Open-Response Items to Assess Geometric Understanding

Jessica Masters (jessica.masters@bc.edu)

Rachel Wing DiMatteo (rwing@edc.org)

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Regional Meeting

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Session Overview

- Introduction and Overview of the Diagnostic Geometry Assessment Project

- Targeted Misconceptions
  - Geometric Measurement
  - Shape Properties
  - Transformations

- Open-Response Items and Representations of Misconceptions

- Prevalence Estimates of Misconceptions
  - Geometric Measurement
  - Transformations

- Conclusions and Future Work

The Diagnostic Geometry Assessment Project (DGA)

- 4-year project funded by the Institute of Education Sciences
  - Technology and Study Assessment Collaborative (inTASC) at Boston College
  - Center for Leadership and Learning Communities at the Education Development Center, Inc. (EDC)
- Develop a free-standing, online system with three core components:
  - Online tests (10-12 items per misconception)
  - Instant feedback for teachers about student performance
    - Correct/Incorrect
    - Possible Misconception
  - Instructional resources to target misconceptions
- First create open-response items to elicit responses indicative of misconceptions
- Future phases of the project :
  - Convert items to closed-response
  - Develop instructional resources
 - Evaluate the effectiveness of the system

A Word about Our Use of the Word *Misconception*...

- Misconception is used to represent:
  - Flawed preconceptions
  - Underdeveloped reasoning
  - Traditional misconceptions

- Any systematic source of difficulty that students have in their reasoning.

- Targeted instruction can be used to help students refine, reorganize, or build upon their knowledge to overcome misconceptions.
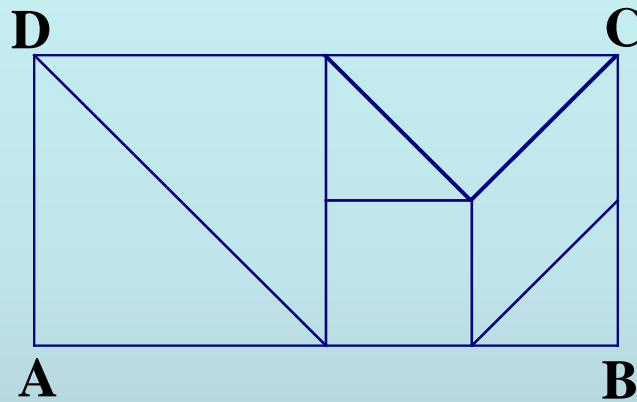
Targeted Misconceptions

inTASC
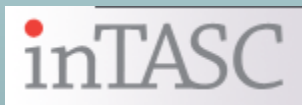
Transcript: Four 6th-graders measure area with tangrams

Students created a non-square rectangle with 7 tangram pieces.

The discussion in the transcript is their response to the prompt:

Describe a way to find the area of your rectangle without using a ruler.

Transcript: Four 6th-graders measure area with tangrams

1. Read transcript.

2. Characterize Marco's understanding of area (with partner).

3. Recharacterize after research about the measurement misconception is shared.
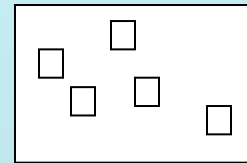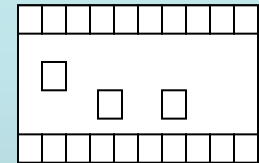
Geometric Measurement

- Students have difficulties in mentally structuring space and connecting structured space to measurement formulas
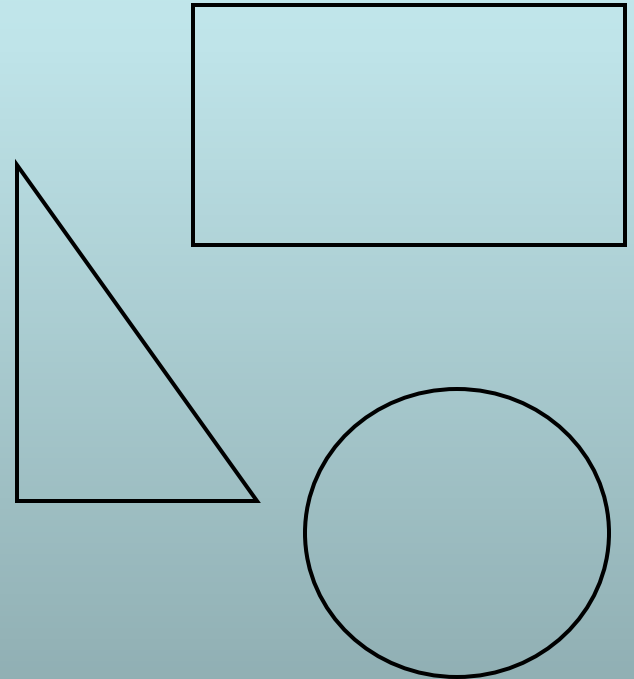
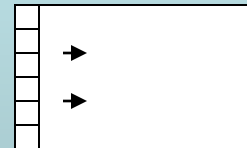# Geometric Measurement

- Students have difficulties in mentally structuring space and connecting structured space to measurement formulas

Area of a rectangle = $\ell \times w$

Area of a triangle = ½bh
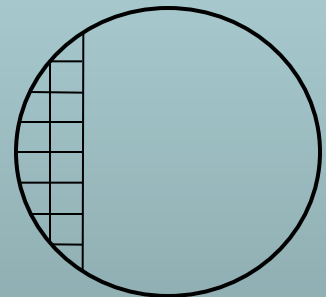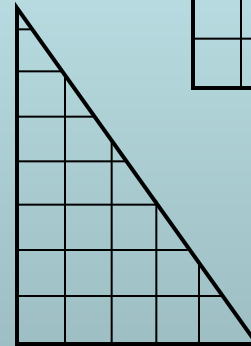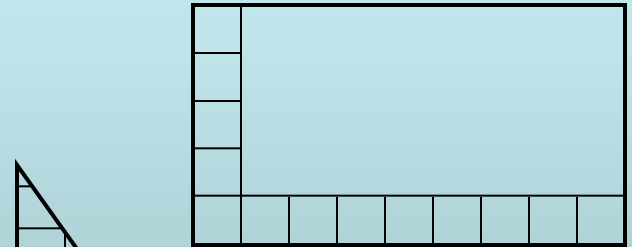
Area of a circle = πr^2

Geometric Measurement

- Students have difficulties in **mentally structuring space** and connecting structured space to measurement formulas
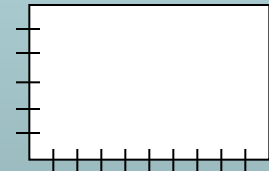
1. No organization as structure space

2. Structure portions of the space

3. Can structure in an organized way but cannot mentally iterate rows or columns
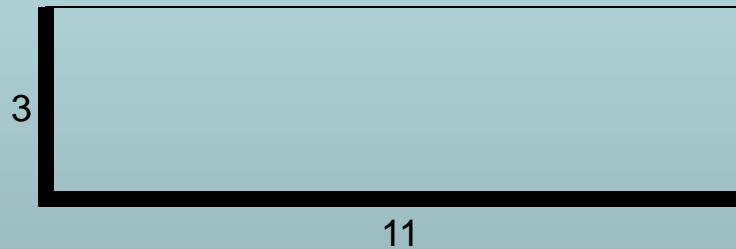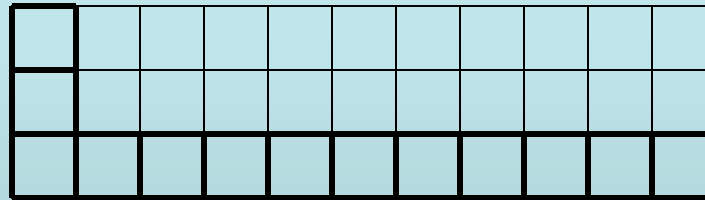
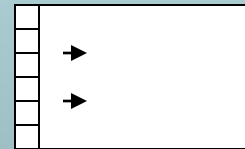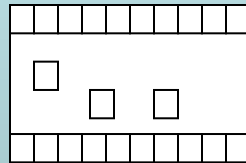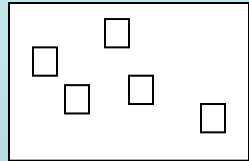4. Can abstract the rows and columns structure

Geometric Measurement

- Students have difficulties in mentally structuring space and **connecting structured space to measurement formulas**

3

11

Area = 3 x 11

Geometric Measurement

- Students have difficulties in mentally structuring space and connecting structured space to measurement formulas

Transcript: Four 6th-graders measure area with tangrams

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Shape Properties

- Students have difficulties when reasoning with *Concept Images* without *Concept Definitions*

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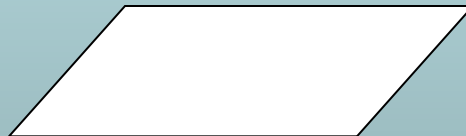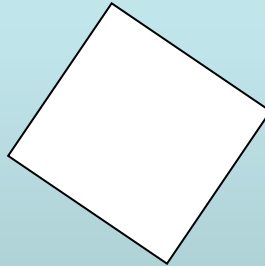
Rectangle

Square

Parallelogram

Shape Properties

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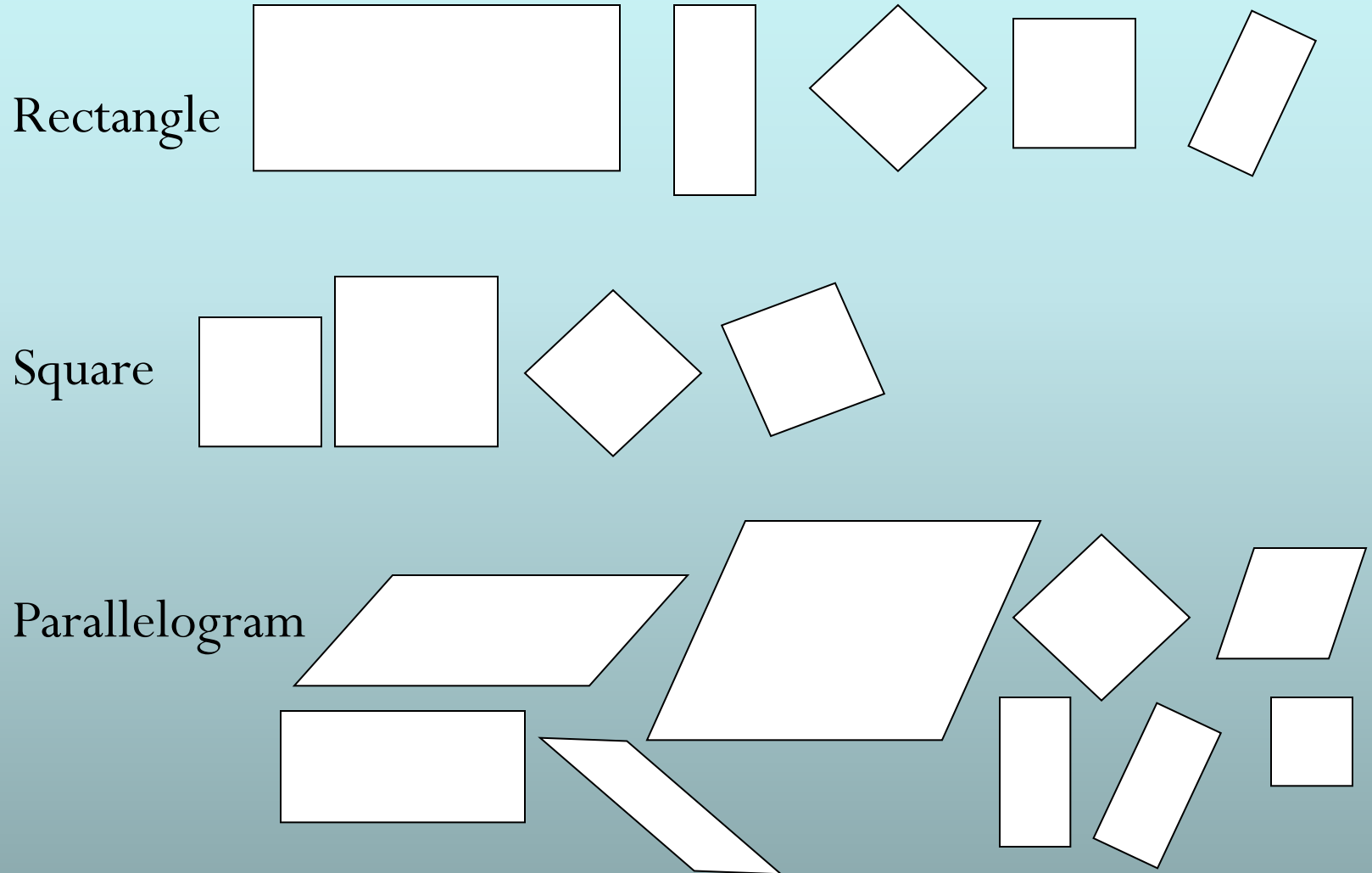
Rectangle

