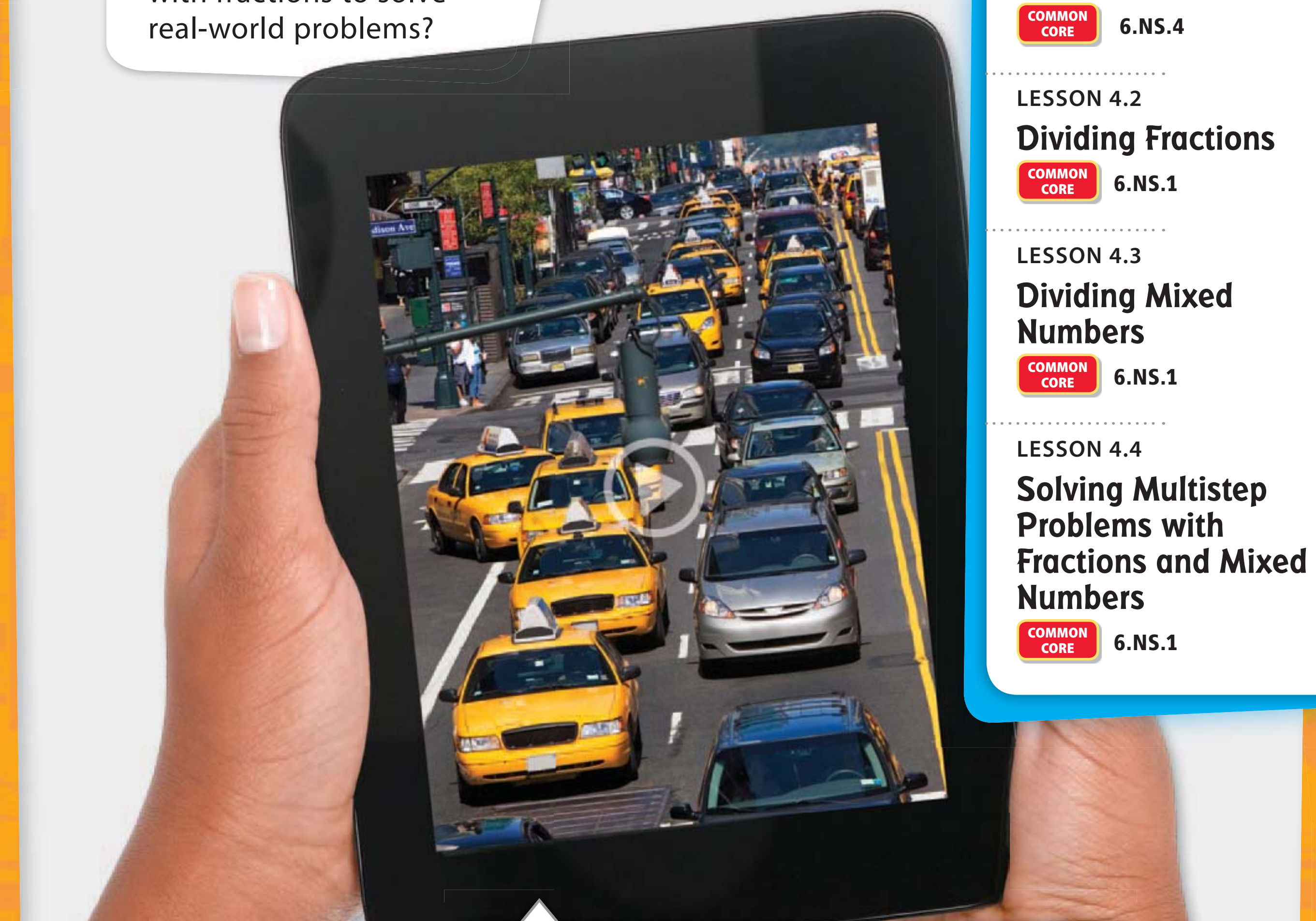
Operations with Fractions

MODULE 4

COMMON CORE

ESSENTIAL QUESTION

How can you use operations with fractions to solve real-world problems?



Real-World Video

To find your average rate of speed, divide the distance you traveled by the time you traveled. If you ride in a taxi and drive $\frac{1}{2}$ mile in $\frac{1}{4}$ hour, your rate was 2 mi/h which may mean you were in heavy traffic.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Write an Improper Fraction as a Mixed Number

EXAMPLE

$\frac{13}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5}$	Write as a sum using names for one plus a proper fraction.
$= 1 + 1 + \frac{3}{5}$	Write each name for one as one.
$= 2 + \frac{3}{5}$	Add the ones.
$= 2\frac{3}{5}$	Write the mixed number.

Write each improper fraction as a mixed number.

1. $\frac{9}{4}$ ______ **2.** $\frac{8}{3}$ ______ **3.** $\frac{23}{6}$ ______ **4.** $\frac{11}{2}$ ______

5. $\frac{17}{5}$ ______ **6.** $\frac{15}{8}$ ______ **7.** $\frac{33}{10}$ ______ **8.** $\frac{29}{12}$ ______

Multiplication Facts

EXAMPLE $7 \times 6 = \blacksquare$

Use a related fact you know.
$6 \times 6 = 36$
Think: $7 \times 6 = (6 \times 6) + 6$
$= 36 + 6$
$= 42$

$7 \times 6 = 42$

Multiply.

9. 6×5 ______ **10.** 8×9 ______ **11.** 10×11 ______ **12.** 7×8 ______

13. 9×7 ______ **14.** 8×6 ______ **15.** 9×11 ______ **16.** 11×12 ______

Division Facts

EXAMPLE $63 \div 7 = \blacksquare$

Think: 7 times what number equals 63?
$7 \times 9 = 63$

$63 \div 7 = 9$

So, $63 \div 7 = 9$.

Divide.

