On explaining the observed pattern of lepton and quark masses

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A paper will emerge soon (and 1606.03292v2)

The Higgs boson observed at CERN supports the validity of the Higgs mechanism all the way up to the Planck scale. The problem is that on this way it is blind to several important phenomena:

1. Neutrinos stay massless.
2. There are no candidates for dark matter.
3. There is not enough CP violation for baryon asymmetry of the Universe
4. Lepton and quark masses stay theoretically arbitrary (my main concern, viz S. Weinberg).

Higgs sector of SM is replaced by quantum flavor dynamics. I quote with admiration Heinz Pagels: (1) coined QFD; (2) A. Carter and H. Pagels, PRL 43, 1845 (1979); (3) H. Pagels and S. Stokar, PRD 20, 2947 (1979)

- $$\mathcal{L}_f = -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} + \bar{q}_L i\not{D} q_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \bar{l}_L i\not{D} l_L + \bar{e}_R i\not{D} e_R + \bar{\nu}_R i\not{D} \nu_R$$

One parameter (scale Λ); asymptotically free; anomaly free; by assumption non-confining

All hard fermion mass terms strictly prohibited by $SU(3)_f \times SU(2)_L \times U(1)_Y$ gauge symmetry of L_f

Important reference: T. Yanagida, Phys. Rev.D20, 2986 (1979):
Fermion content identical with my L_f

$$\mathcal{L}_Y(\nu_R) = G_{\nu_R}(\bar{\nu}_{fR}(\nu_{gR})^C)\Phi^{fg} + h.c.$$

$$\mathcal{L}_Y^{(1)}(e) = G_e^{(1)}(\bar{l}_{afL}e_R^f)\phi^a + h.c.$$

$$\mathcal{L}_Y^{(8)}(e) = G_e^{(8)}(\bar{l}_{afL}\tfrac{1}{2}(\lambda^i)_g^f e_R^g)\phi_i^a + h.c.$$

Elementary Higgs fields: Φ (sextet), ϕ (singlet), ϕ_i (octet).
Weakly coupled: Yukawa couplings and condensates are ARBITRARY

My approach: only QFD, strongly coupled in infrared; ew gauge interactions as weak ext. perturbations; generically quantal

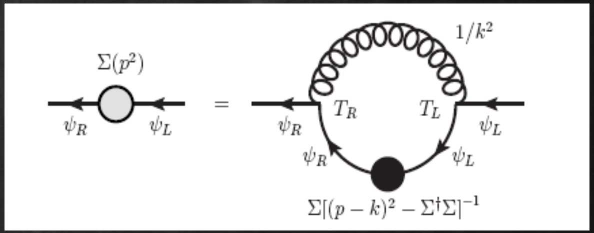
3 Majorana M_R : $3^* \times 3^* = 3 + 6^*$

3 Dirac m_f $3^* \times 3 = 1 + 8$

$$\bar{\nu}_{fR}\Phi^{fg}(\nu_{gR})^C$$

$$m_f = m_{(0)}\lambda_0 + m_{(3)}\lambda_3 + m_{(8)}\lambda_8$$

$$\bar{\psi}_{fR}\phi^{f}_{ag}\psi^{ga}_{L}$$



$$\Sigma(p) = 3\int \frac{d^4k}{(4\pi)^4}\frac{\bar{h}^2_{ab}((p-k)^2)}{(p-k)^2}T_a(f_R)\Sigma(k)[k^2+\Sigma^+(k)\Sigma(k)]^{-1}T_b(f_L)$$

SPONTANEOUS SYMMETRY-BREAKING PATTERN IS FIXED:

- M_R breaks $SU(3)_f$ completely
- $m_{(0)}$ in $SU(3)_f$ singlet breaks $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$
- $m_{(3)}$ and $m_{(8)}$ in $SU(3)_f$ octet breaks $SU(3)_f$ down to $U(1) \times U(1)$

M_R breaks $SU(3)_f$ spontaneously and completely.
Robust prediction I

- All eight flavor gluons acquire masses M_C of order $\Lambda \sim 10^{14}$ GeV
- (phenomenologically unimportant)
  Migdal&Polyakov(1966), Jackiw&Johnson(1973), Cornwall& Norton(1973)
- Eight composite 'would-be' and one pseudo NG bosons belong to complex composite sextet $\bar{\nu}_{fR}(\nu_{gR})^C$
  They are seen as massless poles in WTI.
- Remnant of symmetry (12-8-1=3) are three genuine ν_R – composite Higgs bosons χ_i at the scale Λ.