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Rectangle

Square

Parallelogram

Shape Properties

- Students have difficulties when reasoning with *Concept Images* without *Concept Definitions*

Rectangle ➔ 4 sides; 4 right angles

Square ➔ 4 congruent sides; 4 right angles

Parallelogram ➔ 4 sides; opposite sides are parallel
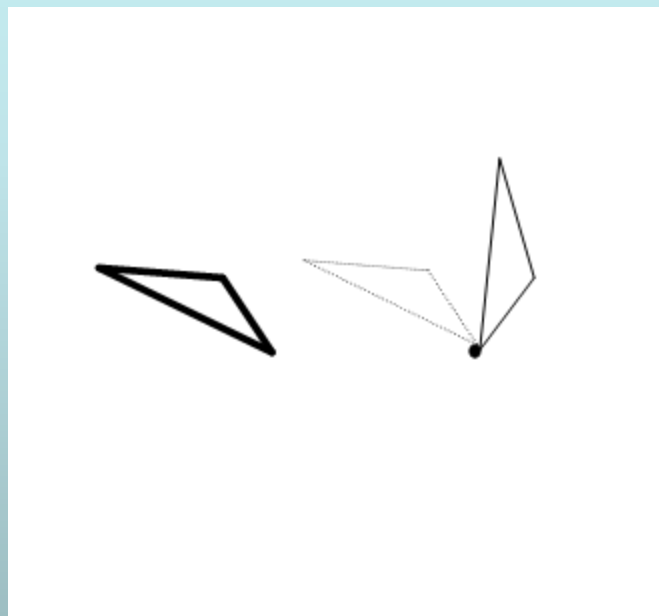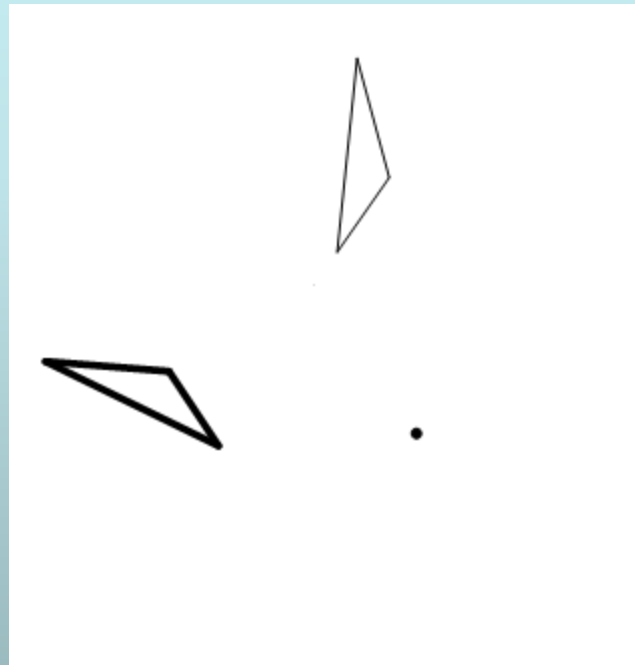
Transformations

- Students struggle with distant points of rotation or distant lines of reflection
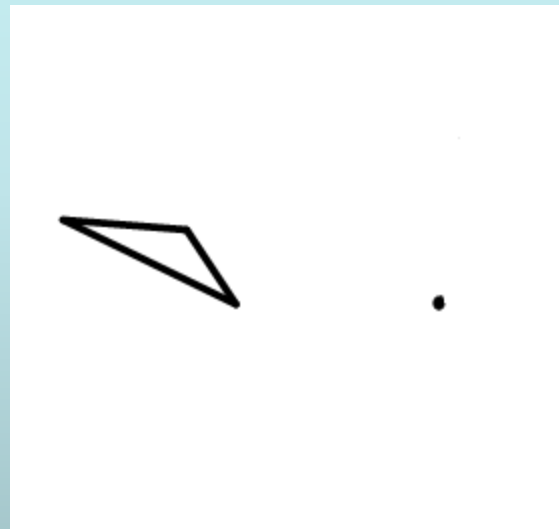
Transformations

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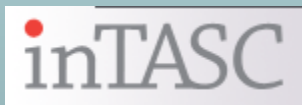
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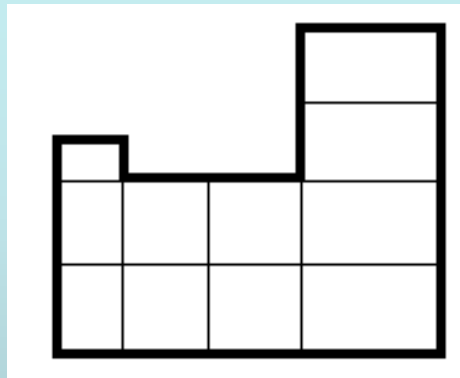
Open-Response Items and Representations of Misconceptions

Geometric Measurement Item

Jack was trying to determine the area of this figure so he decided to draw units in to fill the space.

Do you agree with how Jack drew his units?

If you agree, explain how his units will help him find the area.

If you disagree, explain why you disagree.

Geometric Measurement Item - Responses

Student A

No, I disagree. Because they are all different sizes. You can't find area if they are all different sizes.

Student B

Yes, I agree. He can just count his squares and that's the area.

Student C

I agree. He should count how many units are on the bottom and the right side and times them. Then he'll know the area.

Shape Properties Item

Is this rectangle ⬚ a parallelogram?

If you think it's a parallelogram, how do you know?

If you don't think it's a parallelogram, explain in words what you would have to do to change it into a parallelogram.

Shape Properties Item - Responses

Student A

Yes. I know it's a parallelogram because it has four angles and four sides.
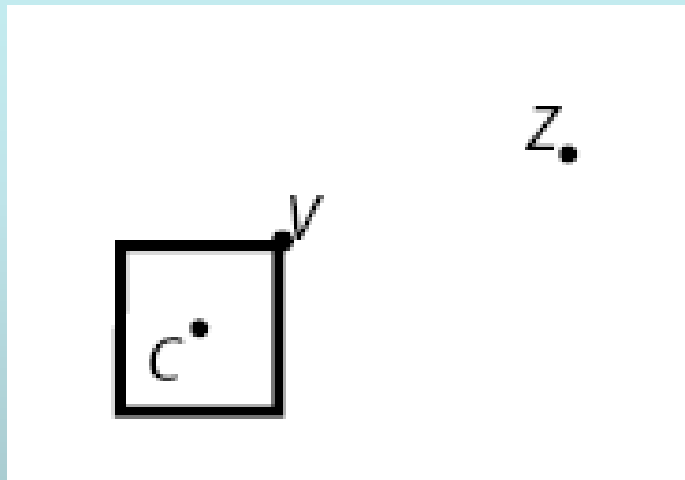
Student B

Yes. I know because the top side is parallel to the bottom and the right side is parallel to the left side.

Student C

No. You have to slant the two short sides to make this shape look like a parallelogram.
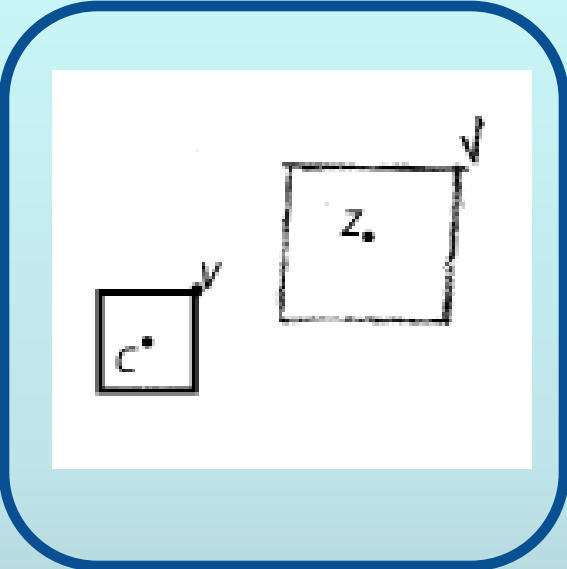
Transformation Item

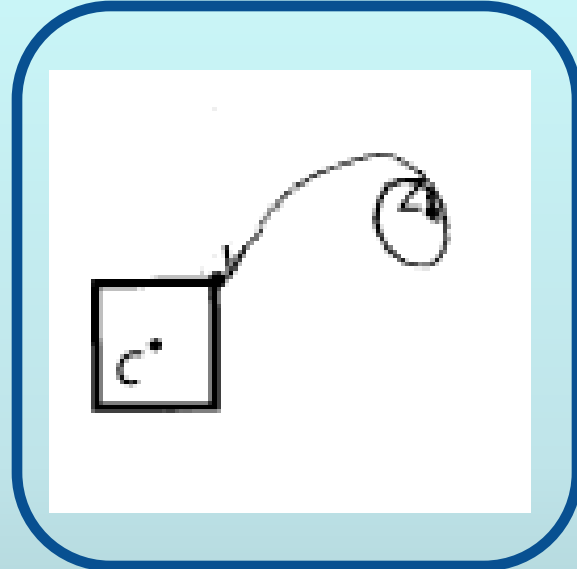Draw the path of point V if the square is rotated (turned) 360 degrees around point Z.

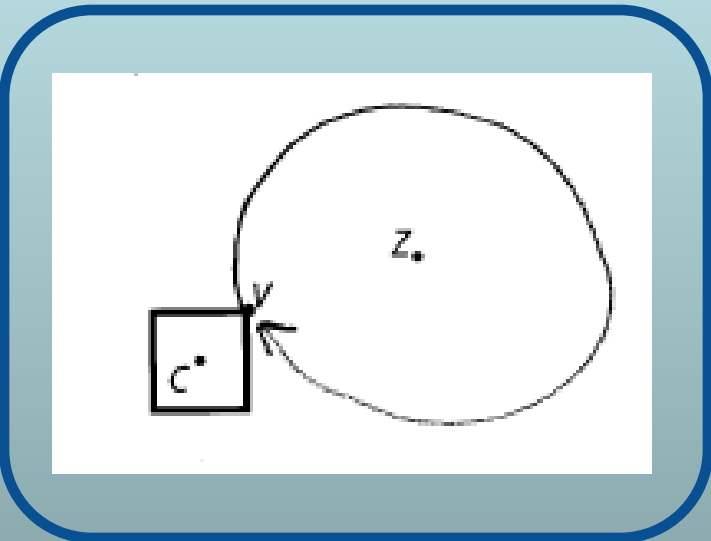# Transformation Item - Responses
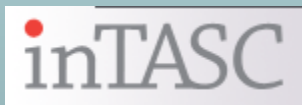
Student A

Student B

Student C

Handout with open-ended items to try out with students

Prevalence Estimates of Misconceptions

DGA: Phase One

- Teachers administered 2 paper test booklets to students:
  - 1 in *either* Geometric Measurement or Transformations
  - 1 in Shape Properties
- Two researchers coded each student response as:
  - Knower
  - Misconceiver
  - Mistaker
  - Blank/Illegible
- Average agreement between researchers was approximately 90%.
- Note: These are initial estimates only- later phases will conduct larger-scale and more scientific evaluations.

