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On entropy generation in thermoelectric devices

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Abstract

In this paper, a comparison between the Entropy Generation Minimization method and the Power Maximization technique is presented. The assessment is performed by analyzing, as a typical example of direct conversion heat engines, the thermoelectric generator. The effects of heat-leak and finite-rate heat transfer on the performance of the generator, which is modeled as a Carnot-like engine with internal irreversibilities, are studied. Even though both methods lead to the same conclusions, the entropy generation minimization method, when applied for such an inherently irreversible device, is shown to be less straight forward than the power maximization technique requiring careful accounting of the different sources of irreversibility. Moreover, the entropy generation rate vs. efficiency behavior of the generator reveals that the efficiency at minimum entropy generation rate and the maximum efficiency are distinct.

Keywords: Thermoelectric generator; Power maximization; Entropy generation minimization; Finite-time thermodynamics

1. Introduction

The basic tools for studying heat engine performance are based on aspects of the first and second laws of thermodynamics. In fact, in terms of the simplest possible considerations, complicated thermal systems can easily be assessed. While detailed predictions of the problem

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Nomenclature

I	Electric current (A)
K, K_C, K_H	Heat transfer coefficients (W/K)
m	Load ratio
$\dot{Q}_1$	Heat entering engine (W)
$\dot{Q}_2$	Heat leaving engine (W)
$\dot{Q}_C$	Heat entering heat sink (W)
$\dot{Q}_H$	Heat leaving heat source (W)
$\dot{Q}_e$	Excess heat bypassing generator (W)
R	Electrical resistance (Ω)
$\dot{S}_{gen}$	Entropy generation rate (W/K)
T_1	Temperature of hot junction (K)
T_2	Temperature of cold junction (K)
T_C	Temperature of heat sink (K)
T_H	Temperature of heat source (K)
$\dot{W}$	Power output (W)
Z	Figure of merit (K^{-1})
α	Seebeck coefficient (V/K)
η	Efficiency of heat engine
η_C	Carnot efficiency
η_{max}	Maximum efficiency
η_{mp}	Efficiency at maximum power

considered are not possible, a rather simplified view of the system fulfilment can be obtained which may be of value to a designer trying to choose among several suggested arrangements before going into detailed analysis.

The performance of heat engines has been the subject of many studies using what appear to be different approaches, while in fact, all these methods are similar and can be classified to fall under the Power Maximization (PM) technique. Bejan [1] recently reviewed the available procedures and suggested a new approach called "entropy generation minimization" (EGM). The method, a combination of thermodynamics, heat transfer and fluid flow essentially unifies what is known in engineering as "thermodynamic optimization" and in physics as "finite-time thermodynamics". Moreover, in the widely used PM method, the heat engine performance is optimized by maximizing power generation, while in the EGM method, the same task is accomplished by minimizing entropy generation.

Several workers used the concept of endoreversibility on a heat engine to predict performance. Among others, Curzon and Ahlborn [2] and DeVos [3] applied finite-time thermodynamics to analyze heat engines operating at maximum power. Gordon and Zarmi [4] and DeVos and Flater [5] modeled the earth and its envelope as a Carnot-like engine with heat input being solar radiation and work output the wind energy generated. Nuwayhid and Moukalled [6] looked further into the addition of a heat-leak term and studied the effect of a planet thermal conductance on the conversion efficiency of solar energy into wind energy. Moukalled et al. [7] developed a generalized model in which a heat-leak term is added to account for friction as well as external thermal losses from the engine to the surroundings in addition to heat exchange between the engine, heat reservoirs and the surroundings via combined modes of heat transfer.

The thermoelectric generator represents a typical example of an inherently irreversible, direct conversion heat engine and entails interesting electrical as well as thermal phenomena. Optimization studies have usually concentrated on power assessment and rarely on entropy tracking criteria. An early paper on the second law evaluation of the thermoelectric generator was that of Lampinen [8]. Lampinen's treatment was of value since it included the Thomson heat, but it did not address finite-rate heat transfer, nor did it account for the power production dependence on the load. Gordon [9], using the thermoelectric generator as an example, clearly demonstrated that real heat engines with sources of irreversibility that include friction and heat-leak exhibit fundamentally different power vs. efficiency curves than those predicted neglecting friction and heat-leak. Nuwayhid and Moukalled [10] extended Gordon's view of the thermoelectric generator and recently reported on the performance assuming different thermodynamic models and conditions. The present work attempts to look into the entropy generation rate suffered by thermoelectric power devices and to compare the PM method vs. the EGM method. The ultimate goal of this work is, however, to further promote utilization of the thermoelectric effect.

In the remainder of this paper, the first and second laws of thermodynamics are first reviewed. Then, the traditional treatment of a thermoelectric generator, as an internally irreversible heat engine, using the power maximization technique is elaborated. This is followed by a detailed study of the generator using the EGM method. The EGM analysis is performed on three different models of the thermoelectric generator. In the first, the generator is taken to be internally irreversible and externally reversible while in the second, the opposite is assumed. The third model is the most general one in which the generator is considered to be internally as well as externally irreversible. For all three models, a comparison between the EGM and PM analyses is performed.

2. The laws of thermodynamics

In general, the first and second laws for any selected "system" may be written as follows:

$$\frac{\partial E}{\partial t} = \sum_{\text{In}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{Out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) + \dot{Q} - \dot{W} \tag{1}$$

and

$$\dot{S}_{\text{gen}} = \frac{\partial S}{\partial t} - \sum_{\text{In}} \dot{m}s + \sum_{\text{Out}} \dot{m}s - \frac{\dot{Q}}{T_0} \tag{2}$$

where the terms are those found in any standard thermodynamics textbook. At steady state (time derivatives are zero) and for a closed system (in/out flow terms zero), the laws become (assuming there may be more than one thermal exchange between the system and surroundings — "N" such contacts):

$$\dot{W} = \sum_{i=1}^{N} \dot{Q}_i \tag{3}$$

and

$$\dot{S}_{\text{gen}} = -\sum_{i=1}^{N} \left(\frac{\dot{Q}}{T} \right)_i \tag{4}$$

The above two equations are applicable to any "system" residing within a temperature difference. Bejan [1] very appropriately referred to a heat engine operating in a finite temperature difference field as simply being an "insulation". This wonderfully simple concept allows very useful observations to be made. In fact, Bejan [1], through the work of Chambadal [11] and Curzon and Ahlborn [2], demonstrates clearly that the maximum power and minimum entropy generation rate points are equivalent. The study of the entropy buildup suffered by a heat engine provides a good demonstration of the useful power one can expect from such devices. This is exploited in this work in an attempt to optimize the performance of that delightfully simple under-used power generating device: the thermoelectric generator.

3. The PM method applied to the thermoelectric generator

The thermoelectric effect was first reported by Seebeck in 1821 when he observed that upon the joining of two dissimilar metal wires and the heating of the junction, an electromotive force developed. Peltier in 1834 found that the crossing of an electric current across a junction of two dissimilar conductors produced a heating or cooling effect depending on the current direction. In 1856, Thomson (Lord Kelvin) showed that there exists a unique relation between the Seebeck and Peltier effects and while so doing he actually discovered a totally new effect — the Thomson effect. In this work the Seebeck and Peltier effects will be utilized in the formulation, while the Thomson effect will be neglected based on the assumption that the properties are constant at the average temperature.

The thermoelectric generator, having a low thermal to electric conversion efficiency has not managed to make its place in the commercial market. Only in temperature measurement has there been a significant application of the thermoelectric phenomenon. Advances in the field have been very slow and limited by material considerations. However, with the introduction of semiconductors and the resultant relative rise in efficiency, renewed interest was stirred but today the practical applications still remain few. Current applications are in remote low level power generation using long-lived heat sources such as radioisotopes, and in small portable refrigeration systems running on low power inputs such as car batteries. As a result of all this, the few existing manufacturers of thermoelectric devices are today finding it hard to survive and make a profit.

The basics of thermoelectric power generation can be found in several available texts, for example, Soo [12]. It is of interest to note that in the thermoelectric generator, the heat-leak irreversibility is a necessary part of the engine, since it is a manifestation of the thermal conductance of the generator legs themselves and, therefore, appears in the modeling directly. Moreover, the thermal conductance is inversely proportional to the electrical resistance and, therefore, the two have conflicting effects — the system is, thus, one that could be described as "inherently irreversible". A simplified Schematic of a thermoelectric generator is depicted in Fig. 1.

In most scoping studies of thermoelectric generation, the usual practice has been to neglect external irreversibilities and to consider only internal effects that are related to the selected material. However, a complete thermodynamic study may consider, in addition to the internally irreversible case, both externally irreversible–internally reversible and fully irreversible cases. Since the intention in this section is to give an overview of the analysis using the PM technique, the fully irreversible case is not considered and attention is focused on the standard internally irreversible–externally reversible and internally reversible–externally irreversible cases.

