On explaining the observed pattern of quark and lepton masses

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Replace the Higgs sector of SM by new strong flavor (family, generation, horizontal) $SU(3)_f$ dynamics (QFD for brevity) with one parameter, the scale Λ.

- For anomaly freedom add one triplet of sterile neutrino right-handed fields.

$$\mathcal{L}_f = -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} + \bar{q}_L i\not{D} q_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \bar{l}_L i\not{D} l_L + \bar{e}_R i\not{D} e_R + \bar{\nu}_R i\not{D} \nu_R$$

$SU(3)_f \times SU(2)_L \times U(1)_Y$ symmetry manifest in the Lagrangian is observed as badly broken down to $U(1)_{em}$

By elementary Higgses: Yanagida, PR D20, 2986(1979)

1. Majorana masses of sterile neutrinos:
   $3 \times 3 = 3^* + 6$ => introduce flavor sextet
   $\Phi = (6, 1, 0)$ $\langle\Phi\rangle_0$ huge ($\sim 10^{14}$ GeV)
   gives also masses M_a to all eight flavor gluons C
   $2 \times 6 = 12 = 8(NG) + 1\ (pseudoNG) + 3(Higgs)$
2. Dirac masses of SM fermions:
   $3^* \times 3 = 1 + 8$ => introduce
   $\Phi = (8, 2, 1)$ to break $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$
3. Neutrino masses by seesaw come out naturally !

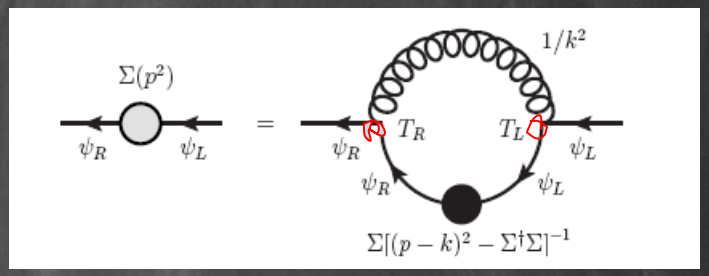
The idea in words:
Dynamical chiral gauge symmetry breakdown à la Nambu :

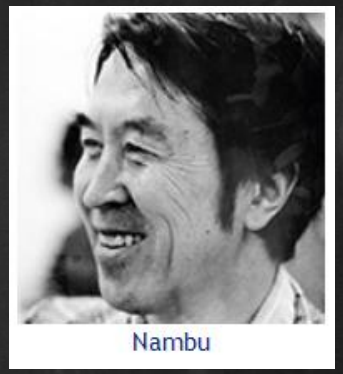
1. Flavor gluon exchanges in 6 of 3x3 in SD equation generate 3 different huge Majorana masses $M_{fR} \sim \Lambda$ of sterile neutrinos => huge masses of all 8 flavor gluons C_a.

2. Flavor gluon exchanges in 3* x 3 channels in SD equation generate 3 different exponentially small Dirac masses m_f same for neutrino, charged lepton, u- and d- quark in given family => masses of W,Z bosons.

3. Mass splitting of charged leptons and quarks in a family is attributed to known different weak hypercharges

Dynamical generation of both M_{fR} and m_f masses by $SU(3)_f$ (rest are inevitable consequences of Goldstone theorem)




Nambu

$$\Sigma(p) = 3\int \frac{d^4k}{(4\pi)^4}\frac{\bar{h}^2_{ab}((p-k)^2)}{(p-k)^2}T_a(f_R)\Sigma(k)[k^2+\Sigma^+(k)\Sigma(k)]^{-1}T_b(f_L)$$

- fermion self-energy (mass) is a bridge between L and R valid both for Majorana and Dirac fermions
- sliding coupling at strong coupling entirely unknown
- IF the idea is warranted ULTIMATELY no free parameters seesaw neutrino mass spectrum truly CALCULABLE

Real life is more prosaic:
We illustrate explicitly $m_f \ll M_R$ by a separable Ansatz for the kernel of SD equation

Integrate only up to Λ. The model thus becomes not asymptotically, but strictly free above Λ. Fix external $p=(p,0,0,0)$ and integrate over angles:

$$\Sigma(p) = \int_0^\Lambda k^3 dk K_{ab}(p,k) T_a(R) \Sigma(k) [k^2 + \Sigma^+ \Sigma]^{-1} T_b(L)$$

For unknown kernel we make the BCS motivated Ansatz

$$K_{ab}(p,k) = \frac{3}{4\pi^2} \frac{g_{ab}}{pk}$$

g_{ab} are the effective low-energy constants (how many ?!)