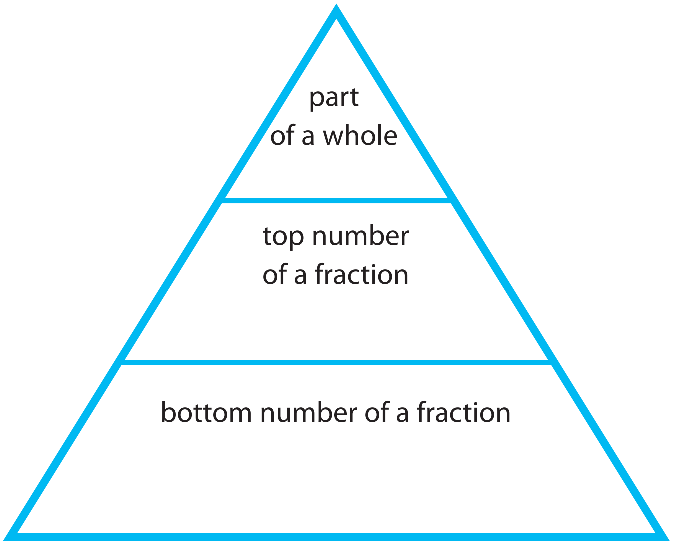
17. $35 \div 7$ ______ **18.** $56 \div 8$ ______ **19.** $28 \div 7$ ______ **20.** $48 \div 8$ ______

21. $36 \div 4$ ______ **22.** $45 \div 9$ ______ **23.** $72 \div 8$ ______ **24.** $40 \div 5$ ______

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the triangle. Write the review word that fits the description in each section of the triangle.



Vocabulary

Review Words

area *(área)*
✔ denominator *(denominador)*
✔ fraction *(fracción)*
greatest common factor (GCF) *(máximo común divisor (MCD))*
least common multiple (LCM) *(mínimo común múltiplo (m.c.m.))*
length *(longitud)*
✔ numerator *(numerador)*
product *(producto)*
width *(ancho)*

Preview Words

mixed number *(número mixto)*
order of operations *(orden de las operaciones)*
reciprocals *(recíprocos)*

Understand Vocabulary

In each grouping, select the choice that is described by the given vocabulary word.

1. reciprocals	Ⓐ 1:15	Ⓑ $\frac{3}{4} \div \frac{1}{6}$	Ⓒ $\frac{3}{5}$ and $\frac{5}{3}$
2. mixed number	Ⓐ $\frac{1}{3} - \frac{1}{5}$	Ⓑ $3\frac{1}{2}$	Ⓒ -5
3. order of operations	Ⓐ $5 - 3 + 2 = 0$	Ⓑ $5 - 3 + 2 = 4$	Ⓒ $5 - 3 + 2 = 6$

Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. Label each flap with lesson titles. As you study each lesson, write important ideas, such as vocabulary and processes, under the appropriate flap. Refer to your finished layered book as you work on exercises from this module.

MODULE 4

Unpacking the Standards

COMMON CORE

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE 6.NS.1

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Key Vocabulary

quotient *(cociente)*
The result when one number is divided by another.

fraction *(fracción)*
A number in the form $\frac{a}{b}$, where $b \neq 0$.

What It Means to You

You will learn how to divide two fractions. You will also understand the relationship between multiplication and division.

UNPACKING EXAMPLE 6.NS.1

Zachary is making vegetable soup. The recipe makes $6\frac{3}{4}$ cups of soup. How many $1\frac{1}{2}$-cup servings will the recipe make?

$$6\frac{3}{4} \div 1\frac{1}{2}$$
$$= \frac{27}{4} \div \frac{3}{2}$$
$$= \frac{27}{4} \times \frac{2}{3}$$
$$= \frac{9}{2}$$
$$= 4\frac{1}{2}$$

The recipe will make $4\frac{1}{2}$ servings.

COMMON CORE 6.NS.4

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

What It Means to You

You can use greatest common factors and least common multiples to simplify answers when you calculate with fractions.

UNPACKING EXAMPLE 6.NS.4

Add. Write the answer in simplest form.

$$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$$ Use the LCM of 3 and 6 as a common denominator.

$$= \frac{2+1}{6}$$ Add the numerators.

$$= \frac{3}{6}$$

$$= \frac{3 \div 3}{6 \div 3}$$ Simplify by dividing by the GCF. The GCF of 3 and 6 is 3.

$$= \frac{1}{2}$$ Write the answer in simplest form.

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LESSON 4.1

Applying GCF and LCM to Fraction Operations

COMMON CORE 6.NS.4

Find the greatest common factor...and the least common multiple of two whole numbers...

ESSENTIAL QUESTION

How do you use the GCF and LCM when adding, subtracting, and multiplying fractions?

Multiplying Fractions

To multiply two fractions you first multiply the numerators and then multiply the denominators.

$$\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}} = \frac{\text{numerator}}{\text{denominator}}$$

The resulting product may need to be written in simplest form. To write a fraction in simplest form, you can divide both the numerator and the denominator by their greatest common factor.

Example 1 shows two methods for making sure that the product of two fractions is in simplest form.

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EXAMPLE 1

COMMON CORE 6.NS.4

Multiply. Write the product in simplest form.

A $\frac{1}{3} \times \frac{3}{5}$

$\frac{1}{3} \times \frac{3}{5} = \frac{1 \times 3}{3 \times 5}$ Write the problem as a single fraction.

$= \frac{3}{15}$ Multiply numerators. Multiply denominators.

$= \frac{3 \div 3}{15 \div 3}$ Simplify by dividing by the GCF. The GCF of 3 and 15 is 3.

$= \frac{1}{5}$ Write the answer in simplest form.

B $\frac{6}{7} \times \frac{2}{3}$

6 in the numerator and 3 in the denominator have a common factor other than one. Divide by the GCF 3.

$\frac{6}{7} \times \frac{2}{3} = \frac{6 \times 2}{7 \times 3}$ Write the problem as a single fraction.

$= \frac{{}^{2}\cancel{6} \times 2}{7 \times \cancel{3}_{1}}$ Simplify before multiplying using the GCF.

$= \frac{2 \times 2}{7 \times 1}$ Multiply numerators. Multiply denominators.

$= \frac{4}{7}$

Math Talk
Mathematical Practices

Compare the methods in the Example. How do you know if you can use the method in **B**?

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YOUR TURN

Multiply. Write each product in simplest form.

1. $\frac{1}{6} \times \frac{3}{5}$ ________

2. $\frac{3}{4} \times \frac{7}{9}$ ________

3. $\frac{3}{7} \times \frac{2}{3}$ ________

4. $\frac{4}{5} \times \frac{2}{7}$ ________

5. $\frac{7}{10} \times \frac{8}{21}$ ________

6. $\frac{6}{7} \times \frac{1}{6}$ ________

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Multiplying Fractions and Whole Numbers

To multiply a fraction by a whole number, you rewrite the whole number as a fraction and multiply the two fractions. Remember to use the GCF to write the product in simplest form.

EXAMPLE 2 Real World

COMMON CORE 6.NS.4

A class has 18 students. The teacher asks how many students in the class have pets and finds $\frac{5}{9}$ of the students have pets. How many students have pets?

STEP 1 Estimate the product. Multiply the whole number by the nearest benchmark fraction.

