Ohm's and Kirchhoff's Laws

Ohm's Law and Kirchhoff's Laws are the most fundamental tools used in the analysis of electric and electronic circuits. The objective of applying these laws to a circuit is to determine the behavior of the voltages and currents throughout the circuit. This handout will provide a summary of three laws as well as examples of their application.

The Basics

To perform circuit analysis, it is important to understand fundamental terms and units.

- Current: $I = \frac{Q}{t}$ (amperes) where Q units are coulombs (C) and represents the charge passing a point in the circuit, and t equals time in seconds (s). One coulomb is 6.238792×10^{18} electrons. This is similar to the amount of water flowing past a point in a garden hose.
- Voltage: $(V) = \frac{W}{Q}$ (volts) is defined as the amount of work (W) in joules required to move a certain amount of charge from one place to another. Another term for voltage is electric potential. Electric current will flow from a region of high potential (voltage) to a region of low potential (voltage). This is similar to water flowing from a region of high pressure to a region of low pressure.

Ohm's Law

The voltage across a circuit element is proportional to the current flowing through that element.

$$V \simeq I$$

Ohm determined that the constant of proportionality was the resistance (R) of that element to the flow of current through it. Therefore, Ohm's Law is simply:

$$V = I * R$$

All real circuit elements (as opposed to ideal) resist the flow of current through them. The amount of that resistance is determined by the physical properties of the material (molecular characteristics and physical dimensions).

The implications of Ohm's law are as follows:

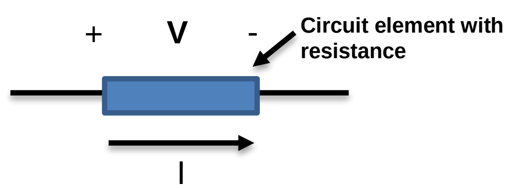
- Current flowing through a real circuit element that has resistance will result in a voltage appearing across the element. This voltage is also considered the voltage drop across the element:

$$\mathbf{V = I * R}$$

- Applying a voltage across a real circuit element will result in a current flowing through the element:

$$\mathbf{I = V/R}$$

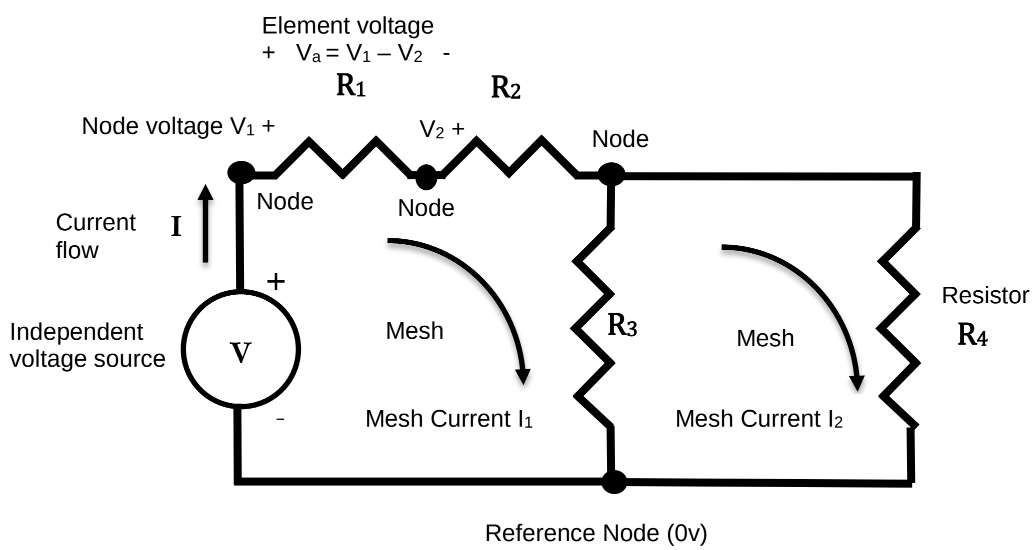
- The direction of the current flow will be from the high voltage (potential) region to the low voltage (potential) region. The higher voltage side of an element is indicated by a plus (+) sign and the lower voltage side is indicated by a minus (-) sign. Current always flows from the plus to the minus. This is called the passive sign convention.



