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# On Falling Neutral Real Rates, Fiscal Policy, and the Risk of Secular Stagnation

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# On falling neutral real rates, fiscal policy, and the risk of secular stagnation*

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**Abstract**

This paper demonstrates that neutral real interest rates would have declined by far more than what has been observed in the industrial world and would in all likelihood be significantly negative but for offsetting fiscal policies over the last generation. We start by arguing that neutral real interest rates are best estimated for the block of all industrial economies given capital mobility between them and relatively limited fluctuations in their collective current account. We show, using standard econometric procedures and looking at direct market indicators of prospective real rates, that neutral real interest rates have declined by at least 300 basis points over the last generation. We argue that these secular movements are in larger part a reflection of changes in saving and investment propensities rather than the safety and liquidity properties of Treasury instruments. We then point out that the movements in the neutral real rate reflect both developments in the private sector and in public policy. We highlight the levels of government debt, the extent of pay-as-you-go old age pensions and the insurance value of government health care programs have all *ceteris paribus* operated to raise neutral real rates. Using estimates drawn from the literature, as well as two general equilibrium models emphasizing respectively life-cycle heterogeneity and idiosyncratic risks, we suggest that the "private sector neutral real rate" may have declined by as much as 700 basis points since the 1970s. Our findings support the idea that, absent offsetting policies, mature industrial economies are prone to secular stagnation. This raises profound questions about stabilization policy going forward. Achievement of levels of deficits and government debt generally considered desirable – especially if complemented by reductions in social insurance – would likely mean negative neutral real rates in the industrial world. Policymakers going forward will need to engage in some combination of greater tolerance of budget deficits, unconventional monetary policies and structural measures to promote private investment and absorb private saving if full employment is to be maintained and inflation targets are to be hit.

**Keywords:** equilibrium real interest rate; R*; secular stagnation; fiscal policy; government debt; social security; healthcare costs; life-cycle; heterogeneity; incomplete markets; precautionary saving.

**JEL Classification:** E0, F3, F4, F6.

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# 1 Introduction

What is the interest rate that is consistent with stable macroeconomic performance of a modern, developed economy? Few questions can rival this one in its difficulty and importance. Equilibrium real interest rates are unobservable; they are affected by a wide swathe of macroeconomic forces, both domestic and global; and are not invariant to policy regimes. All those factors make the assessment difficult and the answers uncertain. And yet a good handle on the equilibrium interest rate is fundamental to our ability to correctly assess the state of the economy, predict the future trends, and set policy appropriately.

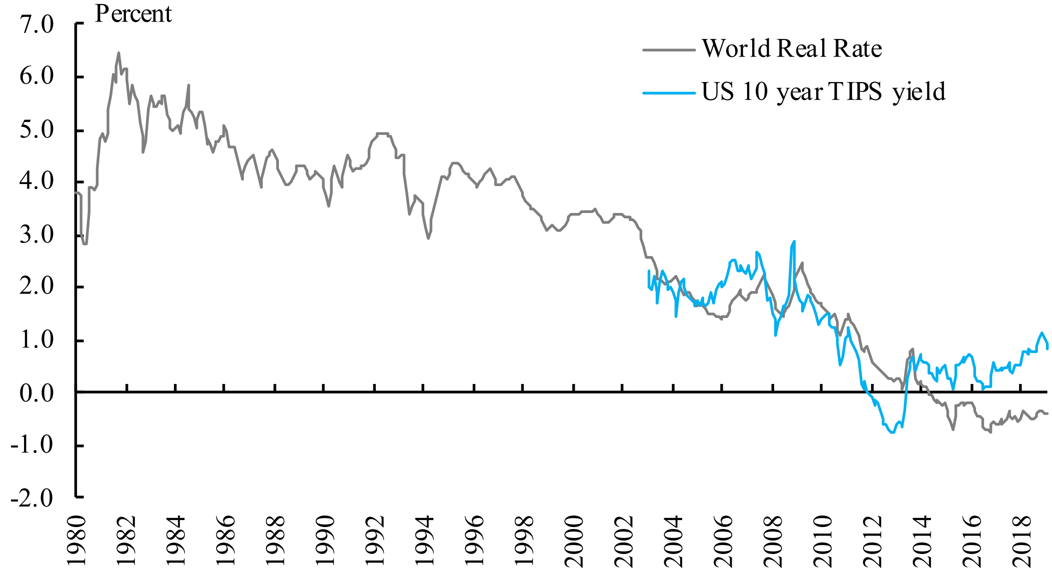
The secular stagnation debate refocused attention on this important issue by pointing towards the downward trend in long-term interest rates across the developed world over the past four decades (Summers (2015), Teulings and Baldwin (2014)). This decline, which started well before the financial crisis, is broad-based, both across countries and across asset classes. Yields on long-maturity inflation-protected government securities have been trending down since at least the early 1990s, and have hovered around their lowest levels on record over the past decade (Figure 1). The 5-year/5-year forward swap rates, which are less likely to be driven by the time-varying liquidity and safety premia, are close to 0% in real terms (Figure 2). More broadly, a large share of the decline in risk-free rates has been mirrored in risky asset returns, such as rates of return on corporate bonds and on equites: notwithstanding some volatility, spreads have remained close to long-run historical averages (Figure 3).

Policymakers have taken notice. Federal Reserve Chairman Jerome Powell's recent remark that the nominal federal funds rate – at the time set at between 2-2.25% – was "*just below the broad range of estimates of the level that would be neutral for the economy*" puts the level of the real neutral rate in the United States at around 0.5% (Powell (2018)). In Japan, faced with very low neutral rates for a long time, the central bank has engaged in aggressive monetary easing including directly targeting long-term interest rates (Kuroda (2016)). Similarly, European policymakers highlighted the equilibrium rate of interest as the key policy variable (Constâncio (2016); Draghi (2016)), while the recent ECB paper concluded that *"most of estimates of R* for the euro area have been negative regardless of the type of model used"*.

The facts that (i) estimates of the decline in neutral short real rates on highly liquid securities track declines in yields on relatively illiquid government indexed bonds and real swaps; (ii) there has been little trend movement in spreads between Treasury securities and corporate securities in given rating classes; (iii) the magnitude of the estimated decline in real rates far exceeds the level let alone the change in spreads leads us to believe that for the purpose of analyzing long-term trend movements in neutral or equilibrium real rates it is appropriate to focus on factors relating to saving and investment propensities rather than issues of liquidity or risk. This is the approach taken in what follows.[1]

[1] Appendix A provides some further discussion of safety and liquidity premium.

**Figure 1:** Real interest rates estimated from the inflation-linked bonds in advanced economies and in the United States



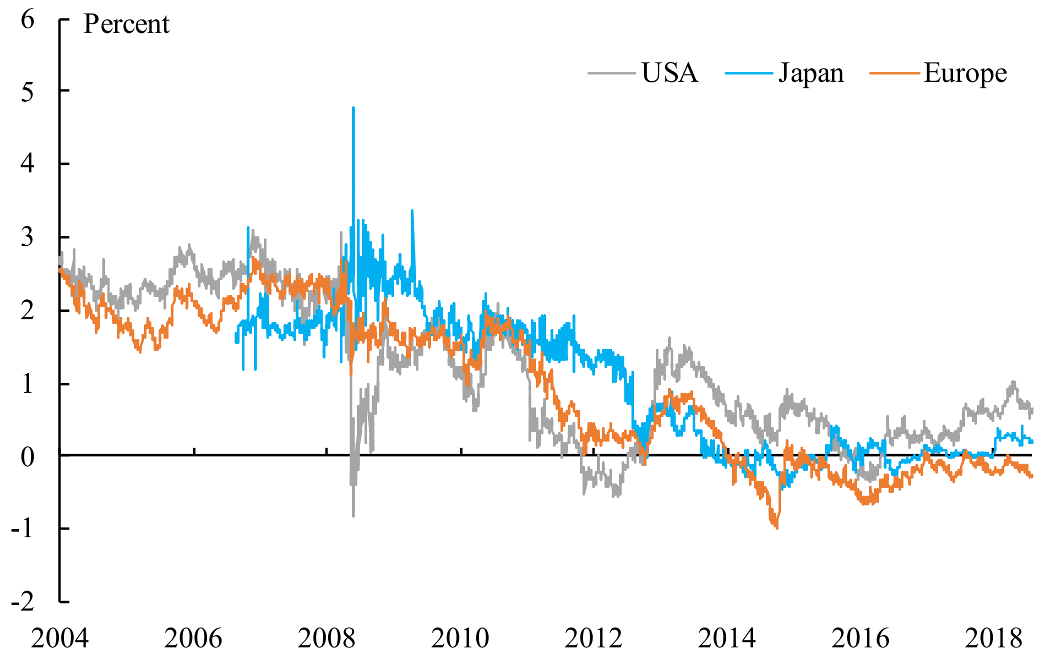

Note: The world real rate is calculated following the methodology in King and Low (2014): it is the average of interest rates on inflation-protected government debt securities across the G7 excluding Italy. Data are from DataStream and form an unbalanced panel. In particular, the Figure relies on the UK inflation-indexed gilts in the early part of the sample. The US TIPS yield is the yield on a constant maturity 10-year Treasury Inflation-Indexed Security, retrieved from FRED, Federal Reserve Bank of St. Louis (code DFII10).

Research to date has largely focused on the analysis of neutral rates for individual countries.[2] In this literature, the decline in the safe interest rates in advanced economies is a robust finding across studies that employ a wide range of tools and methods. For example, Marco Del Negro, Marc Giannoni, Domenico Giannone and Andrea Tambalotti (2017) established that a time-series model and a structural general equilibrium model both detect a downward trend in the equilibrium rate of interest in the United States since the mid-1990s.[3] Internationally, Kathryn Holston, Thomas Laubach and John Williams (2017b) applied the seminal methodology of Laubach and Williams (2003) to four advanced economies (United States, Canada, Euro Area and the United Kingdom), and found that the decline in the real rate of interest is present in each of them. However estimating neutral real rates for individual open economies is a questionable procedure since it implicitly takes as given an endogenous variable—the trade surplus or deficit. A country for example that runs a chronic trade surplus will be found to have a neutral real rate at a level where domestic demand is short of potential output and the reverse will be true for a country running a chronic trade deficit.

[2] A recent exception is the paper by Del Negro et al. (2018) who study trends in world interest rates in an econometric framework. They also find that the global neutral rate has declined significantly over the past three decades.

[3] Their contribution stresses the importance of the convenience yield in driving the low risk-free rates. We discuss their results in the context of our analysis in Appendix A.

**Figure 2:** Real 5-year/5-year swap rates



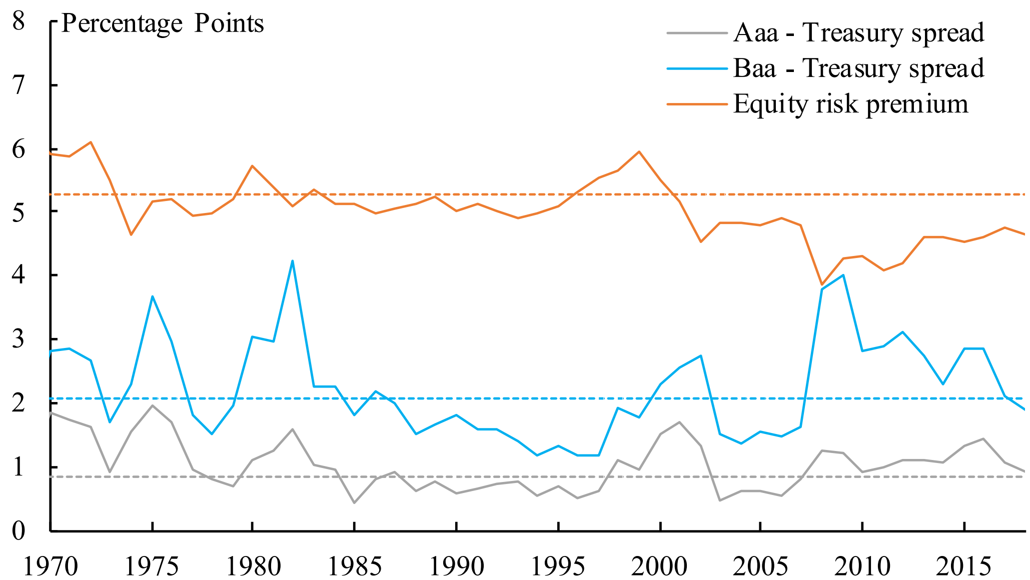

Note: Five- and ten-year nominal and inflation swap rates data are from Bloomberg. Real swap rates are nominal minus inflation.

We therefore estimate the neutral real rate using aggregated data for all of the advanced economies (as if they formed a single, fully integrated economy). We justify the use of the group of advanced economies as our unit of analysis by showing that fluctuations in its current account position with the rest of the world are small. Our results suggest that real rate for the advanced economy block – what we call AE R* for brevity – declined by around 3pp over the past 40 years, and is currently only slightly above zero in real terms. This is consistent with the evidence presented above on the evolution of measures of long term real interest rates.

Researchers have explored a wide range of potential drivers behind the decline in real interest rates.[4] Etienne Gagnon, Benjamin Johannsen and David Lopez-Salido (2016), Carlos Carvalho, Andrea Ferrero and Fernanda Nechio (2015), Noëmie Lisack, Rana Sajedi and Gregory Thwaites (2017) and Gauti Eggertsson, Neil Mehrotra and Jacob Robbins (2019b) all used macroeconomic models with an overlapping generations structure to show that demographic trends can act as powerful forces driving the intertemporal preferences and hence intertemporal prices. Adrien Auclert and Matthew Rognlie (2016) as well as Ludwig Straub (2017) explored different channels linking income inequality and real interest rates in general equilibrium models. The work of Barry Eichengreen (2015) stressed the importance of investment-specific technological change and the resulting decline in the price of capital goods, while the recent study by Emmanuel Farhi and

[4] An older literature considered why interest rates were so high in the 1980s. Blanchard and Summers (1984) conjectured that high interest rates in that period were driven in part by higher profitability of investment. Their conjecture was subsequently supported by Barro and Sala-i Martin (1990) who identified a strong role for stock prices – a proxy of anticipated investment profitability – in affecting world interest rates.

**Figure 3:** Corporate bond and equity spreads in the United States over the long-run




Note: Dashed lines represent the long-run averages, and are calculated over 1919-2018 for bond spreads and 1960-2018 for the ERP. The Aaa and Baa spread data are from Krishnamurthy and Vissing-Jorgensen (2012) up to 2008, and from FRED thereafter (series codes AAA - GS20 and BAA10Y). The equity risk premium estimate is the historical estimate from Professor Aswath Damodaran of New York University: http://www.damodaran.com, constructed as cumulative return differential on S&P500 and 10-year Treasury bond since the 1928.

Francois Gourio (2018) considered other supply-side forces such as intangible capital and market power as the drivers of the real interest rate decline.

While the literature has covered a lot of ground in terms of possible private sector explanations for declines in real rates, it has not highlighted an important issue. One would have expected a period of substantially enlarged government deficits and debt and substantially enhanced pay-as-you-go public pensions and increased health insurance to *ceteris paribus* have substantially raised neutral real rates.[5] That this has not occurred suggests that the economic forces operating to reduce neutral real rates have been stronger than has been generally recognized.

We focus on the role of fiscal policies in influencing neutral real rates. These policies affect the interest rate through a range of channels. We review these mechanisms through the lens of several theoretical paradigms, concentrating in particular on the role of government borrowing, which is the main focus of both theoretical and empirical literatures in macroeconomics. We then survey the existing empirical estimates of the impact of government debt on interest rates. Simple calculations using observed estimates of the impact of deficits on interest rates suggest

[5]One recent study, developed independently and in parallel to ours, which points out the role of government debt is that of Eggertsson et al. (2019b). In comparison to their paper, we focus on a wider set of policies (including social security and old-age healthcare), we consider empirical estimates of the link between debt and R*, and we analyze a fuller extent of theoretical channels through which this link operates.

that the increase from 18% to 68% in the public debt-to-GDP ratio of the advanced economies should *ceteris paribus* have raised real rates by between 1.5 and 2 percentage points over the last four decades. This effect is quantitatively important but is a presumptive underestimate of the impact of fiscal policy changes given the rise in pay-as-you-go government pensions and other social insurance programs. This analysis leads to the conclusion that the fall in real long-term interest rate observed in the data masks an even more dramatic decline in the equilibrium "private sector" real rate.

To incorporate other policies such as the increase in social security and healthcare spending into our analysis, to build further understanding of the mechanisms involved, and to cross-check the magnitudes of these effects, we study these phenomena in a dynamic general equilibrium framework. We construct two tractable models, each one designed to capture different channels through which policies play out in equilibrium.

Building on the work of Mark Gertler (1999), the first model captures the life-cycle behavior, with workers saving for retirement and retirees decumulating their wealth. The resulting heterogeneity in marginal propensities to consume and differences in the implicit discount rates across agents mean that Ricardian Equivalence – the proposition that government borrowing decisions are neutral in equilibrium – does not hold in our model, making the effects of a range of government policies on real rates non-trivial.[6] We simulate the model with the profiles of government debt, government spending, social security and old-age healthcare expenditures that match the experience of developed economies over the past 40 years. These simulations suggest that shifts in these policies pushed equilibrium real rates up by over 3.2pp between the early 1970s and today.[7]

Our second model is of the Bewley-Huggett-Aiyagari tradition. It focuses on idiosyncratic risks and precautionary behavior, channels that are absent from the life-cycle model. When markets are incomplete, government debt is an asset which households can hold to self-insure. Supply of government bonds thus determines the total supply of investable assets, and hence the interest rate in equilibrium. We calibrate this model in a parallel fashion to the life-cycle model, ensuring that the degree of income uncertainty matches the risks estimated from the data on individual household incomes. The realistic calibration of the income process means that the equilibrium of our model features the degree of income inequality that is consistent with what we observe in the data. Our model-based explorations suggest that the increase in the supply of government bonds has pushed interest rates up by about 70bps through this precautionary 'supply of safe assets' channel. Overall, then, we find that the rising government debt accounts for around 1.5pp (0.8pp+0.7pp) upwards pressure on the neutral real interest rate, consistent

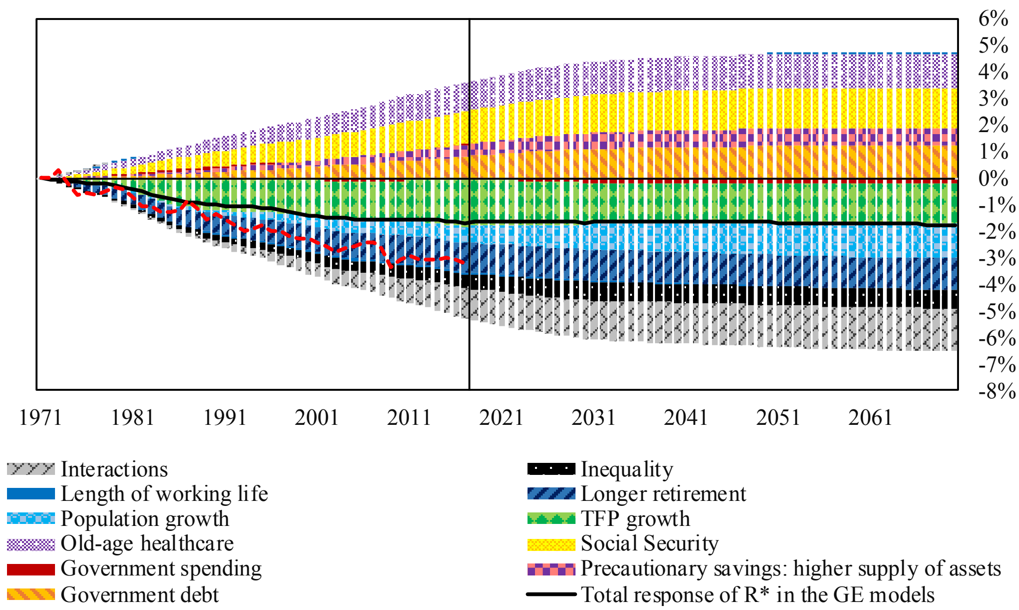
[6]Following a change in government finances, there is some Ricardian offset, but unlike in the representative agent model, this offset is incomplete.

[7]A part of the increase in social security and health spending reflects population aging, with spending per retiree rising less rapidly. This overlap between government policies and demographics shows up in the interaction bars in our decomposition. We discuss this issue in more detail below.

with our calculations based on the empirical elasticities.

Our final contribution is to use the two models to consider a wider range of secular trends in a coherent and unified way. This is motivated by the fact that much of the literature, including the studies cited above, maintain a narrow focus on one secular trend at a time. Some of the exceptions, such as cross-cutting studies of Davide Furceri and Andrea Pescatori (2014) and Łukasz Rachel and Thomas Smith (2015), used a simple reduced-form saving-investment framework to aggregate the different influences and thus suffered from a potential consistency problems – for instance, it was impossible to detect any non-linearities or prevent double-counting.[8] We can speak to this issue because, despite their rich structure, our models are highly tractable and well-suited for an internally consistent analysis of several factors widely regarded as instrumental in explaining the safe rate trends. We show how to use the models to quantitatively assess the impact of the slowdown in productivity growth, the demographic shifts, and the rise in income inequality. As a result, we arrive at a quantitative decomposition of the decline in the real interest rate in advanced economies which takes into account both private and public sector forces (Figure 4). Taken together, all these factors under-explain the decline in advanced economies' neutral real rate that we estimated.

