On a general class of multiple Eulerian integrals II

Frederic Ayant

Vinod Gill
Department of Mathematics Amity University, Rajasthan, Jaipur-303002, India

ABSTRACT
Recently, Raina and Srivastava [5] and Srivastava and Hussain [11] have provided closed-form expressions for a number of a general Eulerian integrals involving multivariable H-functions. Motivated by these recent works, we aim at evaluating a general class of multiple eulerian integrals involving a multivariable I-function defined by Prathima et al [4] with general arguments. These integrals will serve as a key formula from which one can deduce numerous useful integrals.

Keywords :Multivariable I-function, multiple Eulerian integral ,class of polynomials, sequence of polynomials.

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1. Introduction and preliminaries.

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(Re(\alpha)>0, Re(\beta)>0, b>a) \tag{1.1}$$

is a basic result for evaluation of numerous other potentially useful integrals involving various special functions and polynomials. Raina and Srivastava [5], Saigo and Saxena [8], Srivastava and Hussain [11], Srivastava and Garg [10] etc have established a number of Eulerian integrals involving various general class of polynomials, Meijer's G-function and Fox's H-function of one and more variables with general arguments.

The explicit form og the generalized polynomial set [7, p.71, (2.3.4)] is

$$S_n^{\alpha,\beta,\tau}(x) = \sum_{e,p,u,v} C(e,p,u,v) x^R (1-\tau x^{\mathfrak{r}})^{\delta n - v} \tag{1.2}$$

where $C(e,p,u,v) = \dfrac{B^{qn}(-)^p(-p)_e(\alpha)_p(-v)_u(-\alpha-qn)_e\left(-\frac{\beta}{\tau}-sn\right)_v}{u!v!e!p!(1-\alpha-p)_e} l^n(-\tau)^v\left(\dfrac{e+k+\mathfrak{r}u}{l}\right)_n\left(\dfrac{A}{B}\right)^b$ (1.3)

where $\sum_{e,p,u,n} = \sum_{v=0}^{n}\sum_{u=0}^{v}\sum_{p=0}^{n}\sum_{e=0}^{p}$ and $R = ln + \mathfrak{r}v + p$

We recall here the following definition of the general class of polynomials introduced and studied by Srivastava [9]

$$S_V^U(x) = \sum_{\eta=0}^{[V/U]} \frac{(-V)_{U\eta} A_{V,\eta}}{\eta!} x^\eta \tag{1.4}$$

where $V = 0, 1, \cdots$ and U is an arbitrary positive integer. The coefficients $A_{V,\eta}(V,\eta \geqslant 0)$ are arbitrary constants, real or complex.

The multivariable I-function defined by Prathima et al [4] is a extension of the multivariable H-function defined by Srivastava and Panda [12]. It is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r)=I^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.5}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.6}$$

where $\phi(s_1,\cdots,s_r)$, $\theta_i(s_i)$, $i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.7}$$

$$\theta_i(s_i)=\frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.8}$$

For more details, see Prathima et al [4].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i=\sum_{j=1}^{p}A_j\alpha_j^{(i)}-\sum_{j=1}^{q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0, i=1,\cdots,r \tag{1.9}$$

The integral (2.1) converges absolutely if

$|arg(z_k)|<\frac{1}{2}\Delta_k\pi, k=1,\cdots,r$ where

$$\Delta_k=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}>0 \tag{1.10}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.
We will note :

$$\mathbb{A}=(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p}\ :\ (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \tag{1.11}$$

$$\mathbb{B}=(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q}\ :\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r} \tag{1.12}$$

2. Main integral

In this section, we shall establish the following Eulerian multiple integral of multivariable I-function and we shall use the following notations (2.1) and (2.2).