Basic Circuit Terminology

Circuit elements are connected by conductors that have no resistance (ideal wire). Circuit elements and conductors are connected to each other at a point called a node. Multiple elements, but at least two, can be connected to form a node. One node in a circuit must be designated as the reference node and is assigned a voltage of 0 volts. Node voltages are referenced to the reference node. An element voltage for an element not connected to the reference node, for example R_1 and R_2, is the difference of the node voltages at either end of the element. The positive sign of the element voltage is determined by direction of the current flow through the element. For R_1, the current flows from left to right, so the voltage on the left is positive (current flows from + to -). Multiple elements can be connected to form a circuit consisting of nodes and meshes. For this handout, circuit elements that exhibit resistance to current flow are called resistors and are identified as R_x. Another circuit element is a voltage source (such as a battery) that applies a voltage across a circuit. A current source element supplies current to a circuit. Ideal voltage sources are capable of providing a specified voltage with unlimited current. Ideal current sources can provide specified current at any voltage. An element connected between nodes forms a branch. Two elements that share a node exclusively form a series connection (R_1 and R_2); multiple elements that share two nodes form a parallel connection (R_3 and R_4).

The diagram below defines basic circuit topology:

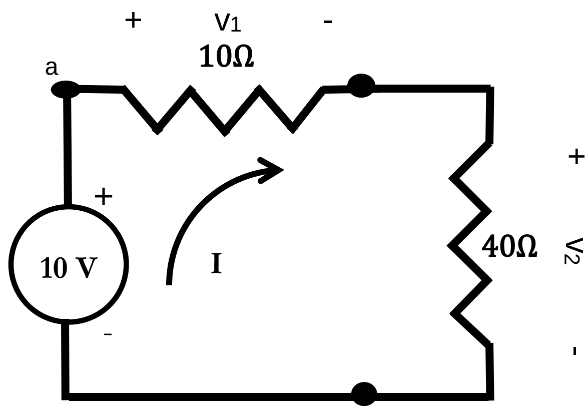


Kirchhoff's Voltage Law (KVL)

KVL states that the sum of the voltages around a loop (or mesh) must equal 0v. A loop is a closed path around a circuit where any node is visited only once. The first step is to assign a voltage variable ($\mathbf{v_x}$ in this case) to each circuit element as well as to designate the sign of the voltage across each element. It does not matter if the sign of the assigned voltage is ultimately correct; the completed analysis will determine the actual polarity.

Starting from node "a" and following the current **I** around the loop, note the polarity of the voltage first encountered as well as the variable name. In this case, the polarity encountered is "+", and the voltage variable is $\mathbf{V_1}$. Write that down, and continue around the loop until complete, writing down each voltage encountered. The resulting equation is:

$$\mathbf{V_1 + V_2 - 10V = 0}$$



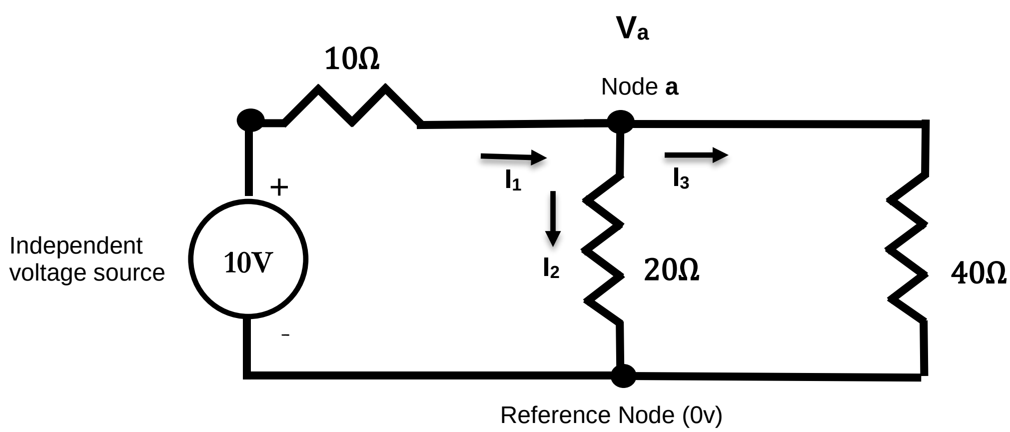
Note that the first polarity encountered of the voltage source is "-", which is reflected in the equation. The third step is applying Ohm's law for each element voltage using the mesh current **I**, and the specific resistor, substituting **I*R$_x$** for each **V** in the equation.

$$\mathbf{I * 10\Omega + I * 40\Omega - 10 = 0}$$

The fourth step is to solve for **I**, which is 1/5 A (amps). And finally, now knowing the current flowing through each resistor, the voltage across each can be calculated using Ohm's law.
So $\mathbf{V_1}$ = 2 volts and $\mathbf{V_2}$ = 8 volts.

