

Old, frail, and uninsured: Accounting for puzzles in the U.S. long-term care insurance market. *

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February 2017

Abstract

Half of U.S. 50-year-olds will experience a nursing home (NH) stay before they die, and a sizeable fraction will incur out-of-pocket expenses in excess of \$200,000. Given the extent of NH risk, it is surprising that only about 10% of individuals over age 62 have private long-term care insurance (LTCI). This market also has a number of other puzzling features. Many applicants are denied coverage by insurers. Coverage of those who have insurance is incomplete. Insurance premia are high relative to an actuarially fair benchmark. Using a model that features agents with private information about their NH entry risk and an insurer who optimally chooses menus of LTCI contracts subject to participation and incentive compatibility constraints, this paper shows that these puzzles can be attributed to adverse selection, overhead costs on the insurer and Medicaid. The model also accounts for the lack of correlation between NH entry and LTCI ownership. This final property is novel because our setup has only one dimension of private information.

Keywords: Long-Term Care Insurance; Medicaid; Adverse Selection; Insurance Rejections.

JEL Classification numbers: E62, H31, H52, H55.

*We thank Taylor Kelley for outstanding research assistance. We are grateful for comments received at the 2015 Workshop on the Macroeconomics of Population Aging, Notre Dame University, the Canon Institute for Global Studies 2015 End of Year Conference, the Keio-GRIPS Macroeconomics and Policy Workshop, the 2016 Workshop on Adverse Selection and Aging held at the Federal Reserve Bank of Atlanta, and the 2016 SED meetings in Toulouse.

1 Introduction

Nursing home expense risk in the U.S. is significant. About half of U.S. 50-year-olds will experience a nursing home (NH) stay before they die, with a sizeable fraction staying three or more years and incurring out-of-pocket expenses in excess of \$200,000. Medicaid, the only form of public insurance for long-term NH care, is only available to individuals with low assets and either low income or impoverishing medical expenses. Given the extent of NH risk and the limited coverage of public insurance, one would expect that many individuals would participate in the private long-term care insurance (LTCI) market. However, the U.S. market is very small: only about 10% of individuals over age 62 have private LTCI. It also has a number of other puzzling features. Many applicants for LTCI are rejected by insurers, and representative policies only provide partial coverage against long-term care (LTC) risk and charge premia well in excess of the actuarially-fair levels.¹

This paper shows that these puzzling features of the U.S. LTCI market can be attributed to two main factors: adverse selection and Medicaid. Our contention that adverse selection is important is surprising and novel because the previous literature has concluded that the conventional theory of adverse selection described in [Rotschild and Stiglitz \(1976\)](#) is inconsistent with the pattern of LTCI coverage in the U.S. This theory predicts that an insurer's optimal menu to a group of individuals with the same observable characteristics always includes a full insurance contract. This contract is chosen by those with private information that they have the highest risk exposure, whereas, those with lower risk exposure choose partial or no insurance. In practice, however, individuals who based on observables are determined to have high frailty are denied coverage by insurers via medical underwriting. A second problem with this theory is that it implies that, if one properly controls for the information set of the insurer, LTCI takeup rates should be higher among NH entrants as compared to non-entrants. In the data, though, LTCI takeup rates among those who enter a NH are either the same as or even lower than LTCI takeup rates among non-entrants. These counterfactual implications of the theory have led most previous researchers to abstract from modeling the supply side of the market, exogenously specifying insurance contracts instead, and/or to posit multiple sources of private information. In contrast to the previous literature, the insurer in our model optimally chooses premia and coverage, and individuals only have one dimension of private information. Yet, our model generates rejections of more frail individuals and correlations between LTCI coverage and NH entry that are consistent with the findings in the empirical literature.

In our model, individuals have private information about whether they face a good (low) or a bad (high) risk of NH entry. Private information, in conjunction with an insurer who has market power and faces variable overhead costs, creates supply-side frictions. The model also features demand-side frictions generated by the presence of a public means-tested LTCI program. This program, which is meant to represent Medicaid NH benefits, crowds out demand for private LTCI. Solving for the equilibrium contracts, we show that each type of friction can, independently, produce (i) no trade equilibria or rejections; (ii) partial coverage of NH costs even for those with the high risk exposure; and (iii) empirically relevant correlations between private risk exposure and LTCI takeup rates, and between LTCI takeup rates

¹Sources for these facts and figures can be found in Section [2.1](#).

and NH entry both unconditional and conditional on observed indicators of risk exposure.

Market power and variable overhead costs generate rejections and partial insurance coverage of both risk types in the presence of private information even when Medicaid is absent. [Chade and Schlee \(2016\)](#) have found that variable overhead costs make bad risk types more costly to insure than good types. As a result, increasing them generates a larger reduction in the coverage of the bad risks and may lead to optimal contracts that pool bad and good types together. Once pooling takes place, further increases in overhead costs result in a decline in the comprehensiveness of the pooling contract until no profitable pooling contract exists and rejection occurs. We show that partial insurance and rejections are more likely when observed risk exposure is high.

Medicaid generates rejections and partial insurance by lowering demand for private LTCI. It lowers demand in two ways. First, by providing individuals with a guaranteed minimum consumption floor in the NH state, Medicaid reduces the extent of NH expense risk faced by individuals. Second, Medicaid is a secondary payer. This means that, for individuals who meet the means test, private insurance benefits reduce Medicaid benefits one-for-one and individuals cannot use private LTCI to “top-up” Medicaid benefits. The presence of Medicaid reduces the set of profitable contracts that poorer individuals, in particular, are willing to take. If no profitable contracts exist then no trade is possible and rejection occurs. When individuals face uncertainty about their resources at the time of NH entry, Medicaid also impacts the extent of coverage offered by traded contracts. This effect, which is novel in the literature, arises for individuals who are partially insured against the NH shock in that they will be eligible for Medicaid in some (low-resource) states. Since the LTCI contract is not contingent on the realization of individual resources at NH entry, these individuals prefer only partial private LTCI coverage. We also establish that the extent of partial coverage and rejections, due to Medicaid, are larger when observed NH risk is high.

These results suggest that adverse selection and Medicaid are promising qualitative explanations of the U.S. LTCI market. To assess their quantitative significance, we develop a detailed quantitative model of the market featuring individuals that face survival and NH entry risk, a monopolist LTC insurer, and means-tested government LTC insurance. Our model has a rich cross-sectional structure, in particular, individuals vary by income, wealth and frailty which are correlated with NH entry risk and observable to the insurer. We refer to individuals with different observable characteristics as different risk groups.² The insurer offers each risk group a menu of contracts that maximize his profits subject to participation and incentive compatibility constraints.

The quantitative model is calibrated to match cross-sectional variation in frailty, wealth, survival risk, NH entry risk, and LTCI take-up rates using data from the Health and Retirement Survey (HRS). To construct a frailty index for HRS respondents, we adapt a methodology from the Gerontology literature such that our index summarizes underwriting criteria used in the LTCI industry.³ Lifetime NH risk of HRS respondents is estimated using an auxiliary model along the lines of [Hurd et al. \(2013\)](#). To assess the baseline calibration, we compare various moments generated from the model to their data counterparts. In

²Note that in the insurance literature it is common to refer to these groups as risk or insurance pools. To avoid confusion with pooling contracts, we instead refer to them as risk groups.

³For papers in this literature see, for example, [Dapp et al. \(2014\)](#), [Ng et al. \(2014\)](#), [Rockwood and Mitnitski \(2007\)](#), [Searle et al. \(2008\)](#) and the references therein.

particular, we demonstrate that the calibrated baseline economy replicates the variation in self-assessed NH entry risk by frailty, the distribution of insurance use across NH entrants, and conditional and unconditional correlations between entry and LTCI take-up rates.

Finally, to identify the relative roles of overhead costs, adverse selection and Medicaid in accounting for various features of the LTCI market, we compare the baseline to various alternative economies. We find that overhead costs and adverse selection play a distinct role relative to Medicaid. Overhead costs and adverse selection allow the model to account for low LTCI take-up rates and rejections among higher income individuals. Medicaid is important in accounting for low take-up rates among the poor. Medicaid also has a particularly important effect on coverage and pricing even for higher income individuals. As a result, Medicaid lowers profits for the insurer even from those who are least likely to receive Medicaid benefits.

The equilibrium framework developed in this paper provides new insights into previous research. [Brown and Finkelstein \(2008\)](#) have found that Medicaid has a large crowding-out effect on private LTCI. Specifically, they find that, due to Medicaid, individuals in the bottom two-thirds of the wealth distribution would not be willing to purchase LTCI even if the contract was actuarially-fair and provided full coverage against LTC risk. Because we solve the insurer’s contract design problem, the optimal LTCI contracts in our model take into account the presence of Medicaid. We find that Medicaid has an impact on both the size of the LTCI market and on the amount of insurance offered to individuals in the market. In particular, we explore a version of our economy that features Medicaid but no private information or overhead costs and thus is fairly similar to the setup in [Brown and Finkelstein \(2008\)](#). We find that about 60% of individuals purchase private LTCI, which is a much higher LTCI take-up rate than the 33% estimated by [Brown and Finkelstein \(2008\)](#). We obtain a higher take-up rate despite the fact that the contracts in our economy are not actuarially-fair due to the insurer’s monopoly power. The large take-up rate occurs because the insurer in our model is able to offer partial-insurance contracts which are preferred by individuals in the presence of Medicaid.

Our results provide a resolution to what [Ameriks et al. \(2016\)](#) refer to as the “LTCI puzzle.” They find that 66% of respondents in the Vanguard Research Initiative survey have a positive demand for an actuarially-fair state-contingent insurance product that pays out when individuals require assistance with activities of daily living (ADLI). However, only 22% of their sample own LTCI. According to our model the reason for low LTCI take-up rates in the U.S. is that for many individuals the gains from trade are exhausted by overhead costs and private information frictions on the supply side and by Medicaid’s partial insurance on the demand side.

Our research bridges the theoretical optimal contracting literature with the empirical literature on adverse selection. Previous research on optimal contracting has found that adverse selection models with a single source of private information generally exhibit a positive correlation between risk exposure and insurance coverage. [Hellwig \(2010\)](#) derives this result in a principal agent setting, [Stiglitz \(1977\)](#) and [Chade and Schlee \(2012\)](#) derive it in a setting with a single monopolistic insurer, and [Lester et al. \(2015\)](#) show that it obtains in a framework that admits varying degrees of market power. Based on this theoretical finding, [Chiappori and Salanie \(2000\)](#) propose testing for adverse selection in an insurance market by estimating the correlation between insurance coverage and insurance claims controlling for the information set of the insurer. If the correlation is positive and significant then that

is evidence of adverse selection in the market. The most relevant empirical paper for the LTCI market, [Finkelstein and McGarry \(2006\)](#), finds evidence that individuals have private information about their NH risk. However, when they implement Chiappori’s test, they fail to find a positive correlation between LTCI ownership and subsequent NH entry despite their best effort to control for information observed by insurers. Similar findings have been documented in other insurance markets. [Chiappori and Salanie \(2000\)](#), for instance, document a negative correlation between the level of insurance and claims in the French auto insurance market and [Fang et al. \(2008\)](#) find that holders of Medigap insurance spend less on medical care as compared to non-holders. These empirical findings have led researchers to conclude that individuals have multiple sources of private information and to construct new adverse selection models with this feature.^{4,5}

To the best of our knowledge, this is the first paper to demonstrate that a quantitative optimal contracting model with asymmetric information and a single source of private information can produce a small positive correlation between insurance ownership and claims when correctly controlling for the insurer’s information set but may produce a negative one if the econometrician’s information is smaller than that of the insurer. This is because the optimal menus offered by the insurer in our model provide only one private type (the bad type) with a positive amount of insurance in only a tiny fraction of risk groups. In all of the other risk groups either both private types have LTCI or neither do. If the econometrician does not perfectly control for the insurer’s information set, the small positive correlation within risk groups may be dominated by the negative correlation between LTCI ownership and average NH entry across risk groups. The negative correlation across risk groups is due to the fact that higher risk groups have higher NH entry on average but are also more likely to be rejected by the insurer. Although we are not able to run the regressions of [Finkelstein and McGarry \(2006\)](#) on data from our model, we find that the correlations arising in our model economy, after controlling to various degrees for observable characteristics, are consistent with those documented in their paper. Our results suggest that the power of tests that rely on LTCI takeup rates (as compared to the size of the LTCI contract) is very low.

A number of papers have investigated the role of demand-side factors in accounting for low LTCI take-up rates. In addition to [Brown and Finkelstein \(2008\)](#), who investigate the role of Medicaid, research has focused on three main channels: bequest motives, homeownership and informal care. [Lockwood \(2016\)](#) argues that bequest motives depress the demand for LTCI and [Davidoff \(2010\)](#) argues that home equity may substitute for LTCI. [Barczyk and Kredler \(2016\)](#) find that the demand for NH care is very elastic to the availability of informal care options in a model with intra-family bargaining. [Mommaerts \(2015\)](#) and [Ko \(2016\)](#) develop non-cooperative structural models of formal and informal care. [Mommaerts \(2015\)](#) finds that informal care reduces the demand for LTCI and that this effect is most pronounced among more affluent individuals. Importantly, she finds that the extent of coverage provided by informal care is largely incomplete and retirees have a substantial residual demand for LTCI. Although we do not model informal care decisions, one can interpret the variation in

⁴For examples of adverse selection models where individuals have multiple sources of private information see [Einav et al. \(2010\)](#) and [Guerrieri and Shimer \(2015\)](#).

⁵However, results in [Chiappori et al. \(2006\)](#) and [Fang and Wu \(2016\)](#) suggest that it is also challenging to produce a negative correlation between insurance coverage and risk exposure in models with multiple sources of private information.

private NH entry probabilities in our model as arising, in part, from the differences in the availability of informal care. This interpretation is supported by [Ko \(2016\)](#) who finds that private information about access to informal care by family members is an important source of adverse selection in the LTCI market. A major theoretical contribution of our analysis, relative to this literature, is that we model both sides of the LTCI market.

The remainder of the paper proceeds as follows. Section 2 provides a set of facts that motivate our analysis. In Section 3 we present a qualitative analysis of our main economic mechanisms using a simplified model. The quantitative model is presented in Section 4. Section 5 describes how we calibrate the quantitative model and assess the baseline calibration. Section 6 contains our results and our concluding remarks are in Section 7.

2 Motivation

Our research is motivated by a number of puzzling features of the U.S. LTCI market. In this section, we describe these features in detail.

2.1 Market Size

The risk of a costly long-term NH stay, which we define as a stay that exceeds 100 days, is large and public health insurance coverage of such stays is limited. Using HRS data and an auxiliary simulation model, we estimate the the lifetime probability of a long-term NH stay is 30%.⁶ On average, those who experience a long-term NH stay spend about 3 years in a NH. According to the U.S. Department of Health, NH costs averaged \$205 per day in a semi-private room and \$229 per day in a private room in 2010. The two main public insurers are Medicare and Medicaid. Medicare, which provides universal coverage of short-term rehabilitative NH stays, partially covers up to 100 days of NH care. Medicaid provides a safety net for those who experience high LTC expenses but it is both income and asset-tested. Consequently, Medicaid is only an option for individuals who either have low wealth and retirement income (categorically needy) or who have already exhausted their personal resources to pay for high medical expenses (medically needy). As a result, many individuals face a significant risk of experiencing large LTC expenses near the end of their life. For example, a NH stay of three years can result in out-of-pocket expenses that exceed \$200,000. Indeed, [Kopecky and Koreshkova \(2014\)](#) find that the risk of large OOP NH expenses is the primary driver of wealth accumulation during retirement.

Given the extent of NH risk in the U.S., one would expect that the market for private LTCI would be large. It is consequently surprising that only 10% of individuals 62 and older in our HRS sample have private LTCI. Moreover, private LTCI benefits account for only 4% of aggregate NH expenses while the share of out-of-pocket payments is 37%.⁷

⁶In comparison, using HRS data and a similar simulation model, [Hurd et al. \(2013\)](#) estimate that the lifetime probability of having any NH stay for a 50 year old ranges between 53% and 59%.

