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On Frame Stability Analysis

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Abstract

The objective of this paper is to evaluate the frame stability analysis methods proposed by the AISC specification (AISC (2010)). The AISC specification prescribes in its main body the direct analysis method for frame analysis, which relies on a second-order elastic analysis. As an alternative analysis method, the AISC specification permits the use of the traditional effective length method combined with additional requirements, or a simplified version of the direct analysis method called the first-order analysis method. Rigorous finite element (FE) models of typical steel buildings were built to evaluate the frame stability analysis methods proposed by the AISC specification. The FE analysis consists of a second-order inelastic analysis that also considers the effects of initial geometric imperfections, partial yielding, and residual stress. Although all the methods in the AISC specification reasonably predict the required strength in the members, their approach to assessing the impact of second-order effects is inadequate. According to the AISC specification, the impact of second-order effects should be measured by the ratio of second-order drift to first-order drift. This ratio, however, especially for the gravity-only load combination, can be greater than the limit defined by the AISC specification, while the second-order effects on the required strength are negligible. Based on the examples explored in this paper and other design standards, alternative methods are proposed to assess the impact of second-order effects.

1. Introduction

A structural design project shall consider factors that may affect frame stability. The AISC specification lists several factors that shall be considered in a structural stability analysis, including: flexural, shear, axial, and connection deformation; geometric imperfection; second-order effects (P-Δ and P-δ); and stiffness reduction due to inelasticity (partial yielding and residual stresses).

Although several structural analysis software packages already allow the user to consider the major factors that may affect frame stability, simplified models are still an option available for

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the designer. For example, the AISC specification in Appendix 7 permits the use of the traditional effective length method combined with additional requirements, or a first-order elastic analysis method. In the AISC specification the simplified methods are an alternative for the direct analysis method for frame analysis found in Chapter C.

The ASCE committee report (Bridge and Clarke (1997)), White, Surovek et al. (2006), Deierlein (2004), Ziemian (2010), and Nair (2009) also discuss frame stability analysis according to the AISC specification. Clarke (1997) provided a thorough comparison of several steel design standards, supplemented by Dória (2007) with a comparison of the final draft of the Brazilian specification for the design of steel structures: NBR 8800 (ABNT (2008)) to other steel design standards.

The main focus of this paper is to evaluate the efficacy of using the ratio of second-order drift to first-order drift ($\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$), as prescribed by AISC (2010), to evaluate the impact of second-order effects on the frame stability analysis. As demonstrated above, ample work exists on accounting for second order effects. This paper focuses on the methods for assessing the need for a rigorous frame stability analysis.

2. Frame stability analysis by AISC (2010) and the implications of large ratio of second-order drift to first-order drift

The goal of conducting a proper frame stability analysis is to accurately find the required strength of a member. The strength of each member subjected to flexure and compression is evaluated in the interaction equations as prescribed in the AISC (2010) specification:

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad When\ \frac{P_r}{P_c} \geq 0.2 \tag{1}$$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad When\ \frac{P_r}{P_c} < 0.2 \tag{2}$$

where P is the axial strength and M is the flexural strength. The subscript c refers to design values, r to required values, and x and y relates to strong and weak axis bending, respectively. Since this paper focuses on frame stability analysis, we are mainly concerned on the determination of the required values. The following sections briefly describe the methods available in the AISC (2010) for frame stability analysis; special consideration is given to the role of the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$.

2.1 Direct analysis method

The direct analysis method is presented in the main body of the AISC (2010) in Chapter C. In short, the method consists of a second-order analysis that considers: flexural, shear, axial, and connection deformations; P-Δ and P-δ effects; initial imperfections via direct modeling (points of intersection displaced from their nominal locations) or by applying lateral loads at all levels (notional loads); the members stiffness are reduced to consider partial yielding and residual stresses; and the effective length factor, K, is taken as unity.

