must give $U > 0$ and $V > 0$	B1	2.2a	Need not show but must be stated	
			Consider putting the load between A and C, x cm from C				
			cw moments about C	M1	3.1b	moments about C	
			If $V \geq 0$ then $W \times 60 \geq 3W \times x$				
			so $x \leq 20$	A1	1.1	allow <	
			Consider putting the load between D and B, y cm from D				
			anti-cw moments about D				
			If $U \geq 0$ then $W \times 30 \geq 3W \times y$	B1	3.1b		
			so $y \leq 10$			allow <	
			The load must be placed not closer than 10 cm to A and 50 cm to B oe	A1	3.2a	Must be clear statement and include CD	
				[6]			

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1	1		1		**2**
2	3				**3**
3	3	1			**4**
4 a	3				**3**
4 b	3				**3**
5 a	1			1	**2**
5 b		1		1	**2**
6 a		1			**1**
6 b		1			**1**
6 c	2				**2**
7 a	1	1		2	**4**
7 b	2		1	1	**4**
7 c				2	**2**
8	6	1			**7**
9 a	1	2		1	**4**
9 b	1	1		2	**4**
10 a	2	2			**4**
10 b				2	**2**
10 c				1	**1**
10 d	2	1	1		**4**
10 e	3	1			**4**
11 a	4	1	1		**6**
11 b	3				**3**
12 a		1	1		**2**
12 b		1		1	**2**
12 c	2	1	2		**5**
13	2	1	1		**4**
14 a	2			2	**4**
14 b	2	1		2	**5**
15	1	2	3		**6**
Totals	**50**	**21**	**11**	**18**	**100**

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Summary of Updates

Date	Version	Change
October 2018	2	We've reviewed the look and feel of our papers through text, tone, language, images and formatting. For more information please see our assessment principles in our "Exploring our question papers" brochures on our website.

OCR
Oxford Cambridge and RSA

Accredited

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics
Printed Answer Booklet

Date – Morning/Afternoon
Time allowed: 2 hours

You must have:
- Question Paper H640/01 (inserted)

You may use:
- a scientific or graphical calculator

First name	
Last name	

Centre number						Candidate number				

INSTRUCTIONS
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

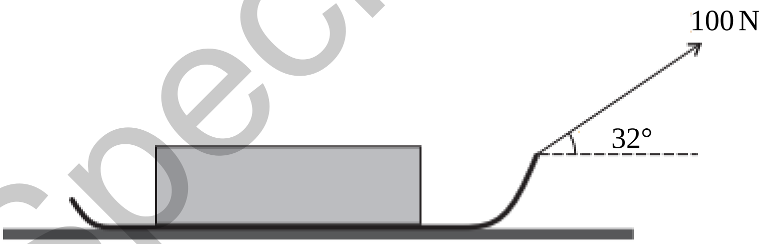
INFORMATION
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Section A (20 marks)

1	
2	

3

4 (a)

4 (b)	
5 (a)	 **Fig. 5**
5 (b)	

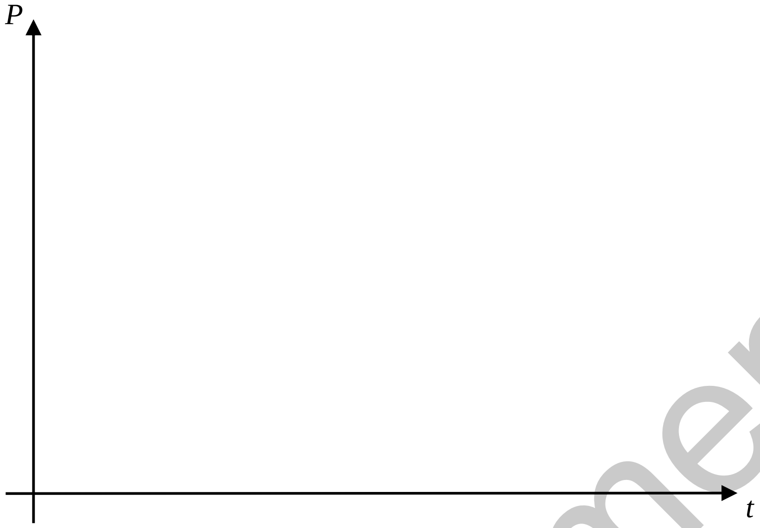
Section B (80 marks)

6 (a)	
6 (b)	
6 (c)	
7 (a)	

7 (b)	
7 (c)	

8	

9 (a)(i)	
9 (a)(ii)	
9 (b)	

10 (a) (i)	
10 (a) (ii)	
10 (b)	

10 (c)	
10 (d)	

10 (e)

11 (a)	

11 (b)	
12 (a)	
12 (b)	

12 (c)

13	

PLEASE DO NOT WRITE IN THIS SPACE

14 (a)

14 (b)	

15

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