$\frac{5}{9}$ is close to $\frac{1}{2}$, so multiply $\frac{1}{2}$ times 18.

$\frac{1}{2} \times 18 = 9$

STEP 2 Multiply. Write the product in simplest form.

$\frac{5}{9} \times 18$

You can write $\frac{5}{9}$ times 18 three ways.

$\frac{5}{9} \times 18 \quad \frac{5}{9} \cdot 18 \quad \frac{5}{9}(18)$

$\frac{5}{9} \times 18 = \frac{5}{9} \times \frac{18}{1}$ Rewrite 18 as a fraction.

$= \frac{5 \times \cancel{18}^{2}}{{}_{1}\cancel{9} \times 1}$ Simplify before multiplying using the GCF.

$= \frac{5 \times 2}{1 \times 1}$ Multiply numerators. Multiply denominators.

$= \frac{10}{1} = 10$ Simplify by writing as a whole number.

10 students have pets.

Math Talk
Mathematical Practices

How can you check to see if the answer is correct?

Reflect

7. **Analyze Relationships** Is the product of a fraction less than 1 and a whole number greater than or less than the whole number? Explain.

____

____

YOUR TURN

Multiply. Write each product in simplest form.

8. $\frac{5}{8} \times 24$ ________

9. $\frac{3}{5} \times 20$ ________

10. $\frac{1}{3} \times 8$ ________

11. $\frac{1}{4} \times 14$ ________

12. $3\frac{7}{10} \times 7$ ________

13. $2\frac{3}{10} \times 10$ ________

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Adding and Subtracting Fractions

You have learned that to add or subtract two fractions, you can rewrite the fractions so they have the same denominator. You can use the least common multiple of the denominators of the fractions to rewrite the fractions.

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EXAMPLE 3

COMMON CORE 6.NS.4

Add $\frac{8}{15} + \frac{1}{6}$. Write the sum in simplest form.

STEP 1 Rewrite the fractions as equivalent fractions. Use the LCM as the denominator of both fractions.

$$\frac{8}{15} \rightarrow \frac{8 \times 2}{15 \times 2} \rightarrow \frac{16}{30}$$

$$\frac{1}{6} \rightarrow \frac{1 \times 5}{6 \times 5} \rightarrow \frac{5}{30}$$

The LCM of 15 and 6 is 30.

STEP 2 Add the numerators of the equivalent fractions. Then simplify.

$$\frac{16}{30} + \frac{5}{30} = \frac{21}{30}$$

$$= \frac{21 \div 3}{30 \div 3}$$

$$= \frac{7}{10}$$

Simplify by dividing by the GCF. The GCF of 21 and 30 is 3.

Reflect

14. Can you also use the LCM of the denominators of the fractions to rewrite the difference $\frac{8}{15} - \frac{1}{6}$? What is the difference?

____

My Notes

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YOUR TURN

Add or subtract. Write each sum or difference in simplest form.

15. $\frac{5}{14} + \frac{1}{6}$ ______

16. $\frac{5}{12} - \frac{3}{20}$ ______

17. $\frac{5}{12} - \frac{3}{8}$ ______

18. $1\frac{3}{10} + \frac{1}{4}$ ______

19. $\frac{2}{3} + 6\frac{1}{5}$ ______

20. $3\frac{1}{6} - \frac{1}{7}$ ______

Guided Practice

Multiply. Write each product in simplest form. (Example 1)

1. $\frac{1}{2} \times \frac{5}{8}$ ______

2. $\frac{3}{5} \times \frac{5}{9}$ ______

3. $\frac{3}{8} \times \frac{2}{5}$ ______

4. $2\frac{3}{8} \times 16$ ______

5. $1\frac{4}{5} \times \frac{5}{12}$ ______

6. $1\frac{2}{10} \times 5$ ______

Find each amount. (Example 2)

7. $\frac{1}{4}$ of 12 bottles of water = ______ bottles

8. $\frac{2}{3}$ of 24 bananas = ______ bananas

9. $\frac{3}{5}$ of \$40 restaurant bill = \$ ______

10. $\frac{5}{6}$ of 18 pencils = ______ pencils

Add or subtract. Write each sum or difference in simplest form.

11. $\frac{3}{8} + \frac{5}{24}$ ______

12. $\frac{1}{20} + \frac{5}{12}$ ______

13. $\frac{9}{20} - \frac{1}{4}$ ______

14. $\frac{9}{10} - \frac{3}{14}$ ______

15. $3\frac{3}{8} + \frac{5}{12}$ ______

16. $5\frac{7}{10} - \frac{5}{18}$ ______

ESSENTIAL QUESTION CHECK-IN

17. How can knowing the GCF and LCM help you when you add, subtract, and multiply fractions?

______

______

______

______

Name ______________________ Class ______________ Date ____________

4.1 Independent Practice

COMMON CORE 6.NS.4

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Solve. Write each answer in simplest form.

18. Erin buys a bag of peanuts that weighs $\frac{3}{4}$ of a pound. Later that week, the bag is $\frac{2}{3}$ full. How much does the bag of peanuts weigh now? Show your work.

____

____

19. **Multistep** Marianne buys 16 bags of potting soil that comes in $\frac{5}{8}$-pound bags.

a. How many pounds of potting soil does Marianne buy?

____

____

b. If Marianne's father calls and says he needs 13 pounds of potting soil, how many additional bags should she buy?

____

____

20. **Music** Two fifths of the instruments in the marching band are brass, one third are percussion, and the rest are woodwinds.

a. What fraction of the band is woodwinds?

____

b. One half of the woodwinds are clarinets. What fraction of the band is clarinets?

____

c. One eighth of the brass instruments are tubas. If there are 240 instruments in the band, how many are tubas?

____

21. Marcial found a recipe for fruit salad that he wanted to try to make for his birthday party. He decided to triple the recipe.

Fruit Salad

$3\frac{1}{2}$ cups thinly sliced rhubarb

15 seedless grapes, halved

$\frac{1}{2}$ orange, sectioned

10 fresh strawberries, halved

$\frac{3}{5}$ apple, cored and diced

$\frac{2}{3}$ peach, sliced

1 plum, pitted and sliced

$\frac{1}{4}$ cup fresh blueberries

a. What are the new amounts for the oranges, apples, blueberries, and peaches?

____

____

____

____

b. **Communicate Mathematical Ideas** The amount of rhubarb in the original recipe is $3\frac{1}{2}$ cups. Using what you know of whole numbers and what you know of fractions, explain how you could triple that mixed number.