With external irreversibilities neglected, the heat source and heat sink temperatures become identical to the hot junction and cold junction temperatures, respectively. The resulting equations for power and efficiency are usually written as:

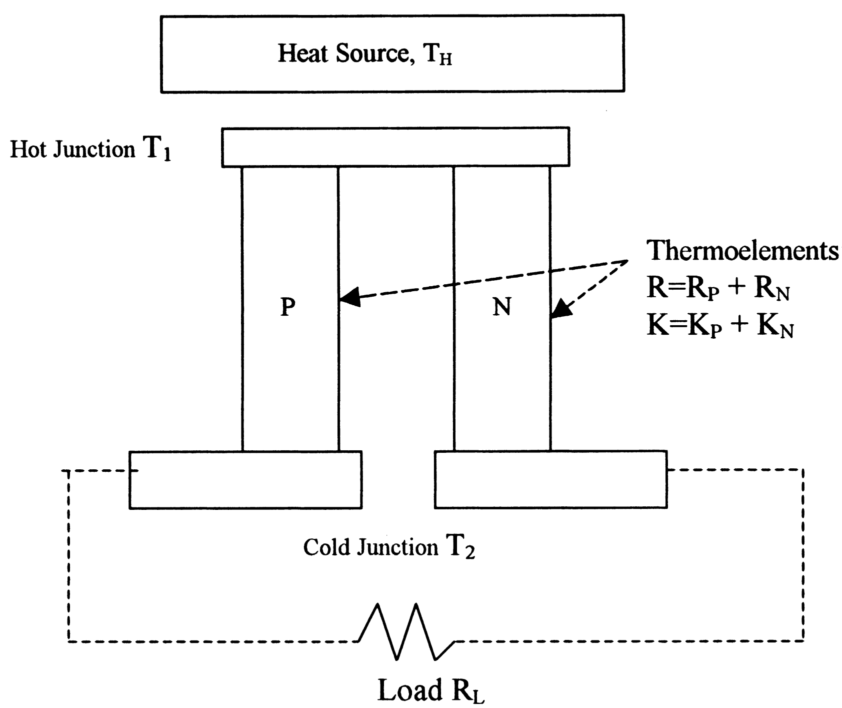

Fig. 1. A schematic of a basic thermoelectric heat engine.

$$\dot{W} = \alpha I(T_H - T_C) - I^2 R, \tag{5}$$

$$\eta = \eta_C \frac{m}{1 + m - \frac{\Delta T}{2T_H} + \frac{(1+m)^2}{ZT_H}} \tag{6}$$

Where $m = R_L/R$, $Z = \alpha^2/RK$, R, K, α and η_C are the load ratio, the figure of merit, the total electrical resistance, the total thermal conductance, the Seebeck coefficient and the Carnot efficiency, respectively.

Eq. (6) is the usual one found in textbooks and the power vs. efficiency behavior is the expected "loop" shape with both efficiency and power rising from zero and returning to zero after passing through distinct maximum power and maximum efficiency points below the Carnot limit. The loss of power is attributed to heat-leak at open circuit (slow operation; no recovered output) and to Joule heat loss at short circuit (fast operation; friction). This observation was first made by Gordon [9].

If the generator is made more and more reversible (say by making R smaller and smaller while holding K artificially constant for simplicity), the graph starts to approach the "dome" shape where the maximum efficiency is the Carnot efficiency and the maximum power and maximum efficiency points become increasingly separated (Fig. 2 shows this behavior). The useful power can be expressed in terms of m as follows:

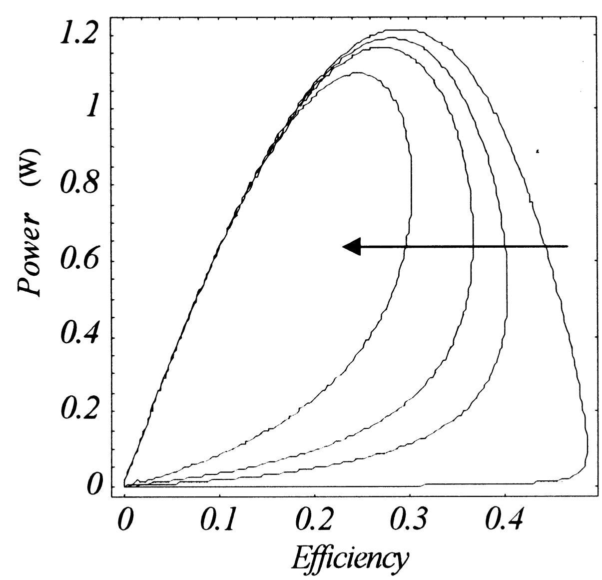

Fig. 2. Typical effect of increasing internal irreversibility (in the direction of the arrow) on the performance of a thermoelectric generator (T_H=600 K, η_C=0.5).

$$\dot{W} = ZK(T_H - T_C)^2 \frac{m}{(1+m)^2} \tag{7}$$

which clearly exhibits a maximum at $m = 1$, as matched load conditions require.

From Eq. (6), with $m = 1$, the efficiency at maximum power is given by:

$$\eta_{mp} = \eta_C \frac{2}{4 - \eta_C + \dfrac{8}{ZT_H}} \tag{8}$$

The previous figure (Fig. 2) shows that as the device becomes more reversible, the maximum power line shifts to the right and approaches a limiting value which is only dependent on the Carnot efficiency. The limit is obtainable by requiring $Z \to \infty$, giving:

$$\lim_{R \to 0} \eta_{mp} = \eta_C \frac{2}{4 - \eta_C} \tag{9}$$

The above equation raises an interesting question about the "maximum" efficiency at maximum powers which appears to be limited to 2/3!

The maximum efficiency is, however, distinct from the efficiency at maximum power and is best written as:

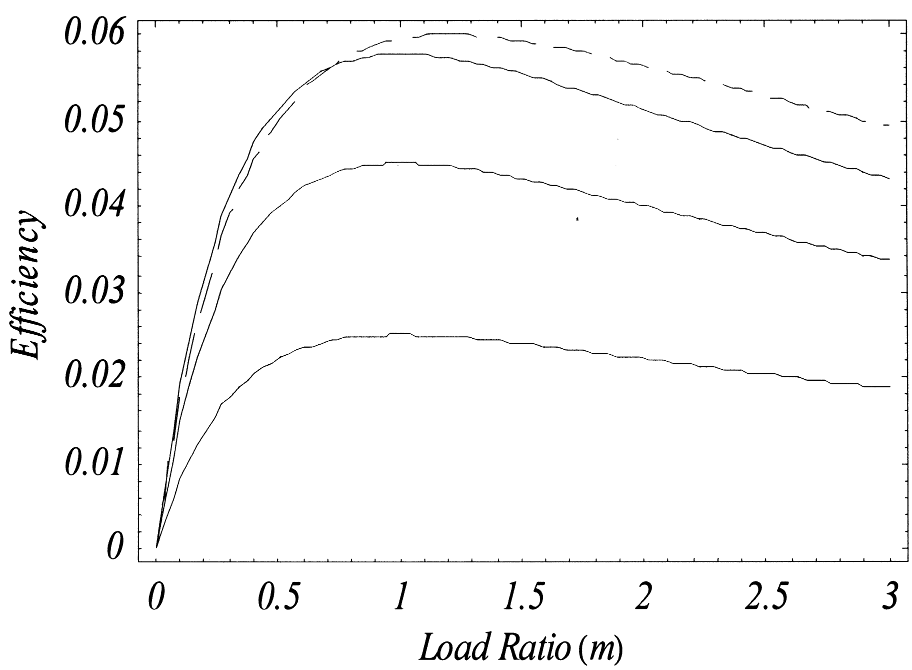

Fig. 3. Renewable efficiency compared to exhaustable efficiency for an internally irreversible generator ($T_H = 600$ K, $\eta_C = 0.5$, $Z = 0.001$ and $K = 0.1$). For the renewable case (solid lines), the inexhaustible heat supply Q decreases from bottom to top as: 90, 50 and 39.

$$\eta_{\max} = \eta_{\rm C}\frac{\sqrt{1+Z\bar{\bar{T}}}-1}{\sqrt{1+Z\bar{\bar{T}}}+(T_{\rm C}/T_{\rm H})} \tag{10}$$

where $\bar{T}$ is the average temperature along the generator. From Eq. (10), it can easily be inferred that the load ratio giving maximum efficiency is not unity but $\sqrt{1+Z\bar{\bar{T}}}$ a value greater than unity. Of course, as $Z \rightarrow \infty$, the Carnot limit is approached. Further, it is obvious that the maximum efficiency is distinct (and larger) than the efficiency at maximum power (see Fig. 2).