In the direct analysis method, if the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ is equal to or less than 1.7 (Limit and Resistance Factor Design (LRFD) load combinations, with stiffness reduction (Section C.2.3 of AISC, 2010)): (i) P-δ can be neglected in the frame analysis, and (ii) notional loads can be applied in gravity-only combinations, disregarding any combination that includes other lateral loads.

2.2 Effective length method

The effective length method is presented in Appendix 7 of the AISC (2010). This method is distinct from the direct stiffness method in that the member stiffness is not reduced, the structure shall support gravity loads through vertical columns, walls, or frames, and the effective length factor, K, in moment frames shall be determined from a side-sway buckling analysis of the structure or alignment charts.

The ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ has major implications in the effective length method. The effective length method cannot be used if the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ is equal to or greater than 1.5 (LRFD load combinations); a note, however, highlights that the B_2 multiplier may be taken as the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$. Another implication is that if the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ is equal to or less than 1.1 (LRFD load combinations) a value of K equal to 1 is permitted in the design of all columns.

The B_2 multiplier, equation (3), is presented in Appendix 8 of the AISC (2010) specification and is used as an alternative to a rigorous second-order analysis to account for P-Δ effects. In the approximate second-order analysis B_2 magnifies the first-order axial force or moment due to lateral translation only.

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1 \tag{3}$$

where α is equal to one for LRFD load combinations. P_{story} is the total vertical load supported by the story, including loads in columns that are not part of the lateral force resisting system. $P_{e\ story}$ is the elastic critical buckling strength for the *story*, which can be determined by sidesway buckling analysis or by an approximate equation that is based on the story height, vertical load, interstory drift, and the story shear.

2.3 First-order analysis method

The first-order analysis method is also presented in Appendix 7 of AISC (2010). The method is considered a simplification of the direct stiffness method, since Kuchenbecker (2004) mathematically manipulated a first-order analysis that provides similar results to a direct stiffness analysis. In the first-order method K still unity, but the notional load is amplified. The method is limited to the analysis of structures that support gravity loads through vertical columns, walls or frames. The required compressive strength whose flexural stiffness contributes to the lateral stability shall be less or equal half of the axial yield strength (LRFD load combinations).

The method can only be used if the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ or B_2 multiplier is equal to or less than 1.5 (LRFD load combinations) – the same requirement is prescribed in the effective length method.

3. Numerical analysis

3.1 Stability analysis using MASTAN2

The computer-based analysis necessary in the direct analysis method, effective length method, and first-order analysis method are performed using the MASTAN2 v3.3 software package (McGuire, Gallagher et al. (2000)). MASTAN2 can perform first and second order, elastic and inelastic analysis, as well as buckling analysis. The stiffness is reduced in all MASTAN2 models analyzed in this paper by multiplying the Young's modulus, E, by a factor of 0.8. In the second-order analysis an additional stiffness reduction factor, τ_b, as prescribed in section C.2.3 of AISC (2010) was included. MASTAN2 provides an option to automatically consider the τ_b factor in the analysis.

The commentary in AISC (2010) explains that a reduced stiffness is used to keep the system available strength within a margin of safety from the elastic stability limit, and also to account for inelastic softening in the members.

3.2 Stability analysis using ABAQUS

Finite element (FE) models were built in ABAQUS 6.5 (Hibbitt (2005)) to evaluate the frame stability analysis methods proposed by the AISC (2010) specification. The material model chosen to represent the steel is non-linear, perfectly plastic (yield stress does not change with plastic strain) and it considers metal plasticity (von Mises yield surface). The Newton-Raphson method is used to solve the nonlinear model. The frame elements were modeled using the 3-node quadratic beam element B32 (Hibbitt (2005)) since it allows the user to define initial stresses (residual stresses) at integration points in the cross-sections.

The residual stress distribution on the cross-section is the same as proposed in Ketter and Galambos (1959) and depicted in Figure 1. Essa and Kennedy (2000) have shown that this distribution is conservative since the entire web is in tension. For column design, however, the most important stress is the compression stress, which is found at the extremities of the flange. The residual stresses are inserted at the integration points of the cross-section via a subroutine developed in FORTRAN.