⁷Medicare and Medicaid account for 18% and 37%, respectively. This breakdown is for 2003 and is from the Federal Interagency Forum on Aging-Related Statistics.

2.2 Rejections

Many applications for LTCI are rejected. [Murtaugh et al. \(1995\)](#) in one of the earliest analyses of LTCI underwriting estimates that 12–23% of 65 year olds, if they applied, would be rejected by insurers because of poor health. Their estimates are based on the National Mortality Followback Survey. Since their analysis, underwriting standards in the LTCI market have become more strict. We estimate rejection rates of as high as 36% for 55–65 year olds by applying underwriting guidelines from Genworth and Mutual of Omaha to a sample of HRS individuals.⁸

To understand how we arrive at this figure, it is helpful to explain how LTCI underwriting works. Underwriting occurs in two stages. In the first stage, individuals are queried about their prior LTC events, pre-existing health conditions, current physical and mental capabilities, and lifestyle. Some common questions include:

1. Do you require human assistance to perform any of your activities of daily living?
2. Are you currently receiving home health care or have you recently been in a NH?
3. Have you ever been diagnosed with or consulted a medical professional for the following: a long list of diseases that includes diabetes, memory loss, cancer, mental illness, heart disease?
4. Do you currently use or need any of the following: wheelchair, walker, cane, oxygen?
5. Do you currently receive disability benefits, social security disability benefits, or Medicaid?⁹

A positive answer to any one of these questions is sufficient for the insurers to reject applicants before they have even submitted a formal application. Many of these same questions are asked to HRS participants. As [Table 1](#) shows, the fraction of individuals in our HRS sample who would respond affirmatively to at least one question is large even for the youngest age group and for the top half of the wealth distribution. Question 3 received the highest frequency of positive responses. If we are conservative and omit question 3 the prescreening declination rate ranges from 18–24%.

If applicants pass the first stage, they are invited to make a formal application. Medical records and blood and urine samples are collected and the applicants cognitive skills are tested. One in five formal applications are denied coverage.¹⁰ Assuming a 20% rejection rate at each round, the resulting overall rejection rate is roughly 36% for 55–66 years old in our HRS sample.

2.3 Pattern of LTCI take-up rates

It is clear from the prescreening questions above that one of the objectives of LTCI underwriting is to screen out individuals with poor health and low wealth. If this screening is

⁸We subsequently refer to this sample of individuals as our HRS sample and details on our sample selection criteria are reported in the Appendix.

⁹Source: 2010 Report on the Actuarial Marketing and Legal Analyses of the Class Program

¹⁰Source: American Association for Long-Term Care Insurance

Table 1: Percentage of HRS respondents who would answer “Yes” to at least one LTCI prescreening question.

	Age		
	55–56	60–61	65–66
All	40.5	43.7	49.6
Top Half of Wealth Distribution Only	31.1	33.6	39.1

Data source: Authors’ calculations using our HRS sample.

successful it will have a depressing effect on both the size and the composition of the market. LTCI take-up rates are likely to be particularly low among those who have low wealth and/or poor health. Medicaid also affects the composition of the market. This program is likely to have the strongest impact on the LTCI take-up rates of the poor because they are most likely to qualify for Medicaid NH benefits. It is consequently interesting to document how LTCI take-up rates vary by wealth and health status in our HRS sample.

To obtain a measure of observable health status for HRS respondents, we summarize all the measures of health collected by LTC insurers and observable in the HRS in a single frailty index. Following the recommendations in the Gerontology literature, all of the measures included in the index are equally weighted. Self-reported health and NH risk are not included since these variables are not observable by insurers.¹¹

Table 2 reports LTCI take-up rates by frailty and wealth quintiles for 62–72 year olds in our HRS sample. We focus on the 62–72 age group because it covers the ages when most individuals purchase LTCI and yields a sample size big enough to have a meaningful number of individuals in each combination of wealth and frailty quintile. The average LTCI take-up rate for this group is 9.4%.¹² The table shows that there is substantial variation in LTCI take-up rates across wealth and frailty quintiles. The take-up rate of individuals in the top wealth quintile and lowest frailty quintile is more than 4 times higher than that of individuals in the lowest three wealth quintiles and highest frailty quintile. Notice that within each frailty quintile, take-up rates increase with wealth. Medicaid, since it has a larger effect on the poor, likely plays an important role in accounting for this pattern. Also notice that, within each wealth quintile, take-up rates decline with frailty. Insurance rejections of high risk individuals are likely an important driver of this pattern, especially for those in wealth quintile 5 who are least affected by Medicaid.

2.4 Pricing and coverage

Those who successfully navigate the LTCI underwriting process face high premia for insurance policies that only provide partial coverage against LTC risk. [Brown and Finkelstein](#)

¹¹More details on the construction of the frailty index, including a list of included variables, can be found in the Appendix.

¹²Note that this number cannot be arrived at by summing across rows or columns of Table 2 because there are not equal numbers of individuals in each node.

Table 2: LTCI take-up rates by wealth and frailty

Frailty Quintile	Wealth Quintile		
	1–3	4	5
1	0.108	0.147	0.233
2	0.103	0.158	0.205
3	0.082	0.131	0.200
4	0.069	0.113	0.157
5	0.056	0.107	0.104

For frailty (rows), quintile 5 has the highest frailty and, for wealth (columns), quintile 5 has the highest wealth. We only report the average of wealth quintiles 1–3 because take-up rates are very low for these individuals. The wealth quintiles reported here are marginal and not conditional on the frailty quintile, so for example only around 7% of the people in the lowest frailty quintile are in the bottom wealth quintile, while 33% are in the top wealth quintile. Data source: 62–72 year olds in our HRS sample.

(2007) estimate individual loads and comprehensiveness for common LTCI products. They find that individual loads, which are defined as one minus the expected present value of benefits relative to the expected present value of premia paid, range from 0.18 to 0.51 depending on whether or not adjustments are made for lapses. In other words, LTCI policies may be twice as expensive as actuarially fair insurance. About two-thirds of policies bought in the year 2000 paid a maximum daily benefit that was fixed in nominal terms over the life of the contract. These policies only covered a fraction of the expected lifetime NH costs. For instance, they estimate that a “representative” LTCI policy only covered about 34% of expected lifetime costs. They also consider how loads vary with comprehensiveness and conclude that loads do not rise systematically with the comprehensiveness of the policy.

Brown and Finkelstein (2011) provide more recent estimates of personal loads and comprehensiveness using data from the year 2010. In 2010 average loads were higher: 0.32 without lapses and 0.50 with lapses. However, coverage was better. A representative policy covered about 66% of expected lifetime LTC costs. The main reason for the improvement in coverage is that later policies included a benefit escalation clause. In addition, the maximum daily benefits tended to be higher and the exemption period tended to be lower as compared to policies issued in 2000.

Even though personal loads increased between 2000 and 2010, sales declined, concentration increased and profits fell. New sales of LTCI in 2009 were below 1990 levels and, according to Thau et al. (2014), over 66% of all new policies issued in 2013 were written by the largest three companies. However, insurers are experiencing losses on their LTC product lines.¹³

¹³The top three insurers are Genworth, Northwest and Mutual of Omaha. For information about losses on this business line see, e.g., *The Insurance Journal*, February 15, 2016, <http://www.insurancejournal.com/news/national/2016/02/15/398645.htm> or Pennsylvania Insurance Department MUTA-130415826.

2.5 Private information

The observations described above are consistent with the hypothesis that individuals have private information about their NH risk exposure, that this information is correlated with frailty and wealth, and that LTC insurers use this information to reject high risk groups. In further support of the hypothesis, [Finkelstein and McGarry \(2006\)](#) find direct evidence of private information in the LTCI market. Specifically, they find that individuals' self-assessed NH entry risk is positively correlated with both actual NH entry and LTCI ownership even after controlling for characteristics observable by insurers. [Hendren \(2013\)](#) shows that Finkelstein's and McGarry's findings are driven by individuals in high risk groups. Specifically, he finds that self-assessed NH entry risk is only predictive of a NH event for individuals who would likely be rejected by insurers. Hendren's measure of a NH event is independent of the length of stay. Since we focus on stays that are at least 100 days, we repeat the logit analysis of [Hendren \(2013\)](#) using our definition of a NH stay and our HRS sample. We get qualitatively similar results. We find evidence of private information at the 10 year horizon (but not at the 6 year) in a sample of individuals who would likely be rejected by insurers. However, for a sample of individuals who would likely not be rejected we are unable to find evidence of private information.¹⁴

Interestingly, even though [Finkelstein and McGarry \(2006\)](#) find evidence of private information in the LTCI market, they fail to find evidence that the market is adversely selected using the positive correlation test proposed by [Chiappori and Salanie \(2000\)](#). When they do not control for the insurer's information set, they find that the correlation between LTCI ownership and NH entry is negative and significant. Individuals who purchase LTCI are less likely to enter a NH as compared to those who did not purchase LTCI. When they include controls for the insurer's information set, they also find a negative although no longer statistically significant correlation. Finally, when they use a restricted sample of individuals who are in the highest wealth and income quartile and are unlikely to be rejected by insurers due to poor health they find a statistically significant negative correlation.

3 A Simple Model with Adverse Selection

In this section we establish formal conditions under which one can account for some of the main qualitative features of the LTCI market described above. The arguments in this section are developed using a simplified one-period version of the baseline model that nests the standard adverse selection setup as a special case. This simplified setup allows us to graphically illustrate some of the most important economic mechanisms driving our quantitative results. First, we demonstrate that supply-side frictions, namely, private information, market power and variable overhead costs, can generate less than full insurance, coverage denials, high loads on individuals, and low profits. We then illustrate how demand-side frictions, namely, reduced demand due to the presence of Medicaid, can independently generate these same qualitative features. Finally, we discuss which parameters are important for producing partial insurance and rejections when both supply-side and demand-side frictions are present.

¹⁴See the Appendix for more details.

To start, suppose the economy consists of a continuum of individuals and a single monopolistic issuer of private LTCI.¹⁵ Each individual has a type $i \in \{g, b\}$. They each receive endowment ω but face the risk of entering a NH and incurring costs m . The probability that an individual with type i enters a NH is θ^i . A fraction $\psi \in (0, 1)$ of individuals are good risks who face a low probability $\theta^g \in (0, 1)$ of a NH stay. The remaining $1 - \psi$ individuals are bad risks whose NH entry risk is $\theta^b \gg \theta^g$. Each individual observes his true NH risk exposure but the insurer only knows the structure of uncertainty. A menu consists of a pair of contracts (π^i, ι^i) , one for each private type $i \in \{g, b\}$. Each contract consists of a premium π^i that the individual pays to the issuer and an indemnity ι^i that the issuer pays to the individual if he incurs NH costs m .

Denote consumption of an individual with risk type i as c_{NH}^i in the NH state and c_o^i otherwise. An individual's utility function is

$$\begin{aligned} U(\theta^i, \pi^i, \iota^i) &= \theta^i u(\omega - \pi^i - m + \iota^i) + (1 - \theta^i) u(\omega - \pi^i), \\ &= \theta^i u(c_{NH}^i) + (1 - \theta^i) u(c_o^i), \end{aligned} \quad (1)$$

and the associated marginal rate of substitution between premium and indemnity is

$$\frac{\partial \pi}{\partial \iota}(\theta^i) = -\frac{U_\iota(\cdot)}{U_\pi(\cdot)} = \frac{\theta^i u'(c_{NH}^i)}{\theta^i u'(c_{NH}^i) + (1 - \theta^i) u'(c_o^i)} \equiv MRS(\theta^i, \pi^i, \iota^i). \quad (2)$$

Assume that the utility function has the property that $MRS(\theta^i, \pi^i, \iota^i)$ is strictly increasing in $\theta^i, i \in \{g, b\}$. Under this assumption, which is referred to as the single crossing property, any menu of contracts that satisfies incentive compatibility will be such that if $\theta^{i'} > \theta^i$ then $\pi^{i'} \geq \pi^i$ and $\iota^{i'} \geq \iota^i$.

The optimal menu of contracts for individuals maximizes the insurer's profits subject to participation and incentive compatibility constraints and can be found by solving

$$\max_{\pi^i, \iota^i} \Pi \quad (3)$$

subject to

$$(PC_i) \quad U(\theta^i \pi^i, \iota^i) - U(\theta^i, 0, 0) \geq 0, \quad i \in \{g, b\}, \quad (4)$$

$$(IC_i) \quad U(\theta^i \pi^i, \iota^i) - U(\theta^i \pi^j, \iota^j) \geq 0, \quad i, j \in \{g, b\}, \quad i \neq j, \quad (5)$$

where

$$\Pi = \psi[\pi^g - \theta^g(\lambda \iota^g)] + (1 - \psi)[\pi^b - \theta^b(\lambda \iota^b)],$$

and $\lambda \geq 1$ reflects variable overhead costs incurred by the issuer.

We now review two classic properties of contracts under adverse selection that are standard in the literature.¹⁶ Our model has these properties when there are no variable overhead

¹⁵See [Chade and Schlee \(2016\)](#) for a discussion of why monopoly power is needed to produce rejections under adverse selection. An open question that we do not pursue here is how much market power is required. [Lester et al. \(2015\)](#) propose a framework that one could, in principal use, to investigate this question.

¹⁶See, for example, [Rothschild and Stiglitz \(1976\)](#) and [Stiglitz \(1977\)](#).

costs ($\lambda = 1$) and bad types do not know for sure that they will enter a NH ($\theta^b < 1$). The first property is that the equilibrium contract is always a separating one. With a single issuer, this equilibrium is such that the participation constraint binds for the good types and the incentive compatibility constraint binds for the bad types. Thus the optimal menu must satisfy the first-order conditions

$$MRS(\theta^g, \pi^g, \iota^g) = \lambda\eta, \quad (6)$$

$$MRS(\theta^b, \pi^b, \iota^b) = \lambda\theta^b, \quad (7)$$

where $\eta \equiv \psi\theta^g + (1 - \psi)\theta^b$ is the fraction of individuals who enter a NH, and must be such that equation (4) holds with equality for the good types and equation (5) holds with equality for the bad types. The second property is that bad types are always offered full insurance. To see this, note that, if $\iota^b = m$ then consumption in the NH state is the same as consumption in the non-NH state. In this case, $MRS(\theta^b, \pi^b, \iota^b) = \theta^b$, which is the optimality condition (7) when $\lambda = 1$. By the same token, it is never optimal to offer full insurance to good types since $MRS(\theta^g, \pi^g, m) = \theta^g > \eta$. Insurance of the good type is always incomplete with $\iota^g < m$.

Figure 1a illustrates a typical optimal menu in this case. The good types get the contract at point G_1 and the bad types get the contract at point B_1 . Note that pooling contracts cannot be equilibria in this setting because starting from a pooling contract at point G_1 , the insurer can always increase total profits by offering the bad types a more comprehensive contract. Separating equilibria where the good types have a $(0, 0)$ contract can occur though. However, the optimal menu will always consist of at least one nonzero contract that offers full insurance and is preferred by bad types. It follows that the standard setup is inconsistent with our motivating observation that U.S. LTCI policies provide only partial coverage. Moreover, the standard setup is inconsistent with denials of coverage because in the model agents are always offered two contracts and one of them is positive. Thus a zero contract is a choice and not a denial in the standard setup. We now describe how to modify the model to make it consistent with these two properties of the U.S. LTCI market.

