On generalized Cosserat-type theories of plates and shells: a short review and bibliography

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Dedicated to Professor Horst Lippmann (1931–2008)

Abstract One of the research direction of Horst Lippmann during his whole scientific career was devoted to the possibilities to explain complex material behavior by generalized continua models. A representative of such models is the Cosserat continuum. The basic idea of this model is the independence of translations and rotations (and by analogy, the independence of forces and moments). With the help of this model some additional effects in solid and fluid mechanics can be explained in a more satisfying manner. They are established in experiments, but not presented by the classical equations. In this paper the Cosserat-type theories of plates and shells are debated as a special application of the Cosserat theory.

Keywords Micropolar continuum · Cosserat continuum · Micropolar shell · Cosserat shell · Micropolar plasticity

1 Introduction

One hundred years ago the Cosserat brothers, Eugèn and François, published the monograph [54], where they presented a new variant of Continuum Mechanics as well as the Mechanics of Rods and Shells based on the idea of taking into account additionally the couple stresses and the rotational degrees of freedom of the material particles as independent variables. This continuum model is later named Cosserat or micropolar continuum. The basic ideas of this approach are presented first time in an earlier publication [53].

The main ideas leading to the micropolar continuum and other generalized media were discussed at the end of nineteenth century by Kelvin, Helmholtz, Duhem, Voigt, Le Roux, the Cosserats and others. In the micropolar continuum, each material particle has six degrees of freedom. From the physical point of view, every material point (or particle) of a micropolar continuum is phenomenologically equivalent to a rigid body. Hence, rotations of the particles are taken into account. Note that by this approach the gap between the General Mechanics and the Strength of Materials is closed in such form that the independent existence of translations

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and rotations (and by analogy of forces and couples or force and moment stresses) is stated, see, e.g., [307]. Historically the first scientist, who obtained similar results, was L. Euler. Discussing one of Langrange's papers he established that the foundations of Mechanics are based on two principles: the principle of momentum and the principle of moment of momentum. Both principles results in the Eulerian laws of motion [307]. In [214] is given the following comment: the independence of the principle of moment of momentum, which is a generalization of the static equilibrium of the moments, was established by Jacob Bernoulli (1686) one year before Newton's laws (1687). It must be noted that Newton's law allows a satisfying description of the motion of material points. If the continuum is presented by material particles with arbitrary shape this is not enough.

Hence the mechanics of generalized continua such as Cosserat continuum which differs from the classical (or Cauchy-type) continuum mechanics is recognized as a branch of mechanics with origins in the seventeenth century, but the first serious theoretical discussions have been started in the mid of the nineteenth century. On the other hand, it is of current and emerging interest of mechanicians, physicians, materials scientists as well as engineers since the limits and possibilities of such theories are not fully known, which is a serious constraint for applications.

At the same time another tendency in Mechanics can be observed. Since Langrange's *Mécanique analytique* (1788) summarizing the state of the art in Mechanics at the end of the eighteenth century Mechanics is split into two branches: the mathematical one and the engineering one. As a result the mathematical branch is developed in a more axiomatic direction while the engineering branch is focussed on technical applications. Examples of the axiomatic approach are, for example, G. Herglotz' Lectures [143] or Cosserats' monograph [54]. The necessity to establish an axiomatic foundation of Mechanics was pointed out by D. Hilbert in 1900 at the 2nd International Congress of Mathematicians in Paris[1] (the 6th Hilbert problem: establishment of the axiomatic structure of Physics, especially Mechanics). The solution of this problem is nontrivial. During the first half of the twentieth century only some scientist worked on this problem. One of them, G. Hamel, published his results from 1908 in [140] (see also [141]). Since the mid fifties of the last century the interest to the axiomatic approaches and Cosserats' ideas is growing again and many publications have appeared. One of the initiators of this direction was W. Noll (see [51] among others).

In the micropolar theory, besides ordinary stresses, couple stresses are introduced, see, for example, [308, 309]. The deformation of the micropolar continuum may be described by the position vector $\boldsymbol{r}$ and the three orthonormal vectors d_i, $i = 1, 2, 3$, so-called directors, which model the translations and the orientation changes of the material particles. Sometimes the properly orthogonal tensor $\mathbf{Q}$ is used for the description of the rotations. In the case of small strains the deformation of the micropolar medium is defined by two independent fields: the displacement vector u and the microrotation vector ϑ. The linear Cosserat theory was developed in the original papers by Günther [137], Aero and Kuvshinskii [3,4], Toupin [304], Mindlin and Tiersten [203], Koiter [161], Palmov [234,235], Eringen [90,92], Schaefer [278], Ieşan [146], etc. Let us mention here the books [68,95,98,147,169,232,295], where many references to other papers can be found. The problems of the micropolar continuum at finite deformations are considered by Grioli [135,136], Toupin [305], Kafadar and Eringen [153,154], Stojanović [296], Besdo [24], Reissner [261,262,265], and Shkutin [286], see also [34,67,95,200,219,228,252,294,321].

Within the framework of the Cosserat continuum many problems are solved which demonstrate the qualitative and quantitative difference from the solutions based on the classical Cauchy continuum model. In particular, in the monographs [95,98,200,232] wave processes in micropolar continua are investigated. The acceleration waves in nonlinear elastic micropolar media are considered in [153]. A generalization is presented in [198], where acceleration waves in elastic and viscoelastic micropolar media are studied. The relation between the existence of acceleration waves and the condition of strong ellipticity of the equilibrium equations is established in [15,74]. The theory of dislocations and disclinations is presented in [328,331,333]. In [79] the theory of superposed small deformations on a large deformation of an elastic micropolar continuum is developed. Three ways of introducing of the Lagrangian strain measures as well as an extended review are given in [245,246]. Variational problems in the micropolar continuum are investigated in [228,229,294].

The main problem of any micropolar theory is the establishment of the constitutive equations. For example, this problem is not discussed in Cosserats' original monograph [54], and this was a reason that the ideas were not recognized by many researchers. But even in the case when this problem is solved in a satisfying manner another problem can be stated: the identification of the material parameters. For example, in the linear micropolar elasticity of isotropic solids one needs six material parameters while only two Lamé moduli are needed in the classical elasticity. The experimental identification of the elastic moduli is discussed in

[1] http://www.mathematik.uni-bielefeld.de/~kersten/hilbert/rede.html and http://en.wikipedia.org/wiki/Hilbert.

[115,116,175,177,204,239], see also the data in [95,98] and the websites by P. Neff[2] and R. Lakes.[3] Another approach to the determination of the micropolar moduli is based on the various homogenization procedures, see, for example, [47,72,178,179,291,318].

