

# **One Dimensional Analysis Program for Scramjet and Ramjet Flowpaths**

Kathleen Tran

**Thesis submitted to the faculty of the Virginia Polytechnic Institute and State  
University in partial fulfillment of the requirements for the degree of**

Master of Science  
In  
Mechanical Engineering

**Walter F. O'Brien, Chair  
Joseph A. Schetz  
Mark R. Paul**

**December 8, 2010  
Blacksburg, VA**

**Keywords: Scramjets, Dual-Mode Scramjets, One Dimensional Modeling,  
Hypersonic Propulsion**

# **One Dimensional Analysis Program for Scramjet and Ramjet Flowpaths**

Kathleen Tran

## **Abstract**

One-Dimensional modeling of dual mode scramjet and ramjet flowpaths is a useful tool for scramjet conceptual design and wind tunnel testing. In this thesis, modeling tools that enable detailed analysis of the flow physics within the combustor are developed as part of a new one-dimensional MATLAB-based model named VTMODEL. VTMODEL divides a ramjet or scramjet flow path into four major components: inlet, isolator, combustor, and nozzle. The inlet module provides two options for supersonic inlet one-dimensional calculations; a correlation from MIL Spec 5007D, and a kinetic energy efficiency correlation. The kinetic energy efficiency correlation also enables the user to account for inlet heat transfer using a total temperature term in the equation for pressure recovery. The isolator model also provides two options for calculating the pressure rise and the isolator shock train. The first model is a combined Fanno flow and oblique shock system. The second model is a rectangular shock train correlation. The combustor module has two options for the user in regards to combustion calculations. The first option is an equilibrium calculation with a “growing combustion sphere” combustion efficiency model, which can be used with any fuel. The second option is a non-equilibrium reduced-order hydrogen calculation which involves a mixing correlation based on Mach number and distance from the fuel injectors. This model is only usable for analysis of combustion with hydrogen fuel. Using the combustion reaction models, the combustor flow model calculates changes in Mach number and flow properties due to the combustion process and area change, using an influence coefficient method. This method

also can take into account heat transfer, change in specific heat ratio, change in enthalpy, and other thermodynamic properties.

The thesis provides a description of the flow models that were assembled to create VTMODEL. In calculated examples, flow predictions from VTMODEL were compared with experimental data obtained in the University of Virginia supersonic combustion wind tunnel, and with reported results from the scramjet models SSCREAM and RJPA. Results compared well with the experiment and models, and showed the capabilities provided by VTMODEL.

# ACKNOWLEDGEMENTS

First I would like to thank my parents, Bai and Minh for their support and encouragement through my entire education. They have supported me through a career change from the biological sciences to engineering. They have taught me the value of hard work. They have taught me that the sky is the limit with a good work ethic and education through their example.

I would like to thank my advisor Dr. O'Brien. Without his support and offer of a position in the Center for Turbomachinery and Research I would not have attended Virginia Tech. I appreciate his support and advice through the many evolutions of thesis topics related to scramjets. Thank you to Dr. Schetz for his advice about modeling and scramjet flow. His insight into scramjet internal flows have helped shaped VTMODEL. I would also like to thank Dr. Paul. He taught some of my favorite yet most difficult classes at Virginia Tech. His chaos class helped prep me for the programming that was required for this piece of work. Finally, I would like to thank Dr. Dancey for acting as a proxy for my thesis defense.

I would like to thank my fiancé, Ken for his love and support through graduate school. He has encouraged me and supported me through all of the trials and tribulations of the last few years. He has provided me a shoulder to cry on and has motivated me to finish my work. He has also made my time in Blacksburg wonderful and special. Last of all I would like to thank all of the friends that I have made here at Virginia Tech. Though I cannot list you all, I want to thank you for making graduate school some of the best years of my life. There were many nights of engineering humor that kept me sane through all stresses of graduate school.

Finally I would like to thank the Aerojet Corporation and ATK-GASL. Their support through various programs during my graduate education aided in the conception and programming of VTMODEL.

# TABLE OF CONTENTS

<b>ABSTRACT</b>	<b>ii</b>
<b>ACKNOWLEDGEMENTS</b>	<b>iv</b>
<b>TABLE OF CONTENTS</b>	<b>vi</b>
<b>LIST OF FIGURES</b>	<b>ix</b>
<b>LIST OF TABLES</b>	<b>xii</b>
<b>NOMENCLATURE</b>	<b>xiii</b>
<b>1. INTRODUCTION AND LITERATURE REVIEW</b>	<b>1</b>
<b>1.1 Introduction to Scramjets and Ramjets</b>	<b>1</b>
1.1.1 History of Ramjets and Scramjets	1
1.1.2 Overview of the Components of Ramjets and Scramjets	7
<b>1.2 Current One Dimensional Models</b>	<b>12</b>
<b>1.3 Motivation for VTMODEL</b>	<b>18</b>
<b>2. DEVELOPMENT OF VTMODEL</b>	<b>20</b>
<b>2.1 Inlet</b>	<b>22</b>
2.1.1 MIL Spec E5007D Inlet Model	22
2.1.2 Kinetic Energy Efficiency of Inlets	24
2.1.3 Comparison of Inlet Models	26
<b>2.2 Isolator</b>	<b>28</b>
2.2.1 Fanno Flow and Oblique Shock System	28
2.2.2 Shock Train Correlation in Rectangular Isolators	32
2.2.3 Comparisons between the Two Isolator Models	33

<b>2.3 Combustor</b>	<b>36</b>
2.3.1 Combustion Modeling- Complete Chemistry Method	36
2.3.2 Combustion Modeling- Non Equilibrium Method	44
2.3.3 Comparison of Combustion Temperatures Produced by the Equilibrium Chemistry Model vs. Non-Equilibrium Chemistry Model	47
2.3.4 Combustion Flow Calculations	49
2.3.5 Use of Other Fuels	52
<b>2.4 Nozzle Modeling</b>	<b>53</b>
<b>3. SUMMARY OF MODEL FEATURES AND OPERATIONS</b>	<b>54</b>
<b>4. PARAMETERIZATION OF FACTORS WITH A GENERIC SCRAMJET GEOMETRY WITH VTMODEL</b>	<b>58</b>
<b>5. COMPARISON WITH RESULTS OF OTHER MODELS AND EXPERIMENTAL DATA</b>	<b>65</b>
<b>5.1 Comparison with Results of Other Models</b>	<b>65</b>
5.1.1 SCCREAM	65
5.1.2 RJPA Comparison	70
<b>5.2 Comparisons with Experiment</b>	<b>76</b>
5.2.1 Analysis of UVA Experimental Results	76
<b>5.3 Discussion of Results</b>	<b>81</b>
<b>6. CONCLUSIONS AND RECOMMENDATIONS</b>	<b>83</b>
<b>6.1 Conclusions</b>	<b>83</b>
<b>6.2 Recommendations</b>	<b>87</b>

<b>7. REFERENCES</b>	<b>89</b>
<b>7.1 Cited References</b>	<b>89</b>
<b>7.2 Uncited References</b>	<b>94</b>



# LIST OF FIGURES

Figure 1-1: Photograph from the 1st ramjet flight (Heiser 1994)	2
Figure 1-2: Engine modes vs Mach Number from the SR 71 Flight Manual (USAF 2002)	4
Figure 1-3: X-43 Captive and Carry (Kazmar 2005)	5
Figure 1-4: Artist Rendering of the X-51 (Warwick 2010)	6
Figure 1-5: Schematic of a Ramjet Engine (Bonanos 2005)	7
Figure 1-6: Scramjet Schematic. Courtesy of NASA Langley)	8
Figure 1-7: RJP A Schematic (Pandolfini 1992)	13
Figure 1-8: University of Maryland Code Validation (O'Brien T, 2001)	16
Figure 1-9: University of Adelaide Code Validation with UVA Tunnel Results (Birzer 2009)	17
Figure 2-1: Station Locations for VTMODEL	21
Figure 2-2: Scramjet Inlet Efficiency from Van Wie (Van Wie 2001)	24
Figure 2-3: Comparison of Pressure Recoveries vs Flight Mach Number for Three Inlet Model Calculations	26
Figure 2-4: Sample Static Pressure Profile with marked Fanno Flow and Oblique Shock Components with VTMODEL Predictions(Rockwell 2010)	31
Figure 2-5: Static Pressure Profile for $\Phi=0.171$ obtained by the University of Virginia (Rockwell 2010) and Static Pressure Predictions by VTMODEL	33
Figure 2-6: Comparison of Shock Train Correlation Model and Fanno Flow with Oblique Shock Model- Mach Number	34

Figure 2-7: Comparison of Shock Train Correlation Model and Fanno Flow with Oblique Shock Model- Static Temperature _____	35
Figure 2-8: Schematic of Growing Combustion Product Sphere Model Concept _____	37
Figure 2-9: Detailed Schematic Combustion Sphere Model _____	38
Figure 2-10: CARS Results of Mean Temperature, Oxygen and Nitrogen Mole Fraction Taken from O’Byrne (O’Byrne2007) _____	39
Figure 2-11: Static Pressure Profile Entered into VTMODEL Equilibrium Combustor	41
Figure 2-12: Combustor Static Temperature calculated by VTMODEL Equilibrium ____	42
Figure 2-13: Mach Number Predicted by VTMODEL with Equilibrium Chemistry ____	42
Figure 2-14: Stagnation Temperature Predicted by VTMODEL with Equilibrium Chemistry _____	43
Figure 2-15: Chemical Kinetics Effects at Mach 8 for $\Phi=1$ Taken from Jachimowski (1988) _____	46
Figure 2-16: Chemical Kinetic Effects Predicted by VTMODEL _____	46
Figure 2-17: Comparison of Non Equilibrium and Equilibrium Chemistry Models ____	47
Figure 3-1: Overall Flow Chart for VTMODEL _____	55
Figure 3-2: Flow Chart of Combustion Modeling for VTMODEL _____	56
Figure 4-1: General Trend of Ideal Nozzle Expansion (Bowcutt 2007)_____	59
Figure 4-2: Geometry of the Generic Combustor _____	60
Figure 4-3: Theoretical ISP for Various Systems (Moses 2003) _____	61
Figure 4-4: ISP vs Flight Mach Number as Predicted by VTMODEL _____	62
Figure 4-5: ISP vs Flight Altitude as Predicted by VTMODEL _____	63
Figure 4-6: ISP vs Equivalence as Predicted by VTMODEL _____	64

Figure 5-1: SCCREAM Combustor Geometry (Bradford 2001) _____	66
Figure 5-2: Bradford Comparison of SCCREAM and SRGULL (Bradford 2001) _____	68
Figure 5-3: VTMODEL Prediction of Bradford Combustor - Mach Number _____	68
Figure 5-4: ISP vs Mach Number as Predicted by RJPA, SCCREAM (Bradford 2001) and VTMODEL _____	71
Figure 5-5: Air Specific Impulse from RJPA (Bonanos 2005) _____	73
Figure 5-6: Air Specific Impulse from RJPA at Various Total Temperatures (Bonanos 2005) _____	74
Figure 5-7: Air Specific Impulse Predicted by VTMODEL Overlay over Bonanos (Bonanos 2005) _____	74
Figure 5-8: Air Specific Impulse Predicted by VTMODEL for $T_o=1010K$ Overlay over Bonanos (Bonanos 2005) _____	75
Figure 5-9: Schematic of UVA Tunnel (Le 2008) _____	77
Figure 5-10: Static Pressure Predicted by VTMODEL for $\Phi=0.341$ Compared to VTMODEL _____	78
Figure 5-11: Static Temperature Predicted by VTMODEL for $\Phi=0.341$ _____	79
Figure 5-12: Mach Number Predicted by VTMODEL for $\Phi=0.341$ _____	79
Figure 5-13: Stagnation Temperature Predicted by VTMODEL for $\Phi=0.341$ _____	80

# LIST OF TABLES

Table 2-1: Sample Inlet Data for Various Flight Mach Numbers versus Altitude	23
Table 4-1: VTMODEL Results for $\Phi=1$ , Mach 7, Altitude= 65,000 ft	61
Table 5-1: SCCREAM Combustor Entrance Properties for VTMODEL	67
Table 5-2: SCCREAM Cycle Analysis Properties (Bradford 2001)	70
Table 5-3: RJPA and VTMODEL Cycle Analysis Properties (Bradford 2001)	72
Table 5-4: UVA Isolator Entrance Parameters (Rockwell 2010)	76

# NOMENCLATURE

A	Area
$c_p$	Specific Heat at Constant Pressure
dx	infinitesimal change in x
$\Delta x$	Finite change in x
f	Coefficient of Friction
H	Enthalpy
$H_d$	Duct Height
h	Step size for 4 <sup>th</sup> order RK Method
$h_o$	Stagnation Enthalpy per Unit Mass
M	Mach Number
M	Non Reactive Species in Chemical Equations
$\dot{m}$	Mass Rate of Flow
P	Pressure
$P_t$	Total Pressure
Q	Net Heat Transfer per Unit Mass of Gas
R	Gas Constant
$Re_\theta$	Reynolds Number Based on Momentum Thickness
S	Shock Length
T	Static Temperature
$T_o$	Total Temperature
$T$	Thrust

TSFC Thrust Specific Fuel Consumption

$u_5$  Exit velocity

$u_m$  Velocity of Combustible Mixture

$u_f$  Combustion Flame Speed

$V_{\text{flow}}$  Bulk Flow Velocity

$V_e$  Nozzle Exit Velocity

$w$  Mass Rate of Flow of Gas Stream

$W$  Molecular Weight

$W_x$  Work

$X$  Drag Force

$x$  Axial Location

### Greek

$\gamma$  Ratio of Specific Heats

$\theta$  Boundary Layer Momentum  
Thickness

$\eta_n$  Nozzle Adiabatic Efficiency

## Subscripts

- a At altitude
- 0 Inlet Entrance
- 1 Isolator Entrance
- 2 Combustor Entrance
- 3 Combustor Exit
- 4 Nozzle Exit
- s Isentropic

# *Chapter 1*

## **Introduction and Literature Review**

### **1.1 Introduction to Scramjets and Ramjets**

#### 1.1.1. History of Ramjets and Scramjets

Ramjets and scramjets are air breathing propulsive engines that rely on the engine's forward movement to compress air at the inlet. Scramjets are similar in basic operating principle to ramjets except that supersonic combustion occurs within the combustor.

The concept of a ramjet has existed for nearly 100 years. The first ramjet was proposed by Rene Lorin in 1913 (Heiser 1994). At the time, Lorin realized that there would be insufficient pressure to operate with subsonic flight. In 1928, a Hungarian engineer by the name of Albert Fono was issued a German patent on a propulsive device that has all of the geometric features of a ramjet. The diagram reproduced by the Applied Physics Laboratory shows a convergent-divergent inlet with a low speed combustor, and a divergent nozzle. In 1935, Rene Leduc was issued a patent in France for a piloted aircraft with a ramjet engine. Leduc was not able to build a prototype until the late 1940s due to the occupation of France during World War II. However, on April 29, 1949, the first ramjet powered flight was accomplished when the Leduc 010 was launched from a parent vehicle and achieved Mach 0.84 at 26,000 ft. This historic aircraft is shown in Figure 1-1.





**Figure 1-1: Photograph from the 1st ramjet flight (Heiser 1994)**

In 1953, the first combined cycle ramjet engine was developed in France. The Griffon II was developed using the SNECMA Alter 101 E3 dry turbojet along with a ramjet that shared the same inlet and nozzle. The Griffon II was able to fly at a Mach 2.1 at 61,000 ft (Heiser 1994). Following all of these firsts in France, there was a movement in the United States and Canada to build and research ramjet and scramjet combustors. In July 1944, the US Navy began to sponsor a research project at the Applied Physics Laboratory to research and develop ramjet powered flight vehicles under the Bumblebee program. The first successful demonstration of a ramjet in supersonic flight under this program was in June 1945 with the Cobra ramjet (Waltrup 1997). In addition to programs at APL, scramjet work was also being start at McGill University in Montreal,

Canada. At McGill, Swithenbank published and reported early work on scramjet inlets, fuel injection, combustion, and nozzles. Swithenbank focuses on hypersonic flight Mach numbers of between 10 and 25. In 1958, Weber and MacKay published an analysis on the feasibility, benefits, and technical challenges to scramjet powered flight (Mach 4-7). In addition to the work on the Bumblebee project at the Applied Physics Laboratory at John Hopkins University, Avery and Dugger started an analytical and experimental study of scramjet engines and the potential in 1957 (Curran 2001). In 1964, Dugger and Billig submitted a patent application for a scramjet that was based on Billig's PhD thesis (UMD 2010).

Ramjets have also been combined with turbine engines for high speed flight. Perhaps the most famous combined cycle aircraft is the SR-71. The SR-71 was developed in the early 1960s at Lockheed Martin's Skunk Works facility (Lockheed Martin 2010). The aircraft had a Pratt and Whitney J58-P4 power system on board. The J58-P4 is a hybrid turbine-ramjet engine. At lower speeds, the engine was flown as a turbojet. At supersonic speeds, the engine then flew in "ramjet mode". The engine was essentially a turbojet inside of a ramjet (Goodall 2002). Figure 1.2 shows the operational modes of the J-58 at increasing flight Mach numbers.

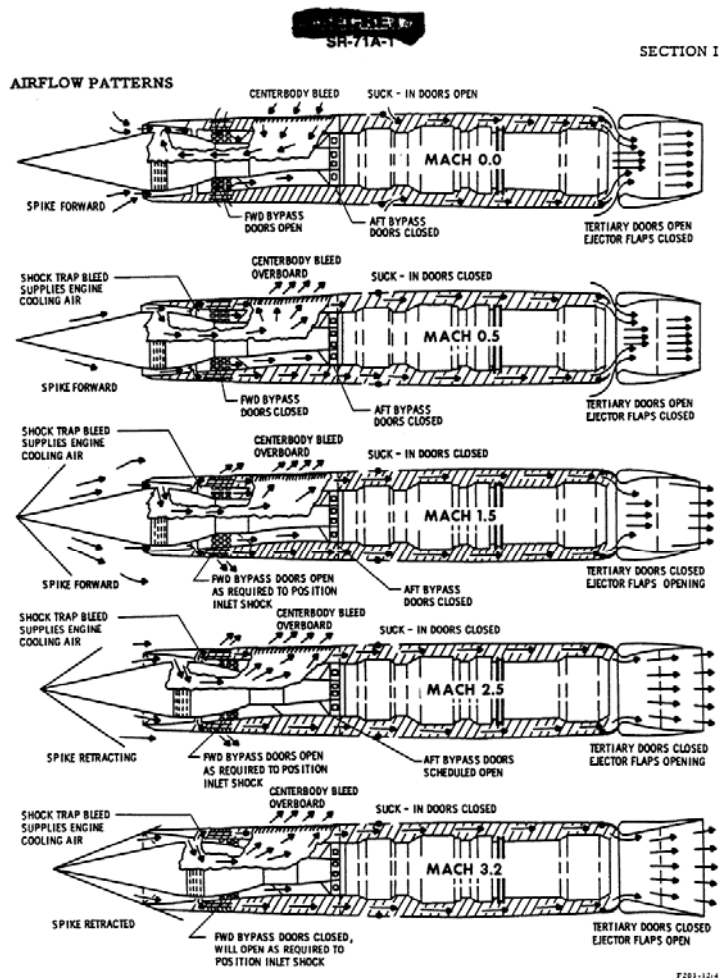


Figure 1-21



1-33

Figure 1-2: Engine modes vs. Mach Number from the SR 71 Flight Manual (USAF 2002)

The evolution of the scramjet engine was to follow the success of ramjets in aircraft and missile systems. To follow the earlier work in scramjet research, the National Aerospace Plane project (X-30) envisioned a single stage space access plane. This project was started in the 1980s and was funded by both NASA and the DOD with additional support from DARPA. The plane was to incorporate a scramjet engine powered by hydrogen. Unfortunately, the National Aerospace Plane project was canceled in 1993 before a

prototype could be built. Some of the research and development for the X-30 was then used for the X-43 hydrogen-fueled hypersonic research aircraft. The X-43 was designed and built to be an unmanned system. A Pegasus booster launched from a B-52 was used to achieve to the correct altitude and speed prior to igniting the X-43 scramjet engine. In 2004, the X-43 was able to reach and maintain a record speed of Mach 9.68 at 112,000 ft (Kazmar 2005).



**Figure 1-3: X-43 Captive and Carry (Kazmar 2005)**

Most recently, an advancement of scramjet-powered vehicle occurred with the successful test of the X-51. The X-51, an integrated rocket-boosted and scramjet vehicle, was developed by Boeing in partnership with the USAF, DARPA, NASA, and Pratt and Whitney Rocketdyne. The scramjet fuel was the hydrocarbon JP-7. On May 26, 2010, the X-51 had a successful first flight. The research vehicle was launched from a B-52. The X-51 broke the record for the longest scramjet-powered flight, operating for over 200 seconds. The X-51 reached Mach 5 in its first flight. The flight was planned to be over

300 seconds, but a sudden deceleration caused the flight to be terminated early (Boeing 2010).



**Figure 1-4: Artist Rendering of the X-51 (Warwick 2010)**

## 1.1.2. Overview of the Components of Ramjets and Scramjets

Ramjets and scramjets, unlike turbomachinery-based engines, have no moving parts and consist of a basic inlet, isolator, combustor, and nozzle. These components are pictured in Figure 1-5 and 1-6. Figure 1-5 is a basic schematic of a ramjet engine, while Figure 1-6 represents a scramjet engine.