$m_{(O)}$ breaks spontaneously $SU(2)_L xU(1)_Y$ down to $U(1)_{em}$; leaves $SU(3)_f$ intact. Robust postdiction II.

- W and Z bosons acquire masses of order Σm_f , the dynamically induced electroweak scale.

- Three composite 'would-be' NG bosons belong to the multi-component complex composite electroweak doublet $\phi^a \sim (\bar{\psi}_{fR}\psi_L^{af})$ They are seen in WTI

$$\Gamma^{\alpha}_{W,pole} = -\frac{g}{2\sqrt{2}}\frac{q^{\alpha}}{q^2}[(1-\gamma_5)\Sigma_f(p+q) - (1+\gamma_5)\Sigma_f(p)]$$

- Remnant of symmetry (4-3=1) is one genuine multi-component
- composite Higgs boson h at the electroweak scale
- Composite Higgses: P. Higgs, F. Englert & R. Brout, H. Pagels, …

$m_{(3)}$ and $m_{(8)}$ break spontaneously $SU(3)_f$ down to $U(1)xU(1)$ (subsidiary breakdown of $SU(2)_L xU(1)_Y$ so far ignored).
Robust prediction III.

- Six flavor gluons acquire negligible contributions to their huge masses.
- Six 'would-be' NG bosons belong to the multi-component composite $SU(3)_f$ real octet $\phi_i \sim [(\bar{\psi}_{fR}(\lambda_i)^f_g \psi^g_L) + h.c.]$ (WTI)
- Remnant of symmetry (8-6=2) are two genuine multi-component composite Higgs bosons h_3 and h_8 at the electroweak scale
- For octet quote P. Higgs !

We illustrate explicitly $m_f << M_R$ by a separable Ansatz for the kernel of SD equation

- Integrate only up to Λ. The model thus becomes not asymptotically, but strictly free above Λ. Fix external p=(p,0,0,0) and integrate over angles:

$$\Sigma(p) = \int_0^\Lambda k^3 dk K_{ab}(p,k) T_a(R) \Sigma(k) [k^2 + \Sigma^+ \Sigma]^{-1} T_b(L)$$

- For unknown kernel we make the BCS motivated Ansatz

$$K_{ab}(p,k) = \frac{3}{4\pi^2} \frac{g_{ab}}{pk}$$

g_{ab} are the effective low-energy constants

With separable Ansatz the SD equation is immediately solved:

$$\Sigma(p^2) = \frac{\Lambda^2}{p} T_a(R)\Gamma_{ab}T_b(L)$$

Difficult part is that Γ has to fulfil non-linear algebraic self-consistency condition (gap equation).

The obtained behavior of

$\Sigma(p^2) = \gamma\Lambda^2/p$

is not without theoretical support: P. Mannheim, Phys. Lett. B773(2017)604, and references therein.