Many publications on the Cosserat continuum are published in the sixties of the last century (in [295] are presented 400 references). In 1969, H. Lippmann has published his famous paper *A Cosserat Theory of Plastic Flow* [184]. He based his theoretical approach on [131,278] and described large strains ignoring the elastic part (rigid-plastic behavior). A motivation for this approach was that the Poynting-effect discussed by Swift in [298] can be presented. Later this approach has been continued by D. Besdo [24]. A possible application is discussed in [23,32]. H. Lippmann continued his investigations in the Cosserat plasticity, see [63,155,187,188,310–312]. Up to now there are many publications on the plastic Cosserat continuum, among them DeBorst [57], Ehlers [69,70], Forest [103], Forest et al. [104–108,282,287], Grammenoudis and Tsakmakis [120–123], Neff [218], Neff and Chełmiński [37,38,221,223], Steinmann [292,293], see also [55,64,65,71,99,100,145,157,166,181,190,225,226,238,266,273,277,283,284]. Another field of interest of H. Lippmann was rock mechanics [185,186]. This field of applied mechanics is closely related to the Cosserat plasticity, see for example [1,2,36,196].

The Cosserat model is used to describe solid materials with a complex microstructure like soils, polycrystalline and composite materials, granular and powder-like materials, see [35,56,60,69,71,72,117,142,164, 165,197,205,218,224,297,299–301,306,314,324,325], nanostructures [150,149], porous media and foams [59,61,62,66,69,70,175–177], and even bones [102,177,239], as well as electromagnetic and ferromagnetic media, see [97,133,199] among others. Starting from the papers by Aero et al. [5] and Eringen [91] the micropolar continuum is applied to model magnetic liquids, polymer suspensions, liquid crystals, and other types of fluids with microstructure, see, for example, [7,14,94,323,332] and the books of Eringen [96], Migoun and Prokhorenko [201] among others. Let us note that the Cosserat approach may be used as a base for the construction of special finite elements [269,271] or as the special regularization procedure.

Since the paper of Ericksen and Truesdell [89] the Cosserat model has found applications in construction of various generalized models for beams, plates, and shells. Within the framework of the direct approach applied in [89], the shell is modeled as a deformable surface at each point of which a set of deformable directors is attached. Hence, in general the deformation of a shell is described by the position vector r and p directors $\boldsymbol{d}_i$, $i = 1, \dots p$. This approach is developed in the original papers by Ericksen [82,83,86,87], Green and Naghdi [124–130], Green et al. [132], Naghdi [209], Naghdi et al. [210–212] and DaSilva and Tsai [58]. This variant of the shell theory is also named Cosserat shell theory or the theory of Cosserat surfaces. Some criticism concerning the direct approach in general and especially the theory of Cosserat surfaces exists, see, for example, [289]. But the theory is developed successfully and there are various applications, see [18,25–31,40,48–50,101,119,138,139,148,156,168,173,174,216,220,253,254,270,272,313] among others. In particular, the theory of symmetry of the constitutive equations is developed in [84,85,206,207]. Finite element formulations of the Cosserat shell theory are presented in [39,152,167,274,322]. Let us mention only the fundamental books by Naghdi [208], Rubin [269], and Antman [19] where the theory of Cosserat shells is presented.

The theory of Cosserat shells contains as a special case the linear theory of Cosserat plates. This theory is mostly formulated with the help of the introduction of one deformable director [124,125]. A variant with various directors is discussed, for example, in [251]. Applications of the Cosserat surface theory to sandwich plates are given in [119,194,195]. In the case of the theory of Cosserat plates with one director the unknown functions are the vector of displacements of the surface, representing the plate, and the vector describing the deformations of the director. Thus one assumes that such theory contains six degrees of freedom, and as a consequence one has to establish six boundary conditions. In the case of Cosserat shells it is also possible to describe the thickness changes. So one can conclude that in this case for each material point six degrees of freedom are assumed: three translational degrees of freedom, two rotational degrees of freedom describing the rotations about the directors and one degree of freedom which is related to the thickness changes.

Independently Eringen has formulated a linear theory of micropolar plates in [93], see also the monograph [95]. The two-dimensional equations of this theory are deduced with the help of the independent integration over the thickness of both the first and the second Euler laws of motion of the linear elastic micropolar continuum. The theories of the zeroth and the first order are presented applying a special linear approximation of the displacement and the microrotation fields. Eringen's theory is based on eight unknowns: the averaged displacements, the averaged macrorotations of the cross-sections and the averaged microrotations. This means that one has to introduce eight boundary conditions. The static boundary conditions in Eringen's plate theory

[2] http://www.mathematik.tu-darmstadt.de/fbereiche/analysis/pde/staff/neff/patrizio/Home.html.

[3] http://silver.neep.wisc.edu/~lakes/home.html.

cannot be presented as forces and moments at the boundaries like in the Kirchhoff-type theories [302]. From the point of view of the direct approach Eringen's micropolar plate is a deformable surface with eight degrees of freedom. Eringen's approach is widely discussed, for example, in [20,33,52,163,170–172,233,250,279–281,303,316,317] and in the monograph [95].

The theories of plates and shells and the theories based on the reduction of the three-dimensional equations of the micropolar continuum are presented in several publications. In [16,17,118,151,259,263] various averaging procedures in the thickness direction together with the approximation of the displacements and rotations or the force and moment stresses in the thickness direction are applied. As a result, one gets different numbers of unknowns and the number of two-dimensional equilibrium equations differs. For example, Reissner [263] presented a generalized linear theory of shells containing nine equilibrium equations. In addition, Reissner worked out the two-dimensional theory of a sandwich plate with a core having the properties of the Cosserat continuum [260]. The variants of the micropolar plate theory based on the asymptotic methods are developed in [6,73,215,275,276]. The nonlinear theory of elastic shells derived from the pseudo-Cosserat continuum is considered in [22]. The linear theory of micropolar plates is discussed in [11] where the discussion on the reduction procedure is given. The Γ-convergence based approach to the Cosserat-type of theory of plates and shells is discussed in [216,217,222].