3.1 Optimal Contracts with Variable Overhead Costs

In this section we show that imposing variable overhead costs on the insurer, i.e., setting $\lambda > 1$, can result in equilibria where both types are offered less than full insurance as well as equilibria where neither type is offered a positive contract. These findings and the line of reasoning follows the analysis of Chade and Schlee (2014) and Chade and Schlee (2016) who study the impact of imposing variable costs on a monopolist insurer in the presence of private information and a continuum of types.¹⁷

To illustrate the impact of variable overhead costs on the optimal menu, consider the impact of slightly increasing λ above 1, i.e., moving from Figure 1a to Figure 1b. Increasing λ increases the slopes of the firm's isoprofit lines. The increased costs of paying out claims are offset by a combination of increased loads on the good types and reduced profits. Indemnities and premia of both types fall and the optimal contracts move southwestward along the

¹⁷Our findings are also related to previous results by Hendren (2013). In his setting, and in ours, even if

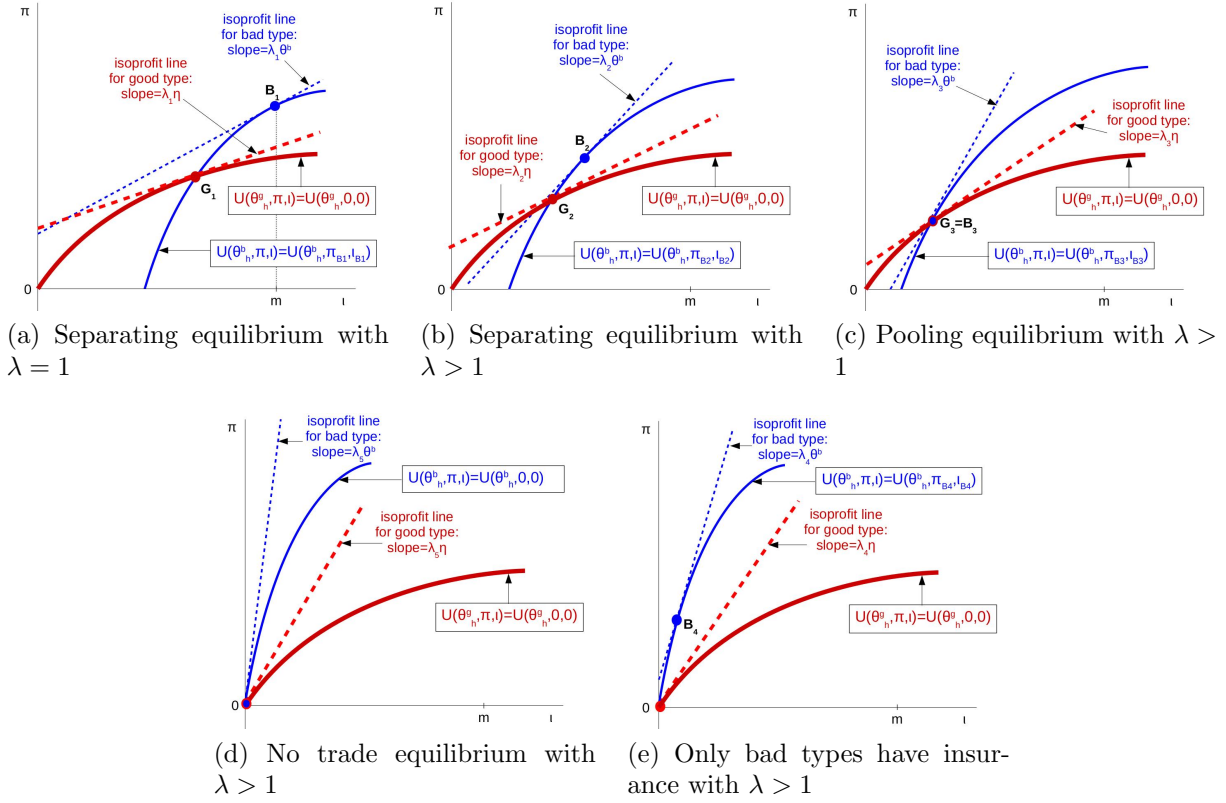


Figure 1: An illustration of the effects of increasing the insurer's proportional overhead costs factor (λ) on the optimal menu. The blue (red) lines are the indifference curves of bad (good) types. The dashed blue lines are isoprofits from contracts for bad types and the red dashed lines are isoprofits from a pooling contract.

individuals' indifference curves. Thus if $\lambda > 1$, the property of the standard model that bad types get full insurance no longer holds as both types are now offered contracts where indemnities only partially cover NH costs.

Proposition 1. *If $\lambda > 1$, then the optimal menu features incomplete insurance for both types, i.e., $i^i < m$ for $i \in \{b, g\}$.*

Proof. See Appendix. □

Since the marginal costs of paying out claims to the bad type are higher than to the good types, as λ increases, the contracts will also get closer together. Once λ is large enough, the insurer will no longer be able to increase profits by offering a separate contract to the bad types as opposed to offering a single (pooling) contract. Figure 1c depicts such a case where both types get the same nonzero contract. Once a pooling contract occurs, the equilibrium under any larger values of λ will also involve pooling. As λ continues to increase, the pooling contract will move along the good types participation constraint with loads on both types rising and profits gradually falling, until λ is so large that no profitable nonzero pooling

contract exists. Figure 1d illustrates the case where the optimal menu consists of the single pooling contract $(\pi, \iota) = (0, 0)$. We adopt the same terminology as Chade and Schlee (2016) and subsequently refer to this case as either a no-trade equilibrium or a rejection. Proposition 2 provides necessary and sufficient conditions for such equilibria to occur in the presence of positive variable overhead costs.

Proposition 2. *There will be no trade, i.e., the optimal menu will consist of a single $(0, 0)$ contract iff*

$$MRS(\theta^b, 0, 0) \leq \lambda \theta^b, \quad (8)$$

$$MRS(\theta^g, 0, 0) \leq \lambda \eta, \quad (9)$$

both hold.

Proof. See Appendix. □

No trade equilibria occur when the amount individuals are willing to pay for even a small positive separating or pooling equilibrium is less than the amount required to provide nonnegative profits to the insurer. Condition (8) rules out profitable separating menus where only bad types have positive insurance, such as the one illustrated in Figure 1e. Condition (9) rules out profitable pooling and separating menus where both types are offered positive insurance.

3.2 Optimal Contracts in the Presence of Medicaid

We have shown that, in the presence of private information, positive variable overhead costs lower profits for the insurer, produce optimal menus in which both types receive less than full insurance, and can generate rejections. We will now show that equilibria with these features can also be generated if there is a means-tested social insurance program in the model that, like the U.S. Medicaid program, guarantees a minimum consumption floor to individuals who incur NH costs. In order to isolate the effects of the Medicaid program on the optimal contracts in this section, we assume that $\theta^b < 1$ and that there are no variable overhead costs ($\lambda = 1$).

Assume that individuals who experience a NH event receive means-tested Medicaid transfers according to

$$TR(\omega, \pi, \iota) \equiv \max \{0, \underline{c}_{NH} - [\omega - \pi - m + \iota]\}, \quad (10)$$

where \underline{c}_{NH} is the consumption floor. Then consumption in the NH state is

$$c_{NH}^{h,i} = \omega + TR(\omega, \pi_h^i, \iota_h^i) - \pi_h^i - m + \iota_h^i. \quad (11)$$

By providing NH residents with a guaranteed consumption floor, Medicaid increases utility in the absence of private insurance thus reducing demand for such insurance. Moreover, Medicaid is a secondary payer which means that, when $\underline{c}_{NH} > \omega - \pi - m + \iota$, marginal increases in the amount of the private LTCI indemnity ι are exactly offset by a reduction in Medicaid transfers, so individual utility stays constant at $u(c_{NH}) = u(\underline{c}_{NH})$. Thus, for

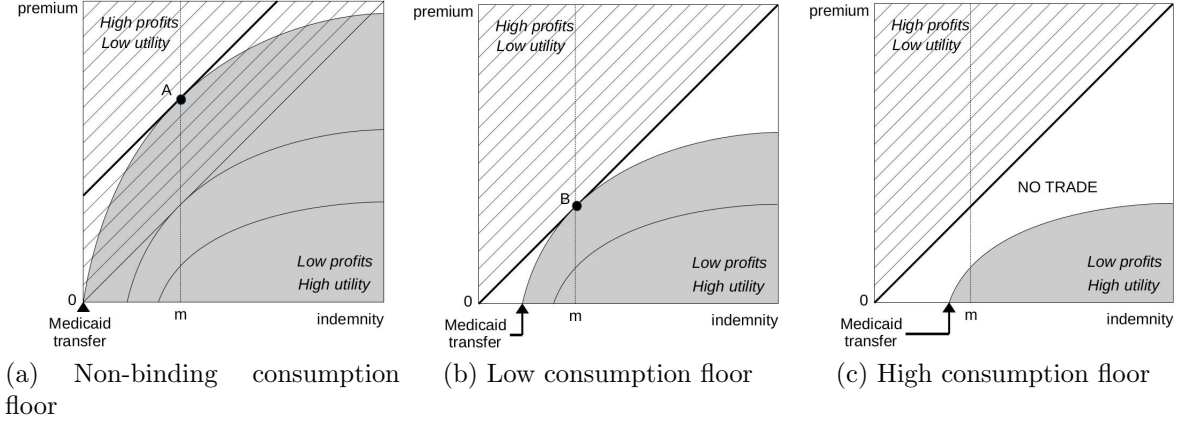


Figure 2: An illustration of the effects of Medicaid on the trading space. The straight lines are the insurer's isoprofit lines and the curved lines are the individual's indifference curves.

individuals that meet the means-test, the marginal utility of the insurance indemnity is zero and only private LTCI contracts in which $\iota - \pi$ exceeds $\underline{c}_{NH} + m - \omega$ are potentially attractive.

Suppose that without Medicaid, the optimal contract of one of the types is given by point A in Figure 2a. Figure 2b illustrates the impact of introducing Medicaid with a small value of \underline{c}_{NH} . Notice that the optimal indemnity is unchanged. However, the individual's outside option has improved, and to satisfy the participation constraint, the premium is reduced. Because the insurer gives the individual the same coverage at a lower price, his profits decline. As \underline{c}_{NH} increases, an equilibrium, such as the one depicted in Figure 2c, will eventually occur. In this case, \underline{c}_{NH} is so large that the insurer can not give the agent an attractive enough positive contract and still make positive profits. The optimal contract is $(0, 0)$.

Now assume that when individuals are choosing their LTCI contract, they face uncertainty about the size of their endowment. Specifically, assume that ω is distributed with density $f(\cdot)$ over the bounded interval $\Omega \equiv [\underline{\omega}, \bar{\omega}] \subset \mathbb{R}_+$. An individual's utility function is

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\bar{\omega}} \left[\theta^i u(c_{NH}^i(\omega)) + (1 - \theta^i) u(c_o^i(\omega)) \right] f(\omega) d\omega, \quad (12)$$

where

$$c_o^i(\omega) = \omega - \pi_h^i, \quad (13)$$

$$c_{NH}^i(\omega) = \omega + TR(\omega, \pi^i, \iota^i) - \pi^i - m + \iota^i, \quad (14)$$

and the Medicaid transfer is defined by (10).

Endowment uncertainty will be used in the baseline model to capture the fact that, in reality, at the time of LTCI purchase, most individuals do not know whether and to what

$\lambda = 1$, the only contract offered is a $(0, 0)$ pooling contract if ψ is sufficiently small.

¹⁸Another way to capture these facts would be to have more model periods and have individuals face uncertainty about the timing and duration of their NH stay.

extent Medicaid will cover their costs if they have a NH event.¹⁸ We introduce it now because it allows the optimal LTCI contracts to achieve qualitative properties that are consistent with the data. In particular, Medicaid can generate rejections and reduce insurer's profits without this uncertainty, but it cannot produce partial insurance for the bad risk type. When endowment uncertainty is absent, due to Medicaid's secondary payer status, a NH event is either insured by Medicaid or private insurance but never both.¹⁹ Thus, if bad types purchase any private insurance, they will prefer full coverage. With endowment uncertainty, in contrast, an individual may rely on both types of insurance. In the case of a NH event, he may be eligible for Medicaid under only some realizations of the endowment and use private LTCI in the other states. However, he will not want full private LTCI coverage because, due to Medicaid, he is already partially insured against NH risk in expectation.

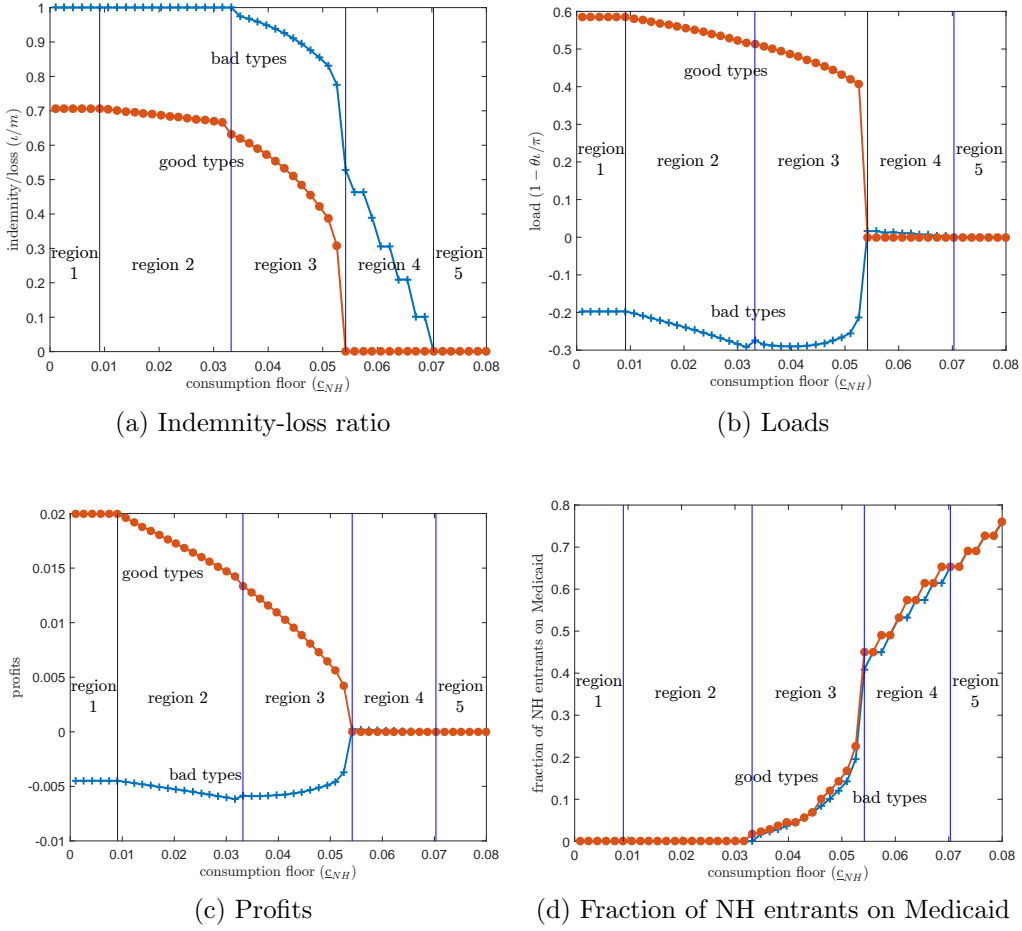


Figure 3: Impact of varying the Medicaid consumption floor, c_{NH} , on the indemnity-loss ratio, loads, profits, and the fraction of NH entrants on Medicaid when the endowment is stochastic.