____

____

____

____

22. One container holds $1\frac{7}{8}$ quarts of water and a second container holds $5\frac{3}{4}$ quarts of water. How many more quarts of water does the second container hold than the first container?

____

23. Each of 15 students will give a $1\frac{1}{2}$-minute speech in English class.

a. How long will it take to give the speeches? ______________

b. If the teacher begins recording on a digital camera with an hour available, is there enough time to record everyone if she gives a 15-minute introduction at the beginning of class and every student takes a minute to get ready? Explain.

____

____

____

c. How much time is left on the digital camera? ______________

H.O.T. FOCUS ON HIGHER ORDER THINKING

24. **Represent Real-World Problems** Kate wants to buy a new bicycle from a sporting goods store. The bicycle she wants normally sells for \$360. The store has a sale where all bicycles cost $\frac{5}{6}$ of the regular price. What is the sale price of the bicycle?

____

25. **Error Analysis** To find the product $\frac{3}{7} \times \frac{4}{9}$, Cameron simplified $\frac{3}{7}$ to $\frac{1}{7}$ and then multiplied the fractions $\frac{1}{7}$ and $\frac{4}{9}$ to find the product $\frac{4}{63}$. What is Cameron's error?

____

____

____

____

26. **Justify Reasoning** To multiply a whole number by a fraction, you can first write the whole number as a fraction by placing the whole number in the numerator and 1 in the denominator. Does following this step change the product? Explain.

____

____

____

Work Area

LESSON

4.2 Dividing Fractions

COMMON CORE 6.NS.1

Interpret and compute quotients of fractions, . . . , e.g., by using visual fraction models. . . .

ESSENTIAL QUESTION

How do you divide fractions?

EXPLORE ACTIVITY 1 Real World

COMMON CORE 6.NS.1

Modeling Fraction Division

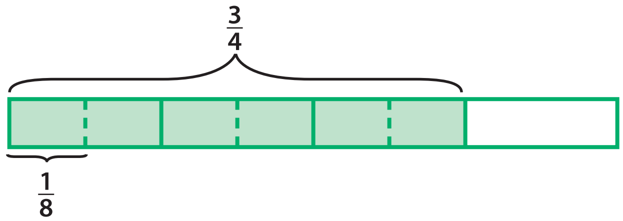
In some division problems, you may know a number of groups and need to find how many or how much are in each group. In other division problems, you may know how many there are in each group, and need to find the number of groups.

A **You have $\frac{3}{4}$ cup of salsa for making burritos. Each burrito requires $\frac{1}{8}$ cup of salsa. How many burritos can you make?**

To find the number of burritos that can be made, you need to determine how many $\frac{1}{8}$-cup servings are in $\frac{3}{4}$ cups. Use the diagram. How many eighths

are there in $\frac{3}{4}$? ______

You have enough salsa to make ______ burritos.



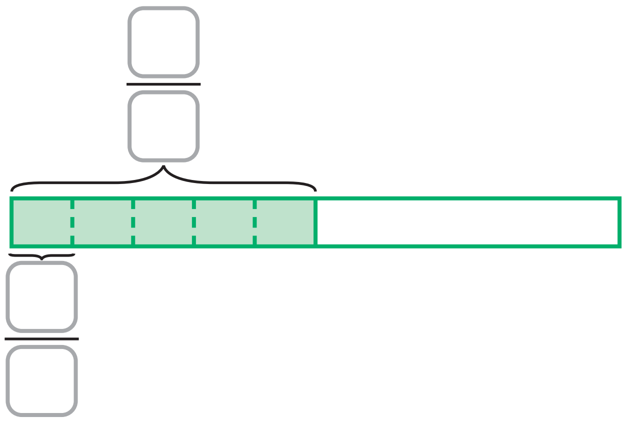
B **Five people share $\frac{1}{2}$ pound of cheese equally. How much cheese does each person receive?**

To find how much cheese each person receives, you need to determine how

much is in each of ______ parts.

How much is in each part? ______

Each person will receive ______ pound.



Reflect

1. Write the division shown by each model.

______

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Reciprocals

Another way to divide fractions is to use *reciprocals*. Two numbers whose product is 1 are **reciprocals**.

$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$ $\qquad$ $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals.

To find the reciprocal of a fraction, switch the numerator and denominator.

$$\frac{\text{numerator}}{\text{denominator}} \cdot \frac{\text{denominator}}{\text{numerator}} = 1$$

EXAMPLE 1

COMMON CORE Prep for 6.NS.1

Find the reciprocal of each number.

A $\frac{2}{9} \quad \frac{9}{2}$ $\qquad$ Switch the numerator and denominator.

The reciprocal of $\frac{2}{9}$ is $\frac{9}{2}$.

B $\frac{1}{8} \quad \frac{8}{1}$ $\qquad$ Switch the numerator and denominator.

The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$, or 8.

C 5

$5 = \frac{5}{1}$ $\qquad$ Rewrite as a fraction.

$\frac{5}{1} \quad \frac{1}{5}$ $\qquad$ Switch the numerator and the denominator.

The reciprocal of 5 is $\frac{1}{5}$.

Math Talk
Mathematical Practices
How can you check your answer?

Reflect

2. Is any number its own reciprocal? If so, what number(s)? Justify your answer.

____

3. **Communicate Mathematical Ideas** Does every number have a reciprocal? Explain.

____

____

4. The reciprocal of a whole number is a fraction with ________ in the numerator.

YOUR TURN

Find the reciprocal of each number.

5. $\frac{7}{8}$ ____________ $\qquad$ **6.** 9 ____________ $\qquad$ **7.** $\frac{1}{11}$ ____________

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EXPLORE ACTIVITY 2 COMMON CORE 6.NS.1

Using Reciprocals to Find Equivalent Values

A Complete the table below.

Division	Multiplication
$\frac{6}{7} \div \frac{2}{7} = 3$	$\frac{6}{7} \times \frac{7}{2} =$
$\frac{5}{8} \div \frac{3}{8} = \frac{5}{3}$	$\frac{5}{8} \times \frac{8}{3} =$
$\frac{1}{6} \div \frac{5}{6} = \frac{1}{5}$	$\frac{1}{6} \times \frac{6}{5} =$
$\frac{1}{4} \div \frac{1}{3} = \frac{3}{4}$	$\frac{1}{4} \times \frac{3}{1} =$

B How does each multiplication problem compare to its corresponding division problem?

____

____

____

C How does the answer to each multiplication problem compare to the answer to its corresponding division problem?

____

____

Reflect

8. **Make a Conjecture** Use the pattern in the table to make a conjecture about how you can use multiplication to divide one fraction by another.

