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OCTREE BASED UNSTRUCTURED GRID COARSENING METHOD FOR 3D MULTIGRID APPLICATIONS

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Abstract. *In this study, a new cell agglomeration technique is developed for Multigrid flow solutions on three dimensional hybrid/unstructured grids. The grid coarsening required by the Multigrid scheme is achieved by agglomerating the unstructured cells based on their distribution on an octree data structure. The grid coarsening strategy developed is first presented on a cubical flow domain. An unstructured grid over the ONERA M6 wing and a hybrid/unstructured viscous type grid over a NACA0012 airfoil section are then taken to demonstrate working examples and variations of the agglomeration technique developed. It is shown that the octree based agglomeration and grid coarsening provides well defined, nested, body fitted and optimum aspect ratio cells at all coarse grid levels.*

1 INTRODUCTION

The multigrid (MG) technique is considered to be the most effective technique to achieve a reduction in the CPU cost of explicit flow solvers [1, 2]. The basic idea behind the MG strategy is to employ multilevel grids which are the coarsening subsets of the original grid. Explicit solvers are known to rapidly reduce the high frequency errors on computational grids. In a MG solution, high frequency errors at each grid level are therefore reduced rapidly. Since high frequency errors on coarse grids correspond to low frequency errors on finer grids, cycling through the coarse grid levels rapidly reduces all the errors ranging from high to low frequency on the original fine grid.

In a MG application on structured grids, a coarse grid can easily be obtained from a given grid by omitting every other grid point in each coordinate direction. A recursive application of this procedure results in a sequence of coarser grids. On the other hand, it is rather difficult to construct coarse grid levels with unstructured grids. A variety of techniques has been developed for unstructured MG coarse grid constructions [3]. The cell agglomeration technique is a widely used method, in which grid cells are fused together to form a smaller set of larger polygonal (or polyhedral in three dimensions) control volumes. Such a process may easily be automated with no geometry loss, provides fully nested coarse level grids, and high solution accuracy [4].

In an agglomeration, the main difficulty is the selection of the cells to be agglomerated so that the new cells formed have acceptable aspect ratios [3]. In order to improve the aspect ratios of the coarser grid level cells, the most of the MG implementations are based on node-centered schemes. In a cell centered MG implementation, Pandya and Frink employ a global agglomeration technique to obtain coarse level grids for unstructured tetrahedral grids [5]. The technique is described in two steps. In the first step, all the fine grid cells attached to the body surface or a far-field boundary are identified and merged with its neighboring cells to form a new coarser cell. After all these prioritized boundary cells are assigned to a coarser cell, an unassigned fine grid cell on the agglomeration front is picked in a random order and merged with its eligible neighbors to form a new coarser cell. The highest level coarse grid is shown in Figure 1, which shows the presence of low quality cells with a significant degradation in the aspect ratio of the cells.

The main objective of the current work is to develop an automated grid coarsening technique suitable for cell-centered based 3D hybrid/unstructured MG applications, which will provide the aspect ratio of the cells about one at all levels. The grid coarsening technique relies on the agglomeration of hybrid/unstructured cells based on their distribution on an octree data structure [6]. Quadtree or octree data structures are well known in storing data associated with location in 2 and 3-dimensional space, respectively. The term octree is used to describe a hierarchical data structure which is based on the successive subdivision of the space into eight equal-sized quadrants. It is shown that the new grid agglomeration technique developed provides well defined, nested and body fitted cells at all coarse grid levels. Finally, because the grid coarsening technique is automated; it can be integrated into a flow solver and may be enabled at the run-time for a MG application.

The remainder of this paper is organized as follows. Section 2 describes the automated grid coarsening procedure. The four level coarsening applications over a cube, ONERA M6 wing and a wing with NACA0012 airfoil section are presented in Section 3. Concluding remarks and an outlook for future work are summarized in Section 4.

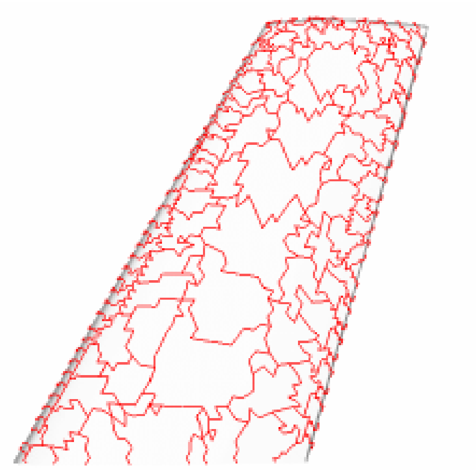


Figure 1. A coarse Level grid on ONERA M6 wing [5].

