Operations with Fractions

Purpose and Audience

This packet can be used in regular classrooms for helping students understand the concepts behind operations with fractions. It is mostly useful for 4^{th}-7^{th} grade. It can also be used to help struggling students in small groups outside of the regular classroom instruction.

Contents

This packet contains the following:

Instructions for using these worksheets

There are several kinds of tasks in these worksheets. Some should be done in groups of two students working together. Some should be done individually and independently. Some are teacher-led. In all cases, there should be discussion about the tasks, but many times the students will be the ones explaining what they did.

Make sure that students are given ample time to work through these problems on their own (or in the groups of two). Do not rush in to give them hints or tell them how to do the problems. Instead, suggest that they use circle fraction pieces or drawings, or that they look at previous problems, to figure things out. They need to find ways to solve these problems that draw on what they know already. This is how they learn for long-term understanding. Let them struggle a little and learn from each other.

The basic approach to learning about fractions is the “concrete – representational – abstract” approach, also known as “objects – pictures – symbols.” Students start with concrete objects to represent fractions, manipulating them to show the operations. Then they use drawings to represent what they’ve done with objects. Drawings are more efficient than objects, but fairly similar. Then they translate what they’ve done with objects and drawings into abstract symbols, finding procedures they can use to be even more efficient with fraction operations.

When students are struggling with the symbolic operations, ask them to represent what they’re trying to do with manipulatives or drawings. This helps many struggling students reconstruct what they’ve learned earlier.

You should read through each section carefully to determine which problems the students will do in small groups, which they will do independently, and when you will draw the whole class together for discussion.

The most important teaching that you will do with this packet is to make sure that students are progressing from objects to drawings to symbolic procedures – that they are making sense of the operations and translating them into procedures. Draw out their ideas about how to do each operation, and focus on it. Don't tell them how to do a procedure: Take what they are thinking and build a class consensus about the procedure. Remember that different procedures are possible for different operations.

You may find that students need more practice than what is given on these pages. Please feel free to add to this work as needed to help your students. This packet is just an outline of what they need to learn – the actual work of teaching is still up to you.

Format: The student pages are formatted with a narrow column so that students can put a full sheet of blank paper up next to the column, to the right of it, overlapping the student page, then do their work and record their answers on the blank paper. This saves the student pages to use as a classroom set, if you don't want to make a copy for every student.

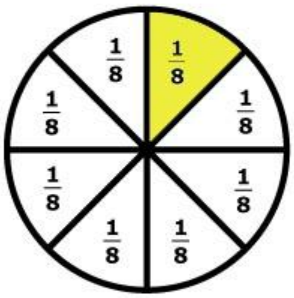
Teacher Background Information

Fraction Basics

By the end of 4th grade, students should have learned these fundamentals of fractions:

1. A fraction is a part of a whole. When a whole is divided into equal-sized pieces, a fraction of the whole is one or more of those pieces. The numerator of the fractions tells how many pieces there are, and the denominator tells how many pieces the whole was divided into.

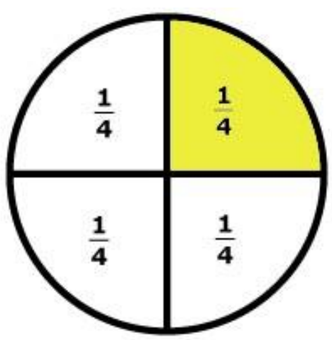
For example, a pizza cut into 8 slices can represent eighths. One slice is 1/8. 4 slices is 4/8. (It's hard to make proper fractions on wikispaces, so please don't be annoyed at the use of the diagonal fraction bar. You should always use the horizontal fraction bar with young students.)

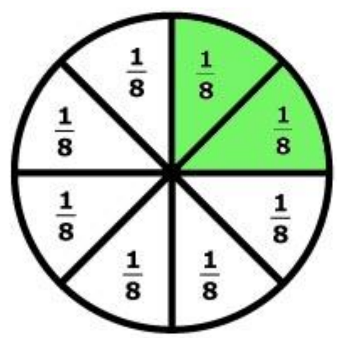


2. Manipulatives and drawings are good ways of representing fractions. Circle fractions can be used as direct models of pizza slices (like the drawing to the left.) Other representations include bar models (think about 1/8 of a Tootsie Roll) and area models (1/8 of a pan of brownies cut 4x2).

3. Rulers are marked to show fractions of one inch. Number lines can be drawn to show that there are numbers (fractions) between integers.

4. Equivalent fractions are the same size (the same total amount of the whole) but they are divided into different numbers of pieces. For example, if you cut a pizza into fourths, one section would be 1/4. If you cut a pizza of the same size into eighths, two sections of that pizza would be 2/8. Both of these sections are the same size.





5. Fractions can also be used to represent parts of a set of object. For example, if there are 10 red M&M's in a bag of 50 M&M's, then the fraction of red M&M's is 10/50.

6. Equivalent fractions can be found by "scaling up" or "scaling down." This makes sense when you think of a fraction as part of a set of objects. In the M&M example above, if there is always the same fraction of red M&M's in every bag, then a bag of 25 (half the original bag) would have 5 red M&M's (half the original amount). The fraction is 5/25, which is equivalent to 10/50. In this case, we have scaled down by a factor of 2, or divided both the part and the whole by 2. In a bag of 100 M&M's, there would be 20 red M&M's, scaling up by a factor of 2 - multiplying both

the part and the whole by 2.

7. "Mixed numbers" are numbers that have both an integer part and a fraction part, like 5 1/3. On the number line, this would be 1/3 of the way from 5 to 6. As a real object, this might be 5 1/3 cups of flour in a cake recipe (5 cups and 1/3 cup more). Mixed numbers can also be represented as "improper fractions," an equivalent number with no integer part. In improper fractions, the numerator is larger than the denominator. For example, 5 1/3 is equivalent to 16/3.

8. Decimal numbers can be written as fractions with denominators of 10, 100, 1000, etc. The first place to the right of the decimal represents tenths, the second decimal place represents hundredths, the third decimal place represents thousandths, etc. For example, the number 7.3 is equivalent to 7 3/10. The number 20.45 is equivalent to 20 45/100.