In both the Cosserat's and the Eringen's micropolar plate theories one has additional kinematic variables—the rotations. It should be noted that in the theories of plates and shells the rotations are introduced as independent kinematic variables before the Cosserat theory was established. The term "angle of rotation" is introduced in Kirchhoff's theory too—but the rotations are expressed by the displacement field. After the pioneering work of Kirchhoff [160] thousands of publications are presented, which try to give the foundations and the methods of deduction of the equations of the Kirchhoff–Love theory, but also of improvements, see, for example, [45,46,110–114,158,159,227,249] among others. Considering sandwich structures with a soft core Reissner worked out a theory by taking into account the transverse shear which was ignored by Kirchhoff [256–258,264]. Similar governing equations (only some effects are not included) were derived by Mindlin introducing additional degrees of freedom for the points of the midplane [202]. The order of the system of the partial differential static equilibrium equations in the case of Reissner-type theories is equal to ten. That means, that the number of boundary equations is equal to five. In the theories of Reissner and Mindlin only two angles of rotations are independent of the displacements, and the transverse shear can be taken into account. The third angle of rotation (rotation about the normal to the surface, so-called drill rotation) is not considered as an independent variable. In Reissner's theory the static boundary conditions are equivalent to the introduction of distributed forces and moments on the contour, the last one has no components in the normal direction. The Reissner's plate as well as the Kirchhoff's plate are not able to react on the distributed moments on the surfaces or boundaries which are directed along the surface normal (so-called drilling moments). That means that Reissner's plate is modeled by a material surface each point having five degrees of freedom (three translation and two rotations) while Kirchhoff's plate is a material surface each point of which has only three degrees of freedom (three translations). The original Kirchhoff's plate theory has only one degree of freedom (the deflection). Now we have thousands of papers and monographs on the Reissner's and Mindlin's approach, see the reviews [134,213] among others.

In the last decades the so-called higher order theories are very popular. Starting with the pioneering contributions of [180,255], new theories are established systematically. If one discusses higher order theories in the point of view of the direct approach one assumes deformable surfaces with additional degrees of freedom. For example, the third order theory presented in [315] can be regarded as a theory with seven degrees of freedom including rotations of the plate cross-sections. Let us mention also the papers [21,109,144,167,241,290] where the rotations in shells are considered, while an extensive discussion of the application of the rotations in Continuum Mechanics is given in [244].

The direct approach in the theory of shells based on Cosserats' ideas is applied also in [326]. In contrast to [89], the shells are regarded as deformable surfaces with material points at which three directors are prescribed. The directors have the following properties: they are orthogonal unit vectors. The deformations of the shell are presented by a position vector and a properly orthogonal tensor. This variant of the shell and plate theories based on the direct approach is developed and continued, for example, in [75,80,81,236,237,285,286,327–330]. It must be noted that this variant is very similar to the one presented within the general nonlinear theory of shells discussed in the monographs of Libai and Simmonds [183], and Chróścielewski et al. [44], see also [41–43,76–78,162,182,189,191–193,243,247,248,288,289]. The two-dimensional equilibrium equations given in [44,183,243] one gets by the exact integration over the thickness of the equations of motion of a

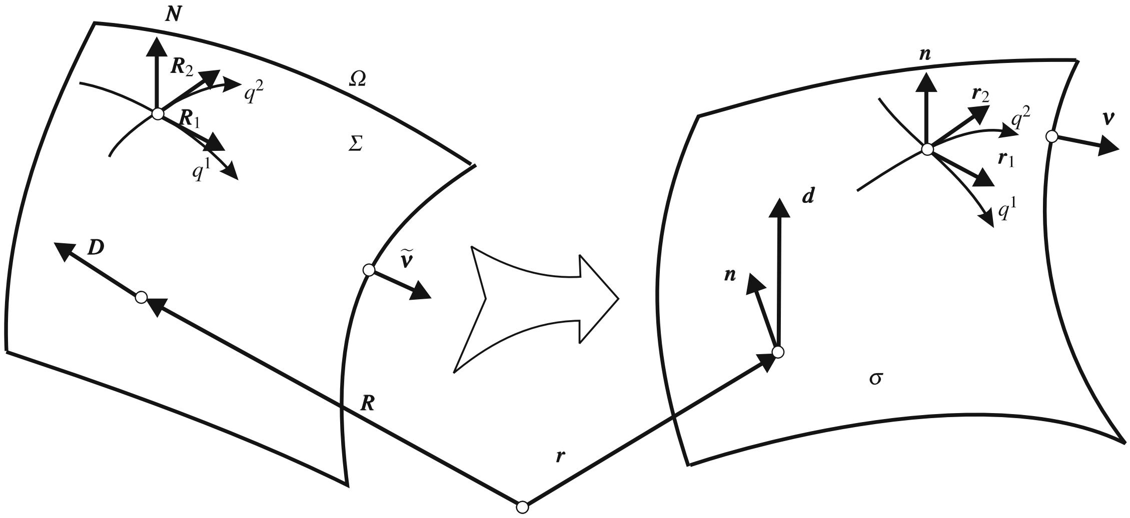

Fig. 1 Kinematics of the Cosserat surface

shell-like body. The deformation measures, which are the same as those introduced within the framework of the direct approach, can be defined as work-conjugate fields to the stress and couple stress tensors.

In [12] the general theory with six degrees of freedom is transformed to a theory of shells with five degrees of freedom (similar to Reissner's theory) introducing some constraints for the deformations. This variant of the theory is discussed in [8,9,13,327]. In [133] the method presented in [12] is applied to the three-dimensional case. It must be noted that the main problem in application of the direct approach is both the establishment and the identification of the constitutive equations. They should be formulated for the two-dimensional measures of stresses and measures of deformations. This means that some effective stiffness properties should be introduced. For anisotropic elastic plates the identification procedure for the effective stiffness properties is discussed in [9,12,327] and for the viscoelastic case in [8,10]. The last one approach has some similarities with [267,268].

It is worth mentioning here the bibliographical papers on the shell theory [230,231,242] and the bibliography collected by Jemielita in [320] since 1767.

The aim of this paper is the discussion of the Cosserat-type theories of plates and shells. The paper is organized as follows. In Sect. 2 we present the basic equations of the theory of the directed surfaces [269]. We restricted ourselves by the variant of the theory with one deformed director. In Sect. 3 we consider a 6-parameter nonlinear theory of shells using the direct approach [44,81,183]. Finally, we present the equations of the linear theory of micropolar plates based on the Eringen's approach [93,95].

2 Cosserat surface

2.1 Kinematics

Following [269] let us introduce the basic relations of the nonlinear theory of Cosserat shells or surfaces. We consider the Cosserat surface as a deformable surface with an attached director. The kinematics of the Cosserat surface may be described as follows. Let Σ be the shell surface in the reference configuration (undeformed state) represented by the Gaussian coordinates q^α $(\alpha = 1, 2)$, and $R(q^1, q^2)$ is the position-vector describing the material points on the surface Σ. The surface σ of the shell after deformation is represented by the coordinates q^α too, the position of the material point on σ is given by $r(q^1, q^2)$ (Fig. 1). Here N and n are the vectors of the unit normals to the shell base surfaces Σ and σ, while ν and $\widetilde{\nu}$ are the unit normal vectors to the shell boundary contours $\omega = \partial\sigma$ and $\Omega = \partial\Sigma$, respectively $(\boldsymbol{\nu} \cdot \boldsymbol{n} = 0, \widetilde{\boldsymbol{\nu}} \cdot \boldsymbol{N} = 0)$. $\boldsymbol{r}_\beta$ and r^α are the co- and contravariant vector bases on σ and R_β, R^α are the co- and contravariant vector bases on Σ

$$\boldsymbol{r}^{\alpha}\cdot r_{\beta}=\delta^{\alpha}_{\beta},\quad \boldsymbol{r}^{\alpha}\cdot n=0,\quad r_{\beta}=\frac{\partial \boldsymbol{r}}{\partial q^{\beta}},\quad R^{\alpha}\cdot R_{\beta}=\delta^{\alpha}_{\beta},\quad \boldsymbol{R}^{\alpha}\cdot \boldsymbol{N}=0,\quad R_{\beta}=\frac{\partial R}{\partial q^{\beta}}\quad (\alpha,\beta=1,2),$$

where δ^{α}_{β} is the Kronecker symbol.