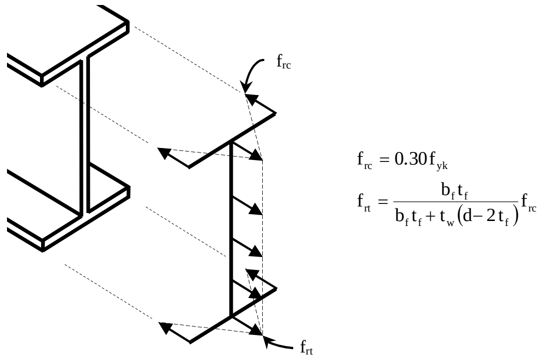
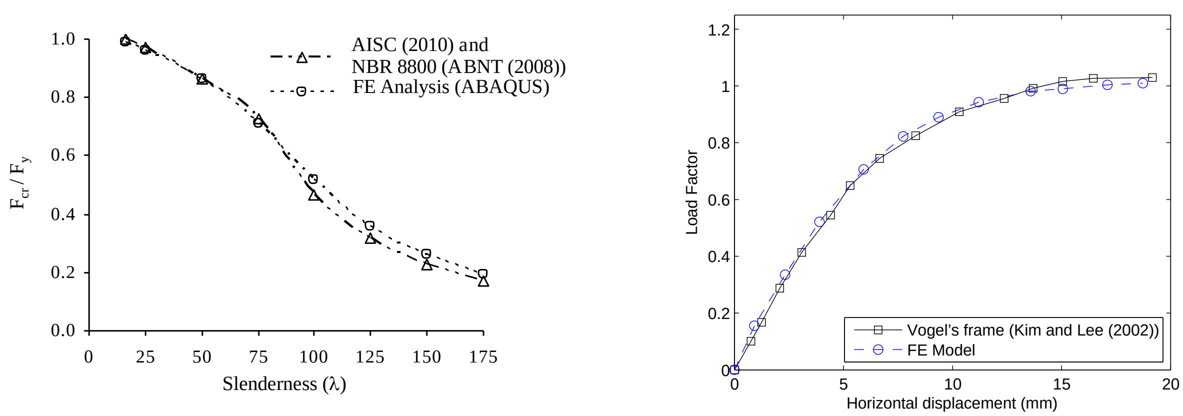


Figure 1 – Residual stress distribution Ketter and Galambos (1959).

3.2.1 Validation of the FE model

In order to validate the FE model, a W14x61 column with initial global imperfection (Δ_0) of H/500 and initial local imperfection (δ_0) of H/1000 was compared to the maximum-strength curve prescribed in the AISC (2010). This is similar to the curve prescribed in the Brazilian specification NBR 8800 (ABNT (2008)). This validation is depicted in Figure 2. The column was divided into 10 finite elements along its length. The comparison shows that the FE model is able to satisfactorily predict the strength of a member.

The FE model of a frame was also validated against the benchmark steel frame – Vogel's portal frame – described and analyzed in Kim and Lee (2002), Figure 2(b). The FE model was able to accurately predict the behavior of the Vogel's portal frame. Since the FE model was able to predict the strength of a member and the behavior of a frame the authors considered it appropriate to use the FE model to determine the adequacy of a frame stability analysis.



a) b)

Figure 2 – Validation of the rigorous analysis in ABAQUS. a) Comparison between design curve for a cross-section W14x61 and a FE model, b) Comparison between Vogel's portal frame P-Δ curve and FE model result.

4. Analysis of typical steel structures

Two steel frames are analyzed in this section. The first is an asymmetric steel frame that was proposed based on the authors' design experience and is intended to reflect a typical low-rise steel frame, Figure 3. The second frame is a typical steel storage rack, which was based on a parametric study reported in Sarawit and Peköz (2003). This frame choice is intended to reflect a typical steel storage rack, Figure 5.