DGA Phase One Data

- Geometric Measurement- Group 1
  - 27 Teachers
  - 1,087 Students
  - 12 Questions (7 used for current analysis)
- Geometric Measurement- Group 2
  - 25 Teachers
  - 936 Students
  - 13 Questions (6 used for current analysis)
- Transformations
  - 28 Teachers
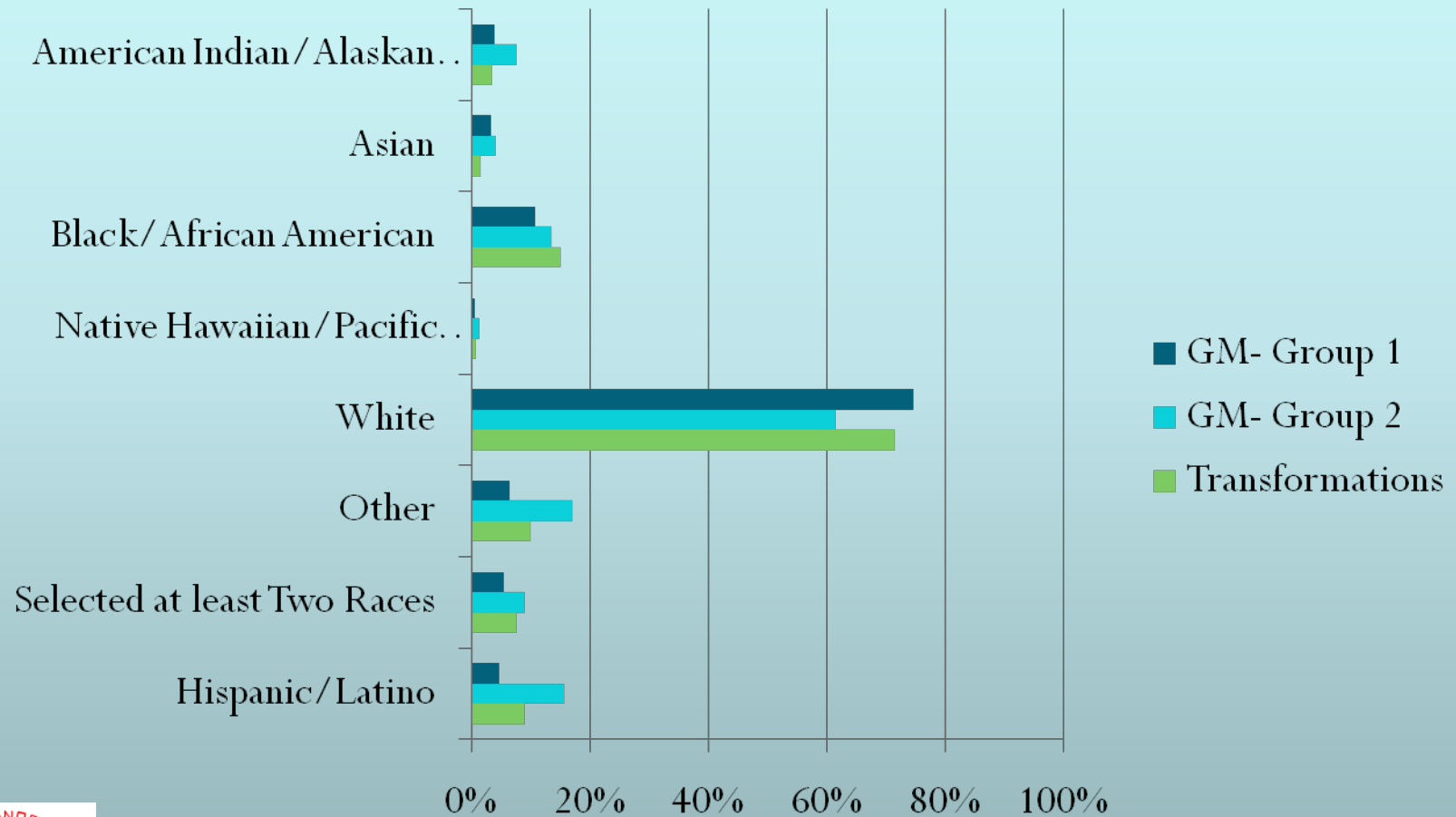 - 1, 096 Students