The Cosserat surface is a material surface, on which is given a director-vector field. By this field the changes of the orientation and length of the material fibres are presented. In this sense the material fibres behave like three-dimensional bodies. In the actual configuration we denote this field by $\boldsymbol{d}$, in the reference configuration by $\boldsymbol{D}$ (Fig. 1). Here we are not assuming, that the vectors $\boldsymbol{d}$ are $\boldsymbol{D}$ tangential vectors with respect to the shell, this means $\boldsymbol{D}\cdot N\neq 0$ and $\boldsymbol{d}\cdot \boldsymbol{n}\neq 0$. Thus, the deformation of the Cosserat surface is given by two vector fields

$$\boldsymbol{r}=R(q^1,q^2),\quad \boldsymbol{d}=d(q^1,q^2). \tag{1}$$

In the general case the director d is not a unit vector and normal to the surface σ. Any material point of the Cosserat surface has six degrees of freedom. Considering various constraints on $\boldsymbol{d}$, one obtains special theories of shells, for example the Kirchhoff–Love theory ($\boldsymbol{d}=n$). A Reissner-type shell theory can be established, if $d\cdot\boldsymbol{d}=1$, see [269] for details. If one takes into account that the director is not a unit vector, but parallel to the normal vector ($\boldsymbol{d}\sim n$), the thickness changes can be modeled.

2.2 Strain energy density of an elastic Cosserat surface

Let us assume an elastic Cosserat-type shell. The specific strain energy W is a function of $\boldsymbol{r}$, $\boldsymbol{d}$ and their gradients

$$W=W(\boldsymbol{r},\mathbf{F},d,\nabla d),\quad \mathbf{F}=\nabla r,\quad \nabla(\ldots)\stackrel{\triangle}{=}r^{\alpha}\otimes\frac{\partial(\ldots)}{\partial q^{\alpha}}. \tag{2}$$

For the transformation of W we apply the principle of material frame-indifference [307,308]. The energy density should be invariant under the superposed rigid-body motion. With other words, W does not change considering the transformations

$$r\rightarrow a+\boldsymbol{r}\cdot\mathbf{O},\quad d\rightarrow\boldsymbol{d}\cdot\mathbf{O},$$

where a is an arbitrary constant vector, and $\mathbf{O}$ is an arbitrary constant orthogonal tensor. From this follows that W does not depend on $\boldsymbol{r}$ and the following equation should be satisfied

$$W\left[\mathbf{F}\cdot\mathbf{O},d\cdot\mathbf{O},(\nabla\boldsymbol{d})\cdot\mathbf{O}\right]=W\left(\mathbf{F},d,\nabla\boldsymbol{d}\right) \tag{3}$$

for the arbitrary orthogonal tensor $\mathbf{O}$.

Let us perform the polar decomposition of the tensor $\mathbf{F}$ [81]. Taking into account that $\mathbf{F}$ is a singular tensor, then one gets the polar decomposition of a nonsingular tensor $\mathbf{F}+\boldsymbol{N}\otimes\boldsymbol{n}$ as it follows

$$\mathbf{F}+N\otimes n=(\mathbf{U}+N\otimes\boldsymbol{N})\cdot\mathbf{A},$$

with $\mathbf{U}$ as a positive definite symmetric tensor on the surface (two-dimensional tensor), which can be computed by $\mathbf{U}=\mathbf{F}\cdot\mathbf{F}^T$. $\mathbf{A}$ is an orthogonal turn-tensor of the surface with $\boldsymbol{N}\cdot\mathbf{A}=\boldsymbol{n}$. Finally, one gets [81]

$$\mathbf{F}=\mathbf{U}\cdot\mathbf{A}.$$

Assuming in Eq. (3) $\mathbf{O}=\mathbf{A}^T$, we obtain the constitutive equation satisfying the principle of the material frame-indifference

$$W=W\left(\mathbf{U},\boldsymbol{d}\cdot\mathbf{A}^T,(\nabla\boldsymbol{d})\cdot\mathbf{A}^T\right). \tag{4}$$

2.3 Principle of virtual work and the equilibrium conditions

The equilibrium equation for the material surface one obtains with the help of the variational calculus. The starting point is

$$\delta \iint_{\Sigma} W \,\mathrm{d}\Sigma - \delta' \mathcal{A} = 0, \tag{5}$$

where $\delta' \mathcal{A}$ is the elementary work of the outer loadings, δ denotes variation. For the sake of simplicity for the variation of the strain energy W we take into account Eq. (2). Then

$$\delta W = \mathbf{T} \bullet \delta \mathbf{F} + \mathbf{M} \bullet \nabla \delta d + \frac{\partial W}{\partial d} \cdot \delta d \tag{6}$$

with

$$\mathbf{T} \triangleq \frac{\partial W}{\partial \mathbf{F}}, \quad \mathbf{M} \triangleq \frac{\partial W}{\partial \nabla d} \tag{7}$$

as the tensors of forces and moments of first Piola–Kirchoff-type (two-point stress measures), $\bullet$ is the scalar product in the space of second-order tensors.

Taking into account Eqs. (6), (7) and the divergence theorem, one gets

$$\delta \iint_{\Sigma} W \,\mathrm{d}\Sigma = \oint_{\Omega} \widetilde{\boldsymbol{\nu}} \cdot (\mathbf{T} \cdot \delta r + \mathbf{M} \cdot \delta d) \,\mathrm{d}s - \iint_{\Sigma} (\nabla \cdot \mathbf{T}) \cdot \delta r \,\mathrm{d}\Sigma - \iint_{\Sigma} \left[\nabla \cdot \mathbf{M} - \frac{\partial W}{\partial d} \right] \cdot \delta \boldsymbol{d} \,\mathrm{d}\Sigma, \tag{8}$$

Considering the variational equation (5) and Eq. (8) the elementary work of the outer loadings $\delta' \mathcal{A}$ acting on the Cosserat surface can be computed. Now we have

$$\delta' \mathcal{A} = \iint_{\Sigma} (\boldsymbol{f} \cdot \delta \boldsymbol{r} + \boldsymbol{\ell} \cdot \delta \boldsymbol{d}) \,\mathrm{d}\Sigma + \int_{\Omega_2} \varphi \cdot \delta r \,\mathrm{d}s + \int_{\Omega_4} \boldsymbol{\gamma} \cdot \delta \boldsymbol{d} \,\mathrm{d}s. \tag{9}$$

In Eq. (9) f, ℓ, φ and γ are given functions on the contour parts Ω_2 and Ω_4, respectively.