9. Not all fractions can be represented by terminating decimal numbers. For example, 1/3 has the decimal equivalent of 0.333333... a repeating, non-terminating decimal (it goes on forever).

10. Fractions are also a way of writing a division statement. 4/5 means 4 divided by 5. You can use this concept to find the decimal equivalent for a fraction: 4 divided by 5 = 0.8.

11. Percents represent parts out of 100. 50% means 50 parts out of 100. This is equivalent to the fraction 50/100, or the decimal 0.5.

Adding and Subtracting Fractions

Students learn to add and subtract fractions by playing with fraction circles*. They have to spend lots of time working with fraction circles to model adding and "taking away." This is how they come to see why the procedure works for adding fractions when the denominators are the same. They recognize that two fractions with the same denominator are just pieces of a whole that has been cut into equal size pieces, so they're just adding (or subtracting) pieces. For example, think of a pizza cut into eighths (8 pieces). 3/8 of a pizza plus 4/8 of a pizza means "3 pieces + 4 pieces = 7 pieces" or 7 eighths.

When one denominator is a multiple of the other, students need to be able to generate equivalent fractions, and then use what they know about adding or subtracting when the denominators are the same. They should learn the process of scaling up or scaling down to create equivalent fractions with denominators that are multiples. To do this, they need to understand conceptually what equivalent fractions are. Their early work with fraction circle pieces involves finding equivalent fractions (other names for the same size pieces). See Fraction Basics and "Equivalent Fractions" in the Adding and Subtracting section of the Student Packet.

When the denominators are not multiples, students first need to learn to estimate the size of the sum or difference. Let students spend considerable time working on #12 in the Basic Fraction Concepts (an assessment) section of the Student Packet. This is where their experience with fraction circles is very valuable.

When the denominators are not multiples, the GLCEs suggest that the easiest starting point is to use the denominator that is the product of the two denominators. However, if they are using fraction bars to model the operation, say 1/4 + 5/6, they might easily find that the 1/12's bars can be used to reconstruct both 1/4 and 5/6, getting 13/12 rather than 26/24 as an answer. Either answer is acceptable. Don't confuse students at this point with LCM while they're still learning to add fractions. (You probably don't need to introduce LCM at all, or GCF either.) If students use the product of the two denominators as the common denominator, then you can focus on scaling down to "reduce" the answer, if you want.

For those students who are ready to move ahead, they can learn to scale up both fractions until they find equivalent fractions with the same denominators.

*Fraction circles can be purchased at The Teacher Store, 6001 S. Pennsylvania Ave., Lansing, or online at sites such as http://www.enasco.com/product/TB16618T.

Multiplying Fractions

An instructional sequence for learning how to multiply with fractions generally takes students through three steps:

1. Multiplying a fraction by a whole number – repeated addition of the fraction

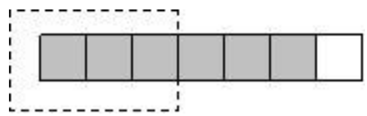
4 · 1/6 means 4 groups of 1/6, or 1/6 + 1/6 + 1/6 + 1/6, which equals 4 sixths, or 4/6.

2. Multiplying a whole number by a fraction – taking a part of a whole number

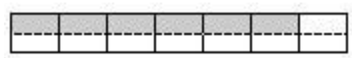
1/2 · 4 literally means "1/2 groups of 4" or just "1/2 of 4," which is found by dividing 4 equally into 2 parts, or 4÷2. To find 2/3 of 9, we first find 1/3 of 9 (3) and then we want 2 of them (6).

3. Multiplying a fraction by a fraction – taking a part of a fraction

1/2 · 6/7 means 1/2 of 6/7, or 1/2 of 6 sevenths, which is 3 sevenths - most easily seen as a drawing.



1/2 · 6/7 can also be represented like this, which by counting, shows 6/14 (which is equivalent to 3/7).



What real-world problems can be solved using each of these ways to multiply fractions?
Match each problem below to a type of multiplication above.

1. A bakery has planned to make cakes today. They need the following ingredients for each cake. They want to bake 12 cakes. How much of each ingredient do they need for all 12 cakes?

3/4 cup of sugar	2 1/3 cups of flour
1/4 teaspoon of salt	2/3 tablespoon of baking powder

2. You have 6 donuts and you want to give 2/3 of them to a friend and keep 1/3 for yourself. How many donuts would your friend get? That is, how much is 2/3 of 6?

3. A pan of brownies was left out on the counter and 1/4 of the brownies were eaten. Then you came along and ate 2/3 of the brownies that were left. How much of the whole pan of brownies was eaten?

How can you calculate answers to multiplications involving fractions?

1. When you multiply a fraction by a whole number, you can see that you multiply the whole number times the numerator, and leave the denominator as it is. This is how you calculated 4 · 1/6 – you're calculating 4 times 1 sixth, which is 4 sixths.

2. When you find a fraction of a whole number, you divide the whole number by the denominator. If the numerator is more than 1, you then multiply the answer by the numerator. This is how you found 2/3 of 9. Of course, you could multiply first, then divide.

3. When you calculate a fraction of a fraction, you can easily see from the example and others like it that you can multiply the numerators and multiply the denominators.

Estimation when multiplying with mixed numbers

Students often find it helpful to estimate answers when finding a fraction of a mixed number, as a first step – just to know what the "ballpark" is for the answer. For the problem $\frac{1}{2} \times 5\frac{3}{4}$, $\frac{1}{2}$ of 5 is $2\frac{1}{2}$, a good estimate.

This might lead to the idea that they can find the fraction of the whole number separately from finding the fraction of the fraction, then add the results (the distributive property).

$\frac{1}{2} \times 5\frac{3}{4} \rightarrow (\frac{1}{2} \times 5) + (\frac{1}{2} \times \frac{3}{4}) \rightarrow (\frac{5}{2}) + (\frac{3}{8}) \rightarrow (\frac{20}{8}) + (\frac{3}{8}) \rightarrow (\frac{23}{8}) = 2\frac{7}{8}$ (which is close to the estimate of $2\frac{1}{2}$)

Or they can use similar graphical methods that they developed when finding a fraction of a fraction.