1. Majorana masses of sterile neutrinos (g_{11}, g_{44}, g_{66}) Fix (for ever) $\Lambda \sim 10^{14}$ GeV

$$M_{fR} \sim \Lambda$$

2. Dirac masses m_f DEGERATE for ν_f, l_f, u_f and d_f in family f

$$m_f = \Lambda \ \exp\ (-1/4\alpha_f)$$

$$m_f = m_0\lambda_0 + m_3\lambda_3 + m_8\lambda_8$$

$$\begin{aligned}
\alpha_1 &= \frac{3}{64\pi^2}\left(g_{33} + \frac{2}{\sqrt{3}}g_{38} + \frac{1}{3}g_{88}\right)\\
\alpha_2 &= \frac{3}{64\pi^2}\left(g_{33} - \frac{2}{\sqrt{3}}g_{38} + \frac{1}{3}g_{88}\right)\\
\alpha_3 &= \frac{3}{64\pi^2}\frac{4}{3}g_{88}
\end{aligned}$$

$$\begin{aligned}
m_{(0)} &= \frac{1}{\sqrt{6}}(m_1 + m_2 + m_3)\\
m_{(3)} &= m_1 - m_2\\
m_{(8)} &= \frac{1}{\sqrt{3}}(m_1 + m_2 - 2m_3)
\end{aligned}$$

- How many parameters ULTIMATELY ???
- We assume that the obtained fermion mass pattern is generic.
- Consequences due to Goldstone theorem are reliable.

Decay rates h-> WW, ZZ, $\gamma\gamma$ (work with P. Benes, in progress)

- In SM the tree-level Lagrangian for H-> WW, ZZ is

$$L = g m_W W^-_\mu W^{+\mu} H + \frac{1}{2\cos\theta_W} g m_Z Z_\mu Z^\mu H$$ and

H -> $\gamma\gamma$ is given by UV finite both fermion and W loops.

- All decay rates h-> WW, ZZ, $\gamma\gamma$ of the composite Higgs are given solely by fermion loops with propagators containing the explicit momentum-dependent self-energies $\Sigma(p)$. This brings peculiarities: (i) Lack of Feynman parametrization . (ii) New fermion-photon vertices enforced by gauge invariance.

- The loop effect of the composite h of order $g^2 m_f = g(g m_f) \sim g\, m_W$ is of the elementary H tree-level order due to the sum rule for m_W

- Transverse two-photon amplitude $h \to \gamma(k_1^{\mu})\, \gamma(k_2^{\nu})$

$$iM^{\mu\nu}(k_1,k_2) = F\, [(k_1.k_2)\, g^{\mu\nu} - k_2^{\mu}\, k_1^{\nu}]$$

is explicitly given by the f. loops with Σ-dependent vertices obtained by solving the underlying vectorial WT identities

$$\Gamma^{\mu}(p',p) = \gamma^{\mu} - (p'+p)^{\mu}\, \Sigma'(p',p)$$

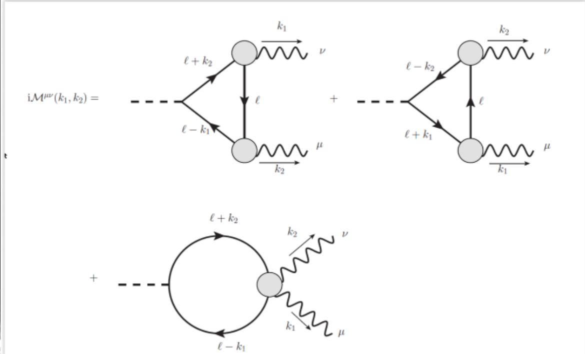


Figure 1.3: $\mathcal{M}^{\mu\nu}(k_1,k_2)$

where $\Sigma'(p',p) = \dfrac{\Sigma(p') - \Sigma(p)}{p'^2 - p^2}$

Pagels-Stokar or Ball-Chiu vertex

Notice that the vectorial WT identities not associated with any breakdown of symmetry are sensitive to derivatives of Σ

$$\Gamma^{\mu\nu}(k_1, k_2, p', p) = [\, (k_1-2p)^{\mu} (k_2+2p')^{\nu}\, \Sigma''(p-k_1, p', p) + (k_1+2p')^{\mu} (k_2-2p)^{\nu}\, \Sigma''(p-k_2, p', p) - 2g^{\mu\nu}\, \Sigma'(p', p)\,]$$

where

$$\Sigma''(p_1, p_2, p_3) = [\Sigma'(p_1, p_3) - \Sigma'(p_2, p_3)]/(p_1^2 - p_2^2)$$

Σ-dependent propagators and vertices define DPT (Pagels).