Figure 3 illustrates how the optimal contracts, profits and Medicaid take-up rates evolve

¹⁹In equilibrium, if an individual is eligible for Medicaid his LTCI contract must be $(0, 0)$ as any nonzero contract would involve the same consumption in the NH state as a $(0, 0)$ contract but lower consumption in

as the Medicaid consumption floor, \underline{c}_{NH} , is increased from zero in the setup with endowment uncertainty. The figure is divided into 5 distinct regions. In region 1, the consumption floor is so low that even if an individual has no private LTCI and the smallest realization of the endowment he will not qualify for Medicaid. In this region, Medicaid has no effect on the optimal contracts. In region 2, Medicaid influences the contracts even though, in equilibrium, neither type receives Medicaid transfers. In this region, Medicaid has a similar effect to that illustrated in Figure 2b. For some realizations of the endowment, good types qualify for Medicaid if the contract is $(0, 0)$. This tightens their participation constraint and the contract offered to them has to be improved. A better contract for good types tightens, in turn, the incentive compatibility constraint for bad types. The insurer responds by reducing premiums for both types, and the indemnity of the good types and loads on both types fall. Since Medicaid's presence has resulted in more favorable contracts for individuals, the insurer's profits fall. In region 3, Medicaid has the same effects as in region 2 but now, in addition, both types receive Medicaid benefits in equilibrium for some realizations of ω . As discussed above, the partial insurance of NH shocks via Medicaid results in optimal contracts that feature partial coverage and, in this region, both types have less than full private insurance. Proposition 3 provides a sufficient condition for this to occur.

Proposition 3. *If $\omega < \underline{c}_{NH}$ for some $\omega \in \Omega$, then the optimal menu features incomplete insurance for both types, i.e., $i^i < m$ for $i \in \{b, g\}$.*

Proof. See Appendix. □

In region 4, the consumption floor is so high that the good types, who's willingness to pay for private LTCI is lower than the bad types, choose to drop out of the private LTCI market. Notice that, even though the average loads are declining as the consumption floor increases, the load on bad types jumps up upon entry into this region. In regions 1–3, the contracts exhibit cross-subsidization with bad types benefiting from negative loads and good types facing positive loads. In region 4, the insurer is able to make a small amount of positive profits by offering a positive contract that is only attractive to the bad types. Finally, in region 5, Medicaid has a similar effect to that depicted in Figure 2c. The consumption floor is so large that there are no terms of trade that generate positive profits from either type. The insurer rejects applicants when the consumption floor is in this region as the optimal menus consist of a single $(0, 0)$ contract.

In the Appendix, we establish that the single-crossing property continues to obtain when Medicaid is present. However, due to the non-convexities Medicaid creates, conditions (8) and (9) in Proposition 2 are no longer sufficient conditions for rejections to occur, and, although still necessary, are not very useful. Proposition 4 provides a stronger set of necessary conditions for rejections in the presence of Medicaid and a stochastic endowment.

Proposition 4. *If the optimal menu is a $(0, 0)$ pooling contract then*

$$U(\theta^b, \lambda\theta^b\iota, \iota) < U(\theta^b, 0, 0), \quad \forall \iota \in \mathbb{R}_+, \quad (15)$$

and

$$U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0), \quad \forall \iota \in \mathbb{R}_+. \quad (16)$$

If condition (15) fails, then one can find a profitable contract that bad types would take, and, if condition (16) fails, then one can find a profitable pooling contract that good types would take. The conditions are not sufficient because, while they rule out profitable pooling contracts and separating contracts where good types get no insurance, they do not rule out separating contracts where both types get positive insurance. Absent Medicaid, there can never exist a separating contract that increases profits if the optimal pooling contract is $(0, 0)$. However, the non-convexities introduced by Medicaid break this property. As a result, even when the optimal pooling contract generates negative profits, a profitable separating contract might still exist.

Figure 3 highlights some important distinctions between our model, where contracts are optimal choices of an issuer, and previous research by, for instance, Brown and Finkelstein (2008), Mommaerts (2015), and Ko (2016), who model demand-side distortions in the LTCI market but set contracts exogenously. In regions 2 and 3, notice that Medicaid's presence only impacts the pricing and coverage of the optimal private contracts. In these regions, the insurer responds to the reduced demand for private LTCI by adjusting the terms of the contracts but still offers positive insurance. In contrast, in regions 4 and 5, Medicaid's presence also impacts the fraction of individuals who have any private LTCI. Notice that the Medicaid reciprocity rates of both types increase as the consumption floor is increased in these regions. This means that, even though good types do not have LTCI in region 4 and no individuals have it in region 5, Medicaid is covering their NH costs only for a subset of the endowment space. For some realizations of ω , they self-insure. Thus, in these regions, Medicaid is crowding-out demand for private LTCI despite providing only incomplete coverage itself. This crowding-out effect is also present in models with exogenous contracts, however, the effects of Medicaid on the terms of positive contracts is not. Thus, allowing the insurer to adjust the contracts in response to the presence of Medicaid is important because, if the terms of the contracts cannot adjust, then the crowding-out effect of Medicaid on the size of the LTCI market will be overstated.

3.3 The Extent of LTCI Coverage

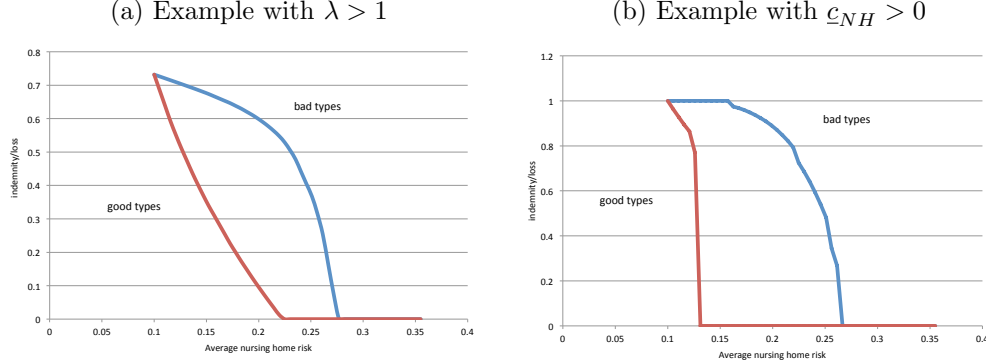
We have illustrated how the presence of either variable overhead costs or Medicaid and endowment shocks impacts the private LTCI market. Namely, they can produce partial coverage and rejections, and thus, low take-up rates and profits. Proposition 5 specifies how varying key parameters affects the extent of coverage when both supply-side and demand-side frictions are modeled.

Proposition 5. *If at least one of the following three conditions hold: $\lambda > 1$, $\theta^b = 1$, or $\underline{c} > 0$, then, in equilibrium, the extent of insurance coverage weakly decreases and the possibility of rejection weakly increases if any of the following occurs:*

1. *the loading factor λ increases,*
2. *the Medicaid consumption floor \underline{c}_{NH} increases,*
3. *the endowment ω decreases when $\omega - m \leq \underline{c}_{NH}$,*
4. *the average NH entry rate, η , increases due to:*

the non-nursing state.

Figure 4: Impact of increasing the NH entry probability of the bad types, θ^b , on the relationship between the average NH entry probability, η and indemnity-loss ratios of each private type.



- a. lower ψ .
 - b. higher θ^b for θ^b sufficiently close to 1.
5. the distribution of true NH entry probabilities θ^i , $i \in \{g, b\}$ becomes more polarized with η not falling too much.

Proof. See Appendix. □

Proposition 5 is important for understanding how our quantitative model, presented next, is able to account for cross-sectional variation in LTCI take-up rates. In our quantitative model, individuals will vary not only in their private types, but also by endowments (earnings) and frailty status. Individuals' private NH entry risk will depend on these characteristics which we assume are observed by the insurer. The insurer will thus condition contracts on them. We refer to individuals with different observable characteristics as different risk groups. It follows that the average NH entry rate, η , will vary across risk groups. The extent of insurance coverage offered to the various risk groups will differ and some risk groups will be rejected by the insurer. This latter property of the model plays an important role in accounting for the cross-sectional patterns of U.S. LTCI take-up rates presented in Table 2. Cases 4–5 of Proposition 5 provide intuition about how the distribution of private NH entry probabilities must vary with endowments and frailty status if the model is to generate these patterns in the data.

Proposition 5 also provides the basis for understanding how our model can generate either small positive correlations or even negative correlations between LTCI coverage and NH entry consistent with the findings documented in Finkelstein and McGarry (2006).^{20,21} Recall that the measure of LTCI coverage they use to estimate these correlations is whether an individual has or does not have LTCI. Their data does not allow them to ascertain how

²⁰We review their findings in Section 2.5.

²¹Note that when they control for information observable by insurers their estimated negative coefficient is not statistically significant. Thus one cannot rule out the possibility of a zero or even a small positive correlation between LTCI coverage and NH entry rates.

the size of LTCI policies varies with NH entry. Thus, if information observed by the insurer is perfectly controlled for, the correlation is only identified off risk groups where one private information type purchases LTCI and the other type does not. Returning to Figure 3, note that this only occurs in region 4. In all of the other regions, either both types hold LTCI or neither type holds LTCI. If region 4 is small then the correlation between LTCI coverage and observed risk exposure will be nonnegative but small.

To introduce the possibility of a negative correlation we need to assume that the information set of the econometrician is smaller than that of the insurer. To see this, suppose that the insurer's information set allows him to assign each individual into one of two risk groups: group 1 and group 2 where the groups are such that $\theta_1^b < \theta_2^b$ but otherwise identical. Clearly, $\eta_1 < \eta_2$, i.e., group 2 has a higher average NH entry rate than group 1. Suppose that the optimal menu for group 1 is nonzero for both types. By item 5 of Proposition 5, group 2 may be rejected. In this case, all members of group 1 will have LTCI and no members of group 2 will have it, and yet, the average NH entry probability of group 2 is higher than that of group 1. If the econometrician's information set does not allow him to discern individuals in group 1 from individuals in group 2, he will measure a negative correlation between LTCI coverage and NH entry.

In general, finding a negative correlation requires that three things occur. First, the econometrician must have less information than the insurer. Second, LTCI coverage needs to be declining in η . Corollary 6 states how this second assumption can occur.

Corollary 6. *Consider two groups of individuals, 1 and 2, such that NH risk is on average higher in group 1 than group 2 ($\eta_1 > \eta_2$). If at least one of the following three conditions hold: $\lambda > 1$, $\theta^b = 1$, or $\underline{c} > 0$, then it is possible that, in equilibrium, either (i) group 1 has less insurance coverage than group 2 ($0 < \iota_1^i < \iota_2^i$, $i \in \{b, g\}$) or (ii) group 1 is rejected and group 2 is not ($\iota_1^i = 0 < \iota_2^i$, $i \in \{b, g\}$).*

Proof. See Appendix. □

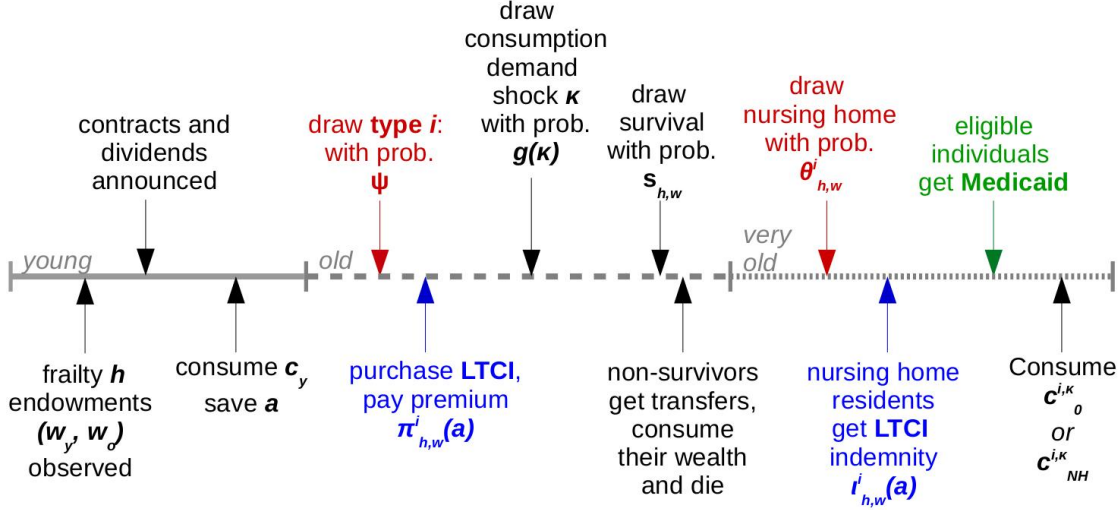
Third, the quantitative features of the optimal contracts must be of a certain form. For instance, suppose that risk group 2 is rejected as before but that risk group 1's optimal menu is a separating one with zero insurance for the good types instead. In this scenario, the differential LTCI take-up rates in group 1 acts as a countervailing force and the econometrician may measure a positive relationship between LTCI coverage and NH entry.

From these examples it is clear that the correlations between LTCI coverage and NH entry in our model have the potential to be either small and positive or even negative. We explore the quantitative model's findings in this respect in Section 6.3.

4 Quantitative model

We consider an endowment economy with two periods, where period 2 is divided into two subperiods. The economy consists of three kinds of actors: a continuum of individuals, a monopolist provider of private LTCI, and a government. We refer to individuals as young in period 1, old in the first subperiod of period 2, and very old in the second subperiod. All individuals become old. At the beginning of the first subperiod, the insurer issues policies

Figure 5: Timeline of events in the baseline model.



and pays out profits to old individuals as dividends. Between the first and second subperiods a survival shock occurs and some old die. At the beginning of the second subperiod, the very old face the risk of experiencing a NH event. The government taxes individuals and uses tax revenue to finance a welfare program for retirees and a Medicaid program for NH residents.

4.1 Individual's problem

Figure 5 lays out the timing of events in the model. At birth an individual draws his frailty status h and lifetime endowment of the consumption good $\mathbf{w} = [w_y, w_o]'$ which are jointly distributed with density $f(h, \mathbf{w})$. Frailty status and endowments are noisy indicators of NH risk. He also observes his probability of surviving from the first to the second subperiod of period 2, $s_{h,w}$, and his period 2 dividend income from ownership of the firm $d\Pi$ where Π is the insurer's profits. Both survival risk and dividend income depend on the individual's frailty status and endowment stream.

A young individual receives w_y and then chooses consumption c_y and savings a . At the beginning of period 2, the individual receives w_o and $d\Pi$ and observes his true risk of entering a NH conditional on surviving to the end of period 2: $\theta_{h,w}^i$, $i \in \{g, b\}$ with $\theta_{h,w}^g < \theta_{h,w}^b$. The individual's true NH entry risk is private information that is not observable by the government or the insurer and the individual realizes a low (good) NH entry probability, $i = g$ with probability ψ . We assume that NH entry probabilities depend on h and \mathbf{w} but that ψ is independent of them. He then chooses a LTCI contract from the menu offered to him by the private insurer. The insurer conditions the menu of contracts offered to each individual on their frailty status, endowments, and asset. Each menu contains two incentive compatible contracts: one for the good types and one for the bad types. A contract consists of a premium $\pi_{h,w}^i(a)$ that the individual pays to the insurer and an indemnity $\iota_{h,w}^i(a)$ that the insurer pays to the individual if the NH event occurs.

After purchasing LTCI, individuals experience a demand shock that induces them to consume a fraction κ of their young endowment where $\kappa \in [\underline{\kappa}, \bar{\kappa}] \subseteq [0, 1]$ has density $g(\kappa)$.

We use this demand shock to capture the following features of NH events in a parsimonious way. On average, individuals have 18 years of consumption between their date of LTCI purchase and their date of NH entry. However, the timing of a NH event is uncertain and individuals who experience a NH event later in life than others are likely to have consumed a larger fraction of their lifetime endowment beforehand.

Very old individuals may experience a NH event at cost m . To capture the fact that many retirees die without ever entering a NH, we assume that with probability $s_{h,\mathbf{w}}$ the retiree survives to this stage and with probability $1 - s_{h,\mathbf{w}}$ he does not. If he does not survive, we assume that he anticipates his death and consumes all his wealth before dying.²²

Individuals who experience a NH event may receive benefits from a public means-tested LTCI program (Medicaid). Medicaid is a secondary insurer in that it guarantees a consumption floor of \underline{c} to those who experience a NH shock and have low wealth and low levels of private insurance.