____

____

____

9. Write a division problem and a corresponding multiplication problem like those in the table. Assuming your conjecture in **8** is correct, what is the answer to your division problem?

____

____

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Using Reciprocals to Divide Fractions

Dividing by a fraction is equivalent to multiplying by its reciprocal.

$\frac{1}{5} \div \frac{1}{4} = \frac{4}{5}$ $\qquad$ $\frac{1}{5} \times \frac{4}{1} = \frac{4}{5}$

EXAMPLE 2

COMMON CORE 6.NS.1

Divide $\frac{5}{9} \div \frac{2}{3}$. Write the quotient in simplest form.

STEP 1 Rewrite as multiplication, using the reciprocal of the divisor.

$\frac{5}{9} \div \frac{2}{3} = \frac{5}{9} \times \frac{3}{2}$ $\qquad$ *The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.*

STEP 2 Multiply and simplify.

$\frac{5}{9} \times \frac{3}{2} = \frac{15}{18}$ $\qquad$ *Multiply the numerators. Multiply the denominators*

$= \frac{5}{6}$ $\qquad$ *Write the answer in simplest form.*

$\frac{15 \div 3}{18 \div 3} = \frac{5}{6}$

$\frac{5}{9} \div \frac{2}{3} = \frac{5}{6}$

YOUR TURN

Divide.

10. $\frac{9}{10} \div \frac{2}{5} =$ ______________

11. $\frac{9}{10} \div \frac{3}{5} =$ ______________

Guided Practice

Find the reciprocal of each fraction. (Example 1)

1. $\frac{2}{5}$ ______________

2. $\frac{1}{9}$ ______________

3. $\frac{10}{3}$ ______________

Divide. (Explore 1, Explore 2, and Example 2)

4. $\frac{4}{3} \div \frac{5}{3} =$ ______________

5. $\frac{3}{10} \div \frac{4}{5} =$ ______________

6. $\frac{1}{2} \div \frac{2}{5} =$ ______________

ESSENTIAL QUESTION CHECK-IN

7. How do you divide fractions?

____

Name ______________________ Class ______________ Date ____________

4.2 Independent Practice

COMMON CORE 6.NS.1

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8. Alison has $\frac{1}{2}$ cup of yogurt for making fruit parfaits. Each parfait requires $\frac{1}{8}$ cup of yogurt. How many parfaits can she make?

____

9. A team of runners is needed to run a $\frac{1}{4}$-mile relay race. If each runner must run $\frac{1}{16}$ mile, how many runners will be needed?

____

10. Trevor paints $\frac{1}{6}$ of the fence surrounding his farm each day. How many days will it take him to paint $\frac{3}{4}$ of the fence?

____

11. Six people share $\frac{3}{5}$ pound of peanuts equally. What fraction of a pound of peanuts does each person receive?

____

12. **Biology** If one honeybee makes $\frac{1}{12}$ teaspoon of honey during its lifetime, how many honeybees are needed to make $\frac{1}{2}$ teaspoon of honey?

____

13. Jackson wants to divide a $\frac{3}{4}$-pound box of trail mix into small bags. Each of the bags will hold $\frac{1}{12}$ pound of trail mix. How many bags of trail mix can Jackson fill?

____

14. A pitcher of contains $\frac{2}{3}$ quart of lemonade. If an equal amount of lemonade is poured into each of 6 glasses, how much lemonade will each glass contain?

____

15. How many tenths are there in $\frac{4}{5}$?

____

16. You make a large bowl of salad to share with your friends. Your brother eats $\frac{1}{3}$ of it before they come over.

a. You want to divide the leftover salad evenly among six friends. What expression describes the situation? Explain.

____

____

____

____

b. What fractional portion of the original bowl of salad does each friend receive?

____

____

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

17. **Interpret the Answer** The length of a ribbon is $\frac{3}{4}$ meter. Sun Yi needs pieces measuring $\frac{1}{3}$ meter for an art project. What is the greatest number of pieces measuring $\frac{1}{3}$ meter that can be cut from the ribbon? How much ribbon will be left after Sun Yi cuts the ribbon? Explain your reasoning.

18. **Represent Real-World Problems** Liam has $\frac{9}{10}$ gallon of paint for painting the birdhouses he sells at the craft fair. Each birdhouse requires $\frac{1}{20}$ gallon of paint. How many birdhouses can Liam paint? Show your work.

19. **Justify Reasoning** When Kaitlin divided a fraction by $\frac{1}{2}$, the result was a mixed number. Was the original fraction less than or greater than $\frac{1}{2}$? Explain your reasoning.

20. **Communicate Mathematical Ideas** The reciprocal of a fraction less than 1 is always a fraction greater than 1. Why is this?

21. **Make a Prediction** Susan divides the fraction $\frac{5}{8}$ by $\frac{1}{16}$. Her friend Robyn divides $\frac{5}{8}$ by $\frac{1}{32}$. Predict which person will get the greater quotient. Explain and check your prediction.

LESSON 4.3

Dividing Mixed Numbers

COMMON CORE 6.NS.1

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions. . . .

ESSENTIAL QUESTION

How do you divide mixed numbers?

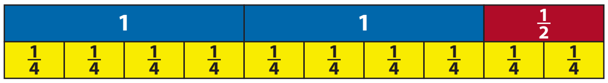
EXPLORE ACTIVITY Real World

COMMON CORE 6.NS.1

Modeling Mixed Number Division

Antoine is making sushi rolls. He has $2\frac{1}{2}$ cups of rice and will use $\frac{1}{4}$ cup of rice for each sushi roll. How many sushi rolls can he make?

A To find the number of sushi rolls that can be made, you need to determine how many fourths are in $2\frac{1}{2}$. Use fraction pieces to represent $2\frac{1}{2}$ on the model below.



B How many fourths are in $2\frac{1}{2}$? ______________

Antoine has enough rice to make ______________ sushi rolls.

Reflect

1. **Communicate Mathematical Ideas** Which mathematical operation could you use to find the number of sushi rolls that Antoine can make? Explain.

____

____

2. **Multiple Representations** Write the division shown by the model.

____

3. **What If?** Suppose Antoine instead uses $\frac{1}{8}$ cup of rice for each sushi roll. How would his model change? How many rolls can he make? Explain.

____

____

____

Using Reciprocals to Divide Mixed Numbers

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Dividing by a fraction is equivalent to multiplying by its reciprocal. You can use this fact to divide mixed numbers. First rewrite the mixed numbers as fractions greater than 1. Then multiply the dividend by the reciprocal of the divisor.