With separable Ansatz the SD equation is immediately formally solved:

$$\Sigma(p^2) = \frac{\Lambda^2}{p} T_a(R)\Gamma_{ab}T_b(L)$$

Difficult part is that Γ has to fulfil non-linear algebraic self-consistency condition (gap equation).

The obtained behavior of $\Sigma(p^2) \sim 1/p$ is not without support: P. Mannheim, Phys. Lett. B773(2017)604, and references therein, and his contribution to this conference.

1. Majorana masses of sterile neutrinos $\bar{\nu}_R \Sigma_R (\nu_R)^C$

$$M_{fR} \sim \Lambda$$

2. Dirac masses of leptons and quarks $\bar{f}_R \Sigma_D f_L$

$$m_f = \Lambda \ \exp\ (-1/4\alpha_{\rm f})$$

$$\begin{aligned}
\alpha_1 &= \frac{3}{64\pi^2}(g_{33} + \frac{2}{\sqrt{3}} g_{38} + \frac{1}{3} g_{88}) \\
\alpha_2 &= \frac{3}{64\pi^2}(g_{33} - \frac{2}{\sqrt{3}} g_{38} + \frac{1}{3} g_{88}) \\
\alpha_3 &= \frac{3}{64\pi^2} \frac{4}{3} g_{88}
\end{aligned}$$

same for all fermion species in a family

Neutrino masses–ultimately exact–true prediction

$$\Sigma_\nu = \begin{pmatrix} \Sigma_L & \Sigma_D \\ \Sigma_D^T & \Sigma_R \end{pmatrix}$$

- $\Sigma_D \sim \Lambda \exp(-1/\alpha)$ and $\Sigma_R \sim \Lambda$ were already computed
- Σ_L is a Majorana mass of the left-handed neutrino of the electroweak left-handed doublet. 2 x 2 = 1 + 3, and the condensing component would be the neutral component of the complex triplet

$$\phi = (\phi^{(0)}, \phi^+, \phi^{++}) \qquad \phi^{ab}(6,3,-2) \sim (\bar{l}_L)^{Ca} i\tau_2 \vec{\tau} l_L^b$$

- Such a suggestion costs nothing provided ϕ is elementary (triplet Majoron).
- We wishfully argue against formation of composite ϕ – strong repulsion without confining force. Hence SEESAW

$$\Sigma_\nu = \begin{pmatrix} 0 & \Sigma_D \\ \Sigma_D^T & \Sigma_R \end{pmatrix} \qquad \boxed{m_\nu \sim \frac{m_D^2}{M_R}}$$

Masses of charged leptons and quarks are distinguished by weak hypercharges

In SD eq. consider besides C also the B exchanges:

$$\bar{g}'(q)^2 \tfrac{1}{4} Y_R Y_L$$

$$\begin{aligned} Y(l_L) &= -1, \quad Y(e_R) = -2, \quad Y(\nu_R) = 0\ , \\ Y(q_L) &= \tfrac{1}{3}, \quad Y(u_R) = \tfrac{4}{3}, \quad Y(d_R) = -\tfrac{2}{3} \end{aligned}$$

We do not know how to incorporate the B exchange into separable Ansatz, and use a primitive parametrization

$$\begin{aligned} \alpha_1(i) &= \tfrac{3}{64\pi^2}\{[g_{33} + \tfrac{2}{\sqrt{3}} g_{38} + \tfrac{1}{3} g_{88}] + g_1^i\} \\ \alpha_2(i) &= \tfrac{3}{64\pi^2}\{[g_{33} - \tfrac{2}{\sqrt{3}} g_{38} + \tfrac{1}{3} g_{88}] + g_2^i\} \\ \alpha_3(i) &= \tfrac{3}{64\pi^2}\{\tfrac{4}{3} g_{88} + g_3^i\}\ . \end{aligned}$$

TABLE I: Fermion masses from experiment [21].