More emphasis is given to the comparison between the different stability analysis methods in the analysis of the first frame. The analysis, however, has brought attention to the difficulty in assessing the second-order effects on the structure by the ratio of ratio of second-order drift to first-order drift ($\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$). The same issue is discussed in the Guide to Stability Design Criteria for Metal Structures (Ziemian, 2010), which shows that a large story drift amplification is not necessarily followed by a large amplification in the second-order forces. Based on the concerns presented in the analysis of the first frame a steel storage rack study is presented in section 4.2 of this paper.

4.1 Asymmetric frame

The asymmetric frame depicted in Figure 3 is intended to reflect a typical steel building frame; dimensions, loads, load combinations, and material used in the stability analysis are summarized in the same figure. Two load combinations are taken in account: a gravity-only load combination (case 2 in the ASCE (2010) specification) and a load combination that includes lateral wind load (case 4 in the ASCE (2010) specification). Although the lateral wind load is depicted in Figure 3 from left to right, wind load from right to left is also considered. The stability analysis considered the initial imperfection by means of an additional lateral-notional load ($N_i=0.002Y_i$) as prescribed in section C2.2b of the AISC (2010).

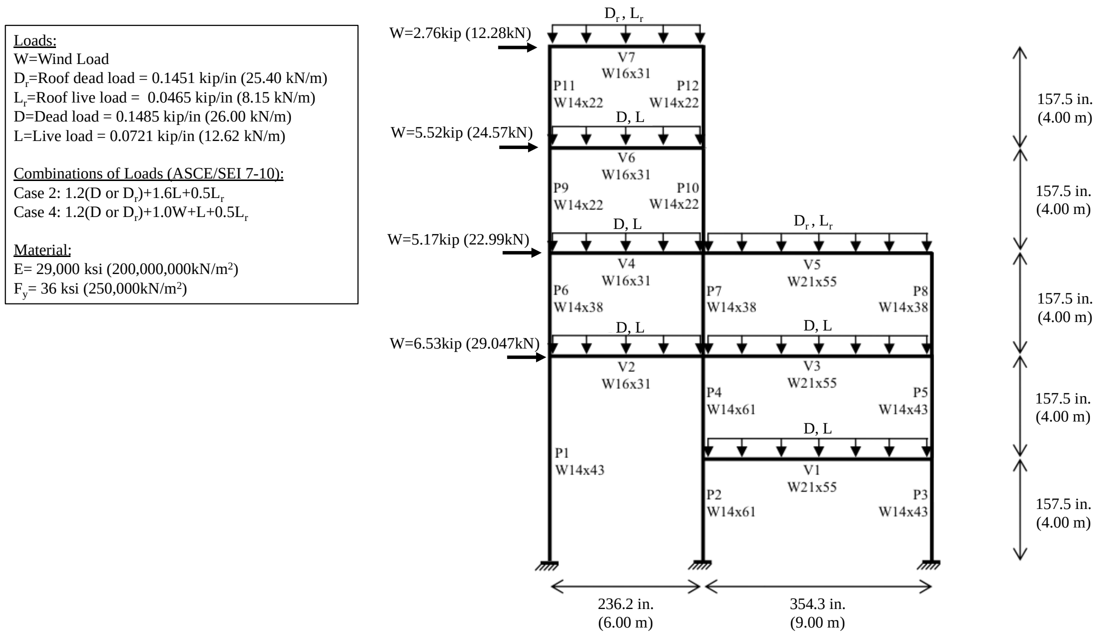
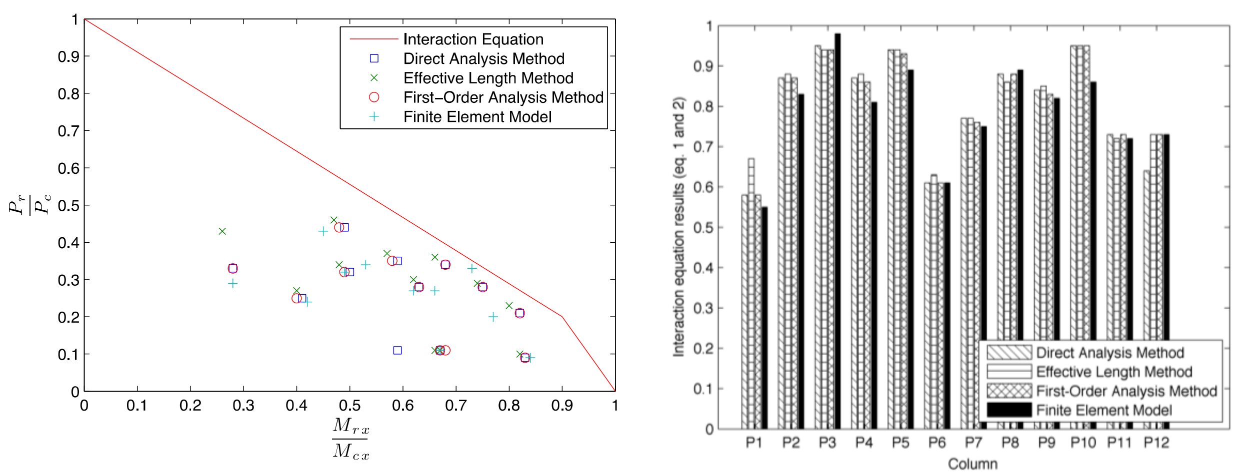