**Figure 4:** Changes in the equilibrium real interest rate as a result of policy, demographic and technological shifts




Our findings suggest that the private sector forces dragging down on interest rates are more powerful than previously anticipated, and that on average across the business cycle, equilibration of private-sector saving and private-sector investment may indeed require very low real rate of interest in advanced economies for years to come. This conclusion is consistent with findings of

[8] Eggertsson et al. (2019b) construct a large quantitative macro model and use it to consider several hypotheses.

Oscar Jorda, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick and Alan Taylor (2017), who established that the current low levels of interest rates are not unusual in historical terms.[9] It is also consistent with the Japanese experience.[10] Our findings raise the possibility that the developed world is at risk of mirroring the experience of Japan, whereby the very low equilibrium rate of interest appears to be a semi-permanent feature of the economic landscape.

The remainder of this paper is structured as follows. Section 2 contains the results of the estimation of the long-term equilibrium real interest rate for advanced economies. Section 3 starts with a discussion of the channels through which government policy influences the equilibrium rate; it then summarizes the results from the existing empirical literature which estimates the size of these effects; and finally it uses these elasticities to calculate some back-of-the-envelope measures of how government borrowing affected R*. In Section 4 we set up the two general equilibrium models and use them to study the impact of government policies on R*. Section 5 contains further simulations, arriving at the full decomposition of the decline in the natural rate, including the impact of secular demographic changes, slowdown in technology and the rise in inequality. Section 6 concludes. Throughout the main text we focus on the results and keep the technical analysis to the minimum, delegating the details to the Appendices.

# 2 Estimating the AE equilibrium real interest rate

We estimate the natural rate of interest for advanced economies adopting what is perhaps the most celebrated applied empirical model designed for this purpose, originally due to Laubach and Williams (2003) and recently re-applied internationally by Holston et al. (2017b). Conceptually, this approach draws on two strands on the literature. By following Wicksell's (1989) definition of the natural rate as the rate consistent with stable inflation and output remaining at equilibrium ("potential") level, it is well aligned with the modern monetary theory, as in Walsh (1998), Woodford (2003) and Gali (2008). That literature is primarily concerned with fluctuations at the business-cycle frequency, where shocks move the economy around a stable steady state. In addition to those business-cycle shocks, the framework employed here is flexible enough to capture secular forces that affect the steady state.

The Laubach and Williams (2003) (henceforth LW) model is particularly appealing in the context of large economies that can be reasonably approximated as closed. In an open economy the procedure may be problematic in that periods of low growth may be associated with exchange rate misalignment rather than with a decline in the equilibrium interest rate, particularly if

[9]Theoretical work focused on the possibility that the real rate remains depressed for a long periods of time or even indefinitely. Alejandro Justiniano and Giorgio Primiceri (2010), Gauti Eggertsson and Neil Mehrotra (2014, 2016, 2019b), Bob Hall (2017), as well as Ricardo Caballero, Emmanuel Farhi and Pierre-Olivier Gourinchas (2016, 2017) provided formal models in which this possibility arises.

[10]For studies of the natural rate in the context of Japan, see Fujiwara et al. (2016), Okazaki and Sudo (2018) and Wynne and Zhang (2018).

coupled with a current account deficit. Our contribution is to carry out the analysis for the advanced economy block, which alleviates that concern somewhat, at least relative to the past literature which proceeded to estimate the model on the individual country data.

There are two implicit assumptions embedded in our exercise. First, our aggregation procedure effectively assumes that the advanced economies block is fully integrated – we simply use aggregated data for advanced economies, 'as if' the block was a single economic entity. As a result, our natural rate and trend growth estimates should be interpreted as the average across the developed world. Because of the high degree of integration across advanced economies, these advanced economy estimates will exert a strong gravitational pull in each of the individual countries. Nonetheless, the individual-country natural rate will depend on its idiosyncratic characteristics and shocks. The second assumption implicit in this exercise is that advanced economy block is a closed economy. While we view this assumption as a gross approximation to reality, we point out that the advanced economy net saving as illustrated by the current account has been an order of magnitude smaller than the current account gaps of individual countries (Figure 5). It also varied much less over the past 30 years fluctuating by less than 1.5 percent of GDP from peak to trough and trending upward through time suggesting that if anything external shocks have raised rather than reduced the estimated neutral real interest rate. These observations give us some confidence that, as a first order approximation, our assumptions are valid.

## 2.1 Sketch of the model and the estimation procedure

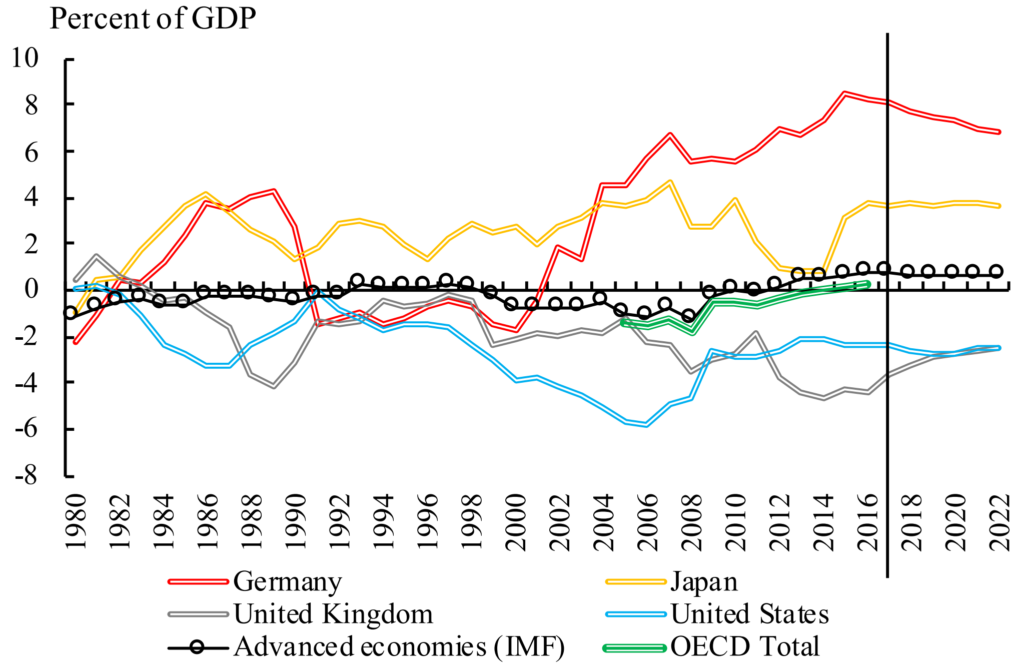
Our approach to estimating the LW model is deliberately off-the-shelf: we use exactly the same procedures as the recent papers in that literature. Out contribution is solely to perform this exercise on the block of advanced economies as a whole. As such, we do not take a stance on the performance of the model, although we discuss some of the issues below.

The philosophy of the LW method is that the natural rate of interest is an endogenous object determined in general equilibrium, and as such it will depend on a host of socio-economic forces, such as trends in preferences, technology, demography, policies and policy frameworks, and so on. It is impossible to know and measure all of the relevant factors. At the same time, a robust prediction of most workhorse macroeconomic models is that the natural rate should vary together with the (expected future) trend growth rate of the economy.[11] To reflect the dependence on growth and on a range of (possibly unknown) other factors, the LW model assumes that the natural rate, denoted $r_t^*$, depends on the estimated trend growth rate of potential output $g_t$ and a time-varying unobserved component $z_t$ that captures the effects of other unspecified influences:

$$r_t^* = g_t + z_t. \tag{1}$$

[11] We discuss the rationale for this link in some detail in Section 5.

**Figure 5:** Current account in advanced economy block and in selected individual economies




Note: The black line shows the current account for advanced economies as defined by the IMF. The green line shows the current account for aggregate OECD block. The OECD data are available only since 2004. Over the period when both data series are available they are very close to each other. Source: IMF and OECD.

The model further assumes that both the growth rate $g_t$ and the unobserved component $z_t$ are random walk processes:

$$g_t = g_{t-1} + \epsilon_{g,t} \quad \epsilon_g \sim N(0, \sigma_g^2) \tag{2}$$

$$z_t = z_{t-1} + \epsilon_{z,t} \quad \epsilon_z \sim N(0, \sigma_z^2) \tag{3}$$

The model specification also admits shocks to the level of potential output. Denoting by $y_t^*$ the natural logarithm of potential output at time $t$:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*,t} \quad \epsilon_{y^*} \sim N(0, \sigma_{y^*}^2). \tag{4}$$

In short, the LW model views the natural rate as the sum of two independent random walks. To achieve identification, LW add two further equations to the model. First, they specify a simple reduced-form equation relating output gap to its own lags, a moving average of the lagged real funds rate gap, and a serially uncorrelated error:

$$y_t = y_t^* + a_1(y_{t-1} - y_{t-1}^*) + a_2(y_{t-2} - y_{t-2}^*) + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \epsilon_{y,t} \quad \epsilon_y \sim N(0, \sigma_y^2) \tag{5}$$

The key in this estimated IS relation is the $a_r$ coefficient, which we expect to be negative. Second, LW add the reduced-form Phillips curve to the model, linking current inflation $\pi_t$ to lagged inflation and the output gap:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y(y_{t-1} - y^*_{t-1}) + \epsilon_{\pi,t} \quad \epsilon_\pi \sim N(0, \sigma^2_\pi), \tag{6}$$

where the standard theory would suggest that coefficient $b_y$ is positive.

The system above can be written in a state-space form, and the Kalman Filter can be used to estimate the unobservable states. To estimate the model, we use data for advanced economies as a block. The data comprise of (log) quarterly real GDP, core inflation and long-term interest rates over 1971Q1:2017Q4 for the aggregated sample of OECD countries. The interest rate series is the arithmetic average of long-term nominal interest rates across an unbalanced panel of 36 OECD economies.[12] To calculate real rates, we subtract from nominal rates a simple measure of expected inflation, constructed as the moving average of past core inflation rates, in line with Holston et al. (2017b). See Appendix B for further details on the data and the estimation procedure.

## 2.2 Results

Table 1 shows the coefficients of the estimated model. Point estimates are all significantly different from zero and have expected signs. In particular, a positive interest rate gap reduces the output gap, while a positive output gap raises inflation. Table 1 also shows the standard errors around the estimated trends, which are large, especially those around the estimates of equilibrium real rate. These wide standard error bands are not specific to our results – indeed, they are a norm in the literature. For instance, Holston et al. (2017b) report similarly large errors for individual economies. These errors are, to an extent, an artifact of the long-sample, as they reflect the cumulative uncertainty of the underlying drivers of equilibrium rates. Nonetheless, these large error bands should act as a reminder of the high uncertainty surrounding the econometric estimates of equilibrium interest rates.

Figure 6 contains the key results. The solid lines are the estimates of the trend growth rate and the natural rate of interest for advanced economies. According to our estimates, AE R* declined steadily from 1980s onwards, and fell sharply during the crisis.[13] It then stabilized at low levels ($\approx 0.5\%$). The estimated growth rate of potential output has been broadly stable up until the crisis, and declined during the crisis by about 1pp. Thus the model suggests that a bulk of the decline in real interest rates is due to factors other than trend GDP growth. This

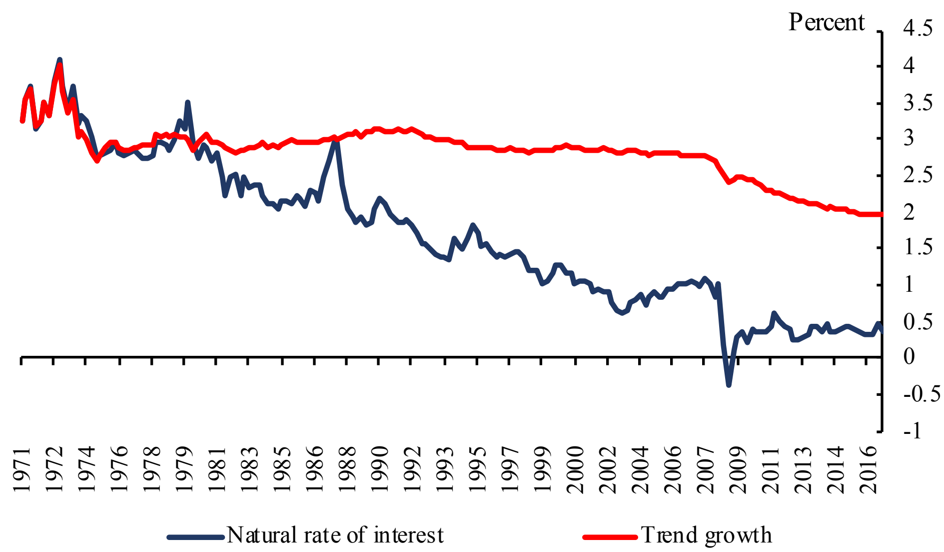
[12] The results are robust to using weighted average or median of the interest rates across countries. Given the strong comovement, these interest rate series are close to each other.

[13] Estimates for the first decade should be taken with a grain of salt, as the model is less accurate during the first few years of the sample while the initial conditions play a larger role.

**Table 1:** State-space model parameter estimates

| Parameter point estimates (t-statistics in parentheses) | | | | | | | |
|---|---|---|---|---|---|---|---|
| $a_1$ | $a_2$ | $a_r$ | $b_\pi$ | $b_y$ | $\sigma_y$ | $\sigma_g$ | $\sigma_z$ |
| 1.71 | -0.79 | -0.04 | 0.90 | 0.09 | 0.25 | 1.03 | 0.31 |
| (21.65) | (10.28) | (2.13) | (17.78) | (2.06) | (5.30) | (29.63) | (9.38) |
| Average standard errors around the estimates | | | | | | | |
| | | $y*$ | $r*$ | $g$ | | | |
| | | 1.19 | 3.12 | 0.16 | | | |

**Figure 6:** AEs R* and trend growth




is consistent with the literature that finds only a loose connection between actual GDP growth and interest rates in historical data (Hamilton et al. (2016)).

These results corroborate other existing findings in the literature. In particular, Holston et al. (2017b) estimated that the declines in real rates for US, Canada, Euro Area and the UK of around 2.3pp between 1990 and 2017; for comparison, the decline over this period for AEs as a whole that we estimate here is around 2pp.

Overall, despite large uncertainty surrounding the point-estimates of these trends, we interpret the results of this exercise as broadly in line with the country-level findings in the literature. Indeed, given the high level of aggregation, we find it encouraging that the estimated unobservables do well at picking up the main events, such as the global financial crisis, during which our estimate of AE R* declines very sharply. We now turn to the discussion of the forces that may be behind this decline.

# 3 Government policy and the equilibrium interest rate

Our focus is on the role of the public sector. Over the past several decades, government policy in the developed world has shifted significantly in at least four respects (Figure 7). First, government debt has risen, from around 20% of GDP to around 70% (government consumption – excluding healthcare – remained relatively stable). Second, old-age payments administered through the social-security and healthcare systems have gone up, from 4% to 7% and from 2.5% to 4% of GDP, respectively, accounting for the lion share of the increase in total social spending (Figure 8). Third, significant changes to tax policies have taken place. The effective corporate tax rates in the rich economies have fallen, from around 32% at the turn of the century to 24% more recently.[14] Wealth taxes, operational in 12 OECD countries in 1990, remain in place only in 4 counties today (OECD (2018)). And, as documented by Thomas Piketty and Emmanuel Saez (2007), the overall progressivity of the tax system has decreased in some jurisdictions – notably in the United States and the United Kingdom. Finally, in some countries, the public provision of credit has increased: in the United States, for example, such provision amounts to around 7% of government debt.

These shifts are likely to have had a profound impact on the economy in general, and on the equilibrium rate of interest in particular. Specifically, all of these shifts – perhaps with the exception of the wealth and personal tax changes[15] – are likely to have pushed interest rates *higher* over the past 30 years. In this and the next section we turn to the analysis of the impact of these policy shifts on the natural rate, with the ultimate goal to inform the counterfactual 'pure' R* that would prevail without government intervention.

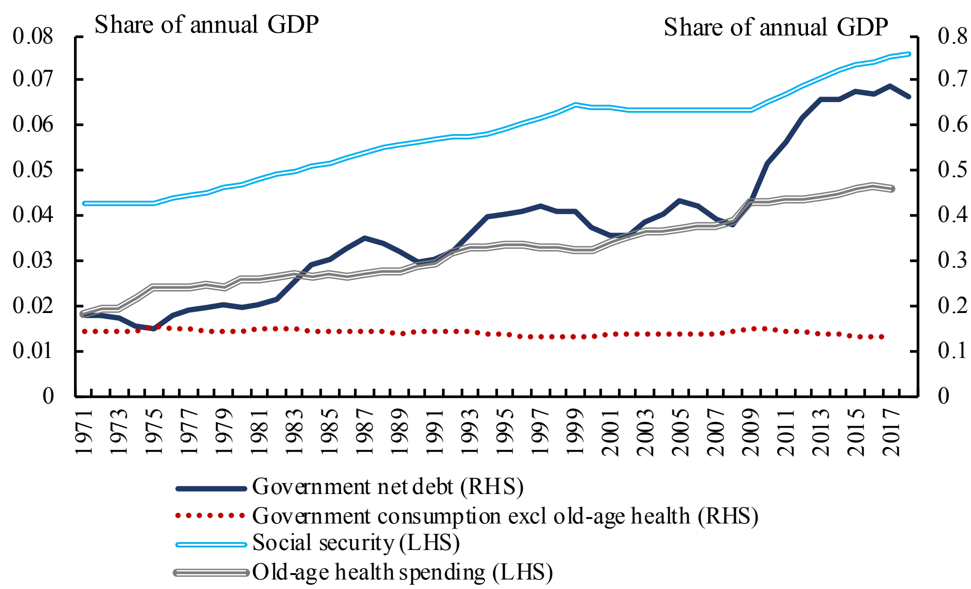
We focus on government debt, social security and healthcare spending, leaving the formal analysis of the impact of tax changes for future work. We find that shifts in government policy have likely pushed up on the equilibrium rates of interest by a significant amount over the period in question. As a rough rule of thumb on the magnitudes involved, our analysis suggests that the tripling of the government debt over the past half century raised rates by 1.5-2pp, while the expansion of social spending of around 5% of GDP added a further 2.5pp. While the precise magnitudes of these multipliers are subject to substantial model and statistical uncertainty, the qualitative conclusion is clear: had the public policy not responded, the advanced world's equilibrium rate would likely be deeply negative.[16]

[14] See http://www.oecd.org/tax/tax-policy/fig7-avg-statutory-tax-rates-by-region-large.png for details.

[15] Smaller incidence of wealth taxation likely reinforced the incentives to accumulate assets, particularly at the top of the wealth distribution. The magnitude of this effect will be governed by the elasticity of capital supply. The nascent but active literature puts these elasticities in the range of -3 to -20 (a one percentage point increase in wealth tax leads to a 3-20% change in wealth). See Zoutman (2015), Jakobsen et al. (2018), Brülhart et al. (2016) and Seim (2017).

[16] A corollary of this link between government debt and interest rates is that a higher value of public debt, compared to market expectations, is likely to raise the natural interest rates. For analysis of this argument, see Kocherlakota (2015).