Or a student might decide to convert the mixed number to an improper fraction first. $\frac{1}{2} \times \frac{23}{4}$

Dividing Fractions

An instructional sequence for learning how to divide with fractions is similar to the one for multiplying.

1. Dividing a fraction by a whole number – "Partitioning" into equal groups

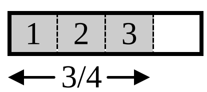
1/4 ÷ 2 means to start with 1/4 of something and divide that quantity equally into 2 groups – in this case, with 1/8 in each group.

2. Dividing a whole number by a fraction – "Measurement division"

6 ÷ 1/2 means "How many 1/2's are in 6?" Since there are 2 halfs in each whole, times six wholes, there are 12 halfs in 6 wholes. (This is a simple origin of "invert and multiply.")

3. Dividing a fraction by a fraction – Also a case of measurement division

3/4 ÷ 1/4 means "How many 1/4's are in 3/4?" A simple drawing can show that there are 3 fourths in 3/4.



This is a little trickier when there isn't an integer number of the divisor in the dividend.

3/4 ÷ 1/2 means "How many 1/2's are in 3/4?" You can see from the drawing above that there is one full 1/2 in the shaded 3/4, plus another half of a 1/2. The answer is 1 1/2. This means there are 1 1/2 halves in 3/4.

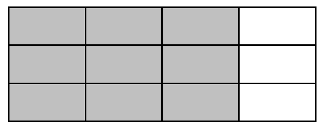
If you use the traditional procedure for calculating the answer (invert and multiply) you will get the same answer.

It's important for students to learn how to estimate answers to division problems to develop a sense of what the answer should be.

Basic Fraction Concepts (an assessment)

1. Three brownies have been eaten out of this pan. What fraction of the pan of brownies is left?

a) 3/9
b) 3/12
c) 9/12
d) 9/3



2. In a bag of 40 M&M's, you count 12 red ones. What fraction of the M&M's are red?

a) 40/12
b) 12/28
c) 6/40
d) 12/40

3. If all bags of M&M's had the same fraction of red ones as in problem 2, how many red ones would you find in a bag that has 80 M&M's in it?

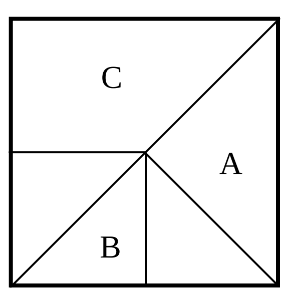
a) 80
b) 40
c) 24
d) 12

4. Which is larger, 3/4 or 3/7? Make a drawing to explain your answer.

5. Look at the drawing below. What fraction of the whole square is region A? __________

region B? __________

region C? __________

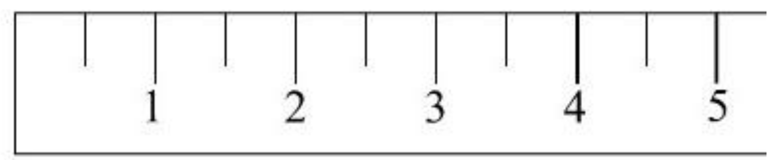


6. Which is larger, $\frac{3}{8}$ or $\frac{3}{12}$? Explain why you think this.

7. Which is larger, $\frac{3}{4}$ or $\frac{2}{3}$? Explain why you think this.

8. Show where these fractions would be on this ruler:

$\frac{1}{4}$ $\quad 1\frac{1}{2}$ $\quad 2\frac{3}{4}$ $\quad \frac{8}{4}$



9. Steve was looking at a plate with 12 brownies. Steve said, "If I carefully cut off 1/4 of each brownie and put them together, I will have 3 full brownies! Is this true? How can you prove it?

10. Order these fractions from smallest to largest:

$\frac{3}{4}, \frac{1}{10}, \frac{5}{12}, \frac{3}{5}, \frac{14}{15}$

smallest ______________________________ largest

11. Which would you rather have, 3/5 of a bag of M&M's that contain 50 pieces, or 2/3 of a bag of M&M's that contains 30 pieces? Explain your answer.

12. For each of the following problems, explain if you think the answer is a reasonable estimate or not.

$$\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$$

$$\frac{11}{12}-\frac{1}{2}=\frac{10}{12}$$

$$\frac{1}{4}-\frac{2}{100}=\frac{1}{3}$$

$$\frac{1}{5}+\frac{2}{3}=\frac{3}{5}$$

$$\frac{2}{3}-\frac{1}{4}=\frac{1}{12}$$

13. Locate and label $\frac{21}{8}$ on a number line. How much is this as a mixed number?

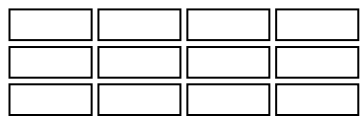
14. Write a fraction that is equivalent to $\frac{9}{12}$

Adding and Subtracting Fractions

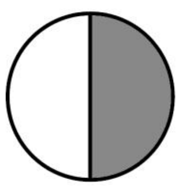
Part 1

Equivalent Fractions

Jackie has 1/3 of a Hershey bar. Steven has 4/12 of a Hershey bar. Who has more?



Use fraction circle pieces to figure out how many different ways you can make 1/2 out of different pieces.



Write at least three combinations of other fractions that are the same as 1/2:

$\frac{1}{2} =$ $\frac{1}{2} =$ $\frac{1}{2} =$

Two fractions that represent the same amount of the whole are called **equivalent fractions**.

Find two equivalent fractions for each of these. Use fraction circles or drawings if you want

$\frac{3}{4} =$ $\frac{1}{4} =$

$\frac{1}{3} =$ $\frac{2}{3} =$

Do you see a mathematical procedure you could use to find equivalent fractions? Explain what the procedure might be.

Mathematically, you can **scale up** 1/2 to each of the other fractions by doubling, tripling or quadrupling the numerators and denominators. Figure out another fraction that is equivalent to 1/2.