It may be of interest to compare the exhaustible efficiency given by Eq. (6) with the so-called "renewable" efficiency which may be given by:

$$\eta_{\rm r} = \frac{mKZT_{\rm H}^2\eta_{\rm C}^2}{(1+m)^2\dot{Q}}, \tag{11}$$

where $\dot{Q}$ is an "inexhaustible" or "renewable" heat source. While a derivative of this with respect to m will show this to maximize at $m = 1$, the exhaustible efficiency maximizes when $m = \sqrt{1+Z\bar{\bar{T}}}$. Fig. 3 shows the exhaustible efficiency contrasted with renewable efficiencies at three different values of $\dot{Q}$. The maximum renewable efficiency which occurs when $\dot{Q} = K(T_{\rm H} - T_{\rm C})(1 + ZT_{\rm C})$ is given by:

$$\eta_{\rm r} = \frac{mZ(T_{\rm H}-T_{\rm C})}{(1+m)^2(1+ZT_{\rm C})}, \tag{12}$$

and is shown to be less than the exhaustible efficiency.

With no external irreversibilities considered, improvement in performance can proceed through material selection. In fact, there has been some work on thermoelectric material selection, through either higher Seebeck coefficients or by optimizing the geometrical configuration of the device, leading to larger Z and, thus, greater efficiencies.

The effect of external irreversibilities, on the other hand, is the same for any internally reversible (i.e. endoreversible) heat engine and has been studied intensely in the literature [1–6]. Although this model is highly unrealistic for the present application, it nevertheless serves as a link among models. For the thermoelectric generator, both R and K are required to simultaneously vanish (in reality they are inversely proportional), and the resulting energy balances become very simple. The power vs. efficiency relation is found to be:

$$\dot{W} = \frac{K_{\rm H}K_{\rm C}\left[(1-\eta)^i T_{\rm H}^i - T_{\rm C}^i\right]}{(1-\eta)\left[K_{\rm H} + (1-\eta)^{(i-1)}K_{\rm C}\right]} \tag{13}$$

where i is the heat input/output power mode (1 for conduction/convection or 4 for radiation). The normalized power ($\dot{W}^* = \dot{W}/KT_{\rm H}^i$) vs. efficiency curves, with the Carnot efficiency and the heat transfer mode as parameters, have the expected "domed" shape with the maximum efficiency terminating at the Carnot limit (by differentiating the above with respect to K and setting to zero one indeed obtains the Carnot limit). The efficiency at maximum power in this "endoreversible" case, obtainable this time by differentiating the above with respect to η and setting to zero, turns out to be the so-called CNCA [3] efficiency given by:

$$\eta_{mp} = 1 - \sqrt{\frac{T_C}{T_H}} \tag{14}$$

The limit of this as $T_H \to \infty$ is unity, compared to the upper limit of 2/3 for the internally irreversible case. Moreover, it turns out that nearly below $\eta_C = 0.75$, the endoreversible (with reversibilities occurring externally) and internally irreversible (with no external irreversibilities) cases produce very similar efficiencies at maximum power as long as Z is greater than about 0.1.

As a general guideline consider a thermoelectric generator with the following parameters: $T_H = 600$ K, $T_C = 300$ K, $\alpha = 10^{-4}$ V/K, $R = 10^{-5}\ \Omega$ and $K = 0.1$ W/K. In the fully reversible limit, the Carnot limit, the efficiency is obviously 50%. The externally reversible assumption gives 20% to 24% efficiency as calculated by Eqs. (8) or (10) respectively. The endoreversible case (CNCA efficiency) gives 30% for the efficiency at maximum power. All these figures are very optimistic and real generators seldom exceed 10% efficiency.

4. The EGM method applied to the thermoelectric generator

An overview of the performance of the thermoelectric generator using the PM technique was given in the previous section. In this section, the generator will be investigated using the EGM method. Entropy generation in a thermoelectric device occurs with heat transfer from the heat source, heat transfer to the surroundings, heat-leak from the hot side to the cold side and dissipative Joule heat. The irreversibilities may occur internally or externally. The analysis will proceed from simplified models to a general one. In specific, three models will be studied: (1) the internally irreversible and externally reversible model, (2) the internally reversible and externally irreversible model and (3) the internally and externally irreversible model. The prime motive behind this work is the apparent lack of studies on entropy generation in thermoelectric devices.

4.1. Entropy generation in the internally irreversible–externally reversible model

The generator is modeled as externally reversible but internally irreversible. The heat transfer to the generator ($\dot{Q}_H$) and from the generator ($\dot{Q}_C$) are:

$$\dot{Q}_H = \alpha I T_H + K(T_H - T_C) - I^2 R/2 \tag{15}$$

and

$$\dot{Q}_C = \alpha I T_C + K(T_H - T_C) + I^2 R/2 \tag{16}$$

where the Joule heat has been split evenly between the two sides of the generator. This usually implies that the Joule heating effect is larger in magnitude than the heat conduction effect along the generator.

The entropy generation rate is, as usual, $\dot{S}_{gen} = -\sum_{i=1}^{n} (\frac{\dot{Q}}{T})_i$, where the i refers to the exchange location (node). So that, for this apparently two node case:

$$\dot{S}_{\text{gen}} = -\frac{\dot{Q}_{\text{H}}}{T_{\text{H}}} + \frac{\dot{Q}_{\text{C}}}{T_{\text{C}}} \tag{17}$$

Applying the above equations gives:

$$\dot{S}_{\text{gen}} = \frac{T_{\text{H}} - T_{\text{C}}}{T_{\text{H}} T_{\text{C}}} \left[K(T_{\text{H}} - T_{\text{C}}) + \frac{1}{2} I^2 R \frac{T_{\text{H}} + T_{\text{C}}}{T_{\text{H}} - T_{\text{C}}} \right] \tag{18}$$

This result is in conformity with Bejan [1] in reference to entropy generation in a general temperature field with no thermoelectric effect and, thus, no electric current flow. The irreversibility in the thermoelectric generator is clearly due to the conduction of heat and to the dissipation of Joule heat (the Seebeck/Peltier effects do not give rise to entropy creation since they are reversible effects). Moreover, the conduction heat given by $\dot{Q}_{\text{K}} = K(T_{\text{H}} - T_{\text{C}})$ generates entropy in the amount $\dot{Q}_{\text{K}}(\frac{T_{\text{H}} - T_{\text{C}}}{T_{\text{H}} T_{\text{C}}})$, while the Joule heat ($\dot{Q}_{\text{J}} = I^2 R$) generates entropy in the amount:$\dot{Q}_{\text{J}}(\frac{T_1 + T_2}{2 T_{\text{H}} T_{\text{C}}})$.

The equation for the total entropy generation rate can best be written in dimensionless form as:

$$\frac{\dot{S}_{\text{gen}}}{K} = \left(\tau^{1/2} + \tau^{-1/2}\right)^2 \left[1 + \frac{Z(T_{\text{H}} + T_{\text{C}})}{2(1 + m)^2} \right] \tag{19}$$

where τ is the temperature ratio ($T_{\text{C}}/T_{\text{H}}$) and Z and m are as defined earlier. It is seen that as Z gets larger, more entropy is generated due to the added effect of the Joulean heating resulting from the induced current. Additionally, as m gets larger, R gets smaller and the generator is less irreversible thereby generating less entropy. There is a problem, however, with what has been done so far! Strictly speaking, what is being assessed is the case where the irreversibilities are only internal, and so, any external sources of entropy generation are not considered. However, to make the study more realistic and consistent with the traditional treatment of thermoelectric performance, a little more thought is required.