$$X_j=(b_j-a_j)+\rho_j(t_j-a_j)+\sigma_j(b_j-t_j) \tag{2.1}$$

$$Y_j=\frac{(t_j-a_j)^{\gamma_j}(b_j-t_j)^{\delta_j}X_j^{1-\gamma_j-\delta_j}}{\beta_j(b_j-a_j)+(\beta_j\rho_j+\alpha_j-\beta_j)(t_j-a_j)+\beta_j\sigma_j(b_j-t_j)} \tag{2.2}$$

for $j=1,\cdots,s$

Lemma ([2] p.287)

$$\int_a^b\frac{(t-a)^{\alpha-1}(b-t)^{\beta-1}}{\{b-a+\lambda(t-a)+\mu(b-t)\}^{\alpha+\beta}}\mathrm{d}t=\frac{(1+\lambda)^{-\alpha}(1+\mu)^{-\beta}\Gamma(\alpha)\Gamma(\beta)}{(b-a)\Gamma(\alpha+\beta)} \tag{2.3}$$

with $t\in[a;b]\quad a\neq b, Re(\alpha)>0, Re(\beta)>0,\ \ \eta+\lambda(t-a)+\mu(b-t)\neq 0$

Theorem

We have the following result

$$\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\prod_{j=1}^s\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}\,S_U^V\left[a\prod_{j=1}^s\frac{(t_j-a_j)^{S_j}(b_j-t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$S_n^{\alpha,\beta,\tau}\left[b\prod_{j=1}^s Y^{\zeta_j};\mathfrak{r},t,q,A,B,k;l\right]I\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^s Y_j^{v_j'} & \mathbb{A}\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\prod_{j=1}^s Y_j^{v_j^{(r)}} & \mathbb{B}\end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_s$$

$$=\left\{\prod_{j=1}^s\left\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,n}\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$C(e,p,u,v)\left\{\prod_{j=1}^s\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^R$$

$$I_{p+3s,q+3s:p_1,q_1;\cdots;p_r,q_r;1,1}^{0,n+3s:m_1,n_1;\cdots;m_r,n_r;1,1}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} & \mathbb{A}'\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(r)}} & \mathbb{B}'\\ \mathrm{b}^{\mathfrak{r}}\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j\mathfrak{r}} & \end{array}\right) \tag{2.4}$$

We obtain a I-function of $(r+1)-$variables

where

$\mathbb{A}'=(1-\tau_j-\zeta_jR;v_j',\cdots,v_j^{(r)},\zeta_j\mathfrak{r};1)_{1,s},(-\lambda_j-KS_j-\gamma_j\zeta_jR-\tau_j;\gamma_jv_j',\cdots,\gamma_jv_j^{(r)},\gamma_j\zeta_j\mathfrak{r};1)_{1,s},$

$(-\mu_j - KT_j - \delta_j\zeta_j R - \tau_j; \delta_j v'_j, \cdots, \delta_j v_j^{(r)}, \delta_j\zeta_j\mathfrak{r}; 1)_{1,s}, (a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}, 0; A_j)_{1,p}$

$: (\mathrm{c}_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r}; (1 - v + \delta\eta, 1; 1)$

$\mathbb{B}' = (-\lambda_j - \mu_j - K(S_j + T_j) - \zeta_j(\gamma_j + \delta_j)R - \tau_j - 1; (\gamma_j + \delta_j)v'_j, \cdots, (\gamma_j + \delta_j)v_j^{(r)}, (\gamma_j + \delta_j)\zeta_j\mathfrak{r}; 1)_{1,s}$

$(1 - \zeta_j R; v'_j, \cdots, v_j^{(r)}, \zeta_j\mathfrak{r}; 1)_{1,s}, (b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}, 0; B_j)_{1,q}$: $(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r}; (0, 1; 1)$ (2.5)

Provided that

(i) $\lambda_j, \mu_j, s_j, t_j, \zeta_j, v_j^{(i)} > 0, \beta_j \neq 0, b_j - a_j \neq 0, \rho_j \neq -1, \sigma_j - 1,$

$(b_j - a_j) + \rho_j(t_j - a_j) + \sigma_j(b_j - t_j) \neq 0, t_j \in [a_j, b_j]$ for $i = 1, \cdots, r, j = 1, \cdots, s$

(ii) $|(\beta_j - \alpha_j)(t_j - a_j)| < |\beta_j\{(b_j - a_j) + \rho_j(t_j - a_j) + \sigma_j(b_j - t_j)\}|; t_j \in [a_j, b_j]$ for $, j = 1, \cdots, s$