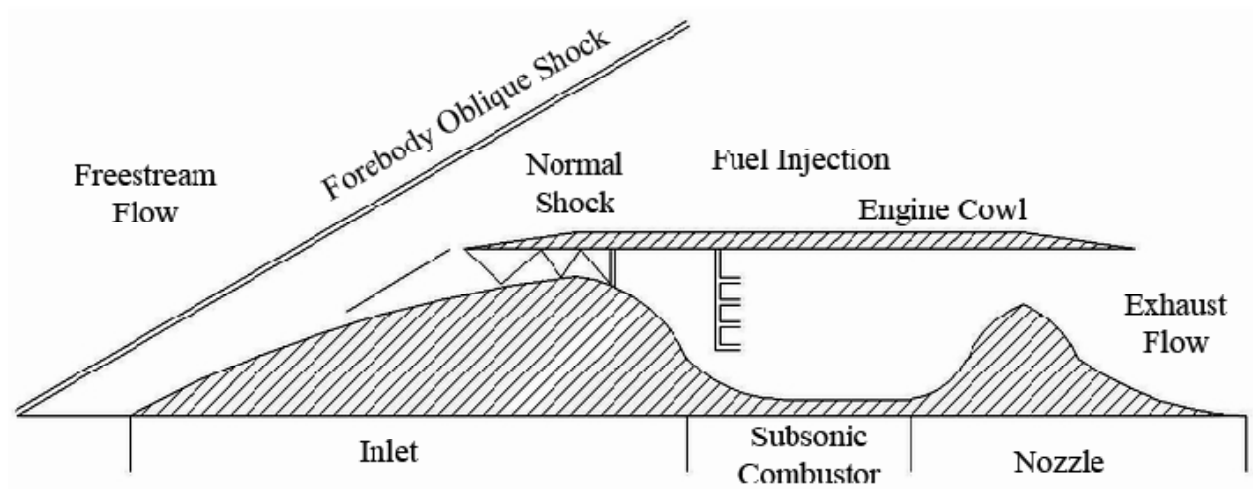


Figure 1-5: Schematic of a Ramjet Engine (Bonanos 2005)

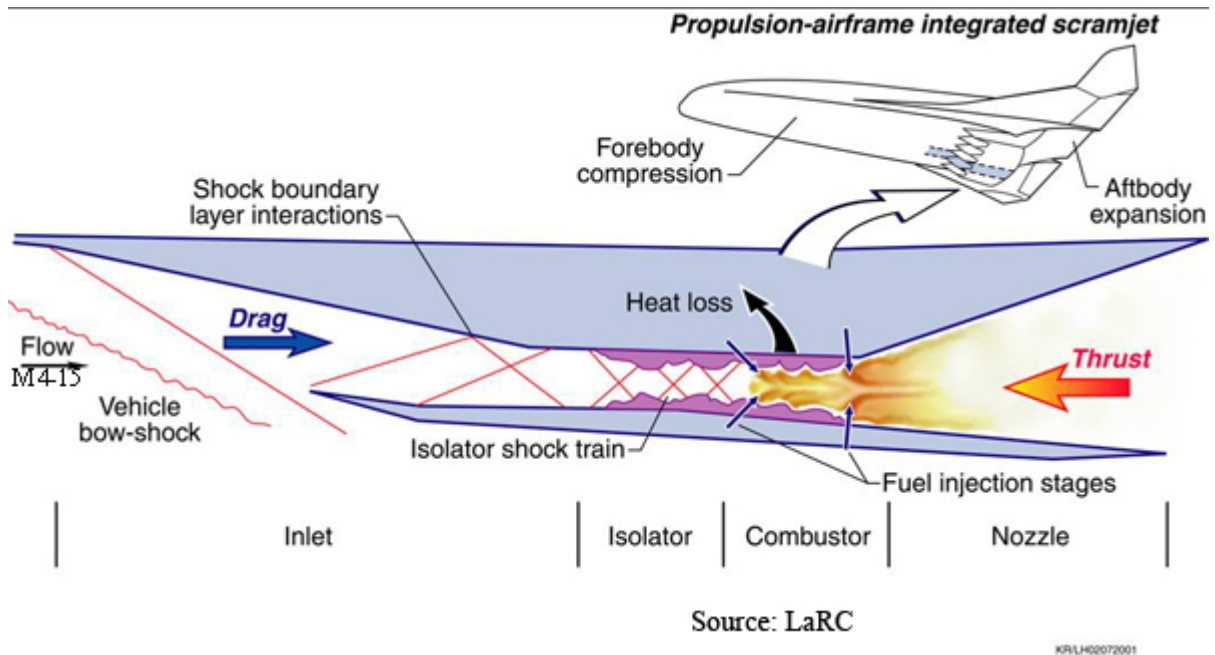


Figure 1-6: Scramjet Schematic Courtesy of NASA Langley

From the figures it can be seen that the first component of a ramjet or a scramjet is the inlet. These supersonic inlets can be of many shapes and designs, but the overall function is the same. The inlet reduces the Mach number and compresses the inlet air to a desired state prior to isolator or combustor entry. According to Segal (Segal 2009), inlets for a scramjet can either be fixed or contain adjustable surfaces. Ramjet inlets for aircraft up to Mach 2 flight conditions can generally be considered to be fixed, however at higher Mach numbers, a variable geometry inlet may be required. An exception to this may be in the case of a missile or a combined cycle missile. In general, a fixed geometry inlet must be of a design that provides adequate flow compression for inlet start. For other vehicle applications, the inlet may require adjustable surfaces for starting and to control the compression of the flow for off- design engine operation (Segal 2009).

There are five features that scramjets inlets will likely contain: (1) All of the design surfaces are used to compress the flow, resulting in a complicated 3D shock system; (2) Adjustable surfaces and variable geometry are used to support flights from supersonic to hypersonic speeds; (3) The inlet through the use of an isolator will have to be “compatible” with the combustion pressure rise; (4) The inlet must be integrated with the fuselage design to accommodate the long compression ramps; (5) Finally, the inlet will be “arranged in a single segment or in several segments” to optimize the frontal area (Segal 2009).

The isolator is an essential part of any scramjet engine. The isolator is a constant cross sectional area duct that is designed to prevent unstart of the inlet. With supersonic combustion, the isolator shock train that is created by the pressure demand of the combustion process can move forward in the inlet, disrupting the inlet function. This can cause failure of the engine. The isolator is designed to contain this shock train, preventing it from unstarting the inlet.

The combustor of the scramjet encloses supersonic combustion. In a ramjet or a dual mode scramjet this process can also occur at or below the local speed of sound. The combustor is generally made up of an igniter, fuel injectors, and a flame holder. The igniter can vary in design with the use of silane, a shock detonator tube, solid propellant igniters, or a plasma torch such as the Virginia Tech Plasma Torch (Bonanos 2005). Some engines have been tested that do not have a definitive igniter, but depend on the fuel auto ignition characteristics, typically of hydrogen fuel. The fuel injectors can be located either upstream or downstream of the igniter depending on the design. The flame holder can be a cavity built into the geometry of the combustor, a flow ramp in the flow



path, or a flush-wall device such as the Virginia Tech Plasma Torch (Bonanos 2005).

The cavity provides flame holding by incorporating a stationary combustion recirculation region for continuously igniting the fuel-air mixture. Combustors are generally expanding in flow area to maintain the flow Mach number at the desired levels.

Within the combustor of a scramjet, there are multiple design issues that must be considered (Schetz 2007):

- Wall shear
- Base pressure drag
- Injector drag
- Heat transfer through the walls
- Isolator pressure rise
- Peak heat flux
- Rayleigh irreversibility
- Incomplete mixing and combustion
- Flow distortion
- Chemical dissociation
- Combustor pressure rise

These design issues are categorized to include momentum, energy, cycle efficiency, and operability effects.

The nozzle of a scramjet has its own requirements for expansion of flow. In a scramjet design, the enthalpy of the flow should have increased enough by the combustion process to produce thrust. The nozzle is generally a divergent duct to expand the flow, typically with a continuing combustion reaction because of the low residence

time of the fuel and air within the combustor. Due to the fact that scramjets require a large nozzle pressure ratio, Segal states that nozzles should be of the “open type” (Segal 2009). This open type is defined as using the aft vehicle surface as part of the nozzle, instead of a separate independent duct. Since the thrust of the engine is only slightly greater than the vehicle’s drag at hypersonic speeds, good efficiency and design of the nozzle is essential to the success of the engine (Segal 2009).

## 1.2 Current One-Dimensional Models

One-dimensional scramjet flowpath analysis codes can be a useful analytical tool for scramjet researchers and designers. There are many advantages to the appropriate use of a one-dimensional code versus a more complex two or three dimensional flow analysis such as CFD. These advantages include faster computational times and easier overall performance-based analysis. Though the analysis cannot predict effects of boundary layers and other multidimensional flow properties, the one-dimensional code can provide reasonable ranges for thermodynamic and performance design criteria.

One of the legacy codes widely used for the one-dimensional simulation of scramjets and ramjets is the Ramjet Performance Analysis Code (RJPA). This code was developed at the Applied Physics Laboratory at John Hopkins University, and is considered the industry standard. The code separates the flow path into major sections designated the Freestream, Diffuser, Combustor, and Nozzle. Each one of these components is modeled as a control volume with data passing across the boundaries. Figure 1-7 below shows the basic schematic of RJPA. The numbers below the schematic are the locations where the program calculates thermodynamic data.

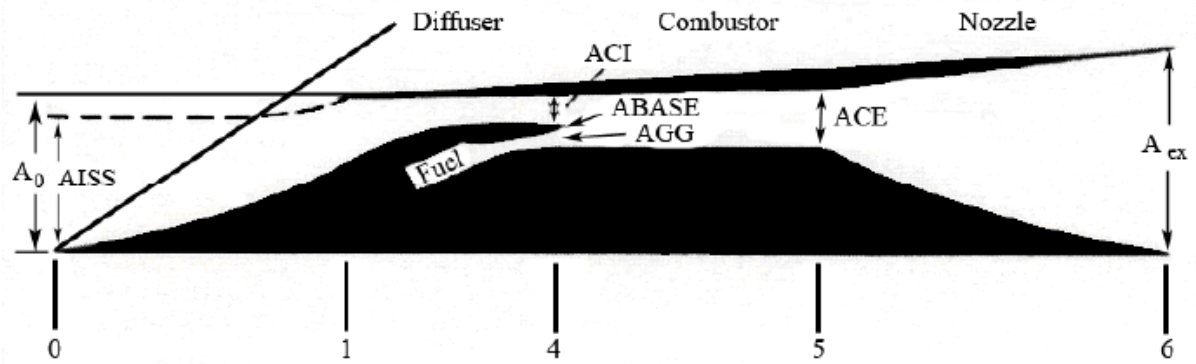


Figure 1-7: RJA Schematic (Pandolfini 1992)

RJPA models the combustion process using thermochemical equilibrium for the selected fuel, the equivalence ratio, a parameter called PSPCI at the combustor entrance (the “precombustion shock pressure at location  $c_i$ ”), the combustion efficiency, a wall friction coefficient, and the wall heat transfer. RJPA also incorporates a “ $\epsilon$ ” factor concept for estimates of the static pressure variation within the combustor. For a given combustor

inlet and exit pressure, it is assumed that  $pA^{\frac{\epsilon}{\epsilon-1}}$  is constant within the combustion process. This epsilon is determined using entropy limits. The constant epsilon assumption relates the wall pressure force,  $P_w$ , the static pressure and the areas in the following relationship, allowing an estimate of overall pressure change in the combustor (Waltrup 1978).

$$P_w A^{\epsilon/(\epsilon-1)} = \text{constant} = \frac{P_3}{P_2} P_2 A_2^{\epsilon/(\epsilon-1)}$$

One of the major capabilities of RJPA is the ability to incorporate flow factors such as the coefficient of friction, heat transfer through the walls, and the pressure ratio due to the combustion shock system (PSPCI). Despite the established capabilities of RJPA, a major limitation on the use of the program in research is the restriction on the program access.

The program is considered ITAR restricted. This restriction makes the access and use of the program out of the public domain.

One scramjet analysis program that is in the public domain is known as HAP (Hypersonic Airbreathing Propulsion). The program accompanies a text by Heiser and Pratt called *Hypersonic Airbreathing Propulsion* (Heiser 1994). This text is part of an AIAA educational series. HAP is written in MS DOS and will run on most computers in the command prompt. Some of the features and analysis capabilities of HAP are the ability to perform trajectory analysis and calculate the overall performance of scramjets, and the use of compressible flow and isentropic flow properties for calorically perfect gasses. The program also assumes a simplified ideal chemical equilibrium in the combustor. These assumptions make HAP inaccurate for use at higher Mach numbers. There is no visual interface in HAP. The inputs are in a text file (Heiser 1994).

A program developed in the late 1980s for one-dimensional scramjet analysis at the NASA Glenn Research Center is called RAMSCRAM. The program uses chemical thermodynamic equilibrium for the combustion modeling (Burkardt 1990). The program allows for multiple fuel injectors and multiple compressors sections (Bradford 2001).

The most recent development of a scramjet performance code at the NASA Langley Research Center is the code SRGULL (Zweber 2002). This program uses multiple subroutines for each section of the combustor. The program uses 1-D, and 2-D modeling for the flow path. The inlet and nozzle subroutines use 2-D modeling. This modeling is called “axisymmetric with 3-D corrections.” For the combustor modeling, SRGULL uses a one-dimensional equilibrium model.

Another one-dimensional code is SCCREAM developed at Georgia Tech (Bradford 1998). SCCREAM stands for Simulated Combined-Cycle Rocket Engine Analysis Module. This program is written in C++, and was developed to provide a conceptual design tool for analyzing rocket and scramjet combined systems. One of the advantages of this code is the fact that it can “run a full range of flight conditions and engine modes in under 60 seconds, and will output a properly formatted POST engine table” (Bradford 1998). The SCCREAM code results from Bradford’s dissertation will be used as a comparison tool for VTMODEL results in Chapter 5.

An addition to combined cycle codes was developed at the University of Maryland by O’Brien et al (O’Brien, T 2001). In this code, finite rate chemistry of hydrogen and Jet A were coupled with flow equations to model combustors in scramjets combustors. Since chemical kinetics is used, fuel ignition and combustion progress can be predicted and modeled. This is a benefit of this code versus chemical equilibrium codes. The Jachimowski reaction was used for hydrogen chemical kinetics (Jachimowski 1988), while the Kundu reaction was used for Jet A (Lee 1991). For modeling of the chemical kinetics, the model uses Chemkin II (Reaction Design 2010). Chemkin is a commercial software product that integrates chemical kinetics into simulations of reacting flow. The University of Maryland code was also validated by comparisons of predicted static pressure versus experimental pressure profiles. Figure 1-8 shows one the comparison with experimental results obtained by Billig.

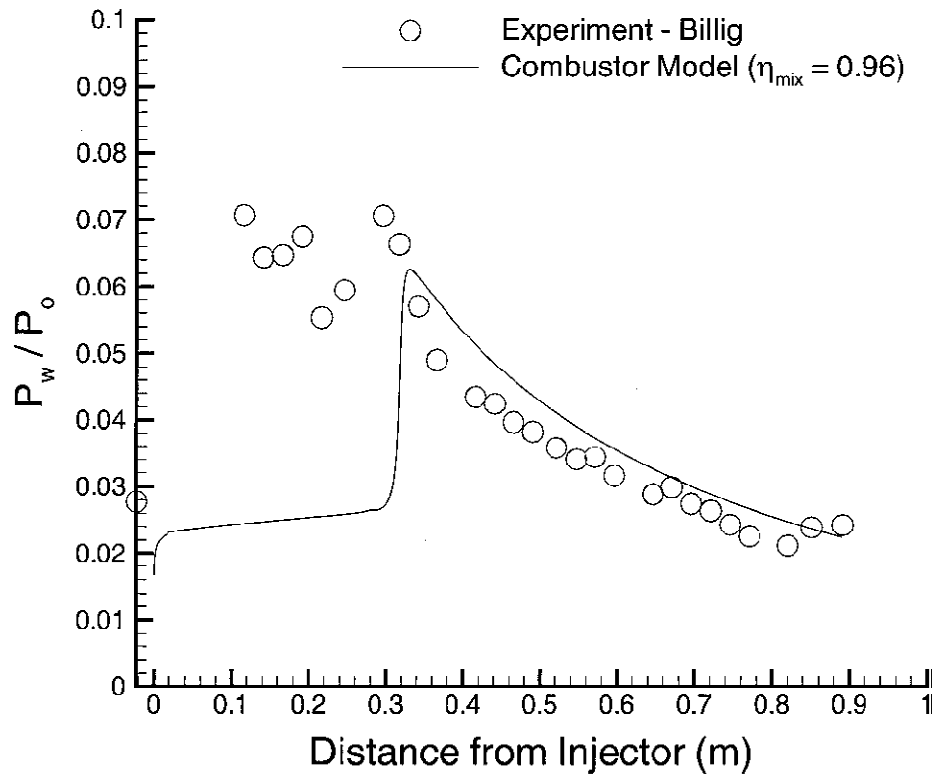


Figure 1-8: University of Maryland Code Validation (O'Brien T, 2001)

Another model created at the University of Adelaide in Australia was used to analyze the University of Virginia Direct Connect Tunnel results (Birzer 2009). The model uses a quasi – one-dimensional solver that assumes that the flow is steady state, ideal gas, and that flow properties are quasi-one dimensional. One of the weaknesses of the program is that the combustion is assumed to be only mixing-limited instead of including kinetics limitations. This assumption lowers the computational time, but the effect on the analysis results is not noted in the paper. The code has been validated by comparisons of predicted pressures to experimental static pressures. Figure 1-9 shows a result with good agreement for hydrogen fuel combustion with  $\phi=0.32$ .

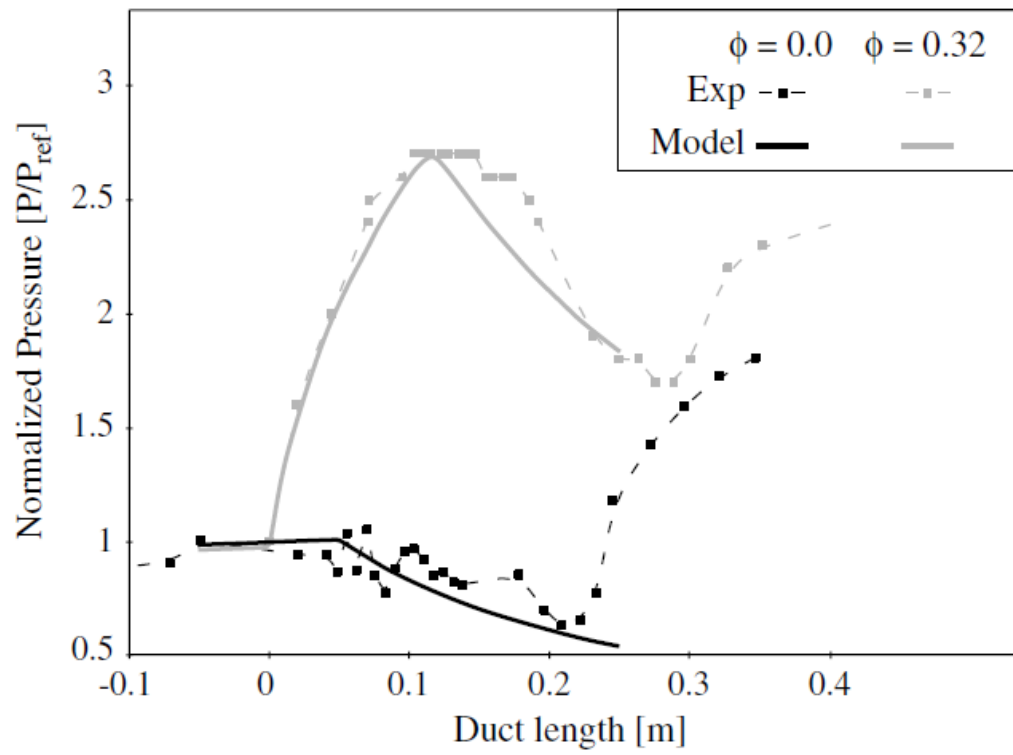


Figure 1-9: University of Adelaide Code Validation with UVA Tunnel Results (Birzer 2009)



## 1.3 Motivation for VTMODEL

From the above review it can be seen that there are many one-dimensional codes available for the analysis of scramjet and ramjet flowpaths. Some of the reviewed codes are combined cycle codes, offering the additional benefit of being able to analyze the performance of such vehicles from launch. The present research provides an additional code for the modeling of scramjet performance, named VTMODEL. The main motivation behind VTMODEL was to provide a capable, user friendly code with thermodynamic analysis capabilities, for use in the public domain.

VTMODEL was developed to make an accessible analysis code that can be modified and improved by the user. Since the program is not a compiled code, the user can add or change the main functions and tailor the code to their requirements. VTMODEL was also developed to make the analysis solution more adaptable to different test requirements. The program can be both predictive and analytic; that is, an internal static pressure profile from experimental data can be entered in and analyzed. The alternative “predictive” solution method iterates on a solution until a specified combustor exit pressure or nozzle exit pressure condition is met.

The analysis modules of VTMODEL provide flow path computations for given flight conditions, and specified Isolator, Combustion, Combustor Flow, and Nozzle parameters. Within these major functions, the user has options of which of the included alternate models to use. For the isolator function, there are two flow models based on correlation data from Sullins and McLafferty (Sullins 1992), and a more analytical Fanno flow model combined with an oblique shock system model. The combustion function also has available two different analyses. The first model is a complete combustion

(equilibrium) combustion solver with an “expanding combustion sphere” efficiency model. The second model is a non-equilibrium hydrogen combustion solver using a mechanism developed by Jachimowski (Jachimowski 1988).

# *Chapter 2*

## **Development of VTMODEL**

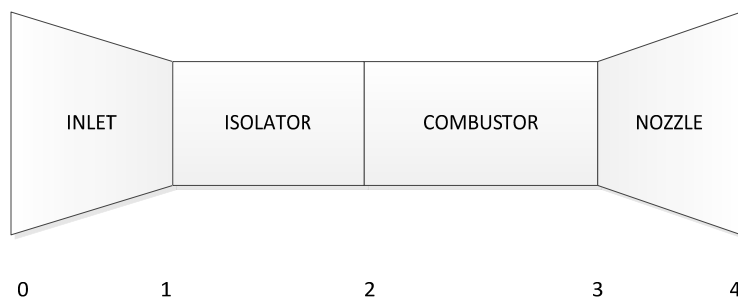
As mentioned, the development of VTMODEL addressed an internal need for a public access one-dimensional ramjet and scramjet analysis code. The model was created to provide flexibility to the user in choosing different models for the inlet, isolator, and combustor. The basis of the model is a segmented approach to the analysis of the flow within each component of the ramjet and scramjet. Each individual module can be modified and adapted, providing flexibility to expand on the model and apply the model to the user's specific criteria. For the flow entering the inlet, the properties are calculated from given flight conditions.

For the isolator with given inlet flow conditions, the model requires a static pressure anchor at an axial location in the combustor or the nozzle exit. This static pressure anchor is necessary for establishing the required static pressure rise in the isolator. The supersonic combustion process, the flow area of the combustor and nozzle, and the friction and heat transfer in the combustor together establish the required combustor inlet Mach number for a given inlet pressure and temperature. This combustor inlet flow state must match the exit condition from the isolator. To calculate and quantify this effect in the isolator, the shock train length or the oblique shock angles are iterated upon to provide the required combustor inlet conditions. It is necessary for a static pressure anchor to be identified and set downstream of the isolator. Example anchor points are the combustor entrance, combustor exit, or nozzle exit. If no other

information is available, the nozzle exit static pressure is equated to atmospheric conditions. This results in a flow prediction based on an ideally-expanded nozzle.

The required isolator flow properties are used by the isolator model calculation process. For one option of the module, VTMODEL assumes the longest shock length possible, which is the isolator length. The program then lowers this value to match the isolator discharge static pressure anchor. This procedure is similar to that for the second option, which iterates on the reflected shock angles. This iteration is necessary for the isolator pressure rise to have a proper match to the flow conditions in the combustor.

The overall model uses the subscripts given in Figure 2-1 for thermodynamic data locations. Positions 0 and 1 designate the entrance and exit of the inlet. The isolator entrance and exit is designated by locations 1 and 2. The combustor entrance is designated as location 2 and the exit is location 3. The exit of the nozzle is location 4. An additional subscript is used for atmospheric conditions.



**Figure 2-1: Station Locations for VTMODEL**

## 2.1 Inlet Modeling

The inlet function requires an input of the flight altitude and Mach number to set the upstream boundary condition. VTMODEL 1 uses the US Standard Atmosphere properties at altitudes specified every 5,000 feet (US Government Printing Office 1976). The incoming flight Mach number ( $M_o$ ) is defined by the user and is used for determining supersonic inlet pressure recovery.

### 2.1.1. MIL Spec E-5007D Inlet Model

As one of two options, the scramjet inlet total pressure recovery is modeled using MIL Spec E-5007D, which provides the following correlations:

$$P_{o2}/P_{o1}=1 \text{ from } M_o=0 \text{ to } 1 \quad (\text{Eq 2-1})$$

$$P_{o2}/P_{o1}=1-0.0776(M_o-1)^{1.35} \text{ from } 1 < M_o < 5 \quad (\text{Eq 2-2})$$

$$P_{o2}/P_{o1}=800/(M_o^4+935) \text{ for } M_o > 5 \quad (\text{Eq 2-3})$$

The inlet module can be disabled by the user to directly input isolator entrance conditions for comparison with direct connect tunnel tests. The module can also be modified to allow the user to enter specified inlet pressure recoveries instead of using MIL Spec E-5007D. Table 2-1 gives example inlet module results for a various test flight Mach numbers and altitudes. This table shows the calculated MIL Spec E-5007D inlet pressure recovery.