Looking back at the previous equation for entropy generation rate, it is immediately apparent that it does not exhibit the appropriate minimum entropy at $m = 1$. This is of concern, since the maximum power and minimum entropy points ought to be one and the same [1]. The problem can be traced back to neglect of the entropy generation due to external heat dumping (Bejan's room to move criteria). Including this effect, one considers that a source of heat $\dot{Q}$ is available at T_{H} and a part of this, $\dot{Q}_{\text{H}}$ enters the generator at T_{H}, while another part $\dot{Q}_{\text{e}}$ is dumped externally towards the sink temperature T_{C}. As such, one may write the entropy generation rate as the sum of those of: (1) internal heat-leak, (2) internal Joule heating, and (3) external heat dumping. The following is, thus, obtained:

$$\dot{S}_{\text{gen}} = K \frac{(T_{\text{H}} - T_{\text{C}})^2}{T_{\text{H}} T_{\text{C}}} + \frac{1}{2} I^2 R \frac{T_{\text{H}} + T_{\text{C}}}{T_{\text{H}} T_{\text{C}}} + \dot{Q}_{\text{e}} \frac{(T_{\text{H}} - T_{\text{C}})}{T_{\text{H}} T_{\text{C}}} \tag{20}$$

The above equation can alternatively be written as:

$$\dot{S}_{gen} = -\frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_C}{T_C} + \dot{Q}_e\left(-\frac{1}{T_H} + \frac{1}{T_C}\right) \quad (21)$$

After writing $\dot{Q}_e = \dot{Q} - \dot{Q}_H$, replacing the current using $\alpha I = KZ(T_H - T_C)/(1+m)$ and performing some algebraic manipulations, either of the previous two equations can be presented in the following form:

$$\dot{S}_{gen} = \dot{Q}\frac{(T_H - T_C)}{T_H T_C} - \frac{mKZ(T_H - T_C)^2}{(1+m)^2 T_C} \quad (22)$$

The above equation can be differentiated with respect to m to show clearly that a minimum occurs at $m = 1$. Thus, maximum power and minimum entropy are one and the same point. For further clarity, the equation is shown plotted in Fig. 4 in the following dimensionless form:

$$\dot{S}^*_{gen} = 1 - \frac{mZT_H}{(1+m)^2}\frac{K(T_H - T_C)}{\dot{Q}} \quad (23)$$

where the last term may be attributed to the fraction of the heat source that goes into the heat-leak. The figure shows clearly that as the heat-leak increases, the entropy generation rate increases and the useful power diminishes. Additionally, Fig. 5 is presented to show how the increase in the thermoelectric figure of merit improves performance (i.e. reduces entropy generation rate).

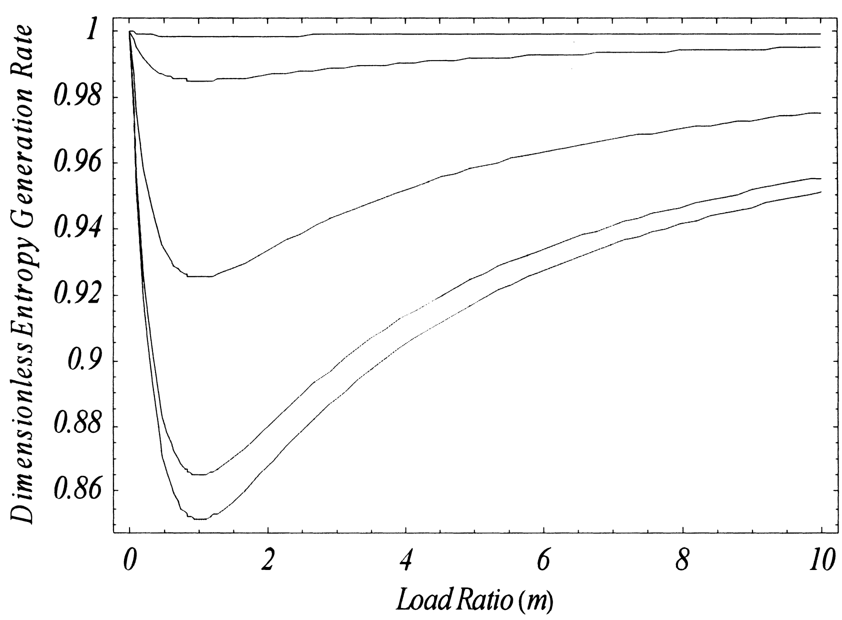

Fig. 4. Dimensionless entropy generation rate vs. load ratio for several values of heat leak in the internally irreversible model ($T_H = 600$ K, $T_C = 300$ K and $Z = 0.001$). The heat leak increment ($K(T_H - T_C)/Q$) increases from top to bottom as: 0.01, 0.1, 0.5, 0.9 and 0.99.

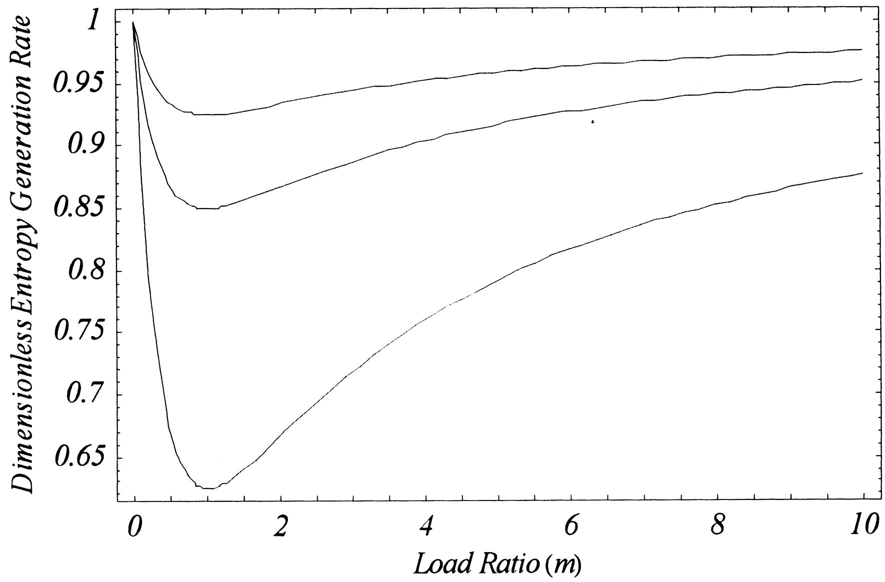

Fig. 5. Dimensionless entropy generation rate vs. load ratio in the internally irreversible model for three values of thermoelectric figure-of-merit Z increasing from top to bottom as: 0.001, 0.002 and 0.005 ($T_H = 600$ K, $T_C = 300$ K and $K(T_H - T_C/Q = 0.5)$.

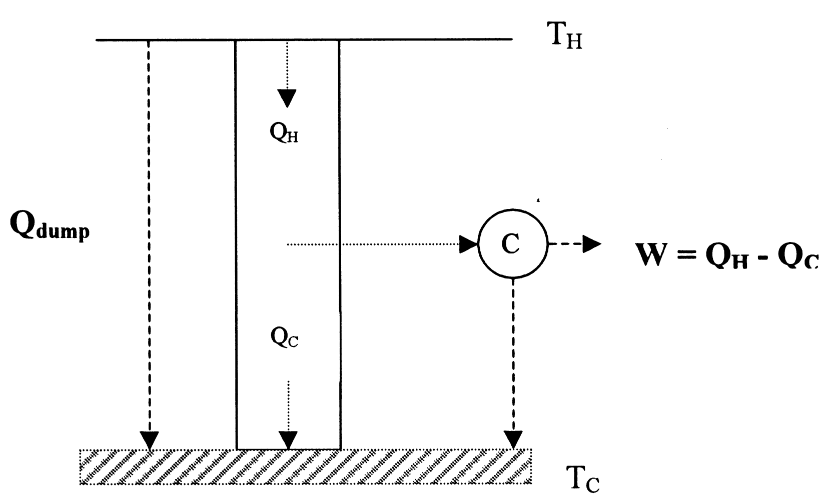

Fig. 6. Equivalent model of the internally irreversible/externally reversible thermoelectric generator.

Combining Eqs. (7) and (22), the following linear relation is found:

$$\dot{W} = \dot{Q}\frac{(T_{\mathrm{H}} - T_{\mathrm{C}})}{T_{\mathrm{H}}} - T_{\mathrm{C}}\dot{S}_{\mathrm{gen}} \tag{24}$$

The equation shows clearly that of the maximum available power (first term on the right), an amount equal to the second term on the right is lost.

Eq. (24) can be recast as:

$$\dot{S}_{\mathrm{gen}} = \dot{Q}\left(\frac{T_{\mathrm{H}} - T_{\mathrm{C}}}{T_{\mathrm{C}}T_{\mathrm{H}}}\right) - \frac{\dot{Q}_{\mathrm{H}} - \dot{Q}_{\mathrm{C}}}{T_{\mathrm{C}}}, \tag{25}$$

which states that in this model, the entropy generation rate, as schematically shown in Fig. 6, is equivalent to an inexhaustible heat source ($\dot{Q}$) proceeding undiminished through a temperature difference ($T_{\mathrm{H}} - T_{\mathrm{C}}$), in parallel with a Carnot engine producing a power ($\dot{Q}_{\mathrm{H}} - \dot{Q}_{\mathrm{C}}$). The result is consistent with the Guoy–Stodola theorem on destruction of availability. Power is entropy free and the diagram shows the produced power dissipated into T_{C}.

Next, a consideration of the case of the internally reversible/externally irreversible thermoelectric generator may afford further insight.

4.2. Entropy generation in the internally reversible–externally irreversible model

Here, irreversibility is assumed to occur externally and one has the so-called endoreversible case. This application makes little real sense, since thermoelectric devices are internally irreversible by nature. It does, however, provide a link with both the fully irreversible case and the general thermal power generation plant models [1].