An individual of type (h, \mathbf{w}) solves the following maximization problem, where the dependence of choices and contracts on h and w is omitted to conserve notation,

$$U_1(h, \mathbf{w}) = \max_{a \geq 0, c_y, \mathbf{c}_{NH}, \mathbf{c}_o} u(c_y) + \beta U_2(a), \quad (17)$$

with

$$U_2(a) = [\psi u_2(a, \theta_{h,\mathbf{w}}^g, \pi^g, \iota^g) + (1 - \psi) u_2(a, \theta_{h,\mathbf{w}}^b, \pi^b, \iota^b)], \quad (18)$$

and

$$u_2(a, \theta^i, \pi^i, \iota^i) = \int_{\underline{\kappa}}^{\bar{\kappa}} \left\{ u(\kappa w_y) + \alpha \left[s_{h,\mathbf{w}} (\theta^i u(c_{NH}^{i,\kappa}) + (1 - \theta^i) u(c_o^{i,\kappa})) \right. \right. \\ \left. \left. + (1 - s_{h,\mathbf{w}}) u(c_o^{i,\kappa}) \right] \right\} g(\kappa) d\kappa, \quad (19)$$

subject to

$$c_y = w_y(1 - \tau) - a, \quad (20)$$

$$c_o^{i,\kappa} + \kappa w_y = (1 - \tau)(w_o + ra + d\Pi) + a - \pi^i(a), \quad i \in \{g, b\}, \quad (21)$$

$$c_{NH}^{i,\kappa} + \kappa w_y = (1 - \tau)(w_o + ra + d\Pi) + a + TR(a, \pi^i(a), \iota^i(a), m, \kappa) - \pi^i(a) - m + \iota^i(a). \quad (22)$$

The Medicaid transfer is

$$TR(a, \pi, \iota, m, \kappa) = \max \left\{ 0, \underline{c}_{NH} - [(1 - \tau)(w_o + ra + d\Pi) + a - \kappa w_y - \pi - m + \iota] \right\}, \quad (23)$$

r denotes the real interest rate and τ is a tax.

²²There is evidence that individuals anticipate their death. [Poterba et al. \(2011\)](#) have found that most retirees die with very little wealth and [Hendricks \(2001\)](#) finds that most households receive very small or

In the U.S. retirees with low means also receive welfare through programs such as the Supplemental Security Income program. We capture these programs in a simple way. After solving the agent's problem we check whether they would prefer to save nothing and consume at the consumption floors: \underline{c}_{NH} in the NH state and \underline{c}_o in the non-NH state. If they do, we allow them to do so and assume that they do not purchase LTCI.²³

4.2 Insurer's problem

The insurer observes each individual's endowments \mathbf{w} , frailty status h , and assets a . He does not observe an individual's true NH entry probability, $\theta_{h,\mathbf{w}}^i$, $i \in \{g, b\}$, but knows the distribution of NH risk in the population and the individual's survival risk $s_{h,\mathbf{w}}$. We assume that the insurer does not recognize that asset holdings depend on \mathbf{w} and h via household optimization. We believe that this is realistic because most individuals purchase private LTCI relatively late in life. Note that the demand shock, κ , is realized after LTCI is contracted.

The insurer creates a menu of contracts $(\pi_{h,\mathbf{w}}^i(a), \iota_{h,\mathbf{w}}^i(a))$, $i \in \{g, b\}$ for each group of observable types that maximizes expected revenues taking into account that individual's face survival risk after insurance purchase. Following [Chade and Schlee \(2014\)](#) and [Chade and Schlee \(2016\)](#), we assume that the insurer faces two costs: a load on contracts, λ , that is proportional to the total payout and a fixed cost, k , of processing claims. His maximization problem is

$$\begin{aligned} \Pi(h, \mathbf{w}, a) = & \max_{(\pi_{h,\mathbf{w}}^i(a), \iota_{h,\mathbf{w}}^i(a))_{i \in \{g, b\}}} \psi \left\{ \pi_{h,\mathbf{w}}^g(a) - s_{h,\mathbf{w}} \theta_{h,\mathbf{w}}^g [\lambda \iota_{h,\mathbf{w}}^g(a) + k I(\iota_{h,\mathbf{w}}^g(a) > 0)] \right\} \\ & + (1 - \psi) \left\{ \pi_{h,\mathbf{w}}^b(a) - s_{h,\mathbf{w}} \theta_{h,\mathbf{w}}^b [\lambda \iota_{h,\mathbf{w}}^b(a) + k I(\iota_{h,\mathbf{w}}^b(a) > 0)] \right\} \end{aligned} \quad (24)$$

subject to

$$(IC_i) \quad u_2(a, \theta_{h,\mathbf{w}}^i, \pi_{h,\mathbf{w}}^i(a), \iota_{h,\mathbf{w}}^i(a)) \geq u_2(a, \theta_{h,\mathbf{w}}^j, \pi_{h,\mathbf{w}}^j(a), \iota_{h,\mathbf{w}}^j(a)), \quad \forall i, j \in \{g, b\}, i \neq j \quad (25)$$

$$(PC_i) \quad u_2(a, \theta_{h,\mathbf{w}}^i, \pi_{h,\mathbf{w}}^i(a), \iota_{h,\mathbf{w}}^i(a)) \geq u_2(a, \theta_{h,\mathbf{w}}^i, 0, 0), \quad \forall i \in \{g, b\} \quad (26)$$

Equation (25) restricts attention to insurance contracts that are incentive compatible and equation (26) requires that insurance contracts deliver at least as much utility to each individual as she can achieve by self-insuring against NH risk. There is no need to impose a non-negativity restriction on profits since, the insurer has the option of offering a contract with a zero indemnity. Let $\mu(h, \mathbf{w}, a)$ denote the measure of agents with frailty status h , endowment \mathbf{w} , and asset holdings a . Total profits for the insurer are given by

$$\Pi = \sum_{\mathbf{w}} \sum_h \sum_a \Pi(h, \mathbf{w}, a) \mu(h, \mathbf{w}, a). \quad (27)$$

no inheritances. This assumption eliminates any desire for agents to use LTCI to insure survival risk. We discuss the implications of alternative assumptions about survival risk in the Appendix.

²³Modelling the Supplemental Security Income program in this way helps us to generate the low levels of savings of individuals in the bottom wealth quintile without introducing additional nonconvexities into the

4.3 Government's problem

In period 1, the government collects taxes on individuals' income when young and saves the revenue at rate r . In period 2, it collects taxes on individuals' income when old and finances the two means-tested welfare programs in the economy. Given the two consumption floors, $\{\underline{c}_{NH}, \underline{c}_o\}$, the government sets the tax rate τ to satisfy the government budget constraint

$$REV = \sum_{\mathbf{w}} \sum_h TR^{h,\mathbf{w}} f(h, \mathbf{w}), \quad (28)$$

where $TR^{h,\mathbf{w}} f(h, \mathbf{w})$ is aggregate government transfers to individuals of type (h, \mathbf{w}) via the two welfare programs and

$$REV = \sum_{\mathbf{w}} \sum_h (1+r)\omega_y f(h, \mathbf{w}) + \sum_{\mathbf{w}} \sum_h \sum_a [\omega_o + ra + d\Pi] \mu(h, \mathbf{w}, a),$$

is aggregate government revenue.

4.4 Equilibrium

We solve for a competitive equilibrium under the assumption that the real interest rate is exogenous. The U.S. economy has strong international financial linkages and it is unlikely that changes in LTCI arrangements would have a large effect on U.S. real interest rates. In order to place private and social insurance for longterm care on an equal footing we recognize the costs of financing Medicaid. Medicaid is financed with an income tax and this distorts savings incentives. We thus solve a fixed point problem that insures that the government budget constraint is satisfied, that insurance markets clear, and that total dividend income received by individuals equals total profits generated by the private LTC insurer.

Definition 1. Competitive Equilibrium. Given a distribution of individuals by frailty and endowments $f(h, \mathbf{w})$, a real interest rate r , and consumption floors $\{\underline{c}_{NH}, \underline{c}_o\}$, a competitive equilibrium consists of a set of insurance contracts $\{\pi_{a,\mathbf{w}}^i, \iota_{a,\mathbf{w}}^i\}$, $i \in \{g, b\}$; profits Π ; a government income tax rate τ ; consumption allocations $\{c_y^{h,\mathbf{w}}, c_o^{h,\mathbf{w},i,\kappa}, c_{NH}^{h,\mathbf{w},i,\kappa}\}$, $i \in \{g, b\}$; and savings policy $a^{h,\mathbf{w}}$ such that the consumption allocations and saving policy solve the individuals' problems and the insurance contracts solve the insurer's problem, total dividend income is equal to total profits of the insurer, the distribution of agents by frailty, endowments and assets is such that

$$\mu(h, \mathbf{w}, a) = \begin{cases} f(h, \mathbf{w}), & \text{if } a = a^{h,\mathbf{w}}, \\ 0, & \text{otherwise,} \end{cases}$$

and the government budget constraint holds.

insurer's maximization problem.

Table 3: Model Parameters

Description	Parameter	Value
Risk aversion coefficient	σ	2
Preference discount factor	β	0.94
Retirement preference discount factor	α	0.21
Interest rate (annualized)	r	0.0062
Frailty distribution	h	BETA(1.53,6.70)
Endowment distribution	$[w_y, w_o]'$	LN(-0.32,0.80)
Copula parameter	ρ_{h,w_y}	-0.4
Demand shock distribution	κ	$1 - \kappa \sim \text{LN}(-1.08, 0.245)$
Fraction of good types	ψ	0.7
Nursing home cost	m	0.0956
Insurer's marginal cost load	λ	1.195
Insurer's fixed cost of paying claims	k	0.016
Medicaid consumption floor	\underline{c}_{NH}	0.02
Welfare consumption floor	\underline{c}_o	0.02
Tax rate	τ	0.011

5 Calibration and Assessment

Solving the model is computationally intensive due to nonconvexities in individual budget sets created by the means-test and the large number of risk groups. We will allow for 101 different income levels and 5 different frailty levels so that there a total of 505 risk groups and thus 505 distinct optimal menus to be computed. When computing the optimal menu of contracts for a given risk group we need to check different possible configurations of contracts and it is not unusual to encounter non-convergence due to the setting of the initial conditions. These computational issues dictate that we parameterize the model by informally calibrating it to data targets.

We assume that individuals work from ages 25 to 64 and retire at age 65. Table 3 lists many of the model's parameters and their calibrated values.

5.1 Preferences and technology

Individuals cover a substantial fraction of NH expenses using their own resources. Given the size of these expenses, it makes sense to assume that households are risk averse and thus willing to pay a premium to avoid this risk. A common choice of the risk aversion coefficient in the macroeconomics incomplete markets literature is $\sigma = 2$. We use this value. The preference discount factor and interest rate in conjunction with σ jointly determine how much people save for retirement. The preference discount factor β is set to reproduce average

²⁴The specific target is 0.222. Our choice of this age group is based on two considerations. First, if we limit attention to those aged 65 we would only have a small number of observations. Second, the average age when individuals purchase LTCI in our sample is 67 and this is the midpoint of the interval we have chosen.

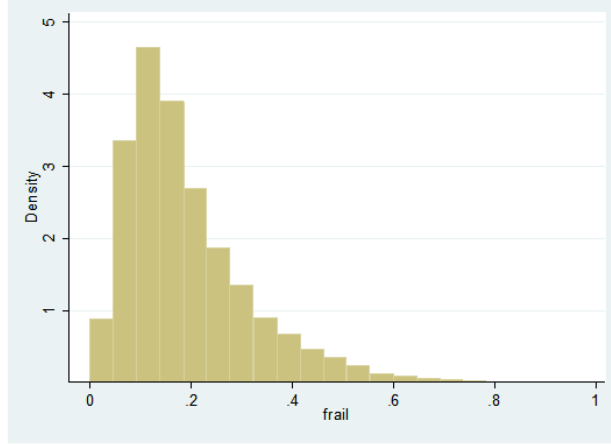


Figure 6: Distribution of frailty for 62–72 year-olds in our HRS sample. Severity of frailty is increasing with the index value and the maximum is normalized to one.

wealth of 62–72 year olds in our HRS sample relative to average lifetime earnings.²⁴ The resulting annualized value of β is 0.94.

On average individuals in our dataset enter a NH at age 83 or about 18 years after they retire. The parameter α captures the discounting between the age of retirement and LTCI purchase, and the age when a NH event is likely to occur. We choose α to reproduce the average wealth of NH entrants immediately before entering the NH relative to the average wealth of 62–72 year olds. This ratio is 1.60 in our dataset and 1.56 in our model.²⁵

We assume that savings earn a risk-free real return of 2% per annum. However, when computing the overall return on savings between the first and second model period, we recognize that savings are not accumulated uniformly during individuals’ working careers. Younger workers have zero or even negative net worth and it is only midway through their working career that they start provisioning for retirement. We capture this in a simple way by assuming that the gross return at retirement of 1 unit of savings for those aged 21–34 is zero. From 35 to 64 we tabulate the total return at retirement of one unit of savings for each age and then average the returns from ages 21–64.²⁶ This procedure results in an effective annualized return on savings of 0.0062.

5.2 Frailty and endowment distributions

The distribution of frailty in the model is calibrated to replicate the distribution of frailty of individuals aged 62–72 in our HRS sample.²⁷ We focus on 62–72 year-old individuals because frailty is observed by the insurer at the time of LTCI purchase. In our HRS sample, the

²⁵To calculate this number in the data, we average the wealth of NH entrants in the wave that precedes their NH entry wave.

²⁶By taking the simple average we are implicitly assuming that individuals are saving the same fraction of their working-age income in each period between 35 and 64.

²⁷The empirical distribution is derived from the frailty index we constructed for HRS respondents. See Section 2.3.

Table 4: Mean frailty by PE quintile in the data and the model.

	PE Quintile				
	1	2	3	4	5
Data	0.23	0.22	0.19	0.16	0.15
Model	0.23	0.20	0.18	0.16	0.13

Data source: Authors' calculations using our HRS sample.

frailty of 62–72 year-old individuals is negatively correlated with their permanent earnings.²⁸ To capture this feature of the data we assume that the joint distribution of frailty and the endowment stream, $f(h, \mathbf{w})$, is a Gaussian copula. This distribution has two attractive features: the marginal distributions do not need to be Gaussian and the dependence between the two marginal distributions can be summarized by a single parameter $\rho_{h, \mathbf{w}}$. The value of this parameter is set to -0.4 so that the model generates the variation in mean frailty by permanent earnings (PE) quintile observed in the data. Table 4 shows the targeted values and model counterparts.

Figure 6 shows the empirical frailty distribution. We approximate it using a beta distribution with $a = 1.53$ and $b = 6.70$. The parameters of the distribution are chosen such that mean frailty in the model is 0.18 and the Gini coefficient of the frailty distribution is 0.35, consistent with their counterparts in the data. When computing the model, we discretize frailty into a 5-point grid. We use the mean frailty of each quintile of the distribution as grid values.

The marginal distribution of endowments is assumed to be log-normal. We equate endowments to the young with permanent earnings and normalize the mean young endowment to 1. This is equivalent to a mean permanent earnings of \$1,049,461 in year 2000 dollars which is approximated as average earnings per adult aged 18–64 in year 2000 multiplied by 40 years.²⁹ The standard deviation of the log of endowments to the young is set to 0.8 because it implies that the Gini coefficient for the young endowment distribution is 0.42. This value is consistent with the Gini coefficient of the permanent earnings distribution for individuals 65 and older in our HRS.