From the variational equation (5) considering Eq. (9) the formulation of the boundary-value problem follows for the equilibrium of a nonlinear elastic Cosserat-type in the reference configuration

$$\nabla \cdot \mathbf{T} + \boldsymbol{f} = \mathbf{0}, \quad \nabla \cdot \mathbf{M} - \frac{\partial W}{\partial d} + \boldsymbol{\ell} = \mathbf{0}, \tag{10}$$

$$\Omega_1 \ : \ r = \boldsymbol{\rho}(s), \tag{11}$$

$$\Omega_2 \ : \ \widetilde{\nu} \cdot \mathbf{T} = \varphi(s), \tag{12}$$

$$\Omega_3 \ : \ \boldsymbol{d} = \boldsymbol{d}^0(s), \tag{13}$$

$$\Omega_4 \ : \ \widetilde{\nu} \cdot \mathbf{M} = \boldsymbol{\gamma}(s). \tag{14}$$

$\boldsymbol{\rho}(s)$, $\boldsymbol{d}^0(s)$ are prescribed functions in the kinematic boundary conditions (11) and (13). $\boldsymbol{\rho}(s)$ defines the position of the part of the shell boundary in the space Ω_1, while $\boldsymbol{d}^0(s)$ defines the director on Ω_1. The functions f and φ in Eqs. (10) and (12) represent the distributed on the shell surface and boundary part Ω_2 forces.

3 Micropolar shells

3.1 Kinematics

Following [12,75,81,326] in this section we consider the equations of the nonlinear elastic micropolar shell theory. An elastic micropolar shell is a two-dimensional analog of the Cosserat continuum, i.e., a micropolar shell is a deformable directed surface each particle of which has six degrees of freedom of the rigid body. Let Σ be again a base surface of the micropolar shell in the reference configuration (for example, in the undeformed state), q^α $(\alpha = 1, 2)$ be Gaussian coordinates on Σ, and $\boldsymbol{R}(q^1, q^2)$ be a position vector of Σ. In

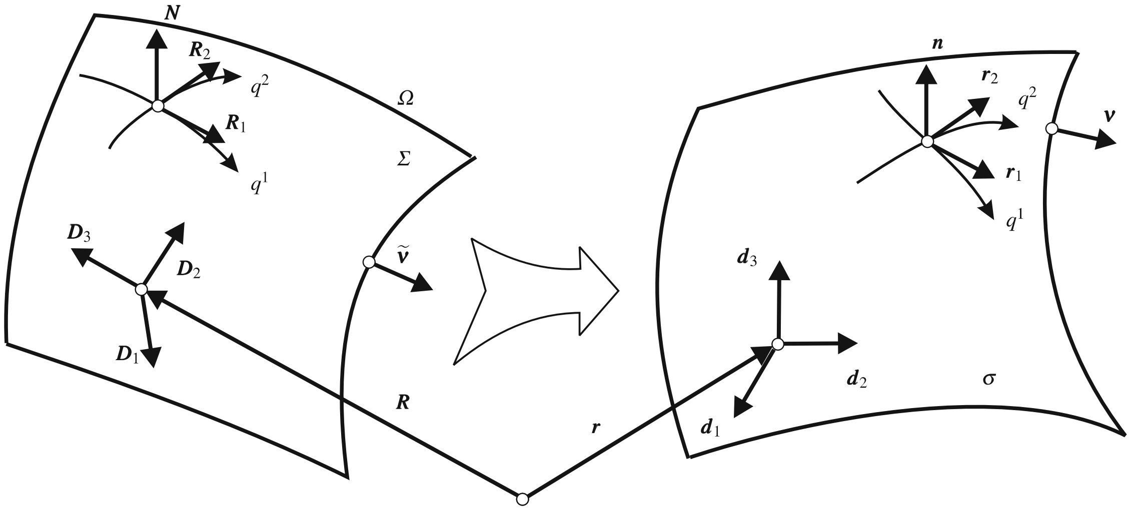

Fig. 2 Deformation of the micropolar shell

the actual (deformed) configuration the surface is denoted by σ, and the position of its particles is given by the vector $\boldsymbol{r}(q^1, q^2)$ (Fig. 2).

In contrast to the Cosserat surface in each point of σ three orthonormal vectors $\boldsymbol{d}_k$, $k = 1, 2, 3$, are attached. The vectors $\boldsymbol{d}_k$ are named directors. We also introduce three orthonormal vectors $\boldsymbol{D}_k$ on Σ. From the physical point of view, d_k and $\boldsymbol{D}_k$ describe the orientation of the shell particles in the actual and the reference configurations, respectively. One may introduce the proper orthogonal tensor $\mathbf{H}(q^1, q^2)$ by the relation $\mathbf{H} = D_k \otimes \boldsymbol{d}_k$. This is the so-called micro-rotation tensor (or turn-tensor). Thus, the micropolar shell is described by two kinematically independent fields

$$\boldsymbol{r} = r(q^1, q^2), \quad \mathbf{H} = \mathbf{H}(q^1, q^2). \tag{15}$$

3.2 Constitutive equations

For the micropolar shell made of an elastic material the strain energy density W exists. By using the principle of local action [307,308] the constitutive equation for the function W is given by

$$W = W(\boldsymbol{r}, \nabla r, \mathbf{H}, \nabla \mathbf{H}). \tag{16}$$

From the principle of material frame-indifference [307,308] we can find that W depends only on two Cosserat-type strain measures $\mathbf{E}$ and $\mathbf{K}$

$$W = W(\mathbf{E}, \mathbf{K}), \tag{17}$$

where

$$\mathbf{E} = (\nabla \boldsymbol{r}) \cdot \mathbf{H}^T, \quad \mathbf{K} = \frac{1}{2} \boldsymbol{R}^\alpha \otimes \left(\frac{\partial \mathbf{H}}{\partial q^\alpha} \cdot \mathbf{H}^T \right)_\times . \tag{18}$$

$\mathbf{T}_\times$ is the vectorial invariant of the second rank tensor $\mathbf{T}$ defined by $\mathbf{T}_\times = (T^{mn} R_m \otimes \boldsymbol{R}_n)_\times = T^{mn} \boldsymbol{R}_m \times R_n$ for any base $\boldsymbol{R}_m$. This operation was originally introduced by J. W. Gibbs, see [319]. An extensive discussion of strain measures in the Cosserat continuum is given in [245,246], where the application of the invariance properties of the strain energy density is discussed.