(iii) When min$(S_j, T_j) > 0$

(a) $Re(\lambda_j + \gamma_j\zeta_j(ln + p)) + \sum_{i=1}^r \gamma_j v_j^{(i)} \min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) + 1 > 0$

(b) $Re(\mu_j + \delta_j\zeta_j(ln + p)) + \sum_{i=1}^r \gamma_j v_j^{(i)} \min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) + 1 > 0$

When max$(S_j, T_j) < 0$

(c) $Re(\lambda_j + S_j[V/U] + \gamma_j\zeta_j(ln + p)) + \sum_{i=1}^r \gamma_j v_j^{(i)} \min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) + 1 > 0$

(d) $Re(\mu_j + t_j[V/U] + \delta_j\zeta_j(ln + p)) + \sum_{i=1}^r \gamma_j v_j^{(i)} \min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) + 1 > 0$

When $S_j > 0, T_j < 0$ inequalities (a) and (d) are satisfied.

When $S_j < 0, T_j > 0$ inequalities (b) and (c) are satisfied.

$|arg(z_k)| < \frac{1}{2}\Delta_k\pi, k = 1, \cdots, r$ Where Δ_k is defined by (1.10)

The multiple series of R.H.S. of (2.4) converges absolutely.

Proof

To establish the multiple integral formula (2.4), we first use the series representations for the polynomials sets $S_V^U(x)$ and $S_n^{\alpha,\beta,\tau}(x)$ respectively in its left hand side. Further, using contour integral representation for the multivariable I-function defined by Prathima et al [4] and then interchanging the order of integration and summation suitably, which is permissible under the conditions stated above, we find that

L.H.S = $\sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \frac{(-V)_{UK} A_{V,K}}{K!} a^K b^R C(e, p, u, v) \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r}$

$$\phi(\xi_1,\cdots,\xi_r)\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j+KS_j}(b_j-t_j)^{\mu_j+KT_j}}{X_j^{\lambda_j+\mu_j+K(S_j+T_j)+2}}Y_j^{\zeta_j R+\sum_{i=1}^{r}\xi_i v_j^{(i)}}$$

$$\left(1-\tau x^{\mathfrak{r}}\prod_{j=1}^{s}Y_j^{\zeta_j q}\right)^{\delta n-v}\mathrm{d}t_1\cdots\mathrm{d}t_s\,\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r \tag{2.6}$$

Now by writing $\left(1-\tau x^{\mathfrak{r}}\prod_{j=1}^{s}Y_j^{\zeta_j q}\right)^{\delta n-v}$ in terms of contour integral and changing the order of integration therein, we obtain

$$\text{L.H.S} = \sum_{K=0}^{[V/U]}\sum_{e,p,u,n}\frac{(-V)_{UK}A_{V,K}}{K!}a^K b^R\,C(e,p,u,v)\,\frac{1}{(2\pi\omega)^{r+1}}\int_{L_1}\cdots\int_{L_r}\int_{L_{r+1}}$$

$$\phi(\xi_1,\cdots,\xi_r)\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\,(-\tau b^{\mathfrak{r}})^{\xi_{r+1}}\,\Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})\left[\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\right.$$

$$\left.\left\{\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j+KS_j}(b_j-t_j)^{\mu_j+KT_j}}{X_j^{\lambda_j+\mu_j+K(S_j+T_j)+2}}Y_j^{\zeta_j R+\sum_{i=1}^{r}\xi_i v_j^{(i)}+\zeta_j\mathfrak{r}\xi_{r+1}}\right\}\mathrm{d}t_1\cdots\mathrm{d}t_s\right]\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r\mathrm{d}\xi_{r+1} \tag{2.7}$$