Figure 3 – Asymmetric frame: dimensions, material, and loads.

Direct analysis, effective length, and first-order analysis method are compared to the FE model in Figure 4. The stability analysis methods considered in this paper and prescribed in the AISC (2010) lead to very similar results; the results are overall slightly higher than the FE model, which does not compromise the safety of the structure. The similarities between the methods and FE model can be justified by the small lateral drift and by the use of columns that are considered of intermediate slenderness; the same may not true for a more slender and flexible structure.



a) b)

Figure 4 – Comparison of different methods for the stability analysis of the asymmetric frame. a) Interaction diagram and results using different methods, b) Direct comparison of interaction equation results of each column using different stability analysis method.

The ratio of second-order drift to first-order drift, especially for the gravity-only load combination, can be large while the second-order effects on the required strength are negligible. The notional loads applied to the frame do not result in large lateral displacements and the story drift for a first or second-order analysis is in fact very small in magnitude (tenths of a inch or less). A large ratio of story drift does not imply that the story has a large lateral displacement, since the ratio can be large while the actual magnitude of the displacements are small.

For the load combination that examines lateral wind load, the ratio of displacements gives a good assessment of the second-order effects on the stability analysis. Based on the same observation, the Brazilian specification (NBR 8800 (ABNT (2008))) prescribes that for the gravity-only load combination, the ratio of second-order drift to first-order drift shall be disregarded in the assessment of the impact of second order effects. NBR 8800 (ABNT (2008)) is intended to guide the structural engineer in the design of typical steel-only and steel-concrete composite structures; the specification is not intended to assist specifications that prescribe design method for special structures, such as steel storage racks. The next section presents a discussion on the stability analysis of steel storage racks.

4.2 Steel storage rack

Sarawit and Peköz (2003) published a research report on design and stability analysis of steel storage rack, which was later partially reproduced in Sarawit and Peköz (2006). Their study focused on the comparison between the effective length method and the direct analysis method for storage rack design. Steel storage rack is currently designed using the ANSI-MH16.1 (2008) specification, nonetheless, ANSI-MH16.1 (2008) prescribes in section 6.3 that stability analysis shall be performed using the effective length method or other rational methods consistent with AISC (2010). The only peculiar requirement in the ANSI-MH16.1 (2008) is that an initial out-of-plumb value of H/240 shall be applied to the structure instead of H/500 as prescribed in the AISC (2010). This requirement complicates the use of the notional load method as currently prescribed in the AISC (2010) since the method was calibrated for a H/500 out-of-plumb value, therefore, we have opted to directly model the out-of-plumb imperfection in our models.