My Notes

EXAMPLE 1 Real World — COMMON CORE 6.NS.1

One serving of Harold's favorite cereal contains $1\frac{2}{5}$ ounces. How many servings are in a $17\frac{1}{2}$-ounce box?

STEP 1 Write a division statement to represent the situation.

$17\frac{1}{2} \div 1\frac{2}{5}$

You need to find how many groups of $1\frac{2}{5}$ are in $17\frac{1}{2}$.

STEP 2 Rewrite the mixed numbers as fractions greater than 1.

$17\frac{1}{2} \div 1\frac{2}{5} = \frac{35}{2} \div \frac{7}{5}$

STEP 3 Rewrite the problem as multiplication using the reciprocal of the divisor.

$\frac{35}{2} \div \frac{7}{5} = \frac{35}{2} \times \frac{5}{7}$ *The reciprocal of $\frac{7}{5}$ is $\frac{5}{7}$.*

STEP 4 Multiply.

$\frac{35}{2} \times \frac{5}{7} = \frac{\overset{5}{\cancel{35}}}{2} \times \frac{5}{\underset{1}{\cancel{7}}}$ *Simplify first using the GCF.*

$= \frac{5 \times 5}{2 \times 1}$ *Multiply numerators. Multiply denominators.*

$= \frac{25}{2}$, or $12\frac{1}{2}$ *Write the result as a mixed number.*

There are $12\frac{1}{2}$ servings of cereal in the box.

Reflect

4. **Analyze Relationships** Explain how can you check the answer.

____

____

5. **What If?** Harold serves himself $1\frac{1}{2}$-ounces servings of cereal each morning. How many servings does he get from a box of his favorite cereal? Show your work.

____

YOUR TURN

6. Sheila has $10\frac{1}{2}$ pounds of potato salad. She wants to divide the potato salad into containers, each of which holds $1\frac{1}{4}$ pounds. How many containers does she need? Explain.

____

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Solving Problems Involving Area

Recall that to find the area of a rectangle, you multiply length × width. If you know the area and only one dimension, you can divide the area by the known dimension to find the other dimension.

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EXAMPLE 2 Real World

COMMON CORE 6.NS.1

The area of a rectangular sandbox is $56\frac{2}{3}$ square feet. The length of the sandbox is $8\frac{1}{2}$ feet. What is the width?

STEP 1 Write the situation as a division problem.

$$56\frac{2}{3} \div 8\frac{1}{2}$$

STEP 2 Rewrite the mixed numbers as fractions greater than 1.

$$56\frac{2}{3} \div 8\frac{1}{2} = \frac{170}{3} \div \frac{17}{2}$$

STEP 3 Rewrite the problem as multiplication using the reciprocal of the divisor.

$$\frac{170}{3} \div \frac{17}{2} = \frac{170}{3} \times \frac{2}{17}$$

$$= \frac{\overset{10}{\cancel{170}} \times 2}{3 \times \underset{1}{\cancel{17}}}$$ Multiply numerators. Multiply denominators.

$$= \frac{20}{3}, \text{ or } 6\frac{2}{3}$$ Simplify and write as a mixed number.

The width of the sandbox is $6\frac{2}{3}$ feet.

Math Talk
Mathematical Practices
Explain how to find the length of a rectangle when you know the area and the width.

Reflect

7. **Check for Reasonableness** How can you determine if your answer is reasonable?

____

____

____

____

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YOUR TURN

8. The area of a rectangular patio is $12\frac{3}{8}$ square meters. The width of the patio is $2\frac{3}{4}$ meters. What is the length? ________

9. The area of a rectangular rug is $14\frac{1}{12}$ square yards. The length of the rug is $4\frac{1}{3}$ yards. What is the width? ________

Guided Practice

Divide. Write each answer in simplest form. (Explore Activity and Example 1)

1. $4\frac{1}{4} \div \frac{3}{4}$

$\frac{\square}{4} \div \frac{3}{4} =$

$\frac{\square}{4} \times \frac{\square}{\square} =$

____

2. $1\frac{1}{2} \div 2\frac{1}{4}$

$\frac{\square}{2} \div \frac{\square}{4} =$

$\frac{\square}{2} \times \frac{\square}{\square} =$

____

3. $4 \div 1\frac{1}{8} =$ ________

4. $3\frac{1}{5} \div 1\frac{1}{7} =$ ________

5. $8\frac{1}{3} \div 2\frac{1}{2} =$ ________

6. $15\frac{1}{3} \div 3\frac{5}{6} =$ ________

Write each situation as a division problem. Then solve. (Example 2)

7. A sandbox has an area of 26 square feet, and the length is $5\frac{1}{2}$ feet. What is the width of the sandbox? ________

8. Mr. Webster is buying carpet for an exercise room in his basement. The room will have an area of 230 square feet. The width of the room is $12\frac{1}{2}$ feet. What is the length? ________

ESSENTIAL QUESTION CHECK-IN

9. How does dividing mixed numbers compare with dividing fractions?

____

____

Name ______________ Class ______________ Date ______________

4.3 Independent Practice

COMMON CORE 6.NS.1

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10. Jeremy has $4\frac{1}{2}$ cups of iced tea. He wants to divide the tea into $\frac{3}{4}$-cup servings. Use the model to find the number of servings he can make.

1				1				1				1				1			
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

____

11. A ribbon is $3\frac{2}{3}$ yards long. Mae needs to cut the ribbon into pieces that are $\frac{2}{3}$ yard long. Use the model to find the number of pieces she can cut.

1			1			1			1		
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

____

12. Dao has $2\frac{3}{8}$ pounds of hamburger meat. He is making $\frac{1}{4}$-pound hamburgers. Does Dao have enough meat to make 10 hamburgers? Explain.

____

____

13. **Multistep** Zoey made $5\frac{1}{2}$ cups of trail mix for a camping trip. She wants to divide the trail mix into $\frac{3}{4}$-cup servings.

a. Ten people are going on the camping trip. Can Zoey make enough $\frac{3}{4}$-cup servings so that each person on the trip has one serving?

____

b. What size would the servings need to be for everyone to have a serving? Explain.

____

c. If Zoey decides to use the $\frac{3}{4}$-cup servings, how much more trail mix will she need? Explain.

____

____

14. The area of a rectangular picture frame is $30\frac{1}{3}$ square inches. The length of the frame is $6\frac{1}{2}$ inches. Find the width of the frame.

____

15. The area of a rectangular mirror is $11\frac{11}{16}$ square feet. The width of the mirror is $2\frac{3}{4}$ feet. If there is a 5 foot tall space on the wall to hang the mirror, will it fit? Explain.

