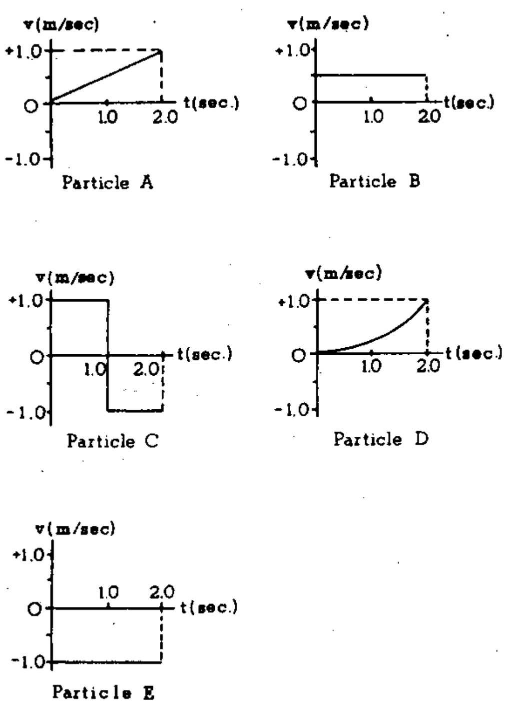
One-Dimensional Kinematics Horizontal Motion ANSWER KEY

1. ____**B.**____ A car travels 30 kilometers at an average speed of 60 kilometers per hour and then 30 kilometers at an average speed of 30 kilometers per hour. The average speed of the car over the 60 kilometers is

(A) 35 km/h **(B) 40 km/h** (C) 45 km/h (D) 50 km/h (E) 53 km/h

Ans. The answer is not 45 km/hr as might seem initially. The problem is that both paths don't take the same amount of time. Find the total time for each. $V = \frac{s}{t} \rightarrow t = \frac{s}{v} = \frac{30\ km}{60\ km/hr} = 0.5$ hrs. and $t = \frac{s}{v} = \frac{30\ km}{30\ km/hr} = 1$ hr. The total time is 1.5 hrs. $V_{avg} = \frac{s}{t} = \frac{60\ km}{1.5\ hrs.} =$ 40 km/hr.

Questions 2-4 relate to five particles that start at x = 0 at t = 0 and move in one dimension independently of one another. Graphs of the velocity of each particle versus time are shown below.



2. ____**E.**____ Which particle is farthest from the origin at 2 s?

(A) A (B) B (C) C (D) D **(E) E**

Ans. Particle E had the largest velocity for the longest time moving the entire time from the origin at –1 m/s.

3. ____**A.**____ Which particle moves with a constant nonzero acceleration?

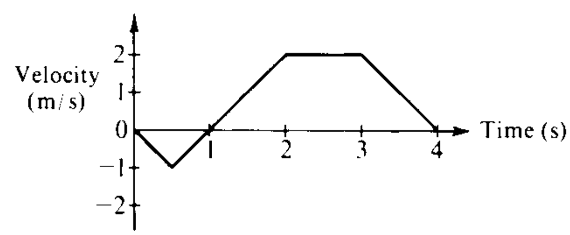
(A) A (B) B (C) C (D) D (E) E

Ans. The acceleration is the slope of the velocity graph. A is the only graph that shows a line with a constant non-zero slope.

4. ____**C.**____ Which particle is in its initial position at t = 2 s?

(A) A (B) B **(C) C** (D) D (E) E

Ans. Particle C has travelled at +1 m/s for one second and –1 m/s for one second. Therefore, he is back where he started.



5. ____**C.**____ The graph above shows the velocity *versus* time for an object moving in a straight line. At what time after time = 0 does the object again pass through its initial position?

(A) Between 0 and 1 s
(B) 1 s
(C) Between 1 and 2 s
(D) 2s
(E) Between 2 and 3 s

Ans. Since we are given the velocity vs. time graph, the area under the curve will give us the distance travelled vs. time. When the negative area under the curve between t = 0 s and t = 1 s is the same as the positive area under the curve from t = 1 to our answer, then the object will have passed once again through the origin.

The distance travelled from t =0 s to t = 1 s is A = ½bh = ½(1)(–1) = –½ m. By t = 2 seconds, the area has already become A = ½bh = ½(1)(2) = 1 m so the car is already moving away from initial position in a positive direction.