The steel storage rack under investigation is based on Sarawit and Peköz (2003) and is intended to represent a typical storage rack, Figure 5; the information necessary for the analysis is summarized in the same figure. An important characteristic of steel storage rack modeling is the stiffness of the connection between column and beam and column and base. The connection stiffness values were chosen according to Sarawit and Peköz (2003) and are also depicted in Figure 5.

The steel storage rack frame is analyzed in details under two gravity-only loads. The first uniform load applied to the steel rack results in a ratio of the critical load of the *frame* to the uniform load in consideration, F_{cr}/F_u, equal to 3.00. An elastic global buckling analysis of the steel storage rack frame defines the critical load of the same (F_{cr}). This ratio is prescribed in Eurocode 3 (CEN (2003)) as the limit to use simplified second-order amplification factors. The second uniform loading under consideration results in a ratio F_{cr}/F_u of 1.35; this ratio is intended to provide more information on the second-order effects by applying an uniform load closer to the critical load of the structure.

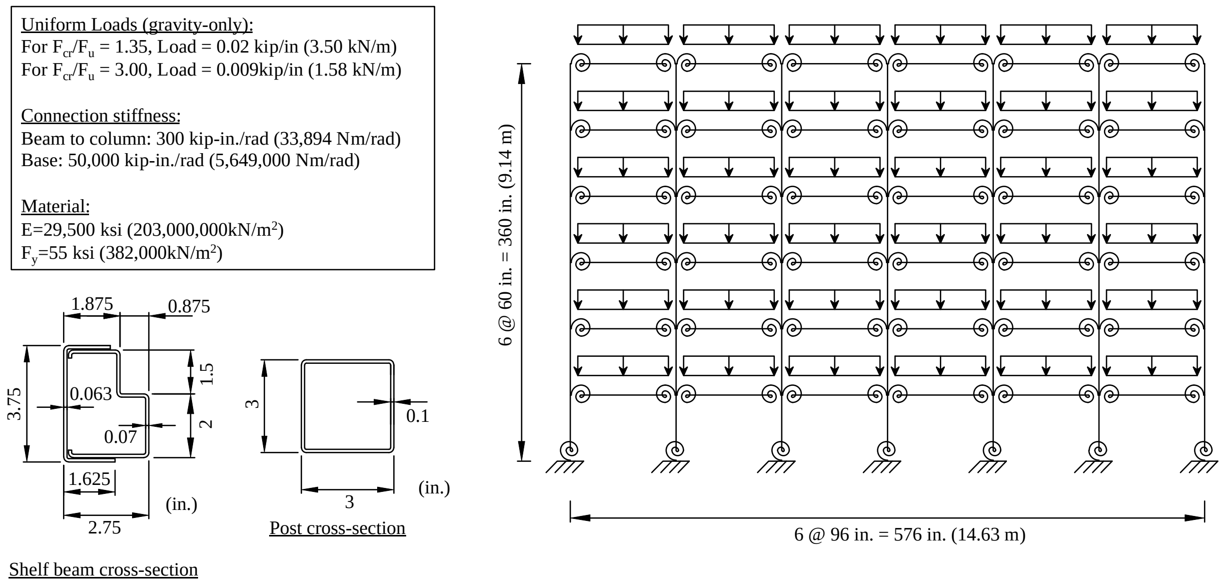


Figure 5 – Steel storage rack: dimensions, loads, connection stiffness, and material.

For the lowest uniform load (F_{cr}/F_u=3) the magnitude of first and second-order displacements are very small, nonetheless the ratio of second-order drift to first-order drift is very large as seen in Table 1. The same result was observed in the last example: for steel storage racks the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ is very large and not a good indicator of second-order effects. Although the AISC (2010) specification only requires a material stiffness reduction in order to perform the first and second-order analysis under consideration, we also calculated the drift ratio for a frame with an out-of-plumbness of H/500. As depicted in Table 1 the drift ratio when an out-of-plumbness is considered is still very large compared to the small impact of second-order effects ($M_{2nd\ order}$ / $M_{1st\ order}$). The large value of the ratio $\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ is explained by the nature of the problem. In a steel storage rack there is a small transfer of moment from the shelf beam to the column and so the column is predominantly loaded axially resulting in small lateral drift. Although the out-of-plumb value is large for steel storage rack analysis, it is not sufficient to trigger large displacements due to P-Δ effects.