**Figure 7:** Advanced economies government policy ratios (in proportion to GDP)



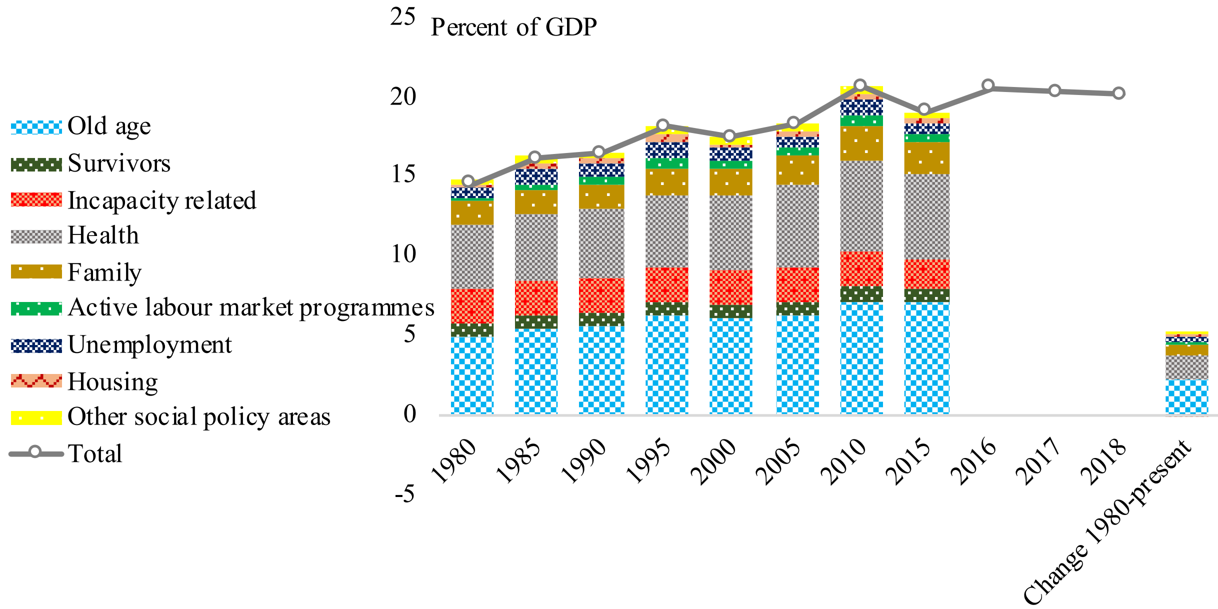

Notes: The Figure shows OECD aggregates in proportion to total OECD GDP. All data are from the OECD. Government net debt line measures the general government net financial liabilities, from the OECD Economic Outlook Database. It includes net government debt held by the public and also other net liabilities of the government. For example, in the United States in 2017, the net financial liabilities as reported by the OECD were 80% of GDP, while net debt held by the public was 75% of GDP. Government consumption figures represent the general government final consumption expenditure, adjusted by subtracting the old-age health spending (note that this series excludes the social security transfers by default). The old-age health spending is calculated as the aggregate health spending on ages 65+. The overall health spending figures are from the OECD/WHO statistics on sources of funding for healthcare. They include healthcare financed directly by the government and from the compulsory schemes. The old-age share is then calculated under the assumption that 60% of total health spending is directed at the older demographic groups, consistent with the evidence available for a handful of OECD countries.

### 3.1 A brief review of the theoretical arguments

We begin by reviewing the effects of government policy on the equilibrium interest rate, focusing on government borrowing, as this has been the main subject of the large literature in macroeconomics which we can draw on. We next describe the key takeaways from three classes of models: the flow-based IS/LM, the neoclassical growth model, and a range of incomplete markets / heterogenous agents models.

In the IS/LM model the interest rate is determined in the flow equilibrium of goods and financial markets. It is thus the flow of government spending and the flow of budget deficits that matter for the determination of the interest rate. A higher deficit – operating through higher government spending or lower net taxes – shifts the IS curve to the right, raising interest rates in the short run. To the extent that in the medium-to-long-run the economy faces a vertical aggregate supply curve, the model predicts that the price level will tend to drift upwards as the economy operates above its potential. This reduces the real money stock (shifting the LM curve to the left) and thus further raises the interest rate until the long-run equilibrium level of output

**Figure 8:** Public social spending in the OECD




Notes: Data are from the OECD Social Expenditure Database (SOCX). The data for 2016-2018 are preliminary and the breakdown by function is not yet available.

is restored.[17] This model thus predicts that in the new equilibrium (with *permanently* higher deficits) the real interest rate is higher. However, a *temporary* increase in deficits will only have a temporary effect on real rates.

In the canonical neoclassical model with complete markets and infinitely-lived agents, Ricardian Equivalence holds and neither deficit nor debt are relevant, as the representative household can fully offset the changes to government's borrowing policy through its saving decisions. Thus independent shocks to government borrowing alone have no effect on the equilibrium interest rate. The neoclassical model instead emphasizes the link between the stock of capital and the interest rate: in equilibrium $r = f'(k) - \delta$.[18] Thus government policies affect the interest rate only to the extent that they impact on the stock of private capital.

A change in government *spending* can affect private sector accumulation of capital in the neoclassical economy. But the model suggests that such policy will raise the real interest rate only temporarily (a classic reference is Baxter and King (1993)). An increase in government spending constitutes a negative wealth effect for the representative consumer: it tightens the resource constraint. The consumer responds by reducing consumption and leisure and by raising labor supply. The marginal product of capital increases, driving up investment. In a new steady state, both capital and labor supply are higher, but their ratio – and hence the interest rate – are

[17] In a richer model, or indeed in the real world, this may be brought about by tightening of monetary policy.

[18] On the balanced growth path, the level of effective capital stock adjusts such that the interest rate simultaneously satisfies the balanced growth version of the representative household's Euler Equation: $r = \frac{1}{IES} \cdot g + \theta$ where $IES$ is the intertemporal elasticity of substitution and $\theta$ is the rate of time preference.

unchanged. The interest rate is higher only along the transition path. The model suggests that this can be quite persistent, with half-life of about 6 years in the classic calibration of Baxter and King (1993).

Finally, in the micro-founded modern macroeconomic models that depart from the representative agent and complete markets assumptions, Ricardian Equivalence does not hold, and government transfer policies affect the equilibrium allocations through several distinct channels.

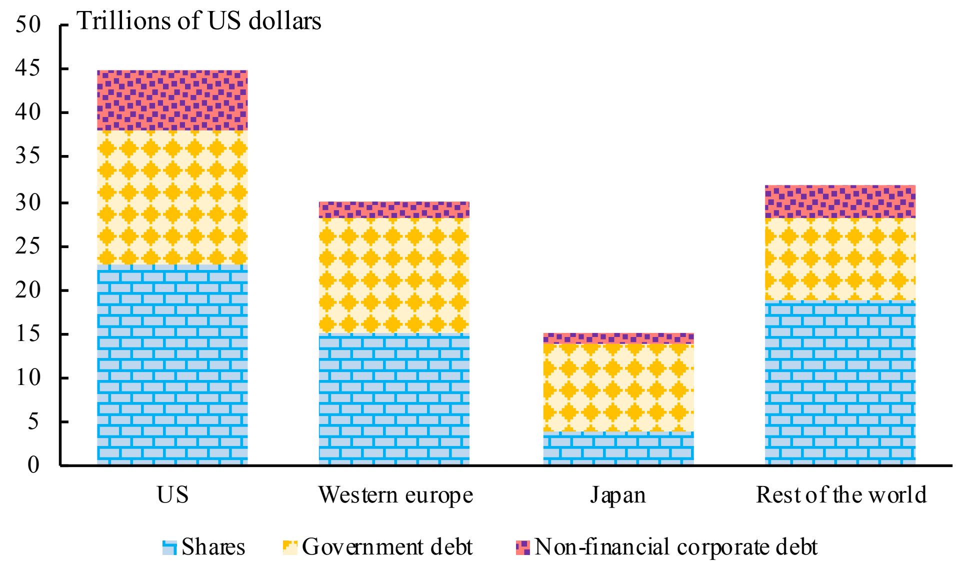
First, the intertemporal transfers – that is, *redistribution across time* – matters if peoples' planning horizons are finite. This could be because of finite lives coupled with less-than-perfect bequest motive, as in the seminal models of Peter Diamond (1965) and Olivier Blanchard (1985), or perhaps due to time-dependent preferences and myopic behavior pioneered by David Laibson (1997). The reason is intuitive: with finite planning horizon, agents currently alive expect to shoulder only a part of the financing burden that comes with today's transfer; the rest is to be serviced by future generations. Such transfers thus affect agents' wealth and their consumption and saving plans.

Second, *transfers across agents* can affect aggregate consumption and saving (and hence the interest rate) if agents have different marginal propensities to consume (MPCs). Differences in MPCs could arise because of several distinct features of the economic environment. They could be a result of uninsurable risks and binding borrowing constraints, as in the works of Rao Aiyagari and Ellen McGrattan (1993, 1997) and the model of Hyunseung Oh and Ricardo Reis (2012). They could emerge because some agents have little to no liquid wealth, preventing them from adjusting their consumption, as in the paper by Greg Kaplan, Giovanni Violante and Justin Weidner (2014). Another reason may be the life-cycle: propensity to consume may differs between workers and retirees, as in Gertler (1999), or may vary with age as in Gagnon et al. (2016) and Eggertsson et al. (2019b). Heterogenous MPCs and distortionary taxes deliver this result in the savers-spenders model of Gregory Mankiw (2000).[19] In all those models, government transfers from a low-MPC agent to a high-MPC agent will boost the aggregate desire to consume and lower desired savings, thereby raising the interest rate.

The third way in which government policy affects interest rates is what may be called a *precautionary saving channel*. One facet of this channel is that government policies can directly reduce the risks faced by the agents. The mechanism is close to the one analyzed by Engen and Gruber (2001). Under imperfect insurance, agents who face some idiosyncratic risks – for example those related to health or unemployment – attempt to self-insure through saving. This precautionary saving motive acts to push the interest rate below the rate that would prevail in a complete markets economy (where all risks are insurable and so do not affect the agents' behavior). Government policies such as social insurance will affect the importance of

[19]Interestingly, in the savers-spenders model of Mankiw (2000), if taxes are levied lump-sum, a deficit-financed transfer that permanently increases the level of debt does not affect the stock of capital or the interest rate in the long run. The reason is that the interest rate is pinned down by the savers, who are infinitely lived and Ricardian.

**Figure 9:** Total global investable financial assets (2013 data)




Source: Kay (2015).

precautionary saving: a stronger social safety net or higher unemployment and disability benefits curtail the associated risks, curbing the desire to save. Conversely, lack of social insurance means that agents need to rely on their own resources when experiencing hardship, making personal saving a priority. However, as illustrated in Figure 8, the overall size of the social safety net across the OECD has changed little over the period in question. We do not attempt to model it here, but leave it as an important direction to be explored in future work.

The other facet of the precautionary saving channel – and the one we focus on in this paper – works through the provision of assets and liquidity which agents use to insure themselves against shocks. This mechanism is at the heart of Aiyagari and McGrattan (1997) and has recently been discussed in the context of secular stagnation in Caballero et al. (2016) and Caballero and Farhi (2014). The intuition we have in mind is simple: a rise in government debt raises the overall supply of assets in the economy, which, all else equal, pushes interest rates up. Indeed, there is evidence in the data that government debt constitutes a non-trivial proportion of the total investable financial assets in the developed world, so that this channel can have a quantitative bite. The estimates of the share of government bonds in total financial assets range from one-third in the US to two-thirds in Japan (Figure 9).

An important issue in the context of these channels is the quantitative easing (QE) policy pursued by advanced economy central banks over the crisis period. It is useful to distinguish between economically two distinct kinds of QE. The first kind encompasses policies that swap risky assets for safe assets and includes policies such as QE1, LTRO, and many other lender of last resort central bank interventions. The second kind is a policy whereby the central bank issues reserves to buy risk-free debt. The policies that belong to the first kind can indeed alleviate safe asset shortage during a crisis, as shown in the formal model of the "safety trap" by Ricardo Caballero and Emmanuel Farhi (2018). Such policies would thus raise the *short-run* natural rate of interest. This logic captures an important transmission channel of these unconventional tools. It is not, however, the focus of our paper, which focuses on the *long-run* real neutral interest rate – that is, the interest rate that prevails once cyclical conditions and policies have washed out.[20] Indeed, the composition of the Fed balance sheet shows that, in the United States, this first kind of QE was very short-lived, with the amount of risky assets on the Fed balance sheet scaled back quickly as the worst period of the crisis has passed. The second kind of QE proved much more persistent, with central banks balance sheets still elevated and close to their peaks today. But QE of the second kind constitutes primarily a maturity transformation of government debt, rather than change in the total availability of investable assets. Consequently, through the lens of our model (or the model of Caballero and Farhi cited above) such policy would not have any effect. It is possible to write down richer frameworks, notably ones where various assets have differing degree of liquidity. In those frameworks QE can indeed have (potentially large) real effects through changing the liquidity composition of households asset holdings. But the literature on this is still in its infancy (Cui and Sterk, 2018).

In summary, macroeconomic theory developed over the past couple of decades enriched the basic model of Frank Ramsey and Robert Barro (Barro (1974)) with several channels that make the government policy a relevant determinant of the long-term interest rate. We now turn to the empirical evidence that has been accumulated in parallel to these theoretical advances.

## 3.2 Empirical evidence on the link between government policy and long-term interest rates

The main challenge when estimating the effect of government borrowing on interest rates is the large number of potentially confounding factors which may make simple regressions of interest rates on debt spurious and uninformative. Shifts in both desired saving and desired investment brought about by secular trends unrelated to fiscal policy will affect the interest rate and will likely be correlated with headline measures of government borrowing. Unaccounted for, these factors will introduce an omitted variable bias into econometric estimates.

We shall not attempt a full-blown empirical assessment in this paper, and instead present a summary of the empirical estimates in the literature. For an interested reader, Appendix C illustrates several challenges of estimating the causal relationship between equilibrium interest rates and government debt through a simple empirical exercise for the US, Canada, Euro Area and the UK. These challenges include the presence of international capital flows and of endogenous responsiveness of policy to excess of private saving over private investment, both of which are

[20]The reader may want to think of the short-run equilibrium real rate as fluctuating cyclically around the long-term real rate.

likely to attenuate the individual-country estimates of the impact of deficits on interest rates. With that caveat, we now present the estimates from a broad literature that attempted to deal with these and other confounding factors in finding the link between government finances and real rates.

Several key studies in the empirical literature focused on the United States. In a chapter of the *Handbook of Macroeconomics* at the turn of the century, Douglas Elmendorf and Gregory Mankiw (1999) reviewed the theoretical and empirical literature on the Ricardian Equivalence proposition, concluding that, while the studies that attempted to estimate the impact of government finances on interest rates cannot reject the null hypothesis of zero impact, they suffer from lack of statistical power.[21] More recent work appears more conclusive. In their literature review of this topic, William Gale and Peter Orszag (2002) conclude that the effect of government deficit on the real rates is positive and economically significant: a 1pp increase in the deficit-to-GDP ratio tends to raise interest rates by around 50-100bps. And the two most authoritative contributions on the topic suggest estimates that are significant, albeit somewhat smaller. Thomas Laubach (2009) studies how forward rates on government securities react to news in CBO's fiscal forecasts. The identifying assumption in his work is that long-term rates and forecasts are not contaminated by current events and shocks at the business cycle frequency. According to his estimates, a rise in government deficit of 1pp of GDP raises interest rates by about 20-30bps; an equal increase in debt/GDP ratio results in a rise of about 3-4bps. He asserts that these flow- and stock-multipliers are broadly consistent, because of the autocorrelation of the deficits observed in the data.[22] Another important contribution to this literature is that of Eric Engen and Glenn Hubbard (2004), who consider a host of specifications linking interest rates or changes in interest rates to government debt or to the deficit, both contemporaneously and in a forward looking setting. Their results suggest that a 1pp rise in government debt / GDP pushes interest rates up by about 3bp, broadly in line with Laubach's findings.[23]

Further evidence is available for advanced economies beyond the United States. In an international setting, Anne-Marie Brook (2003) documents that the range of estimates of the effect of a 1pp increase in government debt/GDP ratio on interest rates is 1-6bps, with the corresponding range for a 1pp increase in deficits in the region of 20-40bps.[24] In an important study of the

[21] They write of the literature that tends to find close to zero effect of government deficit on rates: *"Our view is that this literature [...] is ultimately not very informative. [...] Plosser (1987) and Evans (1987) generally cannot reject the hypothesis that government spending, budget deficits, and monetary policy each have no effect on interest rates. Plosser (1987) also reports that expected inflation has no significant effect on nominal interest rates. These findings suggest that this framework has little power to measure the true effects of policy."*

[22] Specifically, he estimates the autocorrelation of 0.83, implying that the 1pp rise in the deficit should have $\frac{1}{1-0.83} = 6$ times the effect of a 1pp rise in debt – broadly in line with what he finds.

[23] The results vary across different specifications, highlighting that the precise econometric details matter for the conclusions of this line of empirical research.

[24] For the Euro Area, several older papers focused on the impact of fiscal policy on government bond spreads rather than interest rates. These papers tend to find smaller effects: on average, 1pp increase in deficit-GDP ratio raises spreads by around 10bps.

**Table 2:** Impact of government borrowing on the interest rate: summary of the literature

| Study | Country / region | 1pp increase in deficit/GDP | 1pp increase in debt/GDP |
|---|---|---|---|
| Gale and Orszag (2002) | US | 50-100bps | - |
| Laubach (2009) | US | 20-30bps | 3-4bps |
| Engen and Hubbard (2004) | US | 18bps | 3bps |
| FRB/US model | US | 40-50bps | - |
| Faini (2006) | Euro Area | 40bps | - |
| Brook (2003) | Advanced economies | 20-40bps | 1-6bps |
| Kinoshita (2006) | 19 OECD economies | - | 4-5bps |
| *Average* | | *38bps* | *3.5bps* |

Euro Area, Riccardo Faini (2006) finds that a 1pp rise in deficits at the Euro Area level raises long-term rates by around 40bps, close to – and if anything, higher than – the US multipliers. Considering an even wider panel of 19 OECD economies spanning 1971-2004, Noriaki Kinoshita (2006) finds that the effect of a 1pp rise in government debt-to-GDP ratio is to raise interest rates by 4-5bps.

A complementary way to assess the size of these effects is to consider simulations from large-scale models used for quantitative analysis in policy institutions. Because these models are carefully estimated using real-world data, they should be able to provide a steer as to the size of the effects. A well known example is the FRB/US model, used and maintained by researchers at the Federal Reserve Board (Laforte and Roberts, 2014). In a recent speech, Stanley Fischer (2016) uses this model to estimate the impact of a persistent increase in deficit on real rates, and finds that a 1pp increase in deficit raises the equilibrium rate by between 40 and 50bps, depending on whether the deficit increased because of a tax cut (smaller effect) or a rise in government spending (larger effect). These figures are thus slightly larger than the empirical estimates cited above.

In summary, the estimates in the literature paint a fairly consistent picture: a 1pp rise in deficit tends to raise interest rates by around 30-40bps; while a 1pp rise in debt/GDP ratio results in an increase of about 3-5bps (Table 2). We suspect this figure is an underestimate of the impact of an exogenous increase in budget deficits on real rates because fiscal expectations are measured with error, because any one country can import capital and so attenuate rate increases when budget deficits increase, and because there will be a tendency – as fiscal policy is used to stabilize the economy – for periods of low neutral real rates to coincide with periods of expansionary fiscal policy.

### 3.3 The historical impact of government borrowing on R*

The elasticities identified in the empirical work, combined with the historical path of government borrowing, give simple back-of-the-envelope estimates of the historical influence of fiscal policy on real interest rates. The bottom line is that the rise in government borrowing over the past 40 years has likely pushed interest rates higher, perhaps by as much as 1.5pp. We now present the details.

Focusing first on the US, we estimate the historical impact using either the deficit or debt figures and the respective multipliers discussed above. To be consistent with the bulk of empirical papers, we use the CBO definitions of government debt and deficit, and distinguish between the headline government budget deficit and the primary deficit, which excludes net interest payments.[25] The additional benefit of using the CBO figures is that they include the forecasts for the relevant variables, which allows for a peek into the future, all the way to 2027 using the June 2017 projections.

Figure 10 shows three estimates of the impact of government borrowing on interest rates in the United States, with positive values denoting upward pressure on real rates. This simple calculation suggests that public borrowing exerted an upward influence on interest rates in the US for much of the period, perhaps except the early 2000s. On this basis, the private sector neutral rate in the US – the measure that strips down the effect of public debt – would have been about 1pp *lower* on average, and between 1 and 2pp lower in the most recent period.[26]

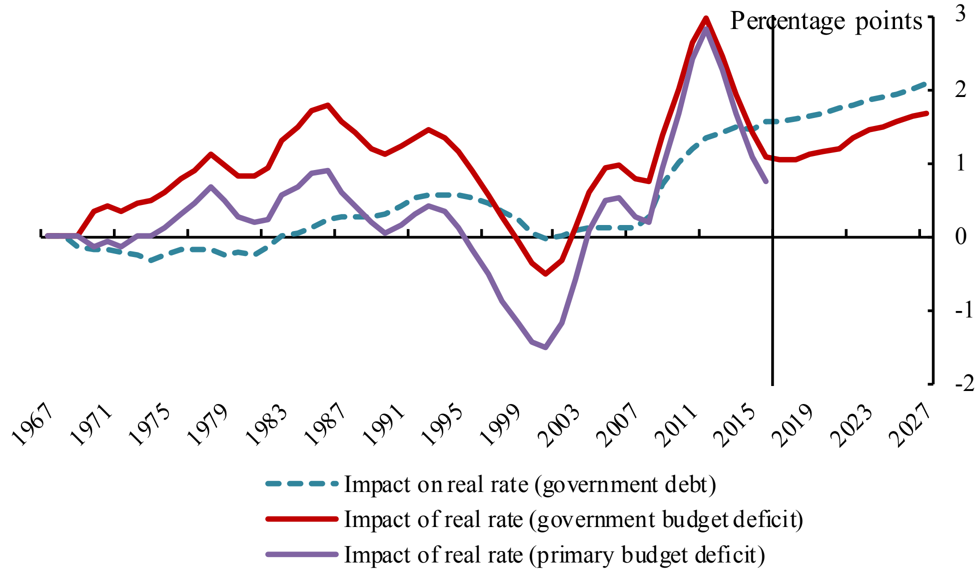
But the rise in government debt was not confined to the US alone. To examine how this force has shaped interest rates across the advanced world, we use the data on government debt-to-GDP ratios in the OECD. Figure 11 shows the estimate of the average fiscal policy impact on real rates across these economies. The effect builds steadily from the start of the 1980s, with the average estimate of the impact of under 1pp up until the crisis and around 1.5pp more recently.