2 GRID COARSENING BASED ON OCTREE HIERARCHY

In literature, it is seen that, the agglomeration coarsening approach is the most powerful technique and is widely used method due to being fully nested, easily automated, no geometry loss and high solution accuracy. In an agglomeration method, grid cells are fused together to form a smaller set of larger polygonal (or polyhedral in three dimensions) control volumes.

The main difficulty in agglomeration is the selection of the cells (or sometimes called point removal) to be agglomerated so that the new cells formed have acceptable aspect ratios [3]. In this study, a new grid coarsening technique is developed. This technique relies on agglomerating hybrid / unstructured cells based on their distribution on an octree data structure for 3D application.

The term octree is used to describe a class of hierarchical data structures whose common property is that they are based on the principle of recursive decomposition of space. Octrees are spatial data structures that successively partition a region of space into 8 equally sized octants. The mostly investigated octree approach for region representation is based on the successive subdivision of the image array into eight equal-sized octants. The octants and octal tree can be also characterized as given in Figure 2. It can be observed that this process is represented by a tree of degree 8 (i.e., each non-leaf node has eight children).

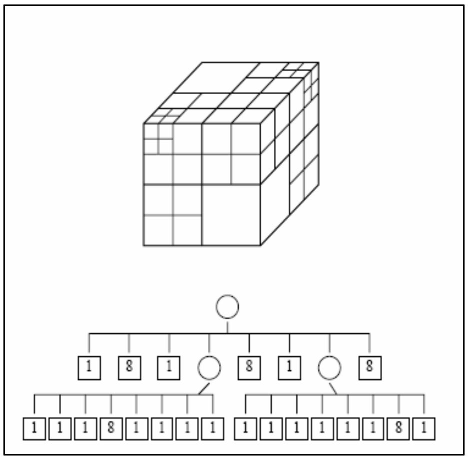


Figure 2 Cells octree example

The octree data structure is used in this study as a cell selection algorithm at agglomeration strategy for coarsening of 3D unstructured / hybrid grid. A computational grid is first represented by the nodes corresponding to the cell centers. The octant cells are created as imaginary cells over the cell domain such a way that each octant covers maximum eight cell center points. It should be noted that the coarse grid cells formed by agglomerating the neighboring cells in such a way provide an aspect ratio of about one. The coarser grids are obtained by agglomerating the cells in the octants into the parent octants successively. In the grid coarsening process, the coarser grids are nested, that is, there is no a newly generated face, edge or nodes between the grid levels. Such a property also assures that the solid boundaries are not removed at the coarser grid levels.

This new agglomeration strategy can be define as globally coarsening method by merging cells according to parent quadrant or sub-groups. The coarsening algorithm is summarized as follows;

If (MG is active go COARSENING);

- Read number of coarse grid level
- Read the fine level grid
- Assign numbers to cell faces
- Assign face/cell connectivity
- Calculate cell centers and store
- Prepare the octal tree structure till all meshes located
- Prepare the 2nd grid level by merging the cells located at the tip of tree
- Prepare coarser grid levels according to parent/child hierarchy defined at octant tree

3 RESULTS AND DISCUSSIONS

In this section, the coarsening algorithm is applied to an unstructured grid over a cube to demonstrate the method developed. Then the coarsening algorithm is applied to an unstructured grid with approximately 1.4 million tetrahedral cells over an ONERA M6 wing in Case 2. A variation of the grid coarsening algorithm in generating isotropic coarse level grids is also presented in Case 2. Finally the coarsening algorithm is implemented to a hybrid viscous grid with 0.34 million cells over a wing with NACA 0012 airfoil section in Case3.

3.1 Case 1: An Unstructured Grid over a Cube

The present cell agglomeration algorithm is based on the spatial localization of the cell centers of an unstructured grid in a hierarchic octree data structure. For demonstration and validation purposes, a cubical unstructured grid over the cube is considered. The volume and surface grid distributions of the cubical grid are shown in Figure 3. The volume grid consists of 17047 nodes and 97451 cells. 1660 cells lie on the cube surface. The four agglomerated coarse grid levels are derived by the present octree based agglomeration coarsening algorithm. The coarse grids contain 37322, 14374, 4676 and 647 cells from second to the fifth grid levels respectively.