All results depend upon the explicit form of $\Sigma(p)=\gamma\Lambda^2/p$!!!

Applies also for decay rates of h_3 and h_8 and for the electromagnetic fermion mass splitting

Fermion masses in f are not degenerate but differ by different photon contributions

$Q_e = -1 \quad Q_u = 2/3 \quad Q_d = -1/3 \quad Q_\nu = 0$

Consequently:

1. The neutrino mass spectrum is given solely by QFD.
2. We conjecture (referring to H. Pagels) that the mass splitting of charged leptons and quarks within families is due to the induced Σ-dependent fermion-photon vertices.

1. Neutrino masses-ultimate prediction of QFD

$Y(\nu_R)=0$ hence $\Sigma_\nu = \begin{pmatrix} \Sigma_L & \Sigma_D \\ \Sigma_D^T & \Sigma_R \end{pmatrix}$ given solely by QFD

- $\Sigma_D \sim \Lambda \exp(-1/\alpha)$ and $\Sigma_R \sim \Lambda$ were already computed
- Σ_L is a Majorana mass of the left-handed neutrino of the electroweak left-handed doublet. 2 x 2 = 1 + 3, and the condensing component would be the neutral component of the complex triplet

$$\phi = (\phi^{(0)}, \phi^{+}, \phi^{++}) \qquad \phi^{ab}(6,3,-2) \sim (\bar{l}_L)^{Ca} i\tau_2 \vec{\tau} l_L^b$$

- Such a suggestion costs nothing provided φ is elementary .
- We argue against formation of composite φ – strong repulsion without confining force. Hence SEESAW

$$\Sigma_\nu = \begin{pmatrix} 0 & \Sigma_D \\ \Sigma_D^T & \Sigma_R \end{pmatrix} \qquad m_\nu \sim \frac{m_D^2}{M_R} \sim 10^{-1}\ \text{eV}$$

2. As a warming up (work in progress) we compute the UV finite electromagnetic correction of order Q_i^2 to the universal fermion self-energy $\Sigma(p)$ with bare vertex

$$\delta_i \Sigma(p) = 3e^2 Q_i^2 \int \frac{d^4k}{(2\pi)^4} \frac{\Sigma(k)}{(p-k)^2[k^2+\Sigma^2(k)]}$$

With $\Sigma(p) = m^2/p$ the integral is easily computed. There is a simplification: We do not consider photon exchanges between families. This computation, pretending to describe the fermion mixing is left for future work.

The full fermion self-energy $\Sigma_i(p) = \Sigma(p) + \delta_i\Sigma(p)$ which defines the fermion mass by solving $m_i = \Sigma_i(p^2{=}m_i^2)$ is

The result of the massless photon exchange with bare vertex is

$$\Sigma_i(p^2) = (1 + \frac{\alpha}{\pi}\frac{Q_i^2}{2})\frac{m^2}{p} - \frac{\alpha}{\pi}\frac{3Q_i^2}{4\sqrt{2}}(F+G)\frac{m^3}{p^2} + \frac{\alpha}{\pi}\frac{3Q_i^2}{4\sqrt{2}}(F-G+\frac{\pi}{2})m$$

Where the slowly varying dimensionless functions F and G are

$$F = \frac{1}{2}\ln\frac{(\mathrm{p/m}+\frac{1}{\sqrt{2}})^2+\frac{1}{2}}{(\mathrm{p/m}-\frac{1}{\sqrt{2}})^2+\frac{1}{2}}$$

$$G = \mathrm{arctg}\frac{(\mathrm{p/m})\sqrt{2}}{1-(\mathrm{p/m})^2}$$

Fermion masses are the poles of the full fermion propagator: $m_i = \Sigma_i(m_i^2)$. Explicitly,

$$\frac{m_i}{m} \equiv (1 + \frac{\alpha}{\pi}\frac{Q_i^2}{2})\frac{m}{m_i} - \frac{\alpha}{\pi}\frac{3Q_i^2}{4\sqrt{2}}(F+G)\frac{m^2}{m_i^2} + \frac{\alpha}{\pi}\frac{3Q_i^2}{4\sqrt{2}}(F-G+\frac{\pi}{2})$$

A hope is that small corrections of order α with new fermion-photon vertices cause substantial observed amplification. For bare vertex the miracle does not happen.