Flight Mach Number	Altitude (ft)	T <sub>a</sub> (K)	M <sub>1</sub>	T <sub>1</sub> (K)	Pressure Recovery P <sub>02</sub> /P <sub>01</sub>
5	50,000	217	4.42	265	0.51
5	60,000	217	4.42	265	0.51
5	70,000	218	4.42	267	0.51
5	80,000	221	4.42	270	0.51
6	50,000	217	5.06	290	0.36
6	60,000	217	5.06	290	0.36
6	70,000	218	5.06	292	0.36
6	80,000	221	5.06	296	0.36
7	50,000	217	5.56	326	0.24
7	60,000	217	5.56	326	0.24
7	70,000	218	5.56	328	0.24
7	80,000	221	5.56	332	0.24
8	50,000	217	5.98	366	0.16
8	60,000	217	5.98	366	0.16
8	70,000	218	5.98	369	0.16
8	80,000	221	5.98	374	0.16
9	50,000	217	6.35	411	0.11
9	60,000	217	6.35	411	0.11
9	70,000	218	6.35	413	0.11
9	80,000	221	6.35	419	0.11
10	50,000	217	6.69	457	0.07
10	60,000	217	6.69	457	0.07
10	70,000	218	6.69	460	0.07
10	80,000	221	6.69	466	0.07

**Table 2-1: Sample Inlet Data for Various Flight Mach Numbers versus Altitude**

## 2.1.2. Kinetic Energy Efficiency of Inlets

An additional inlet model specifically designed for scramjet application was added to VTMODEL based on the compilation of inlet information in the Van Wie section of *Scramjet Propulsion* (Van Wie 2001). Van Wie used experimental correlations for kinetic energy efficiency from various experimental results. The data and his correlations are shown in Figure 2-2 below.

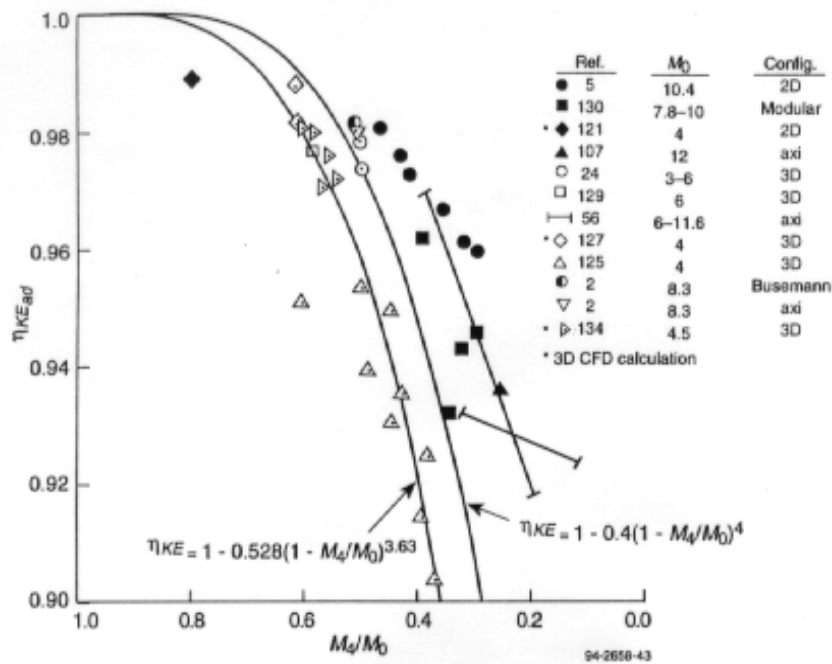


Figure 2-2: Scramjet Inlet Efficiency from Van Wie (Van Wie 2001)

From this efficiency chart, the following equations can be selected as the kinetic energy efficiency correlations:

$$\eta_{KE} = 1 - 0.528(1 - M_1/M_0)^{3.63} \quad (\text{Eq 2-4})$$

$$\eta_{KE} = 1 - 0.4(1 - M_1/M_0)^4 \quad (\text{Eq 2-5})$$

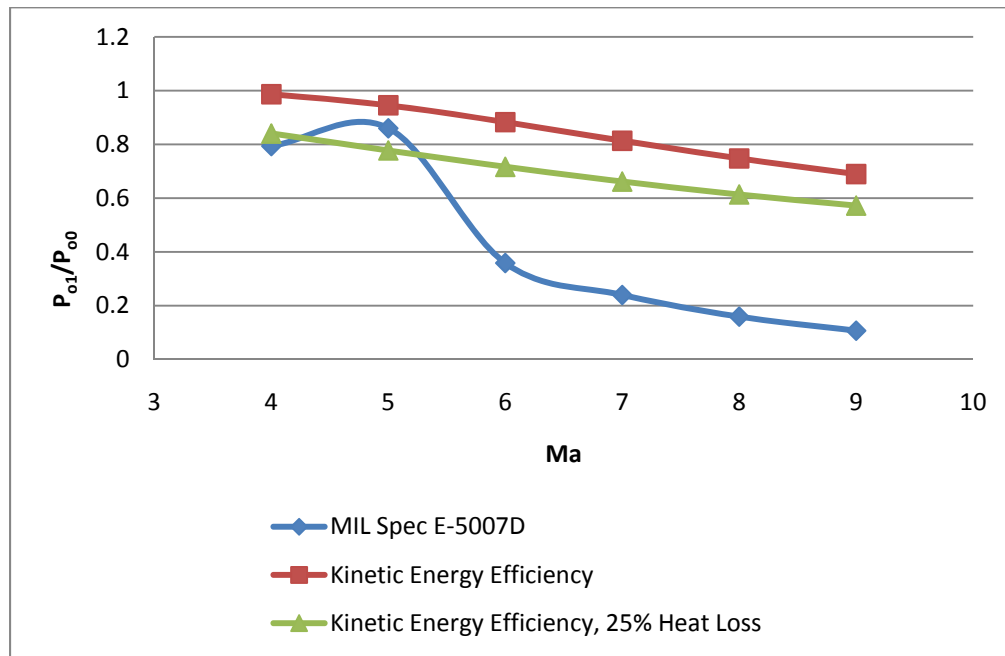
The kinetic energy efficiency correlation is then used to find the pressure recovery in the inlet. The following equation is then used to find the value of the stagnation pressure recovery.

$$\eta_{KE} = \frac{2}{(\gamma - 1)M_0^2} \frac{T_{o1}}{T_{o0}} \left[ 1 + \frac{\gamma - 1}{2} M_0^2 - \left( \frac{P_{o0}}{P_{o1}} \right)^{\frac{\gamma}{\gamma - 1}} \right] \quad (\text{Eq 2-6})$$



### 2.1.3. Comparison of Inlet Models

To provide a comparison of the two VTMODEL inlet models, an example range of flight Mach numbers was selected. The flight Mach number was varied from 4-9, representing a range of scramjet flight conditions. For the kinetic efficiency model, the first efficiency correlation was chosen for illustration.



**Figure 2-3: Comparison of Pressure Recoveries vs Flight Mach Number for Three Inlet Model Calculations**

In Figure 2-3, it can be seen that the MIL Spec E5007D analysis predicts very high losses as flight Mach numbers increase beyond Mach 5. Users report that MIL Spec E5007D predicts a pressure recovery value that is very conservative (that is, very low) (Trefny, 2010). The Van Wie kinetic energy efficiency model with no heat loss predicts the minimum losses in the inlet. This behavior is most likely due to the assumption that total temperature stays constant through the inlet. In modeling real inlets, the heat transfer is important to the flow physics. To study this behavior, a condition of 25% heat

loss ( $T_{02}/T_{01}=0.75$ ) was assumed in the inlet. The effect on the pressure recovery prediction can be seen in Figure 2-3. This solution falls between the ideal kinetic energy efficiency model and the MIL Spec model. The heat transfer can be predicted by using Reynolds Analogy. The model is presented in Chapter 2.3.4.

## 2.2 Isolator Modeling

### 2.2.1. Fanno Flow and Oblique Shock System

Depending on the type of flow solution desired, the VTMODEL isolator models can be used to predict local flow properties in the isolator. If a static pressure profile is given from an experiment (referred to as “user supplied”), the models can be used to predict local temperatures and Mach numbers. If the model is predictive, the isolator flow variables become part of a system iteration which is ultimately anchored by the nozzle exit pressure.

Within VTMODEL, there are two isolator flow models that the user can choose. The first model is comprised of two separate components. The initial component enables modeling of pressure rise due to friction. The model uses the Fanno flow relationships (Hill and Peterson 1992, pp. 77-84) to calculate the pressure rise due to friction in a constant cross sectional area. The following relationship initiates the calculation based on the length  $x$  of the isolator and the lengths  $L^*$  from stations 1 and  $x$ , respectively, to the sonic ( $M=1$ ) state.

$$\left(\frac{4c_f L^*}{D}\right)_x = \left(\frac{4c_f L^*}{D}\right)_1 - \frac{4c_f x}{D} \quad (\text{Eq 2-7})$$

where  $c_f=0.0015$  is the VTMODEL default value and can be changed by the user.

The static pressure rise due to friction can be either determined from the previously mentioned user-supplied static pressure profile or iterated as a contributing factor to the combustor/nozzle exit pressure. Using the Fanno flow model, the values of

$c_f$ ,  $\gamma$ , isolator length  $L$  and hydraulic diameter  $D$ , and inlet  $M$  determine the pressure change.

To calculate the flow, the following equation calculates the length of the duct that is necessary to change the Mach number of the entering flow to a value of 1. The superscript \* designates the sonic station.

$$\frac{4c_f L^*}{D} = \frac{1-M^2}{M^2} + \frac{\gamma+1}{2} \ln \frac{(\gamma+1)M^2}{2\left(1+\frac{\gamma-1}{2}M^2\right)} \quad (\text{Eq 2-8})$$

From this result, a relationship can be derived using the Fanno flow relationship above to determine the Mach number at the desired location. The following equations will then calculate the static pressure, static temperature, and stagnation pressure changes due to Fanno flow with the reference values based on the sonic state.

$$\frac{T}{T^*} = \frac{(\gamma+1)}{2\left(1+\frac{\gamma-1}{2}M^2\right)} \quad (\text{Eq 2-9})$$

$$\frac{P}{P^*} = \frac{1}{M} \left[ \frac{(\gamma+1)}{2\left(1+\frac{\gamma-1}{2}M^2\right)} \right]^{1/2} \quad (\text{Eq 2-10})$$

$$\frac{P_o}{P_o^*} = \frac{1}{M} \left[ \frac{2}{\gamma+1} \left( 1 + \frac{\gamma-1}{2} M^2 \right) \right]^{(\gamma+1)/2(\gamma-1)} \quad (\text{Eq 2-11})$$

With a given pressure profile, the pressure increase due to the oblique shock system can be identified from a significant change in the slope of the pressure graph. This pressure rise in the isolator due to the combustion process is modeled using a system of

two oblique shocks. When the overall static pressure ratio is input by the user, the shock angles of the reflected oblique shock system can be calculated. From these angles, the Mach number and temperature following each shock are calculated.

To demonstrate the segmentation of the static pressure profile into Fanno flow and oblique shock-modeled portions, Figure 2-4 below is given as an example. The static pressure data was obtained from the University of Virginia Direct Connect tunnel (Rockwell 2010). The isolator has a constant cross sectional area. As can be seen from Figure 2-4, approximately 10 kPa of the total pressure rise is modeled by Fanno flow, while the remaining 45 kPa pressure rise is modeled by the oblique shock system.

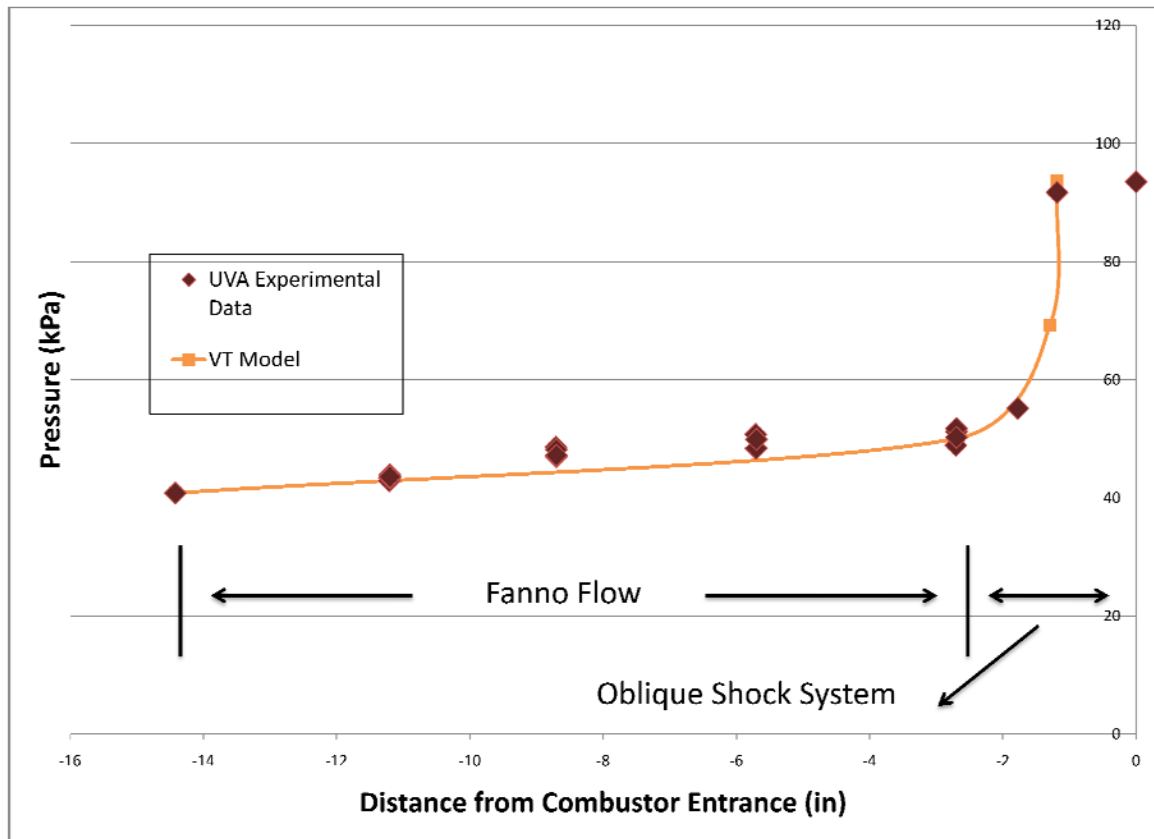


Figure 2-4: Sample Static Pressure Profile with marked Fanno Flow and Oblique Shock Components with VTMODEL Predictions (Rockwell 2010)

When VTMODEL is used in a predictive manner, the pressure rise with the oblique shock system is calculated with an initial value for the shock angles. The overall scramjet modeling program then iterates on a final solution with the combustor or the nozzle exit pressure as defined by the user.

## 2.2.2. Shock Train Correlation in Rectangular Isolators

The second available isolator model in VTMODEL is based on an experimental correlation of shock trains in a rectangular duct by Sullins and McLafferty (Sullins 1992). They based the correlation on work done by Waltrup and Billig (Waltrup 1973) in the 1970's, relating shock train length to flow features in axisymmetric ducts. Sullins and McLafferty produced a single correlation from experimental data for two entrance Mach numbers, 2 and 2.85. The resulting relationship correlates well with experimental data up to 80% of the maximum pressure rise due to a normal shock for the given inlet conditions. This upper limit of the maximum pressure rise will not occur with supersonic combustion (scramjet conditions) since the combustor inlet conditions must be supersonic. The Sullins and McLafferty correlation is given by the following equation:

$$\frac{(M_1^2-1)(S/H_d)Re_\theta^{0.2}}{\sqrt{\theta/H_d}} = 50 \left( \frac{P_2}{P_1} - 1 \right) + 170 \left( \frac{P_2}{P_1} - 1 \right)^2 \quad (\text{Eq 2-12})$$

With this correlation, only the pressures at the end and the beginning of the isolator are considered. Knowing the pressure changes, other flow properties may also be calculated at these end points. Following the valid data range of the correlation ( $2 < M_1 < 3$ ), VTMODEL will select the closest correlation to use depending on the entrance Mach number to the isolator. For isolator entrance Mach numbers greater than 3, the Fanno flow and oblique shock system is always used.

### 2.2.3. Comparisons between the Two Isolator Models

Figure 2-6 and 2-7 below show an example isolator module run based on previously shown experimental pressure data obtained from the University of Virginia dual mode combustion test facility for an equivalence ratio  $\Phi=0.171$  (Rockwell 2010). The input static pressure profile is shown in Figure 2-5 below.

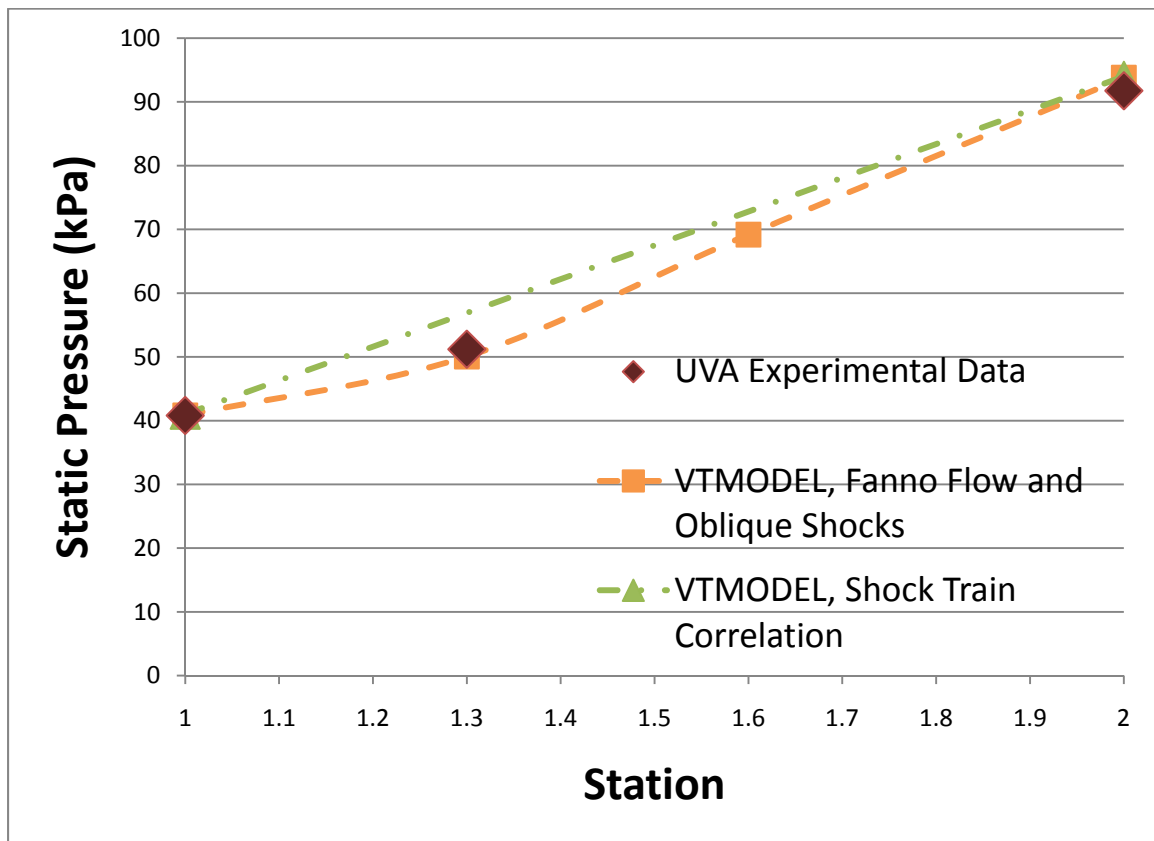
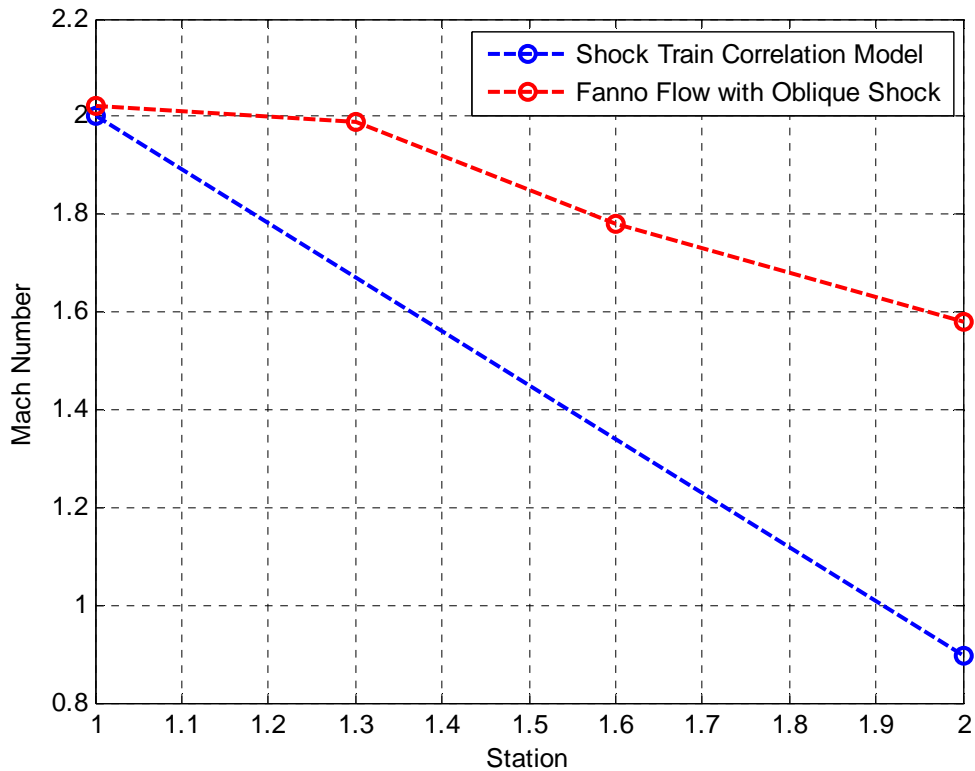


Figure 2-5: Static Pressure Profile for  $\Phi=0.171$  obtained by the University of Virginia (Rockwell 2010) and Static Pressure Predictions by VTMODEL



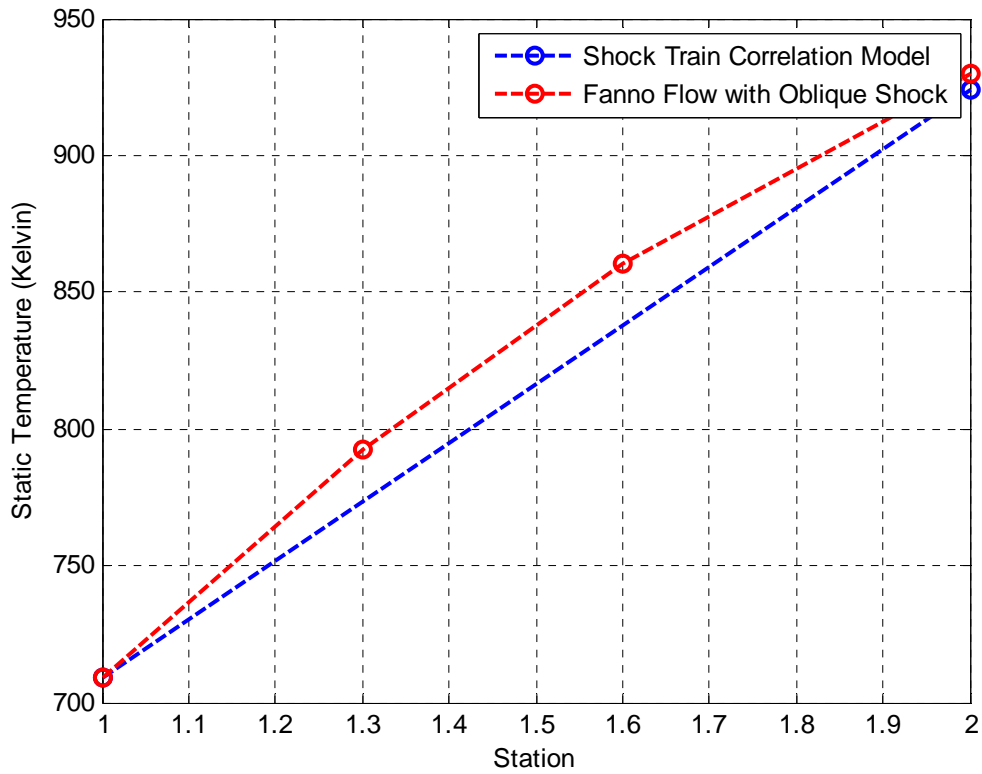


**Figure 2-6: Comparison of Shock Train Correlation Model and Fanno Flow with Oblique Shock Model - Mach Number**

As can be seen from Figure 2-6, the resulting isolator exit Mach numbers vary widely depending on what isolator program is used. For the same given pressure rise, the shock train correlation model gives a subsonic condition, while the Fanno flow and oblique shock correlation predict a supersonic Mach number. For an entrance Mach number of 2, a normal shock would give an exit Mach number of 0.5774 (John 1984). Therefore, despite the subsonic Mach number, the resulting value still shows pressure rise below the strength of a normal shock.