The coarse level surface cell distributions at each coarse level are shown in red color in Figure 3. It is observed that the coarse grid levels gradually form into a Cartesian grid structure as expected due to the special nature of the octree data structure. The grid level 5 shows the distribution of all the cells in the top level octants.

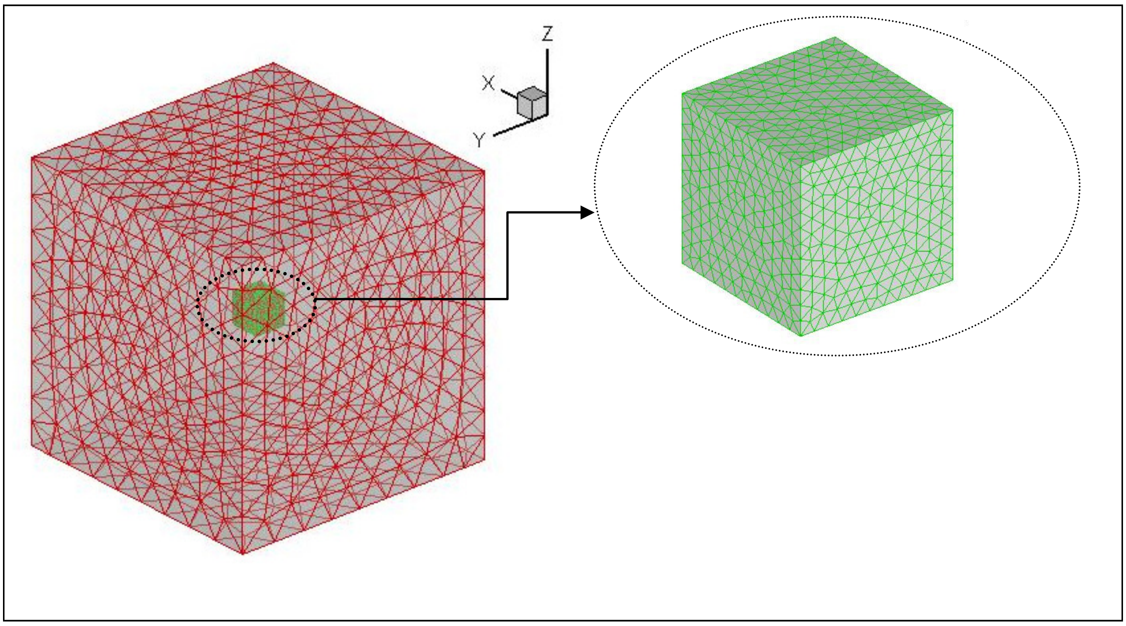


Figure 3. Volume and Surface Meshes Around Cube.