16. Ramon has a rope that is $25\frac{1}{2}$ feet long. He wants to cut it into 6 pieces that are equal in length. How long will each piece be?

17. Eleanor and Max used two rectangular wooden boards to make a set for the school play. One board was 6 feet long, and the other was $5\frac{1}{2}$ feet long. The two boards had equal widths. The total area of the set was $60\frac{3}{8}$ square feet. What was the width?

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

18. **Draw Conclusions** Micah divided $11\frac{2}{3}$ by $2\frac{5}{6}$ and got $4\frac{2}{17}$ for an answer. Does his answer seem reasonable? Explain your thinking. Then check Micah's answer.

19. **Explain the Error** To divide $14\frac{2}{3} \div 2\frac{3}{4}$, Erik multiplied $14\frac{2}{3} \times \frac{4}{3}$. Explain Erik's error.

20. **Analyze Relationships** Explain how you can find the missing number in $3\frac{4}{5} \div \blacksquare = 2\frac{5}{7}$. Then find the missing number.

LESSON 4.4

Solving Multistep Problems with Fractions and Mixed Numbers

COMMON CORE 6.NS.1

...Solve word problems involving division of fractions by fractions...

ESSENTIAL QUESTION

How can you solve word problems involving more than one fraction operation?

Solving Problems with Rational Numbers

Sometimes more than one operation will be needed to solve a multistep problem. You can use parentheses to group different operations. Recall that according to the **order of operations,** you perform operations in parentheses first.

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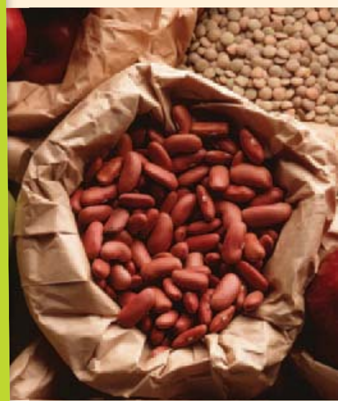
EXAMPLE 1 Problem Solving

COMMON CORE 6.NS.1

Jon is cooking enough lentils for lentil barley soup and lentil salad. The lentil barley soup recipe calls for $\frac{3}{4}$ cup of dried lentils. The lentil salad recipe calls for $1\frac{1}{2}$ cups of dried lentils. Jon has a $\frac{1}{8}$-cup scoop. How many scoops of dried lentils will Jon need to have enough for the soup and the salad?



Analyze Information

Identify the important information.

- Jon needs $\frac{3}{4}$ cup of dried lentils for soup and $1\frac{1}{2}$ cups for salad.
- Jon has a $\frac{1}{8}$-cup scoop.
- You need to find the total number of many scoops of lentils he needs.

Formulate a Plan

You can use the expression $\left(\frac{3}{4} + 1\frac{1}{2}\right) \div \frac{1}{8}$ to find the number of scoops of dried lentils Jon will need for the soup and the salad.

Solve

Follow the order of operations. Perform the operations in parentheses first.

First add to find the total amount of dried lentils Jon will need.

$$\frac{3}{4} + 1\frac{1}{2} = \frac{3}{4} + \frac{3}{2}$$
$$= \frac{3}{4} + \frac{6}{4}$$
$$= \frac{9}{4}$$
$$= 2\frac{1}{4}$$

John needs $2\frac{1}{4}$ cups of lentils.

Jon needs $2\frac{1}{4}$ cups of dried lentils for both the soup and the salad.

To find how many $\frac{1}{8}$-cup scoops he needs, divide the total amount of dried lentils into groups of $\frac{1}{8}$.

$$2\frac{1}{4} \div \frac{1}{8} = \frac{9}{4} \div \frac{1}{8}$$

$$= \frac{9}{4} \times \frac{8}{1}$$

$$= \frac{9 \times \cancel{8}^{2}}{{}_{1}\cancel{4} \times 1}$$

Simplify before multiplying using the GCF.

$$= \frac{18}{1} = 18$$

Jon will need 18 scoops of dried lentils to have enough for both the lentil barley soup and the lentil salad.

Justify and Evaluate

You added $\frac{3}{4}$ and $1\frac{1}{2}$ first to find the total number of cups of lentils. Then you divided the sum by $\frac{1}{8}$ to find the number of $\frac{1}{8}$-cup scoops.

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YOUR TURN

1. Before conducting some experiments, a scientist mixes $\frac{1}{2}$ gram of Substance A with $\frac{3}{4}$ gram of Substance B. If the scientist uses $\frac{1}{8}$ gram of the mixture for each experiment, how many experiments can be conducted? ____________

Guided Practice

1. An art student uses a roll of wallpaper to decorate two gift boxes. The student will use $1\frac{1}{3}$ yards of paper for one box and $\frac{5}{6}$ yard of paper for the other box. The paper must be cut into pieces that are $\frac{1}{6}$ yard long. How many pieces will the student cut to use for the gift boxes? (Example 1) ________________

ESSENTIAL QUESTION CHECK-IN

2. How can you solve a multistep problem that involves fractions?

____

Name ______________________ Class ______________ Date ______________

4.4 Independent Practice

COMMON CORE 6.NS.1

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3. Naomi has earned \$54 mowing lawns the past two days. She worked $2\frac{1}{2}$ hours yesterday and $4\frac{1}{4}$ hours today. If Naomi is paid the same amount for every hour she works, how much does she earn per hour to mow lawns? (Example 2)

____

4. An art teacher has $1\frac{1}{2}$ pounds of red clay and $\frac{3}{4}$ pound of yellow clay. The teacher mixes the red clay and yellow clay together. Each student in the class needs $\frac{1}{8}$ pound of the clay mixture to finish the assigned art project for the class. How many students can get enough clay to finish the project?

____

5. A hairstylist schedules $\frac{1}{4}$ hour to trim a customer's hair and $\frac{1}{6}$ hour to style the customer's hair. The hairstylist plans to work $3\frac{1}{3}$ hours each day for 5 days each week. How many appointments can the hairstylist schedule each week if each customer must be trimmed and styled?

____

6. A picture framer has a thin board $10\frac{1}{12}$ feet long. The framer notices that $2\frac{3}{8}$ feet of the board is scratched and cannot be used. The rest of the board will be used to make small picture frames. Each picture frame needs $1\frac{2}{3}$ feet of the board. At most, how many complete picture frames can be made?