Substituting the value of Y_j from (2.2) and after simplifications, we get

$$\text{L.H.S} = \sum_{K=0}^{[V/U]}\sum_{e,p,u,n}\frac{(-V)_{UK}A_{V,K}}{K!}a^K b^R\,C(e,p,u,v)\,\frac{1}{(2\pi\omega)^{r+1}}\int_{L_1}\cdots\int_{L_r}\int_{L_{r+1}}$$

$$\phi(\xi_1,\cdots,\xi_r)\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\,(-\tau b^{\mathfrak{r}})^{\xi_{r+1}}\,\Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})$$

$$\left[\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\left\{\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j+KS_j+\gamma_j\sum_{i=1}^{r}\xi_i v_j^{(i)}+\gamma_j\zeta_j(R+\mathfrak{r}\xi_{r+1})}}{X_j^{\lambda_j+\mu_j+K(S_j+T_j)+2+(\gamma_j+\delta_j)(R\zeta_j+\sum_{i=1}^{r}\xi_i v_j^{(i)}+\zeta_j\mathfrak{r}\xi_{r+1})}}\right.\right.$$

$$\left.\frac{(b_j-t_j)^{\mu_j+KT_j+\delta_j\sum_{i=1}^{r}\xi_i v_j^{(i)}+\gamma_j\zeta_j(R+\mathfrak{r}\xi_{r+1})}}{\beta_j^{(R\zeta_j+\sum_{i=1}^{r}\xi_i v_j^{(i)}+\zeta_j\mathfrak{r}\xi_{r+1})}}\left(1-\frac{(\beta_j-\alpha_j)(t_j-a_j)}{\beta_j X_j}\right)^{-(\zeta_j R+\sum_{i=1}^{r}\xi_i v_j^{(i)}+\zeta_j\mathfrak{r}\xi_{r+1})}\right\}$$

$$\left.\mathrm{d}t_1\cdots\mathrm{d}t_s\right]\mathrm{d}\xi_1\cdots\mathrm{d}\xi_r\mathrm{d}\xi_{r+1} \tag{2.8}$$

If $\dfrac{(\beta_j-\alpha_j)(t_j-a_j)}{\beta_j X_j}<1, t_j\in[a_j;b_j]$ for $j=1,\cdots,s$

then use the binomial expansion is valid and we thus find that

$$\text{L.H.S} = \sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \sum_{\tau_1,\cdots,\tau_s=0}^{\infty} \frac{(-V)_{UK} A_{V,K}}{K!} a^K b^R C(e,p,u,v) \prod_{j=1}^{s} \left\{ \frac{(\beta_j - \alpha_j)^{\tau_j}}{\beta_j^{\tau_j} \tau_j!} \right\}$$

$$\frac{1}{(2\pi\omega)^{r+1}} \int_{L_1} \cdots \int_{L_r} \int_{L_{r+1}} \psi(\xi_1,\cdots,\xi_r) \prod_{i=1}^{r} \theta_i(\xi_i) z_i^{\xi_i} (-\tau b^{\mathfrak{r}})^{\xi_{r+1}} \Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})$$

$$\prod_{i=1}^{s} \left\{ \frac{\Gamma(\tau_j + R\zeta_j + \sum_{i=1}^{r} \xi_i v_j^{(i)} + \zeta_j \mathfrak{r} \xi_{r+1})}{\Gamma(R\zeta_j + \sum_{i=1}^{r} \xi_i v_j^{(i)} + \zeta_j \mathfrak{r}\xi_{r+1})} \beta_j^{-(R\zeta_j + \sum_{i=1}^{r} \xi_i v_j^{(i)} + \zeta_j \mathfrak{r}\xi_{r+1})} \right\}$$

$$\left[\int_{a_1}^{b_1} \cdots \int_{a_s}^{b_s} \left\{ \prod_{j=1}^{s} \frac{(t_j - a_j)^{\lambda_j + KS_j + \gamma_j \sum_{i=1}^{r} \xi_i v_j^{(i)} + \gamma_j \zeta_j (R + \mathfrak{r}\xi_{r+1}) + \tau_j}}{X_j^{\lambda_j + \mu_j + K(S_j+T_j) + 2 + (\gamma_j+\delta_j)(R\zeta_j + \sum_{i=1}^{r} \xi_i v_j^{(i)} + \zeta_j \mathfrak{r}\xi_{r+1}) + \tau_j}} \right.\right.$$

$$\left.(b_j - x_j)^{\mu_j + KT_j + \delta_j \sum_{i=1}^{r} \xi_i v_j^{(i)} + \delta_j \zeta_j (R + \mathfrak{r}\xi_{r+1})} \mathrm{d}t_1 \cdots \mathrm{d}t_s \right] \mathrm{d}\xi_1 \cdots \mathrm{d}\xi_r \mathrm{d}\xi_{r+1} \quad (2.9)$$