Find equivalent fractions that have smaller denominators for each of these:

$\frac{6}{8} =$ $\frac{6}{12} =$

$\frac{4}{10} =$ $\frac{3}{9} =$

Do you see a mathematical procedure you could use to find equivalent fractions that have smaller denominators? You would **scale down** in this case, because the new fractions use proportionally smaller numbers.

Part 2

Adding and subtracting fractions with the same denominator

Think about this: You have 2/6 of a pizza. Is this less than 1/2 or more than 1/2? Use fraction circles to help figure this out. Explain your answer.

1. **Your class had a pizza party. 3/8 of one pizza was left over, and 4/8 of another pizza was left over. You put them both into one box.**

- Do you have more than 1 whole pizza, or less than 1 whole pizza?

Explain your answer. Use fraction circle pieces or drawings to help explain.

- How much pizza do you have altogether? $\frac{3}{8} + \frac{4}{8} =$

Does this problem make sense if you think of each eighth of the pizza as one slice? Is this how many slices you have altogether? 3 slices + 4 slices = ___ slices

This does make sense because the denominator of a fraction tells how big each piece is. Each pizza is cut into 8 pieces, so each piece is one eighth of the whole. 3/8 of the pizza is the same as 3 slices of the pizza. 4/8 is 4 slices. You're just adding slices.

Try this with a slightly different problem:

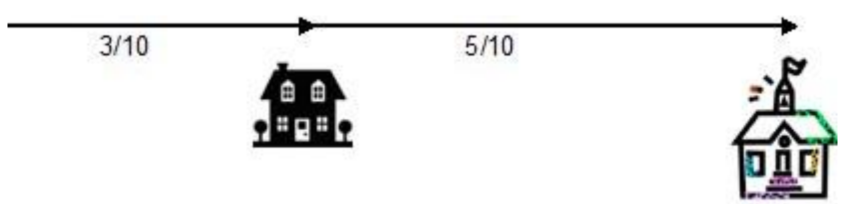
2. A cake recipe requires 3/5 cup of sugar for the frosting and 1/5 cup of sugar for the cake. How much sugar is that altogether?

Explain your answer. Use drawings, fraction bars or fraction circles to explain.

Use fraction circles, fraction bars or drawings to find

$\frac{1}{3}+\frac{4}{3}$ $\qquad\qquad$ $1\frac{2}{3}+\frac{2}{3}$

3. You walk 3/10 of a mile to your friend's house, and then 5/10 of a mile to school. How far did you walk altogether?



Describe a mathematical procedure you can use to add fractions with the same denominators.

Part 3

Adding and subtracting fractions with different denominators

4. There is 3/8 of a pizza in one box and 1/4 of a pizza in another box.

- If you put the leftover pizza into one box, would you have more than 1/2 or less than 1/2 of a whole pizza?

  ____

Explain your thinking. Use a drawing or fraction circles if you want.

- How much do you have altogether? $\frac{3}{8}+\frac{1}{4}=$

What did you do to find the answer?

Try your procedure again to add or subtract these fractions:

$\frac{3}{4}+\frac{5}{8}$ $\frac{2}{3}-\frac{1}{6}$

Try these problems:

5. 1/10 of the M&M's in a bag are red and 1/5 are blue. What fraction of all the M&M's are red and blue? What fraction of the M&M's are NOT red or blue?

6. You give 1/3 of a pan of brownies to Susan and 1/6 of the pan of brownies to Patrick. How much of the pan of brownies did you give away? How much do you have left?

7. You go out for a long walk. You walk 3/4 mile and then sit down to take a rest. Then you walk 3/8 of a mile. How far did you walk altogether?

8. Pam walks 7/8 of a mile to school. Paul walks 1/2 of a mile to school. How much farther does Pam walk than Paul?

9. A school wants to make a new playground by cleaning up an abandoned lot that is shaped like a rectangle. They give the job of planning the playground to a group of students. The students decide to use 1/4 of the playground for a basketball court and 3/8 of the playground for a soccer field. How much is left for the swings and play equipment? Draw a picture to show this.

Explain a procedure you can use to add or subtract fractions that have different denominators.

Part 4

Adding and subtracting with denominators that are not multiples

10. Marty made two types of cookies. He used 2/3 cup of sugar for one recipe and 1/4 cup of sugar for the other.

- Is the total amount of sugar greater than 1/2 cup or less than 1/2 cup? ___________
- Is the total amount greater than 1 cup or less than 1 cup? ___________

Explain your answers.

Some people would say that $\frac{2}{3}+\frac{1}{4}=\frac{3}{7}$.

a) Why would they say this?

b) Do you agree?

c) How would you solve this problem? Work with fraction circles to figure this out. Record your answer below.

$$\frac{2}{3}+\frac{1}{4}=$$

- How is the denominator of the answer related to the two denominators in the problem?

Try these:

$\frac{1}{3}+\frac{1}{2}=$

$\frac{1}{6}+\frac{3}{4}=$

$\frac{2}{6}+\frac{1}{5}=$

If you can add fractions, you can subtract them also.
Try some as a review:

1. After a party, $\frac{5}{8}$ of the cake is left over. That night, big brother eats $\frac{2}{8}$ of the cake. How much is left over after that?

2. You have $7\frac{5}{8}$ feet of yarn to make a bracelet. You only use $4\frac{1}{8}$ yards for the bracelet. How much yarn is left over?

3. Susan swims a race in $29\frac{3}{10}$ seconds. Patty swims the race in $33\frac{9}{10}$ seconds. How much faster was Susan than Patty?

4. A pitcher contains $2\frac{3}{4}$ pints of orange juice. After you pour $\frac{5}{8}$ of a pint into a glass, how much is left in the pitcher?

5. $\frac{7}{8}-\frac{1}{2}=$

6. $\frac{5}{6}-\frac{1}{4}=$

7. $\frac{7}{10}-\frac{3}{4}=$

Multiplying Fractions

Part 1

Multiplying a fraction and a whole number: Repeated addition

A dime is 1/2 inch wide. If you put 5 dimes end to end, how long would they be from beginning to end?