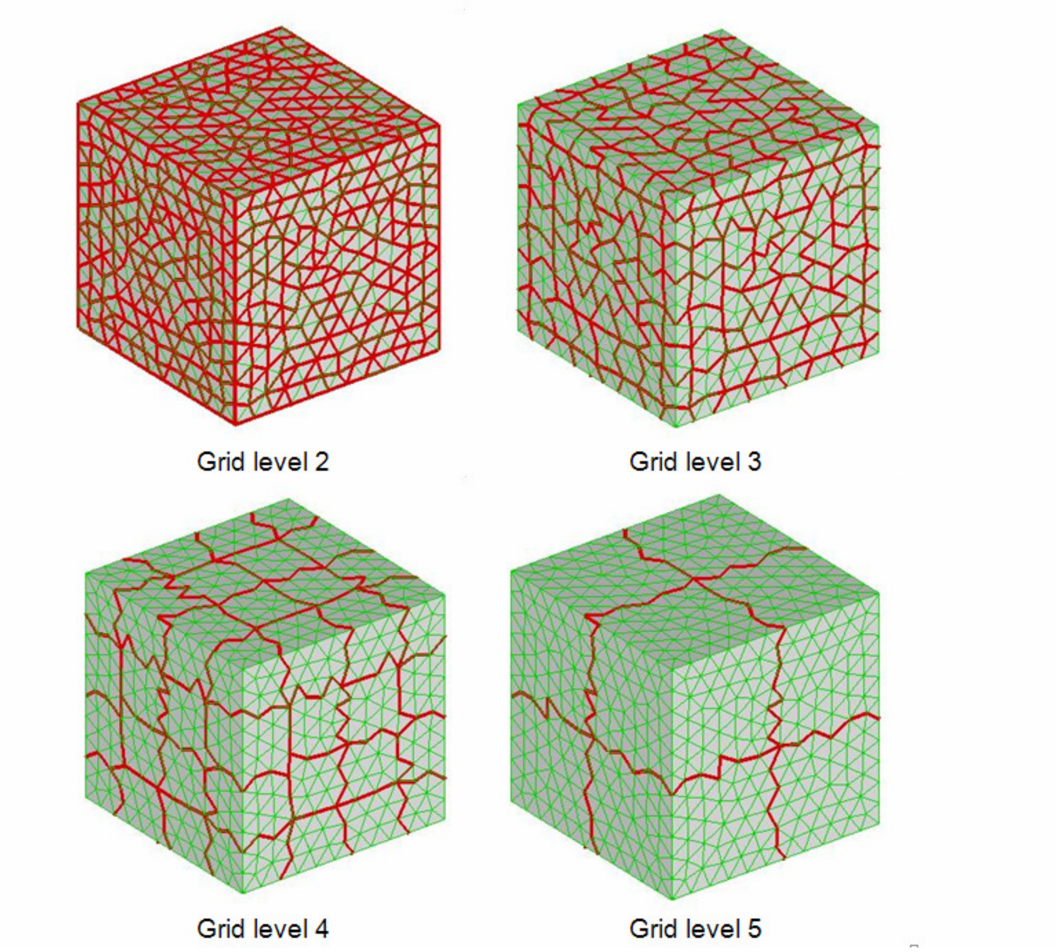


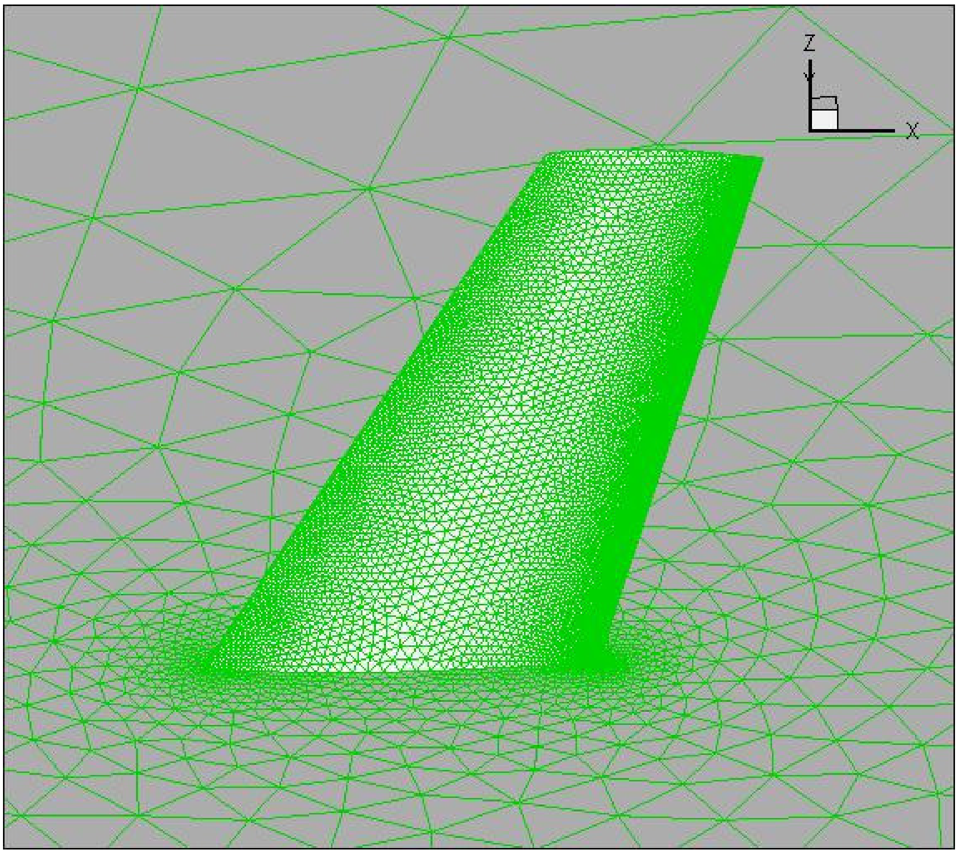
Figure 4. The coarse level grids on surface of cube.