From both Figures 2-6 and 2-7 it can be seen that the predicted combustor entrance flow properties are highly dependent on the isolator modeling method that is

used. Both models are included in the final VTMODEL to allow the user to pick the isolator model that best models their scramjet/ramjet engine flowpath.



**Figure 2-7: Comparison of Shock Train Correlation Model and Fanno Flow with Oblique Shock Model - Static Temperature**

## 2.3 Combustor Modeling

Two options for calculating the combustion process are offered in VTMODEL. A unique method is developed based on a local combustion efficiency model, the complete combustion of the injected fuel in the combustor with the progress of the reaction controlled by the local combustion efficiency, and the use of compressible flow influence coefficients to calculate the local Mach number and other flow properties in the combustor. This method is computationally fast, provides local flow properties at any desired spacing, and can be applied to any fuel.

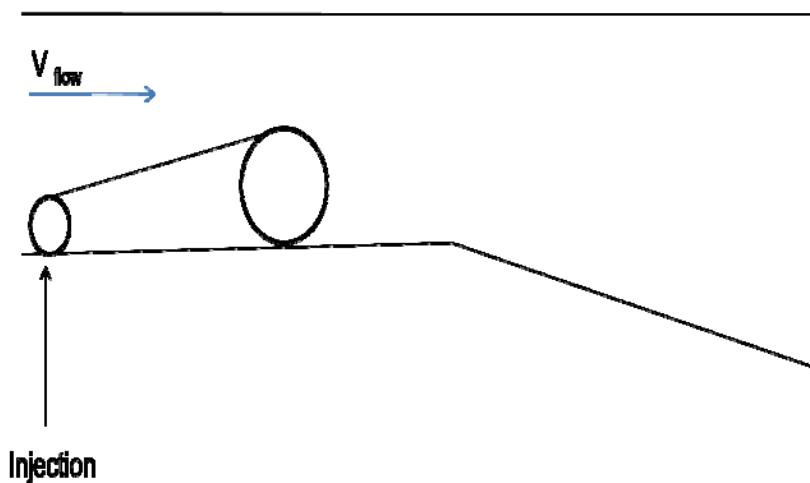
A second method based on the work of Jachimowski (Jachimowski 1998) is implemented. This finite rate combustion model incorporates a set of reduced-order hydrogen combustor chemistry equations, and is therefore limited to the use of hydrogen fuel in the combustor.

### 2.3.1. Combustion Modeling; Complete Combustion Method

The scramjet combustor is modeled using a complete combustion model, a flame speed model, and an influence coefficient compressible flow calculation. Here, complete combustion is defined as an immediate fuel and air reaction yielding only the completely oxidized combustion products and the remaining fuel or oxidizer, with no dissociation of combustion product species.

Using the model, the combustion temperature prediction is the result of successive combustion calculations in sections over the length of the combustor.

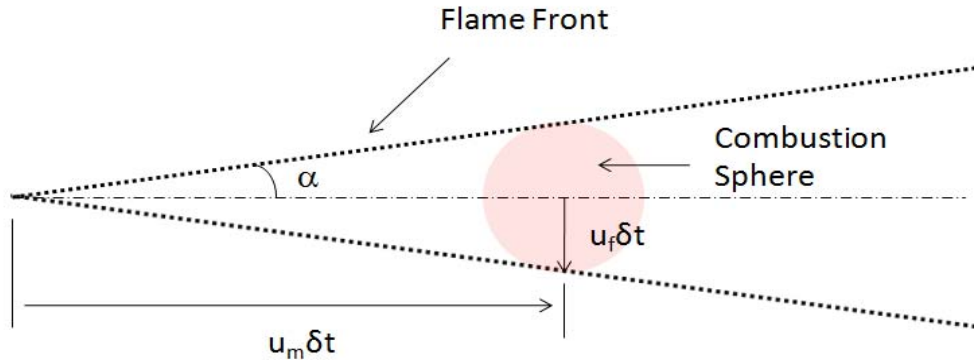
The flame speed model is used to determine the amount of fuel that is burned in each section, providing a combustion efficiency input. The flame speed model calculates the “fuel-air combustion sphere” produced by fuel injection into the flowpath., shown conceptually in Figure 2.8.



**Figure 2-8: Schematic of Growing Fuel-Air Combustion Product Sphere Model Concept**

The model is based on an approach presented in a text by Hill and Peterson (Hill 1975). The combustion flame speed is set to an appropriate level (between 40-80 m/s for hydrogen). Complete mixing of the fuel and air at the injection point is assumed. The growth of the projected area of the combustion sphere relative to the local cross sectional of the combustor is taken as the amount of fuel burned in a section, and therefore the local combustion efficiency. For the combustion calculation, the equivalence ratio along with the combustion sphere model determines the moles of fuel burned in the section.

Figure 2-9 below shows the geometry of the combustion sphere model.

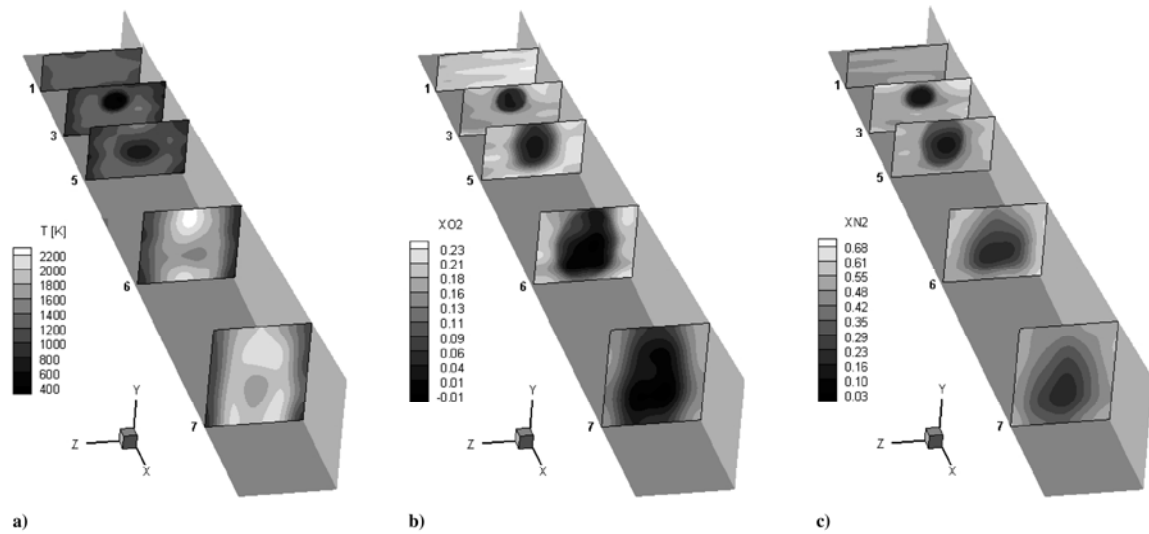


**Figure 2-9: Detailed Schematic of Combustion Sphere Model**

In the figure,  $\alpha$  designates the cone angle of the flame front,  $u_m$  designates the velocity of the combustible mixture, and  $u_f$  designates the flame speed. The two velocities define the cone angle:

$$\alpha = \sin^{-1} \left( \frac{u_f}{u_m} \right) \quad (\text{Eq 2-13})$$

Although the combustion sphere model clearly involves simplifying assumptions, the model is conceptually supported by experimental data obtained using the coherent anti-Stokes Raman scattering (CARS) method (O’Byrne 2007). This method was used to measure temperature and hydrogen combustion species in the NASA Langley Research Center’s Direct Connect Tunnel. The results of the experiment are in Figure 2-10 below.

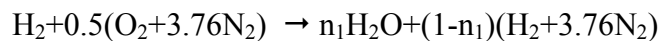


**Figure 2-10: CARS results for Mean Temperature, Oxygen and Nitrogen Mole Fraction Taken From O'Byrne 2007**

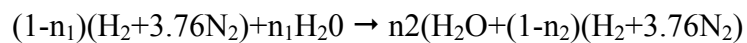
As can be seen from the above figure, the temperature graphs show a growing distribution of heated combustion products as the process proceeds through the combustor. His distribution is suggestive of the spherical model.

The complete combustion calculation assumes no dissociation and does not include any chemical kinetics. For the calculations, a variable specific heat  $c_p$ , and variable enthalpies are calculated based on the temperature of the flow. The overall combustion equation for a stoichiometric case with hydrogen fuel is given below:

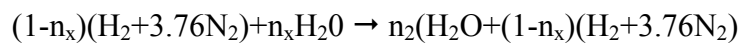
At the initial combustor segment,



At the subsequent combustor segment,



At the end of the combustor,



where  $0 < n_x < 1$ , controlled by the combustion efficiency derived from the combustion sphere model.

At a given combustor segment location, the amount of hydrogen burned and the proportion of air and water for the reactants is thus known. The temperature change of the combustion products in the segment may then be calculated as the adiabatic flame temperature for the segment reaction, using the First Law of Thermodynamics.

$$H_2 - H_1 = 0 \quad (\text{Eq 2-14})$$

expanding the above equation yields

$$\sum_i n_i \left[ (H_{T_2}^o - H_0^o) - (H_{T_0}^o - H_0^o) + (\Delta H_f^o)_{T_0} \right] = \sum_j n_j \left[ (H_{T_1}^o - H_0^o) - H_{T_0}^o - H_0^o + \Delta H_f^o \right] \quad (\text{Eq 2-15})$$

$$\text{where } (H_x - H_y) = c_p(T_x - T_y)$$

where  $T_1$  designates the temperature at the segment inlet, and  $T_2$  designates the segment outlet adiabatic flame temperature. The subscript 0 designates the base reference state of 298K.

Figures 2-11 to 2-14 show an example calculation for a combustor using complete combustion for hydrogen. In this case, an assumed pressure profile was entered into VTMODEL. This pressure profile is shown in Figure 2-11. The Fanno flow and oblique shock isolator model was used for the isolator to match the given combustor inlet pressure, which resulted in the combustor inlet temperature and Mach number. For the equilibrium chemistry calculation, a combustion efficiency of 70% was assumed. This efficiency resulted from a flame speed of 76 m/s.

With the combustor temperature and pressures, the local Mach numbers and other flow variables in the combustor were then calculated using the influence coefficient method to be described in Section 2.3.4. This procedure yielded a complete prediction for the flow states in the combustor as shown in Figures 2-12 to 2-14.

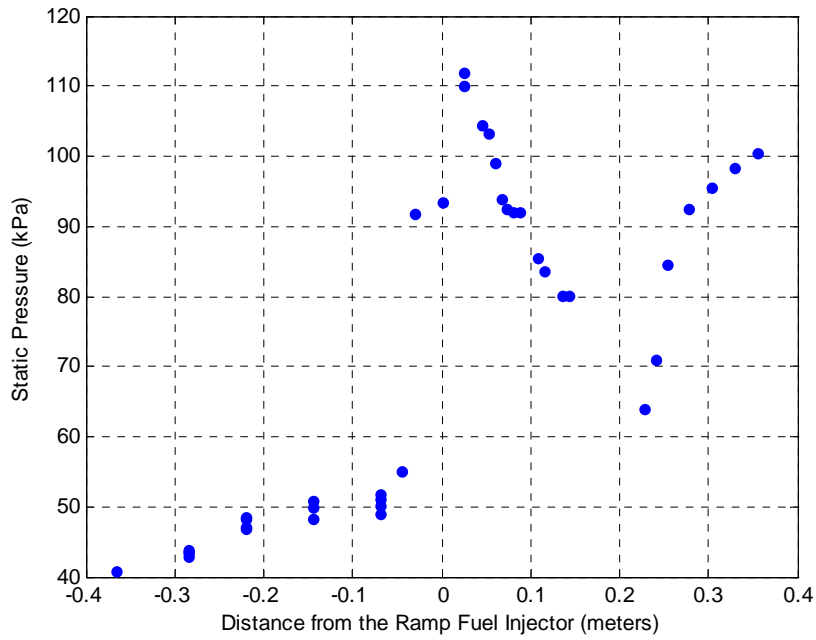
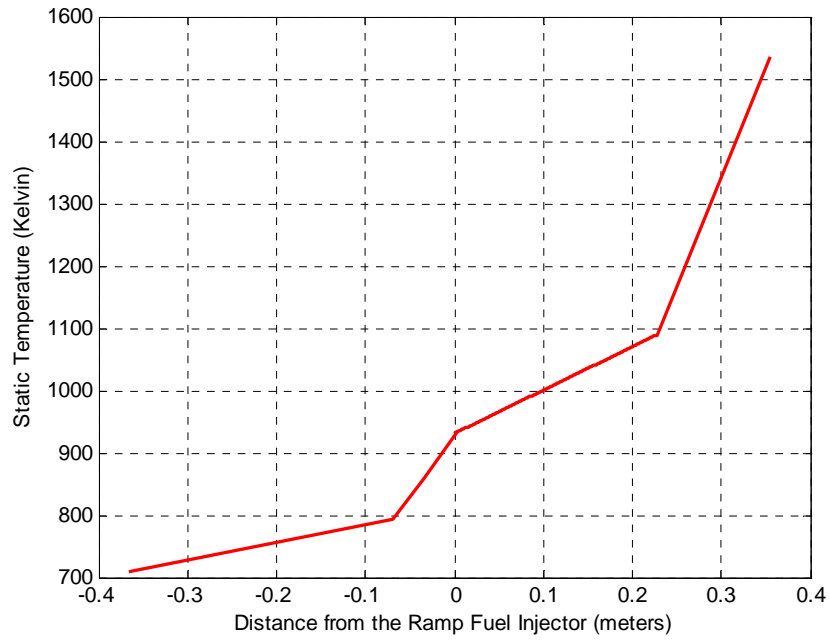
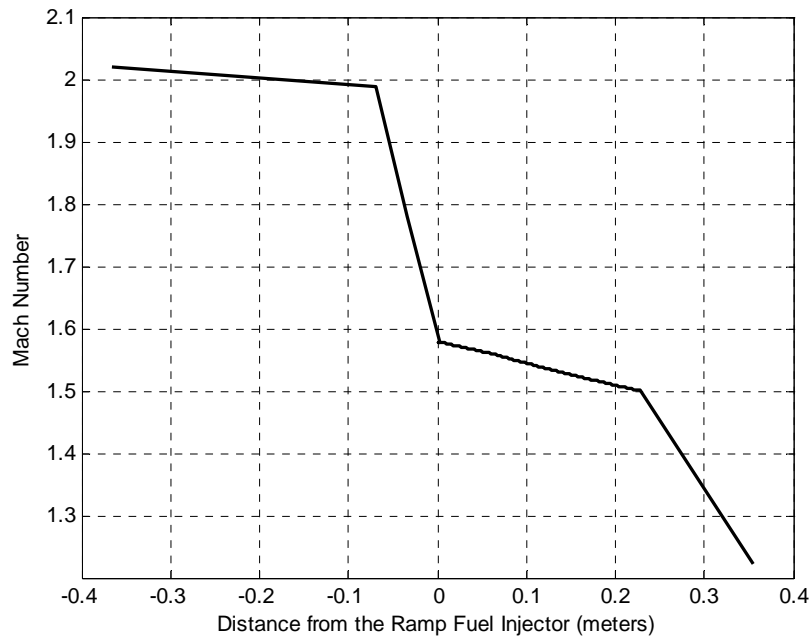


Figure 2-11: Static Pressure Profile Entered into VTMODEL Complete Combustion Model

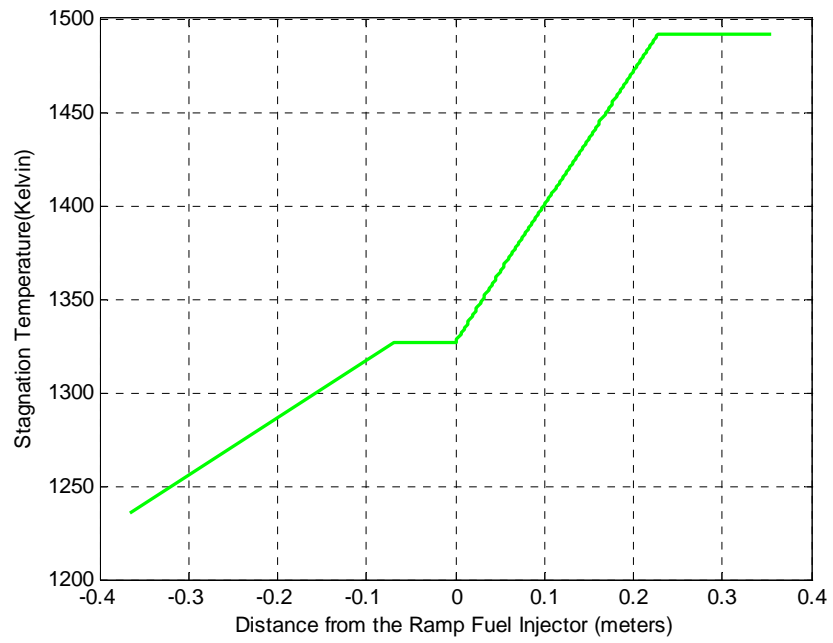




**Figure 2-12: Calculated Combustor Static Temperature with hydrogen fuel calculated by VTMODEL**



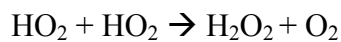
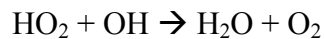
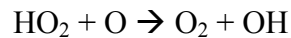
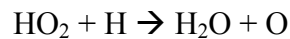
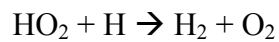
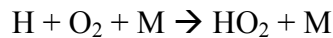
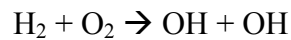
**Figure 2-13: Mach Number with hydrogen fuel predicted by VTMODEL**



**Figure 2-14: Stagnation Temperature Predicted by VTMODEL**

## 2.3.2 Combustion Modeling; Non Equilibrium Method

Due to the limitations of the complete combustion model, a finite rate hydrogen combustion module was implemented in VTMODEL using simplified chemical kinetics and non-equilibrium hydrogen chemistry. This model was based on the work done by Jachimowski on the analytical study of hydrogen combustion (Jachimowski 1998). In his NASA report, Jachimowski determined that the chemical kinetic efficiency, defined as the ratio of non-equilibrium to equilibrium thrust of a modeled scramjet, varied from 83-91%. Jachimowski had determined that the burning velocity of hydrogen in scramjet combustion is more sensitive to some reactions than others. The following 9 equations were used in for the kinetic modeling of hydrogen combustion:



In the above chemical equations M represents a non reactive species. The rate coefficients for each reaction were also obtained from the NASA report by Jachimowski.

The rate coefficient is defined as follows:

$$k = AT^n \exp(-E/RT) \quad (\text{Eq 2-16})$$

To solve the chemical kinetics, a steady state assumption was made (Atkins 2001). In this assumption, the concentration of intermediates in the chemical kinetic equations does not change with time. This assumption simplifies the mathematics and enables calculation without a full kinetic code.

In addition to chemical kinetics, a mixing efficiency was added to the combustion program. This mixing efficiency was developed by Anderson, et al (Jachimowski 1998). The efficiency was given in the form of

$$\eta_{\text{mix}}=1-\exp(-ax) \quad (\text{Eq 2-17})$$

where  $a$  is a constant dependent on Mach number and  $x$  is the distance from the fuel injection in centimeters.

To test the validity and the accuracy of VTMODEL's non-equilibrium chemistry model, a sample case was run for the sample combustors contained in the Jachimowski report. Results were reported for assumed flight Mach numbers of 8, 16, and 25. The Mach 8 condition was selected for the comparison analysis. In Figure 2-15, Jachimowski presents results for his reference along with various calculations with different rate constants. As shown in Figure 2-16, the combustion temperatures predicted by the VTMODEL implementation were within the spread of the values given in the NASA report in Figure 2-15. VTMODEL used the previous 9 kinetic equations and the recommended values for the rate coefficients. The hydrogen kinetic model was also within the spread for both of the other flight Mach numbers.