Write a mathematical expression that shows how you got your answer.

Use fraction circles to figure out 3 times 1/4.

Do the same for these:

$5 \cdot \frac{1}{8}$ $3 \cdot \frac{2}{5}$

$4 \cdot \frac{3}{10}$

What patterns do you see in your results?

The patterns you see in your results are a prediction. Come up with your own examples to see if your prediction holds.

Find the following. Use manipulatives or drawings to help, if you want.

$7 \cdot \frac{2}{5}$ $4 \cdot \frac{1}{2}$

Write the answers to both problems above as improper fractions **and** mixed numbers.

1. **A bakery is making cakes today. They want to bake 12 cakes. They need the following ingredients for each cake. How much of each ingredient do they need for all 12 cakes?**

For 1 cake	**For 12 cakes**
$\frac{3}{4}$ cup of sugar	
$\frac{1}{4}$ teaspoon of salt	
$\frac{2}{3}$ tablespoon of baking	
$2\frac{1}{3}$ cups of flour	

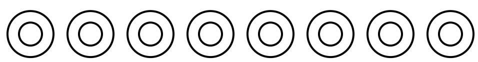
Write a story to go with $5 \cdot \frac{1}{8}$

Part 2

Finding a fraction of a whole number

2. You have 10 cookies and want to give 1/2 of them to a friend. How many do you give to your friend?

3. You have 8 donuts and you want to give 1/4 of them to a friend. How many donuts would your friend get?



(You can think of this as sharing the donuts equally among 4 friends. How many does each friend get?)

We talk about these two problems as 1/2 *of* 10 and 1/4 *of* 8. These are multiplication problems, like saying that 6 x 3 means 6 groups *of* 3.

You can write the donut problem as $\frac{1}{4}$ of 8 or $\frac{1}{4} \cdot 8$

- How much is $\frac{1}{8} \cdot 16$, or 1/8 of 16?

Can you see this problem as a division problem? What division expression would represent this problem?

Write an explanation of a mathematical procedure you can use to multiply a unit fraction times a whole number.

- If you use repeated addition to figure out $8 \cdot \frac{1}{4}$, what do you get?

  Is this the same as $\frac{1}{4} \cdot 8$ Why?

4. You have 6 donuts and you want to give 2/3 of them to a friend and keep the rest for yourself. How many donuts would your friend get?

Draw a picture if it helps. Explain how you found your answer.

- Using what you know, figure out $\frac{3}{4}$ of 12.

When you try each of the multiplications below, say to yourself, this is 3/8 *of* 16, etc.

$\frac{3}{8} \cdot 16$

$\frac{2}{5} \cdot 20$

$\frac{2}{3} \cdot 9$

Explain how you found your answers.

- Solve 4/5 of 200 any way you want to, and explain how you did it.

- Make up another problem like this, using a number over 100. Then solve it.

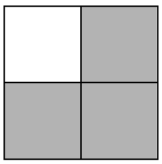
5. Five friends buy a package of 12 cookies and want to share them equally. Each friend will get 1/5 of the cookies. How much will each friend get?

Use a drawing to show this if you want.

Part 3

What does it mean to multiply a fraction times a fraction?

6. 3/4 of a pan of brownies was sitting on the counter. You decided to eat 1/3 of the brownies in the pan. How much of the whole pan of brownies did you eat?

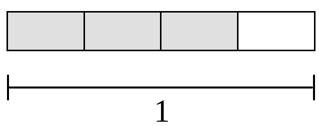


You could write this as $\frac{1}{3}$ of $\frac{3}{4}$. Since we know that "of" means "times," this is a multiplication problem: $\frac{1}{3} \cdot \frac{3}{4}$

To figure this out, look at the drawing. What is 1/3 of the shaded area? Color 1/3 of the shaded area with hatch marks. How much of the whole pan of brownies is that?

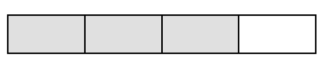
- So what is $\frac{1}{3} \cdot \frac{3}{4}$? This is often confusing, because you have to think about how much of **the whole** you get.

- Find $\frac{2}{3} \cdot \frac{3}{4}$



You could visually see the answer to the last two problems in the drawing. The next ones are not as easy, but drawings can help. Here's how you can find 1/2 · 3/4 (1/2 *of* 3/4)

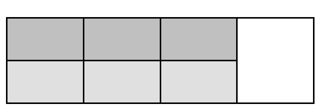
First, make a drawing that shows 3/4.



Actually, make it thick.

Then, to find 1/2 of the 3/4, draw a line across the 3/4 to cut it in half.

Shade 1/2 of 3/4 darker.



Next, ask yourself, how much of the whole is this new, dark shaded part? That is, how much of the whole is 1/2 of 3/4?

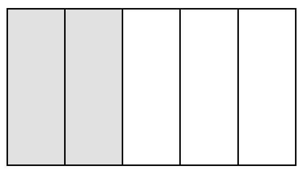
Well, when you cut the original 3/4 in half, you made eighths.

1	2	3	4
5	6	7	8

So how many eighths are shaded darkly? This is your answer.

- So what is $\frac{1}{2} \cdot \frac{3}{4}$?

Now you try it. Copy this picture onto your paper, then draw lines to find 3/4 of 2/5.



- $\frac{3}{4} \cdot \frac{2}{5}$

Do you see a pattern in these answers? Without making a drawing, predict what the answer would be to this: $\frac{2}{5} \cdot \frac{2}{3}$

____

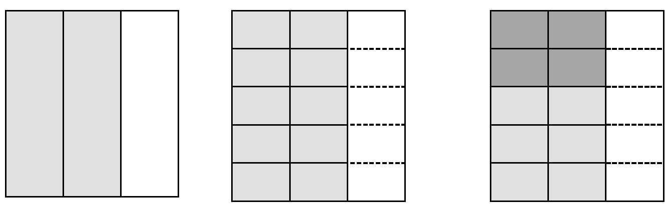
Then check your prediction with a drawing.