The heat flow to the hot junction from the heat source and from the cold junction to the heat sink are:

$$\dot{Q}_{\mathrm{H}} = K_{\mathrm{H}}(T_{\mathrm{H}} - T_1) \quad \text{and} \quad \dot{Q}_{\mathrm{C}} = K_{\mathrm{C}}(T_2 - T_{\mathrm{C}}) \tag{26}$$

Additionally, there is assumed to be no internal irreversibilities ($R = 0, K = 0$) and the heat flows into the generator and out of it are conserved and given by:

$$\dot{Q}_{\mathrm{H}} = \alpha I T_1 \quad \text{and} \quad \dot{Q}_{\mathrm{C}} = \alpha I T_2 \tag{27}$$

The above equations can be solved simultaneously to yield:

$$T_1 = \frac{K_{\mathrm{H}}T_{\mathrm{H}}}{\alpha I + K_{\mathrm{H}}} \tag{28}$$

$$T_2 = \frac{K_{\mathrm{C}}T_{\mathrm{C}}}{K_{\mathrm{C}} - \alpha I} \tag{29}$$

By eliminating αI between the two equations, the following relation between T_1 and T_2 is obtained:

$$T_2 = \frac{K_C T_C}{K_H + K_C - K_H \dfrac{T_H}{T_1}} \tag{30}$$

The total entropy generation rate is equal to the sum of the rates of entropy generated by finite-rate heat transfer at the junctions and that generated by the excess heat dumping ($\dot{Q}_e$) due to the inexhaustible heat source $\dot{Q}_e(\dot{Q}=\dot{Q}_e+\dot{Q}_H)$ and is given by:

$$\dot{S}_{gen} = \dot{Q}_H\left(\frac{T_H - T_1}{T_1 T_H}\right) + \dot{Q}_C\left(\frac{(T_2 - T_C)}{T_2 T_C}\right) + \dot{Q}_e\left(\frac{T_H - T_C}{T_H T_C}\right) \tag{31}$$

or

$$\dot{S}_{gen} = \dot{Q}_H\left(\frac{T_C - T_1}{T_1 T_C}\right) + \dot{Q}_C\left(\frac{(T_2 - T_C)}{T_2 T_C}\right) + \dot{Q}\left(\frac{T_H - T_C}{T_H T_C}\right) \tag{32}$$

This can also be written as:

$$\dot{S}_{gen} = K_H\left(\frac{(T_C - T_1)(T_H - T_1)}{T_1 T_C}\right) + K_C\left(\frac{(T_2 - T_C)^2}{T_2 T_C}\right) + \dot{Q}\left(\frac{T_H - T_C}{T_H T_C}\right) \tag{33}$$

Replacing T_2 by its equivalent expression given by Eq. (30), the entropy generation rate becomes:

$$\dot{S}_{gen} = \dot{Q}\left(\frac{T_H - T_C}{T_H T_C}\right) + \frac{K_H(T_H - T_1)\big(K_C(T_C - T_1) + K_H(T_H - T_1)\big)}{T_C(K_H T_1 + K_C T_1 - K_H T_H)} \tag{34}$$

Taking $\partial S_{gen}/\partial T_1$ and setting to zero, the optimum hot side temperature that minimizes the entropy generation rate is found to be:

$$T_{1,\,opt} = \frac{K_C\sqrt{T_H T_C} + K_H T_H}{K_C + K_H} \tag{35}$$

Substitution into Eqs. (30) and (33) results, respectively, in:

$$T_{2,\,opt} = \frac{K_H\sqrt{T_H T_C} + K_C T_C}{K_C + K_H} \tag{36}$$

and

$$\dot{S}_{gen,\,min} = \dot{Q}\left(\frac{T_H - T_C}{T_H T_C}\right) + \frac{K_H}{K_H + K_C}\frac{\left(\sqrt{T_H} - \sqrt{T_C}\right)^2}{T_C} \tag{37}$$

Based on the obtained values for T_1 and T_2 that minimize the rate of entropy creation, the output of the heat engine is given by:

$$\dot{W}_{max} = \dot{Q}_H - \dot{Q}_C = \frac{K_H K_C}{K_H + K_C}\left(\sqrt{T_H} - \sqrt{T_C}\right)^2 \tag{38}$$

and the efficiency is

$$\eta_{\mathrm{opt}} = 1 - \frac{K_{\mathrm{H}}\sqrt{T_{\mathrm{H}}T_{\mathrm{C}}} + K_{\mathrm{C}}T_{\mathrm{C}}}{K_{\mathrm{C}}\sqrt{T_{\mathrm{H}}T_{\mathrm{C}}} + K_{\mathrm{H}}T_{\mathrm{H}}} \tag{39}$$

It can be seen that as:

$$K_{\mathrm{C}} \to \infty,\ T_{1,\ \mathrm{opt}} \to \sqrt{T_{\mathrm{C}}T_{\mathrm{H}}},\ T_{2,\ \mathrm{opt}} \to T_{\mathrm{C}} \quad \text{and} \quad S_{\mathrm{gen,\ min}} \to Q\left(\frac{T_{\mathrm{H}} - T_{\mathrm{C}}}{T_{\mathrm{C}}T_{\mathrm{H}}}\right)$$

On the other hand, as:

$$K_{\mathrm{H}} \to \infty,\ T_{1,\ \mathrm{opt}} \to T_{\mathrm{H}},\ T_{2,\ \mathrm{opt}} \to \sqrt{T_{\mathrm{C}}T_{\mathrm{H}}} \quad \text{and} \quad S_{\mathrm{gen,\ min}} \to Q\left(\frac{T_{\mathrm{H}} - T_{\mathrm{C}}}{T_{\mathrm{C}}T_{\mathrm{H}}}\right) + \frac{\left(\sqrt{T_{\mathrm{H}}} - \sqrt{T_{\mathrm{C}}}\right)^2}{T_{\mathrm{C}}}$$

Similar conclusions could be made regarding the optimum efficiency.

Comparing the optimum temperatures and power obtained here via the EGM method with those obtained in DeVos [3] using the PM technique, they are found to be equal, confirming that minimum entropy generation rate and maximum power output conditions are coincident.

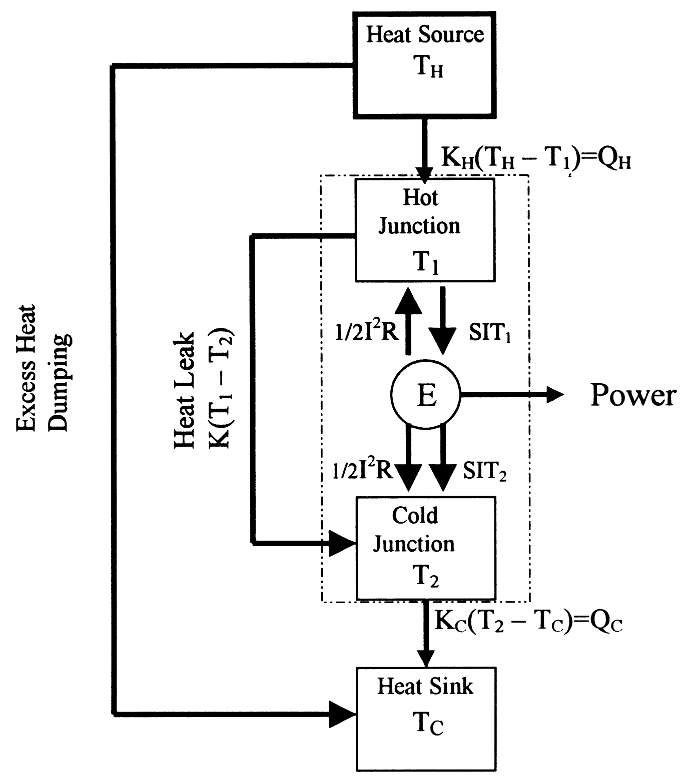

Fig. 7. General schematic diagram for the internally and externally irreversible model of the thermoelectric generator.

4.3. Entropy generation in the internally and externally irreversible model

In the previous two Carnot-like engines used to model the thermoelectric generator, either the internal or the external irreversibility was neglected. In this section, a general model that accounts for both irreversibilities is sought. For that purpose, the Carnot-like engine's model developed by Moukalled et al. [13] is modified to account for internal irreversibilities within the engine and used here to analyze, following the EGM approach, the thermoelectric generator. A Schematic of the modified model is depicted in Fig. 7. As can be seen, in addition to the internal irreversibilities and heat-leak, the model allows for finite-rate heat input and output with heat transfer obeying Newton's law.