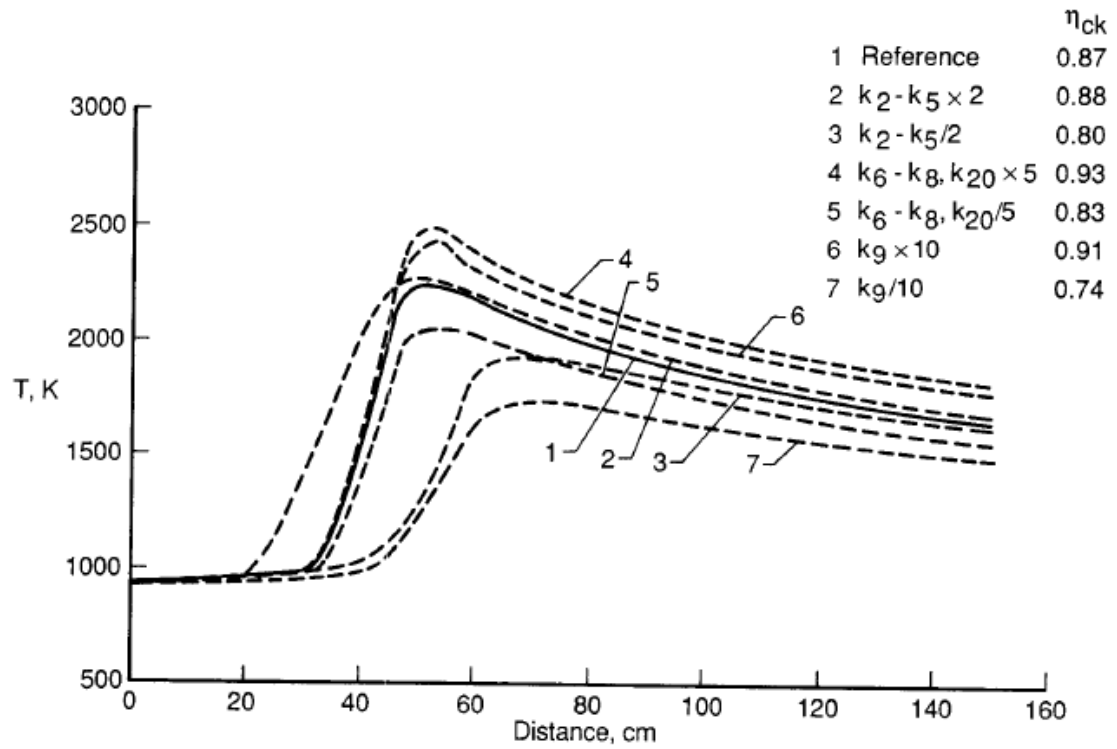


Figure 2-15: Chemical Kinetics Effects at Mach 8 for  $\Phi=1$  Taken from Jachimowski (1988)

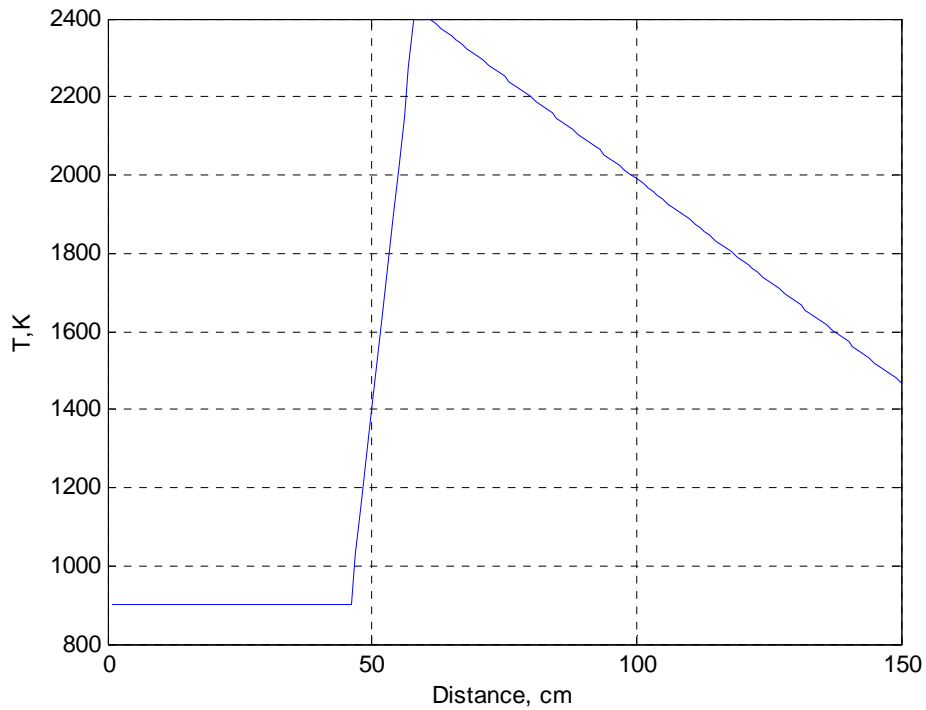


Figure 2-16: Chemical Kinetic Effects Predicted by VTMODEL

### 2.3.3. Comparison of Combustion Temperatures Produced by the Complete Combustion Chemistry Model vs. Non-Equilibrium Chemistry Model

As previously shown, the complete combustion (equilibrium) and non-equilibrium models of VTMODEL can both be used to calculate combustion temperatures. A comparison of the results of the two models for a given pressure profile input is shown in Figure 2-18. The combustor pressure profile was obtained from University of Virginia's direct connect tunnel (Rockwell 2010).

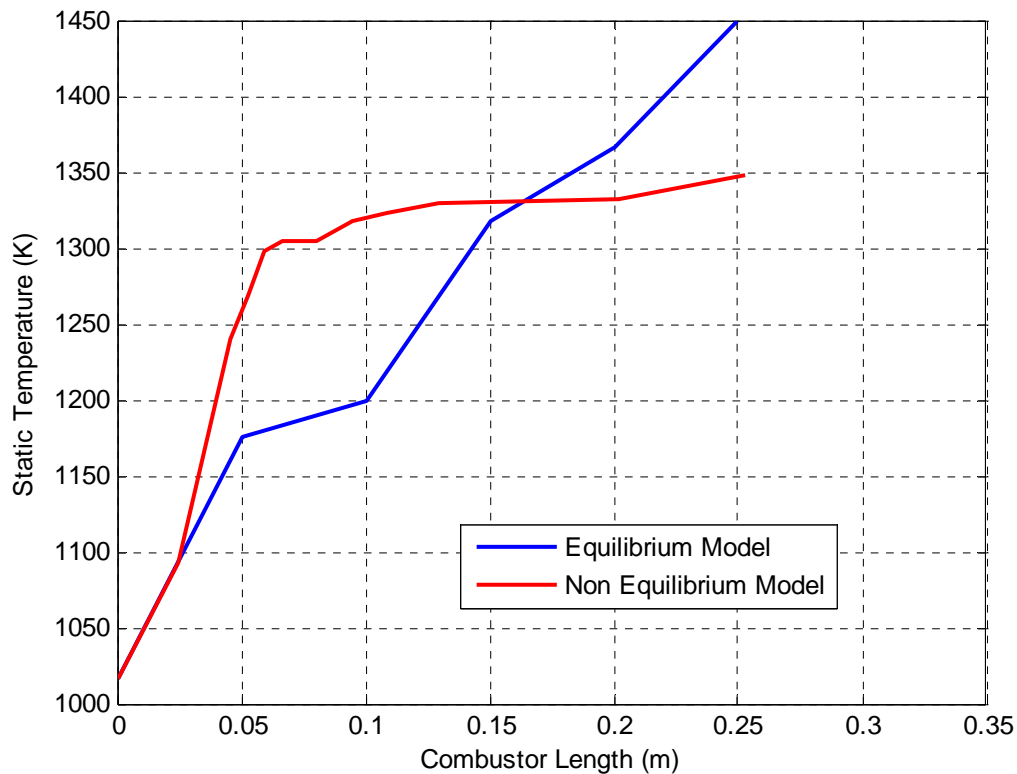


Figure 2-17: Comparison of Non Equilibrium and Equilibrium Chemistry Models

As can be seen in Figure 2-17, the two combustion models predict different static temperature distributions. The differences in slope can be attributed to the difference in the kinetic mixing parameters and the fuel-air combustion product sphere assumptions.

### 2.3.4. Combustor Flow Calculations

Along with the combustion model, an influence coefficient model is provided to determine the change in Mach number at each station. With the Mach number, other desired properties may be calculated such as stagnation pressure and stagnation temperature. The calculation of Mach number using influence coefficients was presented by Shapiro (Shapiro 1953). The equation below calculates the local Mach number based on the change of various parameters.

$$\frac{dM^2}{M^2} = -\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2} \frac{dA}{A} + \frac{1+\gamma M^2}{1-M^2} \frac{dQ-dW_x+dH}{c_p T} + \frac{\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2} \left[ 4f \frac{dx}{D} + \frac{dX}{1/2\gamma P A M^2} - 2\gamma \frac{dw}{w} \right] + \frac{2(1+\gamma M^2)\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2} \frac{dw}{w} - \frac{1+\gamma M^2}{1-M^2} \frac{dW}{W} - \frac{d\gamma}{\gamma} \quad (\text{Eq 2-18})$$

This equation was initially solved with a first order explicit Euler solver. However, due to stability issues, a 4<sup>th</sup> order Runge-Kutta (RK) solver was written for the equation. The RK solver for the Shapiro equations is a separate function within VTMODEL. The solver produces an error on the magnitude of  $h^4$  with an error per step of  $h^5$ . The variable  $h$  is defined as the step size. A basic example of the RK solver is given below (Chapra 2004). For an initial problem of

$$M^{2'} = f(x, M^2), y(x_o), M_o^2 \quad (\text{Eq 2-19})$$

The RK method of solving this problem is

$$M_{n+1}^2 = M_n^2 + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \quad (\text{Eq 2-20})$$

$$x_{n+1} = x_n + h \quad (\text{Eq 2-21})$$

Where  $k_1, k_2, k_3, k_4$  is given below

$$k_1 = f(x_n, M_n^2) \quad (\text{Eq 2-22})$$



$$k_2 = f\left(x_n + \frac{1}{2}h, M_n^2 + \frac{1}{2}hk_1\right) \quad (\text{Eq 2-23})$$

$$k_3 = f\left(x_n + \frac{1}{2}h, M_n^2 + \frac{1}{2}hk_2\right) \quad (\text{Eq 2-24})$$

$$k_4 = f(x_n + h, M_n^2 + hk_3) \quad (\text{Eq 2-25})$$

The default step size  $h$  was determined using an informal initial sensitivity analysis. The step size of  $h=0.001$  was selected as the initial step size for any combustor below 1 meter in length. An initial step size of 0.01 was selected for any combustor above 1 meter in length. The flow equations are solved with decreasing step size until there is no significant change in the results between the step sizes. The solver automatically decreases the step size to obtain a result independent of  $h$ .

From Equation 2-18, there are multiple parameters that effect the calculation of the local Mach number. The program defaults some of these values to zero. These values are the change in body force and change in work. For friction coefficient, a value of 0.0015 is set as the default. The value can be changed by the user.

Basic heat transfer is included in the program as an option to calculate the cooling load and the heat transfer in the combustor. The calculation uses Reynolds Analogy to calculate the heat transfer. The skin friction  $C_f$  can be calculated by Equation 2-26 or if known can be entered in as a parameter for Equation 2-27.

$$C_f = \frac{\tau_w}{1/2\rho U^2} \quad (\text{Eq 2-26})$$

$$St = \frac{C_f}{2} \quad (\text{Eq 2-27})$$

From this relationship, the Stanton number can be used to calculate  $h$  using Equation 2-28.

$$St = \frac{h}{\rho U c_p} \quad (\text{Eq 2-28})$$

The adiabatic wall temperature can be calculated from the following equations

$$T_{aw} = T + r \frac{U^2}{2c_p} \quad (\text{Eq 2-29})$$

where

$$r = \sqrt[3]{Pr} \quad (\text{Eq 2-30})$$

therefore,

$$T_{aw} = T + r(T_o - T) \quad (\text{Eq 2-31})$$

To calculate the heat transfer, the user is required to input a desired or predicted wall temperature ( $T_w$ ). Equation 2-32 and 2-33 are used to calculate the heat transfer.

$$q_w = h(T_w - T_{aw}) \quad (\text{Eq 2-32})$$

$$Q_w = q_w A \quad (\text{Eq 2-33})$$

In addition to Reynolds Analogy, the heat transfer per segment can also be entered into the program as a parameter.

From the combustor flow calculations for the Mach number, the static pressure, stagnation temperature, and stagnation pressure can be calculated. Since the static temperature is the input from the combustion modules, the Mach number can be used to calculate the stagnation temperature. If the static pressure is an input parameter, the static pressure is accepted as the correct value, and other flow property calculations are made.

### 2.3.5. Use of Other Combustion Models and Fuels

In addition to the two implemented combustion models, VTMODEL is written so that other combustion models can be integrated into the program. One of the expansions that are being planned is a hydrocarbon combustion module for predictions of flow properties for a hydrocarbon scramjet combustion test rig being developed at Virginia Tech. Providing the ability to use different kinetic and equilibrium chemistries models was one of the major goals in developing VTMODEL.

## 2.4 Nozzle Modeling

The nozzle module is an optional component of VTMODEL. For analyses with data from a direct connect tunnel like the University of Virginia Facility, the nozzle module is bypassed and not used. However, the module can be added for flight experiments, or full engine wind tunnel experiments. With the nozzle module, the scramjet performance is calculated using a defined nozzle adiabatic efficiency.

$$\eta_n = \frac{h_{03} - h_4}{h_{03} - h_{4s}} \quad (\text{Eq 2-34})$$

Using the efficiency, the exit Mach number and other performance variables are calculated from following equations. Note that the nozzle exit pressure is a required input for the calculation.

$$M_4^2 = \frac{2}{\gamma - 1} \left\{ \frac{\eta_n \left[ 1 - (P_4/P_{03})^{(\gamma-1)/\gamma} \right]}{1 - \eta_n \left[ 1 - (P_4/P_{03})^{(\gamma-1)/\gamma} \right]} \right\} \quad (\text{Eq 2-35})$$

$$V_e = M_4 \sqrt{\gamma R T_4} \quad (\text{Eq 2-36})$$

$$\text{Thrust} = \dot{m} V_e + (P_4 - P_a) A_4 \quad (\text{Eq 2-37})$$

$$\text{TSFC} = \frac{\dot{m}_f}{\text{Thrust}} \quad (\text{Eq 2-38})$$

$$\text{ISP} = \frac{\text{Thrust}}{\dot{m}_f} \quad (\text{Eq 2-39})$$

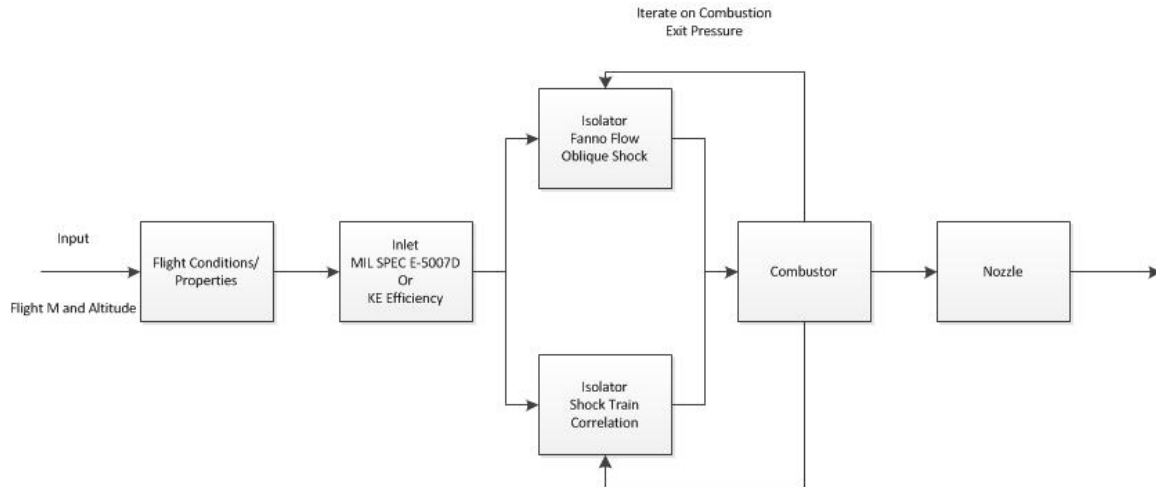
# *Chapter 3*

## **Summary of Model Features and Operations**

VTMODEL is written in MATLAB® as a series of functions. The user inputs required for the full flow path program are the geometry of the flow path, flight Mach number and altitude, combustion efficiency, equivalence ratio, combustion entrance pressure, and combustion or nozzle exit pressure. The user also has the ability to enter in a static pressure distribution for the flow path in the event of a combustion tunnel test validation.

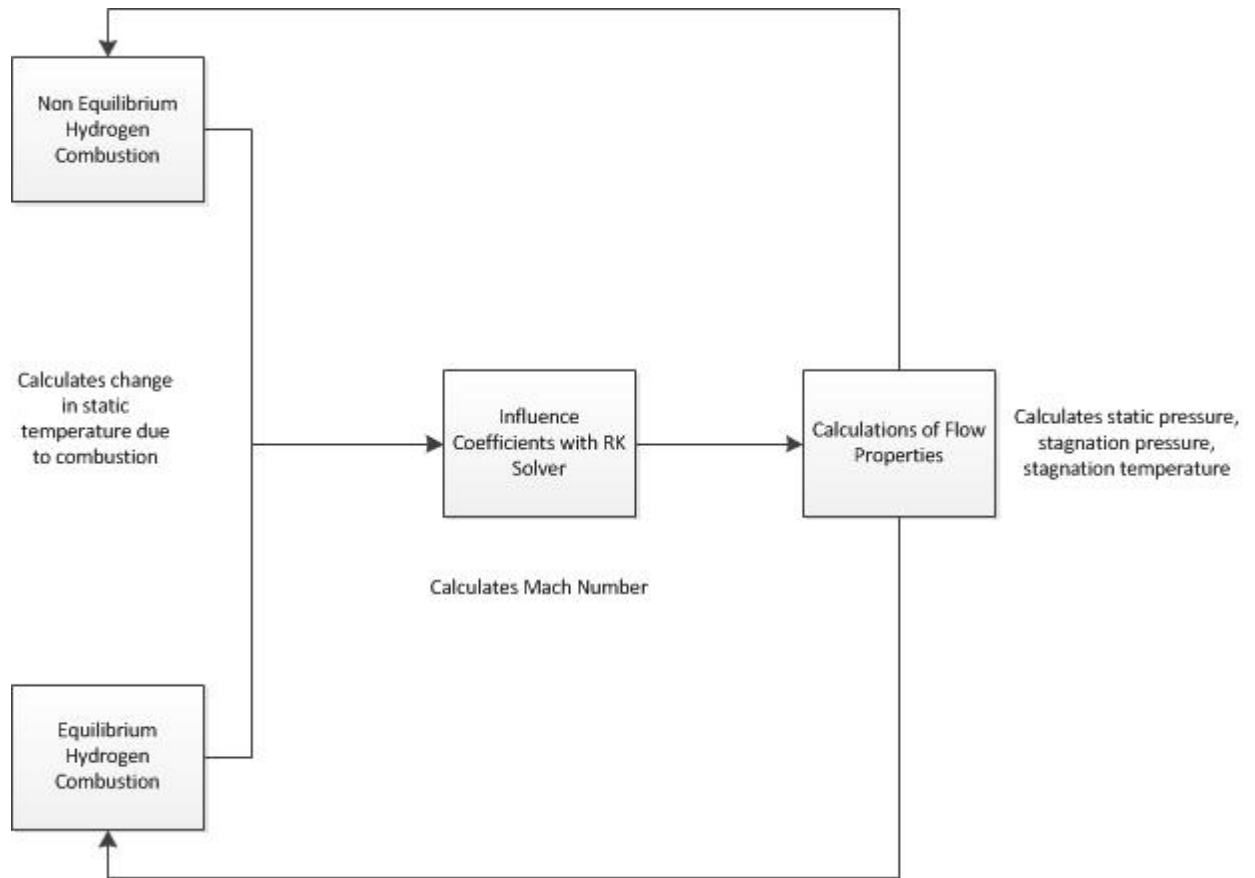
For design or predictive usage, one of the following components must be determined either by estimation or by previous experiments. These components are the combustion entrance pressure, combustion exit pressure or nozzle exit pressure. Once the pressure at one of these points is set as an anchor point, VTMODEL will iterate upon a solution to match this point. When the combustion entrance pressure is set, the combustor and nozzle portions will only be run once, while the isolator shock length or shock angles will be adjusted to match this pressure. When either the combustor exit pressure or the nozzle exit pressure is selected as the anchor point, the isolator shock length or shock angles will be iterated to obtain a matching combustion pressure entrance state.

The full program flow path is given below.



**Figure 3-1: Overall Flow Chart for VTMODEL**

As shown on the flow path, the isolator module is modified in each loop to match a combustion exit pressure or another pressure anchor point. The detail of the combustion program is shown in Figure 3-2. This figure shows the process for each calculation interval in the program. The process is repeated until the end of the combustor. This calculation interval is decreased in each iteration until the changes in predicted temperatures are less than 1%. This assures that the result is not dependent on the step size of the solver.



**Figure 3-2: Flow Chart of Combustion Modeling for VTMODEL**

One of the benefits of VTMODEL is the ability to separate an analysis into components. A beneficial use of this separation would be the analyses of direct connect conditions. Since the isolator entrance condition is generally known, the flow conditions including the local pressure can be entered directly into the model. The nozzle can also be included or ignored for the calculation. For an analytical model of experimental results, VTMODEL is flexible enough to use user input static pressures at any location.

Results of VTMODEL analyses can be saved individually for each run. Since all of the analysis and predictive tools are separate functions, the main program can be modified for each run, and geometries entries need not be repeated. Each of the functions

in VTMODEL has a description of the variables necessary for the function to run and output the results data correctly. The general format for calling up a function is [variables output separated by commas]=function name (inputted variables separated by commas). The individual function can be called up independent of the rest of the program. For instance, to examine just the flow in an isolator, only the isolator function has to be called up.

VTMODEL can be segmented to allow the user to use only the functions that are necessary to solve their problem. The way VTMODEL is written allows the user to calculate and account for shock trains in the isolator that can be caused by the combustion system. The user can also see thermodynamic data at every axial position within the combustor. The code is written so that only the inlet, shock train isolator, and nozzle are calculated by the entrance and outlet conditions. The Fanno flow/oblique shock isolator model has intermediate points where the Fanno flow ends and the area incorporating the reflected shocks begins. The combustor model is written so that temperature, stagnation temperature, stagnation pressure, and pressure are calculated at every segment location across the combustor. Since only the exit pressure for either the combustor or the nozzle has to be defined downstream, VTMODEL allows modeling of systems where a full set of data may be not available.



## *Chapter 4*

# **Parameterization of factors with a generic scramjet geometry with VTMODEL**

To demonstrate the use of the predictive capabilities of VTMODEL in designing a scramjet flow path, an example generic model was created. The flow path was specified with a generic length inlet function with the MIL SPEC inlet pressure recoveries for the flight Mach numbers. The following parameters were used in the program

- Flight Mach numbers were varied between 5 and 10
- Altitudes varied between 50,000 to 80,000 ft
- Combustor equivalence ratio was  $0.1 < \Phi < 1$
- Isolator dimensions: width=0.08 m and length=0.8 m
- Isolator model: McLafferty rectangular shock train correlation
- Combustor length is 1.50 meters
- Constant cross sectional area of section of the combustor was 0.50 square meters
- Combustor diverged at an angle of 3% for the remainder of the combustor length (see Figure 4-2)
- Nozzle parameters were set to have an efficiency of 95% and an area ratio of 1.5. This area ratio was taken from Figure 4-1 for the approximate average value between freestream Mach numbers of 5 -10.