Write a few sentences to explain to someone else how to multiply fractions:

The method of dividing a rectangle in both directions can be generalized to a procedure. The procedure is to multiply the denominators to find the new denominator, and multiply the numerators to find the new numerator.

$\frac{3}{4} \cdot \frac{2}{5} = \frac{6}{20}$ $\frac{2}{5} \cdot \frac{2}{3} = \frac{4}{15}$

Do you see why that works? Consider 2/5 of 2/3. Create a rectangle that shows 2/3. Then divide that 2/3 into fifths. What fraction have you created in the new drawing? You've created 15ths. This is the product of the denominators.



When you shade the 2/5 part of 2/3, you have shaded four 15ths. 4 is the product of the numerators, a 2x2 grid made from 2 parts of the 2/3 by 2 parts of the 2/5.

7. **You have 3/4 of a pizza. You want to divide it equally between two friends. How much do you each get?**

   a. Write this problem as a division statement.

   b. Solve it by using the procedure we described above.

   c. Show how you could get an answer by using fraction circles or drawing a picture.

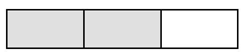
Part 4

Simplifying fractions when multiplying

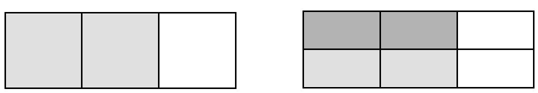
8. You have 2/3 of a pumpkin pie left over from Thanksgiving. You want to give 1/2 of it to your sister. How much of the whole pumpkin pie will this be? (Use fraction circles or drawings to figure this out.)

Here are two drawing methods for this problem:

Find $\frac{1}{2} \cdot \frac{2}{3}$ by looking at this drawing



Now find it using the second method

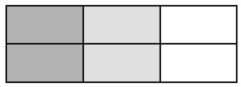


Are your two answers equivalent?

Which answer is simpler? Why do you think it's simpler?

If you use the numerical procedure to find $\frac{1}{2} \cdot \frac{2}{3}$ by multiplying the numerators and multiplying the denominators, you get an answer that isn't as simple as it could be: $\frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6}$. You then can find an equivalent fraction with smaller numbers, from your knowledge of fraction families: $\frac{2}{6} = \frac{1}{3}$. This is what you get using the first method. (We called this procedure for finding equivalent fractions "scaling down" because the numerator and denominator are proportionally smaller numbers. They have been scaled down by a factor of 2.)

Looking at the drawing for the second method, do you see how that is also $\frac{1}{3}$ of the whole? You could move one of the darker colored squares below the other one, then it would show $\frac{1}{3}$.



Try this with a different problem:

- Use a drawing to find $\frac{3}{4} \cdot \frac{4}{5}$

- Then find the answer using the numerical procedure.

How are the two answers to this problem similar?

- Try this: $\frac{1}{2} \cdot \frac{4}{9}$ Use a drawing

- Then find the answer using the numerical procedure.

The first method uses a drawing to find the answer. 1/2 of 4/9 means 4/9 divided into two equal groups, or 4/9 divided in half. From the drawing, you can see that you're taking 1/2 of the 4 shaded parts, or dividing the 4 by 2. Then you get 2/9. Looking at the original problem, divide the 4 (in the numerator of the second fraction) by the 2 (in the denominator of the first fraction), to get 2 over 9, or $\frac{2}{9}$.

So an equivalent numerical procedure is to divide one of the numerators by the other denominator to simplify the result.

Try this numerical procedure with these multiplications.
Use drawings to check your answers.

$\frac{1}{3} \cdot \frac{6}{7}$ $\frac{1}{5} \cdot \frac{20}{27}$

$\frac{1}{4} \cdot \frac{8}{9}$ $\frac{2}{3} \cdot \frac{9}{10}$

Part 5

Working with mixed numbers

Estimate the answer to $\frac{1}{2} \cdot 5\frac{3}{4}$

Now find the actual answer, using any method. Show how you found your answer.

Write a real world situation to go along with this multiplication.

Multiply $2\frac{3}{4} \cdot \frac{2}{3}$ Show how you found your answer.

A general procedure for finding answers to these multiplications is to change the mixed number into an improper fraction, then use the procedure of multiplying numerators and multiplying denominators. Try it with the two problems above (if that's not what you did already). Does it give the same answers?

Dividing Fractions

Part 1

Dividing a fraction by a whole number

One definition of division is “partitioning” a number into equal groups. With whole numbers, this means “6 ÷ 2” is 6 partitioned into 2 groups – with 3 in each group.

O O O	O O O

The partitioning division question is “How many are in each group, if we make 2 equal sized groups?” The procedure is to start passing out the 6 items into 2 groups until all are passed out, and then count the number in each group. We have divided 6 evenly into 2 groups.

It means the same thing with fractions. “2/4 ÷ 2” means to start with 2/4 of something and divide that equally into 2 groups. For example: **You have 2/4 of a pizza and you want to share it equally between 2 people. How much of the pizza does each person get?** Make a drawing to show this.

Try these:

$\frac{4}{12} \div 2$ $\frac{4}{5} \div 2$ $\frac{6}{5} \div 3$

This problem is just a little harder: You have 1/4 of a pizza and want to share it equally between 2 people. How much does each person get?

Try these:

$\frac{3}{4} \div 2$ $\frac{3}{8} \div 2$ $\frac{10}{8} \div 3$

What have you discovered about dividing a fraction by a whole number?

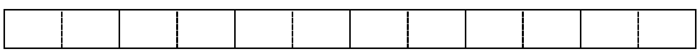
Part 2

Dividing a whole number by a fraction

Write a story to go with this problem: $12 \div 1/2$. We'll come back to it later.

The other definition of division is called "measurement division." It asks the question, how many of the divisor are in the dividend. In the case of $6 \div 2$, ask: How many groups of 2 are in 6? The answer (of course, again) is 3.

Dividing a whole number by a fraction is a case of measurement division. "How many of the fraction are in the whole number?" So $6 \div 1/2$ means "How many 1/2's are in 6?" Use circle fractions if you want to figure this out, or a bar model.