3.2 Case 2: An Unstructured Grid over ONERA M6 Wing

The octree based grid coarsening algorithm developed in this study is next implemented for coarsening an unstructured grid around the well known test case geometry, ONERA M6 wing. The unstructured grid, shown in Figure 5, contains 255156 nodes and 1391537 cells. The four levels of coarse grids are generated automatically with a coarsening ratio of about 40%. The generation of all the coarse level grids takes about 330 CPU seconds on an Intel xeon EM64T processor with 2GHz clock speed operating under Linux.

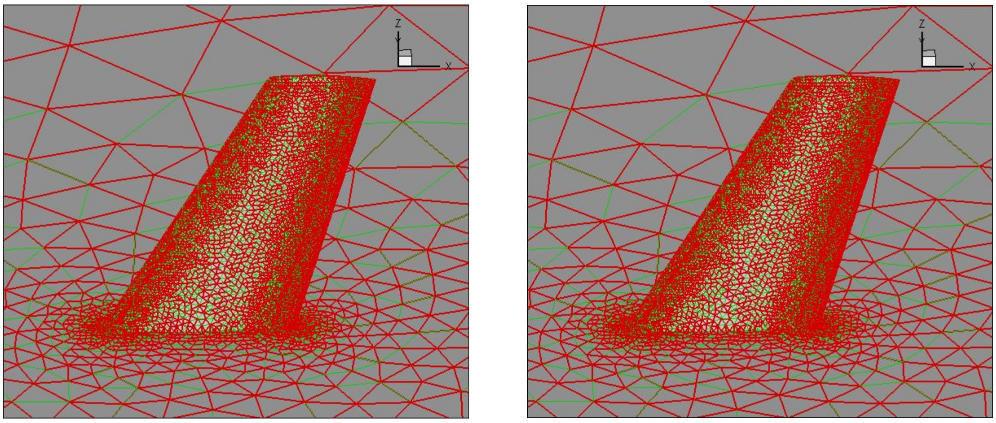
The coarsening algorithm has an option in generating two types of coarse grids. While the first coarsening strategy (Type A) tries to keep the original anisotropic grid distribution at all levels, the second coarsening strategy (Type B) aims at generating uniform Cartesian grids.

The type A and Type B coarse grid sequences generated are shown in Figure 5. Type A and Type B grids may be differentiated starting from the third level as the Type B cell agglomeration algorithm tries to create more isotropic Cartesian grids at all levels. However, it should be noted that both Type A and Type B coarse level grids have higher quality, convex cells with an aspect ratio of around 1 in comparison to the reference study given in [5].