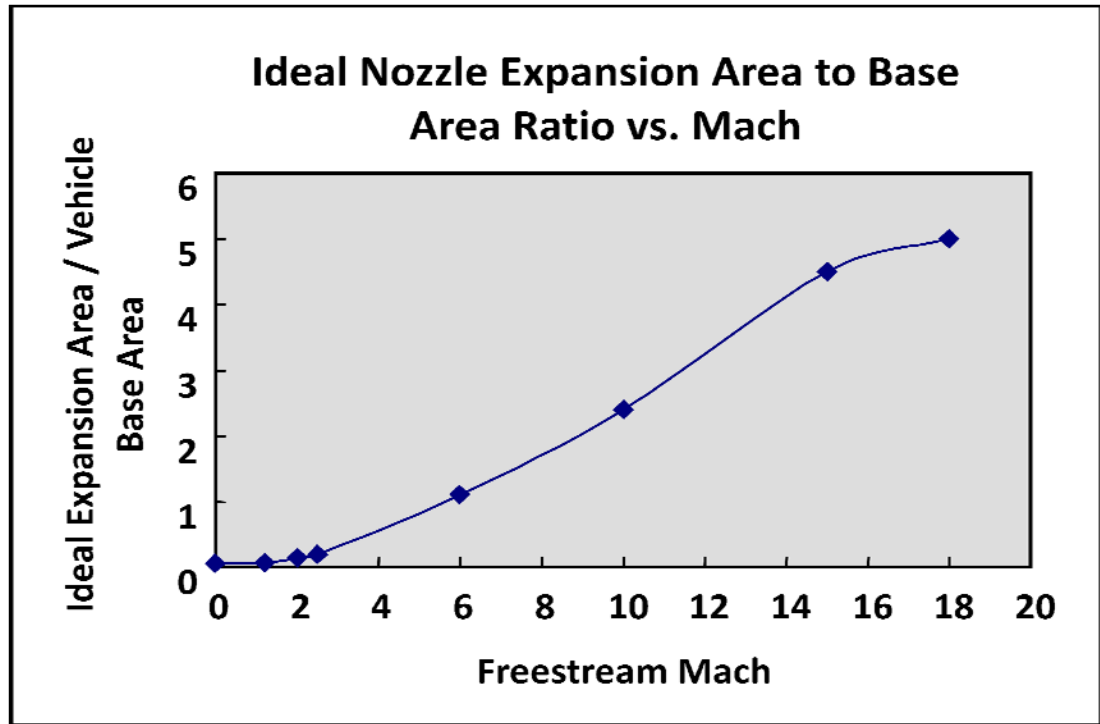
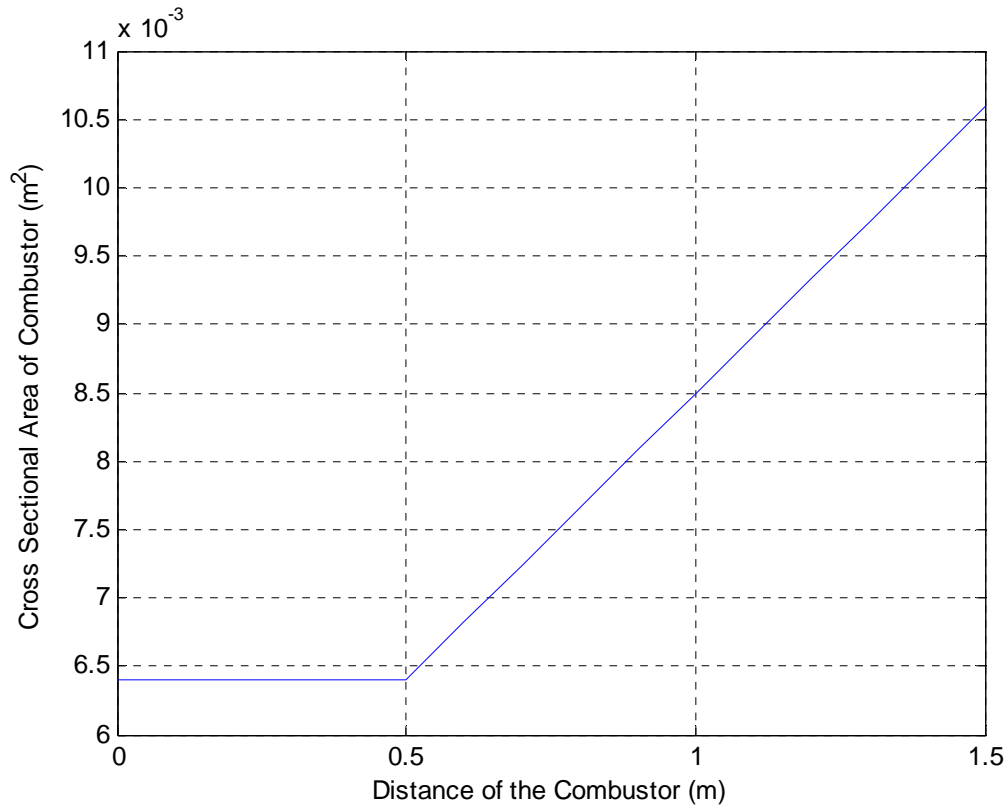


Figure 4-1: General Trend of Ideal Nozzle Expansion (Bowcutt 2007)

The model was set up to use the predictive nature of VTMODEL to determine the effect of altitude, Mach number, and equivalence ratio on the specific impulse of the scramjet (ISP). Default parameters of coefficient of friction equal 0.0012, and hydrogen fuel were entered. In this analysis, no heat transfer through the walls was used. This analysis was performed prior to the addition of the Reynolds Analogy heat transfer calculation to VTMODEL. The combustor cross sectional area is shown in Figure 4-2. The figure is plotted from the geometry data entered into MATLAB.



**Figure 4-2: Geometry of the Generic Combustor**

One of the primary goals of VTMODEL was to provide modeling software that had the ability to be predictive in nature. Using the geometry above and the input parameters listed, a solution was obtained. The nozzle exit pressure was set at atmospheric pressure for the flight altitude (an ideal nozzle assumption). With the combustion calculations, the program iterated the pressure rise due to the isolator shock system to obtain this exit pressure. An example result from the calculation is shown in Table 4.1 below.

Station	P(kPa)	T (K)	M
a	5.69	216	7
0	5.69	216	7
1	23.7	325	5.56
2	103.6	319	5.62
3	49.8	2015	3.84
4	5.69	1392	5.63

Table 4-1: VTMODEL Results for  $\Phi=1$ , Mach 7, Altitude=65,000 ft

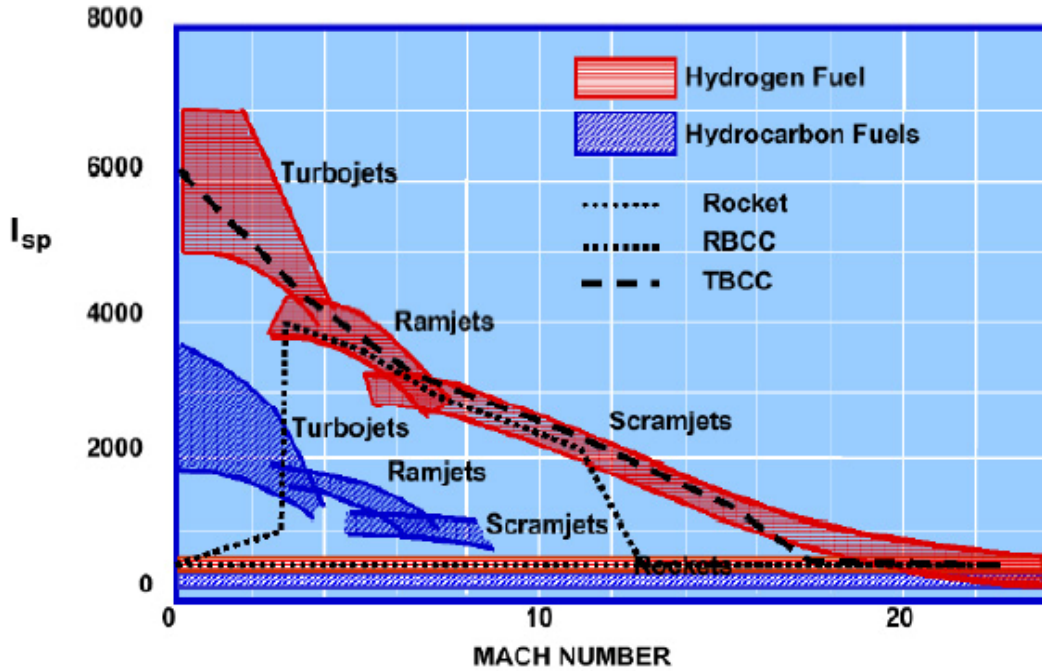
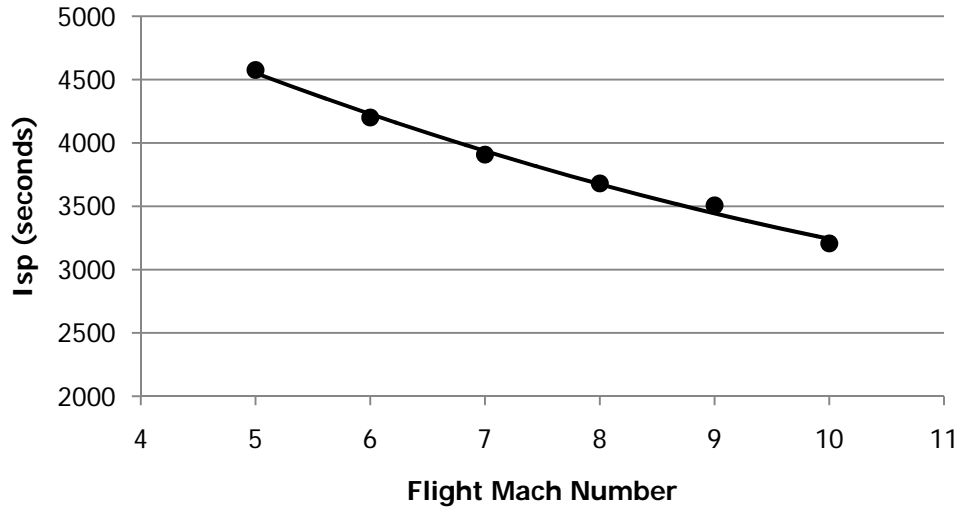


Figure 4-3: Theoretical ISP for Various Systems (Moses 2003)

Moses (2003) obtained the performance results shown in Figure 4-3. As shown, the predicted theoretical  $I_{SP}$  for a scramjet between flight Mach numbers of 5 and 10 is between 2000-3500 s. The  $I_{SP}$  decreases as Mach number increases. A parameterization on flight Mach numbers was performed using the above generic VTMODEL, and  $I_{SP}$  was

calculated and reported. The altitude was held constant at 65,000 feet, and the equivalence ratio was held at 1. Figure 4-4 shows the results of the VTMODEL analysis for the “generic” scramjet combustor.

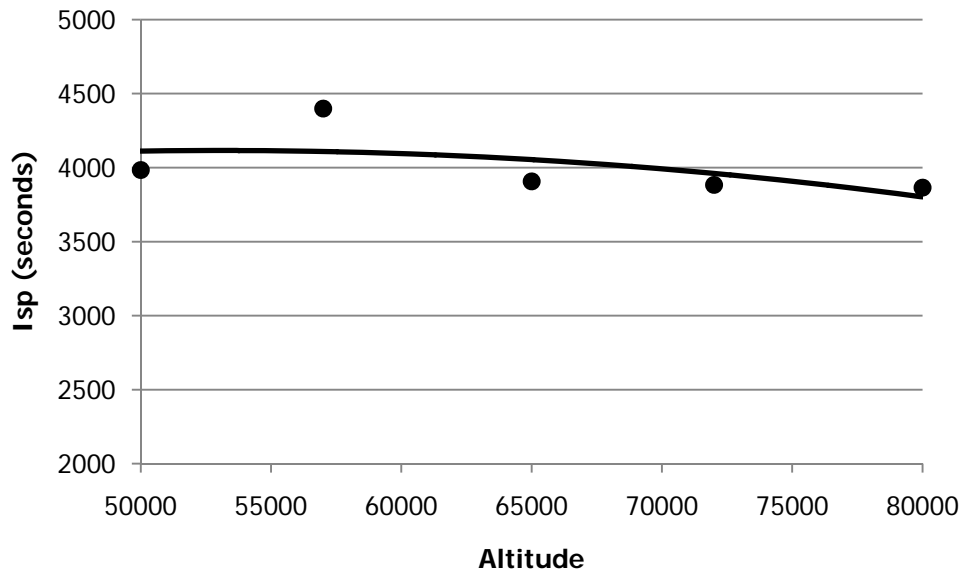


**Figure 4-4: ISP vs Flight Mach Number as Predicted by VTMODEL**

As shown in the above figure, the VTMODEL overpredicted the values of  $I_{SP}$  as compared to Moses. The range of Mach Numbers covers both ramjet and scramjet operations. Moses predicted values between approximately 4300-3500 sec at Mach 5. For Mach 10, the  $I_{SP}$  values ranged between 2700-2100 sec. VTMODEL predicted an  $I_{SP}$  approximately 200-500 sec higher than Moses. The differences might be due to the difference between the parameters and geometry of the “generic scramjet” and the Moses model. The higher  $I_{SP}$  can also be attributed to the lack of a heat transfer model in the analysis. The decreasing trend of  $I_{SP}$  as flight Mach number increases was correctly predicted by VTMODEL.

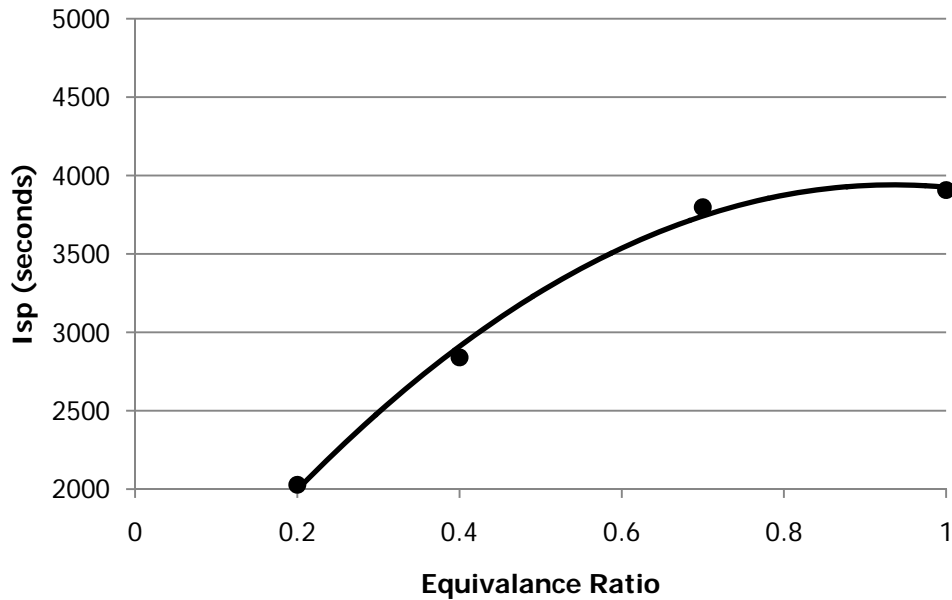
An analysis of the effect of equivalence ratio and of flight altitude was also performed using VTMODEL. The flight altitude was varied between 50,000 ft and 80,000 ft. The flight Mach number was held constant at 7 and the equivalence ratio was

held at 1. Results are shown in Figures 4-5 and 4-6. For the flight altitude variation, there is only 200 sec difference in the value of the  $I_{SP}$ .



**Figure 4-5:  $I_{SP}$  versus Flight Altitude predicted by VTMODEL**

A parameterization to examine the effect of equivalence ratio was performed. The result of this analysis is seen in Figure 4-6. As a general trend, the  $I_{SP}$  increases with increasing equivalence ratio.



**Figure 4-6: ISP versus Equivalence Ratio as predicted by VTMODEL**

From the above analysis, VTMODEL is shown to be a predictive tool for engine cycle analysis of a ramjet or a scramjet engine. In the simulations presented, the internal flow data produced by VTMODEL showed that at Mach 5, with an equivalence ratio of 1, the engine was functioning as a ramjet, with subsonic combustion. At Mach 10, the combustion was supersonic. Even though only the Isp value is graphed and compared to Moses for the generic scramjet combustor, a full thermodynamic data set was calculated by VTMODEL for each case.

# *Chapter 5*

## **Comparison of Results with Other Models and Experimental Data**

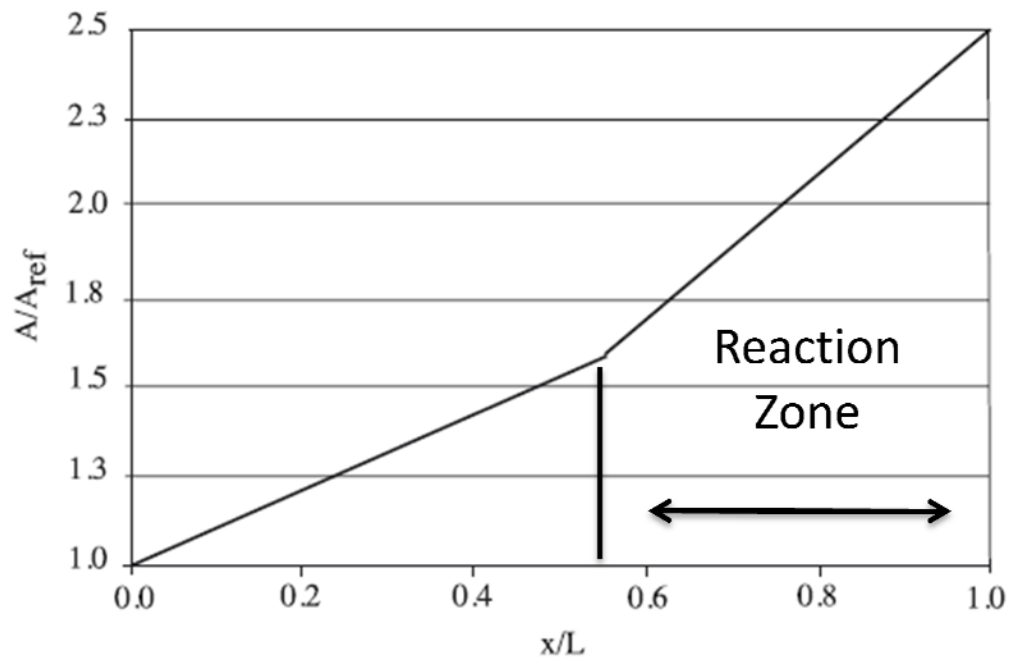
### **5.1 Comparison with Results of Other Models**

To further demonstrate the validity of VTMODEL as a viable option for scramjet and ramjet analysis, VTMODEL was run with various geometries to compare to other published models. The comparisons were based on the primary sources and published materials. The two main comparisons for predictive modeling were taken from the doctorate dissertations of Bradford (Bradford 2001) and Bonanos (Bonanos 2005). The experimental data was taken from the University of Virginia wind tunnel (Rockwell 2010).

#### **5.1.1. SCCREAM**

SCCREAM was developed at Georgia Tech in Atlanta, Georgia as part of a Rocket Combined Cycle Analysis program (Bradford 2001). SCCREAM is the scramjet analysis section of the program. SCCREAM uses an equilibrium solver for hydrogen combustion. The following analysis was taken from the doctoral dissertation by Bradford as part of his comparison of SCCREAM to RJPA. The combustor geometry that he used is given below in Figure 5-1. Note that the geometry is different from the VTMODEL standard generic geometry that was given in Chapter 4.





**Figure 5-1: SCCREAM Combustor Geometry (Bradford 2001)**

As can be seen from the Figure 5-1, the combustor is constantly diverging with a change in slope at  $x/L = 0.55$ . The portion of the combustor for  $0 < x/L < 0.55$  is taken as the isolator in the system. The portion for  $0.55 < x/L < 1.0$  is assumed to be the region of the combustion reaction. Since this geometry was different from the default VTMODEL geometry, to compare results the VTMODEL combustor and isolator geometry were modified to provide an area change. The area change in the beginning of the combustor (prior to the reaction region) was modeled using the RK solver to solve for the change in Mach number due to flow with only friction and change in area (no combustion). This region was then followed by the reflected oblique shock function at the end of the isolator. The “reaction region” was modeled to be the combustor in VTMODEL. In

addition to the area profiles, the length was assumed to be 1.5 meters and the nominal area was assumed to be 0.581 meters. Table 5-1 lists the other parameters entered into VTMODEL to compare results with SCCREAM.

<b>Property</b>	<b>Value</b>
Flight Mach Number	8
Friction Coefficient	0.0018
Fuel Injection	6000 ft/s
Equivalence Ratio	1

**Table 5-1: SCCREAM Combustor Entrance Properties for VTMODEL (Bradford 2001)**

From VTMODEL, the Mach number was calculated in the “isolator” and combustion region. The values were then graphed over a similar axis to Bradford for direct comparison. Figure 5-2 and 5-3 show this comparison. Figure 5-2 is Bradford’s SCCREAM comparison to SRGULL (Zweber 2002). With the above given parameters. Figure 5-3 is VTMODEL’s prediction assuming hydrogen combustion.

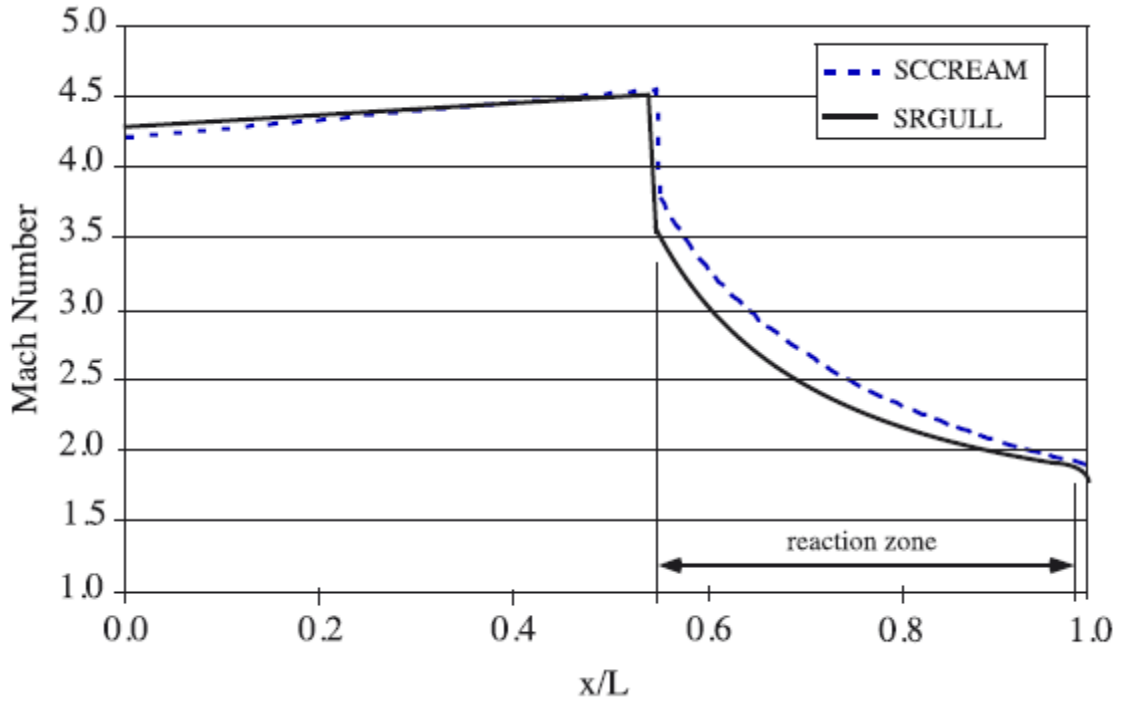


Figure 5-2: Bradford Comparison of SCCREAM and SRGULL (Bradford 2001)

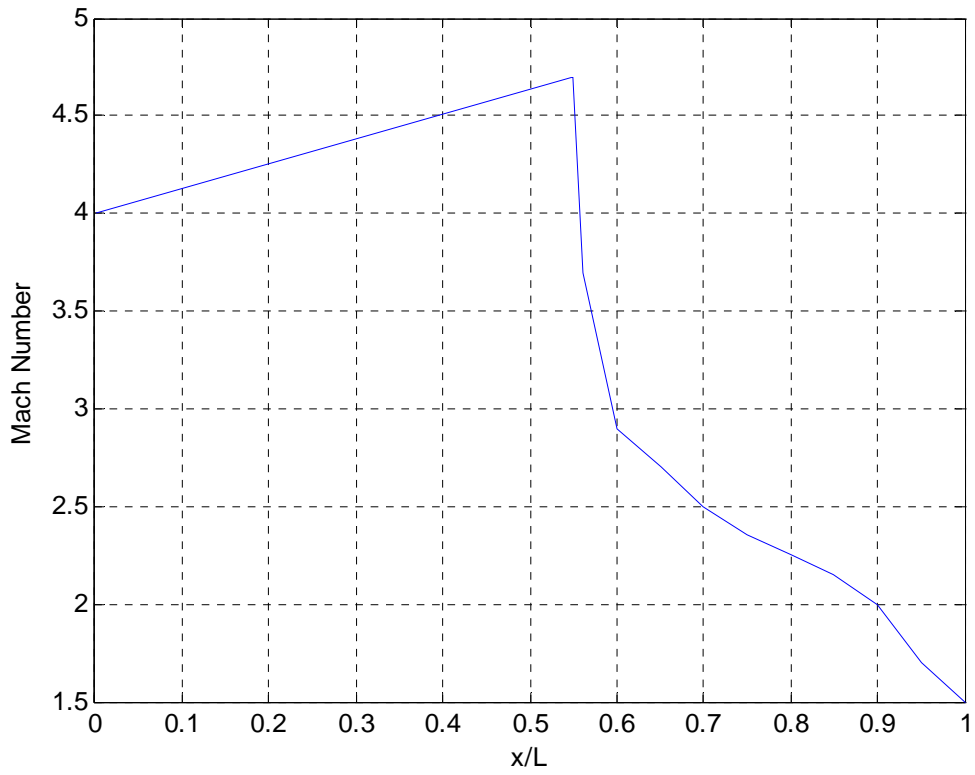


Figure 5-3: VTMODEL Prediction of Bradford Combustor - Mach Number

VTMODEL predicted the Mach number decrease in the combustor to be of the same order of magnitude as Bradford's calculations. The other thermodynamic values such as combustion temperature were not included in his dissertation, so the VTMODEL results are not included for comparison.