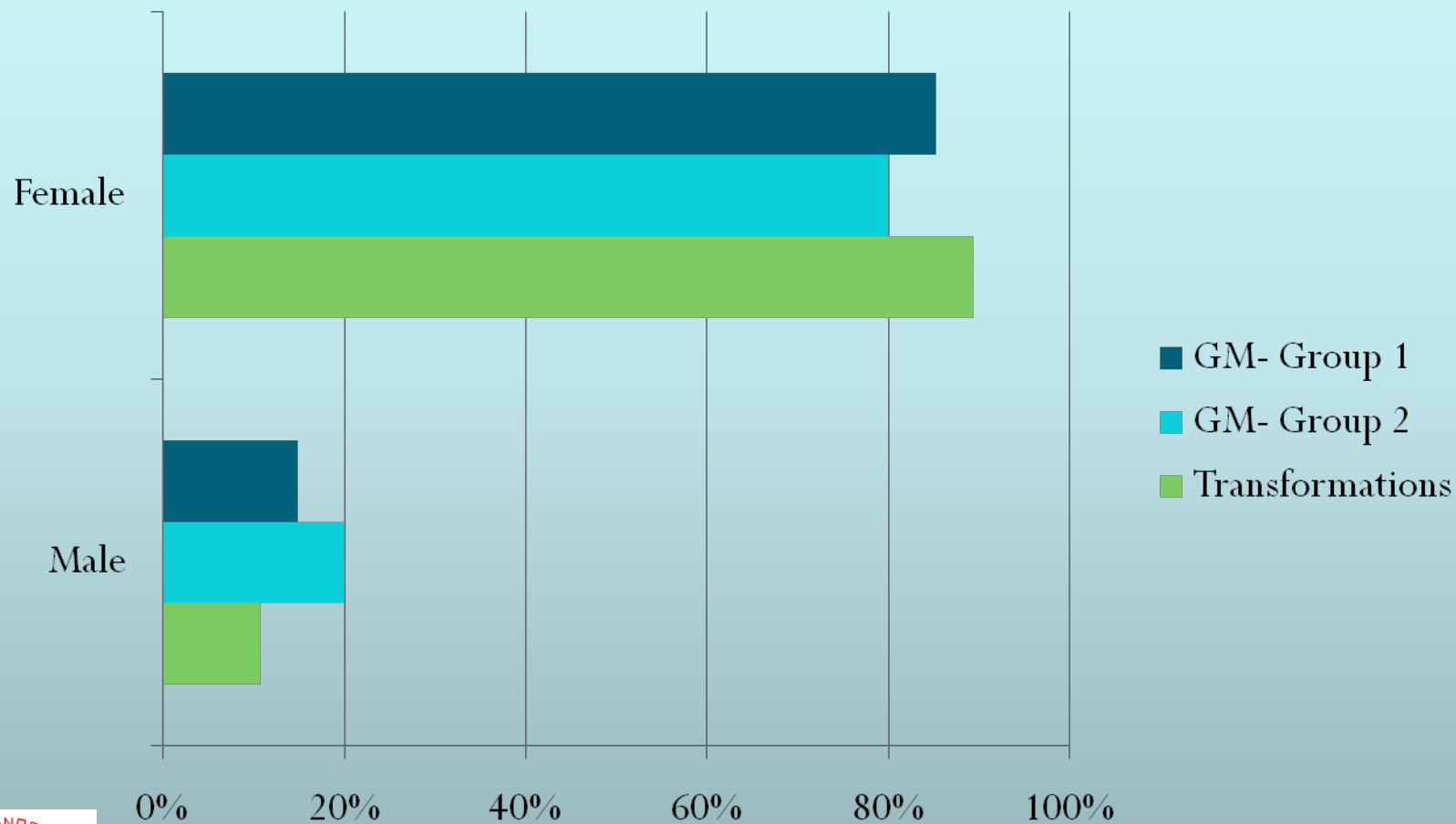 - 10 Questions (9 used for current analysis)
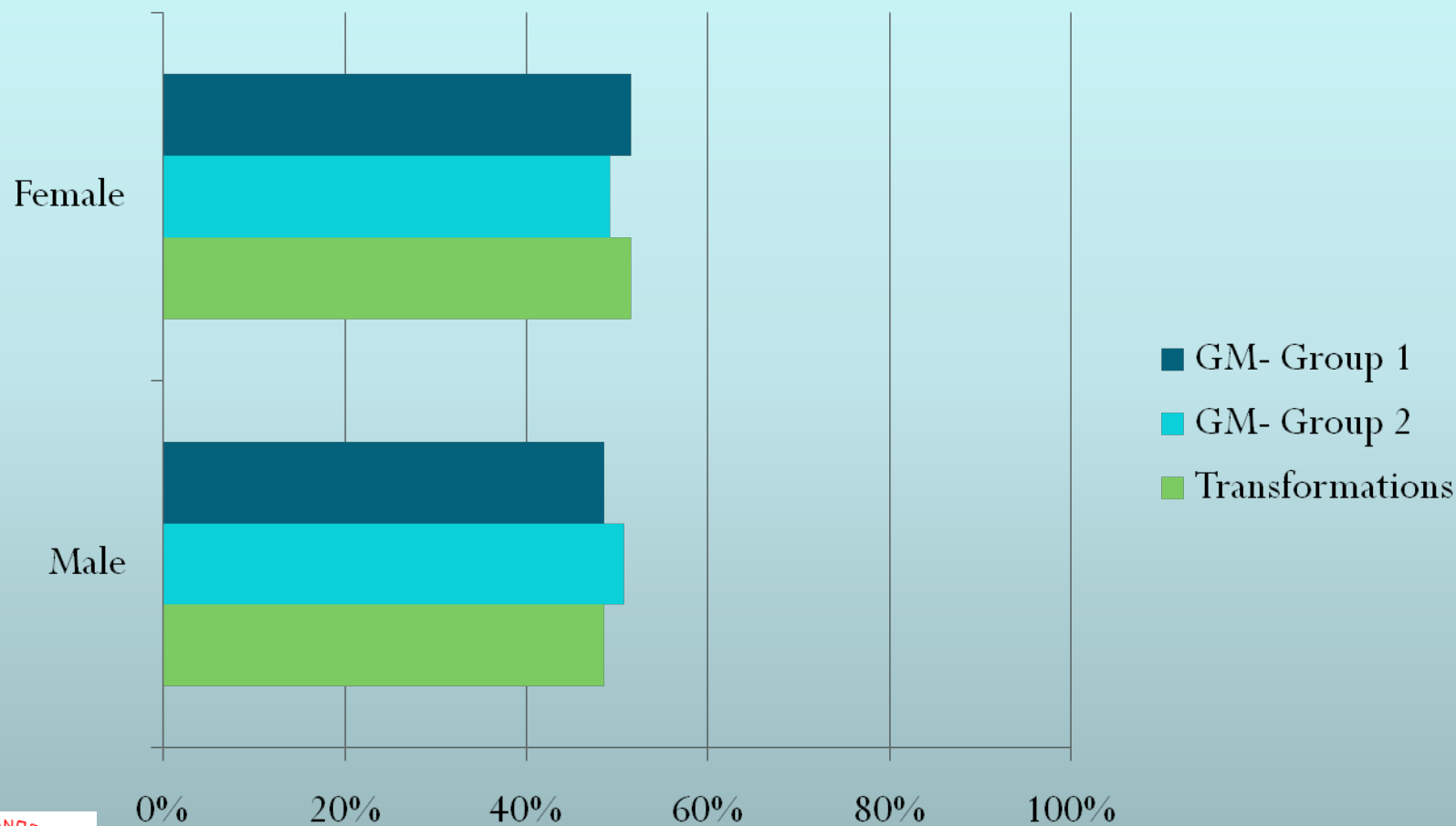
Race of Participating Teachers
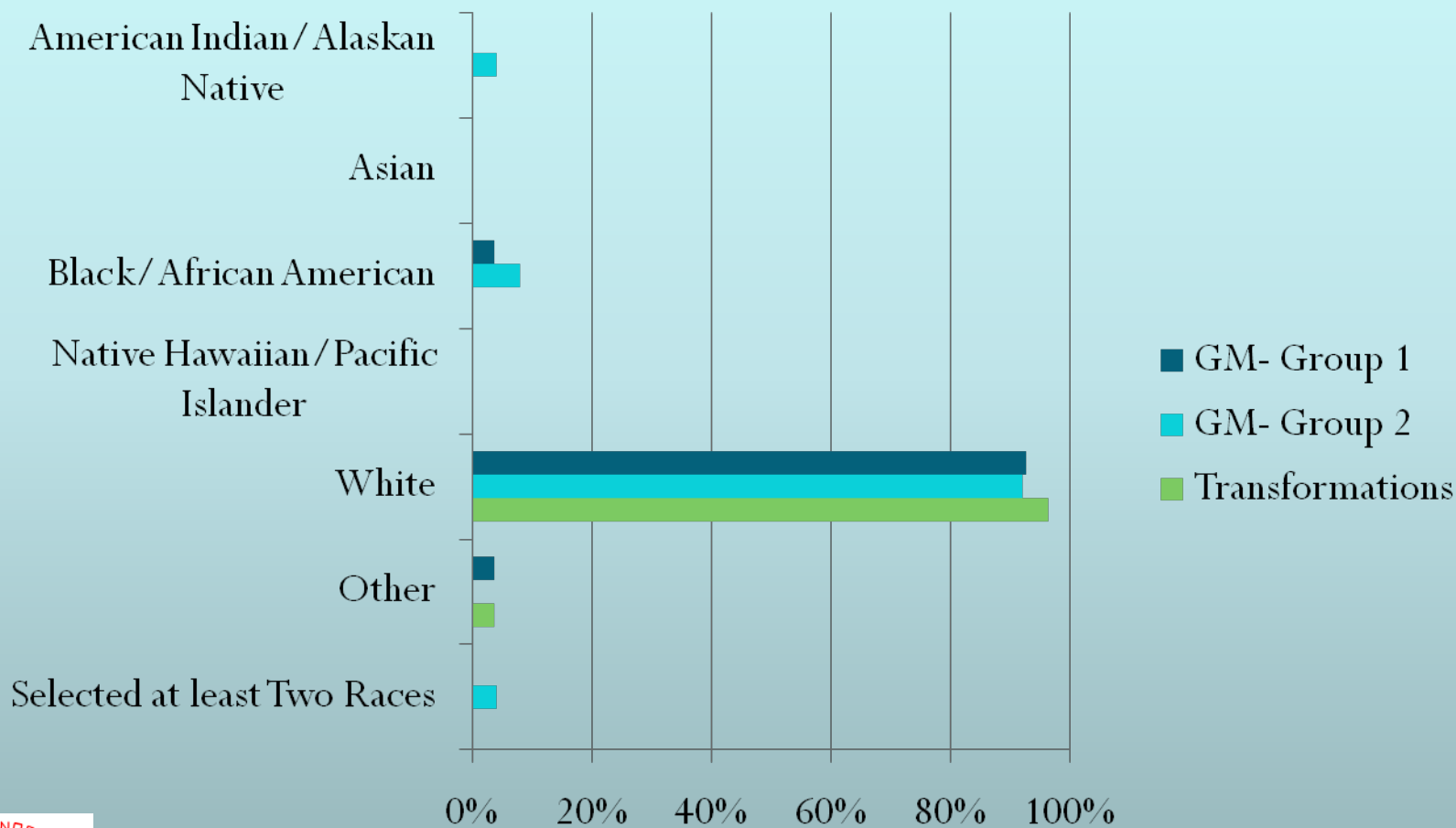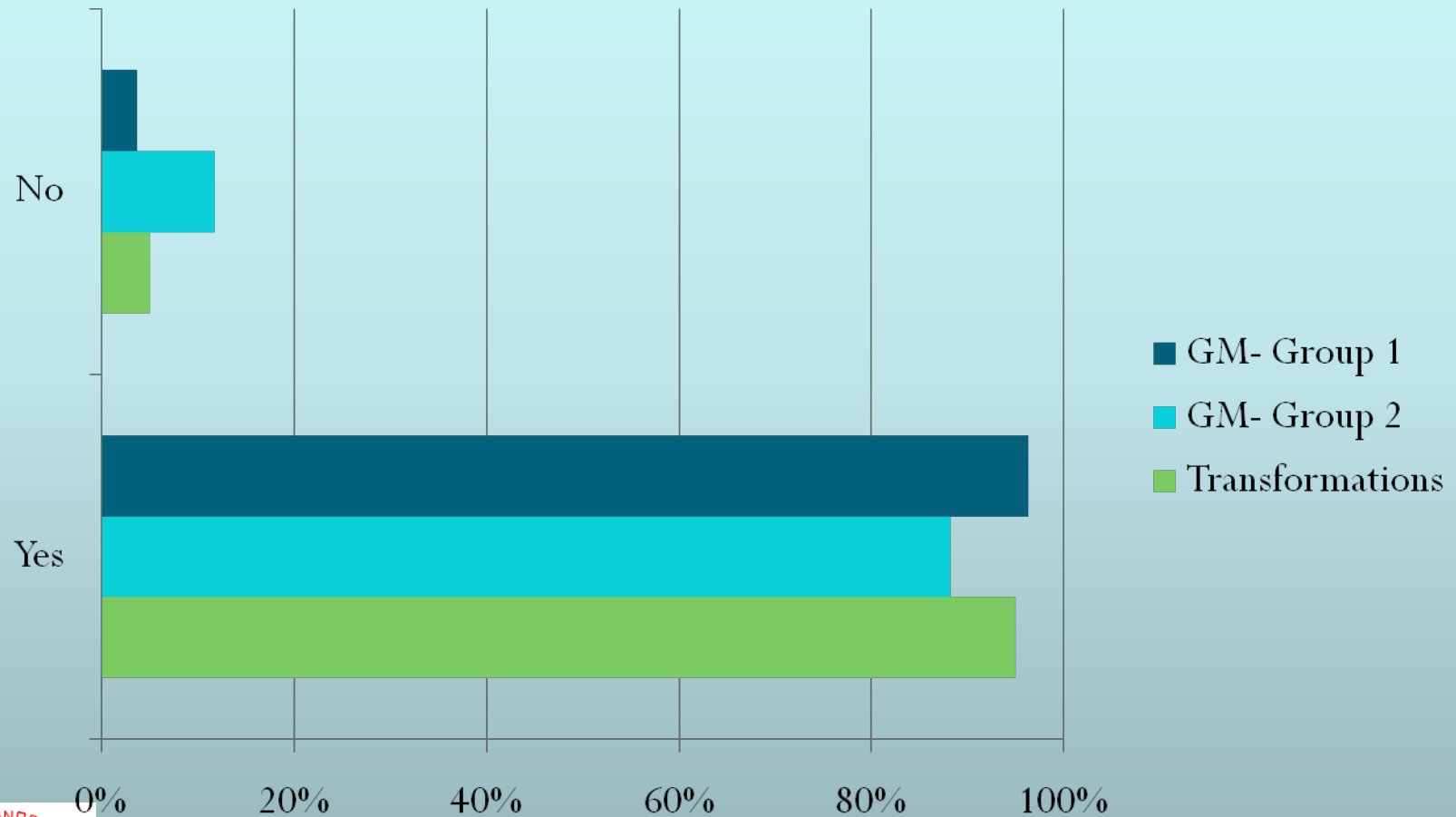
Race/Ethnicity of Participating Students
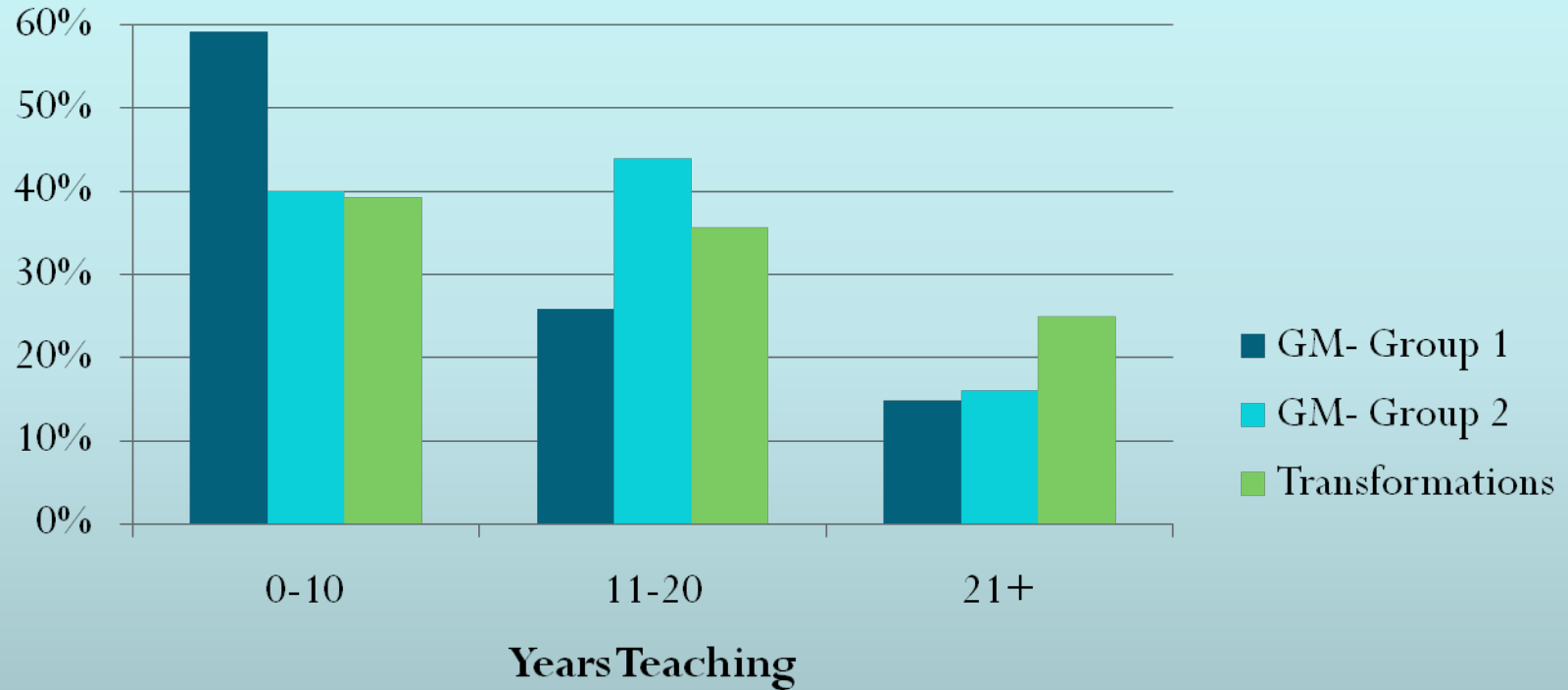
Gender of Participating Teachers