Kirchhoff's Current Law (KCL)
KCL states that the total current entering a node must equal the total current leaving the node. This is also called nodal analysis. Consider node "**a**".

$$\mathbf{I_1 = I_2 + I_3}$$



Now apply Ohm's law to each resistor. Note that since there is a current (**I_2**) flowing through a 20**Ω** resistor, there is a voltage across that resistor of (**V_a** – 0)= **I_2** * 20**Ω** or **I_2** = **V_a**/20**Ω**. Since the voltage across the resistor 40**Ω** is the same as the voltage across the 20**Ω** resistor, **I_3** = (**V_a** -0)/ 40**Ω**. The current through the 10**Ω** resistor can be determined by the voltage across it, which is **$V-V_a$**. Therefore, **I_1** = (10-V_a)/10**Ω**. Substituting these results for each **I_x** into the KCL equation results in:

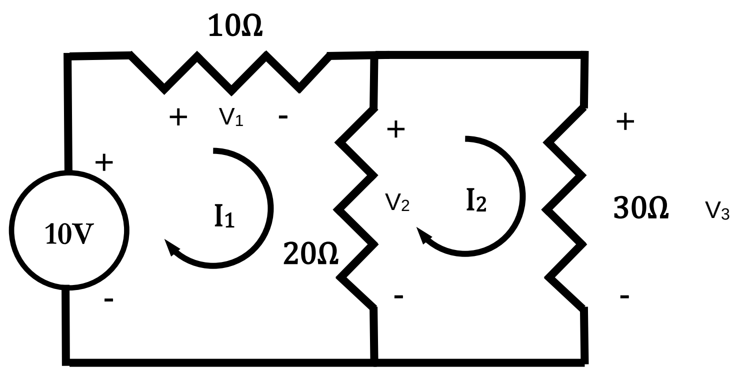
$$(10-V_a)/10\Omega = V_a / 20\Omega + V_a / 40\Omega.$$

Simplifying results in:

$$V_a = 5.454 \text{ volts}$$

Applying KVL to more complex circuits

Circuits can have multiple nodes and meshes. The circuit below contains 2 meshes to which we can apply KVL to each mesh. This process is called mesh analysis.



First, assign voltage variable names and polarities to each circuit element. Then identify the currents in each mesh and assign each current a variable name. Next, starting at a node in the desired mesh, follow each current around the mesh and note the variable name and polarity. Lastly, apply Ohm's Law to each element.

For Mesh 1:

$$V_1 + V_2 - 10v = 0 \text{ or } 10v = V_1 + V_2$$

$$10v = I_1 * 10\Omega + (I_1 - I_2) * 20\Omega$$

For Mesh 2:

$$V_3 - V_2 = 0$$

$$I_2 * 30\Omega + (I_2 - I_1) * 20\Omega = 0$$

Note that for elements that are shared between two meshes, there are two currents identified flowing through the element. In this case, based on the direction of each defined current, the currents oppose each other in the 20Ω element, so the total current would be the difference between the two. When analyzing each mesh, assume the associated mesh current is the larger of the two and assign the "+" voltage sign to the element based on that mesh current. When analyzing Mesh 1, the "+" sign for $\mathbf{R_2}$ is assigned to the top assuming $\mathbf{I_1} > \mathbf{I_2}$, and the total current is ($\mathbf{I_1}$-$\mathbf{I_2}$). When analyzing Mesh 2, the "+" sign for $\mathbf{R_2}$ is assigned to the bottom assuming $\mathbf{I_2} > \mathbf{I_1}$, and the total current is ($\mathbf{I_2}$-$\mathbf{I_1}$). Simplifying each equation:

Mesh 1: $\mathbf{10v = 30I_1 - 20\,I_2}$

Mesh 2: $\mathbf{I_2 = \frac{2}{5}I_1}$

Using the substitution method to solve the system of equations, $\mathbf{I_1} = 0.4545$A and $\mathbf{I_2} = 0.1818$A.