Now using (2.1) and then evaluating the inner-most integral by using the lemma (2.3), we get

$$\text{L.H.S} = \left\{ \prod_{j=1}^{s} \{(b_j - a_j)^{-1} (1+\rho_j)^{-\lambda_j - 1} (1+\sigma_j)^{-\mu_j - 1} \right\} \sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \sum_{\tau_1,\cdots,\tau_s=0}^{\infty} \frac{(-V)_{UK} A_{V,K}}{K!}$$

$$C(e,p,u,v) \left\{ \prod_{j=1}^{s} \frac{(\beta_j - \alpha_j)^{\tau_j} (1+\rho_j)^{-K_j S_j - \gamma_j \zeta_j R - \tau_j} (1+\sigma_j)^{-KT_j - \delta_j \zeta_j R}}{\tau_j! \beta_j^{\tau_j + \zeta_j R}} \right\} a^K b^R$$

$$\frac{1}{(2\pi\omega)^{r+1}} \int_{L_1} \cdots \int_{L_r} \int_{L_{r+1}} \psi(\xi_1,\cdots,\xi_r) \prod_{i=1}^{r} \theta_i(\xi_i) z_i^{\xi_i} (-\tau b^{\mathfrak{r}})^{\xi_{r+1}} \Gamma(-\xi_{r+1})\Gamma(v-\delta n+\xi_{r+1})$$

$$\prod_{j=1}^{s} \left\{ \frac{\Gamma(\tau_j + \lambda_j + KS_j + \gamma_j\zeta_j R + \gamma_j \sum_{i=1}^{r} \xi_i v_j^{(i)} + \gamma_j \zeta_j \mathfrak{r}\xi_{r+1} + 1)}{\Gamma(\lambda_j + \mu_j + K(S_j + T_j) + (\gamma_j + \delta_j)(\zeta_j R + \sum_{i=1}^{r} \xi_i v_j^{(i)} + \zeta_j \mathfrak{r}\xi_{r+1}) + \tau_j + 2)} \right.$$

$$\left. \Gamma(-\xi_{r+1})\Gamma(v - \delta n + \xi_{r+1})\Gamma(\mu_j + K_{tj} + \delta_j \zeta_j R + \delta_j \sum_{i=1}^{r} \xi_i v_j^{(i)} + \delta_j \xi_j \mathfrak{r} \zeta_{r+1} + 1) \right\}$$

$$\prod_{j=1}^{s} \left\{ \frac{(1+\rho_j)^{-\Gamma_j} (1+\sigma_j)^{-\delta_j}}{\beta_j} \right\}^{\sum_{i=1}^{r} \xi_i v_j^{(i)}} \prod_{j=1}^{s} \left\{ \frac{(1+\rho_j)^{-\gamma_j \zeta_j q} (1+\sigma_j)^{-\delta_j \zeta_j q} (-\tau b^{\mathfrak{r}})}{\beta_j^{\zeta_j \mathfrak{r}}} \right\}^{\xi_{r+1}} \mathrm{d}\xi_1 \cdots \mathrm{d}\xi_r \mathrm{d}\xi_{r+1} \quad (2.10)$$

Finally, reinterpreting the multiple Mellin-Barnes contour integral in terms of multivariable I-function, we obtain the result (2.4).

3. Particular cases

The multivariable I-function occurring in the main integral can be suitably specialized to a remarkably wide variety of special functions which are expressible in terms of E, G, H and I-function of one and several variables. Again by suitably specializing various parameters and coefficients, the general class of polynomials and the general sequence of functions can be reduced to a large number of orthogonal polynomials and hypergeometric polynomials. Thus using various special cases of these special functions, we can obtain a large number of others integrals involving simpler special functions and polynomials of one and several variables.

