### **OPEN-CHANNEL FLOW**

- Open-channel flow is a flow of liquid (basically water) in a conduit with a free surface.
- That is a surface on which pressure is equal to local atmospheric pressure.



# **Classification of Open-Channel Flows**

Open-channel flows are characterized by the presence of a liquid-gas interface called the *free surface*.

 <u>Natural flows</u>: rivers, creeks, floods, etc.



 <u>Human-made systems</u>: fresh-water aquaducts, irrigation, sewers, drainage ditches, etc.





### Total Head at A Cross Section:

The total head at a cross section is:

$$H = z + \frac{P}{\gamma} + \alpha \frac{V_{av}^2}{2g}$$

where H=total head
 Z=elevation of the channel bottom
 P/g = y = the vertical depth of flow (provided that pressure distribution is hydrostatic)
 V<sup>2</sup>/2g= velocity head





#### Comparison of Open Channel Flow and Pipe Flow



### Comparison of Open Channel Flow & Pipe Flow

- 1) OCF must have a free surface
- 2) A free surface is subject to atmospheric pressure
- 3) The driving force is mainly the component of gravity along the flow direction.
- 4) HGL is coincident with the free 4 surface.
- 5) Flow area is determined by the geometry of the channel plus the level of free surface, which is likely to change along the flow direction and with as well as time.

- 1) No free surface in pipe flow
- 2) No direct atmospheric pressure, hydraulic pressure only.
- 3) The driving force is mainly the pressure force along the flow direction.
- HGL is coincident with the free 4) HGL is (usually) above the conduit
  - 5) Flow area is fixed by the pipe dimensions The cross section of a pipe is usually circular..

### Comparision of Open Channel Flow & Pipe Flow

- 6) The cross section may be of any from circular to irregular forms of natural streams, which may change along the flow direction and as well as with time.
- 7) Relative roughness changes with the level of free surface
- 8) The depth of flow, discharge and the slopes of channel bottom and of the free surface are interdependent.

6) The cross section of a pipe is usually circular

- 7) The relative roughness is a fixed quantity.
- 8) No such dependence.

- Canal
- Flume
- Chute
- Drop
- Culvert
- Open-Flow Tunnel

 CANAL is usually a long and mild-sloped channel built in the ground.



FLUME is a channel usually supported on or above the surface of the ground to carry water across a depression.





### • CHUTE is a channel having steep slopes.



 DROP is similar to a chute, but the change in elevation is affected in a short distance.



CULVERT is a covered channel flowing partly full, which is installed to drain water through highway and railroad embankments.







 OPEN-FLOW TUNNEL is a comparatively long covered channel used to carry water through a hill or any obstruction on the ground.





### Channel Geometry

- A channel built with constant cross section and constant bottom slope is called a PRISMATIC CHANNEL.
- Otherwise, the channel is **NONPRISMATIC**.

THE CHANNEL SECTION is the cross section of a channel taken normal to the direction of the flow.

THE VERTICAL CHANNEL SECTION is the vertical section passing through the lowest or bottom point of the channel section.





### Geometric Elements of Channel Section

THE DEPTH OF FLOW, y, is the vertical distance of the lowest point of a channel section from the free surface.



### **Geometric Elements of Channel Section**

THE DEPTH OF FLOW SECTION, d, is the depth of flow normal to the direction of flow.



## **Geometric Elements of Channel Section**

#### **THE TOP WIDTH, T,**

is the width of the channel section at the free surface.

#### THE WATER AREA, A,

is the cross-sectional area of the flow normal to the direction of flow.

#### **THE WETTED PERIMETER**, P,

is the length of the line of intersection of the channel wetted surface with a crosssectional plane normal to the direction of flow.

#### **THE HYDRAULIC RADIUS**, R = A/P,

is the ratio of the water area to its wetted perimeter.

#### **THE HYDRAULIC DEPTH, D = A/T**,

is the ratio of the water area to the top width.







(a) Circular channel ( $\theta$  in rad)



- $R_h = \frac{A_c}{p} = \frac{y(b + y/\tan\theta)}{b + 2y/\sin\theta}$ 
  - (b) Trapezoidal channel



(c) Rectangular channel



# Channel Geometry

- The wetted perimeter *does not* include the free surface.
- Examples of R for common geometries shown in Figure at the left.