Excluding the effect of higher government debt, Figure 12 shows the equilibrium rate of interest for advanced economies estimated in Section 2, and then subtracts the calculated impact of fiscal policy in Figure 11. The resulting proxy for the private sector R* that excludes the impact of public debt hovered around zero since the early 2000s, and remains negative at the

[25]Using the primary deficit helps address the concern of reverse causality from the interest rate to projected deficits through higher outlays on debt service. For the US Laubach (2009) estimates that a 1pp increase in deficit/GDP ratio raises interest rates by 25bps when he uses the headline deficit to estimate the effect, and 29bps when he uses the primary deficit. Thus the numbers are fairly close, but the primary deficit seems to have a somewhat larger effect.

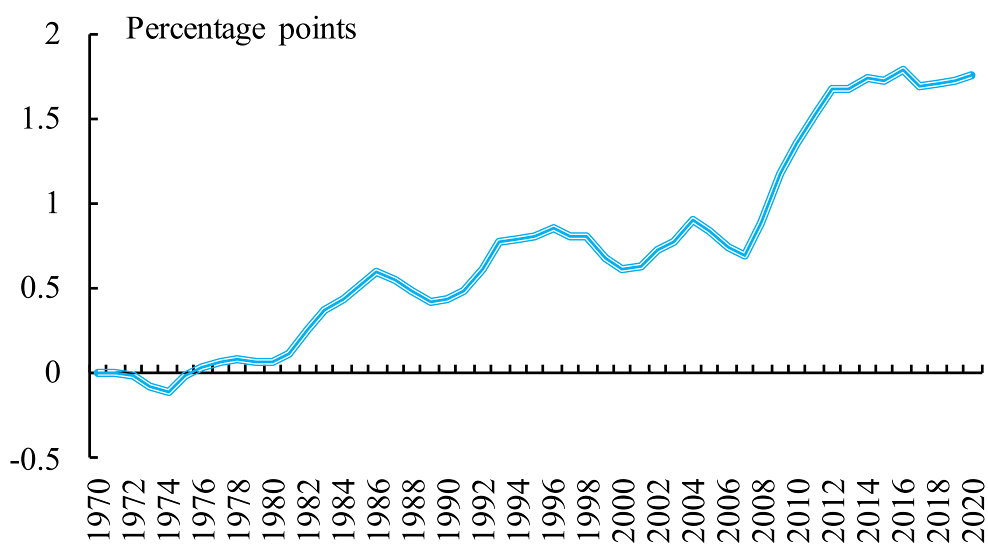
[26]Our calculations exclude the role of public credit provision, guarantees or subsidies (we use net debt held by the public in these calculations). According to the The U.S. Congressional Budget Office (2018), the total obligations of the US government as part of the federal credit programs amount to $1.5trn, which is roughly 7% of US government debt. Deborah Lucas (2016) studies the role of federally backed credit in stimulating the macroeconomy following the 2007/08 financial crash and discusses the data indicating the large increase in loan guarantees and direct loans over the past 40 years (and in particular the historic increase in the crisis). Thus the inclusion of these obligations in the empirical calculations and in the model-based simulations would likely further raise the size of the impact of government policies on real rates.

**Figure 10:** Impact of government borrowing on US interest rate



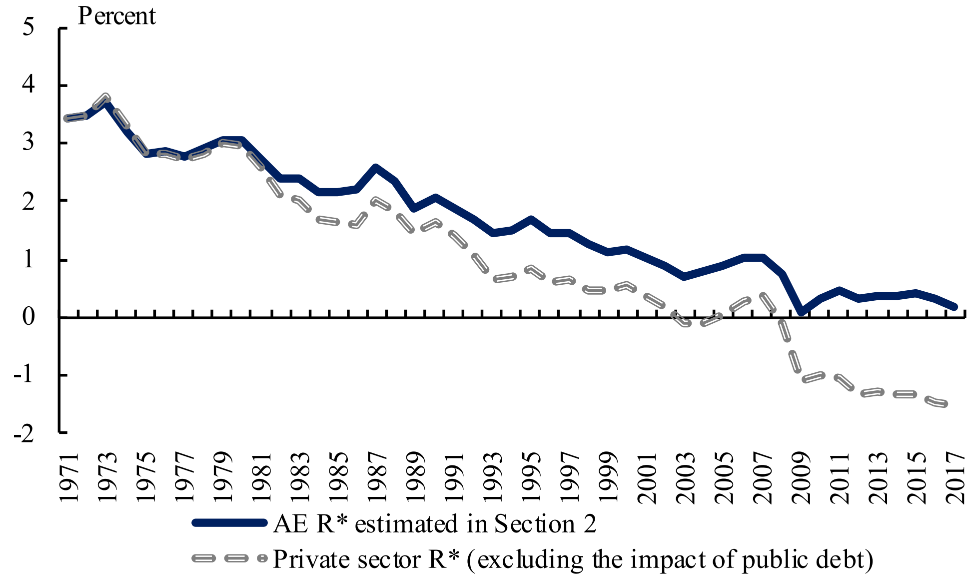

Note: The calculation underlying the deficit line assumes that a 1pp increase in government deficit raises the interest rate by 35bps. The corresponding figure for debt is 3.5bps. Both figures are meant to represent the consensus in the literature discussed above.

**Figure 11:** Impact of government borrowing on the equilibrium rate in advanced economies




Note: Constructed using the increase in government net financial liabilities across the OECD. Assumes that an additional 1pp increase in the ratio of government debt-to-GDP raises interest rates by 3.5bp.

**Figure 12:** Advanced economies R* adjusted for the impact of government debt




Note: The figure shows the estimated equilibrium real interest rate in advanced economies, and an adjusted measure that subtracts the impact of government borrowing in Figure 11.

moment.[27]

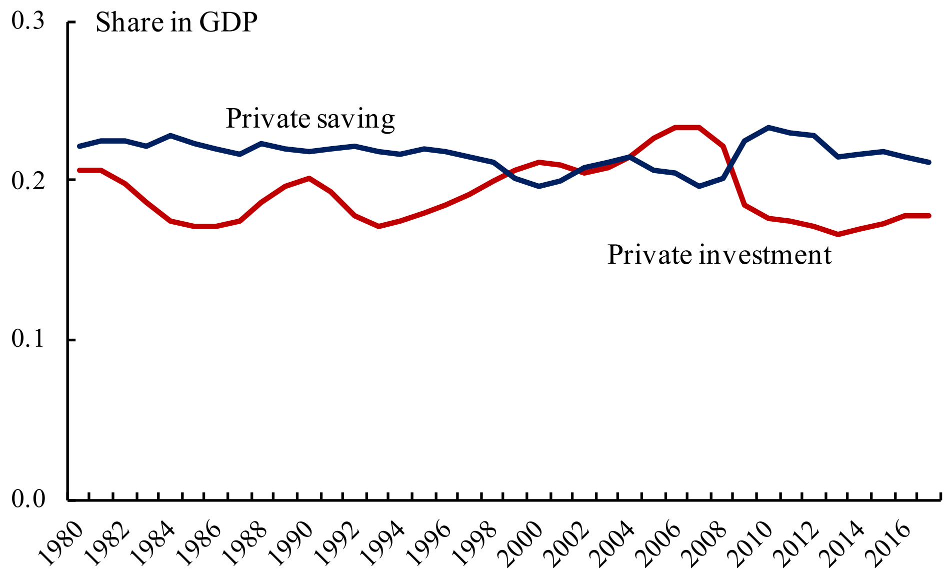
Are these results plausible? We think they are, for two reasons. First, the low levels of the private sector equilibrium rates identified here are consistent with some other estimates, for example with the recent work by Pierre-Olivier Gourinchas and Helene Rey (2016), who study the behaviour of the *private* consumption-to-wealth ratio. They conclude that real rates may be around -2% over the next decade. This is broadly in line with our adjusted measure in Figure 12. Second, the private sector real rate at around -2% is also consistent with the behavior of the *realized equilibrium private sector saving and investment rates.* The ratios of private saving and private investment to GDP have been stable up until the crisis, when they diverged sharply (Figure 13). This behavior is consistent with the path of the private sector real rate in Figure 12: during the great moderation period, cyclical shocks and structural shifts were absorbed by the changes to the interest rate, leaving the quantities of saving and investment little changed. But during the Great Recession the private sector equilibrium rate may have reached what could be its lower bound (with inflation targets anchored at around 2%), meaning that more of the equilibration occurred through quantities rather than prices.[28] Finally, our results are quantitatively consistent with the simulations in Eggertsson et al. (2019b).

To develop further intuition and to consider other mechanisms through which public policy may have affected the interest rate, we now turn to a complementary approach: a general equilibrium modeling framework.

[27]This is an imperfect and incomplete proxy, however, as it misses the potential impact of other government policies such as social security and healthcare provision. We turn to those forces in the following section.

[28]For a formal exposition of a similar argument, see Caballero et al. (2016).

**Figure 13:** Private sector saving-to-GDP and investment-to-GDP ratios in advanced economies




Source: IMF, OECD and authors' calculations.
Notes: The figure shows PPP-weighted gross private saving and gross private fixed capital formation across the following countries: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom, United States.

# 4 Government policy and R*: a model-based assessment

## 4.1 Two general equilibrium models

In Section 3.1 we outlined various channels through which government debt may affect the equilibrium real interest rate; our goal in this Section is to illustrate their quantitative importance within a general equilibrium framework. We want our approach to be simple and transparent, providing a credible complement and a cross-check to the empirical analysis above.

To achieve these goals, we build not one but two general equilibrium models: one capturing the finiteness of life and life-cycle heterogeneity, and another which focuses on precautionary behavior. We judge that two simpler models may be better than one complex one. Our approach allows for a coherent assessment of the importance of different channels while remaining clear and easy to understand and replicate.[29]

The first model, which builds closely on Gertler (1999), highlights life-cycle heterogeneity. In this economy, ex-ante identical individuals are at different points in their lives: some are working, some are already retired. This drives the differences in their consumption and saving behavior. The framework is similar to that of Blanchard (1985) and Yaari (1965) – individuals face constant probability of death and so their horizons are finite – but, in addition to their model, workers retire and finance consumption with savings until death.

[29]We leave combining the two models for future work.

The second model is a Bewley-Huggett-Aiyagari economy with incomplete markets and uninsurable income risk at the level of an individual household. A similar model was considered by Aiyagari and McGrattan (1997) who also studied the role of government debt on the equilibrium allocation in presence of idiosyncratic risk. The main differences between ours and their approach are that: (i) we calibrate the risk component of the income process to deliver a realistic dose of uncertainty, which implies that distributions of income and assets in the model broadly match distributions observed in developed economies such as the United States;[30] (ii) we cast the model in continuous time, taking full advantage of the recent analytical and computational discoveries in macroeconomics.

Here we sketch the main workings of the two models and develop the intuition; a more detailed description of the models is available in Appendix D for the life-cycle model and Appendix E for the incomplete market model.

## 4.2 Model of finite lives and life-cycle heterogeneity

### 4.2.1 Demographics and preferences

There are two stages of life, work and retirement, with exogenous transition probabilities. That is, each worker faces a given probability of retirement $1-\omega$, and, once a retiree, a given probability of death $1-\gamma$. Population grows at a gross rate $1+n$.

There is no aggregate risk; the only sources of uncertainty facing an individual are the risk of retirement while a worker (associated with a loss of labor income) and a risk of death while a retiree. Left unchecked, these sources of risk would affect agents behavior. This would make aggregation problematic, and, more importantly, it would be unrealistic: timing of retirement is, for the most part, known. To deal with this unrealistic feature, we assume that there are perfect annuity markets for the retirees (neutralizing the influence of the risk of death on their behavior), and that workers' preferences have a certainty equivalence property (such that the *risk* of retirement does not affect workers' behavior in equilibrium).[31] These two assumptions are both realistic and convenient, in that they allow for the derivation of the aggregate consumption function, as we illustrate momentarily.

Specifically, we assume that agents have recursive Epstein-Zin preferences defined as follows:

$$V_t^z = [(C_t)^\rho + \beta^z \mathbb{E}_t\{V_{t+1} \mid z\}^\rho]^{1/\rho} \tag{7}$$

where $C_t$ denotes consumption, $V_t^z$ and $\beta^z$ stand for agent's $z \in \{w, r\}$ value function and the

[30] We match the degree of income inequality in the data, but fall short of matching the extreme degree of wealth inequality observed in the real world. We discuss the (standard and well-known) reasons why this is so below.

[31] In particular, workers are assumed to have recursive Epstein and Zin (1991) preferences that generate certainty equivalent decision rules in the presence of income risk.

discount factor respectively, and $\sigma = \frac{1}{1-\rho}$ is the intertemporal elasticity of substitution.

Retirees and workers differ in two crucial respects. First, they have different discount factors. Because of the positive probability of death facing any retiree, their discount factor is the time preference parameter $\beta$ multiplied by the probability of surviving into the next period:

$$\beta^w = \beta \tag{8}$$

$$\beta^r = \beta \cdot \gamma. \tag{9}$$

Second, the expectation of the value function next period differs between a worker and a retiree. In particular, a worker takes into account the possibility of retiring, so that her expectation of the value function next period is a probability-weighted sum of the values in the two states:

$$E_t\{V_{t+1} \mid w\} = \omega V_{t+1}^w + (1-\omega) V_{t+1}^r, \tag{10}$$

while the expectation of the value function of a retiree is simply given by

$$E_t\{V_{t+1} \mid r\} = V_{t+1}^r. \tag{11}$$

We now outline the problems of the two types of agents.

### 4.2.2 Retirees

Retirees consume out of savings and social security payments. Each period, some retirees die. We make the assumption – standard in the literature – that those who survive receive the proportional share of the proceeds. This means that the effective return faced by individual retirees is $R_t/\gamma$, higher than the ongoing interest rate $R_t$.[32]

Because probability of death is independent of age and the government does not discriminate across retirees in its social security transfer policy, each retiree (irrespective of age) solves an identical problem, which is:

$$V_t^r = \max_{C_t^r} [(C_t^r)^\rho + \beta\gamma \mathbb{E}_t\{V_{t+1}^r\}^\rho]^{1/\rho} \tag{12}$$

subject to the flow budget constraint:

$$A_{t+1}^r = (R_t/\gamma) A_t^r - C_t^r + E_t^r, \tag{13}$$

where $A_t^r$ stands for retiree's assets, $C_t^r$ is her consumption expenditure, and $E_t^r$ is the social security and healthcare cost transfer.[33]

[32]For retirees *as a group* wealth accumulates at the interest rate $R_t$, as the higher individual return cancels out with some retirees dying.

[33]Our modeling of healthcare provision is very simple – we treat old-age healthcare cost as a lump-sum transfer,

### 4.2.3 Workers

Individuals are born workers and have no assets at the start of life. They consume out of asset wealth and their labour income net of taxes. Because of the demographic structure (in particular the assumption that probability of retirement is independent of age[34]), worker's problem is effectively the same no matter the age. Each worker solves:

$$V_t^w = \max_{C_t^w}\{(C_t^w)^\rho + \beta[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^\rho\}^{1/\rho} \tag{14}$$

subject to:

$$A_{t+1}^w = R_t A_t^w + W_t - T_t - C_t^w, \tag{15}$$

where $T_t$ are lump-sum taxes levied by the government.[35]

### 4.2.4 Firms

The supply side of the model is extremely simple. Market are competitive. Production is carried out by firms employing capital and labor. The aggregate production function is

$$Y_t = K_t^\alpha (X_t N_t)^{1-\alpha} \tag{16}$$

where $N_t$ is the number of workers in the economy. There is exogenous technological progress and population growth, that is $X_{t+1} = (1+x)X_t$ and $N_{t+1} = (1+n)N_t$. Perfect competition in factor markets means that the wage and the rental rate are equated to the marginal products of the factors: $W_t = \alpha\frac{Y_t}{N_t}$ and $R_t = (1-\alpha)\frac{Y_t}{K_t} + (1-\delta)$. Capital evolves according to the standard law of motion: $K_{t+1} = Y_t - C_t - G_t + (1-\delta)K_t$.

### 4.2.5 Government

The government consumes $G_t$ each period, and pays retirees a total of $E_t$ in social security and healthcare benefits. To finance its expenditures the government levies a lump sum tax $T_t$ on the workers. It can also issue one period government bonds $B_{t+1}$. The government flow budget

subsumed in the variable $E$.

[34] Of course this is an unrealistic assumption. But, as explained above, the effect of this assumption on workers' behavior is neutralized through the structure of preferences which exhibit a certainty equivalence property. The role of this assumption is thus only to simplify the model and achieve aggregation, with little cost to the economics.

[35] There are two key channels through which life-cycle considerations affect workers' behaviour. First, a worker takes into account the fact that with probability $1-\omega$ she becomes a retiree. This means that, relative to the representative agent case, she discounts the future stream of wages by more: effectively, this is the *saving for retirement* effect. Mechanically, a larger discount rate reduces the value of human wealth in the consumption function, thus leading to lower consumption and higher saving. Second, a worker discounts the future stream of wealth more because she anticipates that inevitably there will come a time when she becomes a retiree, facing the sad truth that her life is finite. With finite life, wealth can be smoothed out across fewer periods, so its marginal utility value is lower. This effect shows up as a higher effective discount rate applied to future wealth.

constraint is:

$$B_{t+1} + T_t = R_t B_t + G_t + E_t. \tag{17}$$

Iterating forward gives the intertemporal budget constraint of the government:

$$R_t B_t = \sum_{\upsilon=0}^{\infty} \frac{T_{t+\upsilon}}{\prod_{z=1}^{\upsilon} R_{t+z}} - \sum_{\upsilon=0}^{\infty} \frac{G_{t+\upsilon}}{\prod_{z=1}^{\upsilon} R_{t+z}} - \sum_{\upsilon=0}^{\infty} \frac{E_{t+\upsilon}}{\prod_{z=1}^{\upsilon} R_{t+z}}. \tag{18}$$

That is, the difference between the present discounted valued of government revenue and spending must be exactly equal to the current value of the outstanding debt.

Government policy is exogenous. In particular, it is characterized by the four ratios, $\bar{g}_t, \bar{b}_t, \bar{e}_t, \bar{h}_t$, of government consumption, debt, social security and healthcare spending to GDP, respectively:

$$G_t = \bar{g}_t Y_t \tag{19}$$

$$B_t = \bar{b}_t Y_t \tag{20}$$

$$E_t = (\bar{e}_t + \bar{h}_t) Y_t \tag{21}$$

Given the paths of $G_t, E_t$ and $B_t$, taxes adjust to satisfy the intertemporal budget constraint.

#### 4.2.6 Equilibrium

In this economy, markets are competitive and agents take prices as given. Formally, a competitive equilibrium is a sequence of quantities and prices such that (i) households maximize utility subject to their budget constraints, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes, compatible with intertemporal solvency, to finance debt, spending and transfers, (iv) all markets clear.

Appendix D contains the details of the derivation of the equilibrium conditions of the model. The individual policy functions within the two groups – workers and retirees – aggregate up nicely. Aggregating the two consumption levels, we derive the aggregate consumption function:

$$C_t = C_t^w + C_t^r = \pi_t \left\{ (1 - \lambda_t) R_t A_t + H_t + S_t^w + \epsilon_t (\lambda_t R_t A_t + S_t^r) \right\}. \tag{22}$$

In this consumption function, $\pi_t$ denotes each worker's marginal propensity to consume out of wealth, and $\pi_t \epsilon_t$ is the MPC of each retiree. These MPCs multiply the total wealth of each group of consumers (with a slight abuse of notation, $A_t$ now denotes aggregate financial wealth, $H_t$ is aggregate human wealth (the net present value of future wages), and $S_t$ stands for the aggregate value of social security and healthcare payments). Compared to a standard model, the only additional state variable is the share of wealth held by retirees, $\lambda_t$, which fully captures the heterogeneity in the economy.

The total supply of assets is the sum of capital stock $K_t$ and government debt $B_t$ so that the equilibrium requires:

$$A_t = A_t^w + A_t^r = K_t + B_t, \tag{23}$$

i.e. households asset demand equals the asset supply.

### 4.2.7 Why does government transfer policy matter in this economy?

Unlike in an infinite horizon, representative agent economy where the Ricardian Equivalence holds, in this model, government transfers affect the equilibrium outcomes.[36] It is useful to pause and highlight the key channels through which this operates and to appreciate which features of the real world our model is able to capture.