Conclusions

Consequences of the model which depend on explicit form of $\Sigma(p)=\gamma\Lambda^2/p$ like fermion masses, gauge boson masses, decay rates of composite Higgses etc. must better be taken with care. There are generic observable predictions following from basic dynamical assumptions on QFD :

- Three active neutrinos are the extremely light Majorana fermions. (In accord with Weinberg's prediction based on effective f.t.) Lattice QFD computations ?!
- There are two additional electroweak composite Higgses h_3 and h_8 with characteristic Yukawa couplings.
- There is a natural dark matter candidate $S \sim C_{abc}\nu_R^a\nu_R^b\nu_R^c$

composed of sterile neutrinos and bound by strong-coupling QFD as a dark image of 'luminous' QCD nucleon: $N \sim \epsilon_{abc}q^aq^bq^c$.

Compare 3x3x3 = (3*+6)x3 = 1 + 8 + 8 + 10 for both cases.

The composite 'would-be' NG bosons of $SU(3)_f$ Complete breakdown. Consequence No.1

dynamical neutrino mass term (sextet) : $\bar{\nu}_R \Sigma_R (\nu_R)^C$

neutrino current $j_a^\mu = \bar{\nu}_R \gamma^\mu \frac{1}{2}\lambda_a \nu_R = \frac{1}{2}\bar{n}\gamma^\mu \frac{1}{2}\Lambda_a n$ where

$\frac{1}{2}\Lambda_a = \frac{1}{2}\lambda_a P_R + \frac{1}{2}(-\lambda_a)^T P_L \quad n = \nu_R + (\nu_R)^C \quad (\nu_R)^C = C(\bar{\nu}_R)^T$

WT identity, pole term

$$\Gamma^\mu_{a,pole} = \frac{q^\mu}{q^2}[\hat{\Sigma}(p+q)\Lambda_a - \bar{\Lambda}_a \hat{\Sigma}(p)]$$

Yukawa couplings of 'would-be' NG bosons

$$P_a(p,p) \sim \{\Sigma(p), \tfrac{1}{2}\lambda_a\}\gamma_5, \qquad a = 1,3,4,6,8$$

$$P_a(p,p) \sim [\Sigma(p), \tfrac{1}{2}\lambda_a], \qquad a = 2,5,7$$

flavor gluon masses $M_{aC} \sim M_R$ (Jackiw-Johnson)

The composite 'would-be' NG bosons of $SU(2)_L \times U(1)_Y$ due to QFD. Consequence No.2

$$\Gamma^{\alpha}_{W,pole} = -\frac{g}{2\sqrt{2}}\frac{q^{\alpha}}{q^2}[(1-\gamma_5)\Sigma_f(p+q) - (1+\gamma_5)\Sigma_f(p)]$$

Pagels-Stokar formula and QFD (PRD20,02947(1979)

$$F_f^2 = 8N\int\frac{d^4p}{(2\pi)^4}\frac{\Sigma_f^2(p^2) - \frac{1}{4}p^2(\Sigma_f^2(p^2))'}{(p^2+\Sigma_f^2(p^2))^2} = \frac{5}{4\pi}\sum_f m_f^2$$

$$m_W^2 = \frac{1}{4}g^2\frac{5}{4\pi}\sum_f m_f^2$$

$$m_Z^2 = \frac{1}{4}(g^2+g'^2)\frac{5}{4\pi}\sum_f m_f^2$$

$$m_{3D} \doteq 390\,\mathrm{GeV}$$

1.