Interestingly, the answer to $6 \div 1/2$ is the same as the answer to $6 \cdot 2$. (Why?) Try the following:

$8 \div 1/2$ $6 \div 1/4$ $6 \div 2/3$

What procedure might you use for the last problem? Try some other problems like that one to test out your procedure.

1. **A baker is making cakes for a big party. She uses 1/4 cup of oil for each cake. How many cakes can she make if she has a bottle of oil that has 6 cups in it?**

   Write this problem as a division expression.

2. **The serving size for the granola that Ted likes to eat for breakfast is 3/4 cup. How many servings are there in a box that holds 13 cups?**

   Write this problem as a division expression.

Look back at the story you wrote to go with $12 \div 1/2$. Knowing what you know now, would you change it in any way?

Part 3

Dividing a fraction by a fraction

3. How many quarters are in 75 cents?

Write this problem as a division expression.

Write this problem using fractions.

This is also a case of measurement division. The question is "How many $\frac{1}{4}$'s are there in $\frac{3}{4}$? How do you find the answer?

Try these:

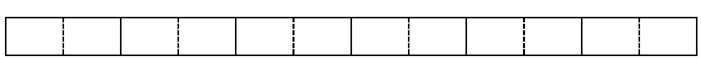
$\frac{4}{5} \div \frac{1}{5}$ $\frac{4}{5} \div \frac{2}{5}$ $2\frac{3}{4} \div \frac{1}{8}$

Say each of these division problems out loud, like the previous example: "How many 1/5's are in …"

4. How many 1/2 cup servings are in a package of cheese that contains 5 1/4 cups altogether?

Write this problem as a division expression.

Draw a picture to help you figure this out. Start with $5\frac{1}{4}$.



Draw a line where $5\frac{1}{4}$ is. Then count how many 1/2's are in this amount.

Is the answer a whole number? Is there anything left over? If so, what part of 1/2 is left over? So the answer is:

Try some others by drawing the original amount and asking
"How many of the divisor is in the original amount?"

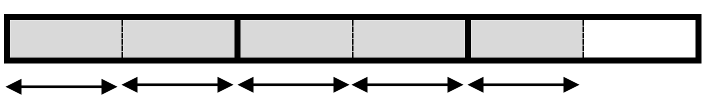
$\frac{5}{4} \div \frac{1}{2}$ $\frac{7}{8} \div \frac{1}{4}$

$3\frac{1}{2} \div \frac{1}{2}$ $2\frac{1}{3} \div \frac{1}{6}$

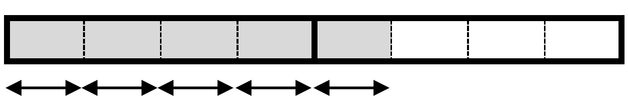
Procedures for dividing fractions

When you have a problem like $\frac{5}{2} \div \frac{1}{2}$ (how many $\frac{1}{2}$'s are there in $\frac{5}{2}$?) you can see that the denominators don't really matter. They are simply the "size of the pieces." You get the same answer for $\frac{5}{4} \div \frac{1}{4}$ *or* $\frac{5}{8} \div \frac{1}{8}$ *or* $\frac{5}{3} \div \frac{1}{3}$.

The answer is 5.



This represents 5/2. How many 1/2's are there?



This represents 5/4. How many 1/4's are there?

This is the number you get when you divide the numerators (and ignore the denominators).

How about $\frac{6}{10} \div \frac{2}{10}$? (How many $\frac{2}{10}$'s are in $\frac{6}{10}$?) Use the drawing below to figure it out:

You really asked: How many 2's are in 6? (What's 6 divided by 2?) Once again, the size of the denominator doesn't matter because it's just the size of each piece. The answer would be the same if you were talking about fourths or thirds or eighths. The answer is 3.

The procedure is this: **When you have a common denominator, simply divide the numerators.**

You can make this work when you don't start with a common denominator, by scaling up one fraction to get common denominators. Try it with these problem:

$\frac{8}{12} \div \frac{1}{3}$

$\frac{6}{4} \div \frac{1}{2}$

$\frac{18}{8} \div \frac{3}{4}$

$2\frac{1}{3} \div \frac{1}{6}$

You could scale up both fractions to a common denominator if you need to. Try it:

$\frac{7}{3} \div \frac{1}{4}$

In this case, do you get a whole number as an answer?

You should recognize #4 and #5 as division problems now. Solve them any way you can.

4. Mrs. Murphy's class is making pillow cases. Each pillow case uses 3/4 of a yard of fabric. How many pillow cases can they make out of 12 1/2 yards of fabric? Will any fabric be left over? If so, how much?

5. A book shelf is 3 1/2 feet long. Each book on the shelf is 5/8 inches wide. How many books will fit on the shelf?

6. How many candy bars can you buy with $9.50 if each candy bar costs $0.75. -or-
   How many candy bars can you buy with 9 1/2 dollars if each candy bar costs 3/4 of a dollar?

"Invert and multiply"

You've probably heard that you can "invert" the second fraction (the divisor) and multiply the two fractions (multiply the numerators and multiply the denominators). Try it with $\frac{7}{3} \div \frac{1}{4}$ to verify that you get the same answer as when you scaled up both fractions to a common denominator and then divided the numerators. Did you see the same processes that you used for scaling up and dividing?

(One process was multiplying 7 times 4. You did this in both procedures. Another was multiplying 3 times 1. The third process was dividing 28 by 3. You do all three processes in both procedures, which is why you get the same answer.)

The reason this old saying works is that multiplication is the opposite of division. So multiplying by the inverse is like doing the opposite twice (like turning left and then turning right – you're going in the same direction again; or like holding up a written word in a mirror and then looking at the reflection in a mirror again – you see the original word).

To review, write what you know about dividing fractions, on a separate piece of paper. Use examples to explain what you know.