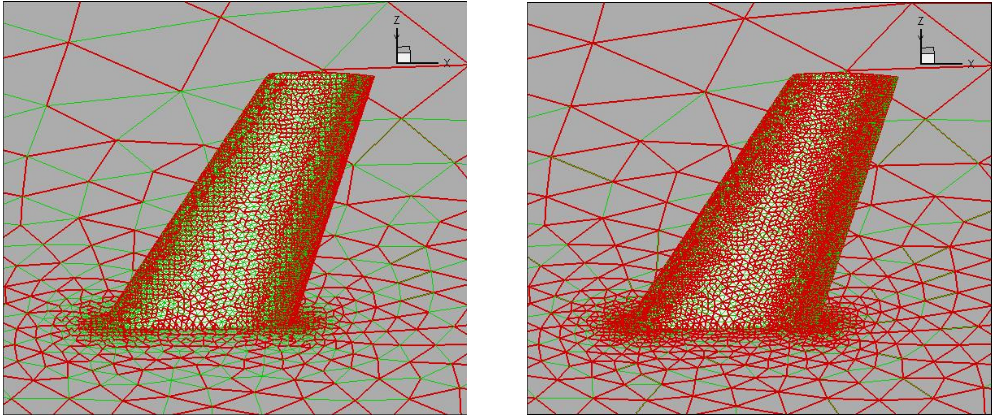


Grid level 1 (Root grid) –1391537 cells

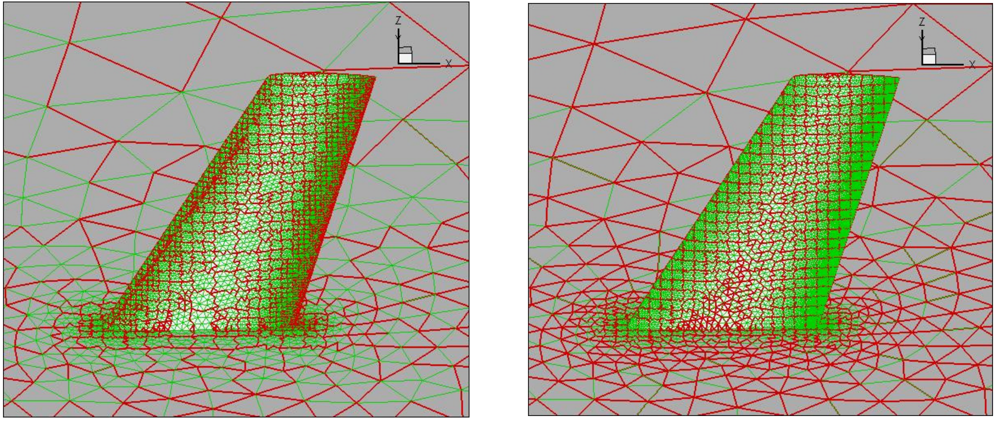
Figure 5. Unstructured grid for ONERA M6 wing.



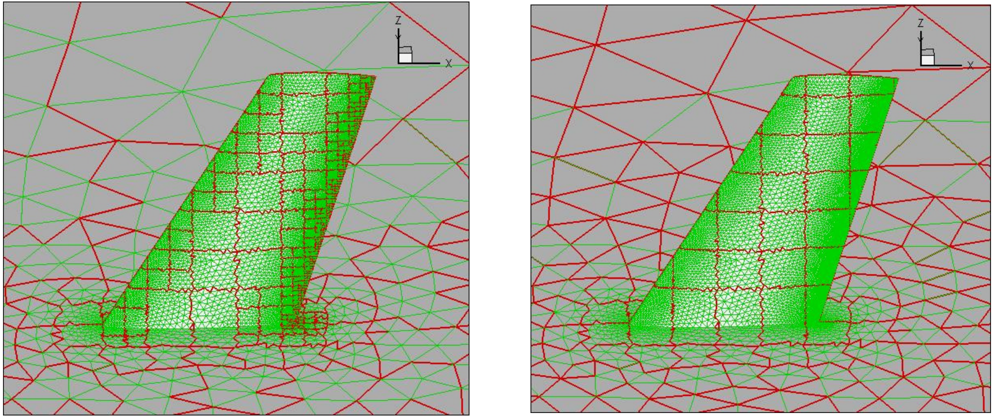
Grid level 2 –546424 (TYPE A) Grid level 2 –546424 (TYPE B)



Grid level 3 – 147047cells (TYPE A) Grid level 3 – 164608 cells (TYPE B)



Grid level 4 – 31065 cells (TYPE A) Grid level 4 – 22841 cells (TYPE B)

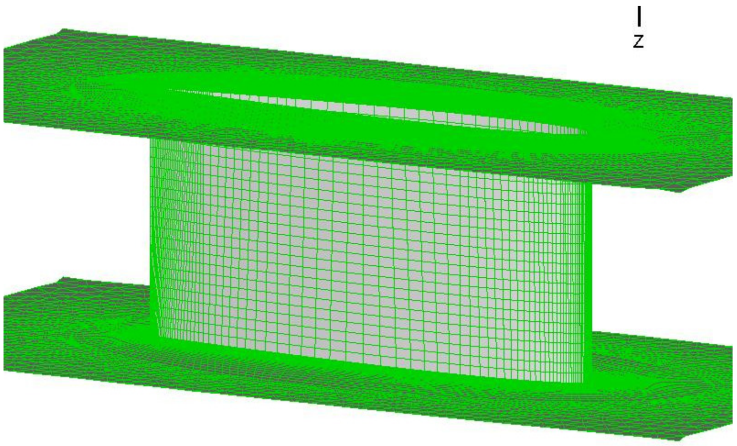


Grid level 5 – 5007 cells (TYPE A) Grid level 5 – 5811 cells (TYPE B)