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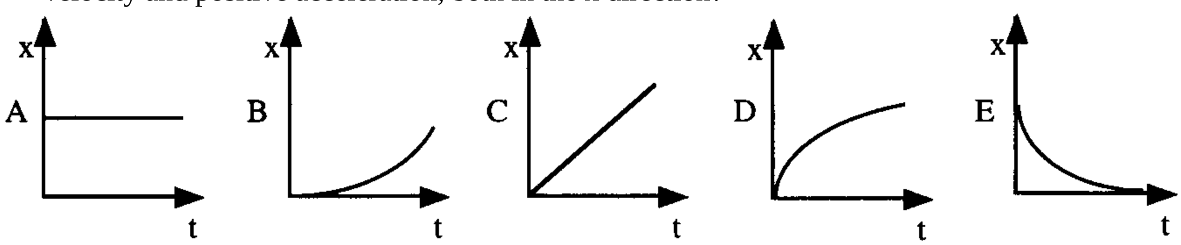
Questions 6-7 relate to five objects that are moving in parallel straight-line paths. The objects all cross a starting line at the instant a clock is started. The distances from the starting line in meters after 1, 2, 3, 4, and 5 seconds are as follows:

	Time (seconds)				
Object	1	2	3	4	5
(A)	1 m	1 m	2 m	2 m	3 m
(B)	1 m	2 m	3 m	4 m	5 m
(C)	1 m	4 m	9 m	16 m	25 m
(D)	4 m	10 m	18 m	28 m	40 m
(E)	6 m	11 m	15 m	18 m	20 m

6. ____**B.**____Which object is moving with zero acceleration?
(A) A **(B) B** (C) C (D) D (E) E
Ans. The distance increases by the same amount in Object C. This means the object is cruising and not accelerating.

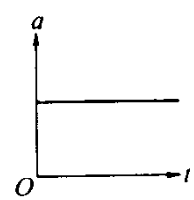
7. ____**D.**____Which object has constant nonzero acceleration and appears to have started from rest?
(A) A (B) B (C) C **(D) D** (E) E
Ans. Object D's distances are increasing by a steady amount every second: First by 4 m, then 6 m, then 8 m. This suggests the velocity is changing at a constant rate, hence, a non-zero acceleration.

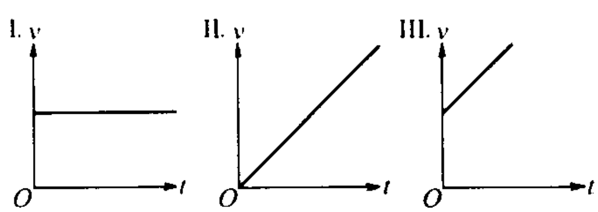
8. _____**B.**_____ Which graph of position x versus time t best represents a moving object with positive velocity and positive acceleration, both in the x direction?



Ans. The object's position is increasing as an upward facing curve. This suggests the distances are getting bigger in every time unit and so the velocity is increasing which means the object has acceleration.

9. _____**D.**_____ The graph at right shows acceleration *a* as a function of time *t* for a particle moving in a straight line. Graphs of speed *v versus* time t that are consistent with the *a versus t* graph above include which of the following?





(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Ans. Both II and III show a non-zero increasing linear velocity. This coincides with a graph of a positive, constant acceleration as shown in the graph at right. Graph I shows a velocity that is constant and not increasing. The acceleration for this graph would be a horizontal line but at a height of zero.

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Problem: Kinematic Equations (CM-1993)

10. ____**D.**____ A 500-kilogram sports car accelerates uniformly from rest, reaching a speed of 30 meters per second in 6 seconds. During the 6 seconds, the car has traveled a distance of

(A) 15 m (B) 30 m (C) 60 m **(D) 90 m** (E) 180 m

Ans. We have to use the **velocity formula** to find the **acceleration** and then substitute the **acceleration** into the **distance formula**:

$\mathbf{v = v_o + at} \rightarrow \mathbf{v = at} \rightarrow 30 = a(6\ s)$

$\mathbf{5\ m/s^2 = a}$

Substituting into the formula for **distance**:

$\mathbf{x = v_o t + \frac{1}{2}at^2 \rightarrow x = \frac{1}{2}at^2}$

$x = \frac{1}{2}(\mathbf{5\ m/s^2})(\mathbf{6\ s})^2 = \mathbf{90\ m}$

Problem: Kinematic Equations (B-1984)

11. ____**C.**____ A body moving in the positive x direction passes the origin at time $t = 0$. Between t = 0 and t = 1 second, the body has a constant speed of 24 meters per second. At $t = 1$ second, the body is given a constant acceleration of 6 meters per second squared in the negative x direction. The position x of the body at $t = 11$ seconds is

(A) +99 m (B) +36 m **(C) –36 m** (D) –75 m (E) –99 m

Expl. In the **first second** the body has travelled **24 meters** to the right **by** $\mathbf{x = v \cdot t = 24\ m/s \cdot (1\ s) = 24\ m}$. So the **formula** for the **distance** covered is

$\mathbf{x = x_o + v_o t - \frac{1}{2}at^2 =}$

$\mathbf{x = 24 + 24(10) - \frac{1}{2}(6)(10)^2 =}$

$\mathbf{x = -36\ m}$

Problem: Kinematic Equations (CM-1988)