Referring to Fig. 7, the heat balance equations at the junctions are given by:

$$K_H(T_H - T_1) = \alpha I T_1 - \frac{I^2 R}{2} + K(T_1 - T_2) \tag{40}$$

$$K_C(T_2 - T_C) = \alpha I T_2 + \frac{I^2 R}{2} + K(T_1 - T_2) \tag{41}$$

where the terms to the left of the equality are $\dot{Q}_H$ and $\dot{Q}_C$, while those to the right are called $\dot{Q}_1$ and $\dot{Q}_2$ and are the internal heat flows (note, of course, that the first law requires $\dot{Q}_H = \dot{Q}_1$ and $\dot{Q}_C = \dot{Q}_2$).

Solving Eqs. (40) and (41) simultaneously, the following equations for T_1 and T_2, as a function of the current I, are obtained:

$$T_1 = \frac{2KK_C T_C + 2KK_H T_H + 2K_C K_H T_H - 2K_H T_H I + (2KR + K_C R)I^2 - R\alpha I^3}{2\left(KK_C + KK_H + K_C K_H + IK_C\alpha - IK_H\alpha - I^2\alpha^2\right)} \tag{42}$$

$$T_2 = \frac{2KK_C T_C + 2KK_H T_H + 2K_C K_H T_C + 2K_C T_C I + (2KR + K_H R)I^2 + R\alpha I^3}{2\left(KK_C + KK_H + K_C K_H + IK_C\alpha - IK_H\alpha - I^2\alpha^2\right)} \tag{43}$$

The entropy generation rate may now be found by noting that this case is a multi-node case. Thus, in addition to the finite-rate temperature difference irreversibilities (first two terms below), there are the internal irreversibilities due to the heat-leak and Joule-heat (the sum of the third and fourth terms) in addition to the irreversibility due to external heat dumping across the imposed temperature field. Thus:

$$\dot{S}_{gen} = \dot{Q}_H\left(\frac{T_H - T_1}{T_1 T_H}\right) + \dot{Q}_C\left(\frac{T_2 - T_C}{T_2 T_C}\right) - \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \dot{Q}_e\left(\frac{T_H - T_C}{T_H T_C}\right) \tag{44}$$

As stated before, the first law requires $\dot{Q}_H$ and $\dot{Q}_C$ to be equal to $\dot{Q}_1$ and $\dot{Q}_2$ respectively. In addition, as in the previous cases, the external dumping is written as $\dot{Q}_e = \dot{Q}_H - \dot{Q}_C$ where $\dot{Q}_H$ is a large heat supply available at T_H. After some manipulation, the resulting entropy generation rate becomes:

$$\dot{S}_{\text{gen}} = \dot{Q}\left(\frac{T_{\text{H}} - T_{\text{C}}}{T_{\text{H}}T_{\text{C}}}\right) - \frac{\dot{Q}_{\text{H}} - \dot{Q}_{\text{C}}}{T_{\text{C}}} \tag{45}$$

As displayed in Fig. 8, the model thus emerges as a parallel connection of an external heat-leak (due to the external heat dumping), a series connection of the two finite-rate heat transfer and a sandwiched Carnot engine producing power. It is interesting to note that Eq. (45), which gives the relation between entropy and available power, has a form precisely similar to the one obtained in the internally irreversible case (Eq. (25)). By replacing the electric current and Seebeck coefficient in terms of the figure of merit and noticing that

$$\dot{W} = \dot{Q}_{\text{H}} - \dot{Q}_{\text{C}} = K_{\text{H}}(T_{\text{H}} - T_1) - K_{\text{C}}(T_2 - T_{\text{C}}) = \alpha I(T_1 - T_2) - I^2 R, \tag{46}$$

the entropy generation rate equation Eq. (45) can be written as:

$$\dot{S}_{\text{gen}} = \dot{Q}\left(\frac{T_{\text{H}} - T_{\text{C}}}{T_{\text{H}}T_{\text{C}}}\right) - \frac{mKZ}{(1+m)^2}\frac{(T_1 - T_2)^2}{T_{\text{C}}} \tag{47}$$

In dimensionless form, the above equation is modified to:

$$\dot{S}^*_{\text{gen}} = 1 - \frac{mZT_{\text{H}}}{(1+m)^2}\frac{(T_1 - T_2)}{(T_{\text{H}} - T_{\text{C}})}\frac{K(T_1 - T_2)}{\dot{Q}} \tag{48}$$

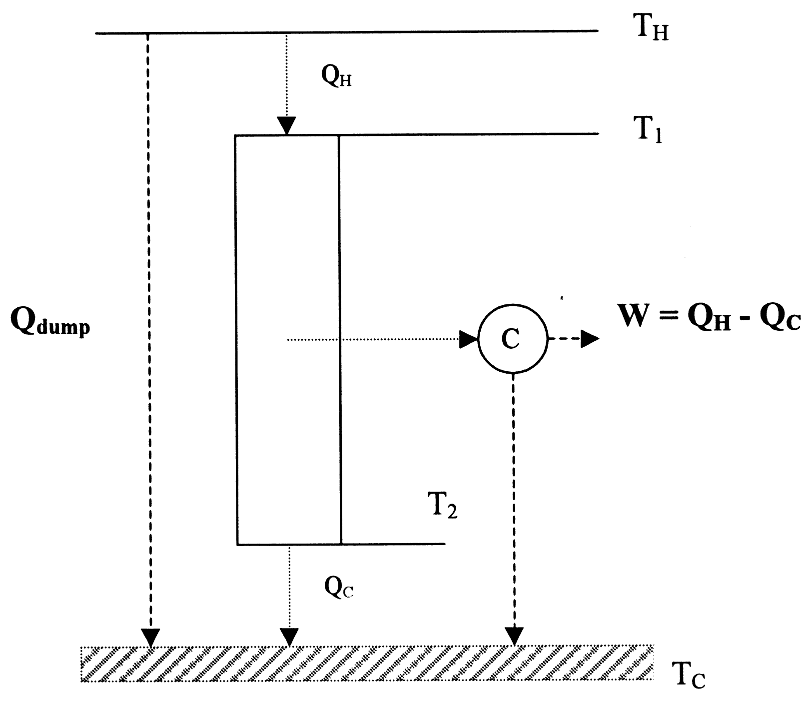

Fig. 8. Equivalent model of the internally and externally irreversible thermoelectric generator.

where the last quotient of the second term can be viewed as the fraction of the heat supply that goes into the heat-leak. If a derivative with respect to m is now taken, one obtains $m_{opt} = 1$ as expected.

If, on the other hand, one writes:

$$\dot{S}_{gen} = \dot{Q}\left(\frac{T_H - T_C}{T_H T_C}\right) - \frac{K_H(T_H - T_1) - K_C(T_2 - T_C)}{T_C} \tag{49}$$

and if the values for T_1 and T_2, as given by Eqs. (42) and (43), are substituted into Eq. (48), the following equation for the entropy generation rate is obtained:

$$\dot{S}_{gen} = \dot{Q}\left(\frac{T_H - T_C}{T_H T_C}\right) - -\frac{(K_C - K_H)R\alpha I^3 + 2K_C K_H(T_C - T_H)\alpha}{2T_C\big(K(K_C + K_H) + (K_C - \alpha I)(K_H + \alpha I)\big)} - \frac{2\big(K_C T_C \alpha^2 + K_H T_H \alpha^2 + K_C K_H R + K(K_C + K_H)R\big)I^2}{2T_C\big(K(K_C + K_H) + (K_C - \alpha I)(K_H + \alpha I)\big)} \tag{50}$$

Now, if the derivative with respect to current I of the entropy generation rate is set to zero, the current minimizing the entropy generation rate emerges. The resultant relation for this current is very long and it would be of little value to report it here. Rather, if the reasonable assumption $K_C = K_H$ is used, then the analytical expression for the optimum current becomes manageable to present and is found to be:

$$I_{opt} = \frac{\sqrt{(2K + K_H)\Big\{(2K + K_H)\big[(T_C + T_H)\alpha^2 + 2KR + K_H R\big]^2 - K_H(T_C - T_H)^2\alpha^4\Big\}}}{(T_C - T_H)\alpha^3} - \frac{(2K + K_H)\big[(T_C + T_H)\alpha^2 + 2KR + K_H R\big]}{(T_C - T_H)\alpha^3} \tag{51}$$