Decimals and Percents (an assessment)

1. Write each of these decimal numbers as a fraction:

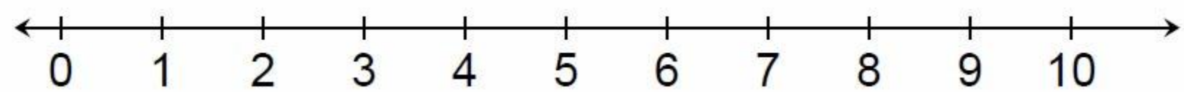
0.1 __________ 0.5 __________ 0.03 __________ 0.25 __________ 0.78 _________

2. Write these decimal numbers:

two tenths ______ 7 hundredths ______ 34 hundredths ______

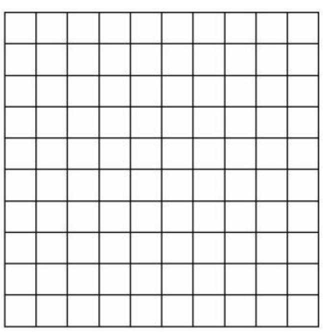
3. What is the place value of the “4” in this number? 23.54 ___________________

4. Locate the number 5.3 on the number line:

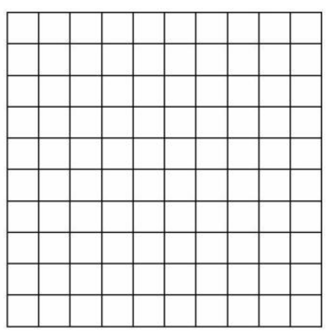


5. Color in the correct number of squares on the hundredths grids below to show each fraction. Then write each fraction as a decimal number.

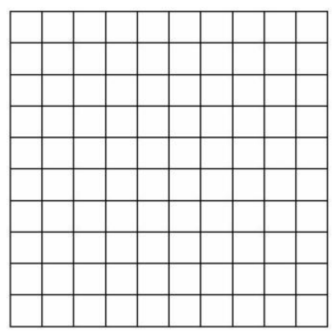
1/5 _________



1/10 _________

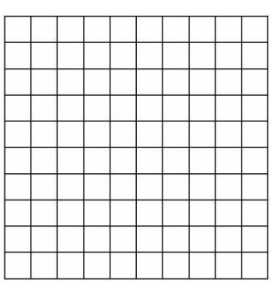


1/20 _________

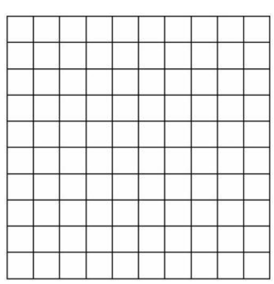


6. Color in the correct number of squares on the hundredths grids below to show each fraction. Then write each fraction as a decimal number.

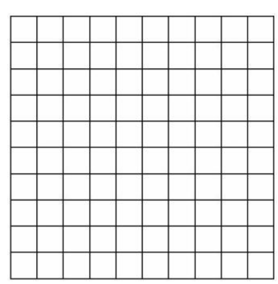
1/2 _________



1/4 _________

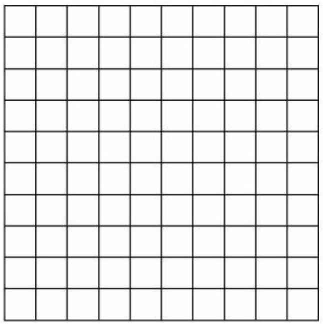


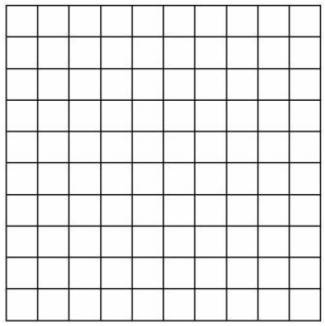
1/8 _________

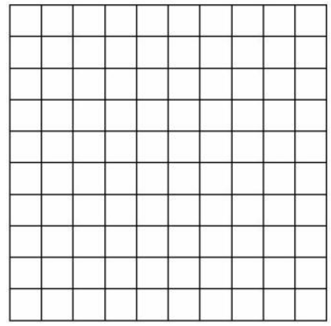


7. Color in the correct number of squares on the hundredths grids below to show each fraction. Then write each fraction as a decimal number and a percent.

4/10 ______________ 5/100 ______________ 45/100 ______________







8. Use a calculator to find the decimal equivalent for each fraction in the first table. Try some of your own in the second table.

$\frac{1}{2}$	0.5	
$\frac{1}{10}$		
$\frac{1}{5}$		
$\frac{1}{4}$		
$\frac{2}{4}$		
$\frac{3}{4}$		
$\frac{3}{10}$		
$\frac{3}{5}$		

9. Cover up the fractions in the first table above so you can only see the decimals. Then list the decimal numbers in the first table in order, from smallest to largest.

10. Write the equivalent percentage for each decimal number in the third column of the table.

11. Which of these is the correct way to add decimal numbers? Circle a, b or c.

a.	b.	c.
22.45	13.8	5.41
+ 3.2	+ 240	+ 27.3
22.77	37.8	32.71

12. In number 11, choose one of the problems that you think is wrong, and show how to do it correctly below.

13. How much change would you get from a $5.00 bill if you bought a candy bar for $0.65 ?
Show how you figured this out.

14. Estimate how much 12 candy bars cost altogether if each one costs $0.65. _________

15. Figure out exactly how much 12 candy bars would cost at $0.65 each. Show how you figured this out.

16. A crate of apples costs $37.25. There are 50 apples in the crate. Estimate how much each one would cost.

17. Figure out how much each apple in number 16 costs, exactly. Show your work.

18. You need to cut a piece of cloth this is $\frac{7}{8}$ of 4.55 meters long.

a. Estimate how long that would be. ______________

b. Convert $\frac{7}{8}$ to a decimal fraction, then multiply to find the exact answer.

18. Find 50% of $24 _________ Find 10% of $120 _________

Find 5% of $36.50 _________ Find 15% of $20 _________

Find 18% of $50 _________

19. A pair of pants is on sale for 20% off the original price of $40.

How much is the sale price? _________

Show two different ways of figuring this out.

20. A store buys a refrigerator for $500 and marks it up by 75% in order to make a profit.

How much would they sell it for? __________ Show all your work.