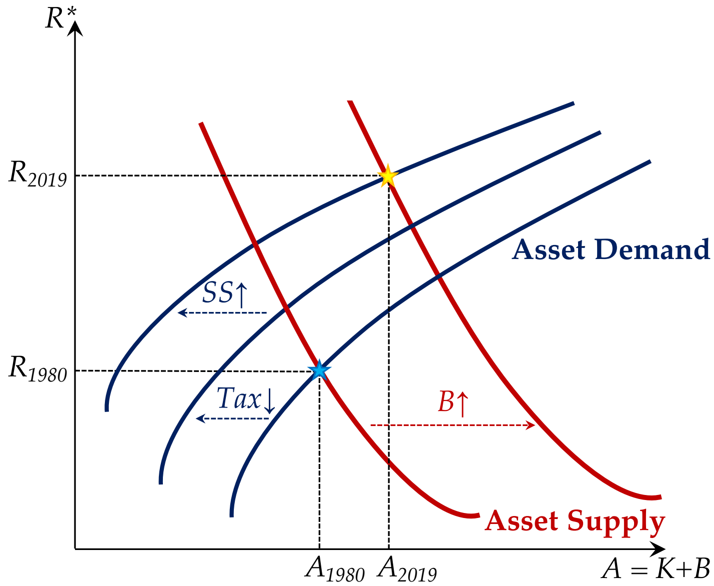
Both the life-cycle model and the incomplete market model of the next section feature an equilibrium which can be diagrammatically represented as an intersection of an upward sloping asset demand schedule (derived from households saving policy functions) and a downward sloping asset supply (representing the capital held by firms plus the value of government debt). Government policies can be represented with shifts of these schedules.

The timing of taxes will matter in this economy because of the finiteness of life. A policy that shifts the tax burden from the present to the future (such as a deficit-financed tax cut) increases human wealth of workers currently alive. There are three reasons why this is so. First, some of the accumulated debt will be paid for by future generations, rather than those currently employed. Second, population growth means that the future per-head bill is smaller than the benefit today. Third, finite life introduces an additional degree of impatience. This is because wealth received in the future – perhaps shortly before death – can only be used to finance consumption across relatively few periods. Workers value today's wealth more, as they can spread their consumption financed with it over longer periods of time. All these forces mean that a tax cut today raises wealth of today's consumers and thus stimulates consumption and lowers saving demand, shifting the asset demand schedule to the left. The rise in deficit and debt shifts the asset supply schedule to the right. To equilibrate the asset market, the market-clearing interest rate increases following a deficit-financed tax cut.

Transfers of wealth across different population groups – such as increased pay-as-you-go social security spending – will also raise aggregate consumption and the interest rate. This is for two reasons. First, retirees have higher marginal propensities to consume than workers (workers save a part of extra resources for retirement). Second, to the extent that current workers anticipate that the higher social security transfers will persist into their own retirement (as they do in our simulations below), this acts to reduce their desire to save. Thus a policy that shifts resources

[36] Similarly, a change in government spending will have a larger effect in this economy than in the standard neoclassical model. In a standard, infinite horizon framework, private consumption is crowded out by deficit-financed government spending, as consumers anticipate that they will have to pay higher taxes at some point in the future. The tax-anticipation effect is weaker in the present framework, as only a part of the future tax burden will be paid for by today's consumers.

**Figure 14:** Asset Market Equilibrium




Notes: The diagrammatic representation of the long-run capital market equilibirum applies to both the life-cycle and the incomplete markets models. The shifts in government policies act to push up the interest rate, from $R_{1980}$ to $R_{2019}$.

from workers to retirees raises consumption demand and the interest rate by shifting the asset demand schedule to the left.

We now proceed to quantify those effects by calibrating our models and simulating the impact of historical policy shifts.

#### 4.2.8 Calibration and the initial steady state of the life-cycle model

Despite the richness of the economics, the model is parsimonious and relatively straightforward to calibrate. We set the preferences and technology parameters at the standard values in the macro literature (Table 3). The growth rate of technological change, the demographics parameters and the government policy ratios are all calibrated to match the data in advanced economies in 1970 (Figure (7) and Table 3).

Because there is population growth and technological progress in this economy, the steady state equilibrium takes the form of a balanced growth path where all variables grow at a constant gross rate equal to $(1+n)(1+x)$. We can characterize the equilibrium by expressing all variables as ratios in units of effective labor (defining, for any variable $Z_t$, $z_t \equiv \frac{Z_t}{X_t N_t}$).

Table 4 shows the key variables along the initial (early-1970s) balanced growth path. The interest rate is 4.5%.[37] As we pointed out above, the key feature of this economy is the heterogeneity in marginal propensities to consume between workers and retirees. Indeed, the endogenous MPC of retirees is over twice that of the workers'. The additional state variable $\lambda$ – the

[37]With growth rate of 2.9% per annum, the economy is dynamically efficient.

**Table 3:** Calibration

| Parameter | Description | Calibration |
|---|---|---|
| Preferences and technology | | |
| $\beta$ | Discount factor | 0.98 |
| $\sigma$ | Intertemporal elasticity of substitution | 0.5 |
| $\alpha$ | Capital share | 1/3 |
| $\delta$ | Depreciation rate | 0.1 |
| $x$ | Rate of technological change | 1.51% |
| Demographics | | |
| $n$ | Gross population growth rate | 1.35% |
| $\frac{1}{1-\omega}$ | Average length of working life (years) | 47.6 |
| $\frac{1}{1-\gamma}$ | Average length of retirment (years) | 10.5 |
| Government | | |
| $\bar{b}$ | Government debt / GDP | 0.18 |
| $\bar{g}$ | Government consumption / GDP | 0.14 |
| $\bar{e}$ | Social security spending / GDP | 0.04 |
| $\bar{h}$ | Old-age healthcare spending / GDP | 0.02 |

ratio of retirees' wealth in total wealth – takes a plausible value of 16%. Ratios of aggregate consumption, investment, capital and assets to output also match the stylized facts from the data well.

#### 4.2.9 The simulation exercise

We now explore how the model economy reacts to changes in government policy. We study four policy levers: government debt, government spending, old-age social security and healthcare transfers.

We carry out the following experiment. Starting the economy in the initial 1970s steady state, we feed the model with the policy profiles depicted in Figure 7. Once announced, the profile of these shifts is fully anticipated by the agents. Beyond the current date, we assume that future policy ratios remain constant at their 2017 values.[38] We then compute the transition path towards this new steady state.

Our focus is on the response of the interest rate to these policy shifts. Figure 15 contains the main result of this Section: the total response of the interest rate to the policy changes discussed above. This response is quantitatively large: according to the model, government policies pushed up on the equilibrium interest rate by around 4pp over the past 50 years. Moreover, the model

[38]This is a conservative assumption, as one may reasonably expect the upwards drift in both debt and social security spending to continue, at least for some time.

**Table 4:** The 1970s steady state

| Variable | Description | Value |
|---|---|---|
| $\psi$ | Ratio of retirees to workers | 0.19 |
| $R$ | Real gross interest rate | 1.045 |
| $\epsilon$ | Ratio of retirees' to workers' MPCs | 2.01 |
| $\pi_w$ | Workers' MPC | 0.06 |
| $\pi_r$ | Retirees' MPC | 0.13 |
| $\lambda$ | Share of retirees' wealth in total wealth | 0.17 |
| $y$ | Output | 1.50 |
| **Ratios (in proportion to output):** | | |
| $c$ | Consumption | 0.57 |
| $c_r$ | Consumption of retirees | 0.11 |
| $c_w$ | Consumption of workers | 0.45 |
| $a$ | Assets | 2.42 |
| $a_r$ | Assets of retirees | 0.40 |
| $a_w$ | Assets of workers | 2.03 |
| $h$ | Human capital | 4.23 |
| $i$ | Investment | 0.27 |
| $k$ | Capital | 2.25 |
| $\tau$ | Taxes | 0.21 |
| $s$ | Social security wealth of the retireees | 0.50 |
| $s_w$ | Social security wealth of the workers | 0.91 |

suggests that further upward pressure is to be expected as the economy settles at the new steady state. All of the policies except government spending – which did not change much – play an important role. The final set of bars, labelled "Interactions", is the additional effect on the interest rate from the (non-linear) synergies between the three different policies.[39]

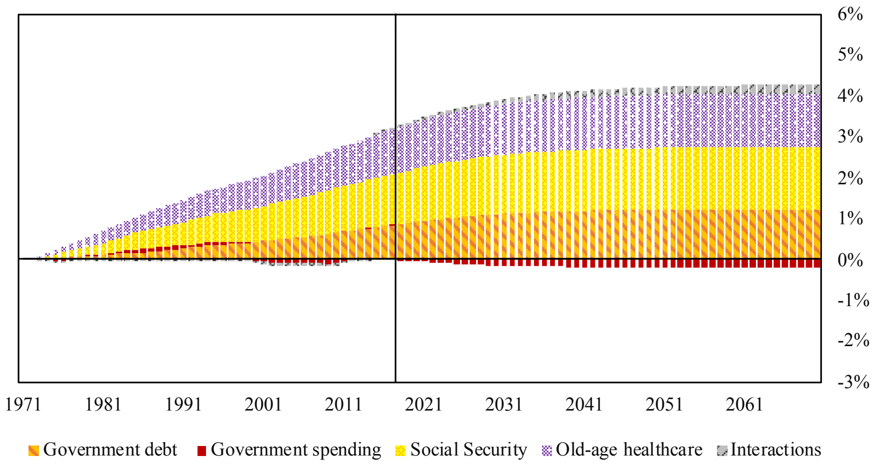
### 4.3 A model of precautionary savings

We now turn to the model of precautionary behavior, which is a continuous time version of the Aiyagari and McGrattan (1997) economy. Population consists of a large number of infinitely-lived individuals of measure 1. Every individual is ex-ante identical, but people face shocks to their income which they cannot fully insure against: markets are incomplete. As a result of this idiosyncratic risk, individuals experience different income histories and thus accumulate different levels of wealth. All the risk is at the individual level: for simplicity, we abstract from aggregate uncertainty.

Our goal here is to assess quantitatively the influence government debt has on precautionary

[39] More precisely, the interaction effect exists because the final steady state is a non-linear system of equations. These non-linearities make the overall effect of several exogenous changes different, in general, from the sum of the parts.

**Figure 15:** Simulated impact of government policies on the equilibrium real interest rate




Notes: The figure shows how the equilibrium real interest rate adjusts to the exogenously given paths of government debt, spending, and old-age social security transfers depicted in Figure 7. The 2017 values in Figure 7 are assumed to be the new steady state values.

behavior. In other words, how different is the prevailing interest rate when government debt-to-GDP ratio is 18% vs. when it is 68%?[40]

### 4.3.1 Brief outline of the model

An individual chooses consumption and asset holdings to maximize her expected utility, subject to the flow budget constraint, the consumption non-negativity constraint, the borrowing constraint and a realization of the idiosyncratic income shock:

$$\max_{\{c_t\}_{t\geq 0}} \mathbb{E} \int_0^\infty e^{-\rho t} u(c_t) dt \tag{24}$$

subject to

$$\begin{aligned}
\dot{a}_t &= (1-\tau)w_t e_t + (1-\tau)r_t a_t - c_t \\
c_t &\geq 0 \\
a_t &\geq \underline{a} \\
e_t &\in \{z_1, ..., z_n\}
\end{aligned}$$

[40]Our model is highly stylized and abstracts from important features present in more advanced and larger models in the literature. We view our model here as an early attempt to quantify the precautionary saving channel of government debt. Richer features may usefully be incorporated in future attempts to answer this question. For example, for analysis of saving rates across the distribution, see Straub (2017) and Fagereng et al., (forthcoming). For evidence on the differential rates of return, see Fagereng et al. (2016). For models with multiple assets or a more careful analysis of the constraint – both of which contribute to a better match to the empirical distribution around the borrowing constraint, see Kaplan et al. (2014), Kaplan et al. (2018) and Achdou et al. (2017). For the state-of-the-art calibration of the income process, see Guvenen et al. (2015).

where $c_t$ is individual consumption, $a_t$ are individual asset holdings (and $\dot{a}_t$ denotes the time derivative, i.e. saving), $r_t$ is the real (net) interest rate, $w_t$ is the wage, $e_t$ is the idiosyncratic shock to household's productivity. The household cannot insure against that idiosyncratic uncertainty. The government levies a proportional tax rate $\tau$ on both labor and capital income.[41]

The supply side is identical to that in the previous model: production function is Cobb-Douglas and there is perfect competition in all markets. Government issues bonds and collects taxes to finance its consumption and transfers. The government budget constraint is

$$\dot{B}_t = G_t + r_t B_t - \tau(w_t + rA_t), \tag{25}$$

which says that the change in government debt is equal to the government funding gap: government consumption $G_t$ plus interest payments $r_t B_t$ minus the tax revenue.

Appendix E presents the definition and solution of the equilibrium of this economy.

#### 4.3.2 Parametrization

We choose the values of the parameters in the precautionary savings model to match the typical values in the literature and to be broadly consistent with the life-cycle model above (Table 3). We set the capital share at $\frac{1}{3}$, the rate of time preference at 0.04, the depreciation rate at 10% and the IES at $\frac{1}{2}$.

The crucial new aspect of the calibration is the idiosyncratic income process. Intuitively, size and persistence of income shocks will determine the strength of the precautionary savings motive, the degree of inequality, and the proportion of households close to or at the borrowing constraint. These outcomes will in turn determine the potency of government financing policy.

In the real world, individual income varies over time for a host of reasons. We do not model these causes here. Instead, we make sure that the income process in our model reflects these uncertainties. Specifically, we follow Ana Castañeda, Javier Díaz-Giménez and José-Víctor Ríos-Rull (2003) and calibrate the income process to match aggregate income inequality in the OECD. There are four productivity and income states:

$$e \in \{0.20,\ 0.55,\ 0.80,\ 5.43\}. \tag{26}$$

Note that to generate the high skewness of the income and wealth distributions observed in the data, individuals in the top state have income an order of magnitude larger than the rest. The

[41] The assumption of a proportional tax rate is natural in a model with income and wealth heterogeneity. With lump-sum taxation, the poorest households would find themselves unable to pay the tax bill. Note that even though the tax is proportional it does not distort the labor supply decisions as the labor supply is inelastic.

corresponding matrix of Poisson intensities is

$$P = \begin{pmatrix} 0.07^{-} & 0.04 & 0.02 & 0.001 \\ 0.03 & 0.13^{-} & 0.01 & 0.001 \\ 0.001 & 0.08 & 0.09^{-} & 0.011 \\ 0.1 & 0.02 & 0.06 & 0.17^{-} \end{pmatrix}, \tag{27}$$

where the values on the main diagonal marked with superscript $^{-}$ indicate the intensity of leaving the current state. For example, the first row indicates that individuals currently in the poorest state $z_1$ leave that state with intensity 0.07, and enter one of the other three states $z_2$, $z_3$, $z_4$ with respective intensities of 0.04, 0.02 and 0.001. Thus states 1 and 3 are the most 'absorbing', in the sense that the out-intensities are the lowest for these states.

Given this income process, the distributional outcomes in the equilibrium of our model are broadly in line with those observed in the data. In particular, the income Gini coefficient in the model is 0.32, close to the OECD average.[42] A question that remains is whether our calibration generates an individual income profile that is realistic. Castañeda et al. (2003) compare the across-the-income-distribution mobility statistics implied by their model with those observed in the data, and claim success: the simple model does reasonably well in capturing the persistence moments. In particular, both in the model and in the data, the income process is highly persistent. Based on that result, we have some confidence that the simple income process does a decent job reflecting the individual income process and transition rates between different points in the income distribution.

#### 4.3.3 Results

We now compare the two stationary equilibria of the model, one with the government debt / GDP ratio set at 18%, and another at 68%.[43] Here we only present the results most relevant to our focus on the government policy and the interest rate; for a further characterization of the two equilibria, in terms of policy functions and the distributions, see Appendix E.

Recall that the equilibrium in this model can be represented in the form of the diagram in Figure 14, similar to the life-cycle model. Different to before, the upward sloping demand for assets is a result of uninsured idiosyncratic risks rather than life-cycle structure. The downward

[42] The wealth Gini delivered by the model is somewhat lower than that in the data (0.55 in the model vs. 0.8 in the data in the US), and the fraction of households at the constraint is also relatively low, at around 9% vs. around a third in the data. These shortcomings are well-known in the literature. Some of the features of the real world that drive the extreme concentration of wealth holdings, such as the systematic differences in the saving rates and the rates of return on investments across the wealth distribution, are missing from this model. Furthermore, the asset market and the constraint are modeled in a crude way. For our purposes, the simple model is sufficient to use it as a laboratory for the effects of government bond supply on the equilibrium allocation. A richer model with some of the above features would likely predict larger effects of government policy.

[43] Analysis of the transition path is not crucial for our purposes and is beyond the scope of this paper.

| | Low Debt Equilibrium | High Debt Equilibrium |
|---|---|---|
| Government debt / GDP | 0.18 | 0.68 |
| Government consumption / GDP | 0.14 | 0.14 |
| Average tax rate | 0.35 | 0.36 |
| Real interest rate | 4.50 | 5.16 |
| Private capital / GDP | 2.56 | 2.40 |
| Income Gini | 0.32 | 0.32 |
| Fraction of individuals at the constraint | 0.09 | 0.09 |

**Table 5:** Equilibria in the precautionary saving model

sloping demand for capital aggregates firms' desired capital holdings and government debt, identical to before. An increase in government debt shifts the asset supply schedule to the right, resulting in a higher real interest rate in equilibrium.

The key quantitative result in Table 5 is that an increase in public debt / GDP ratio observed in the data implies a real interest rate that is 66bps higher in equilibrium. We conclude that the baseline heterogenous agents incomplete markets model suggests that the precautionary saving channel from government policy to the real interest rate is quantitatively important.

## 4.4 Summary and discussion

In summary of this Section, our analysis underscores the importance of secular public policy shifts in accounting for changes in the equilibrium interest rate. The natural corollary of our findings is that government intra- and intertemporal transfer policy is, in principle, an effective tool that can affect equilibrium interest rates in the economy. Similar policy implications have been discussed previously by Kocherlakota (2015) and Caballero and Farhi (2014).

One objection to our analysis might be that economic agents – consumers, investors, firms etc. – may in fact be more Ricardian than we currently assume. Our response to this is four-fold. First, in Section 3 we presented a broad range of empirical evidence that is inconsistent with the Ricardian Equivalence proposition. Second, in our framework, Ricardian Equivalence does not hold despite fully rational expectations and no information asymmetries: indeed, it would be irrational to be Ricardian in the economy we describe. Third, and relatedly, the assumptions that lead to rejection of Ricardian Equivalence are rather natural (i. people retire; ii. people die; iii. some people are credit constrained; iv. some people face risks they find hard to insure). All those considerations make us comfortable with our assumptions that the Ricardian offset is imperfect.

At this point it is also useful to highlight that wide uncertainty bands surround our point estimates, including those coming out of the models discussed above. Like all theory models, these tools are built upon a set of uncertain assumptions, and as such are only rough approximations of reality – this is especially true for models as minimalistic and transparent as ours. Even abstracting from model misspecification, there is a wide range of plausible parameter values with which to calibrate these models. A different combinations of parameters will produce quantitatively different results. We come back to the robustness of our analysis in Appendix F.[44] Having said that, the combination of a range of empirical studies together with directional guidance from the theory (broadly defined) suggest that there are strong reasons to conclude that the government policies we scrutinized here have put significant upward pressure on safe neutral real rate over the past several decades.

# 5 Underlying weakness in R*

Our simulation analysis concluded that the major shifts in governments' policies over the past 50 years facilitated a significant transfer of resources from low-MPC to high-MPC individuals and permitted households to self-insure against idiosyncratic shocks. All else equal, added together these shifts would have pushed interest rates in advanced world up by around 4pp. But of course all else was not equal. In this Section we use our models to complete the picture by quantitatively assessing the impact of other secular forces – demographics, trend productivity growth and income inequality – on the long-term interest rate.[45]

Table 6 documents the major demographic transition that has been underway in advanced economies for the past 50 years. Population growth in the developed economies has fallen rapidly in past decades, from around 1.4% per annum in 1970s to less than 0.4% today. This trend is expected to continue; in fact, the latest UN projections suggest that population in advanced economies will start shrinking around 2050. As population growth decelerated, life expectancy has gone up significantly, and retirement ages did not keep up. As a result, the average length of retirement is nearly twice what it was in the 1970s. This positive development carries significant implications for life-cycle budgeting and thus for the balance of desired saving and investment.

Another force that our life-cycle model is well-equipped to capture is the slowdown in the pace of long-run growth. Our modeling framework inherits the property shared by essentially all dynamic macroeconomic models, namely that the long-run equilibrium interest rate is linked to the expected future consumption growth. This relationship – the Euler Equation or the dynamic IS curve – is the result of intertemporal optimization of households, who choose how much to consume today vs. tomorrow (and hence determining the growth rate of their consumption) based on the interest rate. In general equilibrium, the expectations of future consumption growth in the long-run coincide with the expectations of TFP growth. Hence the theory suggests that

[44] The final draft of the paper will include further robustness analysis.