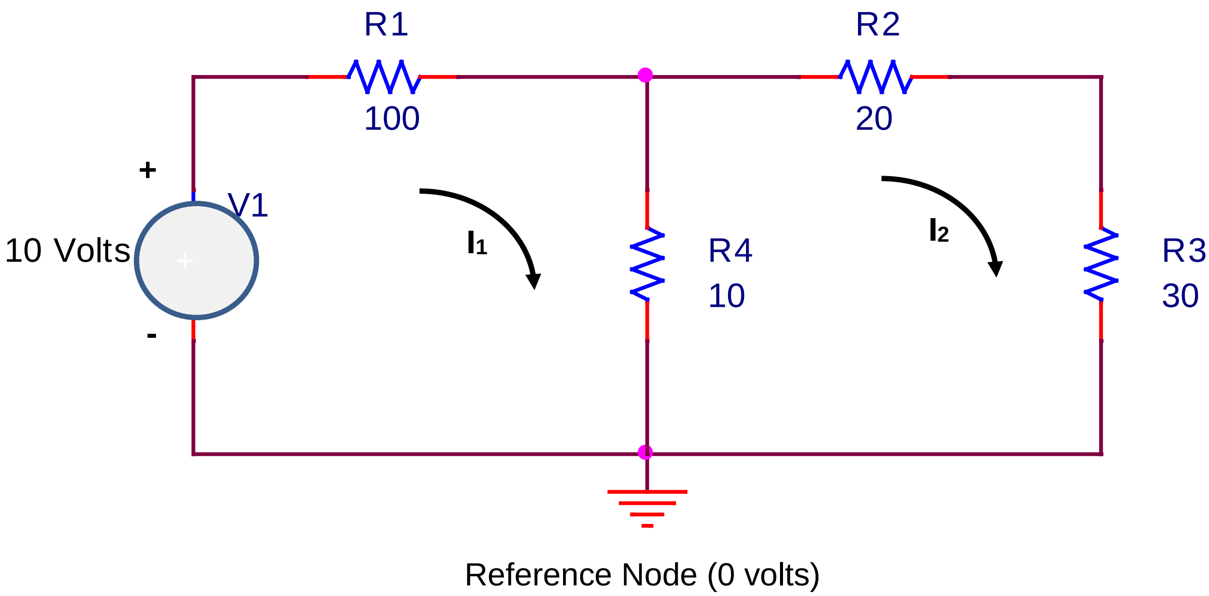
Guidelines for the application of Kirchhoff's Laws

To solve for current and voltages in more complex circuits, the application of both KVL and KCL may be required. Although the application of either will lead to the desired solution, the application of one may provide an easier path to the answer. The following guidelines can be uses to choose an approach to analyzing a circuit.

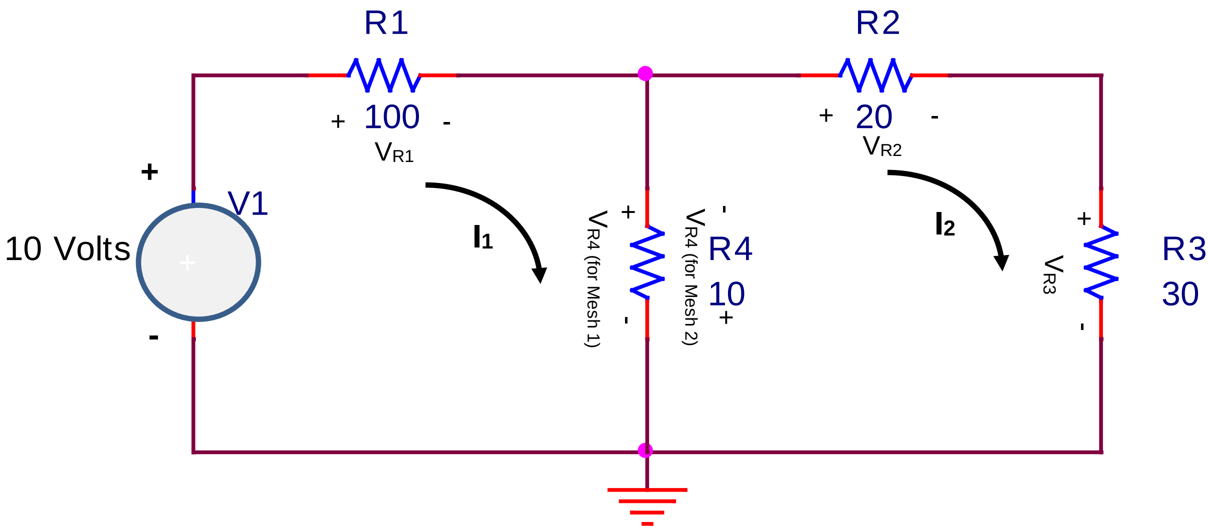
- If determining element voltages is desired, apply KCL, specify mesh currents and solve for voltages
- If determining branch currents is desired, apply KVL, specify node voltages and solve for currents
- When there are many series connected elements and voltage sources, use KVL (mesh analysis)
- When there are many parallel connected elements and current sources, use KCL (nodal analysis)
- If there are fewer nodes than meshes, use KCL
- If there are fewer meshes than nodes, use KVL
- If determining node voltages is required, use KCL
- If determining branch currents is required, use KVL