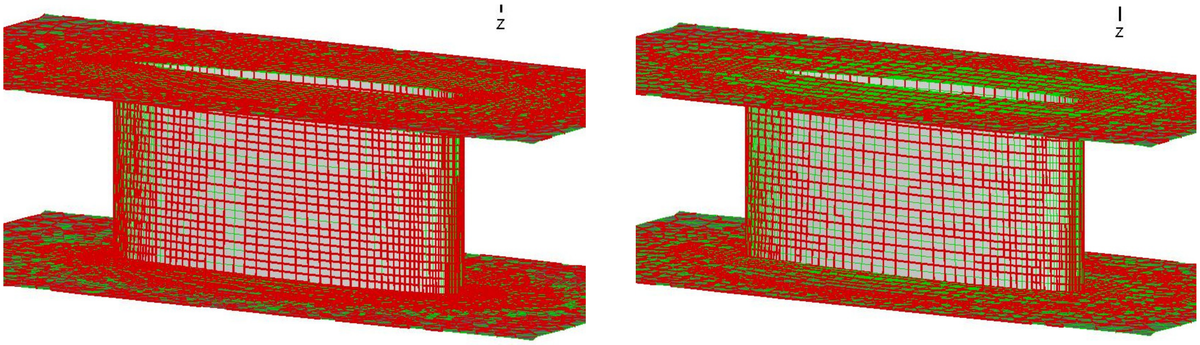
Figure 6. ONERA M6 wing coarse grid levels.

3.3 Case 3: A Hybrid Grid over a Wing with NACA0012 Airfoil Section

In this case, a hybrid viscous grid around a wing with NACA 0012 airfoil section is considered. The hybrid grid consists of hexahedral cells in the boundary layer region around the wing and tetrahedral cells in the remaining inviscid flow region (Figure 7). Four level coarse grid levels are generated with a maximum of 40% coarsening ratio between the coarse grid levels and by employing the Type A algorithm. The fine and coarse level grids are of the size 340693, 125070, 33868, 7144 and 1678 ranging from level 1 to 5 respectively. The grid distribution on the wing surface and on the symmetry planes are shown in Figure 7. It is again observed that the Type A coarsening algorithm tends to create Cartesian grids while preserving the original anisotropic grid distribution in space. Similarly, high quality cells with an aspect ratio of about 1 are generated at all grid levels.

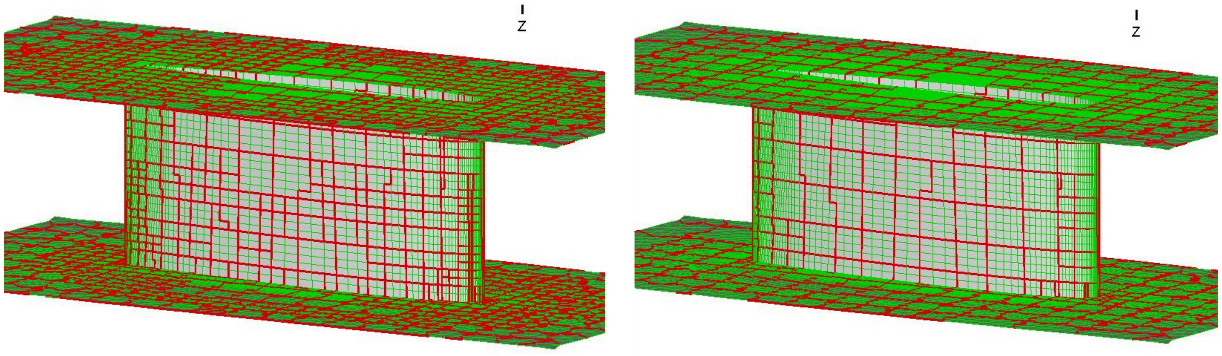


Grid level 1 (Root grid)



Grid level 2

Grid level 3



Grid level 4

Grid level 5

Figure 7. NACA0012 airfoil coarse grid levels (close up view).

4 CONCLUSIONS

In this study, a new grid agglomeration technique based on the localization of hybrid/unstructured grid cells on an octree data structure is presented. It is successfully demonstrated that the coarsening algorithm developed can produce high quality agglomerated cells at all coarse grid levels and preserves cell aspect ratios around one. Due to the nature of the octree localization, the coarse grids assume Cartesian grid structures. A variation in the agglomeration algorithm may preserve the original grid anisotropy or may produce uniform Cartesian grids at coarser grid levels. The multigrid flow solutions to be performed in near future will provide the sensitivity of the convergence acceleration on the cell quality of the coarse grid levels.

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