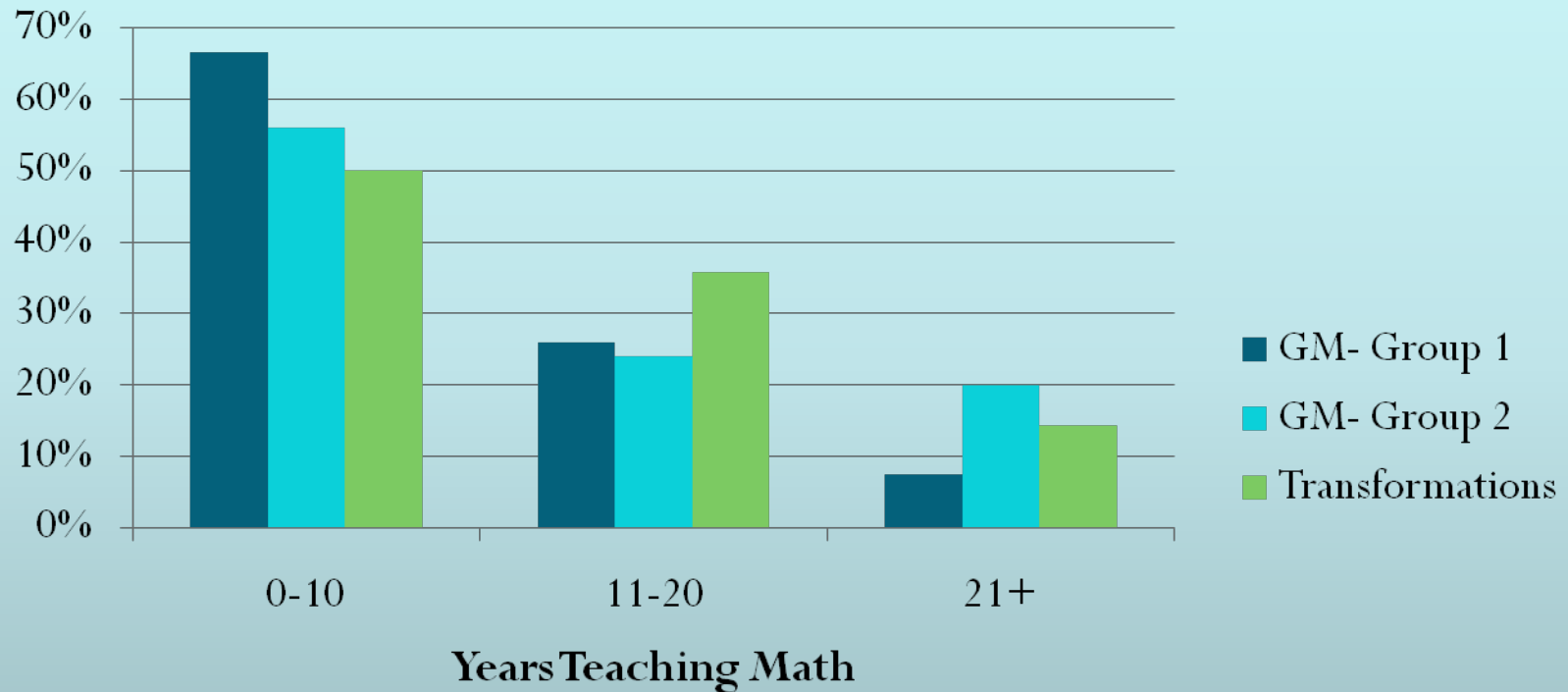# Gender of Participating Students

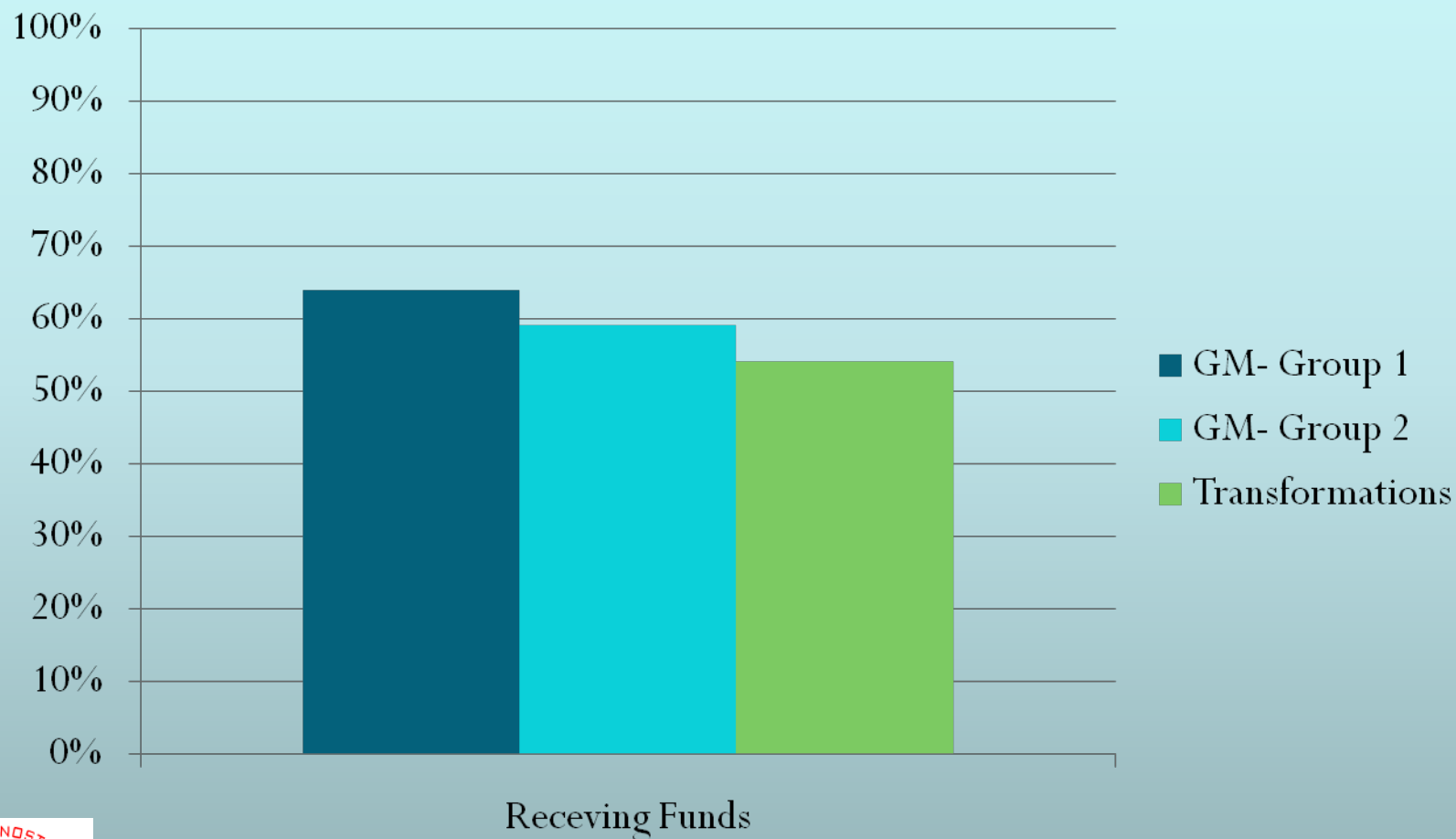# Participating Students- English as their First Language