Using I_{opt}, the following expressions for T_1 and entropy generation rate are obtained:

$$T_{1,\,opt} = \frac{\big\{(T_C - T_H)\alpha^2 + 4KR + 2K_H R\big\}\sqrt{(2K + K_H)\Big\{(2K + K_H)\big[(T_C + T_H)\alpha^2 + 2KR + K_H R\big]^2 - K_H(T_C - T_H)^2\alpha^4\Big\}}}{4(2K + K_H)(T_C - T_H)\alpha^4}$$

$$- \frac{16R^2K^3 + 4RK^2\big\{(3T_C + T_H)\alpha^2 + 6K_H R\big\} + 2K\big\{\big(T_H^2 - T_C^2\big)\alpha^4 + 2K_H R(3T_C + T_H)\alpha^3 + 6K_H^2 R^2\big\}}{4(2K + K_H)(T_C - T_H)\alpha^4}$$

$$- \frac{K_H\big\{2T_H(T_H - T_C)\alpha^4 + K_H R(3T_C + T_H)\alpha^2 + 2K_H^2 R^2\big\}}{4(2K + K_H)(T_C - T_H)\alpha^4} \tag{52}$$

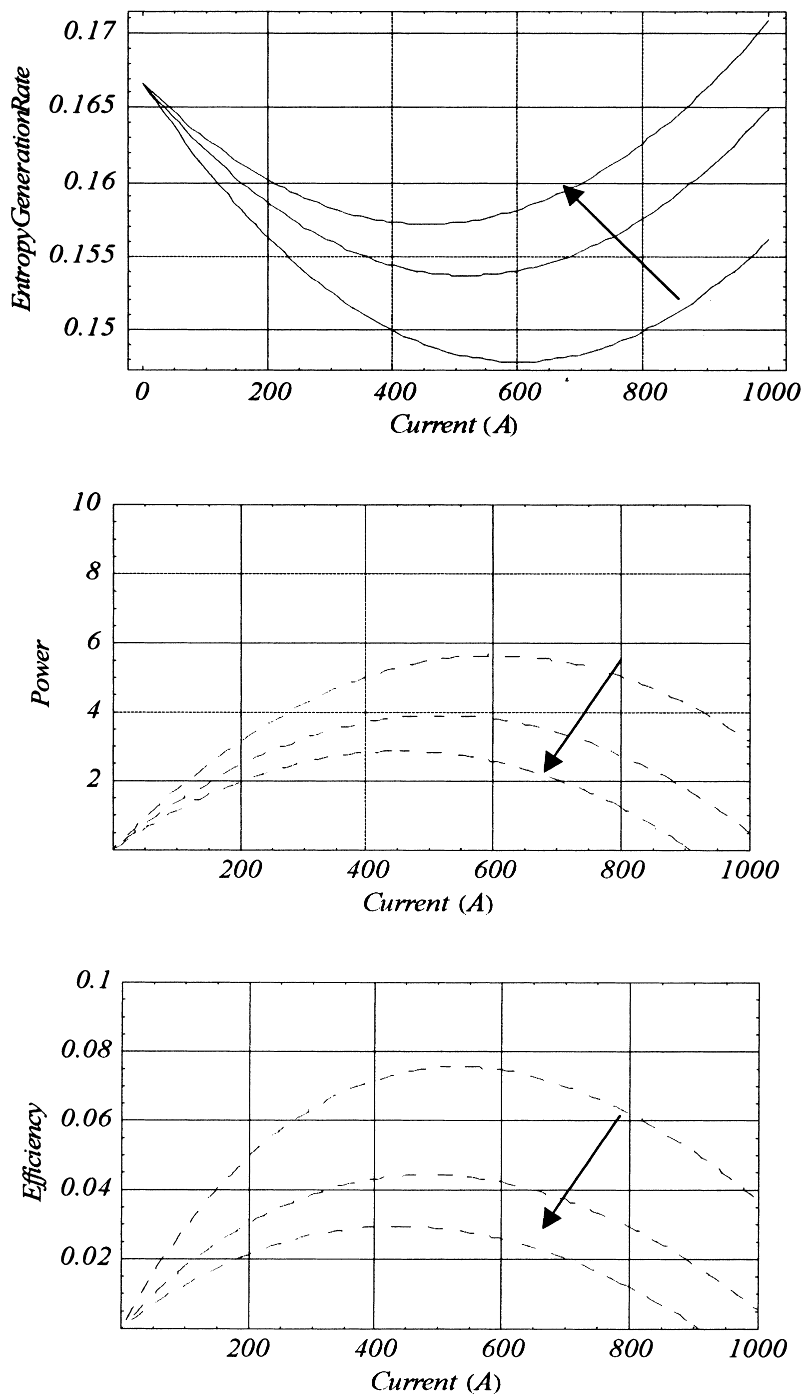

Fig. 9. The effects of varying the heat leak on entropy generation rate, power and efficiency in the fully irreversible thermoelectric model. Arrow shows K to increase as: 0.3, 0.5 and 0.7 W/K. ($T_H = 600$ K, $T_C = 300$ K, $\alpha = 10^{-4}$, $R = 10^{-5}$ and $K_H = K_C = 1$.)

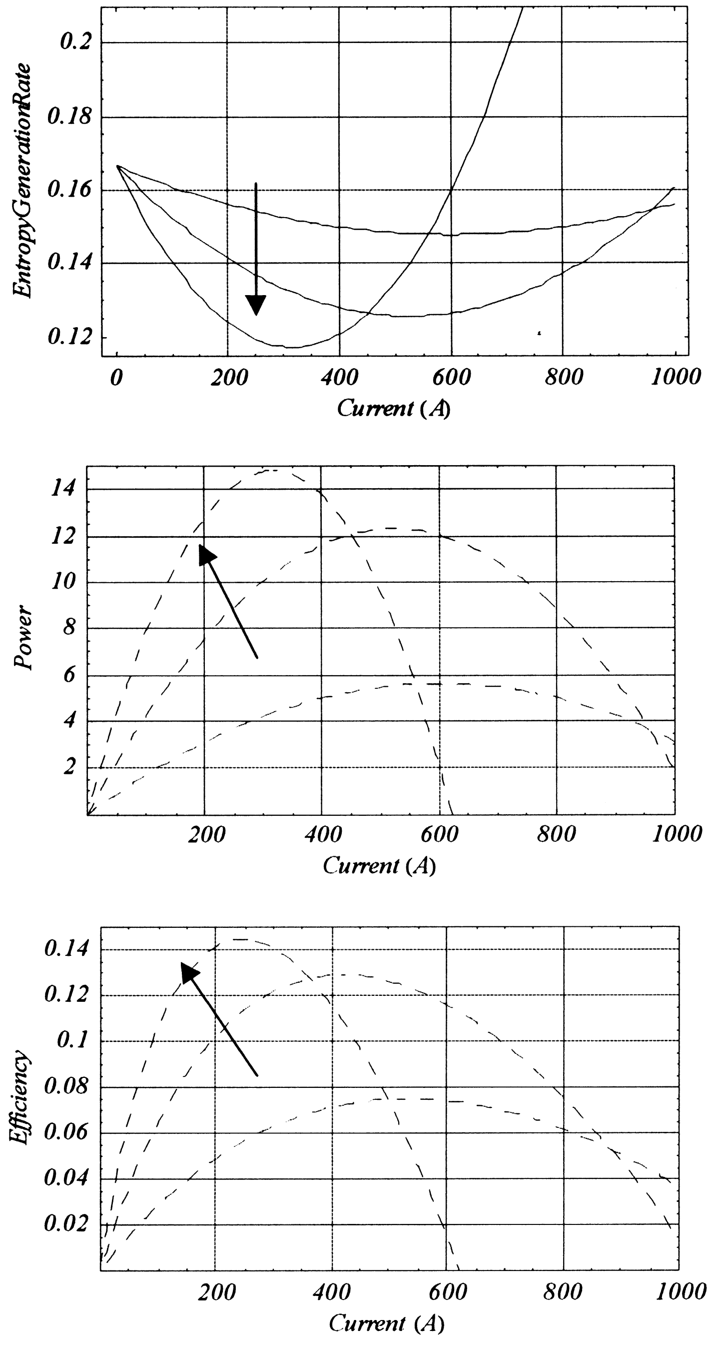

Fig. 10. The effects of varying the Seebeck coefficient on entropy generation rate, power and efficiency in the fully irreversible thermoelectric model. Arrow shows α to increase as: 10^{-4}, 2.5×10^{-4} and 5×10^{-4} V/K. ($T_H = 600$ K, $T_C = 300$ K, $R = 10^{-5}$, $K = 0.3$ and $K_H = K_C = 1$.)