Geometric elements for different channel cross sections

	rectangular	trapezoidal	triangular circular		parabolic		
	$ \begin{array}{c} B \\ \hline  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\  \\ $	B	B	$D \xrightarrow{B} h$			
flow area A	bh	(b+mh)h	$mh^2$	$\frac{1}{8}(\theta-\sin\theta)D^2$	$\frac{2}{3}Bh$		
wetted perimeter P	b+2h	$b+2h\sqrt{1+m^2}$	$2h\sqrt{1+m^2}$	$\frac{1}{2} \theta D$	$B + \frac{8}{3} \frac{h^2}{B} \qquad *$		
hydraulic radius R <sub>h</sub>	$\frac{bh}{b+2h}$	$\frac{(b+mh)h}{b+2h\sqrt{1+m^2}}$	$\frac{mh}{2\sqrt{1+m^2}}$	$\frac{1}{4} \left[ 1 - \frac{\sin \theta}{\theta} \right] D$	$\frac{2B^2h}{3B^2+8h^2}$		
top width B	b	b+2mh	2mh	$or \qquad (\sin \theta / 2)D \\ 2\sqrt{h(D-h)}$	$\frac{3}{2}Ah$		
hydraulic depth D <sub>h</sub>	h	$\frac{(b+mh)h}{b+2mh}$	$\frac{1}{2}h$	$\left[\frac{\theta - \sin\theta}{\sin\theta/2}\right]\frac{D}{8}$	$\frac{2}{3}h$		
* Valid for $0 < \xi \le 1$ where $\xi = 4h/B$							

 $\xi > 1$  then  $P = (B/2) \left[ \sqrt{1 + \xi^2} + (1/\xi) \ln(\xi + \sqrt{1 + \xi^2}) \right]$ 

If

## Types of Flow

 Criterion: Change in flow depth with respect to time and space



## Types of Flow

 Criterion: Change in discharge with respect to time and space



# **Classification of Open-Channel Flows**

- Obstructions cause the flow depth to vary.
- Rapidly varied flow (RVF) occurs over a short distance near the obstacle.
- Gradually varied flow (GVF) occurs over larger distances and usually connects UF and RVF.



#### Steady non-uniform flow in a channel.



### State of Flow



Note that **R** in Reynold number is Hydraulic Radius



# Effect of Gravity

- In open-channel flow the driving force (that is the force causing the motion) is the component of gravity along the channel bottom. Therefore, it is clear that, the effect of gravity is very important in open-channel flow.
- In an open-channel flow Froude number is defined as:

$$F_r = \frac{\text{Inertia Force}}{\text{Gravity Force}}, \text{ and } F_r^2 == \frac{V^2}{gD} \text{ or } F_r = \frac{V}{\sqrt{gD}}$$

- In an open-channel flow, there are three types of flow depending on the value of Froude number:
  - $F_r > 1 \longrightarrow$  Supercritical Flow
  - $F_r=1 \longrightarrow Critical Flow$
  - $F_r < 1 \longrightarrow Subcritical Flow$

In wave mechanics, the speed of propagation of a small amplitude wave is called the celerity, C.

If we disturb water, which is not moving, a disturbance wave occur, and it propagates in all directions with a celerity, C, as:



For a rectangular channel, the hydraulic depth, D=y. Therefore, Froude number becomes:  $F_r = \frac{V}{\sqrt{\sigma v}} = \frac{V}{C}$  Now let us consider propagation of a small amplitude wave in a supercritical open channel flow:

 $C \leftrightarrow C$ 

 $F_r > 1$ , i.e; V > C

Since V > C, it CANNOT propagate upstream it can propagate only towards downstream with a pattern as follows:



Disturbance will be felt only within this region

 This means the flow at upstream will not be affected.
 In other words, there is no hydraulic communication between upstream and downstream flow.



In other words, there is hydraulic communication between upstream and downstream flow. Now let us consider propagation of a small amplitude wave in a critical open channel flow:

$$F_r = 1$$
, i.e;  $V = C$   $C \leftarrow O \rightarrow C$ 

Since V = C, it can propagate only downstream with a pattern as follows:

This means the flow at downstream will be affected.