$m_3(e) \equiv m_\tau$	$m_2(e) \equiv m_\mu$	$m_1(e) \equiv m_e$
1.777 GeV	105.7 MeV	0.511 MeV
$m_3(u) \equiv m_t$	$m_2(u) \equiv m_c$	$m_1(u) \equiv m_u$
173.1 GeV	1.28 GeV	2.2 MeV
$m_3(d) \equiv m_b$	$m_2(d) \equiv m_s$	$m_1(d) \equiv m_d$
4.18 GeV	96.0 MeV	4.6 MeV

TABLE II: The parameters of the theory describing the fermion spectra.

g_{33}	g_{38}	g_{88}
1.27262	-0.0594915	1.502790
g_1^e	g_2^e	g_3^e
-0.382808	-0.315776	-0.341190
g_1^u	g_2^u	g_3^u
-0.332490	-0.196766	-0.060098
g_1^d	g_2^d	g_3^d
-0.305581	-0.320025	-0.295027

Impressionistic conclusion:
Properties of quantum flavor dynamics

- Masses of leptons and quarks are ultimately calculable
- There is a QFD understanding of seesaw origin of neutrino mass spectrum
- No genuine electroweak symmetry scale, $\Lambda \sim 10^{14}$ GeV
- Composite Higgs is similar to but distinct from SM Higgs
- There are three superheavy neutrino-composite inflatons χ
- There are decent candidates for dark matter – axions

Illustration of composite NG bosons by Ward-Takahashi identity (SU_3)

dynamical neutrino mass term (sextet) : $\bar{\nu}_R \Sigma_R (\nu_R)^{\mathcal{C}}$

neutrino current $j_a^\mu = \bar{\nu}_R \gamma^\mu \frac{1}{2}\lambda_a \nu_R = \frac{1}{2}\bar{n}\gamma^\mu \frac{1}{2}\Lambda_a n$ where

$\frac{1}{2}\Lambda_a = \frac{1}{2}\lambda_a P_R + \frac{1}{2}(-\lambda_a)^T P_L \quad n = \nu_R + (\nu_R)^{\mathcal{C}} \quad (\nu_R)^{\mathcal{C}} = C(\bar{\nu}_R)^T$

WT identity, pole term

$$\Gamma^\mu_{a,pole} = \frac{q^\mu}{q^2}[\hat{\Sigma}(p+q)\Lambda_a - \bar{\Lambda}_a \hat{\Sigma}(p)]$$

Yukawa couplings of 'would-be' NG bosons

$$P_a(p,p) \sim \{\Sigma(p), \tfrac{1}{2}\lambda_a\}\gamma_5, \quad a = 1,3,4,6,8$$

$$P_a(p,p) \sim [\Sigma(p), \tfrac{1}{2}\lambda_a], \quad a = 2,5,7$$

flavor gluon masses $\boxed{M_{aC} \sim M_R}$ (Jackiw-Johnson)

Dynamically generated masses of leptons and quarks break spontaneously $SU(2)_L \times U(1)_Y$ to $U(1)_Q$: There are 3 'would-be' NG bosons i.e., massive W and Z.

$$\Gamma^{\alpha}_{W,pole} = -\frac{g}{2\sqrt{2}}\frac{q^{\alpha}}{q^2}[(1-\gamma_5)\Sigma_f(p+q) - (1+\gamma_5)\Sigma_f(p)]$$

compute the W, Z masses in terms of Σ using the Pagels-Stokar formula

$$F_f^2 = 8N\int \frac{d^4p}{(2\pi)^4}\frac{\Sigma_f^2(p^2) - \frac{1}{4}p^2(\Sigma_f^2(p^2))'}{(p^2+\Sigma_f^2(p^2))^2}$$

$$m_W^2 = \frac{1}{4}g^2\frac{5}{4\pi}\sum_f m_f^2 \qquad m_Z^2 = \frac{1}{4}(g^2+g'^2)\frac{5}{4\pi}\sum_f m_f^2$$

$$m_{3D} \doteq 390\,\mathrm{GeV}$$

Thanks for your attention !

"Before I came here I was confused about this subject. Having listened to your lecture I am still confused. But on a higher level."

Enrico Fermi