12____**C.**____ An object released from rest at time t = 0 slides down a frictionless incline a distance of 1 meter during the first second. The distance traveled by the object during the time interval from t = 1 second to t = 2 seconds is

(A) 1 m (B) 2 m **(C) 3 m** (D) 4m (E) 5 m

Ans. We have to use the **distance formula** from t = 0 s to t = 1 s, to find the **acceleration** and then substitute the **acceleration** into the **distance formula** again but from t = 1 s to t = 2 s:

Distance formula: $\mathbf{x = \frac{1}{2}at^2}$

In the first second: $1\ m = \frac{1}{2}a(1\ s)^2 \rightarrow 1 = \frac{1}{2}a \rightarrow \mathbf{2 = a}$

After one second the object's **velocity** will be $\mathbf{v = v_o + at} \rightarrow \mathbf{v = at} \rightarrow v = 2(1\ s) = 2\ m/s$

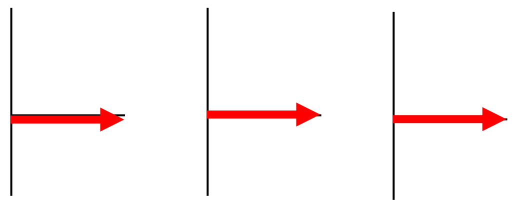
Substituting into the formula for **distance**: $x = v_o t + \frac{1}{2}at^2 \rightarrow x = (\mathbf{2})(\mathbf{1}) + \frac{1}{2}(\mathbf{2})(\mathbf{1})^2$

$\mathbf{x = 3\ m}$

One-Dimensional Kinematics Horizontal Motion ANSWER KEY

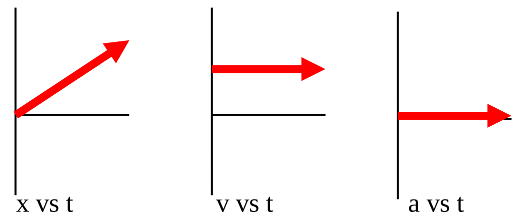
13. Problem: Sketch the Kinematic Graphs for Constant (Uniform) Acceleration

Stationary particle



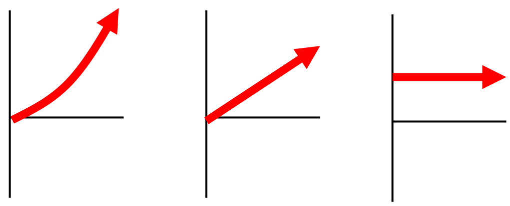
x vs t v vs t a vs t

Particle moving with constant velocity



x vs t v vs t a vs t

Particle moving with constant non-zero acceleration

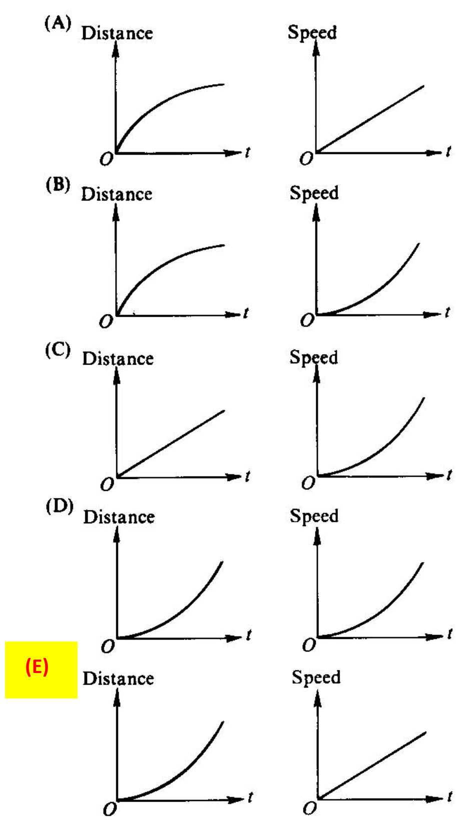


x vs t v vs t a vs t

Problem: Kinematic Graphs (CM-1988)

14. ___(E)___

Which of the following pairs of graphs shows the distance traveled *versus* time and the speed *versus* time for an object uniformly accelerated from rest at time $t = 0$?