On taking $V = 0, U = 1$ and $A_{0,0}$ in (2.4), the general class of polynomials $S_V^U(x)$ reduces to unity an we get

Corollary 1

$$\int_{a_1}^{b_1} \cdots \int_{a_s}^{b_s} \prod_{j=1}^{s} \frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}} S_n^{\alpha,\beta,\tau}\left[b\prod_{j=1}^{s} Y^{\zeta_j}; \mathfrak{r}, t, q, A, B, k; l \right]$$

$$I\left(\begin{array}{c|c} \mathrm{z}_1 \prod_{j=1}^{s} Y_j^{v_j'} & \mathbb{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r \prod_{j=1}^{s} Y_j^{v_j^{(r)}} & \mathbb{B} \end{array} \right) \mathrm{d}t_1 \cdots \mathrm{d}t_s = \left\{ \prod_{j=1}^{s} \{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1} \right\}$$

$$\sum_{e,p,u,n} \sum_{\tau_1,\cdots,\tau_s=0}^{\infty} C(e,p,u,v) \left\{ \prod_{j=1}^{s} \frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-\gamma_j\zeta_j R-\tau_j}(1+\sigma_j)^{-\delta_j\zeta_j R}}{\tau_j!\beta_j^{\tau_j+\zeta_j R}} \right\} b^R$$

$$I_{p+3s,q+3s:p_1,q_1;\cdots;p_r,q_r;1,1}^{0,n+3s:m_1,n_1;\cdots;m_r,n_r;1,1} \left(\begin{array}{c|c} \mathrm{z}_1 \prod_{j=1}^{s} \left\{ \beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j} \right\}^{-v_j'} & \mathbb{A}'_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r \prod_{j=1}^{s} \left\{ \beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j} \right\}^{-v_j^{(r)}} & \mathbb{B}'_1 \\ \mathrm{b}^{\mathfrak{r}} \prod_{j=1}^{s} \left\{ \beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j} \right\}^{-\zeta_j \mathfrak{r}} & \end{array} \right) \quad (3.1)$$

where

$$\mathbb{A}'_1 = (1-\tau_j-\zeta_j R; v_j', \cdots, v_j^{(r)}, \zeta_j\mathfrak{r}; 1)_{1,s}, (-\lambda_j-\gamma_j\zeta_j R-\tau_j; \gamma_j v_j', \cdots, \gamma_j v_j^{(r)}, \gamma_j\zeta_j\mathfrak{r}; 1)_{1,s},$$

$$(-\mu_j-\delta_j\zeta_j R-\tau_j; \delta_j v_j', \cdots, \delta_j v_j^{(r)}, \delta_j\zeta_j\mathfrak{r}; 1)_{1,s}, (a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}, 0; A_j)_{1,p}$$

$$: (\mathrm{c}_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r}; (1-v+\delta\eta, 1; 1)$$

$$\mathbb{B}' = (-\lambda_j-\mu_j-\zeta_j(\gamma_j+\delta_j)R-\tau_j-1; (\gamma_j+\delta_j)v_j', \cdots, (\gamma_j+\delta_j)v_j^{(r)}, (\gamma_j+\delta_j)\zeta_j\mathfrak{r}; 1)_{1,s}$$

$$(1-\zeta_j R; v_j', \cdots, v_j^{(r)}, \zeta_j\mathfrak{r}; 1)_{1,s}, (b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}, 0; B_j)_{1,q}: (\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r}; (0,1;1) \quad (3.2)$$

with the same notations and corresponding validity conditions that (2.4).

Putting $s=1$ in (2.4), we arrive at the following integral form

Corollary 2

$$\int_{a_1}^{b_1}\frac{(t-a_1)^{\lambda}(b_1-t)^{\mu}}{X_j^{\lambda+\mu+2}}\,S_U^V\left[a\frac{(t-a_1)^{S_j}(b_1-t)^{T}}{X^{S+T}}\right]S_n^{\alpha,\beta,\tau}\left[bY^{\zeta};\mathfrak{r},t,q,A,B,k;l\right]$$

$$I\left(\begin{array}{c|c}\mathrm{z}_1Y^{v'} & \mathbb{A}\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_rY^{v^{(r)}} & \mathbb{B}\end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_s=\left\{(b_1-a_1)^{-1}(1+\rho)^{-\lambda-1}(1+\sigma)^{-\mu-1}\right\}$$