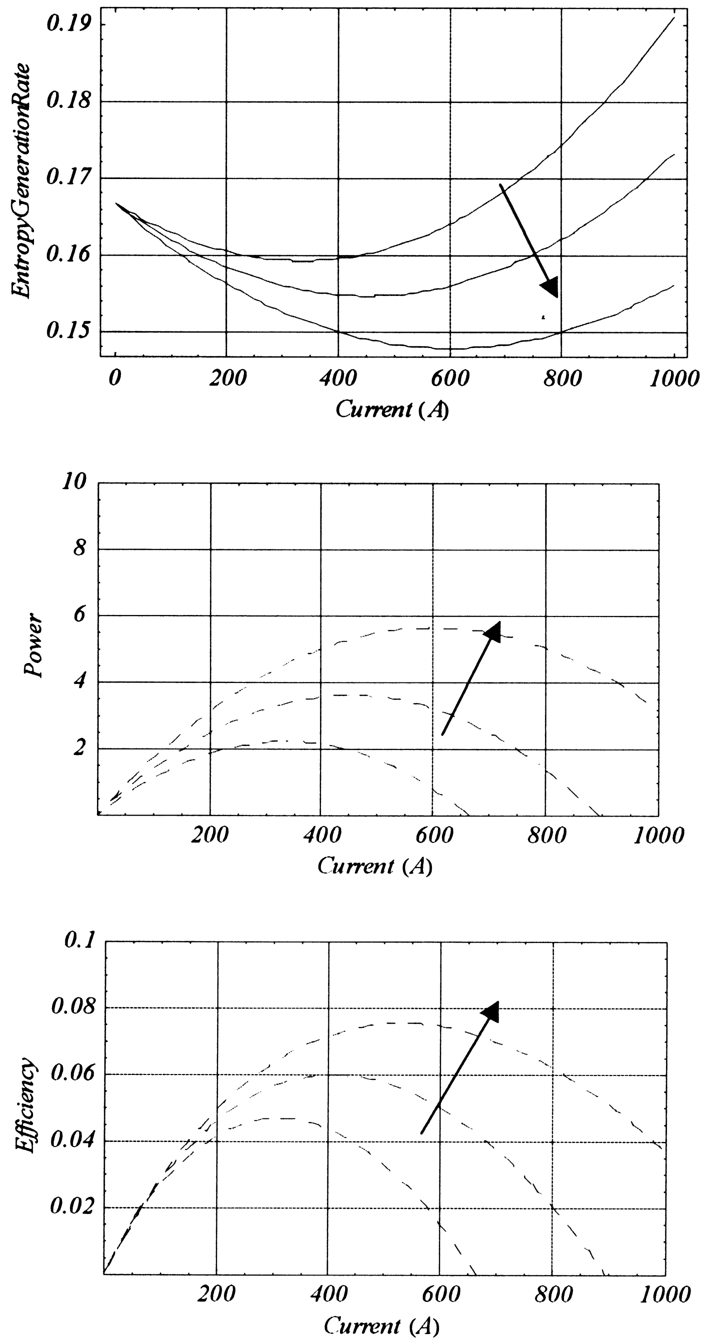

Fig. 11. The effects of varying K_C on entropy generation rate, power and efficiency in the fully irreversible thermoelectric model. Arrow shows K_C to increase as: 0.3, 0.5 and 1 W/K. ($T_H = 600$ K, $T_C = 300$ K, $\alpha = 10^{-4}$, $R = 10^{-5}$, $K = 0.3$ and $K_H = 1$.)

$$\dot{S}_{\text{gen, opt}} = \dot{Q}\left(\frac{T_H - T_C}{T_H T_C}\right) - \frac{K_H(2K+K_H)\big((T_C+T_H)\alpha^2 + 2KR + K_H R\big)}{2(2K+K_H)T_C\alpha^2} + \frac{K_H\sqrt{(2K+K_H)(2K+K_H)(T_C+T_H)\alpha^2 + 2KR + K_H R)^2 - K_H(T_C - T_H)^2\alpha^4)}}{2(2K+K_H)T_C\alpha^2} \quad (53)$$

The equation for the optimum $T_{2,\text{ opt}}$ can also be found but will not be reported here for brevity. Moreover, the value for T_1, as given by Eq. (51), that minimizes entropy is exactly the value that maximizes power which is derived in Ref. [10] using the PM technique. Therefore, once again and for the general model, the maximum power output is associated with the minimum entropy generation rate.

Using the above derived equation, the effects of the various parameters on entropy generation, power output and efficiency of the generator are assessed and results are displayed in Figs. 9–12. By augmenting the heat-leak through increasing the value of the heat transfer coefficient K while holding all other parameters constant (Fig. 9), both efficiency and power outpout decrease, as expected, and the rate of entropy generated increases. Moroever, it is obvious that the optimum current that minimizes the entropy generation rate maximizes power

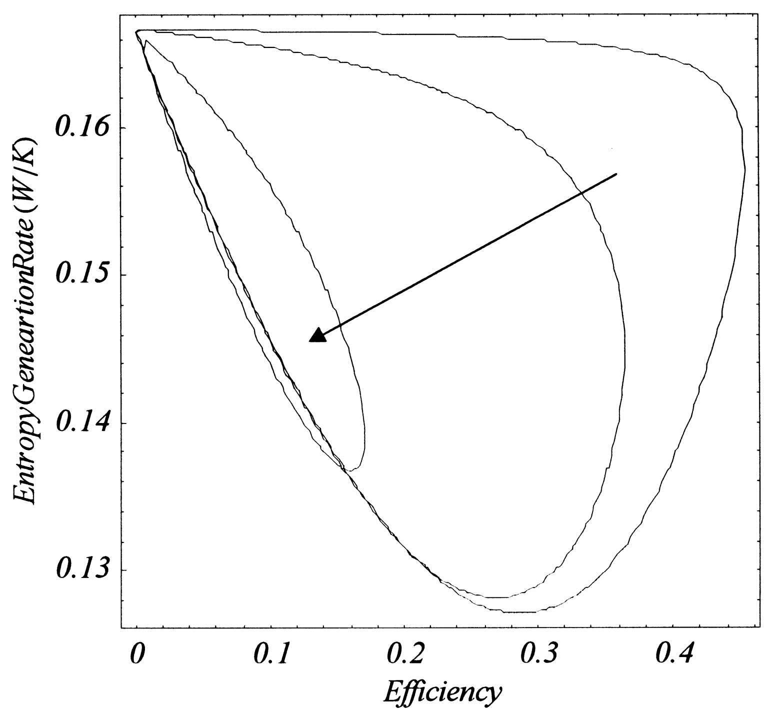

Fig. 12. The effects of varying K on the entropy generation rate vs. efficiency in the fully irreversible model. Arrow shows K to increase as: 0.001, 0.01 and 0.1 W/K. ($T_H = 600$ K, $T_C = 300$ K, $\alpha = 10^{-4}$, $R = 10^{-5}$ and $K_H = K_C = 1$.)

output. Furthermore, the efficiency at minimum entropy generation rate(i.e. maximum power output) and the maximum efficiency of the engine are distinct.

The effects of varying the Seebeck coefficient are depicted in Fig. 10. As α increases, the entropy generation rate decreases and the power and efficiency increase. This is expected since an increase in the Seebeck effect results in augmenting the "figure of merit" Z ($Z = \alpha^2/RK$). Again, the maximum efficiency and minimum entropy generation points are not coincident.

The effects of finite-rate heat transfer on the variations of entropy generation, power, and efficiency are depicted in Fig. 11. As the resistance to heat transfer increases (i.e. K_C decreases), the entropy generation rate increases, and the power output, efficiency at maximum power, and maximum efficiency of the generator diminish. This behaviour is anticipated because decreasing the thermal resistance to finite-rate heat transfer (by increasing K_H and/or K_C) reduces the irreversibilty of the heat engine by forcing the hot and cold junction temperatures to become closer to the heat source and heat sink temperatures (e.g. in this case T_2 becomes closer to T_C). As in the previous plots, the minimum entropy generation is associated with the maximum power output whereas the maximum efficiency and the efficiency at minimum entropy generation rate are distinct.

Finally, the entropy generation rate vs. efficiency curves for different heat-leak values are presented in Fig. 12. As shown, the minimum entropy point and the maximum efficiency point are distinct and by increasing the value of K and thus irreversibilty, they become increasingly separated. This plot can be seen to have the shape of an inverted power vs. efficiency curve. The starting point, however, is not zero, as for power, but is equal to the entropy generation rate due to the heat traverse of the available temperature field.

5. Concluding remarks

A comparative assessment of the performance of the thermoelectric generator using the EGM method and the PM technique was presented. The effects of heat-leak, internal dissipation, and finite-rate heat transfer were investigated. The detrimental effect of these variables on the performance of the generator was demonstrated. A tab on the entropy generation rate suffered by the device was kept and each case was found to have an equivalent model consistent with the Guoy–Stodola theorem. The PM and EGM methods were shown to lead to the same conclusions (i.e. minimum entropy generation rate and maximum power output conditions are coincident) with the EGM method being less straightforward than the PM technique, requiring careful accounting of the different sources of irreversibility. The study provided useful thermodynamic reasoning as applied to a low maintenance device which is under-used in a world of bulky pollution generating monsters.

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