For the nonlinear elastic micropolar shells the structure of the strain energy density is discussed in [77], where various types of anisotropy are considered. In particular, in the case of isotropic behavior, the following quadratic functions may be used

$$\begin{aligned} 2W = {} & \alpha_1 \mathrm{tr}^2 \mathbf{E}_\parallel + \alpha_2 \mathrm{tr}\, \mathbf{E}_\parallel^2 + \alpha_3 \mathrm{tr} \left(\mathbf{E}_\parallel \cdot \mathbf{E}_\parallel^T \right) + \alpha_4 n \cdot \mathbf{E}^T \cdot \mathbf{E} \cdot \boldsymbol{n} \\ & + \beta_1 \mathrm{tr}^2 \mathbf{K}_\parallel + \beta_2 \mathrm{tr}\, \mathbf{K}_\parallel^2 + \beta_3 \mathrm{tr} \left(\mathbf{K}_\parallel \cdot \mathbf{K}_\parallel^T \right) + \beta_4 \boldsymbol{n} \cdot \mathbf{K}^T \cdot \mathbf{K} \cdot \boldsymbol{n}. \end{aligned} \tag{19}$$

Here $\mathbf{E}_\| = \mathbf{E}\cdot\mathbf{A}$, $\mathbf{K}_\| = \mathbf{K}\cdot\mathbf{A}$, $\mathbf{A} = \mathbf{I} - \boldsymbol{N}\otimes\boldsymbol{N}$, $\mathbf{I}$ is the three-dimensional unit tensor, and α_i, β_i are the elastic constants, $i = 1, 2, 3, 4$. The surface strain energy W should be positive definite, from which follow the inequalities [80]

$$\begin{aligned} &2\alpha_1+\alpha_2+\alpha_3>0, \quad \alpha_2+\alpha_3>0, \quad \alpha_3-\alpha_2>0, \quad \alpha_4>0,\\ &2\beta_1+\beta_2+\beta_3>0, \quad \beta_2+\beta_3>0, \quad \beta_3-\beta_2>0, \quad \beta_4>0. \end{aligned} \tag{20}$$

In [44] the following relations for the elastic moduli appearing in (19) are used

$$\begin{aligned} &\alpha_1 = C\nu, \quad \alpha_2=0, \quad \alpha_3 = C(1-\nu), \quad \alpha_4=\alpha_s C(1-\nu),\\ &\beta_1 = D\nu, \quad \beta_2=0, \quad \beta_3 = D(1-\nu), \quad \beta_4=\alpha_t D(1-\nu),\\ &C=\frac{Eh}{1-\nu^2}, \quad D=\frac{Eh^3}{12(1-\nu^2)}, \end{aligned} \tag{21}$$

where E and ν are the Young's modulus and the Poisson's ratio of the bulk material, respectively, α_s and α_t are dimensionless coefficients, while h is the shell thickness. α_s is similar to the shear correction factor introduced in the plate theory by Reissner [256] ($\alpha_s = 5/6$) and by Mindlin [202] ($\alpha_s = \pi^2/12$), see also [134]. α_t plays the same role for the moment stresses. The value $\alpha_t = 0.7$ was proposed by Pietraszkiewicz [240,241].

3.3 Equilibrium equations

As well as in the case of the Cosserat surface, the Lagrangian equilibrium equations of the micropolar shell can be derived from the principle of virtual work

$$\delta \iint_\Sigma W\,\mathrm{d}\Sigma = \delta' A, \tag{22}$$

where

$$\delta' A = \iint_\Sigma (\boldsymbol{f}\cdot\delta\boldsymbol{r} + \boldsymbol{l}\cdot\boldsymbol{\psi})\,\mathrm{d}\Sigma + \int_{\omega_2}\boldsymbol{\varphi}\cdot\delta\boldsymbol{r}\,\mathrm{d}s + \int_{\omega_4}\boldsymbol{\eta}\cdot\boldsymbol{\psi}\,\mathrm{d}s, \quad \mathbf{I}\times\boldsymbol{\psi} = -\mathbf{H}^T\cdot\delta\mathbf{H}. \tag{23}$$

In Eq. (22) $\boldsymbol{\psi}$ is the virtual rotation vector, $\boldsymbol{f}$ is the surface force density distributed on Σ, $\boldsymbol{l}$ is the surface couple density distributed on Σ, φ and $\boldsymbol{\eta}$ are linear densities of forces and couples distributed along corresponding parts of the shell boundary Ω, respectively.

Thus, the Lagrangian shell equations take the form

$$\nabla\cdot\mathbf{D} + f = \mathbf{0}, \quad \nabla\cdot\mathbf{G} + \left[(\nabla R)^T\cdot\mathbf{D}\right]_\times + \boldsymbol{l} = 0, \tag{24}$$

$$\mathbf{D} = \mathbf{P}_1\cdot\mathbf{H}, \quad \mathbf{G} = \mathbf{P}_2\cdot\mathbf{H}, \quad \mathbf{P}_1 \triangleq \frac{\partial W}{\partial \mathbf{E}}, \quad \mathbf{P}_2 \triangleq \frac{\partial W}{\partial \mathbf{K}}, \tag{25}$$

$$\Omega_1 \;:\; r = \boldsymbol{\rho}(s), \tag{26}$$

$$\Omega_2 \;:\; \widetilde{\boldsymbol{\nu}}\cdot\mathbf{D} = \boldsymbol{\varphi}(s), \tag{27}$$

$$\Omega_3 \;:\; \mathbf{H} = \mathbf{h}(s), \quad \mathbf{h}\cdot\mathbf{h}^T = \mathbf{I}, \tag{28}$$

$$\Omega_4 \;:\; \widetilde{\boldsymbol{\nu}}\cdot\mathbf{G} = \boldsymbol{\eta}(s). \tag{29}$$

Here $\boldsymbol{\rho}(s), \boldsymbol{h}(s)$ are given vector functions. The Eqs. (24) are the equilibrium equations for the linear momentum and angular momentum of any shell part. The tensors $\mathbf{D}$ and $\mathbf{G}$ are the surface stress and couple stress tensors of the first Piola–Kirchoff-type, and the corresponding stress measures $\mathbf{P}_1$ and $\mathbf{P}_2$ in Eqs. (25) are the Kirchoff-type tensors, respectively. The boundary Ω of Σ is divided into two parts $\Omega = \Omega_1\cup\Omega_2 = \Omega_3\cup\Omega_4$. The following relations hold true

$$\boldsymbol{N}\cdot\mathbf{D} = \boldsymbol{N}\cdot\mathbf{G} = N\cdot\mathbf{P}_1 = N\cdot\mathbf{P}_2 = \mathbf{0}.$$