### State of Flow



D in Froude Number is Hydraulic Depth

# **Velocity Profiles**

 In order to understand the velocity distribution, it is customary to plot the isovels, which are the equal velocity lines at a cross section.



- Velocity is zero on bottom and sides of channel due to no-slip condition the maximum velocity is usually below the free surface.
- It is usually three-dimensional flow.
- However, 1D flow approximation is usually made with good success for many practical problems.



## **Velocity Distribution**

The velocity distribution in an open-channel flow is quite nonuniform because of :

- Nonuniform shear stress along the wetted perimeter,
- Presence of free surface on which the shear stress is zero.



# Uniform Flow in Channels



Head loss = elevation loss  $h_L = z_1 - z_2 = S_0 L$ 

- Flow in open channels is classified as being *uniform* or *nonuniform*, depending upon the depth *y*.
- Depth in Uniform Flow is called normal depth y<sub>n</sub>
- Uniform depth occurs when the flow depth (and thus the average flow velocity) remains constant
- Common in long straight runs
- Average flow velocity is called uniform-flow velocity V<sub>o</sub>
- <u>Uniform depth</u> is maintained as long as the slope, cross-section, and surface roughness of the channel remain unchanged.
- During uniform flow, the terminal velocity reached, and the head loss equals the elevation drop

## Uniform Flow in Channels



#### Non-uniform gradually varied flow. $S_f \neq S_w \neq S_o$



#### Chezy equation (1768)

Introduced by the French engineer Antoine Chezy in 1768 while designing a canal for the watersupply system of Paris

$$V = C \sqrt{R_h S_f}$$

C = Chezy coefficient

$$60\frac{\sqrt{m}}{s} < C < 150\frac{\sqrt{m}}{s}$$

where

60 is for rough and 150 is for smooth also a function of **R** (like *f* in Darcy-Weisbach)

#### Darcy-Weisbach equation (1840)

$$h_{f} = f \frac{L}{D} \frac{V^{2}}{2g} = f \frac{L}{4R_{h}} \frac{V^{2}}{2g}$$
$$LS_{f} = f \frac{L}{4R_{h}} \frac{V^{2}}{2g}$$

$$R_h S_f = f \frac{V^2}{8g} \implies V = \sqrt{\frac{8g}{f}} \sqrt{R_h S_f}$$

# Manning Equation for Uniform Flow

$$V = \frac{1}{n} R^{2/3} S_{0}^{1/2}$$

Discharge:

$$Q = VA$$

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$$

# Manning Equation (1891)

$$V = \frac{1}{n} R_{h}^{2/3} S_{f}^{1/2}$$
 (SI System)

Notes: The Manning Equation 1) is dimensionally nonhomogeneous 2) is very sensitive to *n* 

Is *n* only a function of roughness? NO!

Dimensions of n? T / L<sup>1/3</sup>

 $V = \frac{1.49}{n} R_h^{2/3} S_f^{1/2}$  (English system)

# Values of Manning n

С. Х	Values of	Mar	nning's	Rough	ness (	Coefficie	ent n
Glass, plastic, machined me	tal	•••	••		• •	••	0.010
Dressed timber, joints flush	••	••	••	• •			0.011
Sawn timber, joints uneven		•••	••		••	• •	0.014
Cement plaster	(*) * (*)	••		• •	••		0.011
Concrete, steel troweled			• •	• •	••	••	0.012
Concrete, timber forms, unf	inished	••	• •	• •	••	••	0.014
Untreated gunite	••		• •	••	••	••	0.015-0.017
Brickwork or dressed masor	nry	••	• •	••	••		0.014
Rubble set in cement			••	••	••	••	0.017
Earth, smooth, no weeds	••	• •			••		0.020
Earth, some stones and wee	ds	••	• •		••	••	0.025
Natural river channels:							
Clean and straight		••	• •	••	••	••	0.025-0.030
Winding, with pools and	shoals				••	• •	0.033-0.040
Very weedy, winding and	overgrown				••		0.075-0.150
Clean straight alluvial cha	nnels	•••		• •	••	•	0.031d1/6
s sector Al						(d=D)	-75 size in ft.)