Moments from the outmost right column of the frame are compared. Because the frame is displaced to the right in order to apply an out-of-plumbness to the frame, the column with higher required strength is the outmost right column in the frame and as such it will govern the design of each story of the rack. Moments due to a second and first-order are very similar if the analysis is performed according to the AISC (2010) specification – reduction of stiffness only. If an out-of-plumbness is also considered in the model, the difference between moment predictions by a second and first-order analysis increases but remains under a 30% difference.

B_2 multiplier is also reported in Table 1. Different from the ratio of second-order drift to first-order drift, B_2 remains at the same magnitude of the ratio $M_{2nd\ order}$ / $M_{1st\ order}$, which indicates that perhaps B_2 is a good indicator of the second-order effects on the structure. B_2 in this case can be easily calculated based on a sidesway buckling analysis. For the Eurocode 3 (CEN (2003)) the load ratio under consideration (F_{cr}/F_u=3) is the limit to use simplified second-order amplification factors, which does not preclude that greater values of F_{cr}/F_u can be employed but that other methods shall be used to consider second-order effects.

Table 1: Comparison of different stability analysis of a steel storage rack at F_{cr}/F_u = 3.

	Story Drift (in.)					Column Moments*	
Story	$\Delta_{1st\ order}$	$\Delta_{2nd\ order}$	$\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$	$\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ out-of-plumbness is considered	B_2	$M_{2nd\ order}$ / $M_{1st\ order}$	$M_{2nd\ order}$ / $M_{1st\ order}$ out-of-plumbness is considered
First	1.42E-04	3.90E-02	275.70	-29.75	1.02	0.82	1.28
						0.93	0.65
Second	-2.61E-05	1.26E-01	-4827.43	-88.34	1.05	1.04	1.22
						1.01	1.12
Third	2.03E-05	2.24E-01	11027.12	-161.00	1.07	0.99	1.10
						1.00	1.17
Fourth	-4.27E-05	3.08E-01	-7216.87	-211.83	1.08	1.00	0.99
						1.00	1.15
Fifth	2.08E-04	3.69E-01	1775.06	-284.96	1.09	1.00	0.93
						1.00	1.12
Sixth	-4.88E-04	4.07E-01	-833.81	-225.58	1.10	1.00	0.95
						1.00	1.02

* First line refers to the relationship between the moments at the bottom of the column in the story, and second line to the relationship at the top of the column in the story.

In the next case study a larger uniform load is applied to the frame (F_{cr}/F_u=1.35), and the analysis is performed for a load closer to the elastic global buckling load of the frame. Similar conclusions – small magnitude of the story drift but high drift ratio – are reached regarding the story drift comparison, Table 2. The ratio of second to first-order moment, however, considerably increases if out-of-plumbness is considered in the model. This aspect is not captured by the B_2 multiplier.

Table 2: Comparison of different stability analysis of a steel storage rack at F_{cr}/F_u = 1.35.