Experience of Participating Teachers

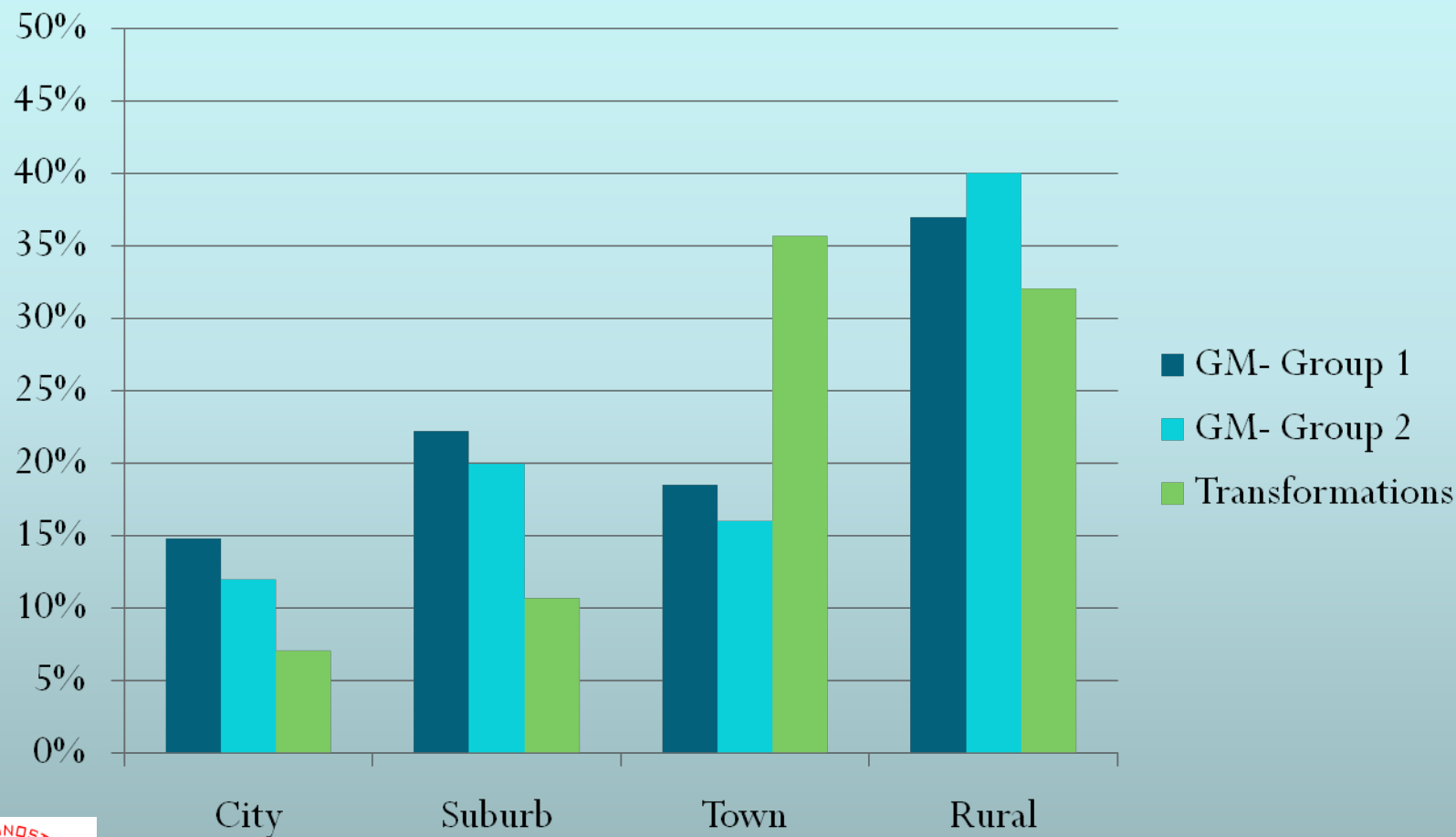# Math Experience of Participating Teachers
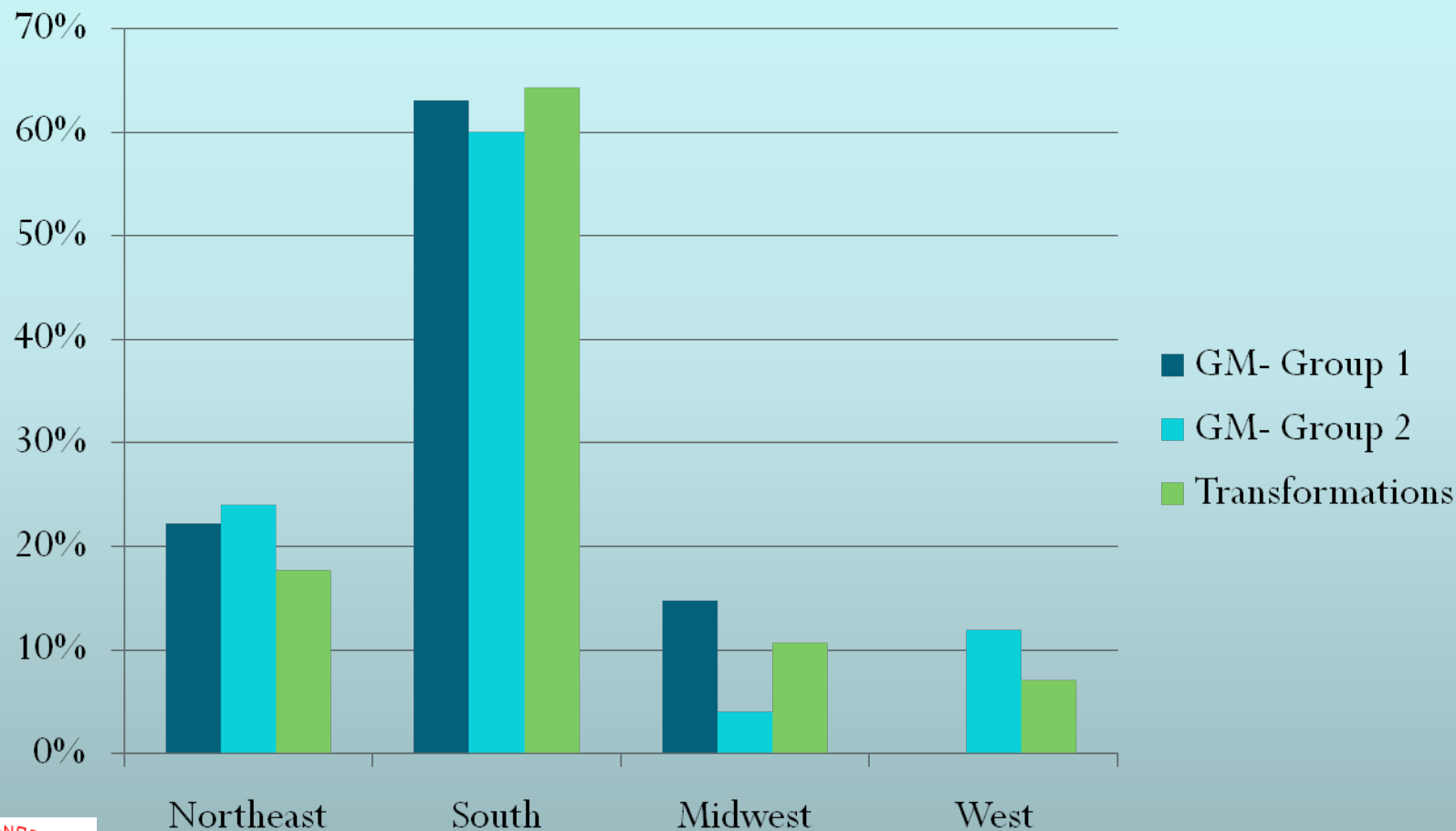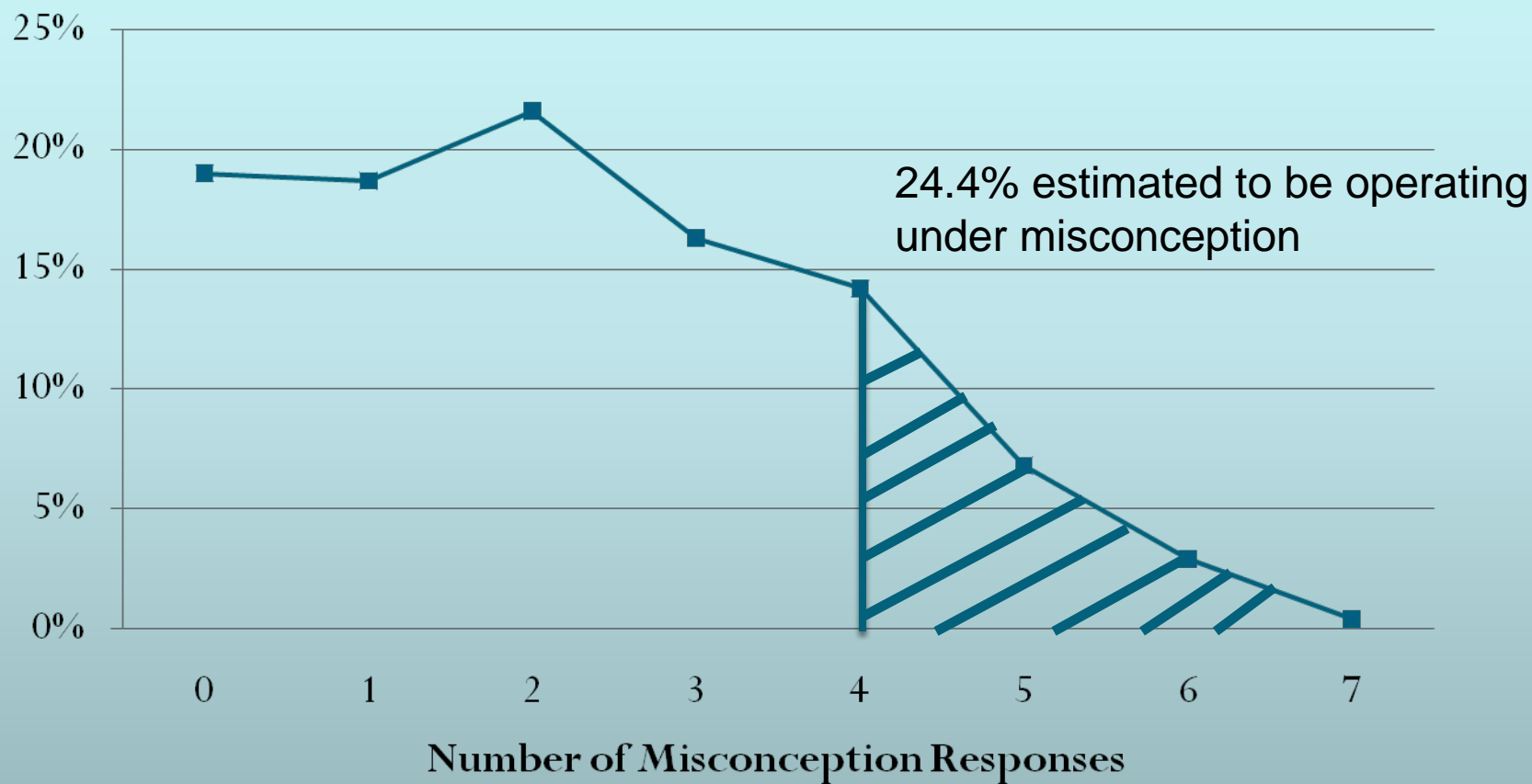
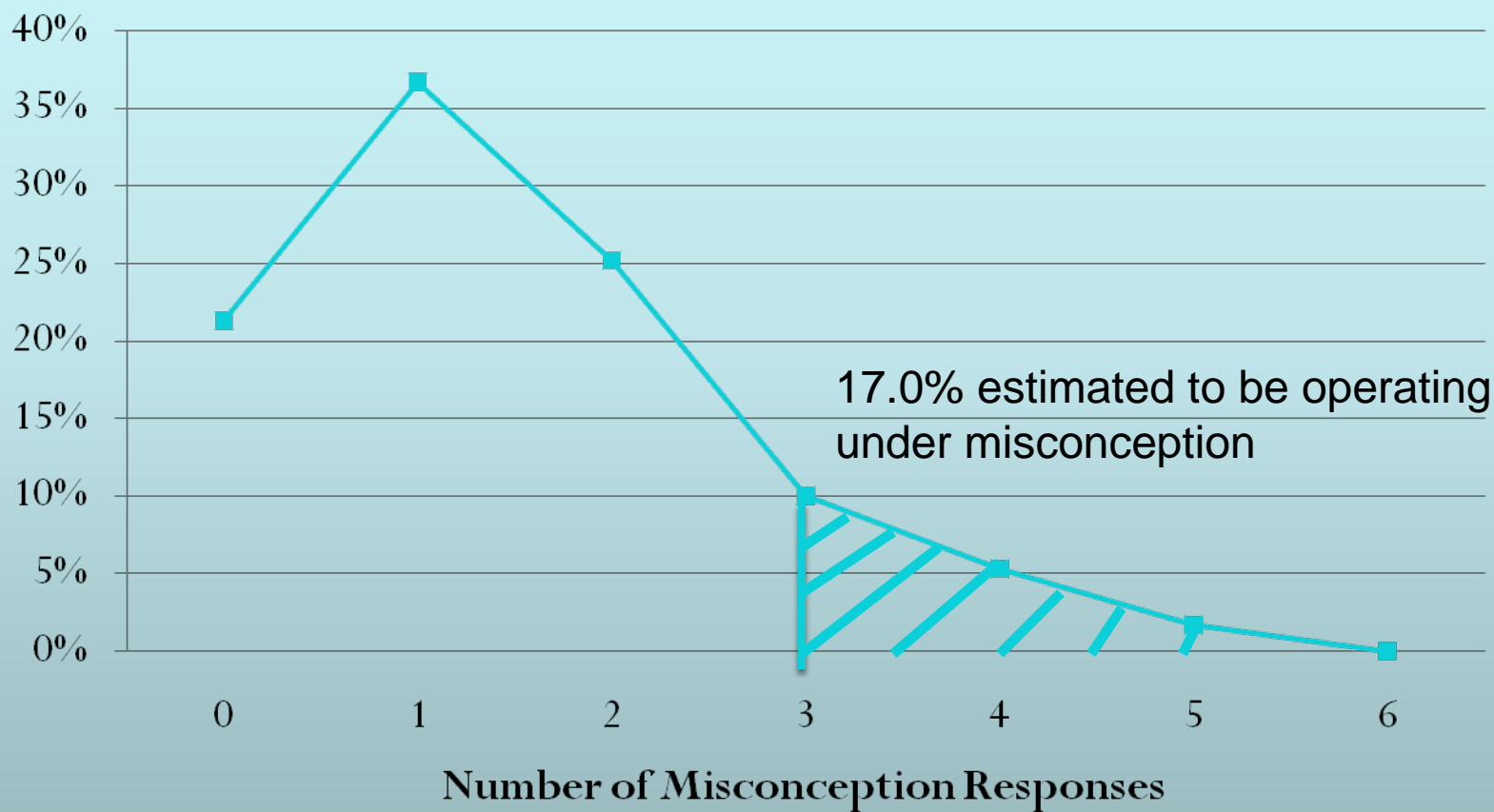# Schools Receiving Title I Funds