$$\sum_{K=0}^{[V/U]}\sum_{e,p,u,n}\sum_{\tau_1=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}C(e,p,u,v)\left\{\frac{(\beta-\alpha)^{\tau}(1+\rho)^{-KS-\gamma-\tau}(1+\sigma)^{-KT-\delta\zeta R}}{\tau!\beta^{\tau+\zeta R}}\right\}a^Kb^R$$

$$I_{p+3,q+3:p_1,q_1;\cdots;p_r,q_r;1,1}^{0,n+3:m_1,n_1;\cdots;m_r,n_r;1,1}\left(\begin{array}{c|c}\mathrm{z}_1\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} & \mathbb{A}'_2\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(r)}} & \mathbb{B}'_2\\ \mathrm{b}^{\mathfrak{r}}\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j\mathfrak{r}} & \end{array}\right) \quad (3.3)$$

where

$\mathbb{A}'_2=(1-\tau_1-\zeta R;v',\cdots,v^{(r)},\zeta\mathfrak{r};1),(-\lambda-KS-\gamma\zeta R-\tau_1;\gamma v',\cdots,\gamma v^{(r)},\gamma\zeta\mathfrak{r};1),$

$(-\mu-KT-\delta\zeta R-\tau;\delta v',\cdots,\delta v_j^{(r)},\delta\zeta\mathfrak{r};1),\ (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},0;A_j)_{1,p}$

$:\ (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r};(1-v+\delta\eta,1;1)$

$\mathbb{B}'_2=(-\lambda-\mu-K(S+T)-\zeta(\gamma+\delta)R-\tau_1-1;(\gamma+\delta)v',\cdots,(\gamma+\delta)v^{(r)},(\gamma+\delta)\zeta\mathfrak{r};1),$

$(1-\zeta R;v',\cdots,v^{(r)},\zeta\mathfrak{r};1),(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)},0;B_j)_{1,q}$: $(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r};(0,1;1)$ (3.4)

with the same notations and corresponding validity conditions that (2.4).

Putting $t_j=b_j(b_j-a_j)v_j;j=1,\cdots,s$ in (2.4), we obtain the following result.

Corollary 3

$$\int_0^1\cdots\int_0^1\prod_{j=1}^{s}\frac{(1-v_j)^{\lambda_j}v_j^{\mu_j}}{X_j'^{\lambda_j+\mu_j+2}}\,S_U^V\left[a\prod_{j=1}^{s}\frac{(1-v_j)^{S_j}v_j^{T_j}}{X_j'^{S_j+T_j}}\right]S_n^{\alpha,\beta,\tau}\left[b\prod_{j=1}^{s}Y^{\zeta_j};\mathfrak{r},t,q,A,B,k;l\right]$$

$$I\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^s Y_j^{v_j'} & \mathbb{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\prod_{j=1}^s Y_j^{v_j^{(r)}} & \mathbb{B} \end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_s = \left\{\prod_{j=1}^s\{(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,n}$$

$$\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}C(e,p,u,v)\left\{\prod_{j=1}^s\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^R$$

$$I_{p+3s,q+3s:p_1,q_1;\cdots;p_r,q_r;1,1}^{0,n+3s:m_1,n_1;\cdots;m_r,n_r;1,1}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} & \mathbb{A}' \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(r)}} & \mathbb{B}' \\ \mathrm{b}^{\mathfrak{r}}\prod_{j=1}^s\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j\mathfrak{r}} & \end{array}\right) \tag{3.5}$$

where

$$X_j' = v_j(\rho_j-\sigma_j)+\rho_j+1 \tag{3.6}$$

and

$$Y_j = \frac{((1-v_j)^{\lambda_j}v_j^{\delta_j}(X_j')^{1-\gamma_j-\delta_j}}{(\alpha_j+\beta_j\rho_j)(1-v_j)+(1+\sigma_j)\beta_jv_j} \tag{3.7}$$

for $j = 1,\cdots,s$

with the same notations and corresponding validity conditions that (2.4).