 $n = 0.031d^{1/6}$  d in ft d = median size of bed material  $n = 0.038d^{1/6}$  d in m

#### **Relation between Resistance Coefficients**



## Example 1

A trapezoidal channel has a base width b = 6 m and side slopes 1H:1V. The channel bottom slope is So = 0.0002 and the Manning roughness coefficient is n = 0.014. Compute a)the depth of uniform flow if Q = 12.1 m3/s b)the state of flow



## Solution of Example 1

a) Manning's equation is used for uniform flow;

$$Q = \frac{A}{n} R^{2/3} \sqrt{S_o}$$

 $A = b.y_o + 2.(y_o^2/2) = y_o(b + y_o)$  $P = b + 2\sqrt{2} y_o = 6 + 2\sqrt{2} y_o$ 

So = 0.0002 n = 0.014 Q = 12.1 m3/s

$$=$$

$$y_0$$

Y(m)	<b>A</b> (m²)	P(m)	R(m)	<b>A</b> R <sup>2/3</sup>
1	7	8.28	0.84	6.23
1.2	8.64	9.39	0.92	8.17
1.4	10.36	9.96	1.04	10.63
1.5	11.25	10.24	1.098	11.976

$$AR^{2/3} = \frac{Qn}{\sqrt{S_o}} = 11.98$$

$$11.98 = y_o (6 + y_o) \left( \frac{y_o (6 + y_o)}{6 + 2\sqrt{2}y_o} \right)^{2/3}$$

by trial & error y<sub>o</sub>=1.5 m

# Solution of Example 1

b) The state of flow

$$Fr = \frac{V_{ave}}{\sqrt{gD}}, D = \frac{A}{T}, T = b + 2y_{o}$$

$$A = 1.5 (6+1.5) = 11.25 m^{2}$$

$$T = 6+2 \times 1.5 = 9 m$$

$$D = 11.25 / 9 = 1.25 m$$

$$V_{ave} = \frac{Q}{A} = \frac{12.1}{11.25} = 1.076 m/s$$

$$Fr = \frac{1.076}{\sqrt{9.81x1.25}} = 0.307 < 1$$
 Subcritical



# **Compound Channel**



#### **Generalized section representation**



### **Composite Section**

A channel section, which is composed, of different roughness along the wetted perimeter is called composite section. For such sections an equivalent Manning roughness can be defined as



### **Compound Channel**

is the channel for which the cross section is composed of several distinct subsections



#### Discharge computation in Compound Channels

- To compute the discharge, the channel is divided into 3 subsections by using vertical interfaces as shown in the figure:
- Then the discharge in each subsection is computed separately by using the Manning equation.
- In computation of wetted perimeter, water-to-water contact surfaces are not included.



## Example 2

Determine the discharge passing through the cross section of the compound channel shown below.

The Manning roughness coefficients are  $n_1 = 0.02$ ,  $n_2 = 0.03$  and  $n_3 = 0.04$ . The channel bed slope for the whole channel is So = 0.008.



### Solution of Example 2

Divide the channel into 3 subsections by using vertical interfaces as shown in the figure:



# Example 2

**For the main channel (subsection I):** 

The main channel is a composite channel too. Therefore, we need to find an equivalent value of n.

$$n_{eq} = \left(\frac{\sum n_i^2 P_i}{\sum P_i}\right)^{1/2}$$

$$n_{eq} = \left(\frac{n_1^2 5 + n_2^2 \sqrt{5} * 2 + n_3^2 \sqrt{5} * 2}{5 + 4\sqrt{5}}\right)^{1/2} = \left(\frac{(0.02)^2 5 + 2\sqrt{5}(0.03^2 + 0.04^2)}{5 + 4\sqrt{5}}\right)^{1/2}$$

$$n_{eq} = 0.03074$$

$$A_1 = \frac{1}{2}(5+13) * 2 + (13 * 1) = 31 \text{ m}^2$$

$$P_1 = 5 + 2x2\sqrt{5} = 13.944 \text{ m}$$

$$Q_1 = \frac{31}{0.03074} \left(\frac{31}{13.944}\right)^{2/3} \sqrt{0.008} = 154.05 \text{ m}^3 / s$$

$$A_2 = 10 * 1 = 10 m^2$$

$$P_2 = 10 + 1 = 11 m$$

$$Q_2 = \frac{10}{0.030} \left(\frac{10}{11}\right)^{2/3} \sqrt{0.008} = 27.97 m^3 / s$$