### 5.1.2. RJPA Comparison

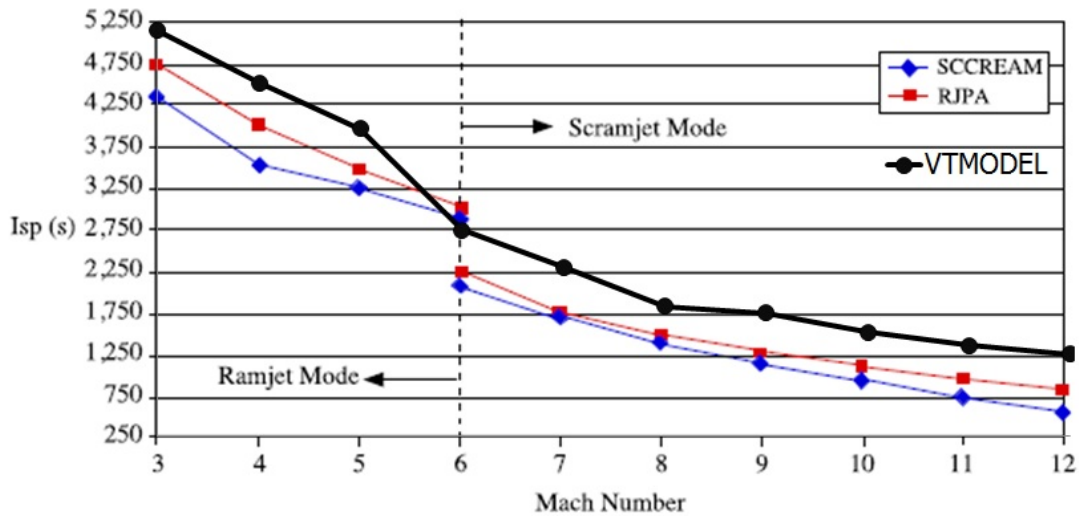
The benchmark program used for scramjet and ramjet performance analysis is RJPA. RJPA was developed by John Hopkins University's Applied Physics Lab. The program is a FORTRAN program that is a control volume cycle analysis. Since the results of RJPA are considered ITAR restricted, available published results from two dissertations were used for comparison with VTMODEL.

The first comparison with RJPA results is included in Figure 5-4. This comparison was performed by Bradford to compare his results from SCCREAM to RJPA. He varied the flight Mach number from 3 to 12. For this analysis he included the additional performance parameters shown in Table 5-2 below.

<b>Property</b>	<b>Value</b>
Efficiency	98.50%
Nozzle Exit Area	65 ft <sup>2</sup>
Diffuser Exit Area	10 ft <sup>2</sup>
Combustor Exit Area	16.8 ft <sup>2</sup>

**Table 5-2: SCCREAM Cycle Analysis Properties (Bradford 2001)**

Using the above properties and the Bradford geometry, VTMODEL was used to calculate the  $I_{SP}$  for a set of flight Mach Numbers from 3 to 12. The results of this analysis are plotted with the RJPA and Bradford results in Figure 5-4.



**Figure 5-4: I<sub>sp</sub> vs Mach number as Predicted by SSCREAM, RJPA (Bradford 2001), and VTMODEL**

Comparing the results, it can be seen that as for the comparisons with the results of Moses in Chapter 4, VTMODEL also consistently overpredicts I<sub>sp</sub> with respect to SSCREAM and RJPA. There are many possible reasons for this overprediction. In his analysis, Bradford stated that he adjusted the equivalence ratio to suit the fuel flow rate from RJPA. The VTMODEL analysis assumed an equivalence ratio of 1. In addition, since heat transfer through the walls was neglected, the calculated I<sub>sp</sub> will be higher. VTMODEL also does not allow the user to specify ramjet or scramjet mode. The model automatically calculates the combustor entrance parameter based on the other parameters used. Therefore, at Mach 6, there is only one I<sub>sp</sub> value calculated for VTMODEL.

Given the above results, another comparison of VTMODEL to RJPA was desired. Bonanos used RJPA to perform cycle analysis and combustion efficiency analysis on experimental data obtained from the University of Virginia wind tunnel. He compared

his results with an “aeroramp” injector and flame holder to those for a physical ramp. Since Bonanos used air specific impulse as a performance factor, the nozzle function of VTMODEL was modified to include this additional calculation. Air specific impulse is calculated by the following equation:

$$AirSpecificImpulse = \frac{Thrust}{\dot{m}_a + \dot{m}_f} \quad (Eq\ 5-1)$$

In addition, the geometry and entrance conditions for VTMODEL were modified to allow for comparison. The combustor parameters used for the VTMODEL comparison with Bonanos’ RJPA results are given below in Table 5-3.

Properties	Values
Combustor Length	0.23 m
Combustor Entrance Area	9.6e <sup>-4</sup> m <sup>2</sup>
Combustor Exit Area	1.2e <sup>-3</sup> m <sup>2</sup>
Friction Coefficient	0.0023
Nozzle Efficiency	98.50%

**Table 5-3: RJPA and VTMODEL Cycle Analysis Properties (Bonanos 2005)**

In addition to the cycle analysis properties presented in Table 5-3, Bonanos used Reynolds Analogy to calculate heat transfer through the walls for each one of his experimental cases. The results of this analysis were included in the VTMODEL I<sub>SP</sub> calculations. Bonanos desired a comparison between performance with ethylene and hydrogen fuels, so he normalized his results based on the following expression:

$$\Phi * (f/a)_{stoich} * heating\ value$$

The results of analysis are shown in Figure 5-5. He shows the variation of Air Specific Impulse for the ramp and aeroramp geometry tested in the UVA wind tunnel.

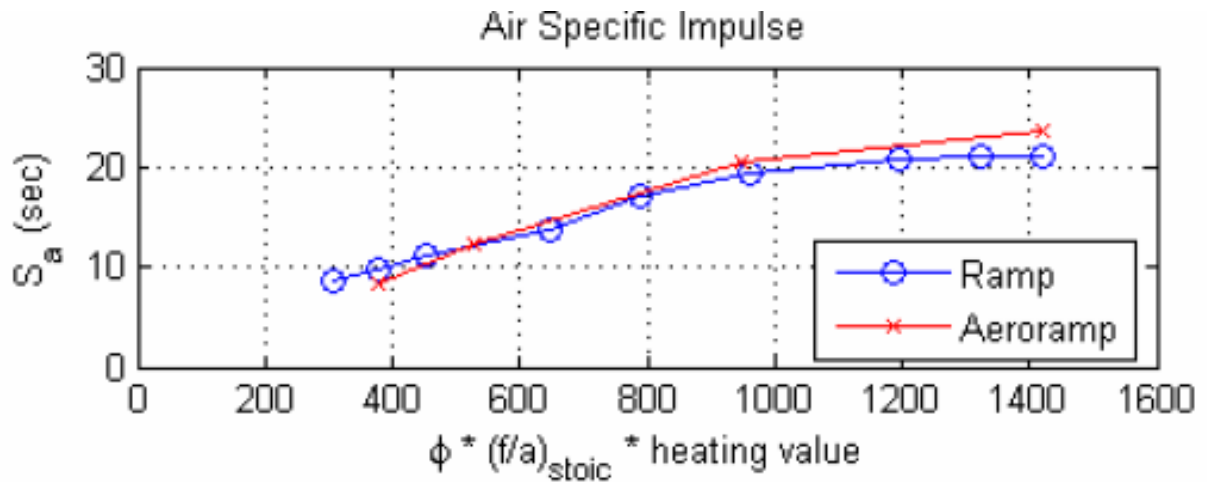


Figure 5-5: Air Specific Impulse from RJPA (Bonanos 2005)

Bonanos also wanted to see the effect of combustor inlet total temperature on air specific impulse. He set up the following parameters for the case of  $T_o=1010K$ .

- $530 < T_o(K) < 1010$
- Flight  $M_o=4.22$
- Altitude=88000 ft
- Inlet Area= 9.65 in<sup>2</sup>
- Area Ratio=6.43

For this set of data, he was able to calculate the results shown in Figure 5-6.



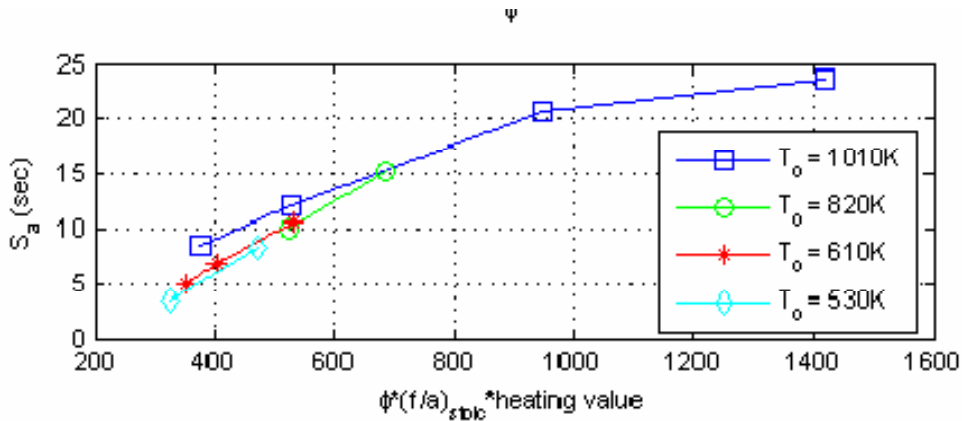


Figure 5-6: Air Specific Impulse from RJPA at Various Total Temperatures (Bonanos 2005)

VTMODEL was used to analyze the Bonanos scramjet flow path for air specific impulse. For the VTMODEL analysis, no aeroramp was assumed. An inlet total temperature of  $T_0=1010\text{K}$  was entered. In Figures 5-7 and 5-8, the VTMODEL results are overplotted on the Bonanos results, and are seen to closely compare with results for both the Aeroramp and the ramp combustor design, and for the  $T_0=1010\text{K}$  case on the variable  $T_0$  graph.

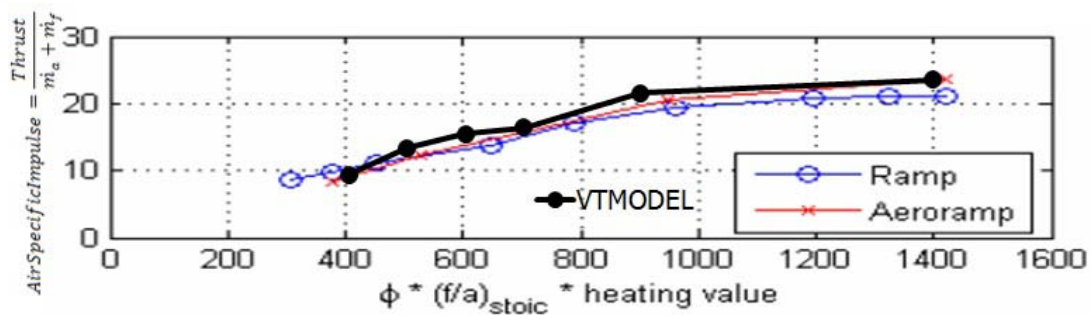
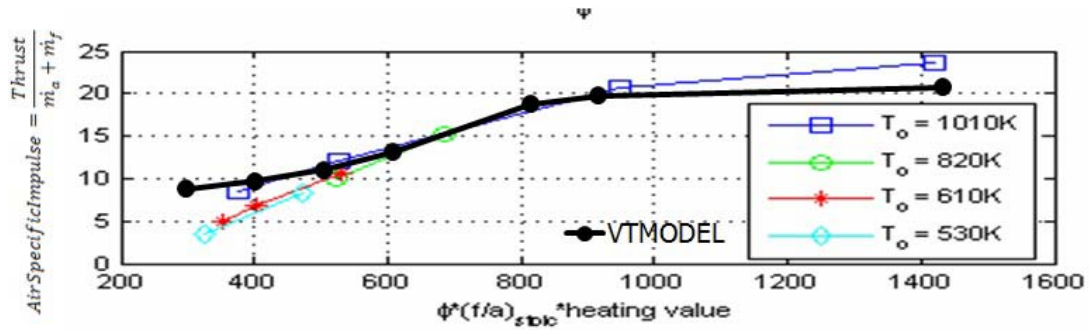


Figure 5-7: Air Specific Impulse Predicted by VTMODEL Overlay over Bonanos (Bonanos 2005)



**Figure 5-8: Air Specific Impulse Predicted by VTMODEL for To=1010K Overlay over Bonanos (Bonanos 2005)**

From the above figures it can be seen that with the addition of heat transfer, VTMODEL was able to predict air specific impulse values close to RJPA predicted values. Without the addition of heat transfer, VTMODEL overpredicted the values of air specific impulse. The values were approximately 11%-17% higher than the numbers presented in Figure 5-7 and 5-8.

## 5.2 Comparisons with Experimental Data

### 5.1.1. Analysis of UVA Experimental Results

VTMODEL was used to compare to results from the University of Virginia Direct Connect Combustion Tunnel. Operating conditions for the tunnel were as shown in Table 5.4. The geometry of the tunnel is as shown in Figure 5-9. The tunnel was set up to simulate Mach 5, 70,000 ft flight. The isolator entrance conditions were as shown in Table 5-4 for clean air. The equivalence ratio analyzed was 0.341.

	Clean Air	Methane Simulation	Hydrogen Simulation
Mole Fraction – H <sub>2</sub> O	0%	8%	17%
Mole Fraction – CO <sub>2</sub>	0%	4%	0%
Mole Fraction – O <sub>2</sub>	0%	3.18%	4.505%
Mole Fraction – Air	100%	84.82%	78.495%
Mach Number	2.03	2.02	2.02
Static Pressure (psia)	5.92	6.02	6.03
Static Temperature (R)	1277	1304	1307
Density (lbm/ft <sup>3</sup> )	0.01249	0.01205	0.01130
a (ft/s)	1711	1747	1804
v (ft/s)	3474	3529	3644
Gamma	1.336	1.322	1.320

**Table 5-4: UVA Isolator Entrance Parameters (Rockwell 2010)**

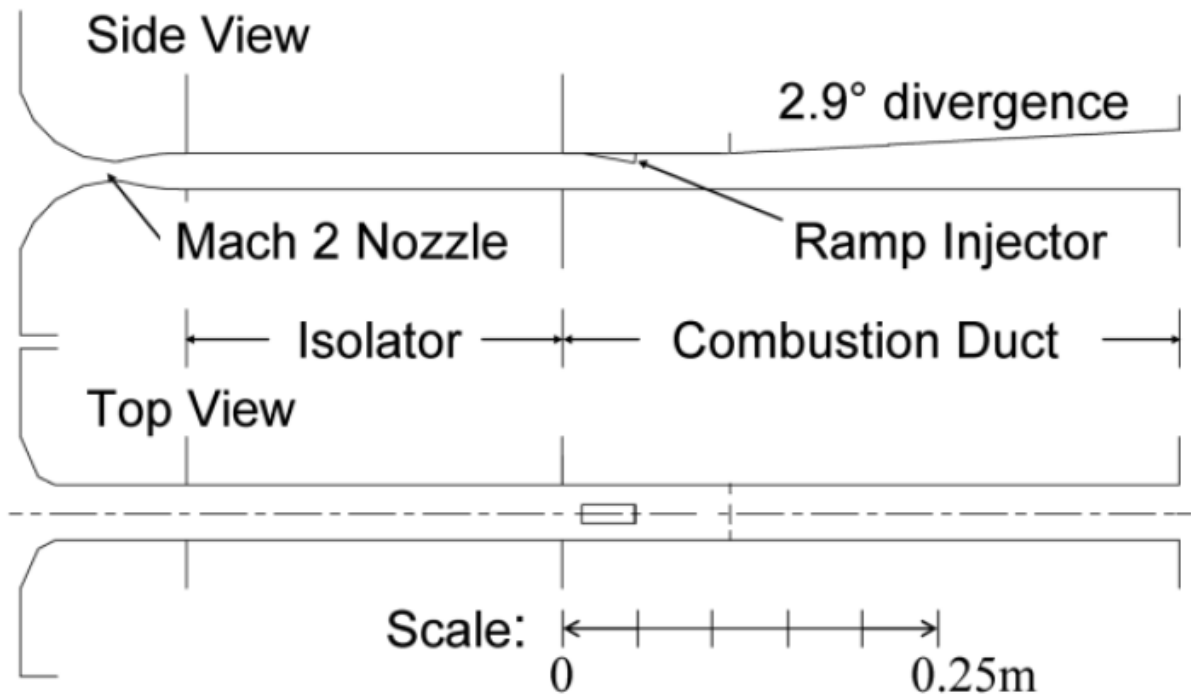
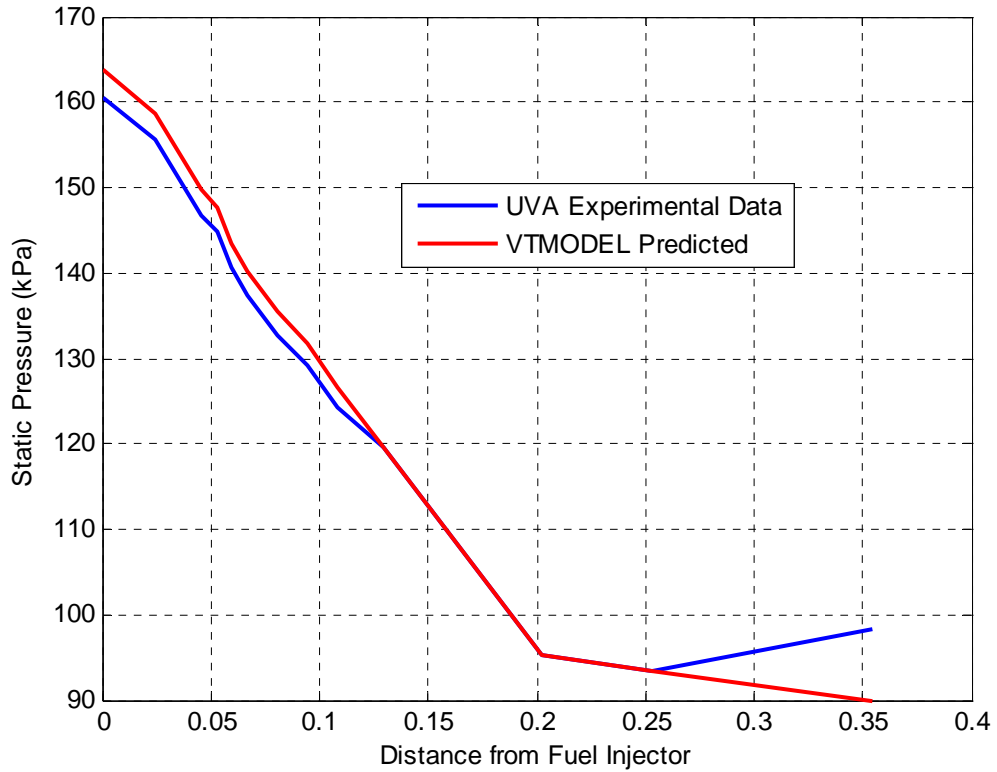


Figure 5-9: Schematic of UVA Tunnel (Le 2008)

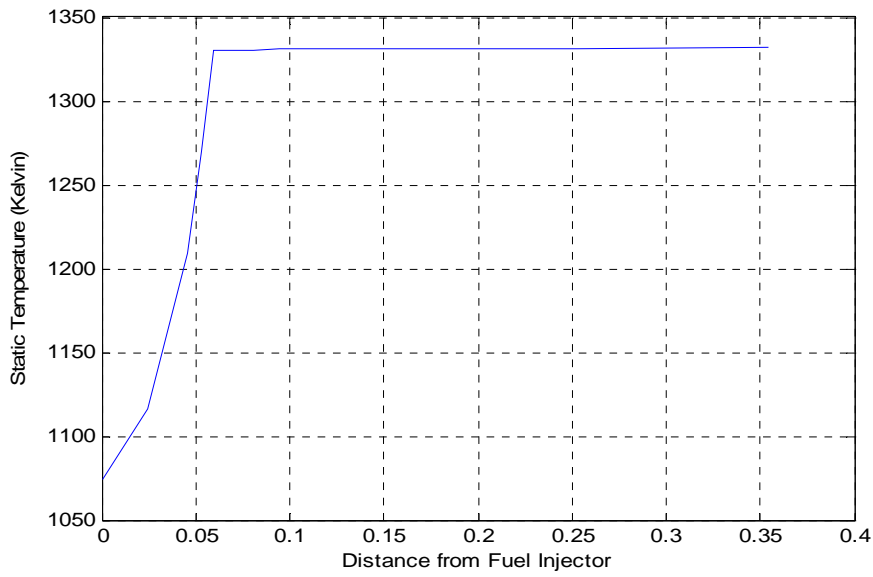
A predictive analysis using VTMODEL was run for this case. The only modules used in this analysis are the Isolator and Combustor. The isolator geometry was fixed based on the schematic of the UVA tunnel. The incoming properties were taken from Table 5-4. The friction coefficient was assumed to be 0.0015 and the Fanno flow and oblique shock model was used for the isolator. The combustor static pressure anchor point was set at 0.25 m upstream of the fuel injector. This anchor point was chosen due to the artifact at the end of the combustor where the static pressure increases. The data set was assumed to be for subsonic combustion. The Mach number entering the combustor from the isolator is predicted to be 0.858 by VTMODEL. The flow path Mach number was reduced to this subsonic value in the isolator. The results of the analysis are shown in the following figures. The predicted static pressure profiles agree within 5% of the

experimental data. The agreement obtained for the pressure profile supports the subsonic combustor entry Mach number assumption.