3.4 Relation to the Cosserat surface theory

The structure of the elementary work for micropolar shells (23) and Cosserat surfaces (9) are similar, but the mechanical sense of $\boldsymbol{\ell}, \boldsymbol{\gamma}$ and $\boldsymbol{l}, \boldsymbol{\eta}$ is different. The definition of the director $\boldsymbol{d}$ does not fix the orientation of an absolute rigid body in the space since any arbitrary rotations about d can be considered. This means that the elementary work of the outer loading acting on the Cosserat shells (9) does not take into account the drilling moments about $\boldsymbol{d}$ since they do not execute work with rotations about $\boldsymbol{d}$. On the other hand we can compute the variation of the director d considering only the length changes $\delta\boldsymbol{d} = (\delta\boldsymbol{d}) \cdot d\,\boldsymbol{d}$. It is obvious that the length changes are not related to any rotation, which means that they are not related to any forces and moments. These variations describe strains (microstrains) of the material points defining the shell. The corresponding loading characteristics are hyper-stresses (force dipoles). In this case in Eqs. $(10)_2$ and (14) the components of the vector functions ℓ and γ have different nature. Let us present these functions as a sum of two terms parallel and normal to the director: $\boldsymbol{\ell} = \boldsymbol{\ell} \cdot \boldsymbol{d}\,\boldsymbol{d} + \boldsymbol{\ell} \times \boldsymbol{d}$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma} \cdot \boldsymbol{d}\,\boldsymbol{d} + \boldsymbol{\gamma} \times \boldsymbol{d}$. The first terms correspond to the force dipoles acting in the $\boldsymbol{d}$-direction, the second terms are the moment loadings without the moments about d.

Within the framework of the Cosserat-surface shell model one can discuss the material surface composed of deformable particles on which forces and moments and some hyper-stresses act. At same time the micropolar shell can be represented by a surface composed of rigid microparticles of arbitrary ellipsoidal shape. The interaction between these particles are given by forces and moments only. Now the Cosserat-surface shell model can be presented by a surface composed of microparticles of beam shape changing during the deformation its length, but they does not reflect the rotation about there axis. Summarizing one can state that the Cosserat-surface shell model does not follows from the micropolar shell and vice versa. It should be noted that the micropolar model seems to be more complete since it is only necessary to prescribe forces and moments. Let us note that the difference between the Cosserat-surface shell model and the micropolar or 6-parametric shell model is analogous to the difference between the Ericksen's liquid crystals theory [88] and the Eringen's micropolar fluids [96] in the continuum mechanics.

4 Theories of shells and plates by reduction of the three-dimensional micropolar continuum

We have mentioned in the Introduction that there are some approaches based on the reduction of the three-dimensional Cosserat continuum to the two-dimensional equations. These two-dimensional theories inherit some Cosserat-type properties from the three-dimensional continuum. The most popular is the Eringen's linear theory of plates [93,95]. Here we discuss briefly the governing equations of this theory in the case of equilibrium.

4.1 Basic equations of three-dimensional linear Cosserat continuum

The small strains of the micropolar media are usually described by using the vector of translation $\boldsymbol{u}$ and the vector of microrotation ϑ, see [95,232]. From the physical point of view, $\boldsymbol{u}$ describes the displacement of a particle of a micropolar body while ϑ corresponds to the particle rotation. The local equilibrium equations of micropolar continuum are [95]

$$\widetilde{\nabla} \cdot \sigma + \rho \boldsymbol{f} = 0, \quad \widetilde{\nabla} \cdot \boldsymbol{\mu} + \sigma_\times + \rho \boldsymbol{\ell} = 0, \tag{30}$$

where σ and $\boldsymbol{\mu}$ are the stress and couple stress tensors, respectively, f and $\boldsymbol{\ell}$ are the mass force and couple vectors, respectively, ρ is the density, $\widetilde{\nabla}$ is the three-dimensional nabla operator. Equation $(30)_1$ is the local form of the balance of momentum while Eq. $(30)_2$ is the balance of moment of momentum.

The static boundary conditions have the following form

$$\boldsymbol{n} \cdot \boldsymbol{\sigma} = t^0, \quad n \cdot \mu = \boldsymbol{m}^0 \quad \text{at } S_f. \tag{31}$$

Here $\boldsymbol{t}^0$ and $\boldsymbol{m}^0$ are the surface forces and the surface couples acting on the corresponding part of the surface S_f of the micropolar body, $S = S_u \cup S_f \equiv \partial V$. The kinematic boundary conditions consist of the following relations

$$\boldsymbol{u} = \boldsymbol{u}^0, \quad \boldsymbol{\vartheta} = \boldsymbol{\vartheta}^0 \quad \text{at} \quad S_u, \tag{32}$$

where $\boldsymbol{u}^0$ and ϑ^0 are given functions at S_u. Other types of the boundary conditions may also be formulated.

The linear stain measures, i.e., the linear stretch tensor ε and the linear wryness tensor æ, are given by the relations

$$\boldsymbol{\varepsilon} = \widetilde{\nabla}\boldsymbol{u} + \boldsymbol{\vartheta} \times \mathbf{I}, \quad \boldsymbol{æ} = \widetilde{\nabla}\boldsymbol{\vartheta}. \tag{33}$$

In [95], $(\widetilde{\nabla}\boldsymbol{\vartheta})^T$ is used as the linear wryness tensor. Here we use the definition $(33)_2$ for the consistency with the definition of ε.

For an isotropic solid the constitutive equations are

$$\boldsymbol{\sigma} = \lambda \mathbf{I} \operatorname{tr} \boldsymbol{\varepsilon} + \mu \varepsilon^T + (\mu + \kappa)\varepsilon, \quad \mu = \alpha \mathbf{I} \operatorname{tr} æ + \beta \boldsymbol{æ}^T + \gamma æ, \tag{34}$$

where $\lambda, \mu, \kappa, \alpha, \beta, \gamma$ are the elastic moduli which satisfy the inequalities [95,219]

$$2\mu + \kappa \geq 0, \quad \kappa \geq 0, \quad 3\lambda + 2\mu + \kappa \geq 0, \quad \beta + \gamma \geq 0, \quad \gamma - \beta \geq 0, \quad 3\alpha + \beta + \gamma \geq 0.$$

4.2 Transition to the two-dimensional equilibrium equations

The Eringen's transition to the two-dimensional equations is based on the linear in z approximation of the translation and rotation together with independent integration of the equilibrium equations (30) through the thickness. Let the plate-like body occupies the volume $V = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in \mathcal{M} \subset \mathbb{R}^2, z \in [-h/2, h/2]\}$. Here h is the plate thickness. For the sake of simplicity we assume that $h = \text{const}$. For the translations and rotations of the plate-like body Eringen introduced the following approximation

$$\boldsymbol{u}(x, y, z) = \boldsymbol{v}(x, y) - z\boldsymbol{\varphi}(x, y), \quad \boldsymbol{\vartheta}(x, y, z) = \boldsymbol{\phi}(x, y), \quad \boldsymbol{\varphi} \cdot \boldsymbol{i}_3 = 0, \tag{35}$$

with two vector fields $\boldsymbol{\varphi}(x, y), \phi(x, y)$. Hence, in Eringen's theory of plates one has 8 kinematically independent scalar fields: $v_1, v_2, v_3, \varphi_1, \varphi_2, \phi_1, \phi_2, \phi_3$.