[45] These trends are a subject of study in a fast growing literature. In particular, for work on demographics, see Carvalho et al. (2015), Gagnon et al. (2016) and Lisack et al. (2017). For analysis of inequality, see Auclert and Rognlie (2016) and Straub (2017).

**Table 6:** Demographic transition in advanced economies

| | Growth of 20+ population | Retirement age | Years working | Years in retirement |
|---|---|---|---|---|
| 1970 | 1.4 | 67.6 | 47.6 | 10.5 |
| 1975 | 1.3 | 66.6 | 46.6 | 12.3 |
| 1980 | 1.2 | 66.1 | 46.1 | 13.4 |
| 1985 | 1.1 | 65.1 | 45.1 | 15.0 |
| 1990 | 0.9 | 64.7 | 44.7 | 16.1 |
| 1995 | 0.8 | 63.8 | 43.8 | 17.5 |
| 2000 | 0.7 | 63.6 | 43.6 | 18.6 |
| 2005 | 0.8 | 64.1 | 44.1 | 18.9 |
| 2010 | 0.7 | 64.8 | 44.8 | 18.8 |
| 2015 | 0.4 | 65.5 | 45.5 | 18.7 |
| *Projection:* | | | | |
| 2020 | 0.2 | 66.1 | 46.1 | 18.6 |
| 2025 | 0.2 | 66.8 | 46.8 | 18.4 |
| 2030 | 0.2 | 67.5 | 47.5 | 18.3 |

Sources: United Nations and OECD.

real rates and expected productivity growth ought to be linked.[46]

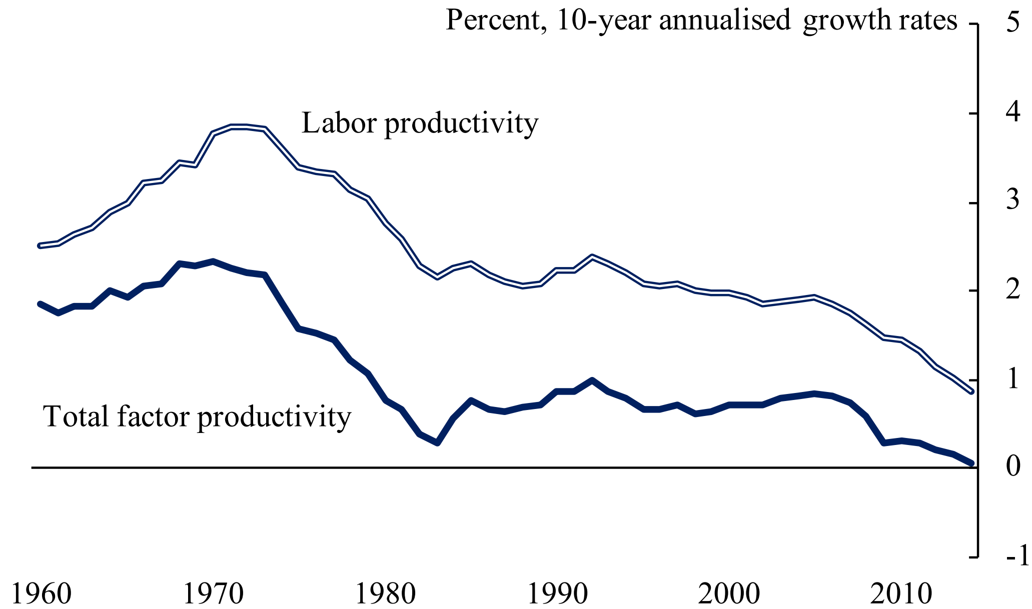
This strong prediction of the theory is, however, more tenuous in practice. In an early contribution to this topic, Christopher Carroll and Lawrence Summers established that, across countries, consumption growth and income growth are tightly linked and follow each other, and that households with more steeply rising income profiles tend to save more, not less. These findings – inconsistent with the standard permanent-income hypothesis / life cycle model – have been rationalized in the literature with buffer-stock models of savings (whereby households face uncertain income process, similarly to our second model) and introducing consumption habits in household preferences.[47] While our models attenuate the link between interest rates and consumption choices in line with these findings, nonetheless we urge a significant degree of caution when interpreting the results on the link between TFP and R*. Our preferred interpretation is that the low interest rates today are chiefly a symptom of a demand-side problem. We return to this issue in the final section of the paper where we discuss policy implications.

These caveats notwithstanding, Figure 16 shows that trend growth rates of both productivity and of TFP have declined significantly in advanced economies, first in the early 1980s when TFP

[46] In a representative agent, infinite-horizon economy, the Euler Equation takes a particularly straightforward form, whereby long-run consumption growth rate and the interest rate are linked linearly, with the coefficient equal to the intertemporal elasticity of substitution. Within our framework, that link is still there, although it is attenuated by finite horizons and borrowing constraints: intuitively, the interest rate is relatively "less important" in driving consumption growth, as other factors (such as possibility of death or credit constraints) come into play. This implies that a given change in the expectations of future consumption growth – driven by news about TFP, say – will require a *larger* response of the interest rate to restore equilibrium.

[47] See Deaton (1991), Carroll (1997) and Carroll et al. (2000) and the literature that followed.

**Figure 16:** Trend productivity and TFP growth in advanced economies



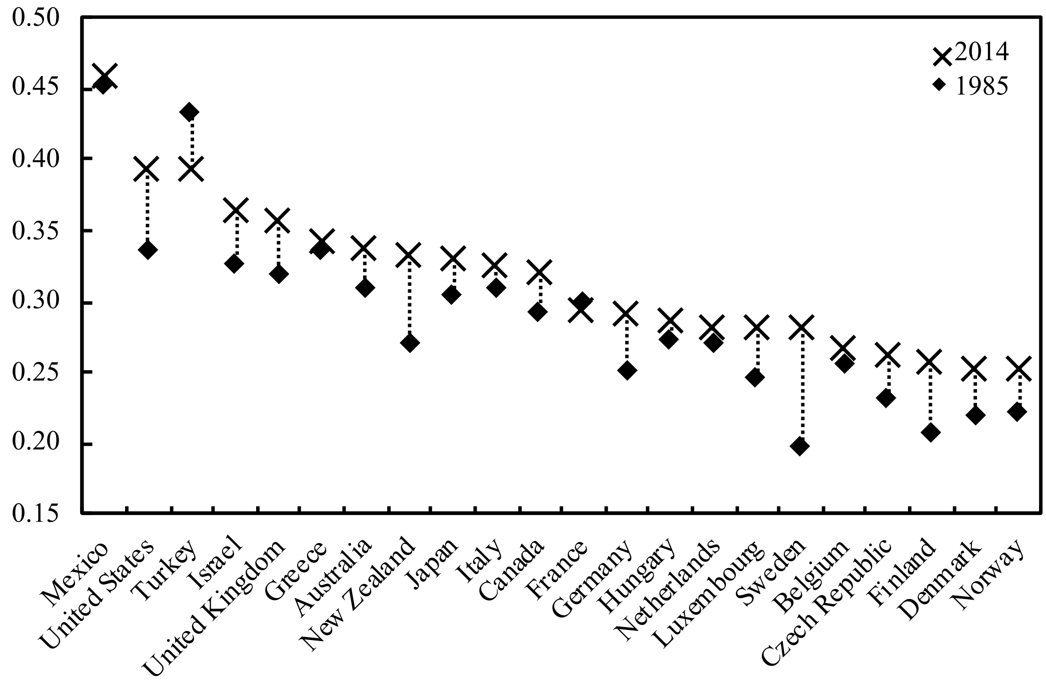

Source: Adler et al. (2017) with data from Penn World Table 9.0; IMF, World Economic Outlook; and IMF staff calculations. Note: Purchasing power parity GDP weighted average of the largest 20 developed economies.

growth halved from about 2% to 1% per annum, and then again in the mid-2000s.

The third force we consider is the rise in income inequality. Inequality has increased in the United States and many other advanced economies (Figure 17). Our second model is well suited to give us an estimate of this shift on the real rate of interest. To trace out the effects of rising inequality in our modeling framework, we recalibrate the income process in such a way as to match the increase in income Gini coefficient in the OECD since the 1970s. Our calculations implicitly assume that ex-post inequality is driven by larger variance of individual income shocks, which constitutes a source of additional uncertainty for individual workers. An alternative view is that the increase in inequality is a consequence of shifts more tightly linked to heterogeneity across households that is known ex-ante. The distinction is important because only the former kind of shift would lead to an increase in precautionary behavior: being predictable, the latter shift is not associated with heightened risk. There is a long-standing debate about the merits of the two formulations in the literature.[48] The recent work by Fatih Guvenen and co-authors has established the large departures of log-normality in the individual income changes: in particular, earnings changes display strong negative skewness and extremely high kurtosis. Important for our interpretation is their finding that large shocks at the top of the income distribution tend to be very persistent. We view these results as supportive of the gist of our exercise, which interprets the increased disparity between the poor and the rich as going hand in hand with an increase in ex-ante uncertainty, particularly at the top of the income distribution.

[48] Classic references include Lillard and Weiss (1979), MaCurdy (1982) and Guvenen (2009).

**Figure 17:** Gini coefficient of disposable household income across the OECD




Source: OECD Database on Household Income Distribution and Poverty. Disposable income adjusted for household size.

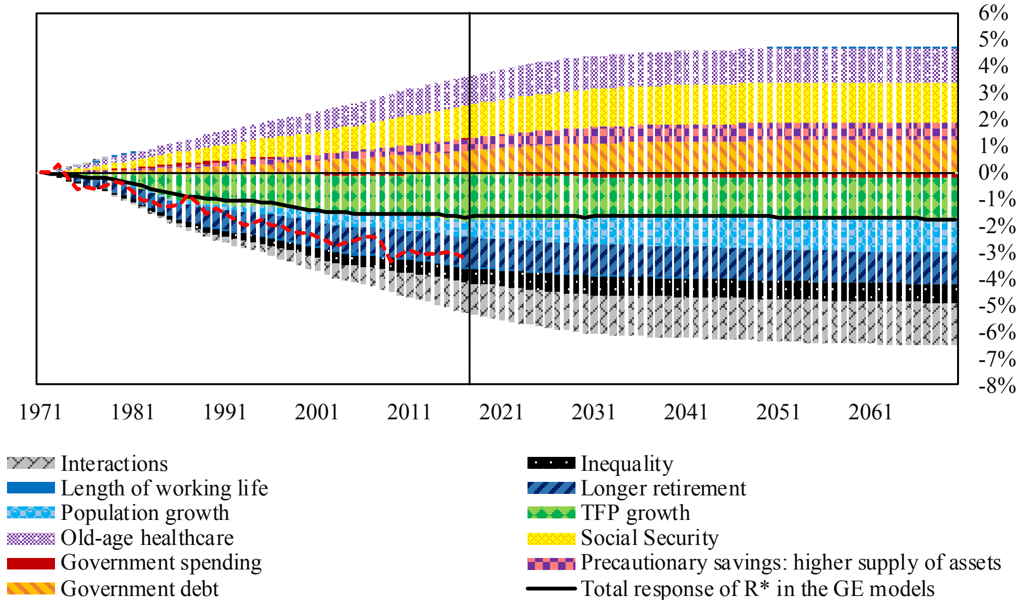
To explore the implications of these trends for the equilibrium real interest rate, we perform the following exercise: in the life-cycle model, we calibrate the changes in demographic transition probabilities, $\omega$ and $\gamma$, to match the trends depicted in the final two columns of Table 6. We then feed in the series for population and TFP growth rates to match the evidence in the first column of Table 6 and in Figure 16. We use the UN demographic projections to inform the path of demographics out to 2050, and assume that the terminal 2050 values are the steady state. We do not have a strong prior as to the path for future TFP growth, and we are well aware of the wide range of existing and plausible views. Aiming for a scenario that reflects the mode of these expectations, we assume that the TFP growth rate picks up from around zero in the latest available data to 0.7% in the long-run.[49] This pick-up in TFP growth is in line with the CBO's assumption for the pickup of TFP growth in the United States (The U.S. Congressional Budget Office (2019)).

In the precautionary saving model, we recalibrate the income process[50] and compare the

[49]There is very large uncertainty around any long-term forecast of the TFP growth rate. In particular, research has shown that current-decade growth of productivity holds little information as to the growth in the following decade. Perhaps naturally, the commentators are split on the prospects for innovation and productivity. See, for example, Brynjolfsson and McAfee (2014) and Gordon (2016) for two perspectives from the opposite ends of a spectrum.

[50]In particular, we change the income received in the highest income state. This is motivated by the fact that the increase in income inequality has been concentrated at the very top of the distribution, as documented by Piketty (2014) and others.

**Figure 18:** Changes in the equilibrium real interest rate as a result of policy, demographic and technological shifts




steady states of the economy under the two calibrations.[51]

To reiterate, within each of the two models, we feed in the (model-specific) set of shocks all at the same time, thereby providing – within each model – an internally consistent laboratory to study this wide range of heterogenous trends. What we miss are the potential interactions across the two models. We assume that the comparable calibration across the two frameworks makes the results comparable and that simply adding the estimates of the impact on R* over the transition across the models results in a consistent picture. But ultimately, only the framework for analysis of all the forces that we consider – and perhaps more – in a single unifying setting would provide a definitive answer to these doubts. This avenue of inquiry is left for future work.

Figure 18 illustrates the full decomposition coming out of our framework under our most preferred calibration (there is substantial uncertainty around these point estimates, and we explore some of the sensitivities in Appendix F). The solid black line marks the changes in the equilibrium interest rate accounted for by our models, starting from the 1970s steady state. Overall, our simulations are able to explain around 1.7pp of the decline in the advanced economy R*. The colored bars decompose this total decline, with changes in government policies pushing interest rates up, and demographics, technology and inequality going the other way.

The big picture message from this part of our analysis is twofold. First, we uncover the presence of strong two-way forces that are, on a gross basis, much larger than the 'net' decline in R* that we and others have estimated. Second, the equilibrium real interest rate today is low but, absent the shifts in government policies that we documented, it is likely that it would have

[51]We obtain the dynamic path by assuming that the effect builds steadily, including over the next decade. Our treatment of the dynamics is thus crude. We leave the analysis of the dynamic adjustment path for future work.

been even lower. These findings are consistent with studies which find that over the long-run neutral real rates vary quite a lot, and that the low levels of neutral rate are relatively common historically (Jordà et al., 2017).

We now comment on the details of our decomposition.

The slowdown in trend TFP growth rate of 0.8pp per year leads, in the model economy, to a 1.8pp decline in the real rate of interest (in the terminal steady state). The key parameter guiding the size of the response is the intertemporal elasticity of substitution. The link between growth and interest rates depends on the inverse of the IES. In our simulation we set IES $= \frac{1}{2}$, which means that the multiplier from growth to interest rates is about 2. We present robustness of our results to an alternative parametrization with higher IES in Appendix F. Taken together, the demographic effects, shown in the blue-shaded bars, are larger than the drag from lower TFP growth: we estimate that to date, demographics have pushed interest rates down by around 1.8pp, and the effect is likely to build going forward. This number is within the range of existing estimates from other the quantitative models.[52] The rise in income inequality leads to a total decline of the natural rate of around 60bps according to our model, broadly in line with the 66bps reported in Auclert and Rognlie (2016). The negative interaction terms (the grey-colored bars) arise partly because of the interplay between government policies and demographic changes in the non-linear steady state.

Our framework cannot account for the full extent of the decline in equilibrium rates, with over 1pp left unexplained in our preferred calibration. Our models miss some of the secular forces that likely pushed neutral rates lower over the past 40 years. One omission is the increasing concentration and the associated increase in market power of firms in the US and other advanced countries (Farhi and Gourio (2018)). Another force is driven by the finding that propensities to save are higher for those with high permanent income (Carroll (2000), Dynan et al. (2004)). In light of these findings, our simulations likely understate the full impact of the increase in permanent income inequality. Using a model which captures this heterogeneity, Ludwig Straub (2017) estimates that the rise in inequality may have pushed down on the real equilibrium interest rate in the US by about 1pp through this channel. Finally, the decline in the price of capital goods may have contributed to lower investment propensities, further decreasing the neutral real rate (Sajedi and Thwaites (2016)). Adding these elements to our framework would in all likelihood add further negative drag to Figure 18.

[52] Smaller estimates – such as 1.3pp those reported by Gagnon et al. (2016) – result from a different parametrization (Gagnon et al. (2016) assume log utility, i.e. unitary IES). Larger estimates, for example those from Lisack et al. (2017) or Eggertsson et al. (2019b) which are in the region of 3pp, take into account the impact of demographics on credit creation and housing markets and model the life-cycle more granularly.

# 6 Conclusion

We draw three main conclusions from the analysis in this paper. First, the neutral real rate for the industrial world has trended downward for the last generation and this is best understood in terms of changes in private sector saving and investment propensities. In the face of neutral real rate estimates, past trends in indexed bond yields, and measures of real swap yields, this conclusion seems inescapable. It is also noteworthy that current real rates appear to be quite well predicted by pre-financial crisis trends. We believe that the these trends are best analyzed in terms of changes in saving and investment propensities or equivalently in terms of trends in desired wealth holdings by consumers and desired capital accumulation by producers. While factors involving liquidity, scarcity and risk no doubt bear on levels of real interest rates we find it highly implausible that they are the main factor accounting for trend movements. The movements are too large and too pervasive across assets and the fluctuations in spreads are too small and lacking in trend for these factors to account for the observed trends in the data.

Second, the neutral real rate would have declined substantially more over the last generation but for increases in government debt and expansions in social insurance programs. Both straightforward extrapolations of existing rules of thumb regarding debt and deficit impacts on interest rates and calculations using workhorse general equilibrium models suggest that fiscal policies have operated to raise real interest rates by several hundred basis points over the last generation. While this conclusion is dependent on our rejection of Ricardian equivalence, we see nothing that leads us to believe that increased government debt automatically calls for increased saving or that pay-as-you-go social security programs alter bequests for most families. The specific magnitudes are very uncertain, but open economy aspects and the possibility suggested by our analysis – that budget deficits emerge in response to excesses of private saving over private investment – lead us to think that we are more likely to understate than overstate the extent of fiscal support for real interest rates in recent years.

Third, the implication of our analysis that but for major increases in deficits debt and social insurance neutral real rates in the industrial world would be significantly negative by as much as several hundred basis points suggests substantial grounds for concern over secular stagnation. From the perspective of our analysis the private economy is prone to being caught in an underemployment equilibrium if real interest rates cannot fall far below zero. Full employment in recent years has been achieved where it has been achieved either through large budget deficits as in the United States or Japan or large trade surpluses as in Germany. It is worth considering that in the United States during the period prior to the financial crisis, negative real short term interest rates, a huge housing bubble, erosion of credit standards and expansionary fiscal policy were only sufficient to achieve moderate growth. Adequate growth in Europe was only maintained through what in retrospect appears to have been clearly unsustainable lending to the periphery.

What does our analysis say about stabilization policy? Most obviously it says that traditional levels of interest rates combined with balanced budgets or even stable debt-GDP ratios are a prescription for recession. Policymakers must, if they wish to avoid output being demand constrained, do some combination of accepting high and rising deficits and government debt levels, living with real interest rates very close to zero or negative, and finding structural policies that promote investment or reduce saving.

Olivier Blanchard (2019) makes the argument that traditional views about fiscal policy likely reflect excessive concern about debt when real interest rates are very low and likely to remain low for a long time to come. The sustainability of a given level of deficit or debt is greater when interest rates are low than when they are high. Nonetheless it has to be acknowledged that the US economy appears to be slowing to below potential growth despite projected primary deficits that will lead even on very favorable interest rate assumptions to steadily growing debt–to-GDP ratios that will ultimately set historical records. There is no guarantee that deficits sufficient to maintain positive neutral real rates will be associated with sustainable debt trajectories. Indeed, the Japanese experience suggests that this may not be the case.

Another possibility is the use of monetary policies that induce significantly negative real rates. This might be achieved through setting negative nominal rates, raising or adjusting inflation targets (e.g. through targeting average rate of inflation and thus "making up" for the past errors), or using unconventional monetary policies such as quantitative easing to achieve the equivalent of reductions in real rates. These approaches raise three issues. First, given that historically rates have been reduced by 500 basis points or more to mitigate recessions in industrial countries there is the question of whether enough room can be generated to stabilize the economy when the next downturn hits. Second, there are questions about whether starting at very low rates further rate reductions are actually stimulative. Eggertsson et al. (2019a) suggest that negative nominal rates actually may interfere with financial intermediation. Third, there is a range of concerns about the possible toxic effects of low rates, including suggestions that they make bubbles and over-leveraging more likely as they encourage risk taking, that they may lead to misallocation of capital by reducing loan payment levels and required rates of return, reinforce monopoly power, benefit the old at the expense of the young, and make the funding of insurance and pension obligations more difficult.