	Story Drift (in.)					Column Moments*	
Story	$\Delta_{1st\ order}$	$\Delta_{2nd\ order}$	$\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$	$\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ out-of-plumbness is considered	B_2	$M_{2nd\ order}$ / $M_{1st\ order}$	$M_{2nd\ order}$ / $M_{1st\ order}$ out-of-plumbness is considered
First	3.14E-04	-9.81E-03	-31.19	3.70	1.05	0.60	2.74
						0.85	-0.59
Second	-5.81E-05	-1.05E-02	180.21	3.79	1.11	1.09	2.11
						1.03	1.67
Third	4.51E-05	-1.03E-02	-227.75	3.70	1.16	0.97	1.49
						0.99	1.98
Fourth	-9.48E-05	-1.04E-02	110.16	3.56	1.20	1.01	0.87
						1.00	1.78
Fifth	4.62E-04	-9.85E-03	-21.30	3.42	1.22	1.00	0.58
						1.00	1.60
Sixth	-1.09E-03	-1.15E-02	10.61	3.34	1.25	1.00	0.73
						1.00	1.12

* First line refers to the relationship between the moments at the bottom of the column in the story, and second line to the relationship at the top of the column in the story.

The B_2 multiplier is in essence a comparison between the load in *each story* and the buckling load in *each story*, while the ratio, F_{cr}/F_u, prescribed in the Eurocode 3 (CEN (2003)) compares the overall buckling load of the *frame* to the buckling load of the *frame*. Because loads at each story interact, the B_2 multiplier and F_{cr}/F_u are conceptually different; the authors consider the ratio F_{cr}/F_u a more adequate indicator of the second-order effects on the structure.

To support the authors' statement a parametric study was carried out to evaluate the impact of other ratios F_{cr}/F_u on the second-order effects ($M_{2nd\ order}$ / $M_{1st\ order}$, out-of-plumbness is considered), Figure 6. The ratio of second- to first-order moment converges to unity for larger load ratios F_{cr}/F_u. While the ratio F_{cr}/F_u accurately follows the impact of second-order effects on the structure, the B_2 multiplier merely changes from 1.25 to 1.10, never reaching the 1.5 limit defined by the AISC (2010) specification.

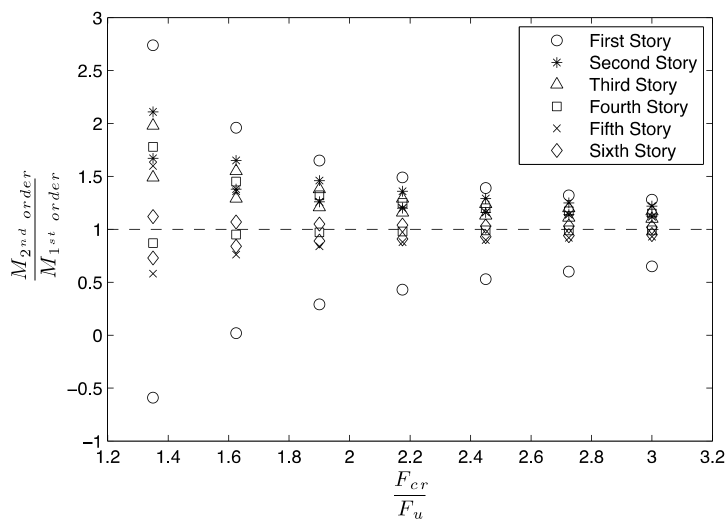


Figure 6 – Vari... ratios of critical load of the frame to the applied uniform load (F_{cr}/F_u).

5. Conclusions

The direct analysis, effective length, and first-order analysis methods lead to strength prediction very similar to the rigorous FE model, but the analysis revealed the difficulty in assessing second-order effects via the ratio of second-order drift to first-order drift. This issue was further explored in the analysis of steel storage racks where the problem is more evident. The B_2 multiplier that is considered in the AISC (2010) specification as a replacement for the second-order drift to first-order drift ratio may also lead to an incorrect assessment of second-order effects. Based on the Brazilian specification, NBR 8800 (ABNT (2008)), the requirement regarding the ratio of second-order drift to first-order drift could be ignored for the gravity-only load combination, but this is only practical for structures that have load combinations that include lateral loads. The ratio F_{cr}/F_u prescribed in the Eurocode 3 (CEN (2003)) is shown in this paper to be a better indicator to assess the need of a second-order analysis than the methods prescribed in the AISC (2010) ($\Delta_{2nd\ order}$ / $\Delta_{1st\ order}$ and B_2).

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