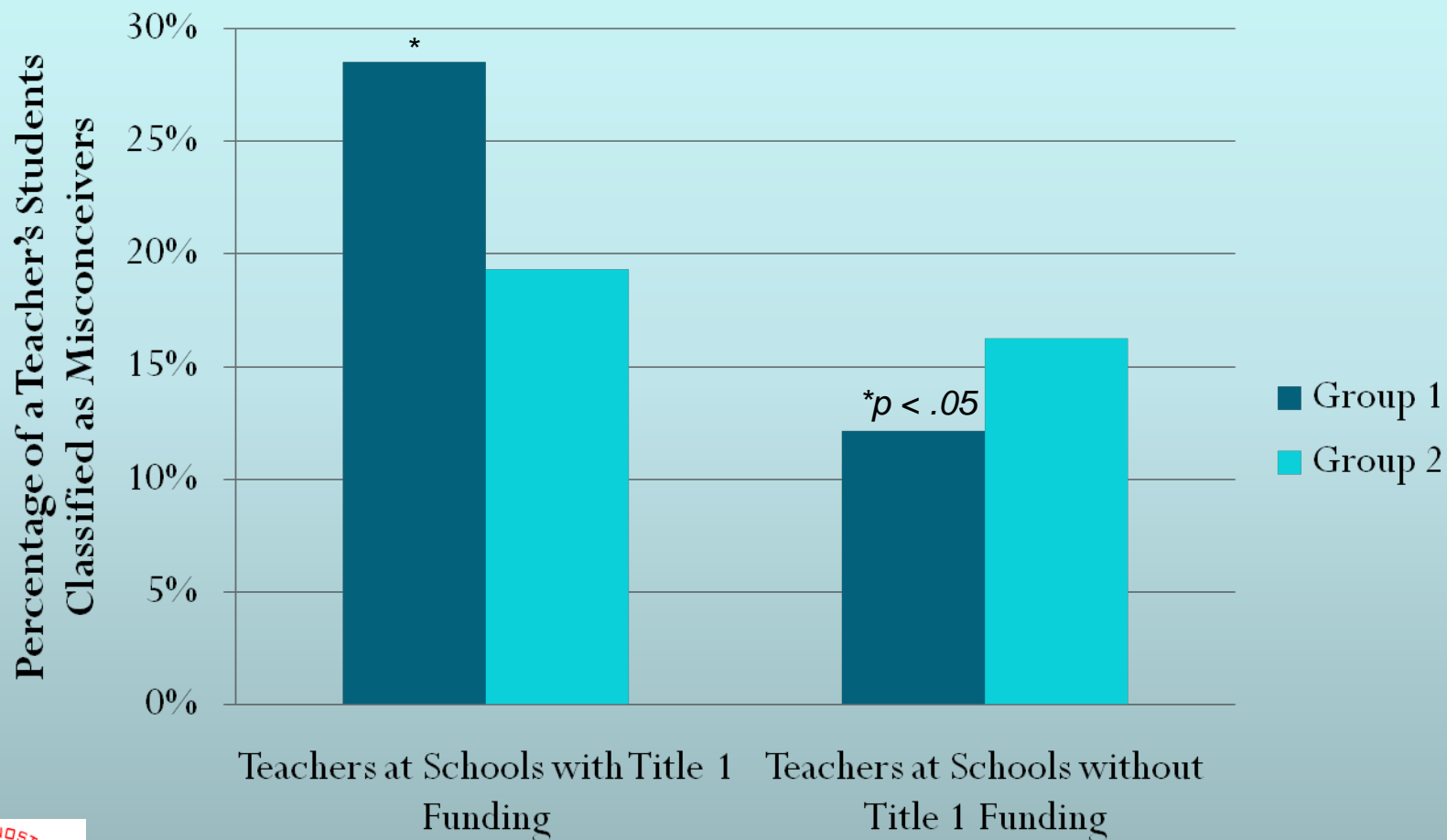# Locale of Participating Teachers

Region of Participating Teachers

GM Group 1- General Prevalence

24.4% estimated to be operating under misconception

Number of Misconception Responses

GM Group 2- General Prevalence

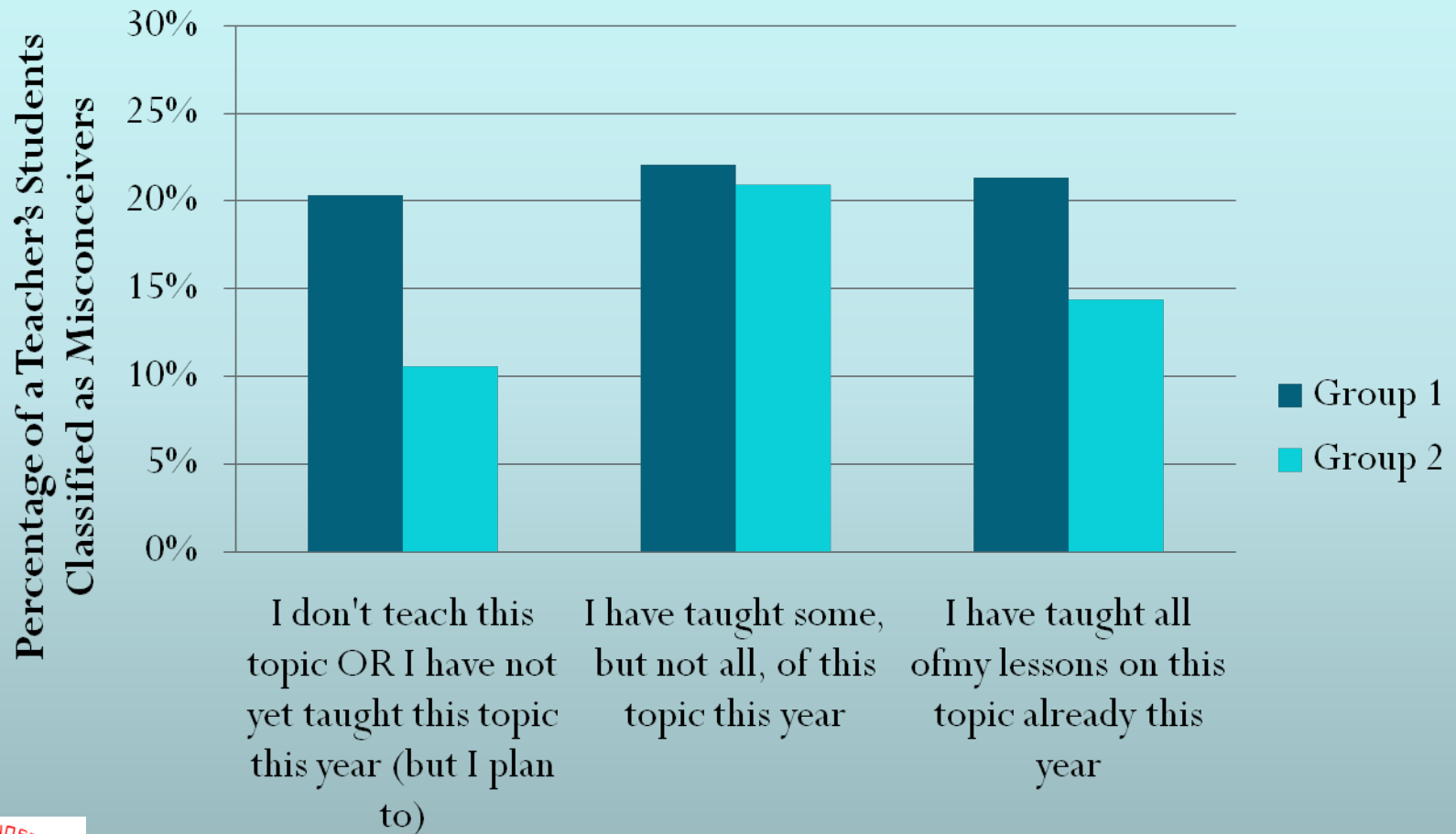
17.0% estimated to be operating under misconception
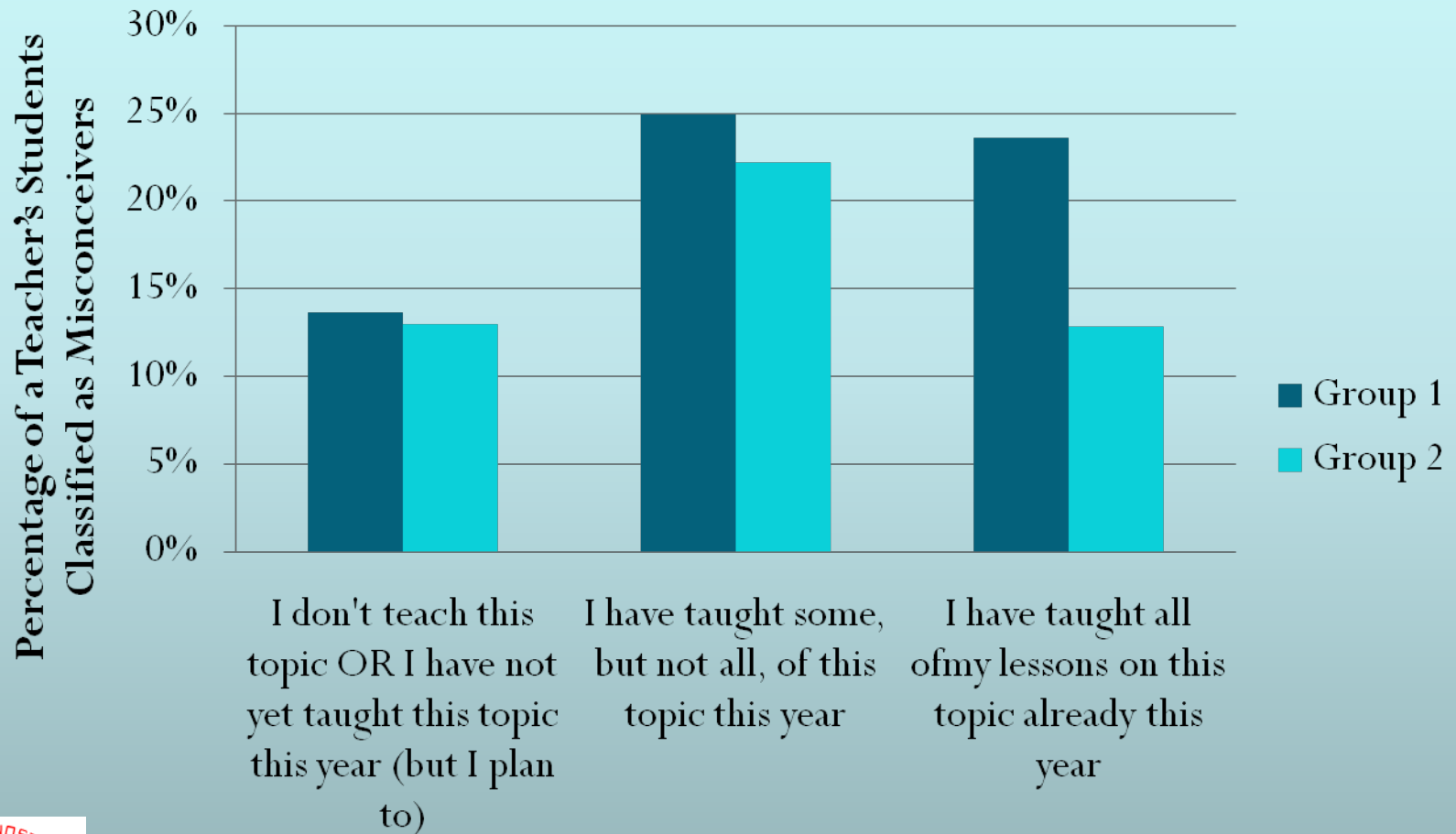
Number of Misconception Responses

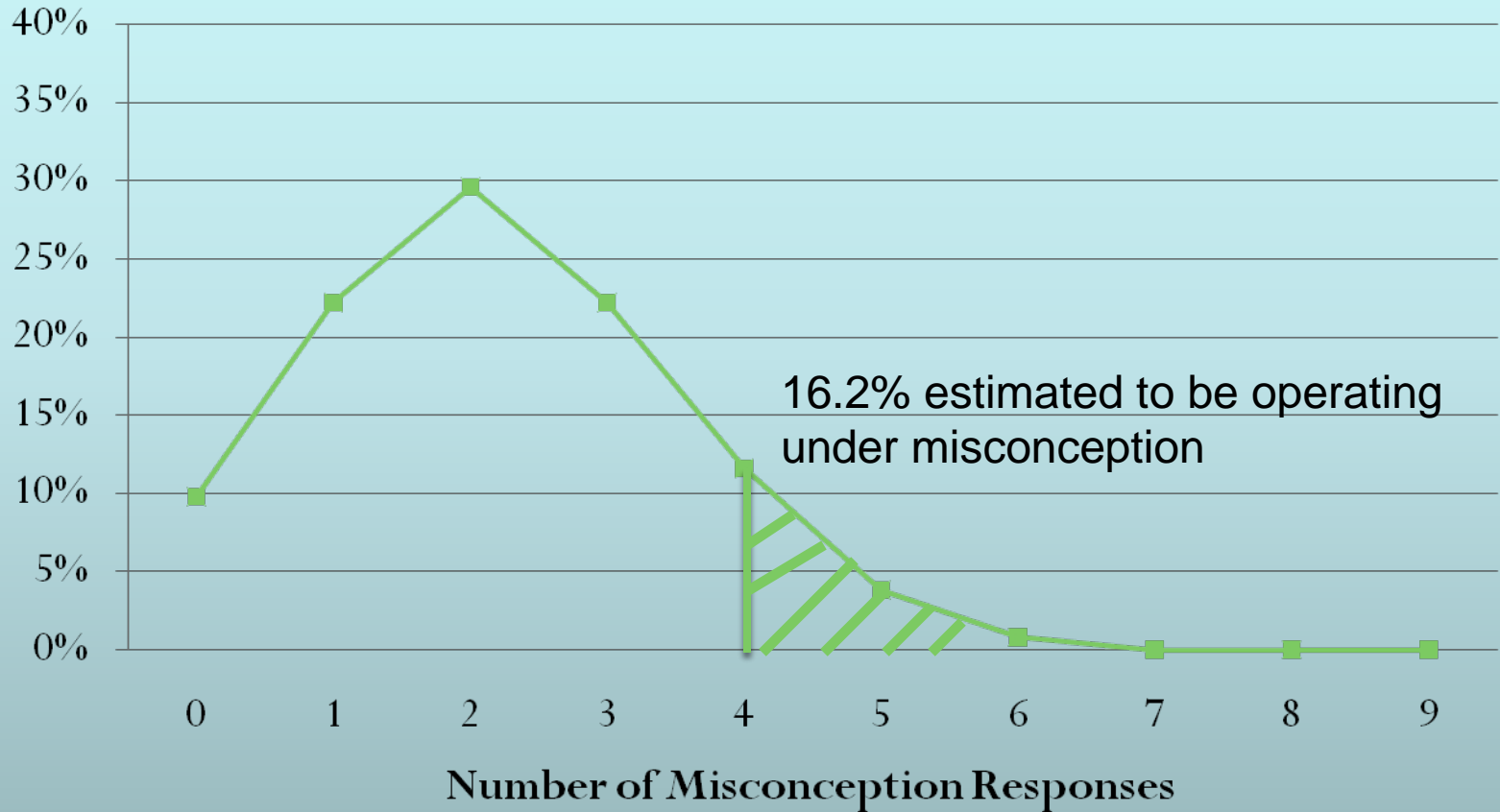# GM Prevalence and Title I Funding

GM Prevalence and When Teacher Teaches Geometry (Generally)