#### For the subsection III:

$$A_{3} = \frac{1}{2}(10+11) * 1 = 10.5 \text{ m}^{2}$$

$$P_{3} = 10 + \sqrt{2} = 11.41 \text{ m}$$

$$Q_{3} = \frac{10.5}{0.040} \left(\frac{10.5}{11.41}\right)^{2/3} \sqrt{0.008} = 22.21 \text{ m}^{3} / s$$

 $Q_{total} = Q_1 + Q_2 + Q_3 = 154.05 + 27.97 + 22.21 = 204.23 \text{ m}^3 / s$ 

# **Energy Concept**

- Component of energy equation
  - 1) z is the elevation head
  - 2) y is the gage pressure head-potential head
  - 3)  $V^2/2g$  is the dynamic head-kinetic head



# **Continuity and Energy Equations**



$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + (S_f - S_o)\Delta x$$

 1D steady continuity equation can be expressed as

 $V_1A_1 = V_2A_2$ 

 1D steady energy equation between two stations

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + h_\ell$$

Head loss h<sub>L</sub>

$$\mathsf{h}_{\ell} = \mathsf{S}_{\mathsf{f}} \Delta \mathsf{X}$$

 The change in elevation head can be written in terms of the bed slope θ

$$S_{o} = \frac{(z_1 - z_2)}{\Delta x}$$

## Example 3

Water flows under a sluice gate in a horizontal rectangular channel of 2 m wide. If the depths of flow before and after the gate are 4 m, and 0.50 m, compute the discharge in the channel.



## Solution:

The energy equation between sections (1) and (2) is:  $H_1 = H_2 + h_f$ 

- The head loss between sections (1) and (2) can be neglected.
- Therefore:

$$z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{V_2^2}{2g}$$

Choose the channel bottom as datum. Then  $z_1 = z_2 = 0$ ,  $\alpha = 1$ 

Substituting above and Q = V \* (b\*y) energy equation between sections (1) and (2) becomes:

$$y_1 + \frac{Q^2}{2g(b^2 y_1^2)} = y_2 + \frac{Q^2}{2g(b^2 y_2^2)}$$

$$\frac{Q^2}{2gb^2} \left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right) = y_1 - y_2$$
  
$$\frac{Q^2}{2g*4} \left(\frac{1}{0.50^2} - \frac{1}{4^2}\right) = 3.5$$
  
solving for Q = 8.352 m<sup>3</sup>/s

**EXAMPLE 4** Water flow with a velocity of 3 m/s, and a depth of 3 m in a rectangular channel of 2 m wide. Then there is an upward step of 30 cm as shown in figure below. Compute the depth of flow over the step.



Energy Eq. Between Sections (1) & (2):

$$z_1 + y_1 + \frac{Q^2}{2gb^2y_1^2} = z_2 + y_2 + \frac{Q^2}{2gb^2y_2^2} \quad Q = V_1(by_1) = V_2(by_2) = 3.2.3 = 18 \text{ m}^3 / s$$

$$3 + \frac{18^2}{2g \cdot 2^2 3^2} = 0.30 + y_2 + \frac{18^2}{2g \cdot 2^2 y_2^2} \Box \ddot{\mathrm{E}} \qquad y_2 + \frac{4.1284}{y_2^2} = 3.1587$$

The last equation contains only one unknown:  $y_2$ . However, it is a third degree polynomial of  $y_2$ .

- Y<sup>3</sup>-3.1587y<sup>2</sup>+4.1284=0 This polynomial has three possible solutions:
- Y<sub>(1)</sub>=2.496≈2.5 m
- Y<sub>(2)</sub>=1.66 m
- Y<sub>(3)</sub>=-0.996 ≈-1 m
- Negative depth is not acceptable
- But both 2.5 m and 1.66 m depths are quite possible.
- Which one will occur on the step????
- Nor Energy equation neither continuity equation will help to decide.
- Luckily, in 1912, Bakhmeteff introduced the concept of SPECIFIC ENERGY, which is the key to even the most complex openchannel flow phenomena.