Endowments to the old are a stand in for social security and private pension benefits. Individuals with the lowest income value get a 60% replacement rate. As income increases, the replacement rate falls in a linear way to 40% at the highest income level. The minimum and maximum levels of the replacement rate are chosen so that the model reproduces the average social security replacement rate in the year 2000 and the ratio of wealth at retirement of quintile 5 to quintile 2. The average replacement rate, taken from Biggs and Springstead

²⁸We use annuitized income to proxy for permanent earnings and assign individuals the annuitized income of their household head. The household head's annuitized income is calculated as the average social security and pension income over all retirement waves observed. For singles, the household head is the respondent, and for couples it is the male.

²⁹To derive average earnings per adult aged 18–64 in year 2000 we divide aggregate wages in 2000 taken from the Social Security Administration by number of adults aged 18–64 in 2000 taken from the U.S. Census.

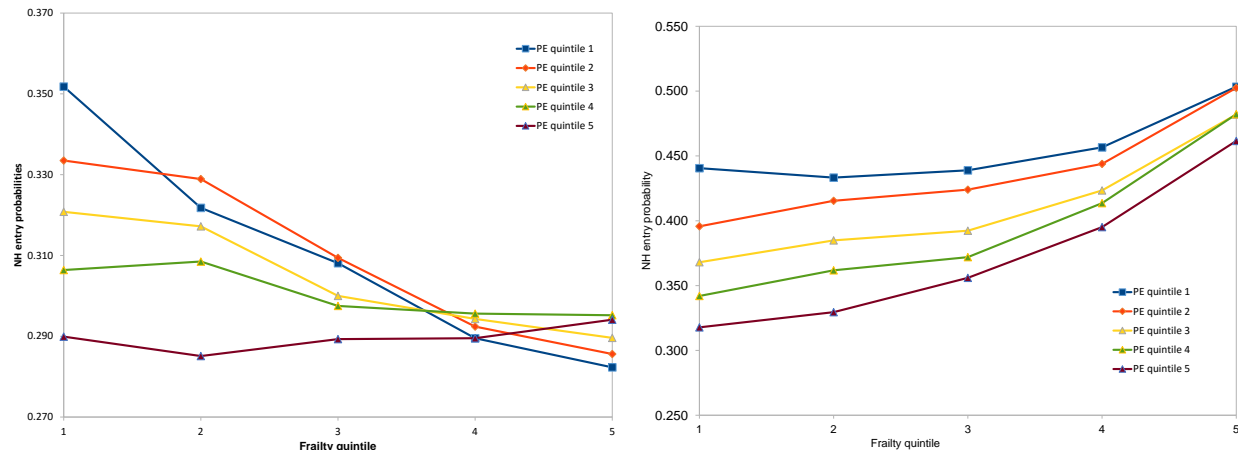


Figure 7: Lifetime NH entry probabilities by frailty and PE quintile: unconditional (left panel) and conditional on surviving to age 80 (right panel). NH entry probabilities are probabilities of a NH stay of at least 100 days.

(2008), is 45% and the ratio of quintile 5's wealth to that of quantile 2's is 18 in our HRS sample.

We want our model to capture the fact that individuals who enter a NH early in their retirement period, on average, have more wealth than individuals who enter later. We thus assume that the distribution of demand shocks κ is such that $1 - \kappa$ is log-normal. We then use the parameters of this distribution to target two data facts. The first data target is that 46% of NH residents are on Medicaid in our HRS sample. In the model, the corresponding value is 45%. The second data target is the level of quintile 5's wealth at the time of NH entry. This number is 0.53 in our dataset and 0.52 in our model. This results in the mean value of κ being 0.55 and the standard deviation of $\log \kappa$ set to 0.245.

5.3 Nursing home entry and survival probabilities

Notice that the moments used to calibrate the joint distribution of frailty and endowments are all exogenous and independent of the survival and NH entry probabilities. Thus, having calibrated this distribution, we can use it to assign individuals in the model to frailty and PE quintiles, and thereby partition the population into 25 groups, one for each frailty/PE quintile combination. To reduce the number of parameters, we assume individuals within the same group have the same survival probability and the same set of true NH entry probabilities.³⁰

To obtain survival and lifetime NH entry probabilities by frailty and PE quintile groups from the data, we use an auxiliary simulation model similar to that in [Hurd et al. \(2013\)](#). All NH entry probabilities are probabilities of experiencing a long-term (at least 100 day) NH stay. We focus on long-term NH stays because stays of less than 100 days are heavily

³⁰We wish to emphasize that these groups are not risk groups because individuals in a given group are not identical to the insurer. The insurer observes 101 distinct levels of permanent earnings and thus will offer different menus to individuals in a given group.

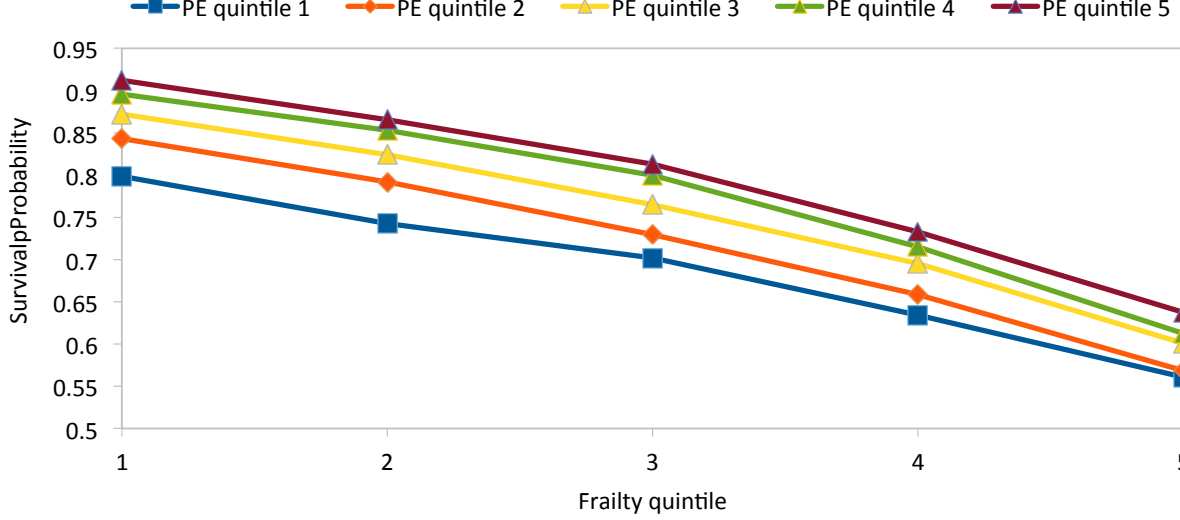


Figure 8: The probability of surviving to age 80 or until experiencing a NH stay by frailty and PE quintile.

subsidized by Medicare. The left panel of Figure 7 shows the probability a 65 year-old will enter a NH before death by frailty and PE quintiles estimated using the simulation model. NH entry risk does not vary much with frailty for PE quintile 5 and is actually decreasing in frailty for PE quintiles 1–4. These patterns occur because frailty is an indicator of both NH entry risk and mortality risk. The right panel of the figure shows that, once we condition on surviving to age 80, the lifetime NH entry probabilities of 65 year-olds increase with frailty. To obtain the survival probabilities, we estimate the probability that a 65 year-old will survive to either age 80 or until a NH event occurs for each of the 25 groups. We use survival until age 80 *or* a NH event because, this way, regardless of which one we target, our calibrated model will be able to match both the unconditional NH entry probabilities and the entry probabilities conditional on survival that are reported in Figure 7. The resulting survival probabilities of each frailty and PE quintile are shown in Figure 8. Not surprisingly, the relationship between frailty and survival is negative in all PE quintiles.

We calibrate the 51 parameters that govern the distribution of NH entry probabilities across private information types in each frailty-income quintile group to reproduce group-specific NH entry probabilities and LTCI take-up rates. We start by normalizing the probability of NH entry of the bad types in quintile 1 of the permanent earnings distribution and quintile 5 of the frailty distribution to 1. This leaves 49 NH type-specific entry probabilities and ψ to be set. The first set of targets we use are the 25 NH entry probabilities reported in the right panel of Figure 7. In our model, the probability of NH entry conditional on survival is $\eta_{h,w}^s = \psi\theta_{h,w}^g + (1 - \psi)\theta_{h,w}^b$. This expression is used to back out the NH entry probabilities of good types, $\theta_{h,w}^g$'s, in each frailty-income quintile group given ψ and that group's $\theta_{h,w}^b$.

The second set of targets are the 15 LTCI take-up rates of individuals in all combinations of quintiles 1-3, 4, and 5 of the wealth distribution and quintiles 1 through 5 of the frailty distribution reported in the lower panel of Table 5. The resulting system is not identified because the number of free parameters ($50 - 25 = 25$), exceeds the number of data moments

Table 5: LTCI take-up rates by wealth and frailty: data and model

Data			
Frailty Quintile	Wealth Quintile		
	1–3	4	5
1	0.168	0.147	0.233
2	0.171	0.158	0.205
3	0.136	0.131	0.200
4	0.115	0.113	0.157
5	0.093	0.107	0.104
Model			
Frailty Quintile	Wealth Quintile		
	1–3	4	5
1	0.145	0.127	0.243
2	0.163	0.179	0.218
3	0.161	0.114	0.183
4	0.083	0.111	0.182
5	0.081	0.111	0.085

For frailty (rows) Quintile 5 has the highest frailty and for wealth (columns) Quintile 5 has the highest wealth. We merge wealth quintiles 1–3 because take-up rates are very low for these individuals. Data source: 62–72 year olds in our HRS sample.

(15). This gap is bridged by restricting the shape of the NH entrance probabilities of bad types, $\theta_{h,w}^b$ in wealth quintiles 1–3. In particular, we assume that the NH entry probabilities of bad types in these wealth quintiles vary with frailty at the same rate as average NH entry rates vary with frailty. This produces 10 restrictions on the $\theta_{h,w}^b$ ’s and reduces the number of free parameters to 15 so that the system is exactly identified. Our decision to restrict the parameters in this way is based on two considerations. First, only a very small number of individuals in quintiles 1 and 2 have LTCI in our dataset. Second, in the model, no individuals in these quintiles buy LTCI because they are guaranteed to get Medicaid if they incur a NH event.³¹

Table 5 reports the 15 LTCI take-up rates implied by the model using our calibration scheme and the data targets. The fit of the model is not perfect due to the fact that we discretize the state space to compute the model. Note, however, that the take-up rates generated by the model increase with wealth and decline with frailty for both the rich and poor. Our calibration scheme also does a good job of reproducing the average LTCI take-up rate. In our HRS sample, 9.4% of retirees aged 62–72 have LTCI and in the model 9.3% of 65 year-olds have a nonzero LTCI contract. The fact that we are able to reproduce the average LTCI take-up rate suggests that the restrictions we have imposed on the $\theta_{h,w}^b$ ’s for

³¹This difference between the model and the data is present for a variety of reasons including measurement error, our parsimonious specification of the Medicaid transfer function, and the fact that we have not modeled all shocks faced by retirees such as spousal death.

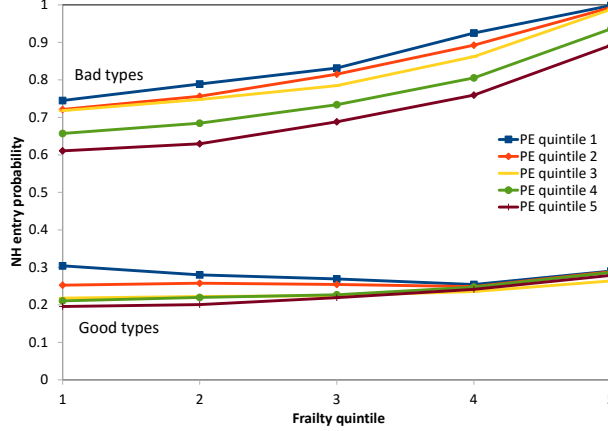


Figure 9: Nursing home entry probabilities conditional on surviving in the model for good and bad types by frailty and PE quintile.

Table 6: Standard deviation of self-reported (private) NH entry probabilities by frailty: data and model

	Frailty Quintile				
	1	2	3	4	5
Data	1.00	0.99	1.03	1.27	1.47
Model	1.00	1.09	1.21	1.36	1.52

The standard deviations are normalized such that the standard deviation of frailty quintile 1 is 1. Data values are standard deviations of self-reported probabilities of entering a NH in the next 5 years for individuals aged 65–72 excluding observations where the probability is 50%. The pattern in the data is robust to variations in the way we construct the standard deviations including how we handle those reporting a probability of 0, 100% or 50%. Data source: Authors’ calculations using our HRS sample.

wealth quintiles 1-3 are broadly consistent with our data.

This calibration scheme yields a value for ψ of 0.7 and type-specific NH entry probabilities conditional on survival that are reported in Figure 9. Notice that the dispersion in private NH entry risk increases with frailty. The increase in dispersion is needed for the model to reproduce the negative relationship between LTCI takeup rates and frailty that is observed within each PE quintile in the data. To see this recall from case 5 of Proposition 5 that increasing the dispersion of the distribution of type-specific NH entry probabilities increases the possibility of rejection and, thus, puts downward pressure on the take-up rates. Also notice that none of the others cases mentioned in the proposition can work. The parameters λ , \underline{c}_{NH} and ω do not vary with frailty ruling out cases 1–3. To get lower takeup rates via case 4 requires the average NH entry rate to increase with frailty. The counterpart to the average NH entry rate in our baseline model is the unconditional lifetime NH entry rates

³²More details on the mapping from the simple model without survival risk and the baseline model is provided in the Appendix.

shown in the left panel of Figure 7.³² Notice that these rates do not increase with frailty: they are flat for PE quintile 5 and decline with frailty in all the other quintiles.

One way to assess this aspect of our model is to provide independent evidence that dispersion in private NH entry probabilities, and thus the severity of the private information friction, increases with frailty. The first row of Table 6 reports normalized standard deviations of self-reported NH entry probabilities for 65–72 year-old HRS respondents by frailty quintile. These probabilities are not exactly comparable to the private NH entry probabilities in the model for two reasons. First, they are self-reported probabilities of NH entry in the next 5 years whereas the model values are lifetime NH entry probabilities. Second, the self-reported probabilities are very noisy with 1/3 of respondents reporting 0.5 and another third reporting either 0 or 1. The second row of Table 6 reports the distribution of private NH entry probabilities by frailty quintile that emerge from our calibration. Despite the noise, a comparison of the first and second rows of the table show that the dispersion of private information is increasing in frailty in both the data and the model.

5.4 Nursing home and insurance costs

In practice NH care expenses have two components. The first component is nursing and medical care and the second component is room and board. We interpret this second component as being part of consumption and thus a choice and not a medical expense shock. We estimate the medical expense component of the cost of a NH stay in the following way. Stewart et al. (2009) estimate that the average total cost of a NH stay was \$60,000 in the year 2003. Barczyk and Kredler (2016) estimate that the medical expense component accounts for 56.2% of the total cost. Using this factor yields \$33,720 per year for the medical component of a NH stay. Braun et al. (2015) estimate that the average duration of NH stays that exceed 90 days is 3.25 years. Medicare provides NH benefits for up to the first 100 days. To account for this, we subtract 100 days resulting in an average benefit period of 2.976 years. Multiplying the annual medical cost by the average length of a NH stay yields total medical expenses of \$100,351 or a value of $m = 0.0956$ when scaled by average lifetime earnings.

Private LTCI products are costly to administer. Insurers need to verify that those claiming benefits qualify under the terms of the contract.³³ Regulatory rules for setting rates on LTCI policies have changed during our sample period. Prior to 2000, a common requirement was that insurers fix the minimum percentage of premium revenue earmarked for losses (benefits to the insured) at 60% leaving 40% to cover administrative overhead and profits. This scheme resulted in large increases in premia as the group of insured aged and claims increased. In 2000, the rules were changed to set caps on losses over the lifetime of the policy. Since, 2014 new policies are expected to embed a 10% margin in their pricing of premia to reflect the risk of adverse claims experiences over the life of the policy.³⁴ We interpret these

³³Since health status can improve, insurers also need to monitor those receiving benefits on an ongoing basis. It is not unusual for individuals to have multiple qualified events. In recent years insurers have also gotten involved in care management. In some cases, the insured or the family of the insured prefer to keep the insured out of a nursing home. Thus there may be opportunities to coordinate with the insured and their family to come up with alternative accommodations that are preferred by the family and reduce claims.