Many times, however, the application of both laws is required.

Practice Problem:



Find **V_{R1}, V_{R2}, V_{R3}, V_{R4}, I_1, I_2**



Reference Node (0 Volts)

Solution:

Applying KVL for Mesh 1:

$$\mathbf{V_{R1} + V_{R4} - 10V} = 0$$

Where $\mathbf{V_{R1}}$ and $\mathbf{V_{R4}}$ are the voltages across the respective resistors.

Applying Ohms Law for each resistor in the mesh:

$$\mathbf{V_{R1} = I_1 * R1}$$

and

$$\mathbf{V_{R4} = (I_1 - I_2) * R4} \qquad \textbf{(1)}$$

And substituting these back into the KVL equation for Mesh 1 yields:

$$\mathbf{I_1 * R1 + (I_1 - I_2) * R4 - 10V = 0}$$

or

$$\mathbf{I_1 * (R1 + R4) - I_2 * R4 = 10V}$$

Applying KVL to Mesh 2:

$$\mathbf{V_{R2} + V_{R3} + V_{R4} = 0}$$

Applying Ohms law for each resistor in the mesh:

$$\mathbf{V_{R2} = I_2 * R2}$$

$$\mathbf{V_{R3} = I_2 * R3}$$

$$\mathbf{V_{R4} = (I_2 - I_1) * R4} \qquad \textbf{(2)}$$

Referring to equations 1 and 2, the difference in the order of the mesh currents. Substituting these voltage equations back into the KVL equation for Mesh 2 yields:

$$I_2 * R2 + I_2 * R3 + (I_2 - I_1) * R4 = 0$$

or

$$-I_1 * R4 + I_2 * (R2 + R3 + R4) = 0$$

Substituting the actual values into both mesh equations yields:

$$I_1 * 110\Omega - I_2 * 10\Omega = 10V$$

$$-I_1 * 10\Omega + I_2 * 60\Omega = 0$$

These two simultaneous equations can be solved using either elimination or substitution. The result is:

$$I_1 = 0.092308A \text{ or } 92.31mA$$

$$I_2 = 0.01539A \text{ or } 15.39mA$$

Given these currents, the voltage across **R1** using Ohm's Law is:

$$V_{R1} = 9.231V.$$

Using the KVL equation from Mesh 1:

$$V_{R4} = 0.7692V \text{ or } 769.2mV.$$

Given the current through **R3** and **R4**, the voltages across those resistors, using Ohm's Law, are:

$$V_{R2} = 0.3077V \text{ or } 307.7mV$$

$$V_{R3} = 0.4615V \text{ or } 461.5mV.$$

As we had determined from the KVL equation for Mesh 2, we can now show that:

$$V_{R2} + V_{R3} - V_{R4} = 0$$

Note that there is more than one approach to solve this problem. For example, all the resistors could have been combined into one resistance value and Ohm's law could then be used to determine I_1. Given that $R_{total} = 108.333\Omega$:

$$I_1 = 10V/R_{total} = 10V/108.33\Omega = 92.31 \text{ ma}.$$