____

7. Jim's backyard is a rectangle that is $15\frac{5}{6}$ yards long and $10\frac{2}{5}$ yards wide. Jim buys sod in pieces that are $1\frac{1}{3}$ yards long and $1\frac{1}{3}$ yards wide. How many pieces of sod will Jim need to buy to cover his backyard with sod?

____

8. Eva wants to make two pieces of pottery. She needs $\frac{3}{5}$ pound of clay for one piece and $\frac{7}{10}$ pound of clay for the other piece. She has three bags of clay that weigh $\frac{4}{5}$ pound each. How many bags of clay will Eva need to make both pieces of pottery? How many pounds of clay will she have left over?

____

9. Mark wants to paint a mural. He has $1\frac{1}{3}$ gallons of yellow paint, $1\frac{1}{4}$ gallons of green paint, and $\frac{7}{8}$ gallon of blue paint. Mark plans to use $\frac{3}{4}$ gallon of each paint color. How many gallons of paint will he have left after painting the mural?

____

10. Trina works after school and on weekends. She always works three days each week. This week she worked $2\frac{3}{4}$ hours on Monday, $3\frac{3}{5}$ hours on Friday, and $5\frac{1}{2}$ hours on Saturday. Next week she plans to work the same number of hours as this week, but will work for the same number of hours each day. How many hours will she work on each day?

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

11. **Represent Real-World Problems** Describe a real-world problem that can be solved using the expression $29 \div \left(\frac{3}{8} + \frac{5}{6}\right)$. Find the answer in the context of the situation.

____

____

____

____

____

12. **Justify Reasoning** Indira and Jean begin their hike at 10 a.m. one morning. They plan to hike from the $2\frac{2}{5}$-mile marker to the $8\frac{1}{10}$-mile marker along the trail. They plan to hike at an average speed of 3 miles per hour. Will they reach the $8\frac{1}{10}$-mile marker by noon? Explain your reasoning.

____

____

____

____

____

13. **Multiple Representations** You are measuring walnuts for banana-walnut oatmeal and a spinach and walnut salad. You need $\frac{3}{8}$ cup of walnuts for the oatmeal and $\frac{3}{4}$ cup of walnuts for the salad. You have a $\frac{1}{4}$-cup scoop. Describe two different ways to find how many scoops of walnuts you will need.

____

____

____

____

____

MODULE QUIZ

Ready to Go On?

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4.1 Applying GCF and LCM to Fraction Operations

Solve.

1. $\frac{4}{5} \times \frac{3}{4}$ ______

2. $\frac{5}{7} \times \frac{9}{10}$ ______

3. $\frac{3}{8} + 2\frac{1}{2}$ ______

4. $1\frac{3}{5} - \frac{5}{6}$ ______

4.2 Dividing Fractions

Divide.

5. $\frac{1}{3} \div \frac{7}{9}$ ______

6. $\frac{1}{3} \div \frac{5}{8}$ ______

7. Luci cuts a board that is $\frac{3}{4}$ yard long into pieces that are $\frac{3}{8}$ yard long. How many pieces does she cut? ______

4.3 Dividing Mixed Numbers

Divide.

8. $3\frac{1}{3} \div \frac{2}{3}$ ______

9. $1\frac{7}{8} \div 2\frac{2}{5}$ ______

10. $4\frac{1}{4} \div 4\frac{1}{2}$ ______

11. $8\frac{1}{3} \div 4\frac{2}{7}$ ______

4.4 Solving Multistep Problems with Fractions and Mixed Numbers

12. Jamal hiked on two trails. The first trail was $5\frac{1}{3}$ miles long, and the second trail was $1\frac{3}{4}$ times as long as the first trail. How many miles did Jamal hike? ______

ESSENTIAL QUESTION

13. Describe a real-world situation that is modeled by dividing two fractions or mixed numbers.

______

______

______

COMMON CORE

MODULE 4 MIXED REVIEW

Assessment Readiness

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Selected Response

1. Two sides of a rectangular fence are $5\frac{5}{8}$ feet long. The other two sides are $6\frac{1}{4}$ feet long. What is the perimeter?

Ⓐ $11\frac{7}{8}$ feet   Ⓑ 13 feet

Ⓒ $23\frac{3}{4}$ feet   Ⓓ $35\frac{5}{32}$ feet

2. Which shows the GCF of 18 and 24 with $\frac{18}{24}$ in simplest form?

Ⓐ GCF: 3; $\frac{3}{4}$

Ⓑ GCF: 3; $\frac{6}{8}$

Ⓒ GCF: 6; $\frac{3}{4}$

Ⓓ GCF: 6; $\frac{6}{8}$

3. A jar contains 133 pennies. A bigger jar contains $1\frac{2}{7}$ times as many pennies. What is the value of the pennies in the bigger jar?

Ⓐ \$1.49   Ⓑ \$1.52

Ⓒ \$1.68   Ⓓ \$1.71

4. Which of these is the same as $\frac{3}{5} \div \frac{4}{7}$?

Ⓐ $\frac{3}{5} \div \frac{7}{4}$

Ⓑ $\frac{4}{7} \div \frac{3}{5}$

Ⓒ $\frac{3}{5} \times \frac{4}{7}$

Ⓓ $\frac{3}{5} \times \frac{7}{4}$

5. Andy has $6\frac{2}{3}$ quarts of juice. How many $\frac{2}{3}$-cup servings can he pour?

Ⓐ $4\frac{4}{9}$ Ⓑ 6

Ⓒ 7   Ⓓ 10

6. What is the reciprocal of $3\frac{3}{7}$?

Ⓐ $\frac{7}{24}$ Ⓑ $\frac{3}{7}$

Ⓒ $\frac{7}{3}$ Ⓓ $\frac{24}{7}$

7. A rectangular patio has a length of $12\frac{1}{2}$ feet and an area of $103\frac{1}{8}$ square feet. What is the width of the patio?

Ⓐ $4\frac{1}{8}$ feet

Ⓑ $8\frac{1}{4}$ feet

Ⓒ $16\frac{1}{2}$ feet

Ⓓ 33 feet

8. Which number is greater than the absolute value of $-\frac{3}{8}$?

Ⓐ $-\frac{5}{8}$

Ⓑ $-\frac{1}{8}$

Ⓒ $\frac{1}{4}$

Ⓓ 0.5

Mini-Task

9. Jodi is cutting out pieces of paper that measure $8\frac{1}{2}$ inches by 11 inches from a larger sheet of paper that has an area of 1,000 square inches

a. What is the area of each piece of paper that Jodi is cutting out?

____

b. What is the greatest possible number of pieces of paper that Jodi can cut out of the larger sheet?

____