³⁴See King (2016) for more details.

Table 7: Distribution of insurance across NH residents: data and model

	LTCI	Medicaid	Both	Neither
Data	8.2	45.6	2.7	43.4
Model	9.2	44.8	0.1	45.9

Percent of NH residents covered by LTCI only, Medicaid only, both, or neither in the data and the model. Data source: Authors' calculations using our HRS sample.

regulations as suggesting that about 30% of premium revenue is for overhead expenses and about 10% is for (risky) profits.

Our model has two parameters that govern overhead costs of paying claims. The parameter λ is a proportional cost of paying claims and k is a fixed cost. We have seen that λ affects the type of equilibrium (separating or pooling) and also plays a central role in producing rejections. The fixed cost k affects the average size of contracts and also plays a role in generating rejections. As k increases, risk groups with less comprehensive contracts are more likely to be rejected because in these less profitable groups the insurer's net revenue will not be large enough to cover the higher fixed costs. We choose λ and k by targeting two moments. The first moment is total costs incurred by the insurer as a fraction of total premium revenue which is 30%. The second moment is an average load on individuals of 0.40 which is in the middle of the range of average loads documented in the literature.³⁵

5.5 Government programs and taxes

We set the consumption floors \underline{c}_{NH} and \underline{c}_o to 0.02 which corresponds to \$7,053 per year under the assumption that an average stay lasts 2.976 years. This value was chosen to match estimates of consumption floors from the previous literature.³⁶ The tax rate that is needed to clear the government budget constraint is 0.011.

5.6 Insurance Distribution

We have not explicitly targeted the distribution of insurance use (private, public, none) across NH residents. It follows that comparing the predictions from the model with data along this dimension is another way to assess our model. Table 7 shows that the model does an excellent job generating the distribution of insurance across NH residents in the HRS data. The biggest difference is that the fraction of individuals who end up with both LTCI and Medicaid is positive in the model but smaller than in the data. In the model, these are individuals who, ex-ante, bought LTCI because they were not covered by Medicaid for all realizations of the demand shock but, ex-post, drew a realization of κ that resulted in Medicaid eligibility.

³⁵See Section 2.4 for details.

³⁶See Kopecky and Koreshkova (2014) for more details.

Table 8: Rejection rates in the Baseline, the No Overhead Costs, the No Medicaid, and the Full Information economies

Scenario Description	Baseline	No Overhead Costs $\lambda = 1, \kappa = 0$	No Medicaid $\underline{c}_{nh} = 0.001$	Full Information θ_h^i public
Average	90.5	34.9	15.0	50.5
By PE Quintile				
1	100	100	20.0	100
2	100	74.3	0.0	100
3	87.5	0.0	0.0	38.1
4	87.1	0.0	0.0	0.0
5	78.0	0.0	54.9	14.5
By receiving Medicaid NH benefits conditional on surviving				
Would	44.5	34.2	4.0	41.1
Would not	46.0	0.7	11.0	9.4

Rejection rates are percentage of individuals who are only offered a single contract of $(0, 0)$ by the insurer. Note that for PE quintiles that the figures are expressed as a percentage of individuals in that quintile. However, the bottom two rows of the table are a decomposition of the average rejection rate for that economy.

6 Results

In Section 2, we motivated our analysis by describing some of the main features of the U.S. private LTCI market. We now turn to discuss how well our model accounts for these observations and why. The results from Section 3 demonstrated that, at a qualitative level, the main features of the market could be accounted for, independently, by either supply-side frictions due to variable overhead costs and adverse selection or demand-side frictions due to Medicaid. One of our main objectives here is to understand the role of each of these mechanisms. To help distinguish between them, we compare the baseline economy with three other scenarios. In each scenario, the endowments and pre-tax interest rate are the same, and the proportional income tax rate is adjusted to balance the government budget constraint. In the No Overhead Costs economy we remove the insurer’s variable and fixed costs by setting $\lambda = 1$ and $\kappa = 0$. In the No Medicaid economy, the NH consumption floor \underline{c}_{NH} is reduced to 0.001.³⁷ The Full Information economy, which is designed to understand the effects of private information, assumes that the insurer can directly observe each individual’s true NH risk exposure, $\theta_{h,w}^i$.

6.1 LTCI take-up rates and rejections

We explained in Section 2.3 that LTCI take-up rates decline with frailty for both poor and rich individuals in our HRS sample and we have calibrated the baseline model to reproduce

³⁷It is not possible to reduce the consumption floor all the way to zero because then some individuals

Table 9: LTCI take-up rates by wealth and frailty: Baseline and Full Information economies

Baseline					
Frailty Quintile	Wealth Quintiles				
	1	2	3	4	5
1	0.000	0.000	0.145	0.127	0.243
2	0.000	0.000	0.163	0.179	0.218
3	0.000	0.000	0.161	0.114	0.183
4	0.000	0.000	0.083	0.111	0.182
5	0.000	0.000	0.081	0.111	0.085
Full Information					
Frailty Quintile	Wealth Quintile				
	1	2	3	4	5
1	0.000	0.000	0.624	1.000	0.811
2	0.000	0.000	0.745	1.000	0.875
3	0.000	0.000	0.668	1.000	0.890
4	0.000	0.000	0.537	1.000	0.911
5	0.000	0.000	0.545	1.000	0.918

these observations (see Table 5). Our model has two mechanisms for generating low LTCI take-up rates. Some risk groups are rejected by the insurer because there is no basis for trade. The menu of contracts offered to these risk groups consists of a single $(0, 0)$ contract. As we explained in Section 3, for other risk groups the menu consists of two contracts, a non-zero one and a $(0, 0)$ one, and the good type chooses the $(0, 0)$ one.³⁸ It turns out that these latter menus are very rare in the Baseline economy. Only 0.2% of individuals choose not to purchase private LTCI.³⁹ Thus, the low LTCI take-up rates produced by our model are almost exclusively due to rejections. Consistently, as the first column of Table 8 shows, the rejection rate in our Baseline economy is 90.5%. It is 100% for individuals in PE quintiles 1 and 2 and declines with permanent earnings in quintiles 3–5.

Recall from Section 2.2, that we estimated that roughly 36% of 55–66 year old HRS respondents would be rejected based on medical underwriting if they applied for LTCI. Some care must be exercised in relating rejections or denials based on this data to our model concept of rejections. In our model, a rejection is a no-trade result. For there to be a basis for trade, the insurer must be able to offer a profitable contract that is attractive to the purchaser. Our data measure of rejections primarily captures no trade that arises because from the insurer’s perspective insuring a particular risk group is not profitable.

would experience negative consumption. Also note that the non-NH consumption floor \underline{c}_o does not vary across economies.

³⁸This result follows from Lemma 6 (see the Appendix) which establishes that the single crossing condition obtains in our setup with Medicaid and stochastic endowments.

³⁹The small fraction of individuals who choose not to purchase LTCI is partly due to frictions from Medicaid and overhead costs and partly due to private information. When we run a counterfactual simulation that removes Medicaid and overhead costs the fraction of individuals who choose not to purchase LTCI increases to 2.4%.

Unfortunately, our HRS dataset does not include questions that would allow us to ascertain how many individuals do not have private LTCI because they perceive that it is too expensive. However, survey results in [Ameriks et al. \(2016\)](#) suggest that pricing is also an important reason for low take-up rates. It follows that our estimate of a rejection rate of 36% using HRS data is best thought of as providing a lower bound on situations where no trade might occur.

We now turn to analyze the roles of overhead costs on the insurer and Medicaid in producing rejections and thus low LTCI take-up rates. To get at their individual roles, compare columns 2 and 3 of Table 8, which report rejection rates in the No Overhead Costs economy and the No Medicaid economy, to column 1. Removing either overhead costs or Medicaid has a big effect on the average rejection rate. However, removing overhead costs has a larger impact on the rejection rates of the rich, whereas removing Medicaid has a larger impact on the rejection rates of the poor. In particular, notice that removing overhead costs eliminates rejections of individuals in quintiles 3–5. When overhead costs are removed profits from offering insurance increase. This has a bigger effect on contracts for higher income risk groups whose demand is not as greatly reduced by the presence of Medicaid. In contrast, when Medicaid is removed, rejections are eliminated in quintiles 2–4 and substantially reduced in quintile 1. They do not fall to zero in quintile 1 because some individuals in quintile 1 are so poor that they cannot afford NH care and must rely on Medicaid even though the Medicaid consumption floor is extremely low. Interestingly, while removing Medicaid is needed to substantially reduce rejections of the poor (those in quintile 1 and 2) and removing insurer costs is needed to substantially reduce rejections of those in quintile 5, rejections in quintiles 3–4 can be eliminated by removing either one.

Individuals who are rejected by the insurer can be divided into two groups: those who, if they survive to the very old age and enter a NH, will receive Medicaid NH benefits and those who will not. In the baseline economy 46.0% of individuals are rejected by the insurer but ultimately are too affluent to satisfy the Medicaid means test. This latter group pays all of their NH expenses out of pocket. When either overhead costs or Medicaid are removed the fraction of individuals in this group declines. When overhead costs are removed, only 0.7% of individuals are rejected and end up paying out of pocket for NH care. These are individuals in PE quintile 2 whose demand for LTCI was low because their likelihood of ending up on Medicaid if NH entry occurred was high, but who ultimately ended up with too much wealth to meet the Medicaid means test. When Medicaid is removed, 11.0% of individuals are rejected and end up paying out of pocket for NH care. These are individuals in PE quintile 5 who were rejected by the insurer because of poor health.

Finally, to understand the role that private information plays, the final column of Table 8 shows the rejection rates in the Full Information economy. Absent private information, rejection rates fall from 90.5% to 50.5%. This decline is due to a decline in rejections of individuals in PE quintiles 3–5. Removing private information increases profitability and, like removing overhead costs, this has a larger effect on the rejection rates of higher income individuals. Table 9 reports LTCI take-up rates for the Baseline economy and the Full Information economy. Notice that LTCI take-up rates in the Full Information economy are not only too large but also have the wrong pattern in wealth quintiles 4–5. It is clear from these results that private information plays an essential role in generating the decline in LTCI take-up rates with frailty within the upper wealth quintiles.

Previous research by [Braun et al. \(2015\)](#) and [De Nardi et al. \(2013\)](#) has found that Medicaid and other means-tested social insurance for retirees is highly valued. It insures against a range of risks faced by retirees including lifetime earnings risk, NH risk, spousal death risk and longevity risk. It is thus interesting to consider how effective this form of social insurance is in providing for those who are rejected due to the supply-side distortions. A comparison of the Baseline with the No Overhead Costs and the Full Information economies using Table 8 suggests that Medicaid NH benefits are not very effective in insuring against supply-side frictions. When going from the No Overhead Costs economy to the Baseline economy, the fraction of individuals who are rejected but receive Medicaid NH benefits increases from 34.2% to 44.5%, while the fraction of individuals who are rejected but pay out-of-pocket for NH care goes from 0.7% to 46%. The pattern of changes is similar when moving from the Full Information economy to the Baseline economy. In other words, most of the individuals who are rejected due to overhead costs or private information end up paying for NH care out of pocket.

6.2 Coverage, loads and profits

6.2.1 Coverage and loads in the baseline economy

A second feature of the U.S. market for LTCI that we discussed in Section 2 is that individuals who purchase LTCI only receive partial coverage. The most common policies cover between one-third and two-thirds of lifetime NH expenses and policies that offer comprehensive lifetime coverage have essentially disappeared from the market. Insurance contracts in our Baseline economy capture this feature of the market. Indemnities cover 58% of NH medical costs on average. Table 10 shows how the average fraction of NH costs covered varies by private information type, wealth and frailty. Notice that, conditional on having a positive amount of coverage and on private type, comprehensiveness of the policies does not vary much across observable characteristics. Comprehensiveness does, however, vary significantly across private types. The indemnity covers about 75% of NH costs for bad types and about 50% of NH costs for good types. Combining these results with the previous results on rejections indicates that the insurer is reacting to adverse selection in two ways. He is screening out risk groups that are not profitable by offering $(0, 0)$ contracts, and he is incentivizing individuals in profitable risk groups to reveal their private type by offering a menu featuring a less and a more comprehensive contract.

How does our model generate partial coverage and, in particular, partial coverage for bad risks? In Section 3, we described three different ways to break the classic adverse selection result that bad risks receive full coverage. One way to break it is by generating pooling contracts with positive levels of insurance. However, it can also be broken with separating contracts if the insurer faces overhead costs, or if Medicaid is present and endowments are stochastic. It turns out that positive pooling contracts do not arise in our Baseline Economy. Thus, partial coverage of bad types is due to the second and third factors.

The pricing of LTCI in our Baseline economy is also broadly consistent with pricing in the U.S. LTCI market. Recall that [Brown and Finkelstein \(2007\)](#) and [Brown and Finkelstein \(2011\)](#) find that the average load in the LTCI market is in the range 0.18 and 0.5, depending on whether or not the loads are adjusted for policy lapses and the sample period. They also

Table 10: Comprehensiveness and individual loads by private type and frailty and wealth quintiles in the Baseline economy.

	Wealth Quintile				
	1	2	3	4	5
Good risks (θ^g)					
Fraction of NH costs covered	NA	NA	0.489	0.538	0.497
Load	NA	NA	0.621	0.591	0.599
Bad risks (θ^b)					
Fraction of NH costs covered	NA	NA	0.701	0.766	0.764
Load	NA	NA	-0.089	-0.082	-0.018
	Frailty Quintile				
	1	2	3	4	5
Good risks (θ^g)					
Fraction of NH costs covered	0.507	0.517	0.503	0.497	0.489
Average load	0.598	0.598	0.607	0.609	0.617
Bad risks (θ^b)					
Fraction of NH costs covered	0.744	0.753	0.749	0.748	0.741
Load	-0.061	-0.042	-0.057	-0.057	-0.061

The fraction of NH costs covered is the average indemnity divided by the medical expense cost of a nursing-home stay or (ι/m) for individuals with a positive amount of insurance. NA denotes cases where the denominator is zero.

find that the relationship between loads and comprehensiveness is non-monotonic and that for some individuals loads are negative. The average load was a calibration target (0.40). However, loads by wealth and frailty level were not calibration targets. Table 10, shows that there is considerable cross-subsidization. In all wealth and frailty quintiles, bad types have negative loads as they are subsidized by good types. The loads are also nonlinear in wealth. For good types, wealth quintile 4 receives the most coverage with the lowest load. Coverage is lower and loads are higher in both wealth quintiles 3 and 5. For bad risk types, in contrast, loads and coverage are both increasing in wealth. Still, most individuals in the model are good types and the average pattern of loads by wealth exhibits the same shape as that of the good types.

In the model economy, the insurer creates a separate menu for each risk group and thus, in equilibrium, offers hundreds of menus. However, in reality, insurers usually pool individuals into two or three risk groups. One remedy to this discrepancy between the model and the data would be to assume that the insurer faces a fixed cost of creating each menu. A fixed menu cost gives the insurer an incentive to reduce the number of risk groups by combining individuals with different observable characteristics into the same group. We do not model this fixed cost because it significantly complicates the problem of finding the optimal set of menus.⁴⁰ However, as Table 10 shows, the extent of coverage and loads on each private

⁴⁰Finding the profit maximizing optimal set of menus with fixed menu costs is a challenging combinatorics problem in our setting because there are a very large number of contracts that have to be considered.