A final possibility is structural measures that reduce saving or promote investment. Clearly regulatory policies that encourage investment without sacrificing vital social objectives are desirable. The extent to which these are available is very much open to question. Investment incentives will also operate to raise demand. Policies that reduce the need for retirement saving, such as strengthening social security, or that improve social insurance, will increase aggregate demand even if operated on a balanced budget basis. So will policies that redistribute income from those with lower to those with higher propensities to consume.

It is tempting to suggest that any measure that increases productivity growth will operate to raise neutral real rates as consumers seek to spend more out of higher expected future incomes and firms increase their investment demand. Effects of this kind are indeed suggested by our formal model. We are not sure of their validity in practice. As Carroll and Summers (1991) point out, growth accelerations internationally have typically been associated with declining not increasing real rates and there is not much evidence that consumers are that forward looking, especially if the reforms are associated with transitional costs and heightened short-term uncertainties. Moreover in policy discussions central bankers usually cite stronger productivity as an antidote to inflation and therefore as a reason not to raise rates. Short-term productivity gains which reduce costs and inflation may act to elevate realized interest rates above the neutral rate, further worsening the demand imbalance.

All of this is to suggest that if secular stagnation is avoided in the years ahead it will not be be because it is somehow impossible in a free market economy, but instead because of policy choices. Our conclusions thus underscore the urgent priority for governments to find new sustainable ways of promoting investment to absorb the large supply of private savings and to devise novel long-term strategies to rekindle private demand.

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# Appendix A: Additional discussion of the role of safety and liquidity premium

Our analysis abstracts from aggregate uncertainty and differing levels of liquidity of various assets. Instead, in the empirical part of the paper we focus on the yield on practically risk-free,[53] highly liquid government bonds, and our theoretical framework makes no distinction across asset classes (meaning there is only one interest rate in our models). In this Appendix we provide some additional discussion of this issue.

In our paper we are interested in (i) the trajectory of the equilibrium interest rate over the past few decades; (ii) the counterfactual trajectory which would have occurred had the government supply of safe assets been significantly tighter; (iii) other secular drivers of this trajectory and (iv) the associated policy implications. Given these objectives, one may pose two questions: first, is the safe and liquid rate an interesting object of study? Second, does the existence of time-varying liquidity and safety premia pose a challenge to our conclusions on issues (i)-(iv)?

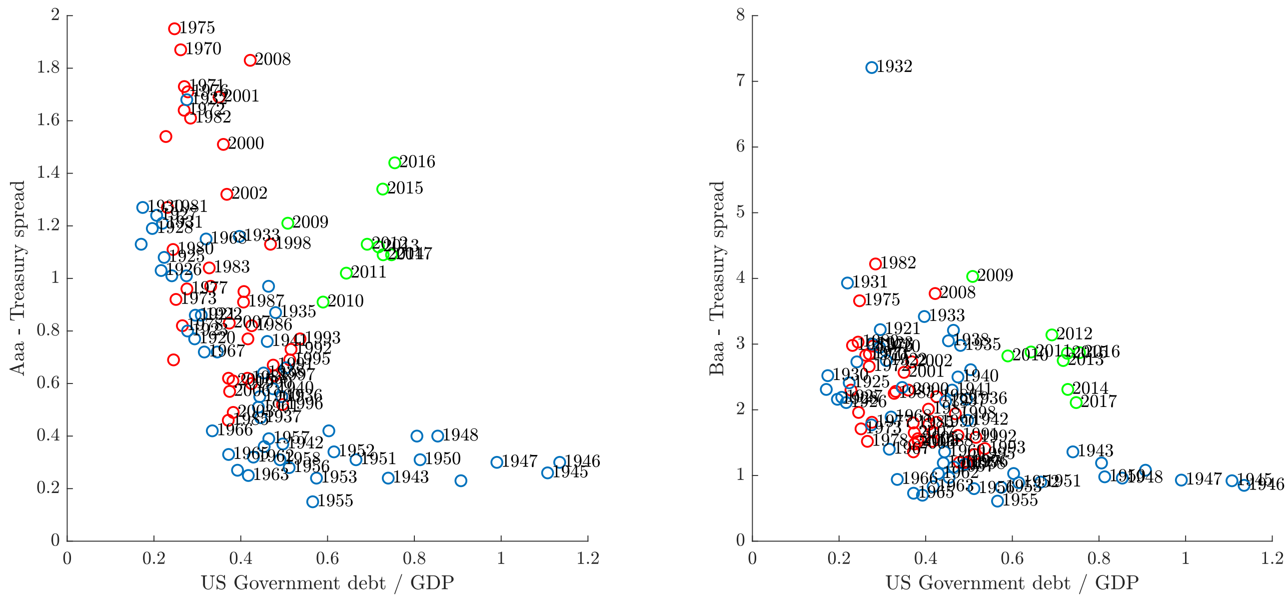
On the first of these questions, we think the answer is affirmative. In the narrow sense, the safe and liquid natural rate is central for monetary policy setting; it is also, quite naturally, relevant for government debt management, fiscal headroom, debt rollovers, etc. As elegantly explained by Olivier Blanchard in his AEA presidential address (2019), it is the rate that is informative about the risk-and-illiquidity-adjusted required rate of return – which in turn is a useful gauge as to the prevailing preferences and hence relevant for assessing welfare implications of policies. In practice, the safe and liquid rate is the benchmark rate for many financial institutions like insurance companies and pension funds.

More importantly, our read of the evidence is that the risk-free rate gives a powerful steer on the equilibrium interest rate that brings desired saving and desired investment into balance. In the US, the long-term swap and TIPS rates have declined in tandem with yields on Treasury debt, suggesting that liquidity can only play a limited role. The Baa-Treasury spread has recently declined towards its long-run average, indicating that risk is unlikely to be a major part of the story either. We view this evidence as broadly consistent with the state-of-the-art econometric estimates from the "safe asset" literature, which suggest that at the global level, the convenience yield accounts for roughly 70 basis points of the decline in the safe real rate (Del Negro et al. (2018)).[54] Another way to cross check the importance of the spread is to consider

[53] Government bonds in advanced economies are free of default risk. However, holdings these assets is risky, as the overall return is determined not only by the coupon but also by the changes in the valuation of the bond. For example, the standard deviation of the total annual return on a 10-year US Treasury bond is 9.5pp over the period 1970-2018.

[54] This is consistent with previous work which suggests that the bulk of the decline in long-term rates reflects the revisions to the expected short-term interest rates. For decompositions of the decline in nominal and real interest rates, see Adrian et al. (2013) and D'Amico et al. (2018), respectively. Using data spanning several asset classes and a stylized saving-investment framework, Rachel and Smith (2017) concluded that the rise in the spread between risky and risk-free rates accounted for about 70bps of the decline in risk-free rates, in line with

**Figure 19:** US government debt and bond spreads




Source: The data through 2008 are from Krishnamurthy and Vissing-Jorgensen (2012). The red markers correspond to the period that we study in this paper (1970 onwards). The data post-2008 – in green – are from FRED.

the relationship between government debt and corporate bond spreads, as in the classic work of Arvind Krishnamurthy and Annette Vissing-Jorgensen (2012). Using data up to the global financial crisis, they showed that there has been a stable relationship between public debt and the spread in the United States, one that traces out the demand schedule for US Treasury debt. An update of their analysis that includes the data covering the past decade shows that the recent outturns defied the pre-crisis relationship (green points in Figure 19). One interpretation of this finding is that the demand for US government debt has indeed shifted outwards. The difference between the realized spread and the spread that one may expect based on the historical relationship is in the region of roughly 60 basis points, supporting the conclusion that changing safety and liquidity premia account for a minority of the total decline over the past several decades.[55]

We also believe that our conclusions are robust to the lessons coming out of the "safe asset" literature. The key focus of our paper is on supply of government assets, which we argue has increased dramatically as government debt has risen. The point we make is simple: had the supply of safe assets been scarcer, the safe and liquid rate would have declined further. As far as this conclusion is concerned, we do not see any real tension here with the existing literature. One economic mechanism that is lacking in our exercise is that, when the economy is at the zero-lower bound and the equilibrium is achieved through lower levels of output and investment

the econometric estimates reported above.

[55] Some degree of caution is warranted when comparing the recent outturns to the historical demand schedule, given that the appropriate comparison period includes the years during and around the WW2.

(when the economy finds itself in a "topsy-turvy" equilibrium as in Eggertsson and Krugman (2012)), a higher supply of safe assets can increase private investment. Instead, our models focus on the long-run equilibrium where the long-term rate adjusts freely, so that formally an increase in supply of safe assets crowds out private capital. But this observation would only further strengthen our policy conclusions which we discuss at the end of the paper, that given the low interest rate environment, government policy must play an active role in reinvigorating demand. On a more conceptual level, it seems plausible that some of the real-economy forces we discuss in the final section of the paper may have acted to put pressure on safe rates relative to risky rates, therefore potentially accounting for some of the 60bps increase in the convenience yield. For example, people may have preference for saving in safe assets for retirement, or they may demand safe assets to insure against idiosyncratic uncertainty. In short, we do not think that there is tension between the "excess saving" explanation (one present in this paper) and "excess demand for safe assets" one, although we note that it is of course possible that any increase in liquidity premium in the past decade was associated with cyclical and policy developments during and in the aftermath of the global financial crisis.

# Appendix B: Estimation of the state-space system

### Data

All data are of quarterly frequency, spanning 1971Q1:2017Q4. All series used in this exercise are for a sample of OECD economies and are produced by the OECD and can be downloaded from the OECD website. For more details on the OECD data, see the OECD Economic Outlook data inventory.

To construct the observed advanced economy interest rate, we use long-term interest rates (10-year government bond yields; database here). As explained in the main text, we calculate the arithmetic average of these interest rates across the unbalanced panel of 36 OECD countries. Our results are unchanged if a median interest rate is used.

The inflation series is the non-food, non-energy consumer price index for OECD-total sample (database metadata available here). We construct the measure of inflation expectations as a moving average of observed core inflation rates over the past four quarters. This, of course, is not an ideal measure, but it follows past efforts in this literature and allows for estimation using the data from a large block of countries (alternative measures of inflation expectations for a large sample of countries going this far back are not available).

Finally, the GDP data cover the OECD-total sample. These are seasonally adjusted real (constant prices) measures, calculated using fixed 2010 PPPs, and expressed in 2010 US dollars. The series we use are calculated using the expenditure approach (OECD series code VPVOBARSA).

### Estimation

The estimation is a recursive process that starts with a guess for the unconditional mean and variance of the unobservable state variables (such as the equilibrium real rate $r^*$ or the level of potential output $y^*$) in the initial period. These are used to produce forecasts for the observables (such as inflation $\pi$ or actual output $y$) next period. For every $t > 1$, the procedure consists of two steps. First, the update step changes the best guess for the unobservable states based on the forecast error on the set of observables. The direction of the revision is determined by the covariance of the observable and unobservable states, and the size of the update is determined by the variance of the observable (higher variance means there is more noise relative to signal on average, which reduces the size of the update). In the second step, this latest information is used to produce a new forecast for next period.

Following the classic notation of Hamilton (1994), we can write equations 5-3 in the state-space form as follows:

$$\mathbf{y}_t = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\xi_t + \mathbf{v}_t \tag{28}$$

$$\xi_t = \mathbf{F}\xi_{t-1} + \epsilon_t. \tag{29}$$

In this system, $\mathbf{x}_t$ is the vector of exogenous or lagged state variables; $\xi_t$ is the vector of endogenous states; and $\mathbf{v}_t$ and $\epsilon_t$ are the vector of uncorrelated Gaussian disturbances. Specifically in this model, after substituting equation (1) into (5), (28) contains two observation equations (equations (5) and (6)) and (29) are the three state equations: (2), (3) and (4).

Following Holston et al. (2017b) and as explained in detail in Holston et al. (2017a), the estimation is carried out in three stages, building up to the full model. This is done to avoid the downward bias in the estimates of the standard deviations of the shocks to $z$ and the trend growth rate. Instead of estimating these parameters directly, they are constructed from the first and second stages and imposed in the final estimation stage. For more details, see the referenced papers.

# Appendix C: Illustration of the difficulties estimating the link between government debt and R*

In this Appendix we showcase the difficulties of estimating the causal link between government debt and R* described in the main text. For the purpose of this illustration, we construct a panel of equilibrium interest rates and government debt / GDP ratios for four large developed economies: the US, Canada, Euro Area and the UK. We use the estimates of R* estimated by Holston et al. (2017b), and study how these estimated equilibrium interest rates vary with the headline measure of government debt in these four economies. We run three regression specifications: the pooled regression, equivalent to OLS on the entire dataset, which effectively ignores the panel-structure of the data; the fixed effects or within regression, which controls for constant unobserved heterogeneity at the economy-level; and a 'between' regression, based on economy-level averages.[56]

Figures 20 and 21 contain the results. The difference between the two figures is the dependent variable: Figure 20 uses the estimate of R* from Holston et al. (2017b) as a dependent variable, while Figure 21 uses their estimate of the unobserved component $z$ (which excludes the effects of declining trend growth). The latter specification is motivated by the idea that it may be easier to deduce the relationship between R* and debt after accounting for trends in productivity. In each figure, the different-colored squares are the data points for the four economies, and the large blurbs denote the economy-level averages. The three different kinds of lines show the estimated relationships: the broken line is the pooled regression, the solid colored lines show the fixed effects model, and the upward sloping dash/dot line shows the 'between' model. The notable result is that both in the pooled and fixed effects models the regression lines are downward sloping: the secular trend in interest rates, which coincided with increasing government debt, dominates these econometric estimates. Only the 'between' model detects a positive relationship between debt and R*, but the inference is extremely limited as the 'between' regression uses only four data points, each corresponding to one economy.[57]

Our simple exercise brings to the fore the difficult challenge that the empirical literature needs to overcome to uncover the link between debt and interest rates, namely that the secular fall in interest rates coincided with a rapid increase in advanced nations' public debt, making identification using measured macro data problematic. In the main text we discuss the papers that overcome the identification difficulties by including detailed measures of the output gap, inflation expectations and portfolio shifts in their regressions or use fiscal forecasts rather than the realized debt/GDP ratios to alleviate the concern that cyclical variation drives the results.

# Appendix D: The life-cycle model derivations

### Retirees

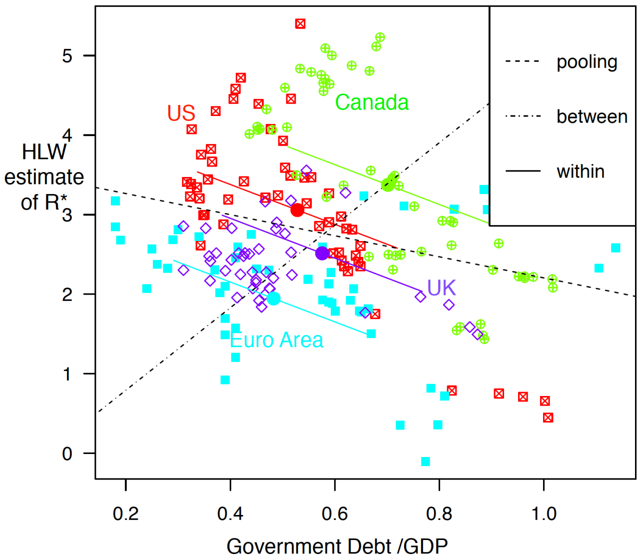
The first order conditions of the retiree's problem yield the Euler Equation:

$$C_{t+1}^{rjk} = (R_{t+1}\beta)^{\sigma} C_t^{rjk} \tag{30}$$

[56]Our panel comprises of annual observations spanning 1961-2013 for the US, Canada and the UK, and 1972-2013 for the Euro Area. We proxy for the government debt in the EA using data for Germany.

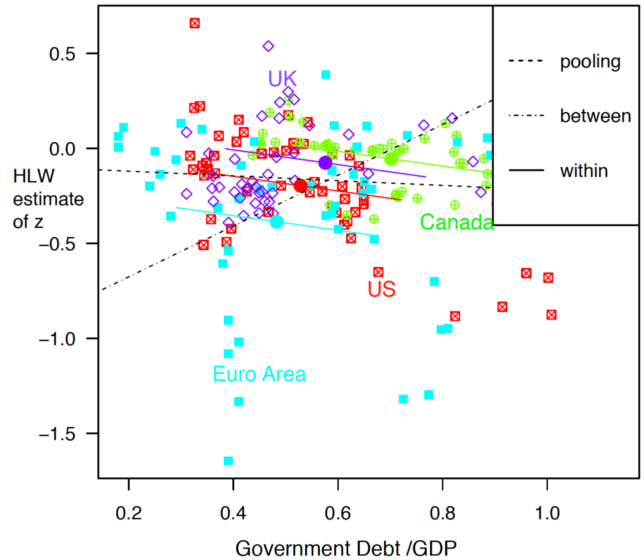
[57]Taken with an appropriately sized pinch of salt, the 'between' estimate suggests that a 1pp increase in government debt / GDP ratio raises the equilibrium rate of interest by about 5bps.

**Figure 20:** Panel regressions of R* and government debt




Source: Holston et al. (2017b) and Jorda-Schularick-Taylor database Jordà et al. (2016).

**Figure 21:** Panel regressions of the z component of R* and government debt




Source: Holston et al. (2017b) and Jorda-Schularick-Taylor database Jordà et al. (2016).

Denoting by $\epsilon_t \pi_t$ the retiree's marginal propensity to consume out of wealth[58], we can write down retiree's consumption function as:

$$C_t^{rjk} = \epsilon_t \pi_t (R_t/\gamma) A_t^{rjk} \tag{31}$$

Plugging this expression into the Euler Equation yields the expression for the evolution of the retiree's MPC:

$$\epsilon_t \pi_t = 1 - (R_{t+1}^{\sigma-1} \beta^\sigma \gamma) \frac{\epsilon_t \pi_t}{\epsilon_{t+1} \pi_{t+1}}. \tag{32}$$

### Workers

The Euler Equation from the worker's problem is:

$$\omega C_{t+1}^{rjk} + (1-\omega) \Lambda_{t+1} C_{t+1}^{rj(t+1)} = (R_{t+1} \Omega_{t+1} \beta)^\sigma C_t^{wj} \tag{33}$$

where $\Lambda$ is the marginal rate of substitution across consumption while being a worker and a retiree, and $\Omega$ is a weighing factor which captures the fact that workers discount future more: $\Omega_{t+1} = \omega + (1-\omega)\epsilon_{t+1}^{\frac{1}{1-\sigma}}$.[59]

Denoting the MPC of the worker by $\pi$, and conjecturing that the consumption function takes

[58]Reasons for this notation will become clear momentarily.

[59]For the complete derivation of worker's Euler Equation, see the Appendix of Gertler (1999).

the form:

$$C_t^{wj} = \pi_t(R_t A_t^{wj} + H_t^j + S_t^j) \tag{34}$$

(where $H$ stands for human wealth and $S$ is social security wealth, given respectively by $H_t^j = \sum_{\nu=0}^{\infty} \frac{W_{t+v} - T_{t+\nu}}{\prod_{z=1}^{v} R_{t+z}\Omega_{t+z}/\omega}$ and $S_t^j = \sum_{\nu=0}^{\infty} \frac{E_{t+\nu}^j}{\prod_{z=1}^{v} R_{t+z}\Omega_{t+z}/\omega}$), we obtain the time path of worker's MPC :

$$\pi_t = 1 - (R_{t+1}\Omega_{t+1})^{\sigma-1}\beta^{\sigma}\frac{\pi_t}{\pi_{t+1}}. \tag{35}$$

**Aggregation**

Marginal propensities to consume are the same across all retirees, so we can just add up their consumptions to get the aggregate consumption function. With slight abuse of notation, denoting now by $C_t^r$, $A_t^r$ and $S_t$ the aggregate variables, we have that:

$$C_t^r = \epsilon_t \pi_t (R_t A_t^r + S_t) \tag{36}$$

where social security wealth is given by the discounted sum of social security payments:

$$S_t = \sum_{\nu=0}^{\infty} \frac{E_{t+\nu}}{\prod_{z=1}^{v}(1+n)R_{t+z}/\gamma}. \tag{37}$$

The evolution of $\epsilon_t \pi_t$ is governed by the Euler Equation of the retiree, given by equation (32).

Marginal propensities to consume are the same across all workers, so we can add individual worker consumption across individuals to obtain their aggregate consumption function. This will depend not only on the aggregate asset wealth but also on the aggregate human wealth and aggregate social security wealth:

$$C_t^w = \pi_t(R_t A_t^w + H_t + S_t^w). \tag{38}$$

The aggregate human wealth is given by

$$H_t = \sum_{\nu=0}^{\infty} \frac{N_{t+\nu}W_{t+\nu} - T_{t+\nu}}{\prod_{z=1}^{v}(1+n)R_{t+z}\Omega_{t+z}/\omega}. \tag{39}$$

Human wealth is a discounted sum of the economy-wide net-of-tax wage bill. The discount rate that is applied to the aggregate wage bill is the product of the gross population growth rate and the rate at which individual workers discount their labor income. The importance of the generation currently alive declines over time, however – they get replaced by newly born generations. So from the point of view of the current generation, the human wealth is discounted more heavily than in the infinite horizon case – the gross population growth rate $(1 + n)$ enters the discount factor. In total, therefore, there are three distinct factors in the life-cycle setting that raise the discount rate on future labor income (relative to the infinite horizon case). They are: (1) finite expected time spent working (reflected by the presence of $\omega$ in the discount rate); (2) greater discounting of the future owing to expected finiteness of life (reflected by the presence of $\Omega$); and (3) growth of the labor force (reflected by the presence of $(1+n)$).