GM Prevalence and When Teacher Teaches Geometry- Measurement

Transformations- General Prevalence

16.2% estimated to be operating under misconception

Number of Misconception Responses

Transformations Prevalence and Title I Funding
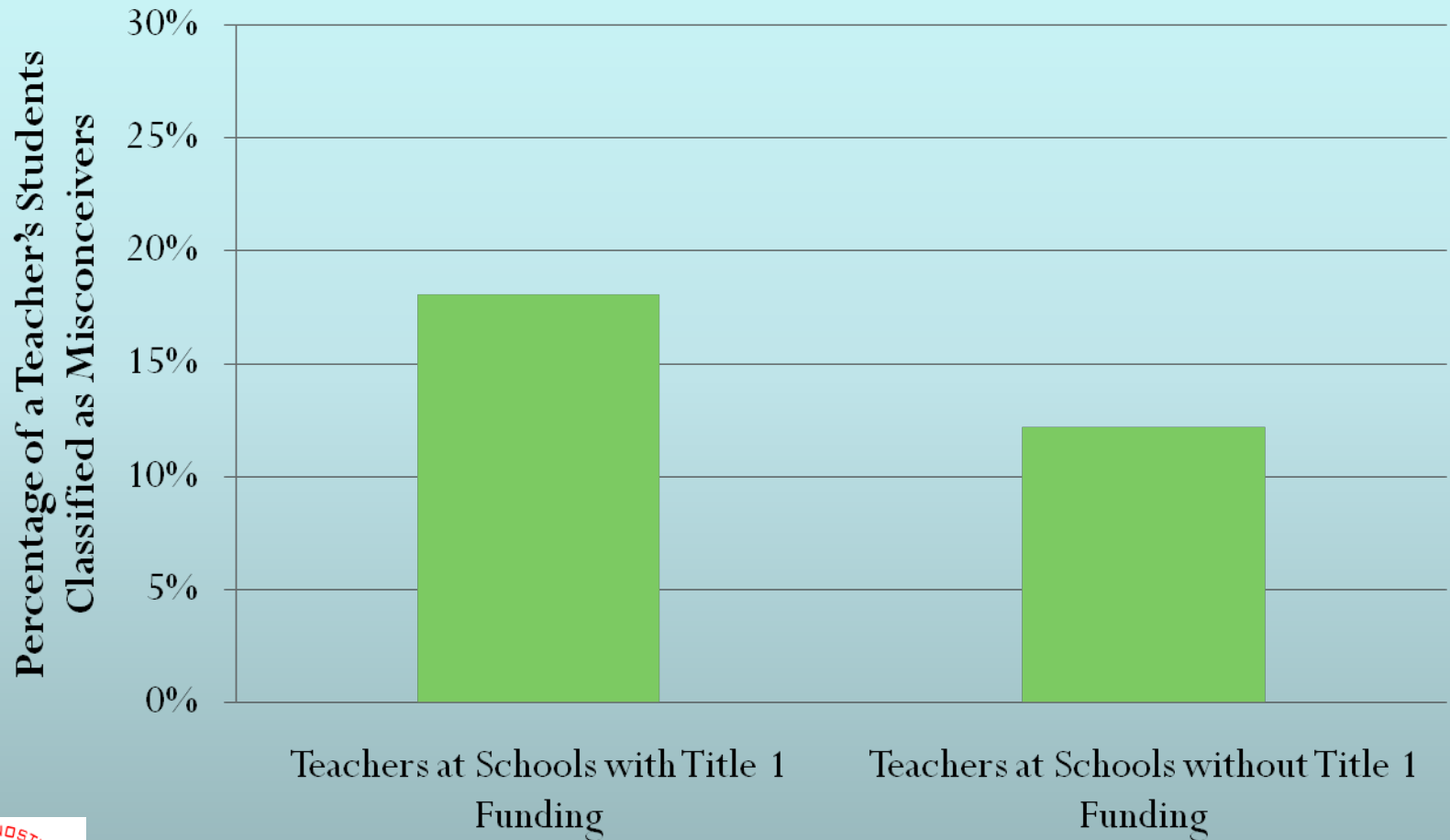
Transformations Prevalence and When Teacher Teaches Geometry (Generally)

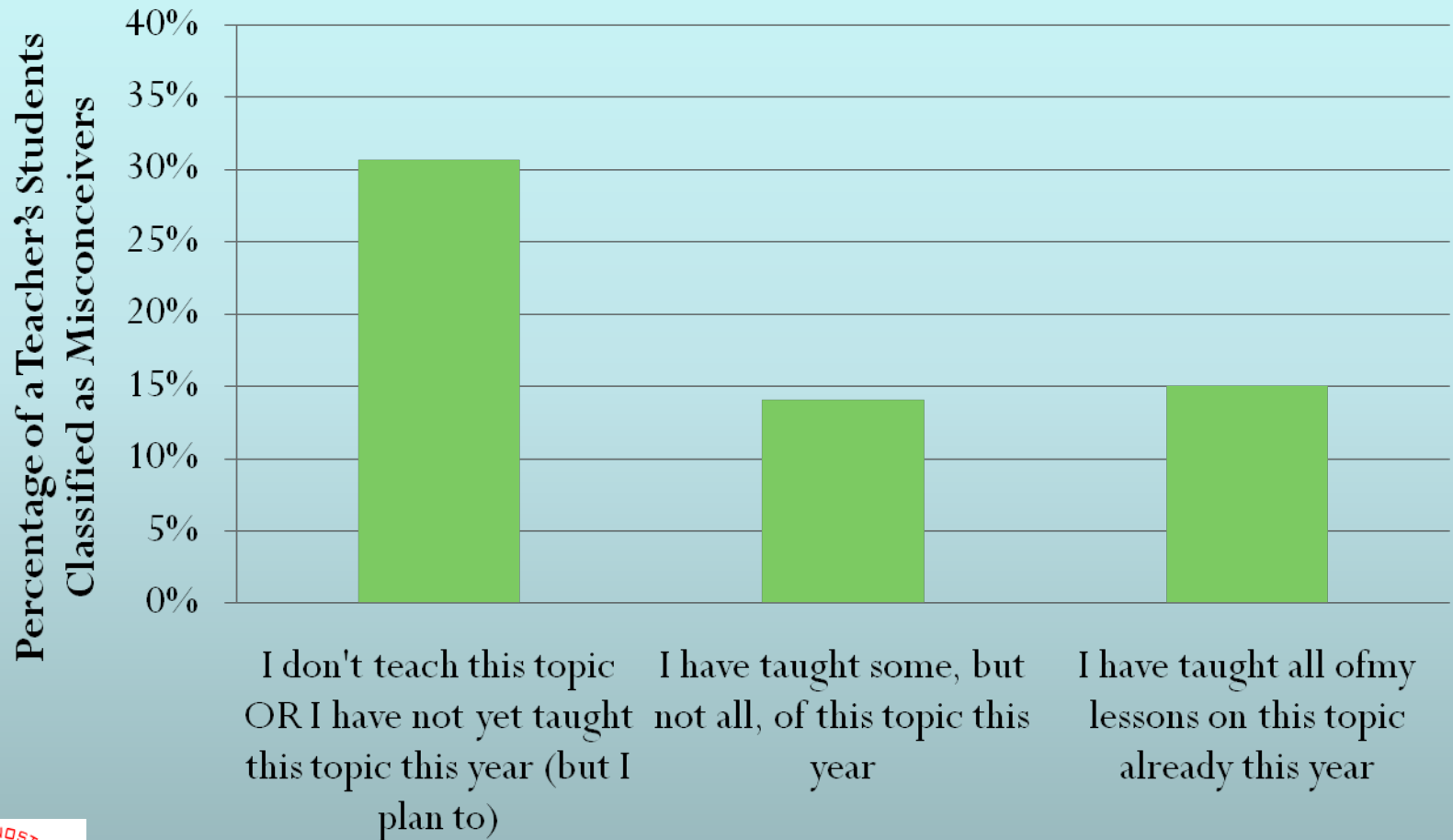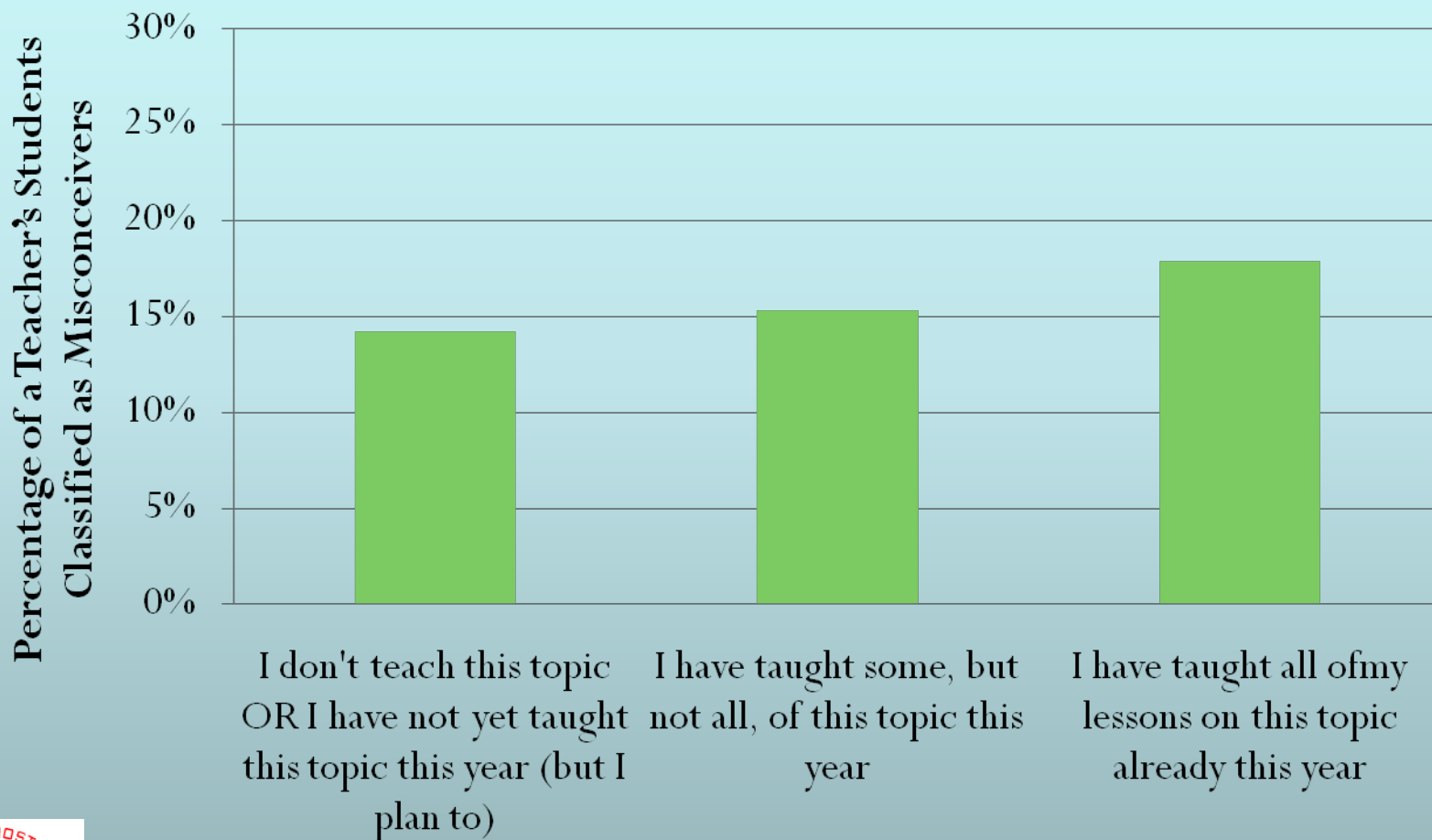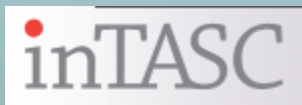# Transformation Prevalence and When Teacher Teaches Geometry- Transformations

Conclusions and Future Work

Conclusions

- DGA is developing an online integrated system with diagnostic test items, instant feedback for teachers, instructional resources to address misconceptions.

- Misconceptions
  - Geometric Measurement
  - Shape Properties
  - Transformations

- Open-ended items can elicit answers that can alert teachers to possible misconceptions.

- Initial *estimates* of the prevalence of misconceptions:
  - Geometric Measurement: 17-24% of students.
  - Transformations: 16% of students
  - Geometric Measurement misconceptions might be more prevalent in students at schools receiving Title I funding- mixed evidence.
  - Misconceptions appear to be resistant to traditional instruction.

Phase Two! Be a Part of It!

- Phase Two: Online Subject Tests begins January, 2010.

- Applications will be available starting December 1, 2009.

- We are seeking 6th, 7th, and 8th grade teachers to administer closed-response online tests to their students in 1, 2, or 3 of the misconception areas.

- You will receive a stipend for your participation!

- Sign up after this presentation to receive information or e-mail us:

dga@bc.edu