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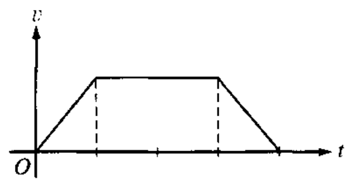
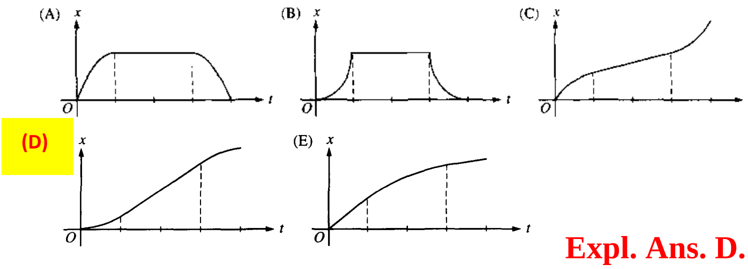


Problem: Kinematic Graphs (CM-1998)

15. ____**D.**____ The graph at left shows the velocity v as a function of time t for an object moving in a straight line. Which of the following graphs shows the corresponding displacement x as a function of time t for the same time interval?



Expl. Ans. D. The velocity is **increasing** in the **first** segment so the object is **accelerating**. This corresponds to a **curved** displacement graph **opening upwards**. In the **second** segment the **velocity** is **staying the same**. This corresponds to a **displacement graph** that is **increasing** in a **linear fashion**. The **third** segment is **decreasing velocity**, or **negative acceleration**. This corresponds to a **curved** graph but opening **downwards**.

FREE RESPONSE PRACTICE PROBLEMS

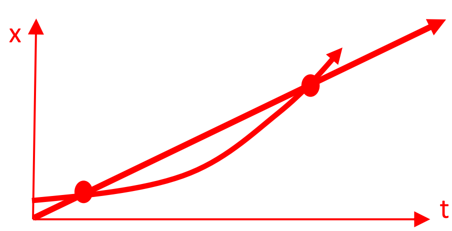
DIRECTIONS: ANSWER EACH OF THE FOLLOWING PROBLEMS! BE SURE TO SHOW ALL WORK CAREFULLY!

I. A student is running to catch the campus shuttle bus, which is stopped at the bus stop. The student is running at a constant velocity of 6 m/s; she cannot run any faster. When the student is still 80 m from the bus, it starts to pull away. The bus moves with a constant acceleration of 0.2 m/s^2.

a) For how much time and how far will the student have to run before she overtakes the bus?
Ans. The displacement of the student is given by $x_{student} = v_o t = 6t$.
The displacement of the bus is given by $x_{bus} = x_o + ½at^2 = 80 + ½(0.2\ m/s^2)t^2 = 80 + 0.1t^2$.
If we want to know when the student catches up with the bus, we should set the two displacement equations equal to each other and solve for t.
$x_{student} = x_{bus}$
$6t = 80 + 0.1t^2$
$0 = 0.1t^2 – 6t + 80$
Solving for t using your graphing calculator: **t = 20 s**
At **t = 20 seconds** the student has moved x = = $v_o t$ = 6(20 s) = **120 m.**

b) When she reaches the bus, how fast will the bus be traveling?
Ans. The **velocity formula** for the bus is **v_{bus} = a·t = 0.2 m/s^2(20 s) = 4 m/s**

c) Sketch a graph showing *x(t)* for both the student and the bus. Take x = 0 as the initial position of the student. (The same axis would be useful.)



d) The equations you used in (a) to find the time have a second solution, corresponding to a later time for which the student and bus will again be at the same place if they continue their specified motions. Explain the significance of this second solution. How fast will the bus be traveling at this point?

Ans. The second solution corresponds to when the bus accelerates enough to catch up with the student again and pass him up.

The second intersection point is t = 40 sec (obtained from the graph in graphing calculator)

The **velocity formula** for the bus is $\mathbf{v_{bus} = a \cdot t = 0.2\ m/s^2(40\ s) = 8\ m/s}$

e) If the student's constant velocity is 4 m/s, will she catch the bus?

No. In that case, the student's distance would be x = 4t.

Setting the equations equal to each other:

$$x_{student} = x_{bus}$$
$$4t = 80 + 0.1t^2$$

Solving for t. $4t = 80 + 0.1t^2$

$$0 = 0.1t^2 - 4t + 80$$

Solving for t using your graphing calculator: **no positive solutions.** The parabola has **no positive x-intercepts so the two will never meet.**

f) What is the *minimum* speed the student must have just to catch up with the bus? How long and how far will she have to run in that case?

So we want to know When there is exactly *one solution* to the quadratic $0 = 0.1t^2 - 4t + 80$.

What we are asking is what value for *v* will yield a single positive value for x:

Which term of the formula $0 = 0.1t^2 - 4t + 80$ moves the parabola left or right on a graph?

Ans. The *bx* term in the parabola $y = x^2 + bx + c$, moves the parabola over.

Using the **discriminant** of the **quadratic formula**, $b^2 - 4ac$: $b^2 - 4(0.1)(80) = 0$

$b^2 - 32 = 0 \rightarrow b^2 = 32 \rightarrow$ **b = 5.7 m/s**