Social security wealth of the workers is:

$$S_t^w = \sum_{\nu=0}^{\infty} \frac{(1-\omega)\omega^\nu N_t \left( \frac{\epsilon_{t+\nu+1} \frac{S_{t+\nu+1}}{\psi N_{t+\nu+1}}}{R_{t+\nu}\Omega_{t+\nu}} \right)}{\prod_{z=1}^{\nu} R_{t+z}\Omega_{t+z}}. \tag{40}$$

The numerator of the sum on the right hand side is a time-$t+\nu$ capitalized value of the social security payments to all the individuals who were in the workforce at $t$ and retire at $t+\nu+1$. The total social security wealth is just the infinite sum of the discounted value of these capitalized payments.

Denoting by $\lambda$ the share of assets held by retirees, we can add the two aggregate consumptions above to get aggregate consumption in the main text:

$$C_t = C_t^w + C_t^r = \pi_t \left\{ (1-\lambda_t)R_t A_t + H_t + S_t^w + \epsilon_t(\lambda_t R_t A_t + S_t^r) \right\} \tag{41}$$

The novel feature is the presence of $\lambda$. Because the MPC of retirees is higher than MPC of workers ($\epsilon > 1$), higher $\lambda$ raises aggregate consumption. So transferring resources across the demographic groups changes overall demand.

The evolution of total wealth of retirees is the sum of return on their wealth from last period plus what the newly retired bring in:

$$\lambda_{t+1}A_{t+1} = \lambda_t R_t A_t - C_t^r + (1-\omega)[(1-\lambda_t)R_t A_t + W_t - C_t^w] \tag{42}$$

From this we get the explicit expression for the evolution of the retiree share:

$$\lambda_{t+1} = \omega(1-\epsilon_t\pi_t)\lambda_t R_t \frac{A_t}{A_{t+1}} + (1-\omega). \tag{43}$$

# Appendix E: The model of precautionary savings: derivations and equilibrium

## Equilibrium in the asset market

Equilibrium in the asset market requires that asset demand (households' desired asset holdings) equals asset supply (firms' capital plus government bonds):

$$A_t = K_t + B_t. \tag{44}$$

Because of exogenous technological progress, the equilibrium in this economy will be characterized by a balanced growth path along which the aggregate variables – $K_t$, $w_t$ and $Y_t$ – grow at rate $\eta$. Below we show how to rewrite the model with variables normalized by GDP, thus making it stationary.

## Transformation into a stationary model

Growth is exogenous, driven by increases in labor augmenting technology: $x_t = e^{\eta t} x_0$. In the balanced growth equilibrium $w_t$, $Y_t$ and $K_t$ will be growing at rate $\eta$ whereas the interest rate will be constant.

Let $k_t = K_t/Y_t$, $\tilde{w}_t = w_t/Y_t$, $\tilde{c}_t = c_t/Y_t$, $\tilde{a}_t = a_t/Y_t$, $\tau_t = T_t/Y_t$, $b_t = B_t/Y_t$ , $\bar{a}_t = A_t/Y_t$, $tran_t = \frac{TR_t}{Y_t}$ denote the normalized variables.

***Households.*** We begin by rewriting the consumer problem. First, note that $c_t = Y_t \tilde{c}_t$ and $Y_t = e^{\eta t} Y_0$. We can rewrite the integral as:

$$\int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt = \int_0^\infty e^{-\rho t} \frac{(e^{\eta t} Y_0 \tilde{c}_t)^{1-\gamma}}{1-\gamma} dt = \frac{Y_0^{1-\gamma}}{1-\gamma} \int_0^1 e^{-(\rho-(1-\gamma)\eta)t} \tilde{c}_t^{1-\gamma} dt. \tag{45}$$

The original budget constraint is:

$$\dot{a}_t = (1-\tau) w_t e_t + (1-\tau) r_t a_t - c_t. \tag{46}$$

Dividing through by $Y_t$:

$$\frac{\dot{a}_t}{Y_t} = \frac{(1-\tau) w_t e_t}{Y_t} + \frac{(1-\tau) r_t a_t}{Y_t} - \frac{c_t}{Y_t}. \tag{47}$$

Note that

$$\dot{a}_t = \frac{\partial a_t}{\partial t} = \frac{\partial \tilde{a}_t}{\partial t} Y_t + \tilde{a}_t \frac{\partial Y_t}{\partial t} = \dot{\tilde{a}}_t Y_t + \tilde{a}_t Y_0 \eta e^{\eta t}. \tag{48}$$

It follows that

$$\frac{\dot{a}_t}{Y_t} = \dot{\tilde{a}}_t + \tilde{a}_t \eta. \tag{49}$$

Thus the budget constraint in transformed variables is:

$$\dot{\tilde{a}}_t = (1-\tau) \tilde{w}_t e_t + ((1-\tau) r - \eta) \tilde{a}_t - \tilde{c}_t \tag{50}$$

And the transformed problem of the household is:

$$\max_{\{\tilde{c}_t\}} \mathbb{E} \frac{Y_0^{1-\gamma}}{1-\gamma} \int_0^\infty e^{-(\rho-(1-\gamma)\eta)t} \tilde{c}_t^{1-\gamma} dt \tag{51}$$

subject to

$$\begin{aligned}
\dot{\tilde{a}}_t &= (1-\tau)\tilde{w}_t e_t + ((1-\tau)r - \eta)\tilde{a}_t - \tilde{c}_t \\
\tilde{c}_t &\geq 0 \\
\tilde{a}_t &\geq 0.
\end{aligned}$$

This is a standard optimal control problem. Because the individual problem is recursive, its stationary version can be summarized with a Hamilton-Jacobi-Bellman (HJB) equation:[60]

$$(\rho-(1-\gamma)\eta)v_j(\tilde{a}) = \max_{\tilde{c}} \left\{ \frac{\tilde{c}^{1-\gamma}}{1-\gamma} + v_j'(\tilde{a})((1-\tau)\tilde{w}e_j + ((1-\tau)r - \eta)\tilde{a} - \tilde{c}) + \sum_{i\neq j} P_{j,i}v_i(\tilde{a}) - P_{j,j}v_j(\tilde{a}) \right\} \tag{52}$$

where the variables with a tilde are normalized by GDP, and $P_{j,i}$ is the Poisson intensity of a change from state $e = z_j$ to state $e = z_i$. This equation has a natural economic interpretation, related to the intuition from the asset pricing literature: the required return to an asset equals the dividend plus the change in value. The left hand side of the equation is the instantaneous required return to holding assets $\tilde{a}$ in state $j$: it is the effective discount rate (i.e. the return $\rho-(1-\gamma)\eta$) times the value function. The first term on the right is the 'dividend': the stream of consumption utility sustained by the given level of asset holdings. The remaining terms denote the instantaneous changes in value, due to asset accumulation and a possibility of a Poisson event that changes the state from $j$ to $i$.

We solve the problem summarized in equation (52) by deriving the first order and the boundary conditions. The first order condition is obtained by differentiating (52) with respect to $\tilde{c}$:

$$\tilde{c_j}^{-\gamma} = v_j'(a). \tag{53}$$

One of the advantages of the continuous time formulation is that equation (53) holds at the constraint. We can use this to derive the boundary condition:[61]

$$v_j'(\underline{a}) \geq ((1-\tau)\tilde{w}e_j + ((1-\tau)r - \eta)\tilde{a})^{-\gamma}. \tag{54}$$

Using these two conditions we solve the HJB equation (52) utilizing the methods described in detail in Achdou et al. (2017).

[60] We focus our attention on stationary equilibria, so that the value function or any other variable in the HJB equation do not depend on time.

[61] To see this, note that equation (53) holding at the constraint implies that $\tilde{c}_j(\underline{a})^{-\gamma} = v_j'(\underline{a})$ where the notation $c_j(\underline{a})$ stands for the policy function in state $e_j$ and assets $\underline{a}$. At the borrowing constraint saving cannot be negative (otherwise the constraint would be breached), so that we must have $\dot{\underline{a}} = (1-\tau)\tilde{w}e_j + ((1-\tau)r - \eta)\underline{\tilde{a}} - \underline{\tilde{c}} \geq 0$, which implies $(1-\tau)\tilde{w}e_j + ((1-\tau)r - \eta)\underline{\tilde{a}} \geq \tilde{c}(\underline{\tilde{a}})$. This, together with the concavity of utility function and equation (53), imply the boundary condition (54).

*Production.* All markets are competitive. The aggregate production function is

$$Y_t = AK_t^{\alpha}(x_t L_t)^{1-\alpha} \tag{55}$$

where $x_t = e^{\eta t}x_0$ is the process for exogenous labor augmenting technical progress, which grows at rate $\eta$.[62] Because we normalized population to be of measure one, we have $L_t = 1$. Capital and labor demands are pinned down by the usual first order conditions:

$$\alpha AK_t^{\alpha-1}x_t^{1-\alpha} = r + \delta \tag{56}$$

$$(1-\alpha)AK_t^{\alpha}x_t^{1-\alpha} = w_t. \tag{57}$$

Capital demand is

$$K_t = \left(\frac{\alpha A}{r+\delta}\right)^{\frac{1}{1-\alpha}} x_t. \tag{58}$$

Dividing through by $Y_t$ we get

$$k_t = \left(\frac{\alpha A}{r+\delta}\right)^{\frac{1}{1-\alpha}} \frac{x_t}{Y_t}. \tag{59}$$

Because $x$ and $Y$ both grow at rate $\eta$, this is the same as:

$$k_t = \left(\frac{\alpha A}{r+\delta}\right)^{\frac{1}{1-\alpha}} \frac{x_0}{Y_0}. \tag{60}$$

Similarly, labor demand is given by:

$$(1-\alpha)Ak_t^{\alpha}\left(\frac{x_0}{Y_0}\right)^{1-\alpha} = \tilde{w}_t. \tag{61}$$

When solving the model, we normalize the starting values $x_0$ and $Y_0$ to unity.

***Government.*** Using the homogeneity of the production function, we can rewrite the government budget constraint (25) as follows. By Euler's Theorem we have:

$$w_t = Y_t - (r+\delta)K_t. \tag{62}$$

This and the asset market clearing $K + B = A$ together imply that (25) becomes:

$$\dot{B}_t = G_t + TR_t + rB_t - \tau(Y_t - (r+\delta)K_t + rK_t + rB_t). \tag{63}$$

Simplifying:

$$\dot{B}_t = G_t + TR_t + (1-\tau)r_t B_t - \tau(Y_t - \delta K_t). \tag{64}$$

[62] To see this, note that $\frac{\dot{x}_t}{x_t} = \frac{\partial x_t}{\partial t}\frac{1}{x_t} = \eta$.

Dividing through by $Y_t$ and rearranging we get the transformed government budget constraint:[63]

$$g_t + tran_t + ((1-\tau)r_t - \eta)b_t - \dot{b_t} = \tau(1 - \delta k_t) \tag{65}$$

In steady state, government debt / GDP ratio is constant, so that for given values of $r$, $b$ and $g$, the government budget constraint pins down the tax rate:

$$\tau = \frac{g + tran + b(r-\eta)}{1 - \delta k + rb}. \tag{66}$$

In the main text we set $TR_t = 0 \ \forall t$.

## Stationary Equilibrium

The following set of equations fully characterizes the stationary equilibrium:

- The HJB equation summarizing the household's problem:

$$(\rho - (1-\gamma)\eta)v_j(\tilde{a}) = \max_{\tilde{c}} \left\{ \frac{\tilde{c}^{1-\gamma}}{1-\gamma} + v_j'(\tilde{a})((1-\tau)\tilde{w}e_j + ((1-\tau)r - \eta)\tilde{a} - \tilde{c}) + \sum_{i \neq j} P_{j,i}v_i(\tilde{a}) - P_{j,j}v_j(\tilde{a}) \right\} \tag{67}$$

- The Kolmogorov Forward Equations which characterize the distributions of workers in the four income states. In stationary equilibrium these take the following form:

$$0 = -\frac{d}{da}[s_j(a)g_j(a)] - \lambda_j g_j(a) + \sum_{i \neq j} \lambda_i g_i(a) \tag{68}$$

  where $s_j(a)$ is the saving policy function from the HJB equation and $g_j(a)$ denotes the distribution (density) of type-$j$ worker, so that

$$\int_{\underline{a}}^{\infty} (g_1(a) + g_2(a) + g_3(a) + g_4(a))\, da = 1, \qquad g_j \geq 0 \ \forall j. \tag{69}$$

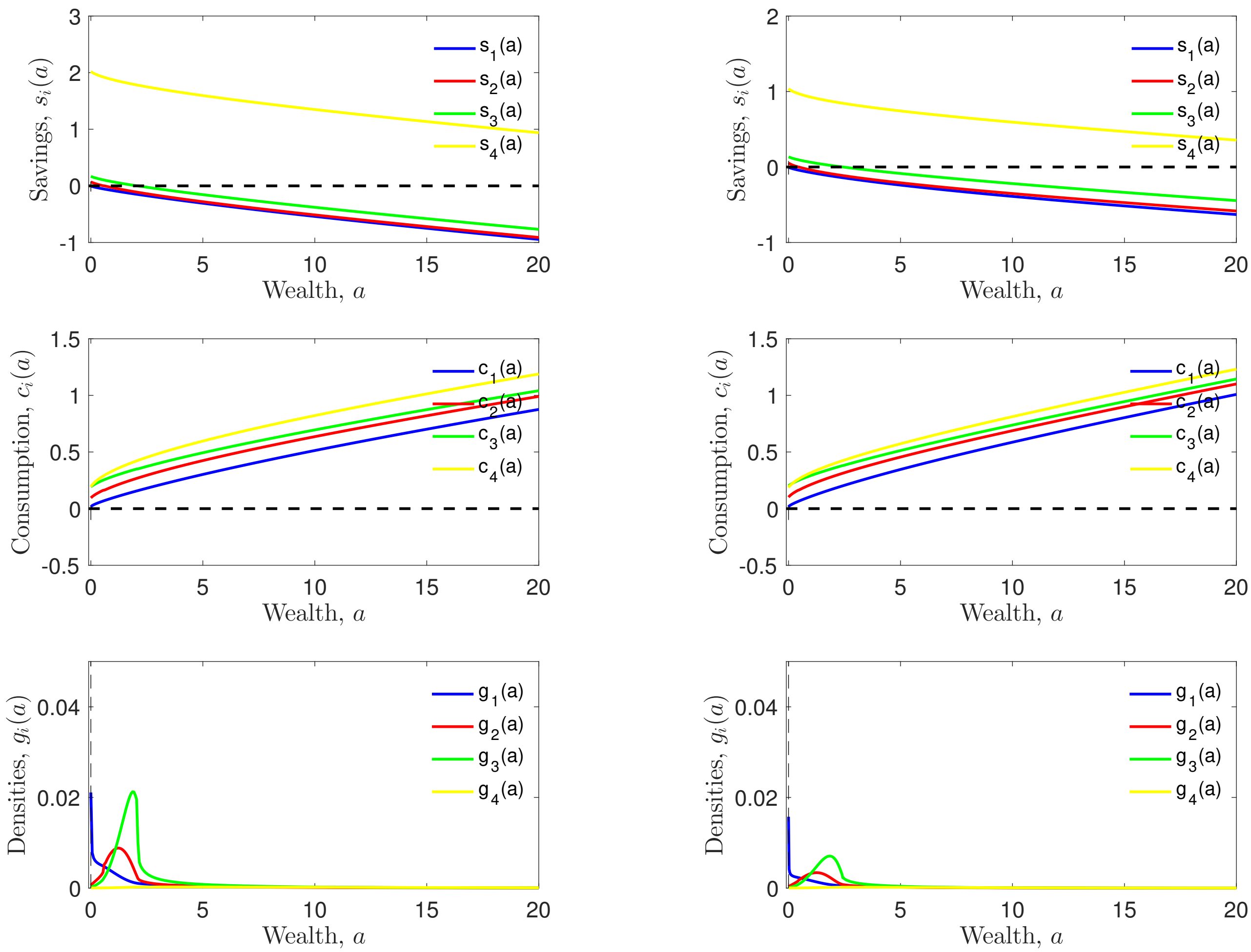
- The asset market clearing condition (expressed using the transformed variables):

$$\bar{a} = \sum_j \int_{\underline{a}}^{\infty} a g_j(a) da = k + b. \tag{70}$$

Figure 22 shows the consumption and saving policy functions as well as the stationary distributions of agents across the asset space in the low- and high-debt equilibria described in the main text.

[63] Note that $\frac{\dot{B}_t}{Y_t} = \frac{\frac{\partial \dot{B}_t}{\partial t}}{Y_t} = \frac{\dot{b}_t Y_t + b_t \dot{Y}_t}{Y_t} = \dot{b}_t + b_t \eta$.

**Figure 22:** Stationary equilibrium with low (left) and high debt (right)

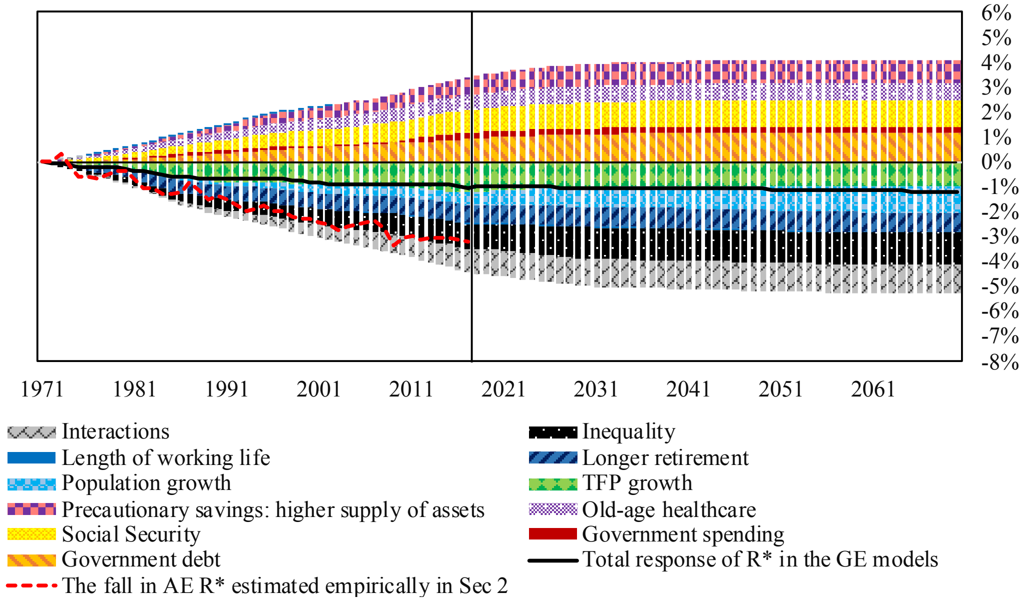



# Appendix F: Sensitivity of the model-based results to alternative parametrization

The key parameter that determines the overall sensitivity of the interest rate to long-term fiscal stance as well as other secular trends, such as technological and demographic change, is the intertemporal elasticity of substitution, $\frac{1}{\gamma}$. In general, the higher this elasticity, the smaller the impact of a change in the macroeconomic environment on the interest rate. The intuition is that, if consumers are very willing to substitute consumption across time, smaller changes in the interest rate will be sufficient to induce them to do so. So less of a change in the interest rate will be required to restore equilibrium.

Our simulations reported in the main text are based on the parametrization in which this elasticity is set equal to $\frac{1}{2}$, which is a standard value used in many macroeconomic models. It is also the average value of the estimated elasticity across a large number of studies described in a comprehensive review by Havranek et al. (2015). Still, some models assume a higher elasticity. For example, the calibration of the Smets and Wouters (2007) model assumes that $IES = \frac{2}{3}$.

**Figure 23:** Decomposition of the changes in the equilibrium real interest rate with a higher IES




To illustrate the sensitivity of our results, we now present the results under this alternative assumption.

Figure 23 illustrates how our decomposition would look like under this higher IES. Our framework is able to account for a smaller proportion of the decline in R* under this calibration: the given set of trends has a smaller impact on the interest rate with a higher IES, in line with the intuition above. Changes in the IES matter more for the life-cycle model than for the incomplete markets model, as intertemporal substitution plays a smaller role in the latter, given the presence of borrowing constraints.

In the future version of this paper, we plan for a more complete examination of the sensitivities. For a more detailed approach to assessing the parameter uncertainty in the context of these models, see Ho (2018).