Table 11: LTCI take-up rates, comprehensiveness and individual loads by private type in the Baseline, the No Overhead Costs, the No Medicaid, and the Full Information economies

Scenario	Baseline	No Overhead Costs	No Medicaid	Full Information
Description		$\lambda = 1, \kappa = 0$	$\underline{c}_{nh} = 0.001$	θ_h^i public
Good risks (θ^a)				
LTCI take-up rate	0.092	0.612	0.847	0.495
Fraction of NH costs covered	0.503	0.543	0.620	0.858
Load	0.603	0.551	0.709	0.492
Bad risks (θ^b)				
LTCI take-up rate	0.095	0.651	0.850	0.495
Fraction of NH costs covered	0.747	0.859	0.807	0.588
Load	-0.054	-0.156	0.118	0.263

The fraction of NH costs covered is the average indemnity divided by the medical expense cost of a nursing-home stay or (ι/m) for individuals with a positive amount of insurance.

type do not vary much across observable characteristics in our Baseline economy. This fact suggests that a small fixed cost of creating menus could substantially reduce the number of menus offered by the insurer.

6.2.2 Coverage and loads in other economies

We next analyze the individual roles of private information, Medicaid, and overhead costs in producing partial coverage, high loads and low takeup rates. The first three columns of Table 11 show the average LTCI take-up rates, fractions of NH costs covered, and loads on good and bad risk types in the Baseline economy, the No Overhead Costs economy, and the No Medicaid economy. Notice that removing either overhead costs or Medicaid not only increases take-up rates but also the comprehensiveness of contracts offered with the removal of Medicaid having the larger effect. Loads, in contrast, may increase or decline depending on which friction is removed. Without overhead costs, loads on individuals are lower than in the Baseline economy. Without Medicaid, demand for LTCI increases and the insurer can increase loads and trade still occurs. The final column of the table presents the same statistics for the Full Information economy. Under full information, not only are take-up rates higher, but contracts are, on average, more comprehensive. However, the impact of going from the Baseline economy to full information differs for good and bad risk types. Good risks experience an increase in coverage and a decline in loads. Bad risks, on the other hand, experience exactly the opposite. The intuition for this finding dates back to [Arrow \(1963\)](#) who demonstrates that the amount of insurance available to those with high risk exposures falls if insurance markets open after their risk exposure is observed.

[Brown and Finkelstein \(2008\)](#) and [Ameriks et al. \(2016\)](#) use a different strategy to assess the roles of high loads, incomplete coverage and Medicaid in accounting for low LTCI takeup

rates. Both of these papers consider counterfactuals in which individuals are offered full insurance against NH risk at an actuarially fair price. [Brown and Finkelstein \(2008\)](#) find that the bottom two-thirds of individuals, when ranked by wealth, do not purchase a full-coverage LTCI policy when Medicaid is present. The results in [Table 11](#) (see also [Section 3.2](#)) show that Medicaid affects not only the fraction of individuals who are offered non-zero contracts but also the comprehensiveness of the contracts. These results suggest that counterfactuals that do not allow the insurer to adjust the size of contracts offered, may overstate the crowding-out effect of Medicaid. Indeed, the crowding out effect of Medicaid is much weaker in our model as compared to [Brown and Finkelstein \(2008\)](#). To see this, consider a version of our Baseline economy in which two supply-side frictions — private information and overhead costs — are removed. Medicaid is present with the consumption floor set at the baseline level. Insurance is not actuarially fair in this scenario, the average load is 0.35, due to the fact that the insurer is a monopolist. Nevertheless, only 36% of individuals do not purchase LTCI. [Table 12](#) reports LTCI take-up rates, comprehensiveness of coverage and average loads by wealth and frailty quintiles in this alternative economy. LTCI takeup rates are 100% in wealth quintiles 3–5. Medicaid crowds out most private insurance in wealth quintile 2 and all private insurance in quintile 1. Wealth quintile 2 is particularly interesting because the load on insurance for this group is only 0.073 and thus close to the actuarially-fair benchmark. These individuals are not interested in a full-coverage private LTCI product because for some values of the demand shock they will be eligible for Medicaid NH benefits.⁴¹ Indeed, 80% of individuals in this group prefer to rely exclusively on Medicaid while 20% purchase a LTCI policy that covers 29% of NH costs.

In contrast to individuals in the lower wealth quintiles, those in quintile 4 receive extensive coverage (93%) and those in quintile 5 receive full coverage. For the latter group, the chance of receiving Medicaid NH benefits is particularly low and full coverage is attractive. This final property of the model is related to results in [Ameriks et al. \(2016\)](#). They find that 66% of individuals in a sample of affluent individuals with median wealth of \$543,000 have demand for an ideal state-contingent LTCI product that is priced in an actuarially-fair manner. However, only 22% of their respondents hold LTCI and they refer to this as a “LTCI puzzle.” For purposes of comparison, in our model, average wealth in wealth quintile 5 is \$821,000 and average wealth in quintile 4 is \$364,000.⁴² Recall that in our baseline economy individuals in wealth quintiles 4–5 have LTCI takeup rates of 13% and 21%, respectively. Thus, we find that the LTCI puzzle that [Ameriks et al. \(2016\)](#) document for wealthy individuals can be attributed to the supply-side distortions induced by private information and overhead costs.

6.2.3 Profits

In [Section 2.4](#) we documented that profits in the U.S. private LTCI market are low. Profits are also low in our Baseline economy. They are less than 0.001% of average lifetime earnings. The left panel of [Figure 10](#), which reports profits by frailty and PE quintiles, reveals that most profits come from insuring healthy rich individuals as most of the other risk groups are rejected and profits are thus zero. Medicaid, overhead costs and private information all work

⁴¹Recall that the optimal contracts have this same property in the simple model considered in [Section 3](#). See [region 4](#) of [Figure 3](#).

⁴²Both figures are expressed in terms of year 2000 dollars.

Table 12: LTCI take-up rates, comprehensiveness, and individual loads in the economy with no private information and no overhead costs.

	Wealth Quintile				
	1	2	3	4	5
LTCI take-up rates	0	0.20	1	1	1
Fraction of loss covered	NA	0.29	0.73	0.93	1.0
Average load	NA	0.073	0.34	0.41	0.35
	Frailty Quintile				
	1	2	3	4	5
LTCI take-up rates	0.81	0.74	0.65	0.57	0.42
Fraction of loss covered	0.95	0.93	0.88	0.89	0.84
Average load	0.36	0.35	0.34	0.35	0.33

NA denotes cases where the denominator is zero.

to reduce profits. Medicaid, however, has the largest impact. When it is removed profits rise to 1.1% of average lifetime earnings.⁴³ The right panel of Figure 10 shows profits by frailty and PE quintiles in the No Medicaid economy. In this economy, in contrast to the Baseline, the insurer generates most of his profits from the poor. Profits fall monotonically with permanent earnings and do not vary much by frailty. Notice that Medicaid has a large effect on profits for two reasons. First, Medicaid’s presence dramatically reduces the fraction of profitable risk groups and when Medicaid is removed the fraction of zero-profit rejected pools declines. Second, Medicaid substantially lowers the profits the insurer receives from risk groups that are getting a positive amount of insurance. For example, when Medicaid is removed, the profits on individuals in PE quintile 5 increase by a factor of 20.

6.3 Insurance ownership and NH entry

We now illustrate that our baseline economy generates, at least at a qualitative level, a broad range of correlations between self-assessed NH entry risk, NH entry and LTCI ownership documented in Finkelstein and McGarry (2006).⁴⁴ These results are interesting because we have posited a single source of private information. We wish to emphasize that none of the statistics that we are about to report are calibration targets and that the results from the model are population moments. First, they find a positive correlation between self-assessed NH entry risk and NH entry and interpret this as evidence of private information, Figure 9 shows that, in the baseline economy, individuals’ private NH entry probabilities are positively correlated with actual NH entry. This is true both on average and within risk groups. Second, they find that individuals act on their private information by documenting a positive correlation between self-assessed NH entry risk and LTCI ownership. The baseline economy has this property too. The LTCI take-up rate of bad types is 9.5% while the take-up

⁴³Profits are 0.38% of average lifetime earnings in the No Overhead Costs economy and 0.25% in the Full Information economy.

⁴⁴See Section 2.5 for an overview of their findings.

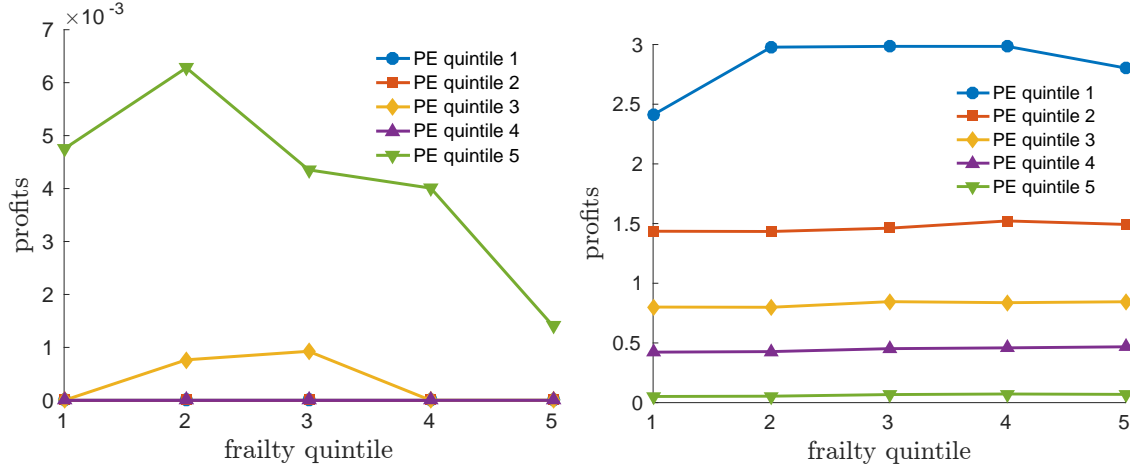


Figure 10: Profits Baseline and No Medicaid Specifications.

The left (right) panel reports profits as a percent of average lifetime earnings by frailty and PE quintiles in the Baseline economy (No Medicaid) economy.

rate of good types is 9.2%. Moreover, bad types have a higher LTCI takeup rate than good types no matter whether or how we control for the information set of the insurer, or whether or not we condition on survival. Third, the correlations between NH entry and LTCI takeup rates in the baseline economy are consistent with those they report. Table 13 reports NH entry rates of LTCI holders and non-holders. Only 37.5% of LTCI holders in the baseline economy enter a NH whereas 40.9% of non-holders enter consistent with the negative correlation they find when they do not control for the insurer's information set.

Chiappori and Salanie (2000) ascertain that to properly test for the presence of private information one must fully control for the information set of the insurer. Finkelstein and McGarry (2006) control for the information set of the insurer by computing the correlation between NH entry and LTCI ownership with two different sets of controls. The first set only controls for observable variation in health. The second set controls for both observable health variables and individuals' wealth and income quartiles. In both cases, they find a small negative but not statistically significant correlation. Only when they consider a special sample of individuals who are in the fourth quartile of the wealth and income distributions and have no health issues that would likely lead them to be rejected by insurers do they find a statistically significant negative correlation. Consistently, as Table 13 shows, if we only control for frailty, we continue to find a negative correlation but the size of the differential between the entry rates of LTCI non-holders and holders is now smaller in each group.⁴⁵ If, in addition to frailty, we also control for wealth and income quartile the differences in the takeup rates between non-holders and holders becomes even smaller.⁴⁶ In addition, the correlation is negative for precisely half of the groups and positive for the other half, and the average differential is essentially zero.⁴⁷ However, like Finkelstein and McGarry (2006), if

⁴⁵We choose to check for conditional independence of NH entry and LTCI ownership because as pointed out by Chiappori and Salanie (2000) this strategy, which is the basis of their χ^2 statistic, is more robust to nonlinearities.

⁴⁶These results are not reported in the table because the number of groups is so large.

⁴⁷Using equal weights for each group the average LTCI ownership rate of entrants is 0.03 percentage

we focus on individuals in the top wealth and income quartile and the lowest frailty quintile we find a negative correlation between LTCI take-up rates and NH entry. The NH entry rate of LTCI holders in this group is 32.1% while the entry rate of non-holders is 32.4%. Taken together these results demonstrate that our model with a single source of private information encompasses the main empirical results that have led the previous literature to conclude that multiple sources of private information are essential for understanding this insurance market.

The reason that the correlations between LTCI take-up rates and NH entry are as small as in [Finkelstein and McGarry \(2006\)](#) is because the differential in LTCI take-up rates between good and bad types only occurs in a tiny fraction of risk groups. It follows that tests based on LTCI take rates will have low power in detecting adverse selection in samples of data generated by our model. Moreover, as we illustrated in [Section 3.3](#), our model generates a pattern of rejections that is increasing in observed risk exposure. Thus, when risk groups are bunched together by the econometrician, the negative correlation between average NH entry and LTCI take-up rates across risk groups within a bunch dominates the small positive correlation between NH entry and LTCI take-up rates that arises within some risk groups.

It is worth noting that these results partly reflect a data limitation. HRS data only reports LTCI ownership rates and not the size of coverage. If data was available on the size of LTCI policies, it would be easier to detect adverse selection in correctly measured risk-groups. However, the bunching effect would still be operative as long as there is a difference between the information set of the econometrician and the insurer.

Finally, there are important differences between the moments generated by our model and the statistics reported in [Finkelstein and McGarry \(2006\)](#). They compute correlations between LTCI ownership and NH entry risk over a short horizon. Specifically, they look at NH entry risk within 5 years of observing LTCI ownership. NH entry risk in our model is the lifetime risk.⁴⁸ Given the short time horizons that their empirical findings are based on, we believe that the most comparable objects from the model are the NH entry rates conditional on survival that we report in [Table 13](#). That said, with no controls, our model still generates a negative correlation even if we do not condition on survival. However, the size of the differential is smaller and the differentials in the upper frailty quintiles flip sign. The reason survival impacts the size of the differentials is because survival impacts the correlation between average NH entry and LTCI take-up rates across risk groups since, as [Figure 7](#) shows, survival changes the way that NH entry varies with both frailty and wealth.

Medicaid, private information and overhead costs all play important roles in producing these results. Abstracting from them increases the fraction of risk groups where the optimal contract for the good type is (0,0) but positive for the bad type. However, we have seen above that abstracting from any one of these mechanisms also changes the size and pattern of rejections. With smaller rejection rates, the size of the bias associated with bunching risk groups together declines.

points higher than the ownership rate of non-entrants.

⁴⁸Alternatively, one could construct an empirical measure of lifetime NH entry risk and compare this to the results from our quantitative model. However, this is not straightforward because lifetime NH risk of HRS respondents is not directly observable and would have to be estimated using an auxiliary model. This creates an additional source of noise and specification error. In our view it is best to compare our model

Table 13: NH entry rates of LTCI holders and non-holders in the Baseline economy

	Frailty Quintile					
	Average	1	2	3	4	5
LTCI holders	36.4	33.2	35.5	37.5	40.9	47.7
Non-holders	40.5	35.5	37.3	39.8	43.1	49.3

Numbers are percent of survivors to the very old stage of life who enter a NH.

7 Conclusion

In this paper we have developed a structural model of the U.S. LTCI market that posits a single source of private information and yet is remarkably successful in accounting for a range of puzzling observations about this market. In particular, we find that low take-up rates of private LTCI and contracts that provide only partial coverage of NH costs are to be expected when overhead costs and the crowding out effects of Medicaid are recognized. Insurers optimally respond to these frictions by rejecting individuals who have poor health or low income via medical underwriting, and by both increasing the price and reducing the comprehensiveness of insurance offered. Our analysis also suggests that tests for adverse selection that are based on correlating insurance ownership and insurance claims may give misleading results. In our model, NH entry by owners of LTCI is about the same as NH entry by those who do not own LTCI, even though the insurer contends with private information by offering optimal menus of contracts that honor participation and incentive compatibility constraints.

Appendix

Coming soon.

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