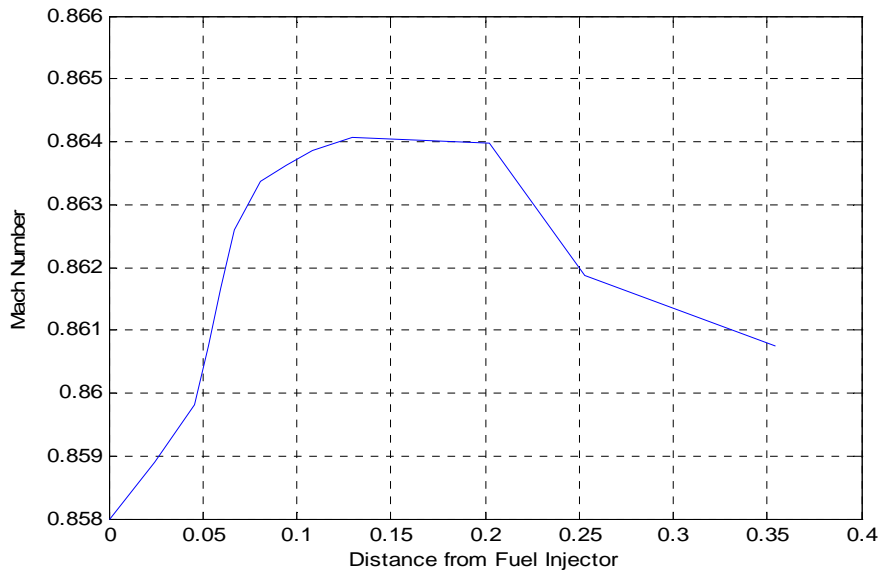
In the following figures, the distance from the fuel injectors is given in meters.



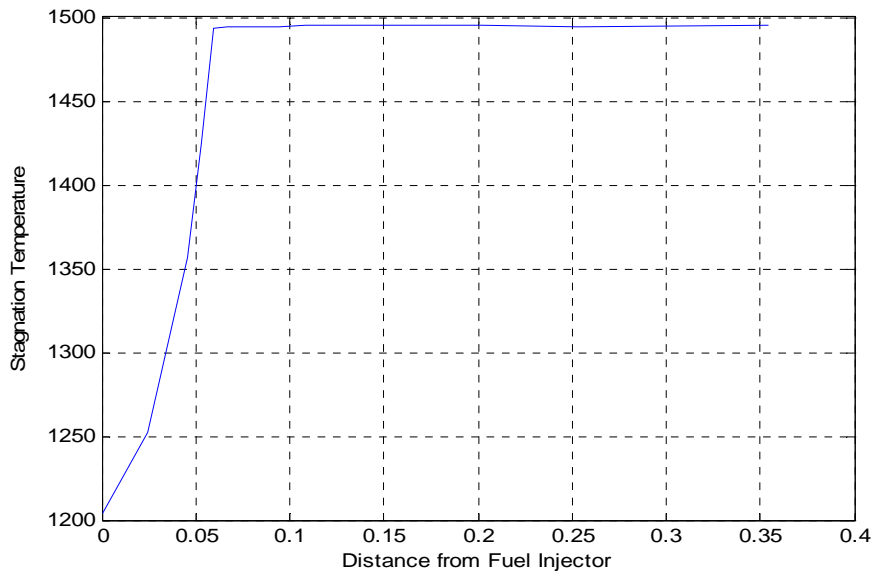
**Figure 5-10: Static Pressure Predicted by VTMODEL for  $\Phi=0.341$  Compared to UVA Experimental Data**



**Figure 5-11: Static Temperature Predicted by VTMODEL for  $\Phi=0.341$**



**Figure 5-12: Mach Number Predicted by VTMODEL for  $\Phi=0.341$**



**Figure 5-13: Stagnation Temperature Predicted by VTMODEL for  $\Phi=0.341$**

## 5.3 Discussion of Results

VTMODEL was used to analyze two different combustor geometry variations for a comparison of results from current scramjet and ramjet analysis programs. The combustor variations were based on original analyses done by Bradford (Bradford 2001) and Bonanos (Bonanos 2005). The predictive abilities of VTMODEL were compared to results from SCCREAM and the Ramjet Performance Analysis Code (RJPA).

VTMODEL was also used to compare to an experimental data set obtained at the University of Virginia Direct Connect Tunnel at  $\Phi=0.341$  (Rockwell 2010). The following comments on the results are noted::

- VTMODEL overpredicted specific impulse without any guidance for combustion efficiency, friction, or heat transfer. The comparisons of SCCREAM and RJPA to specific impulse were done without knowledge of the actual equivalence ratio. VTMODEL also over predicted the specific impulse results from the theoretical model from Moses (Moses 2003).
- VTMODEL was able to closely predict specific impulse as compared to RJPA analysis with the addition of heat transfer analysis in the combustor.
- VTMODEL can be used in two different modes: analytic and predictive.
  - The model can be used to analyze static pressure data from wind tunnel tests as shown in Chapter 2
  - The model can also be predictive with given inlet and exit boundary conditions.
- When given isolator entrance pressure data from a combustion experiment, VTMODEL will calculate the combustor inlet subsonic or supersonic Mach



number required to satisfy the static pressure value at the combustor outlet or at the nozzle outlet. No assumption is needed regarding the subsonic or supersonic condition at the combustor inlet. The model will iterate on the shock length in the combustor to determine the isolator discharge Mach number for the given pressures.

# *Chapter 6*

## **Conclusions and Recommendations**

### **6.1 Conclusions**

An one dimensional scramjet and ramjet analysis code called VTMODEL was written in MATLAB. VTMODEL was created to be a modular code that can be improved upon and expanded. The code was constructed with four main sections: the inlet, isolator, combustor, and nozzle. For the isolator section, the user has a choice of two different isolator flow models. One of the models is a Fanno flow/ oblique shock system combination that iterates on the shock angles of the shock system to match an exit pressure. The second model is a shock train correlation. This correlation iterates on the shock length to match the exit pressure of the combustor or the nozzle depending on the system.

Of the four sections of VTMODEL, the combustor is the most involved and complicated. The complete combustion model includes three functions: a mixing/combustion efficiency function, a combustion calculation function, and a flow properties calculation function. The mixing/combustion efficiency function can either be an efficiency correlation including with the mixing efficiency of the fuel-air mixture, or a “combustion sphere” model that permits the adjustment of the flame speed of the combustion, resulting in a prediction of combustion efficiency. The combustion functions can either a complete combustion model, or a non equilibrium mechanism specifically for hydrogen combustion. For the complete combustion method, the concentration of fuel and combustion products in each section of the combustor is

determined by the combustion sphere model. The non-equilibrium function uses the mixing correlation and also a reduced hydrogen combustion chemistry model presented in Jachimowski (Jachimowski 1988). This reduced hydrogen combustion model is made up of a system of 8 reactions. The model uses some assumptions of steady state intermediates to make calculations possible without an additional chemical kinetics code (Atkins 2001). The third function in the VTMODEL combustor model is a Mach number calculation using influence coefficients as presented in Shapiro (Shapiro 1953). The solver for this differential equation was written in MATLAB, and is a 4<sup>th</sup> order RK solver. From these three modules, all of the thermodynamic properties at each combustor section can be calculated.

To illustrate the predictive capability of VTMODEL, the program was used to analyze a “generic” scramjet flow path. This example was developed to show the cycle analysis capabilities of VTMODEL. The flow path was analyzed for varying flight altitudes, Mach numbers, and equivalence ratios. The specific impulse was calculated for each of these runs. It was determined that VTMODEL correctly predicted the trends of the  $I_{SP}$  vs flight Mach number. The predicted value of the  $I_{SP}$  was 200-500 seconds higher than the “theoretical” values presented in Moses (Moses 2003). Among the many factors that could influence the predications, the discrepancy could be due to the assumption of 100% combustion efficiency in VTMODEL, increasing the specific impulse. The increase could also be due to having no heat transfer through the walls.

VTMODEL was created to be able to analyze and predict thermodynamic data in both scramjet and ramjet flow paths. To demonstrate use in experiment analysis, VTMODEL was used to predict thermodynamic data based on an experimental data

series furnished by the University of Virginia. The results of this analysis showed that VTMODEL had the ability to predict temperatures and Mach number along the combustor. The required inputs were an isolator entrance condition, geometry, equivalence ratio, and static pressure profiles.

VTMODEL also has a predictive capability. The predictive ability of VTMODEL was tested against the models RJPA and SCCREAM. VTMODEL predicted fuel specific impulse and air specific impulse to match the analyses done in Bradford and Bonanos. The results show that the values predicted by VTMODEL were within 5% of the values taken from their results. The predictive model capability of VTMODEL was also shown in one test using data from the University of Virginia Direct Connect combustor. For an equivalence ratio where the static pressure profile was already experimentally obtained, VTMODEL was used to predict this profile and also the Mach numbers, static temperatures, and stagnation temperatures along the combustor. The results show that VTMODEL was able to predict this profile with a high degree of accuracy.

VTMODEL currently continues in development as a function based/modular program to analyze one dimensional flow through a scramjet or ramjet. The benefits of VTMODEL include an ability to predict flow conditions, or to enter known data for analysis. Modules of the code can be removed or modified to the user's specification.

There are limitations to the use of VTMODEL. The most inherent limitation is the use of a one-dimensional model to model 3D flow. The model also does not take into account boundary layer effects or separation. The inlet function of the model is a simplistic model of an actual scramjet inlet. The inlet performance and pressure recovery

is most likely somewhere in-between the supplied kinetic energy model and the MIL Spec model presented in Chapter 2.

Another limitation to VTMODEL is the singularity due to the influence coefficient calculation for Mach number in the combustor. Since the quantity  $(1-M^2)$  is in the denominator, the program currently will not calculate flow through  $M=1$ .

VTMODEL is therefore currently useful for modeling subsonic or supersonic combustion and not for dual mode predictions. This ability for the Mach number to transition from a supersonic to subsonic value in the combustor is essential for dual mode combustion calculations.

## 6.2 Recommendations

Several improvements can be made to improve VTMODEL. The program is currently written in a manner that requires programming knowledge to run the program. A visual graphical interface can be developed to make the program more user friendly. The program also is currently in open MATLAB files. The program can be further refined to allow easier data entry and function call-up. One of the first improvements for VTMODEL should be to use l'Hopital's rule or another mathematical technique to be able to calculate through the  $M=1$  condition. Currently, there is a singularity in the program at this Mach number due to the influence coefficient method of solving for Mach number in the combustor. Heat transfer analysis is extremely important for ramjet and scramjet design and analysis. The current heat transfer analysis model uses Reynolds Analogy. This heat transfer analysis can be expanded on and improved.

Another improvement for VTMODEL can be the integration of a chemical kinetic program such as Chemkin. With the development of the complete combustion model with the combustion sphere combustion delay mechanism, and the uses of the Jachimowski method for hydrogen, the program will run with rate-controlled combustion under certain specifications of Mach number and pressures. With the addition of Chemkin, the development of VTMODEL into a multi fuel code would be quicker and more accurate. The addition of hydrocarbon fuel capability will also aid in the development of a comprehensive scramjet and ramjet analysis code.

Currently, there is an Aerojet-sponsored senior design team at Virginia Tech that is designing and manufacturing a hydrocarbon scramjet test rig for test at the Aerojet facility in Orange, VA. Since the team is designing for hydrocarbon fuel, there was no

benefit in using the current VTMODEL for analysis and for aiding the design. However, with the development of a hydrocarbon chemistry solver, VTMODEL can be used as both a design tool and an analysis tool for hydrocarbon combustion testing.

One advantage of VTMODEL is the modular design. Coding VTMODEL in MATLAB can help expand VTMODEL into a combined cycle code. Since VTMODEL is already modular, different components of the model can be used to prove the scramjet/ramjet flow path analysis of a combined cycle code. A combined cycle code development will enable the model to both analyze scramjet and ramjet flowpaths and combine these flowpaths with either turbomachinery or rockets. This addition of combined cycle analysis will enable VTMODEL to aid in the design and analysis of a system that span sea level to flight altitude analysis.

# REFERENCES

---

1. Atkins , P. and dePaula, J. (2001). Physical Chemistry. 7<sup>th</sup> Ed. New York, W.H Freeman.
2. Boeing (2010, May 26, 2010). "Boeing X-51A WaveRider Breaks Record in 1st Flight." 2010, from <http://boeing.mediaroom.com/index.php?s=43&item=1227>.
3. Bonanos, A. M. (2005). Scramjet Operability Range Studies of an Integrated Aerodynamic- Ramp-Injector/Plasma-Torch Igniter with Hydrogen and Hydrocarbon Fuels. Aerospace Engineering. Blacksburg, Virginia Polytechnic Institute and State University. **Doctor of Philosophy**.
4. Bowcutt, K. G. (2007). Scramjet Vehicle Propulsion-Airframe Integration. Hypersonic Propulsion Education Initiative, NIA.
5. Bradford, J. E. (2001). A Technique for Rapid Prediction of Aftbody Nozzle Performance for Hypersonic Launch Vehicle Design. Aerospace Engineering. Atlanta, Georgia Institute of Technology. **Doctor of Philosophy**.
6. Bradford, J. E. and Olds, J.R. (1998). Improvements and Enhancements to SCCREAM, A Conceptual RBCC Engine Analysis Tool. 34th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit. Cleveland, OH, American Institute of Aeronautics and Astronautics.
7. Burkardt, L. A. and Franciscus, L.C. (1990). RAMSCRAM-A Flexible Ramjet/Scramjet Engine Simulation Program. L. R. Center. Cleveland, NASA.
8. Chapra, S. C. (2004). Applied Numerical Methods with MATLAB for Engineers



- and Scientists. New York, McGraw Hill.
9. Curran, E.T (2001). "Scramjet Engines: The First Forty Years." Journal of Propulsion and Power. **17**(6):1138-1148.
  10. Glassman, I. (1977). Combustion. New York, Academic Press.
  11. Goodall, J. and Miller, J. (2002). Lockheed's SR-71 'Blackbird' Family A-12, F-12, M-21, D-21, SR-71. Hickley, England, AeroFax-Midland Publishing.
  12. Handbook, P. W. MIL-E-5007D
  13. Heiser, W. H. and Pratt, D.T, Ed. (1994). Hypersonic Air Breathing Propulsion. AIAA Education Series. Washington D.C, American Institute of Aeronautics and Astronautics.
  14. Hill, P and Peterson, C. (1975). Mechanics and Thermodynamics of Propulsion 1st Edition. New York, Addison Wesley Longman.
  15. Hill, P and Peterson, C. (1992). Mechanics and Thermodynamics of Propulsion 2nd Edition. New York, Addison Wesley Longman.
  16. Jachimowski, C. (1988). An Analytical Study of the Hydrogen-Air Reaction Mechanism with Application to Scramjet Combustion. NASA Langley Research Center. Hampton.
  17. John, James. (1984). Gas Dynamics 2<sup>nd</sup> Edition. New York, Prentice Hall.
  18. Kazmar, R. R. (2005). Airbreathing Hypersonic Propulsion at Pratt & Whitney – Overview. AIAA/CIRA 13th International Space Planes and Hypersonic Systems and Technologies.
  19. Le, D. B., C.P Goyne, R.H Krauss, and J.C. McDaniel (2008). "Experimental Study of a Dual-Mode Scramjet Isolator." Journal of Propulsion and Power **25**(5):

- 1050-1057.
20. Lee, C, Kundu K., and Acosta W. (1991). Jet-A Reaction Mechanism Study for Combustion Application. NASA Glenn Research Center. Cleveland.
  21. Lockheed Martin (2010). "SR-71 Blackbird." 2010, from <http://www.lockheedmartin.com/aboutus/history/SR71Blackbird.html>.
  22. Moses, P. L. (2003). X-43C Plans and Status. 12th AIAA International Space Planes and Hypersonic Systems and Technologies Conference. Norfolk, VA.
  23. O'Brien, T. F., Ryan P. Starkey, and Mark J. Lewis (2001). "Quasi-One-Dimensional High-Speed Engine Model with Finite-Rate Chemistry." Journal of Propulsion and Power **17**(6): 1366-1374.
  24. O'Byrne, S. and Cutler, A.D. (2007). "Dual-Pump Coherent Anti-Stokes Raman Scattering Measurements in a Supersonic Combustor." AIAA Journal **45**(4): 922-933.
  25. Pandolfini, P. P., and Friedman, M.A. (1992). Instructions for using Ramjet Performance Analysis (RJPA) IBM-PC Version 1.24. Baltimore, John Hopkins University/Applied Physics Lab.
  26. Reaction Design (2010). "Chemkin: Leading Solutions for the Rapid Simulation of Complex Chemistry," from [http://www.reactiondesign.com/images/CHEMKIN-BRO\\_0910.pdf](http://www.reactiondesign.com/images/CHEMKIN-BRO_0910.pdf).
  27. Rockwell, R. (2009-2010). Experimental Data. Personal Communication
  28. Schetz, J. A. (2007). Scramjet Combustors. Hypersonic Propulsion Education Initiative, NIA.
  29. Segal, C. (2009). The Scramjet Engine: Processes and Characteristics. New York,

- Cambridge University Press.
30. Shapiro, A. H. (1953). The Dynamics and Thermodynamics of Compressible Fluid Flow in Two Volumes. New York, Ronald Press Company.
  31. Sullins, G. and McLafferty . (1992). Experimental Results of Shock Trains in Rectangular Ducts. AIAA Fourth International Aerospace Planes Conference. Orlando, FL.
  32. Trefny, C. (2010). Personal Communication.
  33. United States Government Printing Office (1976). US Standard Atmosphere.
  34. UMD (University of Maryland) (2010). "Innovation Hall of Fame: Frederick S. Billig PhD (1933-2006)." <http://www.eng.umd.edu/ihof/inductees/billig.html>.
  35. USAF, Ed. (2002). SR-71 Flight Manual. Manuals of Flight Series, Government Reprints Pr.
  36. Van Wie, D. M. (2001). Scramjet Inlets. Scramjet Propulsion (Progress in Astronautics and Aeronautics). P. Zarchan. **189**.
  37. Waltrup, P. J., and F.S. Billig (1973). "Prediction of Precombustion Wall Pressure Distributions in Scramjet Engines." Journal of Spacecraft **10**(9).
  38. Waltrup, P. J., Billig F.S., and Stockbridge, R.D. (1978). A Procedure for Optimizing the Design of Scramjet Engines. AIAA/SAE 14<sup>th</sup> Joint Propulsion Conference. Las Vegas, American Institute of Aeronautics and Astronautics.
  39. Waltrup, P.J., White, M., Zarlingo, F. and Gravlin, E. (1997). "History of Ramjet and Scramjet Propulsion Development for U.S. Navy Missiles." John Hopkins APL Technical Digest. **18**(2): 234-244.
  40. Warwick, G. (2010). First X-51A Hypersonic Flight Deemed Success. Aviation

Week.

41. Zweber, J. V., Hane Kabis, William W. Follett, and Narayan Ramabadrnan (2002). Towards an Integrated Modeling Environment for Hypersonic Vehicle Design and Synthesis. AIAA/AAAF 11th International Space Planes and Hypersonic Systems and Technologies Conference. New Orleans, American Institute of Aeronautics and Astronautics.

# UNCITED REFERENCES

---

1. Aithal, S. M., V.V. Subramaniam and V. Babu (1996). Numerical simulation of plasma and reacting flows. 27th AIAA Fluid Dynamics Conference. New Orleans, LA, American Institute of Aeronautics and Astronautics.
2. Baurle, R. "VULCAN: Viscous Upwind Algorithm for Complex Flow Analysis, User Manual." from <http://vulcan-cfd.larc.nasa.gov/>.
3. Billig, F. S., Dugger G.L, Waltrup P.J (1972). Inlet-Combustor Interface Problems in Scramjet Engines. 1st International Symposium on Air Breathing Engines. Marseille, France, Institut de Mecanique des Fluides.
4. Bussing, T. R., Murman E.M (1983). A one-dimensional unsteady model of dual mode scramjet operation. 21st Aerospace Sciences Meeting. Reno, NV, American Institute of Aeronautics and Astronautics.
5. Clark, A., Chivey Wu, Maj Mirmirani, Sangbum Choi (2006). Development of an Airframe-Propulsion Integrated Generic
6. Cutler, A.D, Danehy, P.M., Springer, R.R, O'Bryne, S., Capriotti, D.P., and Deloach, R." Coherent Anti-Stokes Raman Spectroscopic Thermometry in a Supersonic Combustor." AIAA Journal **41**(12): 2451-2459.
7. Hypersonic Vehicle Model. 44th AIAA Aerospace Sciences Meeting and Exhibit. Reno, NV, American Institute of Aeronautics and Astronautics.
8. Doolan, C. (2008). A Quasi-One-Dimensional Mixing and Combustion Code for Trajectory Optimization and Design Studies. 15th AIAA International Space Planes and Hypersonic Systems and Technologies Conference. Dayton, Ohio,

- American Institute of Aeronautics and Astronautics.
9. Kato, R. a. I. K. (1996). Numerical Simulation of Flame-Stabilization and Combustion Promotion by Plasma Jets in Supersonic Air Streams. Twenty-Sixth Symposium on Combustion, The Combustion Institute
  10. Minato, R., Takashi Niioka, Hiromu Sugiyama, and Kazuhide Mizobata (2005). Numerical Analysis of Supersonic Combustion by a Plasma Torch. AIAA/CIRA 14th International Space Planes and Hypersonic Systems and Technologies, American Institute of Aeronautics and Astronautics.
  11. Roberts, K. (2008). Analysis and Design of a Hypersonic Scramjet Engine with a Starting Mach number of 4.00. Aerospace Engineering. Arlington, University of Texas at Arlington. **Master of Science**.
  12. Ruscic, B., Reinhardt E. Pinzon, Melita L. Morton, Nanda K. Srinivasan, and J. W. S. a. J. V. M. Meng-Chih Su (2006). "Active Thermochemical Tables: Accurate Enthalpy of Formation of Hydroperoxyl Radical, HO<sub>2</sub>." Journal of Physical Chemistry A **110**: 6592-6601.
  13. Starkey, R. P. (2004). Scramjet Optimization for Maximum Off-Design Performance. 40th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit. Fort Lauderdale, Fl, American Institute of Aeronautics and Astronautics.
  14. Tomioka, S., Tetsuo Hiraiwa, Kan Kobayashi, Muneo Izumikawa, Tomoyuki Kishida, and Hiroyuki Yamasaki (2007). "Vitation Effects on Scramjet Engine Performance in Mach 6 Flight Conditions." Journal of Propulsion and Power **23**(4): 789-796.
  15. Turns, S. R. (2000). An Introduction to Combustion: Concepts and Applications.

New York, McGraw-Hill Higher Education.

16. Weissman, D. (1990). Representation of two-dimensional hypersonic inlet flows for one-dimensional scramjet cycle analysis 28th Aerospace Sciences Meeting. Reno, NV, American Institute of Aeronautics and Astronautics.