If $r = 2$, the multivariable I-function reduces to I-function of two variables defined by Kumari et al [3]. We obtain

Corollary 4

$$\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\prod_{j=1}^s\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}S_U^V\left[a\prod_{j=1}^s\frac{(t_j-a_j)^{S_j}(b_j-t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$S_n^{\alpha,\beta,\tau}\left[b\prod_{j=1}^s Y^{\zeta_j};\mathfrak{r},t,q,A,B,k;l\right] I\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^s Y_j^{v_j'} & \mathbb{A}'' \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_2\prod_{j=1}^s Y_j^{v_j^{(2)}} & \mathbb{B}'' \end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_s$$

$$= \left\{\prod_{j=1}^s\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,n}\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$C(e,p,u,v)\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^R$$

$$I_{p+3s,q+3s:p_1,q_1;p_2,q_2;1,1}^{0,n+3s:m_1,n_1;m_2,n_2;1,1}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} & \mathbb{A}''' \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_2\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(2)}} & \mathbb{B}''' \\ \mathrm{b}^{\mathfrak{r}}\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j\mathfrak{r}} & \end{array}\right) \quad (3.8)$$

where $\mathbb{A}'', B'', A''', \mathbb{B}'''$ are equal to $\mathbb{A}, B, A', \mathbb{B}'$ respectively for $r=2$ and we have the same conditions that (2.4) with $r=2$.

Corolary 5

If $r=1$, the multivariable I-function reduces to I-function of one variable defined by Rathie [6]. We obtain

$$\int_{a_1}^{b_1}\cdots\int_{a_s}^{b_s}\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}S_U^V\left[a\prod_{j=1}^{s}\frac{(t_j-a_j)^{S_j}(b_j-t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$S_n^{\alpha,\beta,\tau}\left[b\prod_{j=1}^{s}Y^{\zeta_j};\mathfrak{r},t,q,A,B,k;l\right]I_{p,q}^{0,n}\left(\mathrm{z}_1\prod_{j=1}^{s}Y_j^{v_j'}\ \middle|\ \begin{array}{c}(\mathrm{c}_1,\gamma_1;C_1),\cdots,(a_p,\gamma_p,;C_p)\\(\mathrm{d}_1,\delta_1;D_1),\cdots,(d_q,\delta_q,;D_q)\end{array}\right)$$

$$=\left\{\prod_{j=1}^{s}\left\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,n}\sum_{\tau_1,\cdots,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$C(e,p,u,v)\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\zeta_jR}}{\tau_j!\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^R$$

$$I_{p+3s,q+3s:1,1}^{0,n+3s:1,1}\left(\begin{array}{c|c} \mathrm{z}_1\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} & \mathbb{A}'_3 \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{b}^{\mathfrak{r}}\prod_{j=1}^{s}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_j\mathfrak{r}} & \mathbb{B}'_3 \end{array}\right) \quad (3.9)$$

where

$$\mathbb{A}'_3=(1-\tau_j-\zeta_jR;v_j',\zeta_j\mathfrak{r};1)_{1,s},(-\lambda_j-KS_j-\gamma_j\zeta_jR-\tau_j;\gamma_jv_j',\gamma_j\zeta_j\mathfrak{r};1)_{1,s},$$

$$(-\mu_j - KT_j - \delta_j\zeta_j R - \tau_j; \delta_j v'_j, \delta_j\zeta_j \mathfrak{r}; 1)_{1,s} : \; (\mathrm{c}_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r}; (1 - \upsilon + \delta\eta, 1; 1)$$

$$\mathbb{B}'_3 = (-\lambda_j - \mu_j - K(S_j + T_j) - \zeta_j(\gamma_j + \delta_j)R - \tau_j - 1; (\gamma_j + \delta_j)v'_j, (\gamma_j + \delta_j)\zeta_j\mathfrak{r}; 1)_{1,s}$$

$$(1 - \zeta_j R; v'_j, \zeta_j \mathfrak{r}; 1)_{1,s}: \; (\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r}; (0, 1; 1) \quad (3.10)$$

we have the same conditions that (2.4) with $r = 1$.

5. Conclusion

Our main integral formula is unified in nature and possesses manifold generality. It acts a key formula and using various special cases of the multivariable I-function, general class of polynomials and a general sequence of functions, one can obtain a large number of other integrals involving simpler special functions and polynomials of one and several variables.

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