To illustrate the transformations of (30) we assume the homogeneous boundary conditions at $z = \pm h/2$, $(x, y) \in \mathcal{M}$

$$n^{\pm} \cdot \boldsymbol{\sigma} = 0, \quad n^{\pm} \cdot \mu = 0, \tag{36}$$

where $\boldsymbol{n}^{\pm} = \pm i_3$. Then the integration of $(30)_1$ over the thickness leads to the equation

$$\nabla \cdot \mathbf{T} + \boldsymbol{q} = 0, \quad \text{where} \quad \mathbf{T} = \langle \mathbf{A} \cdot \boldsymbol{\sigma} \rangle, \quad q = \langle \rho \boldsymbol{f} \rangle, \quad \langle (\ldots) \rangle = \int_{-h/2}^{h/2} (\ldots)\, \mathrm{d}z. \tag{37}$$

Integration of $(30)_2$ gives us the relation

$$\nabla \cdot \mathbf{M}^{\mu} + \mathbf{T}_{\times} + \langle \sigma_{3\alpha} \rangle i_3 \times \boldsymbol{i}_{\alpha} + c = \mathbf{0}, \tag{38}$$

where

$$\mathbf{M}^{\mu} = \langle \mathbf{A} \cdot \boldsymbol{\mu} \rangle, \quad \boldsymbol{c} = \langle \rho \boldsymbol{\ell} \rangle.$$

Equations (37) and (38) constitute the balance equations of the zeroth-order theory of micropolar plates.

Additionally, cross-multiplying $(30)_1$ by zi_3 and integrating over thickness we obtain the equation of the first-order theory

$$\nabla \cdot \mathbf{M}^{\sigma} + \mathbf{T}_{\times} + \widetilde{c} = \mathbf{0}, \tag{39}$$

where

$$\mathbf{M}^{\sigma} = -\langle \mathbf{A} \cdot z\boldsymbol{\sigma} \times \boldsymbol{i}_3 \rangle, \quad \widetilde{\boldsymbol{c}} = i_3 \times \langle \rho z f \rangle.$$

In this theory the stress resultant $\mathbf{T}$, the force–stress resultant $\mathbf{M}^{\sigma}$, and the moment–stress resultant $\mathbf{M}^{\mu}$ are present. Hence, in the case of static boundary conditions on need to assign the values of $\widetilde{\boldsymbol{\nu}} \cdot \mathbf{T}$, $\widetilde{\boldsymbol{\nu}} \cdot \mathbf{M}^{\mu}$, $\widetilde{\boldsymbol{\nu}} \cdot \mathbf{M}^{\sigma}$ at the plate boundary contour. Using Eqs. (34) and (35) one may obtain the constitutive equations for $\mathbf{T}$, $\mathbf{M}^{\sigma}$, and $\mathbf{M}^{\mu}$, which are presented in [95] in the component form.

Let us mention that Eringen's approach is not unique. For example, Gevorkyan [118] obtained the constitutive equations for the shell using the linear pseudo-Cosserat continuum. The drilling moments in his theory are generated by the couple stress tensor $\boldsymbol{\mu}$ only. Reissner [263] introduced the stress resultants $\mathbf{T}$, the force–stress resultant $\mathbf{M}^{\sigma}$, and the moment–stress resultant $\mathbf{M}^{\mu}$ taking into account the transverse shear forces and the drilling moment.

The derivation of the micropolar plate theory proposed in [11] leads to the 6-parametric theory with the force–stress and couple–stress resultant tensors defined as $\mathbf{T} = \langle \mathbf{A} \cdot \boldsymbol{\sigma} \rangle$, $\mathbf{M} = \langle \mathbf{A} \cdot \boldsymbol{\mu} \rangle - \langle \mathbf{A} \cdot z\sigma \times \boldsymbol{i}_3 \rangle$. It is obvious that the couple stress tensor $\mathbf{M}$ is the sum of Eringen's tensors $\mathbf{M}^{\sigma}$ and $\mathbf{M}^{\mu}$.

It is obvious that Eringen's plate theory is not coincide with the linear variant of Cosserat surface as well as with the linear variant of micropolar shell theories discussed above. The vector φ is an analog of the rotation vector used in the Reissner-type theories while $\boldsymbol{\phi}$ is an analog of the microrotation vector used in the linear theory of micropolar continuum. In contrast to the Cosserat surface theory Eringen's micropolar plates theory take into account the drilling moment but can not take into account the force dipoles. On the other hand, this theory differs from the 6-parametric shell theory because it has only six degrees of freedom. In the first order micropolar plate theory by Eringen the two-dimensional couple stress tensor splits into two independent parts, i.e., the force–stress and moment–stress resultants.

5 Conclusion

In this paper we briefly present some applications of the Cosserat continuum and the contribution of Professor Horst Lippmann. Further we restrict ourselves by two-dimensional Cosserat-type theories, i.e., the Cosserat-based theories of plates and shells. We present the most popular in the literature theories: the Cosserat-surface shell model, the micropolar or 6-parametric shell model, and the Eringen's micropolar plate theory. In this paper we also attempt to collect the corresponding papers and monographs related to the considered theories.

Personal supplement

The first author (J. A.) met Horst Lippmann for the first time in the early fifties of the last century. As a young graduate of Mathematics of the University of Greifswald he started his scientific career at a research institute of deformations of metallic materials (Institut für bildsame Formung der Metalle) in Zwickau. Myself, as a student of Mathematics of the University of Leipzig, passed through an internship at this institute and was supervised by Horst Lippmann. So, I could participate in the first steps of H. L. in the field of plasticity, one of the important scientific research fields all over his life. Later, we both changed from Mathematics to Mechanics and in spite of the difficult political situation between East and West Germany we continued as full professors at the Universities of Technology of Munich and Magdeburg our scientific and personal discussions in a quite friendly atmosphere. I will ever remember the nice meetings with Horst Lippmann and his wife Martina. *Johannes Altenbach*

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